Assignment 4: A shower of cosmic rays

Jurjan Bootsma

November 27, 2023

1 Introduction

For centuries, humankind has observed the universe by looking at all the light received from the cosmos. However, the era of multi-messenger astronomy has recently begun and gives us many new opportunities to observe outer space with more than photons, but also neutrinos, gravitational waves and cosmic rays. Those cosmic rays can travel directly to us, but they can also originate from gamma rays interacting with the atmosphere. When those highly energetic photons interact, many more electrons, positrons and photons arise in a massive cascade effect, which is also called a "shower". These products give us another window to study the most energetic events and places in the universe.

This analysis focuses on a simulation of a shower of cosmic rays. An incoming gamma ray photon enters the atmosphere, with a changeable initial energy, and interacts with the atoms in the atmosphere. This is simulated by using statistical distributions, which will be elaborated more in the next section. With this simulation, we studied several characteristics of the shower, such as the location of the first interaction, the shape of the showers for different initial energies, the number of charged particles vs height, the height of the maximum number of charged particles vs initial energy and how many electrons and positrons hit the ground. The following section describes the methodology used, like used functions and distributions. After that the results are shown and discussed. Finally, the conclusions are given.

2 Methods

The simulation starts with a gamma ray photon with an energy, which can be changed by the user, traveling straight down through the atmosphere. The starting position is set at an height of 100 km, because there the interaction effects of the atmosphere can be neglected. Section 3.1 explains more about this. The photon can interact with the nuclei in the atmosphere to form an electron and positron, which is called pair production. Those charged particles can also undergo a process called Bremsstrahlung. After the interactions, the new particles are evaluated. This loop is repeated until particles either hit the ground or fall below the energy of 85 MeV. At this energy, other energy-loss processes become more important and the particles quickly loose all their energy in a short distance.

Interaction processes

The main process that high energy photons undergo when traveling through matter is pair production. This happens when an highly energetic photon interacts with a nucleus to create a subatomic particle and its anti-particle, which is in our case an electron and positron. Equation 2.1 and figure 1 show this.

$$\gamma(+Z) \rightarrow e^- + e^+ \tag{2.1}$$

Where γ is the photon, e^- the electron, e^+ the positron and Z the nucleus. Z is included to show that the photon is not suddenly decaying in the electron and positron, but is actually interacting with the nucleus.

The charged particles undergo Bremsstrahlung where they emit high energy photons. This is a process where charged particles emit photons when they decelerate when deflected by other charged particles, which is shown visually in figure 2. The photons are emitted because the charged particles loose energy. When this happens, the simulation splits the charged particle in a new charged particle and a new photon.

The more matter the particles encounters, the greater the probability that it has interacted at some point. Therefore we look at the column density X [g/cm^2], an amount of matter the particle traverses. The value of X is determined randomly for both processes by taking a sample from an exponential distribution. This is shown in equation 2.2.

$$f(X) = \frac{1}{X_{mean}} e^{-X/X_{mean}} \tag{2.2}$$

Here is X_{mean} the mean interaction length for a process in g/cm^3. For pair production this is X_{pp} = 38 g/cm^2 and for Bremmstrahlung X_{brem} = 26 g/cm^2. The sample is drawn randomly from this distribution using the ROOT framework.

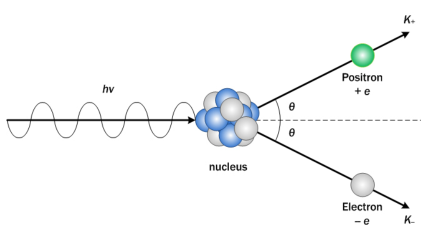

Figure 1: An example of pair production

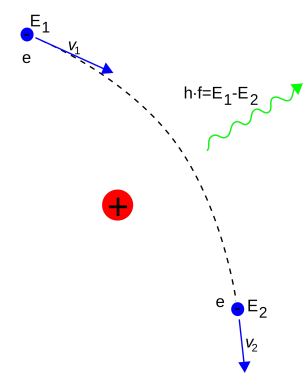

Figure 2: Process of Bremsstrahlung

Positions

The column density has to be translated to an actual physical height, so we know at what particles the new particles form. For this we need to use the density profile of the atmosphere. This is given by equation 2.3.

$$\rho(h) = \rho_0 e^{-h/a} \tag{2.3}$$

Where $\rho(h)$ is the density at height h, ρ_0 the density of air at sea level, with a value of 1.225 kg/m^3 and a is the height scale of the atmosphere, which is 8.42km. This can be translated to a column density by integrating this function for the end position of the particle to the start position, shown by equation 2.4.

$$X = a\rho_0(e^{-h_{end}/a} - e^{-h_{start}/a}) \tag{2.4}$$

Where h_{start} is the higher starting height of the particle and h_{end} the lower height where the particle interacts. The inverse of this function is implemented in the simulation, to obtain a random value for h_{low}, given by equation 2.5.

$$h_{end} = -aln(e^{-h_{start}/a} + \frac{X}{a\rho_0}) \tag{2.5}$$

Furthermore, the x- and y-positions also need to be determined. We approximate the calculation by saying that the traveled distance is much bigger in the z-direction than in the x- and y-position. This leads to equation 2.6.

$$x = \frac{p_x}{|p_z|}(h_{start} - h_{end}) \tag{2.6a}$$

$$y = \frac{p_y}{|p_z|}(h_{start} - h_{end}) \tag{2.6b}$$

Where p is defined as the direction vector.

When a particle hits the ground, its height is set to 0. When a particle falls below the energy 85 MeV, no new positions are computed.

Energy transfer

When an interaction has happened, the energy does not have to be equally distributed for both particles, but it obeys a certain probability density function, shown in equation 2.7.

$$f(u) = \frac{9}{7} - \frac{12}{7}u(1-u) \tag{2.7}$$

Where u is the fraction of the energy carried by the first particle, so u = $E_{daughter}/E_{initial}$. Because of conservation of energy, the other particle gets transferred (1-u) of the energy. The mean of the PDF is exactly at 0.5, which we expect since there should be no preferred energy transfer. However, note that the probability density is maximal at u = 0 and u = 1, so often the energy is not evenly distributed. The simulation draws samples from this distribution using the acceptance-rejection method.

Directions

The angles between the initial particle and its daughter particles are completely determined by the energy of the daughter particle, as is shown in equation 2.8.

$$\theta = m_e c^2/E \tag{2.8}$$

Where θ is this angle, E is the energy of the particle and m_e is the electron mass. This equation applies to both processes. There is still one degree of freedom left, namely the angle ϕ. This is chosen randomly between 0 and 2π for one particle. To conserve momentum, the other particle has the angle $\phi - \pi$. The simulation then rotates the directions for the new particles using those angles.

3 Results & Discussion

3.1 Height of the first interaction

The analysis started by only simulating the location of the first interaction. This was done by randomly simulating a column density using equation 2.2 and calculating the height of first interaction using equation 2.5, which is explained more in the methods section. This led to figure 3, showing a histogram of the height of the first interaction. The photon starts in the simulation at 100 km, but until 70 km there are only a small number of interaction yet. This can be seen from the very low right-tail of the histogram. Below 70 km the distribution starts to become bigger, with a peak at 28 km. This is exactly at the height of the mean column density 38 g/cm^2. Below this peak, the exponential distribution of the column density starts to take over again and the first interactions start to drop. Between 0 and 10 km there are barely any interaction left. This is something we would expect, since otherwise we people would be bombarded by these high-energy particles, which would be deadly.

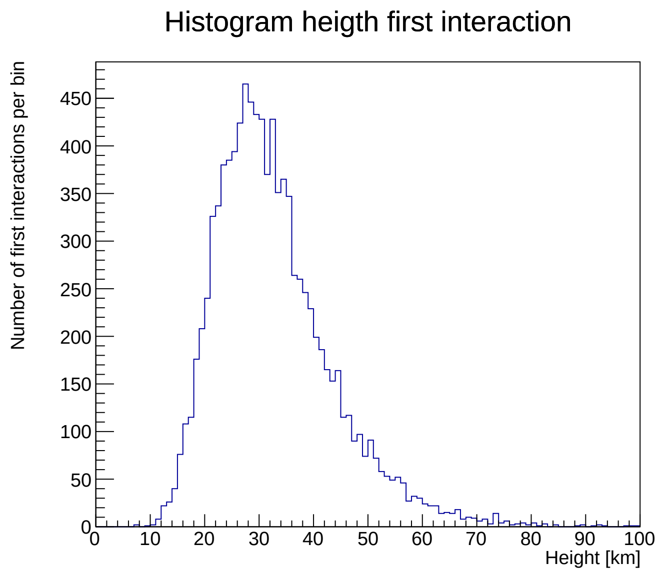

Figure 3: Histogram showing the simulated height of the first interaction between the initial photon and an atmospheric nucleus. The numbers of first interaction per bin is shown as a function of the height above sea level in kilometres. The histogram contains the result of 10000 random simulations

3.2 Shower simulations

We repeated the shower simulation several times while varying the energy of the initial gamma ray photon. This subsection explores the showers with energies 100 GeV, 1 TeV and 10 TeV visually. The rest of the section will discuss more features of those showers. Figure 4 shows the shower for an incoming photon of 100 GeV. Although the first interaction was already high in the atmosphere, most daughter particles are under 20 km. Reasons for this are that the atmosphere is thicker at lower heights, and that there are more particles that can create even more. Furthermore, the particles barely leave the 30m x 30m square, showing that the shower is very concentrated. This is because only particles with higher energies make it further to the ground (because below 85 MeV a particle stops), and equation 2.8 shows that the angles are very small in that case, so the movement in the x- and y-direction is also relatively small.

Figure 5 shows the shower with an incoming energy of 1 TeV. Compared to figure 4, more products can be seen in a slightly broader range. This is because when the first particle starts with an higher energy, the particles will be able to split more often before reaching the lower energy of 85 MeV. This will lead to more particle finally at lower energies, so also with greater angles to the z-direction, resulting in a broader range. Figure 6 shows the 10 TeV shower, which shows even more particles with a broader range.

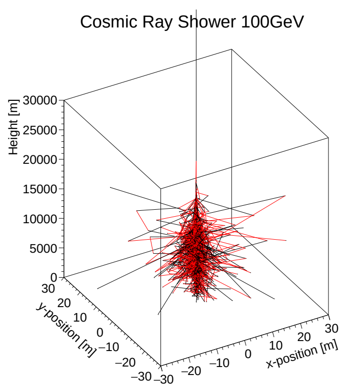

Figure 4: 3D visualization of a simulation of a cosmic ray shower with an incoming photon with energy 100 GeV.

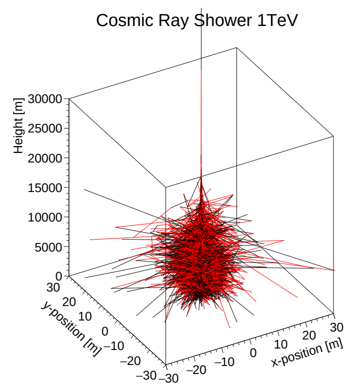

Figure 5: 3D visualization of a simulation of a cosmic ray shower with an incoming photon with energy 1 TeV.

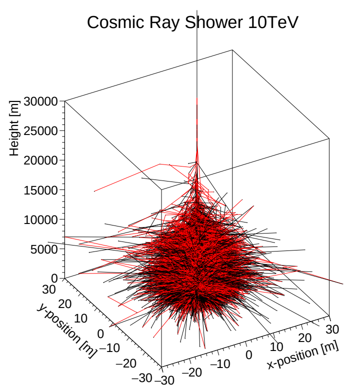

Figure 6: 3D visualization of a simulation of a cosmic ray shower with an incoming photon with energy 10 TeV.

We also looked at the fractional energy distribution of the particles. Figure 7 shows this histogram. There we can see the expected shape from equation 2.7, with a mean of roughly 0.5 and maxima at 0 and 1.

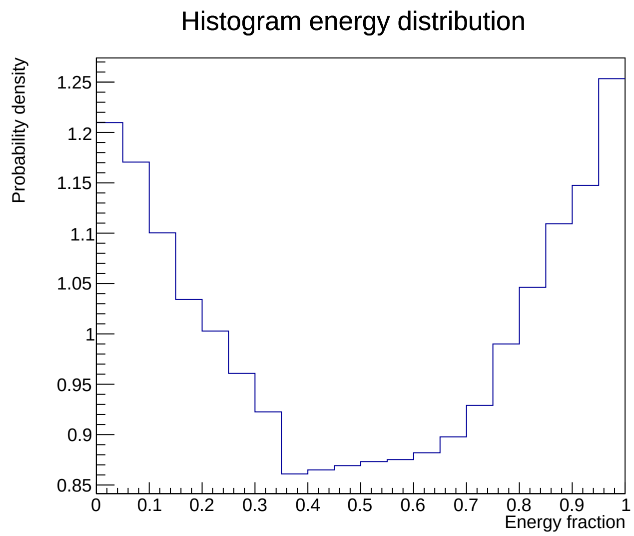

Figure 7: Histogram showing the fraction of energy u transfered to the daughter particle. The histogram is normalized. This fractional energy is defined as u = $E_{daughter}/E_{initial}$.

3.3 Height of charged particles

The simulation starts with 0 charged particles, but there are increasingly more when going to lower altitudes, due to the processes pair production and Bremsstrahlung. This subsection explores the distribution of charged particles over different heights and also looks how the maximum of those distributions are dependent on the energy.

Figure 8 shows the distribution of number of charged particles as a function of the height. We can see that there are barely any in the end position above an height of 20 km. However, below 20 km there are many charged particles, with a peak at 8 km. Heights just above the ground do not have many charged particles left anymore. When repeating the simulation, we see that often no charged particles make it to the ground. This is because they have sent off so much Bremsstrahlung that their energies fell below the 85 MeV threshold.

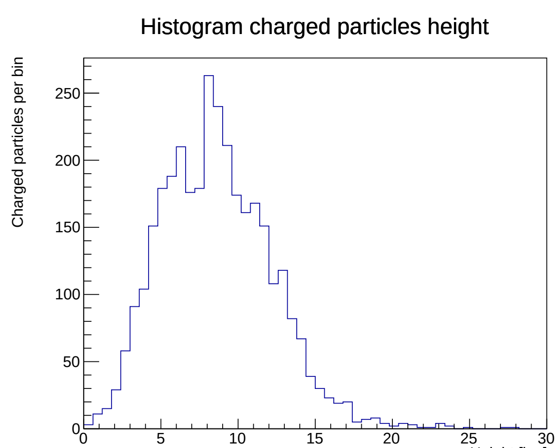

Figure 8: Histogram showing the number of end positions of charged particles per bin as a function of the height in kilometres for a shower with an initial energy of 100 GeV. Note that this is only one example and other simulations may randomly vary.

The peak in figure 8 is often used by experiments to measure the shower energy. This maximum is called H_{max}. We simulated cosmic ray showers in the range 100 GeV - 10 TeV and located the H_{max} for every energy. This resulted in figure 9. This plot shows that the higher the energy, the lower H_{max}. The figure has a logarithmic x-axis and we see a rough straight line in the plot. This points at an exponential dependence between H_{max} and the energy. This negative dependence from H_{max} as a function of the energy is expected, because when the initial photon has an high energy, it is able to split more often, resulting in more charged particles in total. These charged particles take longer to reach the lower boundary energy of 85 MeV and get therefore closer to the ground. The errorbars show that this is still a random process, and the different probabilities affect the outcome.

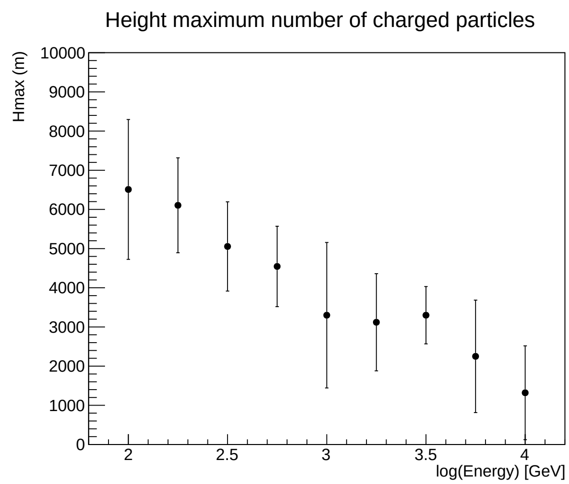

Figure 9: The height of the maximum number of charged particles in the cosmic ray shower. The maximum height H_{max} in metres is on the y-axis and the logarithm of the energy in GeV is on the x-axis.

3.4 Detecting the charged particles

As shown in the sections before, the maximum number of charged particles (H_{max}) is significantly above sea level, especially for the lower energies. However, detectors are built on Earth and therefore it is interesting to analyze how many particles hit the ground. We analyzed this for the case of the HAWC experiment. This experiment uses a large water tank and when a charged particle flies through it, it gives a flash of light. This experiment is built on a mountain of 4100 m. The reason for this is that you want your detector to be as close to H_{max} as possible, because then you have the most flashes of light (and hence charged particles) to analyze. This gives the most information about the initial gamma ray. In figure 9 , the H_{max} 4100 m is roughly around an initial gamma ray energy of 1 TeV. Therefore the HAWC experiment is the most suitable for this energy. It is also sensitive to higher energies, although then it would be more ideal to build the detector on a lower height.

It is also important to know from which initial energy this experiment is sensitive enough to measure the effects of the shower. Figure 10 shows how many charged particles hit the ground at 4100 m as a function of the energy. We see that in this energy range, there is a positive linear relation between the number of charged particles that hit the ground and the energy. The reason for this is because at higher energies, less particles have fallen below the lower energy of 85 MeV.

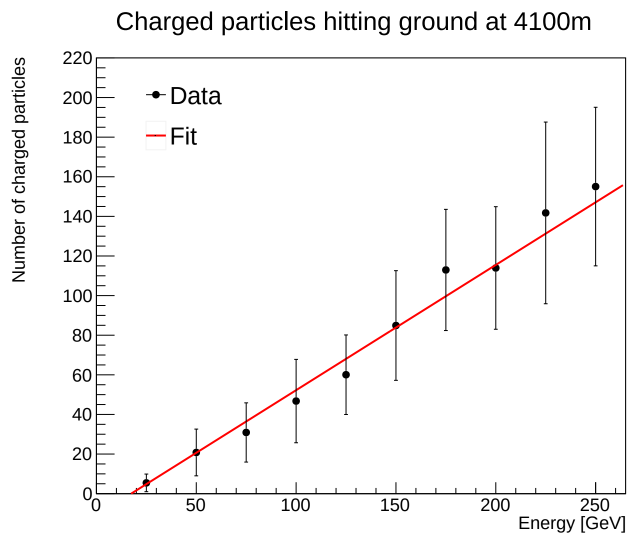

Figure 10: Number of charged particles that hit the ground at 4100m as a function of the energy in GeV. This is the height of the HAWC experiment. The black points with error bars show the outcome of the simulation, repeated 20 times per energy. The red line represents a fitted straight line.

In the HAWC experiment, 100 charged particles are needed for a significant detection. To find the lower energy for which this is the case, we fitted a line through the data points. The linear relation is described by equation 3.1.

$$N = 0.68E - 19.91 \tag{3.1}$$

Where N is the number of charged particles that hit the ground and E is the energy in GeV. When you look for a certain energy, the error is determined by the propagation of errors as in equation 3.2.

$$\sigma_E = \sqrt{(\frac{100-b}{a^2})\sigma_a^2 + \frac{1}{a}\sigma_b^2} \tag{3.2}$$

Where a is the slope of the line and b the offset. The energy for which the detector detects at least 100 charged particles on average, is then given by the following value:

E = 176 $\pm$ 13 GeV

Under this energy, the gamma rays would not be detected significantly on average. Finally, we looked at the typical areal size at this energy and height. This was done by looking at the distance from the center for charged particles that landed on the ground, as in equation 3.2.

$$d = \sqrt{x^2 + y^2} \tag{3.2}$$

Where d is how far the particle landed from the middle in metres, x the x-position in metres and y the y-position in metres. By calculating 50 distances, a standard deviation can be found. This standard deviation is the typical length L on the ground of the shower. However, this is still in units metres, so to find the areal size we assume the shape to be a circle. The arael size is now given by equation 3.3.

$$A = \pi L^2 \tag{3.3}$$

Where A is the typical areal size. The propagation of errors give equation 3.4.

$$\sigma_A = \sqrt{2\pi L}\sigma_L \tag{3.4}$$

This leads for a initial shower energy of 176 GeV for a detector at an height of 4100 m to the following areal size:

A = 19.6 ± 2.1 m^2

4 Conclusions

We conclude that when a gamma ray enters the atmosphere, the first interaction happens almost always well above sea-level, with a peak at 28 km. Secondly, we can see a rough exponential relation between H_{max} and the energy, where H_{max} becomes lower as the energy increases. Furthermore, the HAWC experiment, which is located at 4100 m, is able to significantly detect the cosmic ray showers from an initial photon energy of E = 176 ± 13 GeV. Also, there is a linear relation between the number of charged particles that hit the ground at 4100 m and the energy. Finally, the areal size of the shower at the ground at 4100 m at this lower energy, is A = 19.6 ± 2.1 m^2.