# Pseudo Gradient-Adjusted Particle Swarm Optimization for Accurate Adaptive Latent Factor Analysis Supplementary File

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This is the supplementary file for the paper entitled *Pseudo Gradient-Adjusted Particle Swarm Optimization for Accurate Adaptive Latent Factor Analysis*. It contains a) the symbol appointment table, b) four supplementary figures including flowchart of PPL, the PPL model's RMSE as its swarm size varies, the PPL model's time cost per iteration and total time cost with RMSE as its swarm size varies and M1-11's training curves on D1-6, and c) six supplementary tables recording the Lowest RMSE/MAE, converging iterations and the total time cost of compared models.

## PART. I. SYMBOL APPOINTMENT

TABLE S(I). Adopted Symbols.

| **Symbol** | **Description** |
|---|---|
| $U$, $I$ | The node sets from the concerned application. |
| $R$, $\hat{R}$ | The target matrix and its low-rank approximation. |
| Κ, Λ | The known and unknown entry sets of $R$. The former also denotes the training set when building an LFA model. |
| Ω, Γ | The validating and testing datasets disjoint with Κ. |
| Κ($u$), Κ($i$) | The subsets of Κ related to $u\in U$ and $i\in I$. |
| $r_{u,i}$, $\hat{r}_{u,i}$ | The single elements from $R$ and $\hat{R}$. |
| $f$ | The LF space dimension. |
| $X$, $Y$ | The LF matrices. |
| $x_{u,\cdot}$, $y_{i,\cdot}$ | The row vectors from $X$ and $Y$. |
| $x_{k,\cdot}$, $x_{h,\cdot}$ | Arbitrary independent row vectors of $X$. |
| $R^f$ | A real vector of size $f$. |
| $E_f$ | An $f\times f$ identity matrix. |
| $\varepsilon$ | The learning objective. |
| $\eta$, $\lambda$ | The learning rate and regularization constant. |
| $N$ | The maximum iteration count. |
| $\tau$, $\upsilon$ | The last update points for $x_{u,\cdot}$, $y_{i,\cdot}$. |
| $<,>$ | Computing the inner product of two vectors. |
| $\|\cdot\|_2$ | The $L_2$ norm of an enclosed vector. |
| $\lvert\cdot\rvert_{abs}$ | The absolute value of an enclosed number. |
| $D$, $q$ | The search dimension and swarm size for the PSO-based algorithm. |
| $s_j$, $h_j$ | The velocity and position vectors of the $j$-th particle. |
| $pbest_j$ | The best position of the $j$-th particle. |
| $gbest$ | The best position of the whole swarm. |
| $w$, $\rho$ | The inertia weight and balancing coefficient for the PSO-based update. |
| $c_1$, $c_2$ | The learning factors for the PSO-based update. |
| $r_1$, $r_2$ | Uniformly random numbers in the (0, 1) interval for the PSO-based update. |
| $\hat{m}$, $\hat{v}$ | The first and second-moment estimate for the Adam-based refinement. |
| $\beta_1$, $\beta_2$ | The exponential decay rates for the Adam-based refinement. |
| $\nabla t\, j$ | The estimated gradient for the Adam-based refinement. |
| $\eta_{\min}$, $\eta_{\max}$ | The upper and lower searching bounds of $\eta$ for its adaptation. |
| $\lambda_{\min}$, $\lambda_{\max}$ | The upper and lower searching bounds of $\lambda$ for its adaptation. |
| $F(\cdot)$ | The fitness function for the hyper-parameter adaptation. |
| $f(\cdot)$ | A continuously differentiable function |

# PART. II. SUPPLEMENTARY FIGURES

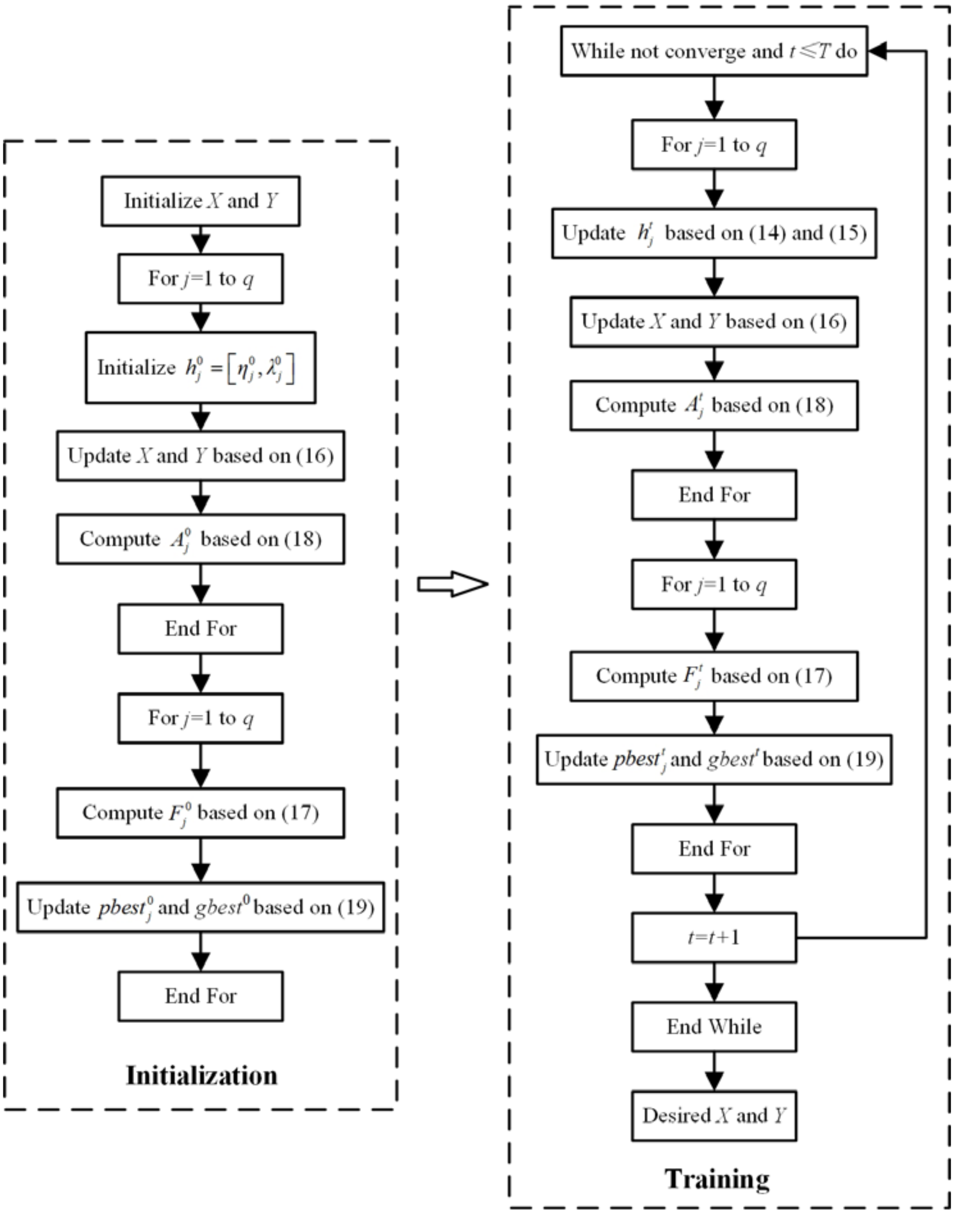
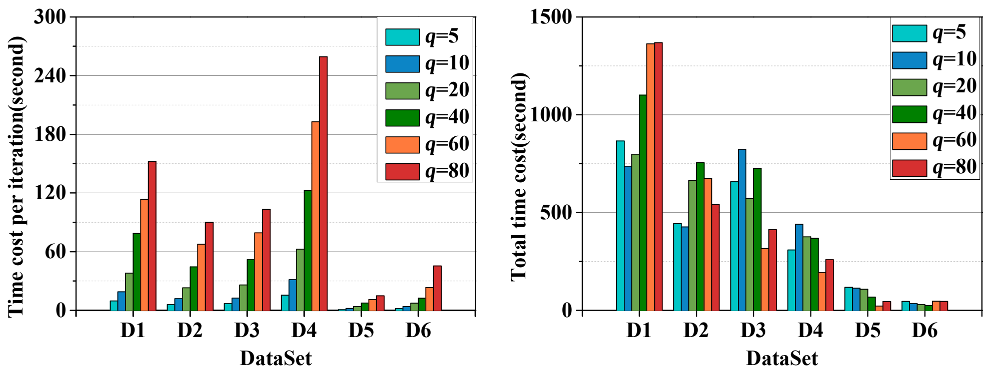
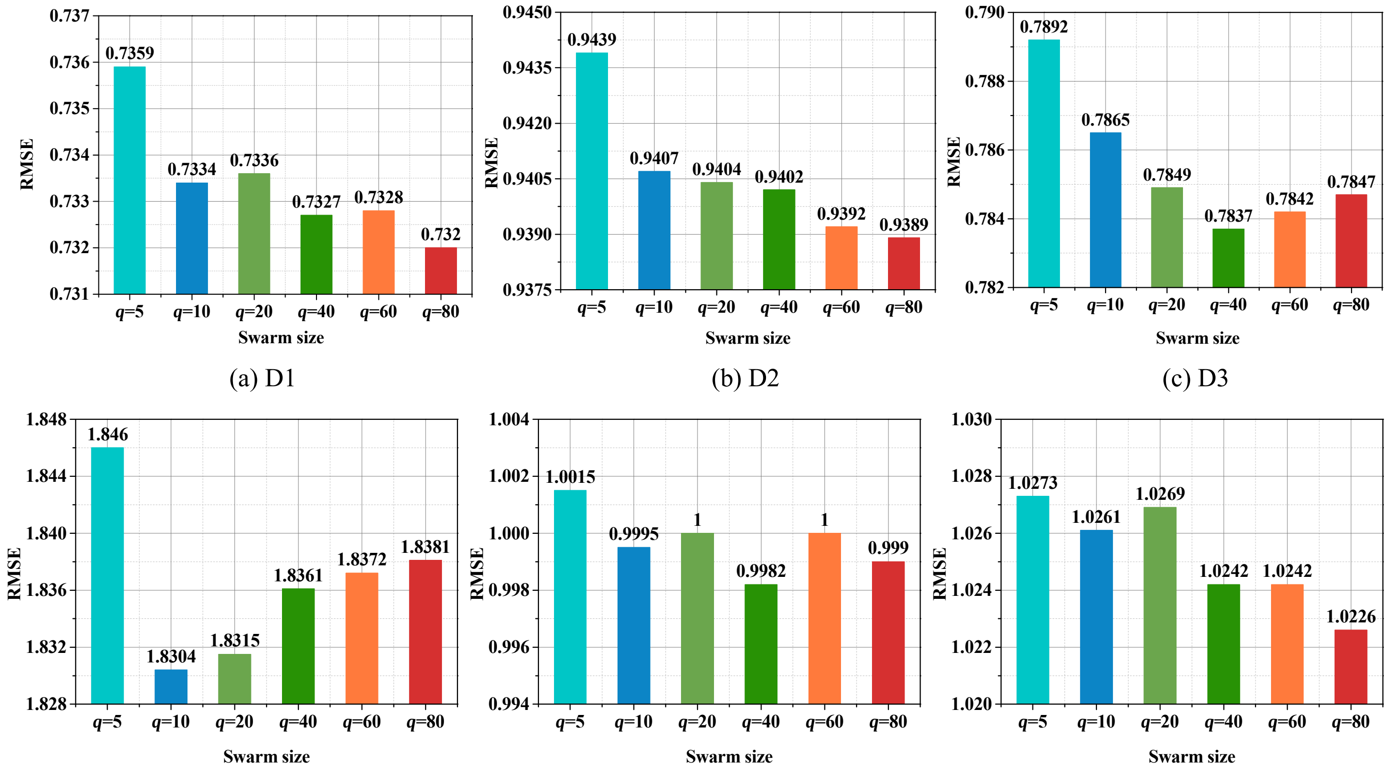



Fig. S.1. The flowchart of the PPL model.




(a) Time cost per iteration (b) Total time cost

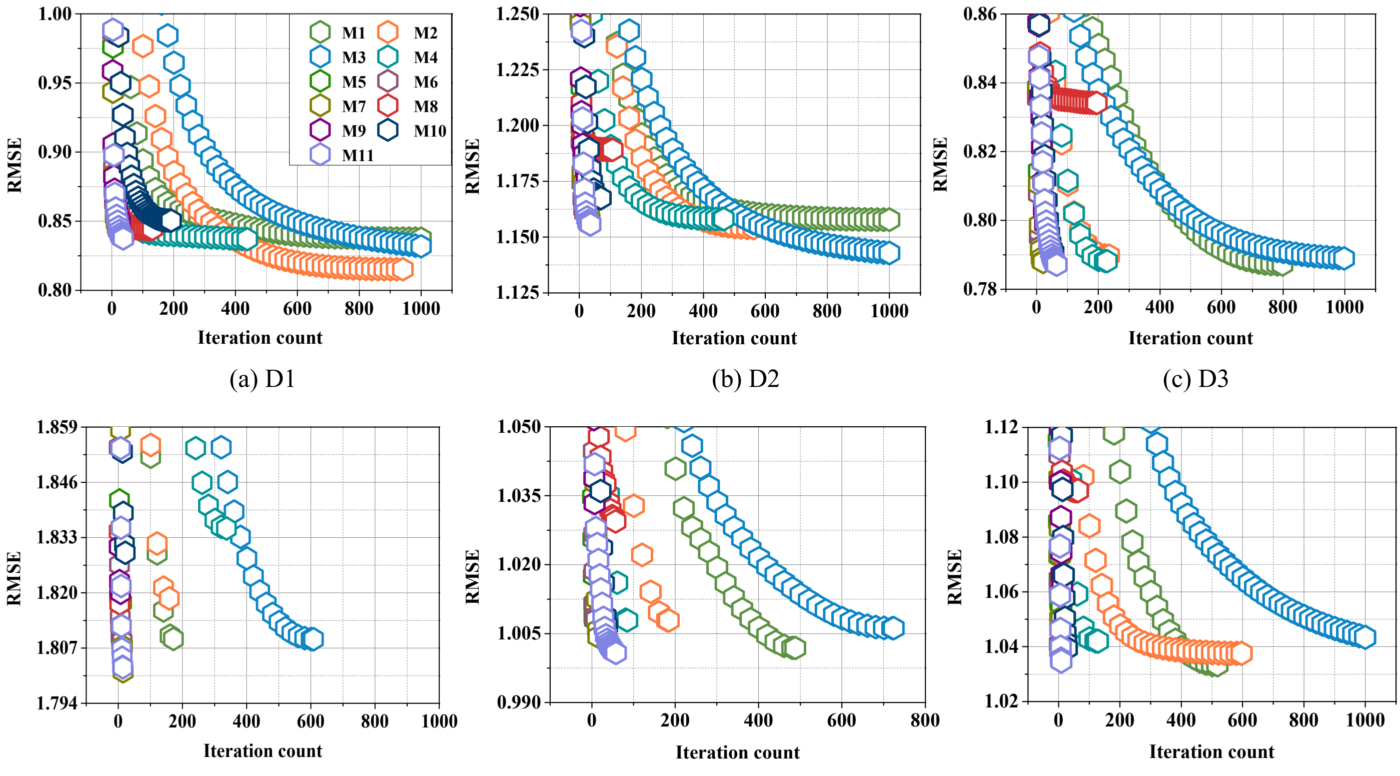
Fig. S.2. The PPL model's time cost per iteration and total time cost with RMSE as the swarm size $q$ varies.




(a) D1 (b) D2 (c) D3

(d) D4 (e) D5 (f) D6

Fig. S.3. The PPL model's RMSE as the swarm size $q$ varies




(a) D1 (b) D2 (c) D3

(d) D4 (e) D5 (f) D6

Fig. S.4. M1-11's training curves on D1-6. Note that all panels share the same legend in panel (a).

TABLE S(II). The Lowest RMSE±std, including win/loss counts for M1-11 on D1-6, where ✪ indicates that the corresponding model obtains the lower RMSE than M11 in the corresponding testing case.

| Model | Lowest RMSE | | | | | | ✪Win/Loss |
|---|---|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | D5 | D6 | - |
| M1 | $0.7337_{\pm 4.8E-4}$ | $0.9445_{\pm 2.4E-4}$ | $0.7872_{\pm 4.5E-4}$ | $1.8381_{\pm 2.8E-3}$ | $1.0001_{\pm 6.1E-5}$ | $1.0263_{\pm 1.0E-3}$ | 6/0 |
| M2 | ✪$\mathbf{0.7187_{\pm 1.2E-4}}$ | $0.9503_{\pm 4.3E-4}$ | $0.7894_{\pm 1.2E-4}$ | $1.8491_{\pm 7.2E-3}$ | $1.0072_{\pm 4.3E-4}$ | $1.0273_{\pm 1.0E-3}$ | 5/1 |
| M3 | $0.7344_{\pm 6.4E-4}$ | ✪$\mathbf{0.9365_{\pm 1.8E-4}}$ | $0.7883_{\pm 2.5E-4}$ | $1.8347_{\pm 6.6E-3}$ | $1.0051_{\pm 3.4E-4}$ | $1.0337_{\pm 2.7E-4}$ | 5/1 |
| M4 | $0.7364_{\pm 8.8E-4}$ | $0.9581_{\pm 8.1E-4}$ | $0.7879_{\pm 1.8E-4}$ | $1.8678_{\pm 1.9E-3}$ | $1.0063_{\pm 5.1E-5}$ | $1.0355_{\pm 3.1E-3}$ | 6/0 |
| M5 | $0.7421_{\pm 6.6E-4}$ | $0.9877_{\pm 9.7E-4}$ | $0.7953_{\pm 1.4E-4}$ | $1.8452_{\pm 4.9E-3}$ | $1.0069_{\pm 3.4E-4}$ | $1.0445_{\pm 5.1E-3}$ | 6/0 |
| M6 | $0.7367_{\pm 1.3E-3}$ | $0.9477_{\pm 6.9E-4}$ | $0.7887_{\pm 1.7E-3}$ | $1.8372_{\pm 4.3E-3}$ | $1.0063_{\pm 3.1E-3}$ | $1.0306_{\pm 3.5E-3}$ | 6/0 |
| M7 | $0.7387_{\pm 1.9E-3}$ | $0.9454_{\pm 1.7E-3}$ | $0.7893_{\pm 2.2E-3}$ | $1.8425_{\pm 3.5E-3}$ | $1.0161_{\pm 3.1E-3}$ | $1.0541_{\pm 4.5E-3}$ | 6/0 |
| M8 | $0.7428_{\pm 2.0E-5}$ | $0.9861_{\pm 3.5E-5}$ | $0.7925_{\pm 1.3E-5}$ | $1.8447_{\pm 3.3E-4}$ | $1.0278_{\pm 9.5E-3}$ | $1.0864_{\pm 6.9E-5}$ | 6/0 |
| M9 | $0.7433_{\pm 3.0E-3}$ | $0.9563_{\pm 8.0E-3}$ | $0.7984_{\pm 5.7E-3}$ | $1.8470_{\pm 4.9E-3}$ | $1.0323_{\pm 2.8E-3}$ | $1.0342_{\pm 1.3E-3}$ | 6/0 |
| M10 | $0.7470_{\pm 3.9E-4}$ | $0.9501_{\pm 2.6E-4}$ | $0.7906_{\pm 3.0E-4}$ | $1.8591_{\pm 2.3E-3}$ | $1.0043_{\pm 7.0E-4}$ | $1.0334_{\pm 2.9E-3}$ | 6/0 |
| M11 | $0.7334_{\pm 7.7E-4}$ | $0.9407_{\pm 4.7E-4}$ | $\mathbf{0.7865_{\pm 4.4E-4}}$ | $\mathbf{1.8304_{\pm 4.9E-3}}$ | $\mathbf{0.9995_{\pm 1.7E-3}}$ | $\mathbf{1.0261_{\pm 2.1E-4}}$ | - |

TABLE S(III). The Lowest MAE±std, including win/loss counts for M1-11 on D1-6, where ✪ indicates that the corresponding model obtains the lower MAE than M11 in the corresponding testing case.

| Model | Lowest MAE | | | | | | ✪Win/Loss |
|---|---|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | D5 | D6 | - |
| M1 | $0.3585_{\pm 9.4E-5}$ | $0.6712_{\pm 1.5E-4}$ | ✪$\mathbf{0.6035_{\pm 2.2E-4}}$ | $1.2475_{\pm 8.1E-4}$ | $0.7725_{\pm 3.8E-4}$ | $0.3648_{\pm 1.1E-4}$ | 5/1 |
| M2 | $0.3460_{\pm 6.8E-4}$ | $0.6717_{\pm 2.2E-4}$ | $0.6057_{\pm 1.5E-4}$ | $1.2520_{\pm 1.4E-3}$ | $0.7760_{\pm 2.2E-4}$ | $0.3665_{\pm 1.4E-3}$ | 6/0 |
| M3 | $0.3758_{\pm 1.5E-4}$ | $0.6624_{\pm 2.6E-4}$ | ✪$0.6046_{\pm 1.3E-4}$ | $1.2462_{\pm 1.7E-3}$ | $0.7731_{\pm 3.1E-4}$ | $0.3568_{\pm 6.8E-5}$ | 5/1 |
| M4 | $0.3663_{\pm 4.7E-4}$ | $0.6871_{\pm 4.4E-4}$ | $0.6057_{\pm 5.6E-4}$ | $1.2702_{\pm 1.3E-3}$ | $0.7813_{\pm 1.6E-4}$ | $0.3682_{\pm 1.3E-3}$ | 6/0 |
| M5 | $0.3489_{\pm 5.4E-4}$ | $0.7197_{\pm 2.4E-4}$ | $0.6095_{\pm 7.3E-4}$ | $1.2795_{\pm 2.6E-3}$ | $0.7817_{\pm 7.4E-4}$ | $0.3719_{\pm 5.6E-3}$ | 6/0 |
| M6 | $0.3513_{\pm 9.2E-4}$ | $0.6658_{\pm 7.4E-4}$ | ✪$0.6048_{\pm 3.6E-3}$ | $1.2483_{\pm 1.7E-3}$ | $0.7748_{\pm 7.6E-4}$ | $0.3661_{\pm 3.5E-3}$ | 5/1 |
| M7 | $0.3484_{\pm 8.3E-4}$ | $0.6663_{\pm 2.4E-3}$ | $0.6085_{\pm 2.2E-3}$ | $1.2576_{\pm 8.2E-3}$ | $0.7818_{\pm 1.2E-3}$ | $0.3708_{\pm 5.9E-3}$ | 6/0 |
| M8 | $0.3437_{\pm 1.4E-3}$ | $0.7199_{\pm 7.9E-3}$ | $0.6079_{\pm 9.5E-5}$ | $1.2789_{\pm 2.0E-3}$ | $0.8273_{\pm 4.1E-5}$ | $0.3888_{\pm 2.5E-5}$ | 6/0 |
| M9 | $0.3675_{\pm 1.7E-3}$ | $0.6698_{\pm 7.9E-3}$ | $0.6075_{\pm 9.9E-3}$ | $1.2635_{\pm 4.7E-3}$ | $0.7752_{\pm 4.0E-3}$ | $0.3659_{\pm 3.1E-3}$ | 6/0 |
| M10 | $0.3491_{\pm 4.0E-4}$ | $0.6661_{\pm 2.0E-4}$ | $0.6059_{\pm 2.2E-4}$ | $1.2677_{\pm 1.3E-4}$ | $0.7729_{\pm 5.8E-4}$ | $0.3706_{\pm 1.1E-3}$ | 6/0 |
| M11 | $\mathbf{0.3430_{\pm 1.4E-4}}$ | $\mathbf{0.6609_{\pm 2.1E-4}}$ | $0.6052_{\pm 5.2E-4}$ | $\mathbf{1.2451_{\pm 2.2E-3}}$ | $\mathbf{0.7723_{\pm 1.6E-3}}$ | $\mathbf{0.3628_{\pm 9.8E-4}}$ | - |

TABLE S(IV). Iteration count corresponding to TABLE S(II).

| Model | Iteration count | | | | | |
|---|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | D5 | D6 |
| M1 | $1000_{\pm 0}$ | $1000_{\pm 0}$ | $798_{\pm 2.9}$ | $172_{\pm 2.1}$ | $488_{\pm 8.2}$ | $518_{\pm 0.4}$ |
| M2 | $945_{\pm 11.2}$ | $562_{\pm 10.5}$ | $235_{\pm 3.9}$ | $159_{\pm 4.6}$ | $185_{\pm 2.5}$ | $597_{\pm 22.5}$ |
| M3 | $1000_{\pm 0}$ | $1000_{\pm 0}$ | $998_{\pm 0.9}$ | $608_{\pm 2.3}$ | $724_{\pm 5.6}$ | $1000_{\pm 0}$ |
| M4 | $438_{\pm 3.4}$ | $466_{\pm 9.8}$ | $228_{\pm 2.9}$ | $337_{\pm 1.4}$ | $85_{\pm 0.5}$ | $128_{\pm 6}$ |
| M5 | $\mathbf{28_{\pm 0.5}}$ | $31_{\pm 2.4}$ | $\mathbf{10_{\pm 2.1}}$ | $\mathbf{6_{\pm 1.2}}$ | $\mathbf{8_{\pm 0.6}}$ | $9_{\pm 4.9}$ |
| M6 | $41_{\pm 4.8}$ | $\mathbf{19_{\pm 2.0}}$ | $18_{\pm 3.6}$ | $12_{\pm 1.6}$ | $11_{\pm 1.6}$ | $\mathbf{6_{\pm 1.3}}$ |
| M7 | $39_{\pm 4.1}$ | $33_{\pm 4.9}$ | $21_{\pm 3.8}$ | $15_{\pm 1.2}$ | $16_{\pm 2.6}$ | $9_{\pm 1.9}$ |
| M8 | $131_{\pm 2.1}$ | $107_{\pm 4.2}$ | $195_{\pm 7.3}$ | $5_{\pm 0}$ | $57_{\pm 25.7}$ | $63_{\pm 9.7}$ |
| M9 | $27_{\pm 4.7}$ | $27_{\pm 6.3}$ | $27_{\pm 4.9}$ | $7_{\pm 1.5}$ | $7_{\pm 2.1}$ | $15_{\pm 1.7}$ |
| M10 | $192_{\pm 1.4}$ | $69_{\pm 1.5}$ | $56_{\pm 0.8}$ | $22_{\pm 0.8}$ | $37_{\pm 1.6}$ | $28_{\pm 1.2}$ |
| M11 | $39_{\pm 1.9}$ | $36_{\pm 1.25}$ | $65_{\pm 4.6}$ | $14_{\pm 0.8}$ | $58_{\pm 6.3}$ | $9_{\pm 2.9}$ |

TABLE S(V). Iteration count corresponding to TABLE S(III).

| Model | Iteration count | | | | | |
|---|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | D5 | D6 |
| M1 | $998_{\pm 0.8}$ | $936_{\pm 2.6}$ | $841_{\pm 1.3}$ | $188_{\pm 1.6}$ | $533_{\pm 3.6}$ | $116_{\pm 3.1}$ |
| M2 | $1000_{\pm 0}$ | $539_{\pm 11.5}$ | $248_{\pm 2.9}$ | $167_{\pm 1.5}$ | $204_{\pm 1.4}$ | $67_{\pm 3.4}$ |
| M3 | $1000_{\pm 0}$ | $1000_{\pm 0}$ | $1000_{\pm 0}$ | $683_{\pm 2.5}$ | $824_{\pm 11.6}$ | $1000_{\pm 0}$ |
| M4 | $1000_{\pm 0}$ | $453_{\pm 7.8}$ | $399_{\pm 2.1}$ | $348_{\pm 3.4}$ | $97_{\pm 2.1}$ | $44_{\pm 1.2}$ |
| M5 | $\mathbf{17_{\pm 1.2}}$ | $35_{\pm 4.2}$ | $\mathbf{9_{\pm 1.6}}$ | $6_{\pm 0.8}$ | $9_{\pm 0.7}$ | $\mathbf{7_{\pm 0.8}}$ |
| M6 | $23_{\pm 3.8}$ | $68_{\pm 7.4}$ | $56_{\pm 3.8}$ | $13_{\pm 2.2}$ | $\mathbf{7_{\pm 0.4}}$ | $13_{\pm 3.1}$ |
| M7 | $39_{\pm 4.6}$ | $27_{\pm 4.1}$ | $18_{\pm 4.2}$ | $18_{\pm 3.7}$ | $12_{\pm 2.2}$ | $16_{\pm 2.2}$ |
| M8 | $39_{\pm 5.6}$ | $37_{\pm 5.5}$ | $176_{\pm 5.5}$ | $\mathbf{5_{\pm 0.5}}$ | $85_{\pm 6.2}$ | $52_{\pm 5.3}$ |
| M9 | $25_{\pm 5.4}$ | $\mathbf{17_{\pm 4.9}}$ | $31_{\pm 6.9}$ | $\mathbf{13}_{\pm 4.6}$ | $12_{\pm 0.9}$ | $8_{\pm 1}$ |
| M10 | $200_{\pm 0.8}$ | $73_{\pm 2.4}$ | $57_{\pm 1.4}$ | $\mathbf{26}_{\pm 1.1}$ | $40_{\pm 1.2}$ | $24_{\pm 1.5}$ |
| M11 | $42_{\pm 3.4}$ | $35_{\pm 1.9}$ | $46_{\pm 2.4}$ | $11_{\pm 1.2}$ | $30_{\pm 4.4}$ | $9_{\pm 2.3}$ |

TABLE S(VI). Total time cost corresponding to TABLE S(II) (Secs)±std.

| Model | Total Time Cost of RMSE | | | | | |
|---|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | D5 | D6 |
| **M1#** | 1774.8±14.8 | 1060.3±17.5 | 932.1±11.2 | 445.3±13.3 | 107.5±2.0 | 172.2±3.1 |
| **M1*** | 55230.4 | 19645.9 | 20976.6 | 11120.4 | 5304.1 | 5544.9 |
| **M2#** | 5168.1±41.1 | 1872.9±9.1 | 857.5±18.5 | 1224.2±5.4 | 145.9±3.8 | 652.3±23.2 |
| **M2*** | 31757.4 | 12247.6 | 8902.7 | 23063.9 | 1734.9 | 4378.1 |
| **M3#** | 7346.5±55.8 | 4252.6±20.6 | 5264.6±38.1 | 7609.8±77.8 | 745.7±15.7 | 1747.3±21.6 |
| **M3*** | 55501.1 | 28591.5 | 25345.7 | 73342.7 | 4991.9 | 6740.9 |
| **M4#** | 3498.7±33.9 | 2076.4±28.3 | 1242.4±22.7 | 4435.5±62.4 | 94.8±2.6 | 210.2±17.7 |
| **M4*** | 31445.2 | 22452.3 | 12026.8 | 31598.5 | 1421.1 | 4369.1 |
| **M5** | **540±10.1** | 321.3±7.5 | **125.3±21.8** | 189±12.2 | **19.2±2.3** | 30.6±3.3 |
| **M6** | 769.3±87.3 | **216.8±63.4** | 285.3±15.6 | 342.6±30.5 | 20.1±1.9 | **20.9±3.8** |
| **M7** | 1659.3±121.4 | 861.3±67.8 | 603.5±62.7 | 1066.3±53.2 | 64.9±4.2 | 68.4±6.5 |
| **M8** | 2584.6±88.9 | 1257.4±31.9 | 2596.7±54.1 | **164.4±1.8** | 134.1±23.3 | 234±19.5 |
| **M9** | 1702.5±255.2 | 956.8±174.5 | 1031.7±115.4 | 617.5±138.5 | 53.1±5.6 | 175.3±10.2 |
| **M10** | 3175.7±44.8 | 775.2±27.4 | 753.5±11.6 | 584.4±23.4 | 80.1±1.8 | 99.8±2.6 |
| **M11** | 736.7±25.8 | 426.8±12.1 | 822.9±8.6 | 440.3±16.3 | 113±12.3 | 33.7±4.3 |

**#Time cost with the optimized hyper-parameters, *Total time cost by the manual grid search for the optimal hyper-parameters. Note that in practice, the grid search is commonly conducted for only once since it is indeed expensive. So the time cost of grid search is not accompanied with std.**

TABLE S(VII). Total Time Cost Corresponding to TABLE S(III) (Seconds)±std.

| Model | Total Time Cost of MAE | | | | | |
|---|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | D5 | D6 |
| **M1#** | 1788.4±23.2 | 933.3±14.9 | 1010±9.8 | 460.6±11.0 | 127.3±1.2 | 38.4±4.3 |
| **M1*** | 56621.4 | 21098.9 | 23089.5 | 13208.1 | 5432.3 | 5614.3 |
| **M2#** | 5750.8±66.9 | 1756.1±17.3 | 1021.9±11.6 | 1517.5±27.3 | 184.5±2.5 | 71.8±2.2 |
| **M2*** | 33437.4 | 13055.8 | 10053.3 | 27220.1 | 1989.6 | 4293.9 |
| **M3#** | 8296.9±63.9 | 4606.9±59.9 | 5488.9±90.9 | 9315.2±216.8 | 859.9±10.8 | 1668.9±32.5 |
| **M3*** | 57759.2 | 29697.6 | 26372.8 | 67306.2 | 5057.8 | 6611.4 |
| **M4#** | 7896.9±54.5 | 2024.7±54.3 | 2233.3±38.0 | 4749.1±32.8 | 119.4±4.9 | 84.1±3.6 |
| **M4*** | 31086.9 | 22521.6 | 12353.8 | 32763.2 | 1529.7 | 5085.3 |
| **M5** | **353.1±25.3** | 397.9±36.7 | **117.3±17.2** | 183.1±19.7 | 22.5±2.0 | **24.8±1.8** |
| **M6** | 417.1±45.6 | 817.6±43.7 | 933.1±54.2 | 382.9±48.5 | **16.4±2.2** | 44.8±4.9 |
| **M7** | 1726.4±158.7 | 648.9±60.8 | 519.6±65.7 | 1270.3±185.6 | 60±3.6 | 134±9.8 |
| **M8** | 736.6±80.5 | 426.3±38.6 | 2363.9±55.4 | **169.7±10.2** | 248.4±5.8 | 192.3±20.7 |
| **M9** | 1563.4±159.7 | 610.9±123.5 | 1206.4±133.6 | 1094.2±173.5 | 100.2±5.8 | 99.7±10.3 |
| **M10** | 3322.5±73.3 | 898.8±21.7 | 748.5±15.9 | 661.2±37.6 | 88.7±1.6 | 82.5±3.2 |
| **M11** | 820.7±54.2 | **393.7±9.2** | 603.1±12.6 | 335.6±16.1 | 73.8±6.9 | 32.9±5.6 |

**The symbols #,* denote the same meanings with those in TABLE S(VI).**