Accurate and Adaptive Latent Factor Analysis for Efficient Representation to High-Dimensional and Incomplete Data: Supplementary File

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This is the supplementary file for the paper entitled *Accurate and Adaptive Latent Factor Analysis for Efficient Representation to High-Dimensional and Incomplete Data.* It mainly contains a) the symbol appointment table, and b) four supplementary figures including flowchart of PPL, a PPL model's RMSE as swarm size varies, a PPL model's time cost per iteration and total time cost with RMSE as swarm size varies and M1-11's training curves on D1-10. c) six supplementary tables recording the Lowest RMSE/MAE, converging iterations and the total time cost of compared models.

PART. I. SYMBOL APPOINTMENT

TABLE S(I). Adopted Symbols.

Symbol	Description
U, I	Node sets from the concerned application.
R, $\hat{R}$	Target matrix and its low-rank approximation.
Κ, Λ	Known and unknown entry sets of R. The former also denotes the training set when building an LFA model.
Ω, Γ	Validating and testing datasets disjoint with Κ.
Κ(u), Κ(i)	Subsets of Κ related to $u\in U$ and $i\in I$.
$r_{u,i}$, $\hat{r}_{u,i}$	Single entry from R and $\hat{R}$.
f	LF space dimension.
X, Y	LF matrices.
$x_{u,\cdot}$, $y_{i,\cdot}$	Row vectors from X and Y.
$x_{k,\cdot}$, $x_{h,\cdot}$	Arbitrary independent row vectors of X.
R^f	Real vector of size f.
E_f	An $f\times f$ identity matrix.
ε	Learning objective.
η, λ	Learning rate and regularization constant.
N	Maximum iteration count.
τ, υ	Last update points for $x_{u,\cdot}$, $y_{i,\cdot}$.
< , >	Inner product of two vectors.
$\lVert\cdot\rVert_2$	L_2 norm of an enclosed vector.
$\lvert\cdot\rvert_{abs}$	Absolute value of an enclosed number.
D, q	Search dimension and swarm size.
s_j, h_j	Velocity and position vector of each particle j.
$pbest_j$	Best position of particle j.
$gbest$	Best position of the swarm.
w	Inertia weight.
c_1, c_2	Acceleration constants.
r_1, r_2	Uniformly random numbers in (0, 1) interval.
$\hat{m}$, $\hat{v}$	First and second-moment estimate.
β_1, β_2	Exponential decay rates.
∇_j^t	Estimated gradient.
$\eta_{\min}$, $\eta_{\max}$	Upper and lower searching bound of η.
$\lambda_{\min}$, $\lambda_{\max}$	Upper and lower searching bound of λ.
$F(\cdot)$	Fitness function.
$f(\cdot)$	A continuously differentiable function

PART. II. SUPPLEMENTARY FIGURES

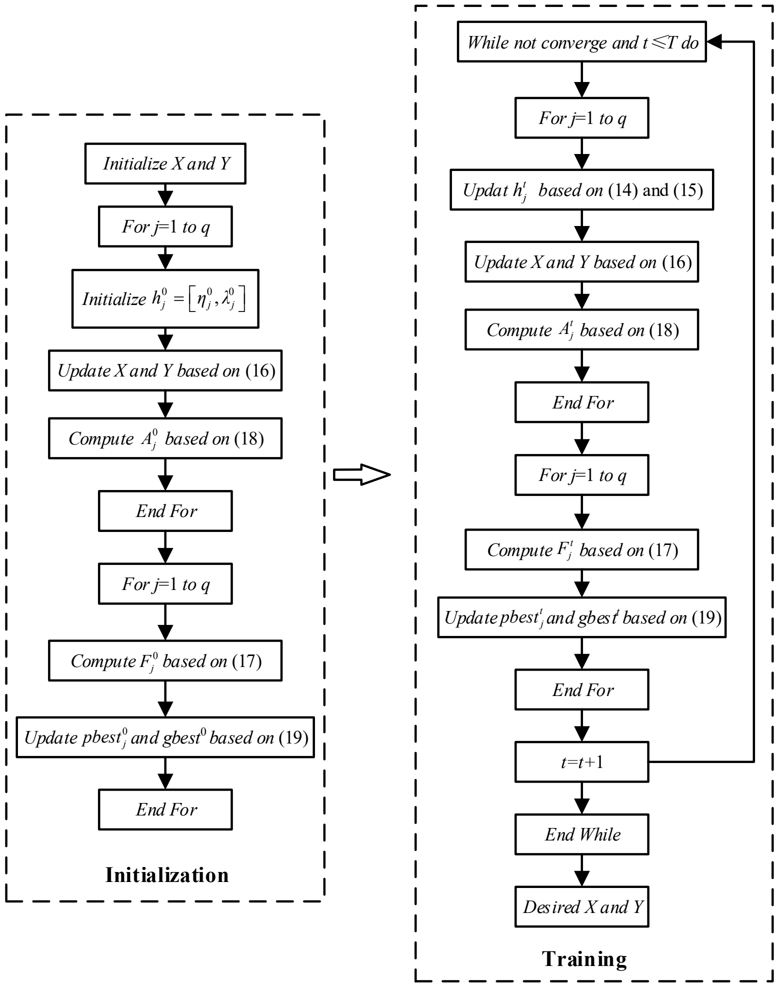
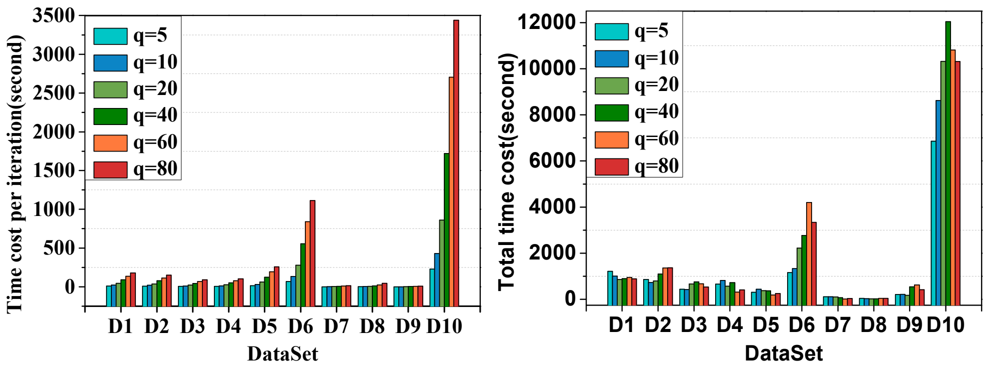


Fig. S.1. Flowchart of PPL.



(a) Time cost per iteration (b) Total time cost

Fig. S.2. A PPL model’s time cost per iteration and total time cost with RMSE as swarm size varies.

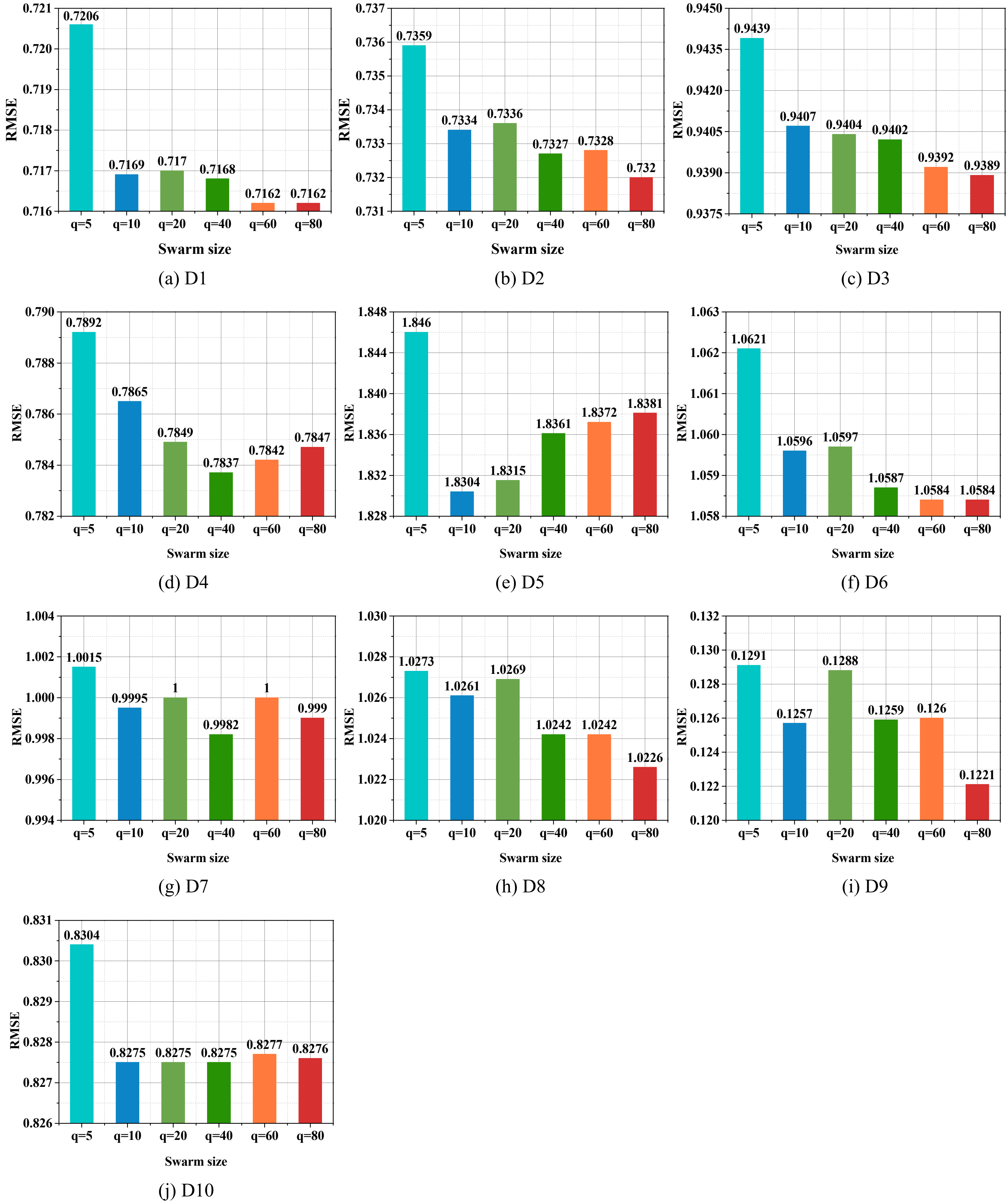


Fig. S.3. A PPL model’s RMSE as swarm size varies

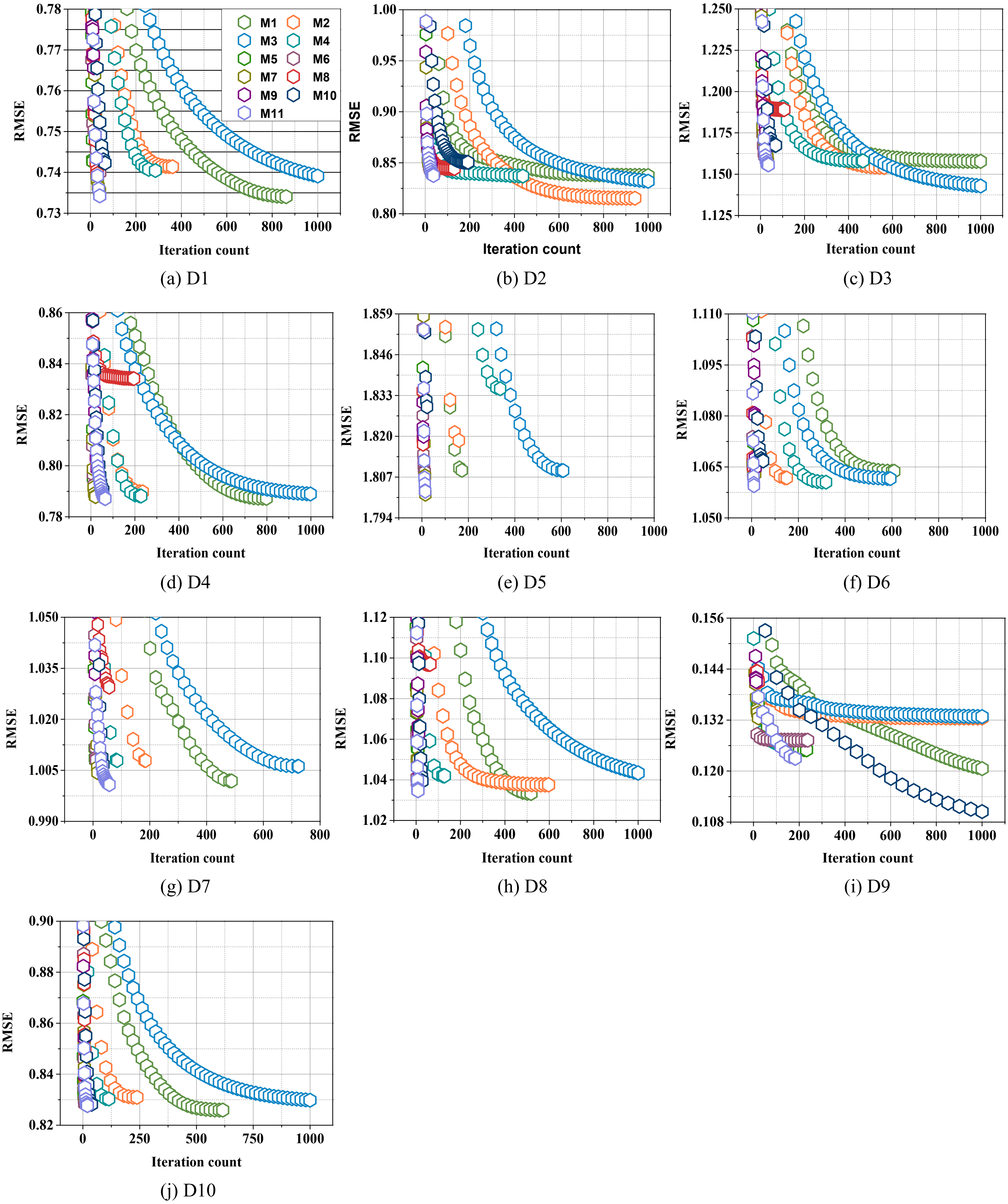


Fig. S.4. M1-11's training curves on D1-10. Note that all panels share the same legend in panel (a).

TABLE S(II). Lowest RMSE±std, including win/loss counts,
where ✪ indicates that M11 has a lower RMSE than the comparison models.

Model	Lowest RMSE										✪Win/Loss
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	-
M1	**0.7168**$_{\pm 1.1E-4}$	✪0.7337$_{\pm 4.8E-4}$	✪0.9445$_{\pm 2.4E-4}$	✪0.7872$_{\pm 4.5E-4}$	✪1.8381$_{\pm 2.8E-3}$	✪1.0638$_{\pm 1.7E-4}$	✪1.0001$_{\pm 6.1E-5}$	✪1.0263$_{\pm 1.0E-3}$	0.1227$_{\pm 2.6E-5}$	**0.8257**$_{\pm 1.0E-4}$	7/3
M2	✪0.7238$_{\pm 1.3E-4}$	**0.7187**$_{\pm 1.2E-4}$	✪0.9503$_{\pm 4.3E-4}$	✪0.7894$_{\pm 1.2E-4}$	✪1.8491$_{\pm 7.2E-3}$	✪1.0617$_{\pm 2.1E-4}$	✪1.0072$_{\pm 4.3E-4}$	✪1.0273$_{\pm 1.0E-3}$	✪0.1348$_{\pm 1.4E-5}$	✪0.8308$_{\pm 2.1E-4}$	9/1
M3	✪0.7198$_{\pm 2.6E-4}$	✪0.7344$_{\pm 6.4E-4}$	**0.9365**$_{\pm 1.8E-4}$	✪0.7883$_{\pm 2.5E-4}$	✪1.8347$_{\pm 6.6E-3}$	✪1.0615$_{\pm 3.5E-4}$	✪1.0051$_{\pm 3.4E-4}$	✪1.0337$_{\pm 2.7E-4}$	✪0.1351$_{\pm 5.7E-5}$	✪0.8296$_{\pm 5.1E-5}$	9/1
M4	✪0.7252$_{\pm 4.8E-4}$	✪0.7364$_{\pm 8.8E-4}$	✪0.9581$_{\pm 8.1E-4}$	✪0.7879$_{\pm 1.8E-4}$	✪1.8678$_{\pm 1.9E-3}$	✪1.0604$_{\pm 8.5E-4}$	✪1.0063$_{\pm 5.1E-5}$	✪1.0355$_{\pm 3.1E-3}$	✪0.1280$_{\pm 8.9E-6}$	✪0.8300$_{\pm 2.8E-5}$	10/0
M5	✪0.7255$_{\pm 2.0E-4}$	✪0.7421$_{\pm 6.6E-4}$	✪0.9877$_{\pm 9.7E-4}$	✪0.7953$_{\pm 1.4E-4}$	✪1.8452$_{\pm 4.9E-3}$	✪1.0657$_{\pm 5.1E-4}$	✪1.0069$_{\pm 3.4E-4}$	✪1.0445$_{\pm 5.1E-3}$	✪0.1271$_{\pm 7.4E-4}$	✪0.8311$_{\pm 4.1E-3}$	10/0
M6	✪0.7192$_{\pm 8.2E-3}$	✪0.7367$_{\pm 1.3E-3}$	✪0.9477$_{\pm 6.9E-4}$	✪0.7887$_{\pm 1.7E-3}$	✪1.8372$_{\pm 4.3E-3}$	✪1.0638$_{\pm 7.9E-4}$	✪1.0063$_{\pm 3.1E-3}$	✪1.0306$_{\pm 3.5E-3}$	✪0.1299$_{\pm 1.2E-4}$	✪0.8281$_{\pm 1.8E-3}$	10/0
M7	✪0.7186$_{\pm 1.3E-3}$	✪0.7387$_{\pm 1.9E-3}$	✪0.9454$_{\pm 1.7E-3}$	✪0.7893$_{\pm 2.2E-3}$	✪1.8425$_{\pm 3.5E-3}$	✪1.0633$_{\pm 2.4E-3}$	✪1.0161$_{\pm 3.1E-3}$	✪1.0541$_{\pm 4.5E-3}$	✪0.1397$_{\pm 4.0E-3}$	✪0.8320$_{\pm 5.8E-3}$	10/0
M8	✪0.7228$_{\pm 2.9E-3}$	✪0.7428$_{\pm 2.0E-5}$	✪0.9861$_{\pm 3.5E-5}$	✪0.7925$_{\pm 1.3E-5}$	✪1.8447$_{\pm 3.3E-4}$	✪1.0650$_{\pm 1.4E-3}$	✪1.0278$_{\pm 9.5E-3}$	✪1.0864$_{\pm 6.9E-5}$	✪0.1429$_{\pm 6.5E-4}$	✪0.8320$_{\pm 1.8E-3}$	10/0
M9	✪0.7398$_{\pm 5.5E-3}$	✪0.7433$_{\pm 3.0E-3}$	✪0.9563$_{\pm 8.0E-3}$	✪0.7984$_{\pm 5.7E-3}$	✪1.8470$_{\pm 4.9E-3}$	✪1.6050$_{\pm 8.9E-3}$	✪1.0323$_{\pm 2.8E-3}$	✪1.0342$_{\pm 1.3E-3}$	✪0.1427$_{\pm 1.2E-3}$	✪0.8391$_{\pm 2.7E-4}$	10/0
M10	✪0.7248$_{\pm 4.7E-4}$	✪0.7470$_{\pm 3.9E-4}$	✪0.9501$_{\pm 2.6E-4}$	✪0.7906$_{\pm 3.0E-4}$	✪1.8591$_{\pm 2.3E-3}$	✪1.0668$_{\pm 5.0E-4}$	✪1.0043$_{\pm 7.0E-4}$	✪1.0334$_{\pm 2.9E-3}$	**0.1125**$_{\pm 1.5E-4}$	✪0.8279$_{\pm 1.0E-4}$	9/1
✪**M11**	0.7169$_{\pm 1.8E-4}$	0.7334$_{\pm 7.7E-4}$	0.9407$_{\pm 4.7E-4}$	**0.7865**$_{\pm 4.4E-4}$	**1.8304**$_{\pm 4.9E-3}$	**1.0596**$_{\pm 2.8E-4}$	**0.9995**$_{\pm 1.7E-3}$	**1.0261**$_{\pm 2.1E-4}$	0.1257$_{\pm 4.2E-3}$	0.8275$_{\pm 7.3E-4}$	-

TABLE S(III). Lowest MAE±std, including win/loss counts,
where ✪ indicates that M11 has a lower MAE than the comparison models.

Model	Lowest MAE										✪Win/Loss
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	-
M1	0.5572$_{\pm 2.5E-4}$	✪0.3585$_{\pm 9.4E-5}$	✪0.6712$_{\pm 1.5E-4}$	**0.6035**$_{\pm 2.2E-4}$	✪1.2475$_{\pm 8.1E-4}$	✪0.8082$_{\pm 6.1E-4}$	✪0.7725$_{\pm 3.8E-4}$	✪0.3648$_{\pm 1.1E-4}$	✪0.0758$_{\pm 2.6E-6}$	**0.6395**$_{\pm 4.2E-5}$	7/3
M2	✪0.5611$_{\pm 8.2E-4}$	✪0.3460$_{\pm 6.8E-4}$	✪0.6717$_{\pm 2.2E-4}$	✪0.6057$_{\pm 1.5E-4}$	✪1.2520$_{\pm 1.4E-3}$	**0.8011**$_{\pm 1.3E-4}$	✪0.7760$_{\pm 2.2E-4}$	✪0.3665$_{\pm 1.4E-3}$	✪0.0791$_{\pm 1.5E-4}$	✪0.6438$_{\pm 6.4E-5}$	9/1
M3	✪0.5588$_{\pm 3.4E-4}$	✪0.3758$_{\pm 1.5E-4}$	✪0.6624$_{\pm 2.6E-4}$	0.6046$_{\pm 1.3E-4}$	✪1.2462$_{\pm 1.7E-3}$	✪0.8079$_{\pm 6.6E-4}$	✪0.7731$_{\pm 3.1E-4}$	✪0.3568$_{\pm 6.8E-5}$	✪0.0791$_{\pm 1.8E-5}$	✪0.6433$_{\pm 2.7E-4}$	9/1
M4	✪0.5619$_{\pm 8.6E-4}$	✪0.3663$_{\pm 4.7E-4}$	✪0.6871$_{\pm 4.4E-4}$	✪0.6057$_{\pm 5.6E-4}$	✪1.2702$_{\pm 1.3E-3}$	✪0.8092$_{\pm 3.5E-4}$	✪0.7813$_{\pm 1.6E-4}$	✪0.3682$_{\pm 1.3E-3}$	✪0.0762$_{\pm 1.1E-5}$	✪0.6454$_{\pm 1.5E-4}$	10/0
M5	✪0.5639$_{\pm 8.2E-4}$	✪0.3489$_{\pm 5.4E-4}$	✪0.7197$_{\pm 2.4E-4}$	✪0.6095$_{\pm 7.3E-4}$	✪1.2795$_{\pm 2.6E-3}$	✪0.8108$_{\pm 5.8E-4}$	✪0.7817$_{\pm 7.4E-4}$	✪0.3719$_{\pm 5.6E-3}$	✪0.0935$_{\pm 7.7E-5}$	✪0.6545$_{\pm 3.8E-3}$	10/0
M6	**0.5571**$_{\pm 3.5E-4}$	✪0.3513$_{\pm 9.2E-4}$	✪0.6658$_{\pm 7.4E-4}$	0.6048$_{\pm 3.6E-3}$	✪1.2483$_{\pm 1.7E-3}$	✪0.8107$_{\pm 9.5E-4}$	✪0.7748$_{\pm 7.6E-4}$	✪0.3661$_{\pm 3.5E-3}$	✪0.0873$_{\pm 4.0E-4}$	✪0.6491$_{\pm 1.6E-3}$	8/2
M7	✪0.5586$_{\pm 4.3E-3}$	✪0.3484$_{\pm 8.3E-4}$	✪0.6663$_{\pm 2.4E-3}$	✪0.6085$_{\pm 2.2E-3}$	✪1.2576$_{\pm 8.2E-3}$	✪0.8133$_{\pm 7.6E-3}$	✪0.7818$_{\pm 1.2E-3}$	✪0.3708$_{\pm 5.9E-3}$	✪0.0837$_{\pm 2.0E-3}$	0.6423$_{\pm 9.1E-4}$	9/1
M8	✪0.5611$_{\pm 3.0E-4}$	✪0.3437$_{\pm 1.4E-3}$	✪0.7199$_{\pm 7.9E-3}$	✪0.6079$_{\pm 9.5E-5}$	✪1.2789$_{\pm 2.0E-3}$	✪0.8093$_{\pm 3.0E-4}$	✪0.8273$_{\pm 4.1E-5}$	✪0.3888$_{\pm 2.5E-5}$	✪0.0834$_{\pm 1.7E-3}$	0.6427$_{\pm 1.7E-5}$	9/0
M9	✪0.5702$_{\pm 7.2E-3}$	✪0.3675$_{\pm 1.7E-3}$	✪0.6698$_{\pm 7.9E-3}$	✪0.6075$_{\pm 9.9E-3}$	✪1.2635$_{\pm 4.7E-3}$	✪0.8086$_{\pm 2.6E-3}$	✪0.7752$_{\pm 4.0E-3}$	✪0.3659$_{\pm 3.1E-3}$	✪0.0874$_{\pm 9.8E-4}$	0.6419$_{\pm 4.3E-3}$	9/1
M10	✪0.5601$_{\pm 3.2E-4}$	✪0.3491$_{\pm 4.0E-4}$	✪0.6661$_{\pm 2.0E-4}$	✪0.6059$_{\pm 2.2E-4}$	✪1.2677$_{\pm 1.3E-4}$	✪0.8094$_{\pm 2.6E-4}$	✪0.7729$_{\pm 5.8E-4}$	✪0.3706$_{\pm 1.1E-3}$	**0.0719**$_{\pm 9.0E-5}$	0.6400$_{\pm 1.1E-4}$	8/2
✪**M11**	0.5584$_{\pm 1.5E-3}$	**0.3430**$_{\pm 1.4E-4}$	**0.6609**$_{\pm 2.1E-4}$	0.6052$_{\pm 5.2E-4}$	**1.2451**$_{\pm 2.2E-3}$	0.8075$_{\pm 4.6E-4}$	**0.7723**$_{\pm 1.6E-3}$	**0.3628**$_{\pm 9.8E-4}$	0.0735$_{\pm 4.8E-3}$	0.6427$_{\pm 2.1E-3}$	-

TABLE S(IV). Iteration Count Corresponding to TABLE S(II).

Model	Iteration count									
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**
M1	$877_{\pm 4.1}$	$1000_{\pm 0}$	$1000_{\pm 0}$	$798_{\pm 2.9}$	$172_{\pm 2.1}$	$610_{\pm 3.3}$	$488_{\pm 8.2}$	$518_{\pm 0.4}$	$1000_{\pm 0}$	$616_{\pm 10}$
M2	$365_{\pm 6.3}$	$945_{\pm 11.2}$	$562_{\pm 10.5}$	$235_{\pm 3.9}$	$159_{\pm 4.6}$	$150_{\pm 3.5}$	$185_{\pm 2.5}$	$597_{\pm 22.5}$	$1000_{\pm 0}$	$238_{\pm 1.2}$
M3	$1000_{\pm 0}$	$1000_{\pm 0}$	$1000_{\pm 0}$	$998_{\pm 0.9}$	$608_{\pm 2.3}$	$593_{\pm 1.6}$	$724_{\pm 5.6}$	$1000_{\pm 0}$	$1000_{\pm 0}$	$1000_{\pm 0}$
M4	$291_{\pm 1.3}$	$438_{\pm 3.4}$	$466_{\pm 9.8}$	$228_{\pm 2.9}$	$337_{\pm 1.4}$	$318_{\pm 2.5}$	$85_{\pm 0.5}$	$128_{\pm 6}$	$221_{\pm 7.5}$	$114_{\pm 1.5}$
M5	**$11_{\pm 2.1}$**	**$28_{\pm 0.5}$**	$31_{\pm 2.4}$	**$10_{\pm 2.1}$**	**$6_{\pm 1.2}$**	**$8_{\pm 0.2}$**	**$8_{\pm 0.6}$**	$9_{\pm 4.9}$	$232_{\pm 10.5}$	**$6_{\pm 0.9}$**
M6	$41_{\pm 3.3}$	$41_{\pm 4.8}$	**$19_{\pm 2.0}$**	$18_{\pm 3.6}$	$12_{\pm 1.6}$	**$8_{\pm 0.9}$**	$11_{\pm 1.6}$	**$6_{\pm 1.3}$**	$237_{\pm 11.9}$	$10_{\pm 2.5}$
M7	$40_{\pm 5.6}$	$39_{\pm 4.1}$	$33_{\pm 4.9}$	$21_{\pm 3.8}$	$15_{\pm 1.2}$	$15_{\pm 3.9}$	$16_{\pm 2.6}$	$9_{\pm 1.9}$	$22_{\pm 4.3}$	$11_{\pm 2.7}$
M8	$31_{\pm 5.9}$	$131_{\pm 2.1}$	$107_{\pm 4.2}$	$195_{\pm 7.3}$	$5_{\pm 0}$	$6_{\pm 2.1}$	$57_{\pm 25.7}$	$63_{\pm 9.7}$	$21_{\pm 2.4}$	$13_{\pm 2.3}$
M9	$17_{\pm 4.9}$	$27_{\pm 4.7}$	$27_{\pm 6.3}$	$27_{\pm 4.9}$	$7_{\pm 1.5}$	$17_{\pm 2.4}$	$7_{\pm 2.1}$	$15_{\pm 1.7}$	**$12_{\pm 0.6}$**	$8_{\pm 0.5}$
M10	$64_{\pm 1.0}$	$192_{\pm 1.4}$	$69_{\pm 1.5}$	$56_{\pm 0.8}$	$22_{\pm 0.8}$	$49_{\pm 1.1}$	$37_{\pm 1.6}$	$28_{\pm 1.2}$	$1000_{\pm 0}$	$38_{\pm 1.2}$
M11	$45_{\pm 4.2}$	$39_{\pm 1.9}$	$36_{\pm 1.25}$	$65_{\pm 4.6}$	$14_{\pm 0.8}$	$10_{\pm 2.2}$	$58_{\pm 6.3}$	$9_{\pm 2.9}$	$183_{\pm 5.6}$	$20_{\pm 4.3}$

TABLE S(V). Iteration Count Corresponding to TABLE S(III).

Model	Iteration count									
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**
M1	$892_{\pm 1.3}$	$998_{\pm 0.8}$	$936_{\pm 2.6}$	$841_{\pm 1.3}$	$188_{\pm 1.6}$	$728_{\pm 0.9}$	$533_{\pm 3.6}$	$116_{\pm 3.1}$	$999_{\pm 0.5}$	$725_{\pm 5.4}$
M2	$324_{\pm 5.9}$	$1000_{\pm 0}$	$539_{\pm 11.5}$	$248_{\pm 2.9}$	$167_{\pm 1.5}$	$168_{\pm 2.9}$	$204_{\pm 1.4}$	$67_{\pm 3.4}$	$1000_{\pm 0}$	$285_{\pm 6.7}$
M3	$1000_{\pm 0}$	$1000_{\pm 0}$	$1000_{\pm 0}$	$1000_{\pm 0}$	$683_{\pm 2.5}$	$691_{\pm 0.9}$	$824_{\pm 11.6}$	$1000_{\pm 0}$	$1000_{\pm 0}$	$1000_{\pm 0}$
M4	$479_{\pm 1.6}$	$1000_{\pm 0}$	$453_{\pm 7.8}$	$399_{\pm 2.1}$	$348_{\pm 3.4}$	$377_{\pm 3.4}$	$97_{\pm 2.1}$	$44_{\pm 1.2}$	$195_{\pm 2}$	$131_{\pm 1.5}$
M5	**$9_{\pm 1.5}$**	**$17_{\pm 1.2}$**	$35_{\pm 4.2}$	**$9_{\pm 1.6}$**	$6_{\pm 0.8}$	**$8_{\pm 0.4}$**	$9_{\pm 0.7}$	**$7_{\pm 0.8}$**	$74_{\pm 1.4}$	**$8_{\pm 0}$**
M6	$57_{\pm 4.6}$	$23_{\pm 3.8}$	$68_{\pm 7.4}$	$56_{\pm 3.8}$	$13_{\pm 2.2}$	$9_{\pm 1.2}$	**$7_{\pm 0.4}$**	$13_{\pm 3.1}$	**$18_{\pm 3.2}$**	$13_{\pm 2.6}$
M7	$31_{\pm 5.2}$	$39_{\pm 4.6}$	$27_{\pm 4.1}$	$18_{\pm 4.2}$	$18_{\pm 3.7}$	$13_{\pm 1.7}$	$12_{\pm 2.2}$	$16_{\pm 2.2}$	$34_{\pm 6.2}$	$11_{\pm 0.5}$
M8	$21_{\pm 2.8}$	$39_{\pm 5.6}$	$37_{\pm 5.5}$	$176_{\pm 5.5}$	**$5_{\pm 0.5}$**	$9_{\pm 3.6}$	$85_{\pm 6.2}$	$52_{\pm 5.3}$	$24_{\pm 2.9}$	$9_{\pm 0}$
M9	$24_{\pm 2.5}$	$25_{\pm 5.4}$	**$17_{\pm 4.9}$**	$31_{\pm 6.9}$	**13**$_{\pm 4.6}$	$19_{\pm 2.8}$	$12_{\pm 0.9}$	$8_{\pm 1}$	$35_{\pm 3.9}$	$12_{\pm 2.5}$
M10	$65_{\pm 1.6}$	$200_{\pm 0.8}$	$73_{\pm 2.4}$	$57_{\pm 1.4}$	**26**$_{\pm 1.1}$	$61_{\pm 1.4}$	$40_{\pm 1.2}$	$24_{\pm 1.5}$	$1000_{\pm 0.8}$	$43_{\pm 1.1}$
M11	$27_{\pm 3.4}$	$42_{\pm 3.4}$	$35_{\pm 1.9}$	$46_{\pm 2.4}$	$11_{\pm 1.2}$	$18_{\pm 2.7}$	$30_{\pm 4.4}$	$9_{\pm 2.3}$	$216_{\pm 18}$	$21_{\pm 5.1}$

TABLE S(VI). Total Time Cost Corresponding to TABLE S(II) (Seconds)±std.

No.	Total Time Cost of RMSE									
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**
M1#	1865.6±16.1	1774.8±14.8	1060.3±17.5	932.1±11.2	445.3±13.3	8348.1±13.2	107.5±2.0	172.2±3.1	130.9±3.6	27088±2684.7
M1*	40220.2	55230.4	19645.9	20976.6	11120.4	192784.6	5304.1	5544.9	5500.6	516705.6
M2#	2381.9±15.0	5168.1±41.1	1872.9±9.1	857.5±18.5	1224.2±5.4	7007.4±118.7	145.9±3.8	652.3±23.2	406.9±25.5	33717.4±311.3
M2*	25448.1	31757.4	12247.6	8902.7	23063.9	67556.9	1734.9	4378.1	1616.6	562702.1
M3#	9581.4±20.8	7346.5±55.8	4252.6±20.6	5264.6±38.1	7609.8±77.8	41084±377.6	745.7±15.7	1747.3±21.6	677.6±14.8	208810±3467.9
M3*	59859.8	55501.1	28591.5	25345.7	73342.7	311300.6	4991.9	6740.9	1339.5	1256679.2
M4#	2855.3±49.0	3498.7±33.9	2076.4±28.3	1242.4±22.7	4435.5±62.4	21654±192.1	94.8±2.6	210.2±17.7	146.3±20.3	24093.8±202.9
M4*	24490.7	31445.2	22452.3	12026.8	31598.5	76992.4	1421.1	4369.1	379.0	342999.1
M5	**256.3±26.1**	**540±10.1**	321.3±7.5	**125.3±21.8**	189±12.2	1057.3±4.2	**19.2±2.3**	30.6±3.3	308.9±21.3	**2898.9±183.5**
M6	913.8±55.6	769.3±87.3	**216.8±63.4**	285.3±15.6	342.6±30.5	1074.7±84.1	20.1±1.9	**20.9±3.8**	317.7±38.4	4765.8±362.4
M7	1915.5±275.8	1659.3±121.4	861.3±67.8	603.5±62.7	1066.3±53.2	3884.5±953.2	64.9±4.2	68.4±6.5	52.7±5.3	9012.1±869.9
M8	745.6±92.1	2584.6±88.9	1257.4±31.9	2596.7±54.1	**164.4±1.8**	**930.8±279.4**	134.1±23.3	234±19.5	**34.5±2.1**	5812.8±450.2
M9	1307.3±184.4	1702.5±255.2	956.8±174.5	1031.7±115.4	617.5±138.5	7900.6±850.1	53.1±5.6	175.3±10.2	60.6±7.8	11463±2064.3
M10	1372.5±18.8	3175.7±44.8	775.2±27.4	753.5±11.6	584.4±23.4	6755.4±315.4	80.1±1.8	99.8±2.6	1264.6±14.2	16779.5±605.2
M11	1012.1±42.6	736.7±25.8	426.8±12.1	822.9±8.6	440.3±16.3	1341.5±124.6	113±12.3	33.7±4.3	220.2±10.7	8618.1±763.9

#Time cost of optimal hyper-parameters, *Total time cost by manual grid search for hyper-parameters. Note that in practice, the grid search is commonly conducted for only once since it is indeed expensive. So the time cost of grid search is not accompanied with std.

TABLE S(VII). Total Time Cost Corresponding to TABLE S(III) (Seconds)±std.

No.	Total Time Cost of MAE									
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**
M1#	1850.6±17.2	1788.4±23.2	933.3±14.9	1010±9.8	460.6±11.0	9818.9±15.7	127.3±1.2	38.4±4.3	122.4±1.8	31907±852.4
M1*	39530.6	56621.4	21098.9	23089.5	13208.1	20789.3	5432.3	5614.3	6210.3	504731.7
M2#	2243.2±25.9	5750.8±66.9	1756.1±17.3	1021.9±11.6	1517.5±27.3	7967.4±132.2	184.5±2.5	71.8±2.2	387.1±9.1	43222±995.1
M2*	26999.5	33437.4	13055.8	10053.3	27220.1	68582.2	1989.6	4293.9	1537.9	491166.1
M3#	9838.3±18.5	8296.9±63.9	4606.9±59.9	5488.9±90.9	9315.2±216.8	48822±553.8	859.9±10.8	1668.9±32.5	627.6±3.3	215410±3545.6
M3*	59063.7	57759.2	29697.6	26372.8	67306.2	309651.5	5057.8	6611.4	1240.6	1218175.4
M4#	4701.4±75.6	7896.9±54.5	2024.7±54.3	2233.3±38.0	4749.1±32.8	25702±192.1	119.4±4.9	84.1±3.6	109.3±2.6	28181.0±794.0
M4*	24498.2	31086.9	22521.6	12353.8	32763.2	77083.7	1529.7	5085.3	447.6	362572.8
M5	**208.2±31.9**	**353.1±25.3**	397.9±36.7	**117.3±17.2**	183.1±19.7	**1034.7±39.1**	22.5±2.0	**24.8±1.8**	101.9±2.7	**3919.8±153.8**
M6	1199.6±66.2	417.1±45.6	817.6±43.7	933.1±54.2	382.9±48.5	1114.3±145.5	**16.4±2.2**	44.8±4.9	**26.2±3.4**	6101.2±1453.9
M7	1584.6±172.2	1726.4±158.7	648.9±60.8	519.6±65.7	1270.3±185.6	3457.7±420.3	60±3.6	134±9.8	79.6±7.8	9471.4±93.1
M8	488.7±44.6	736.6±80.5	426.3±38.6	2363.9±55.4	**169.7±10.2**	1442±465	248.4±5.8	192.3±20.7	38.3±2.4	4257.2±37.1
M9	1642.1±132.3	1563.4±159.7	610.9±123.5	1206.4±133.6	1094.2±173.5	9121.9±674.9	100.2±5.8	99.7±10.3	152.3±14.4	17595.8±3424.1
M10	1452.5±30.3	3322.5±73.3	898.8±21.7	748.5±15.9	661.2±37.6	7921.5±662.1	88.7±1.6	82.5±3.2	1265.5±4.9	19383.3±2521
M11	602.9±12.3	820.7±54.2	**393.7±9.2**	603.1±12.6	335.6±16.1	2414.7±167.6	73.8±6.9	32.9±5.6	248.1±48.5	9140.4±1128.5

#,* have the same meanings with those in TABLE S(VI).