Causal Inference, Homework 3

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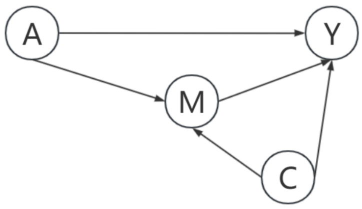
1. Mediation Analysis

Question. Assume that there is a randomized experiment, where participants were assigned to the different treatment groups. The DAG underlying this study is provided below (A: treatment; M: mediator; Y: outcome; C: confounders). (a) What do you notice about the DAG structure and why does this make sense? (b) What assumptions are we making when estimating direct and indirect effects? (c) Based on assumptions in (b), do you agree with this DAG? What should be updated?



Solution. (a) In the DAG underlying this experiment, the treatment A affects the outcome Y through 2 paths: $A \to Y$ and $A \to M \to Y$, which are both causal. The confounders C lie between M (the mediator) and Y (the outcome), as is shown in the DAG. However, since the path $A \to M \leftarrow C \to Y$ is blocked by mediator M (as a collider), there is no confounding between A and Y. Recall that this is a randomized experiment, the causal DAG makes sense.

(b) Definitions. Consider treatment A, covariates C, mediator M and outcome Y. Let Y^a denote the counterfactual outcome Y when intervening to set $A = a$, M^a the counterfactual mediator M when intervening to set $A = a$, and Y^{am} the counterfactual outcome Y when intervening to set $A = a, M = m$. Here are definitions of different causal effects:
Controlled direct effect: $CDE(m) = Y^{1m} - Y^{0m}$;
Natural direct effect: $NDE = Y^{1M^0} - Y^{0M^0}$;
Natural indirect effect: $NIE = Y^{1M^1} - Y^{1M^0}$.
Total effect: $TE = Y^1 - Y^0 = Y^{1M^1} - Y^{0M^0} = NIE + NDE$;

To estimate direct and indirect effects, we consider NDE and NIE. Four assumptions are needed to identify them:
(i) There are no unmeasured treatment-outcome confounders given C, i.e. $Y^{am} \perp A \,|\, C$;
(ii) There are no unmeasured mediator-outcome confounders given (C, A), i.e. $Y^{am} \perp M \,|\, C, A$;

(iii) There are no unmeasured treatment-mediator confounders given C, i.e. $M^a \perp A \mid C$;
(iv) There is no unmeasured mediator-outcome confounder affected by treatment A, i.e. $Y^{am} \perp M^{a^*} \mid C$;

(c) Based on assumptions in (b), I would agree with this DAG. Let's check the four assumptions:
(i) Conditioning on C, Y and A are unconfounded. Plus the randomization assumption (also referred to as exchangeability), we have $Y^{am} \perp A \mid C$.
(ii) Consider M and Y, $M \leftarrow A \rightarrow Y$ and $M \leftarrow C \rightarrow Y$ are two backdoor paths. Conditioning on C and A blocks them and eliminates confounding. Plus the randomization assumption, $Y^{am} \perp M, \mid A, C$.
(iii) Conditioning on C, M and A are unconfounded. Like in (i), $M^a \perp A \mid C$.
(iv) All paths between A and C are naturally blocked, and there is no arrow from A to C. Hence, there is no unmeasured mediator-outcome confounders affected by A.
The DAG satisfies the four assumptions put forward in **(b)**. But, it is the case for only completely randomized experiments. In some other experiments or observational studies, this DAG may lead to model misspecification and systematic bias. To fix the problem, we can add exposure-outcome confounders C' and exposure-mediator confounders C^* to the previous DAG. If C, C' and C^* suffice to control for confounding, then the mediation analysis will have a good interpretability.

2. Instrumental Variable (IV) Analysis

Question. There are four core assumptions for an IV analysis: (1) Relevance; (2) Exclusion restriction; (3) Randomized assignment; (4) Monotonicity. Please explain why the first three assumptions (instrumental conditions) are needed for IV estimation using d-separation ideas.

Solution. Suppose Z is an instrumental variable for investigating the causal effect of exposure A on Y. The first three assumptions for an IV analysis are:
IV.1. (Relevance) Instrument Z and exposure A are associated;
IV.2. (Exclusion Restriction) Instrument Z affects the outcome Y only through exposure A, and there is no direct effect of Z on Y.
IV.3. (Exogeneity / Randomized assignment) Z does not share common causes with the outcome Y.

Under the relevance assumption (IV.1), either there is be a causal path connecting Z and A, or they share a common cause. Hence, **Z and A is not d-separated**, and Z can be used to estimate the effect of Z. Under the exogeneity assumption (IV.3), there is no confounding between Z and Y, corresponding to the DAG situation, **no backdoor path exists between them**. Plus the exclusion assumption (IV.2), there is no direct causal path from Z to Y. Hence,**Z and Y is d-separated conditional on A**. Since Z and A are d-connected, Z affects the outcome Y only through its impact on A. Therefore, IV estimation helps to analyze the causal effect of A on Y, even though there exists unmeasured confounding between them.