

Assignment 7

Junyi Liao 20307110289

December 8, 2022

1 Free-Form Deformation

1.1 Algorithm

Free-form deformation (FFD) is a common method for non-linear transformation in image processing. To operate the shape of objects in an image or a 3D-model, FFD employs a B-spline mesh to estimate the deformation field, which can be controlled by grid points in the mesh. These grid points scattered uniformly in the deformation field are referred as control points, whose effect on surrounding pixels (in 3D model, voxels) is defined by the cubic B-spline function:

$$\beta^{(3)}(a) = \begin{cases} \frac{1}{6}(4 - 6a^2 + 3|a|^3), & 0 \leq |a| < 1, \\ \frac{1}{6}(2 - |a|)^3, & 1 \leq |a| < 2, \\ 0, & |a| \geq 2; \end{cases}$$

thus one pixel or voxel is controlled by 4 control points along one orthogonal direction.

Now we concentrate on image transformation under 2D case. Suppose there're $n_x n_y$ control points

$$\{p_{ij} = (il_x, jl_y) \mid i = 0, 1, \dots, n_x - 1, j = 0, 1, \dots, n_y - 1\},$$

which are uniformly spaced in the field, with length of vertical intervals l_x and of horizontal intervals l_y .

An FFD \mathcal{T} drags the control point p_{ij} with offset ϕ_{ij} , a 2-dimension vector indicating the moving distances along $+x$ and $+y$ directions, then the transformation of point $\mathbf{z} = [x, y]$ is given by

$$\mathcal{T}(\mathbf{z}) = \mathbf{z} + \sum_{i=0}^{n_x-1} \sum_{j=0}^{n_y-1} \phi_{ij} \beta^{(3)} \left(\frac{x - p_{ij}^{(x)}}{l_x} \right) \beta^{(3)} \left(\frac{y - p_{ij}^{(y)}}{l_y} \right);$$

Notice that \mathbf{z} is only controlled by its 4 proximal control points along one direction, denote

$$i = \lfloor x/l_x \rfloor, \quad j = \lfloor y/l_y \rfloor,$$

then \mathbf{z} is controlled by $\{p_{i+k,j+l} \mid k = -1, 0, 1, 2, j = -1, 0, 1, 2\}$; meanwhile the lattice coordinate of \mathbf{z} is

$$[u, v] = \left[\frac{x}{l_x} - i, \frac{y}{l_y} - j \right] \in [0, 1] \times [0, 1],$$

thus \mathcal{T} can be simplified as

$$\mathcal{T}(\mathbf{z}) = \mathbf{z} + \sum_{k=-1}^2 \sum_{l=-1}^2 \phi_{i+k,j+l} B_{k+1}(u) B_{l+1}(v),$$

where B is modified B-spline family:

$$B_0(u) = \beta^{(3)}(1+u) = (1-u)^3/6, \quad B_1(u) = \beta^{(3)}(u) = (4-6u^2+3u^3)/6,$$

$$B_2(u) = \beta^{(3)}(u-1) = (1+3u+3u^2-3u^3)/6, \quad B_3(u) = \beta^{(3)}(2-u) = u^3/6;$$

Define the local offset term

$$Q_{local}(x, y) = \sum_{k=-1}^2 \sum_{l=-1}^2 \phi_{i+k, j+l} B_{k+1}(u) B_{l+1}(v),$$

then transformed image $X' = X + Q_{local}$.

1.2 Code

In implement, we use matrix operation to accelerate the computation. We define a object FFD to execute the transformation. The code for FFD transformation is shown below.

```

1 class FFD:
2     def __init__(self, lx=64, ly=64, ncpx=9, ncpy=9):
3         # Number of pixels in each grid. (length of intervals)
4         self.lx = lx
5         self.ly = ly
6         # Number of control points in each direction.
7         self.ncpx = ncpx
8         self.ncpy = ncpy
9         # Original coordinates of control points.
10        x = np.linspace(0, lx * (ncpx - 1), ncpx)
11        y = np.linspace(0, ly * (ncpy - 1), ncpy)
12        xx, yy = np.meshgrid(x, y, indexing='ij')
13        self.cp_org = np.stack((xx, yy), axis=-1)
14        # Offset of control points.
15        self.cp_offset = np.zeros((ncpx, ncpy, 2))
16
17    @staticmethod
18    def Bspline(u: np.float, idx: int):
19        # B-spline function.
20        if idx == 0:
21            return (1 - u) ** 3 / 6
22        elif idx == 1:
23            return (3 * u ** 3 - 6 * u ** 2 + 4) / 6
24        elif idx == 2:
25            return (-3 * u ** 3 + 3 * u ** 2 + 3 * u + 1) / 6
26        else:
27            return u ** 3 / 6
28

```

```

29     def update_offset(self, mat):
30         # Read in the offset of control points from a text file.
31         print('Read in offset data.')
32         with open(mat, 'r') as fp:
33             txt = fp.read()
34             rows = txt.split('\n')
35             i = 0
36             for row in rows:
37                 j = 0
38                 cols = row.split(',')
39                 for col in cols:
40                     self.cp_offset[i, j, :] = np.array(col.split()).astype(
41                         float)
42                     j += 1
43                     i += 1
44
45     def FFDtransform(self, coord: np.float):
46         """
47             2D FFD transform.
48             Input: Coordinate matrix of shape 2 * N, each column corresponds to a
49             pixel point.
50             Output: Transformed coordinate matrix.
51         """
52
53         x, y = coord[0, :], coord[1, :]
54         flr_x, flr_y = np.floor([x / self.lx, y / self.ly]).astype(int)
55
56         # Lattice coordinates.
57         u, v = x / self.lx - flr_x, y / self.ly - flr_y
58
59         q_local = np.zeros_like(coord)
60         # Get the proximal control point.
61         get_cp = lambda a, b: self.cp_offset[a.clip(0, self.ncpx - 1), b.clip(
62             0, self.ncpy - 1)].T
63         # Local offset.
64         for i in range(-1, 3):
65             for j in range(-1, 3):
66                 bunch = self.Bspline(u, idx=i+1) * self.Bspline(v, idx=j+1)
67                 bunch = np.array([bunch] * 2)
68                 q_local += bunch * get_cp(flr_x + i, flr_y + j)
69
70     return coord + q_local

```

2 Inverse Transformation

2.1 Algorithm

To generate the transformed image from \mathcal{T} , we need to find the value of each pixel (x, y) in transformed image X' . A general idea is to find the corresponding position in the original image X for each pixel in X' based on the inverse transformation

$$X = \mathcal{T}^{-1}(X'),$$

thus the value of pixel in transformed image X' is

$$I'(x, y) = I(\mathcal{T}^{-1}(x, y)).$$

That's the method of generate transformed image based on inverse transformation.

Notice that the inverse $\mathcal{T}^{-1}(x, y)$ is not always integer, we use linear interpolation to estimate the value of position $(x^*, y^*) = \mathcal{T}^{-1}(x, y)$ in the original image based on 4 proximal pixels:

$$\begin{aligned} I(x^*, y^*) &= w_{11}I(\lfloor x^* \rfloor, \lfloor y^* \rfloor) + w_{12}I(\lfloor x^* \rfloor, \lceil y^* \rceil) + w_{21}I(\lceil x^* \rceil, \lfloor y^* \rfloor) + w_{22}I(\lceil x^* \rceil, \lceil y^* \rceil) \\ w_{11} &= (\lceil x^* \rceil - x^*)(\lceil y^* \rceil - y), \\ w_{12} &= (\lceil x^* \rceil - x^*)(y^* - \lfloor y^* \rfloor), \\ w_{21} &= (x^* - \lfloor x^* \rfloor)(\lceil y^* \rceil - y^*), \\ w_{22} &= (x^* - \lfloor x^* \rfloor)(y^* - \lfloor y^* \rfloor); \end{aligned}$$

Also, sometimes the inverse $\mathcal{T}^{-1}(x, y)$ is out of range in the original image. In such case we set it directly as the closest pixel positioned at the boundary of original image.

Thus we can generate the transformed image by inverse transformation and linear interpolation. To accelerate computation we also apply matrix operations. The code is shown below.

```
1 def generate_coord(shape):
2     """
3         Generate the coordinates of pixels with a given shape.
4         Input: the shape of image (h, w), height and width;
5         Output: an array of dimension 2 * N, N = h * w, with each column the
6             coordinate of a pixel.
7     """
8     h = np.linspace(0, shape[0] - 1, shape[0]).astype(float)
9     w = np.linspace(0, shape[1] - 1, shape[1]).astype(float)
10    hh, ww = np.meshgrid(h, w, indexing='ij')
11    coord = np.stack((hh, ww), axis=-1)
12    return coord.reshape((-1, 2)).T
13
```

```

14 def bilinear_interp(img: np.ndarray, coord: np.ndarray):
15     """
16     Based on inverse transformation, compute pixel values.
17     Input: image with shape h * w, transformed coordinate matrix with shape 2 *
18         hw;
19     Output: transformed image with shape
20     """
21
22     h, w = img.shape
23     coord_floor = np.floor(coord).astype(int)
24     offset = coord - coord_floor
25     get_pixel = lambda a, b: img[a.clip(0, h - 1), b.clip(0, w - 1)]
26     la = (1 - offset[0, :]) * (1 - offset[1, :]) * get_pixel(coord_floor[0, :], coord_floor[1, :])
27     lb = offset[0, :] * (1 - offset[1, :]) * get_pixel(coord_floor[0, :] + 1, coord_floor[1, :])
28     ra = (1 - offset[0, :]) * offset[1, :] * get_pixel(coord_floor[0, :], coord_floor[1, :] + 1)
29     rb = offset[0, :] * offset[1, :] * get_pixel(coord_floor[0, :] + 1, coord_floor[1, :] + 1)
30
31     return (la + lb + ra + rb).reshape((h, w))

```

2.2 Estimate of Inverse

An inverse transformation is required in the method above, however the inverse of FFD is unknown. Note that FFD is a non-linear transformation, it's difficult even impossible to derive an inverse transformation of analytical form. Thus we need to find some methods to estimate the inverse.

2.2.1 A Rough Approximation

We can assume that the transformed image X' doesn't diverge much from the original image X . In fact, this assumption can be often satisfied, since many topological structures aren't damaged after transformation. Under the assumption, $Q_{local}(X) = X' - X = o(1)$ is a first-order small quantity; Moreover, $Q_{local}(X) - Q_{local}(X') = J^\top(X - X')$, where Jacobian matrix $J = o(I)$, hence

$$Q_{local}(X) - Q_{local}(X') = o(X - X')$$

is a second-order small quantity, and

$$X = X' - Q_{local}(X') - o(X - X'),$$

$$\mathcal{T}^{-1}(X) \approx X - Q_{local}(X) = 2X - \mathcal{T}(X);$$

2.2.2 Optimization Method

The assumption above is satisfied in varying degrees under different case, and the approximation $2X - \mathcal{T}(X)$ can be inaccurate when the original image is severely deformed.

To improve the accuracy, we apply optimization methods to estimate the inverse of FFD.

For a latent solution (x, y) and the objective point (α, β) , it implies $(x, y) = \mathcal{T}^{-1}(\alpha, \beta)$, where RHS is not accessible. An easy idea is to compare $\mathcal{T}(x, y)$ and (α, β) , therefore the loss function can be defined as the squared distance between them, and the problem can be written as:

$$\min_{x, y} \|\mathcal{T}(x, y) - (\alpha, \beta)\|_2^2;$$

The objective function is approximately convex, so we can solve it by gradient descent or Newton's method. Denote the objective function as f , in gradient method, the descent direction at point x is

$$p_{gd} = -\nabla f(x),$$

while in Newton's method,

$$p_{nt} = -[\nabla^2 f(x)]^{-1} \nabla f(x);$$

Here the gradient $\nabla f(x)$ and Hessian matrix $\nabla^2 f(x)$ can't be written in a closed form, so we estimate them using finite-difference approach:

$$\frac{\partial}{\partial x} f \approx \frac{f(x + h) - f(x - h)}{2h},$$

where h is a relatively small quantity.

Set the learning rate as t , the step size along descent direction, the renewal formula in k -th iteration is

$$x^{(k)} = x^{(k-1)} + tp^{(k-1)}, \quad k = 1, 2, \dots;$$

A properly large learning rate often leads to quick convergence, while too large rate may "leap over" the optimal solution. To guarantee that the value of objective function is descending, we introduce the Armijo line search rule:

$$f(x + tp) \leq f(x) + \alpha t p^\top \nabla f(x)$$

Here α is a small positive constant. Judge the condition above before update the current solution x as $x + tp$. If the condition is not satisfied, set the step size as $t \leftarrow t/2$ and repeat until it's satisfied.

Now we consider the terminate condition. It's known that the optimal value of objective function $f(x, y) = \|\mathcal{T}(x, y) - (\alpha, \beta)\|_2^2$ is 0. Therefore we set a small constant $\varepsilon > 0$, and accept (x^*, y^*) as a solution if $f(x^*, y^*) < \varepsilon$, say the transformed point $\mathcal{T}(x^*, y^*)$ is sufficiently close to the goal point (α, β) .

2.2.3 Implement: A heuristic method

We combine the two methods above and derive a heuristic method to solve the inverse of FFD. We know the selection of initial point $(x^{(0)}, y^{(0)})$ significantly affects the efficiency of optimization method, even the result. Since we can get an approximate inverse by $\mathcal{F}^{-1}(X) \approx 2X - \mathcal{T}(X)$, we can set this as the initial point. In contrast to random initialization, it reduces the number of iterations needed to find the optimal point.

For the hyperparameters, we set the learning rate $t = 1$, the Armijo constant $\alpha = 10^{-4}$, and the acceptable error $\varepsilon = 1$. In implement, $\varepsilon = 1$ means the error from transformed point $\mathcal{T}(x, y)$ to goal point (α, β) is no more than one pixel, which is not observable by naked eye.

For the optimization method, we choose Newton's method, which is a second-order method and converges faster than gradient descent.

The code for inverse estimation is shown below.

- (i) Methods `compute_gradient` and `compute_hessian` estimate the gradient and Hessian matrix of loss function using finite-difference approach, with difference $h = 5 \times 10^{-3}$.
 - (ii) Methods `optim_gd` and `optim_newton` are optimization algorithms (gradient descent and Newton's method).
 - (iii) Method `FFDinvtransform` solves the inverse of the defined FFD.

```

34         f = lambda x, y, x0, y0: ((self.FFDtransform(np.array([[x, y]]).T) - np
35             .array([[x0, y0]]).T) ** 2).sum()
36         d2fdx2 = (f(x + 2 * h, y, xt, yt) + f(x - 2 * h, y, xt, yt) - 2 * f(x,
37             y, xt, yt)) / (4 * h * h)
38         d2fdy2 = (f(x, y + 2 * h, xt, yt) + f(x, y - 2 * h, xt, yt) - 2 * f(x,
39             y, xt, yt)) / (4 * h * h)
40         d2fdxdy = (f(x + h, y + h, xt, yt) - f(x + h, y - h, xt, yt) - f(x - h,
41             y + h, xt, yt) + f(x - h, y - h, xt, yt)) / (4 * h * h)
42         return np.array([[d2fdx2, d2fdxdy], [d2fdxdy, d2fdy2]])
43
44
45     def optim_gd(self, origin, goal, lr=2, epsilon=1, alpha=1e-04):
46         """
47             Use gradient descent method for optimization.
48             Compute the point, which is mapped onto the goal point after FFD
49             transformation.
50
51             Input: origin -> start point, goal -> a,
52                   lr -> learning rate, epsilon: acceptable error;
53             Output: optimal solution (x*, y*), error, number of iterations.
54         """
55
56         # Initialization: compute an approximate solution.
57         origin, goal = origin.reshape((2, 1)), goal.reshape((2, 1))
58         error = ((self.FFDtransform(origin) - goal) ** 2).sum()
59         p = origin
60         ite = 0
61
62         # Iterations.
63         while error > epsilon and ite < 1e3:
64             grad = self.compute_gradient(p[0, 0], p[1, 0], goal[0, 0], goal[1,
65             0])
66             s = 1
67             p_new, error_new = np.zeros_like(p), error
68             # Use Armijo line search.
69             while error_new - error + alpha * lr * s * (grad ** 2).sum() > 0:
70                 p_new = p - lr * grad * s
71                 error_new = ((self.FFDtransform(p_new) - goal) ** 2).sum()
72                 s *= 0.5
73             if np.linalg.norm(p - p_new) < 1e-2:
74                 break
75             p, error = p_new, error_new
76             ite += 1
77
78         return {'sol': p, 'error': error, 'ite': ite}
79
80     def optim_newton(self, origin, goal, lr=1, epsilon=1, alpha=1e-04):

```

```

70      """
71      Use newton method for optimization.
72      Compute the point, which is mapped onto the goal point after FFD
73      transformation.
74      Input: origin -> start point, goal -> a,
75          lr -> learning rate, epsilon: acceptable error;
76      Output: optimal solution (x*, y*), error, number of iterations.
77      """
78
79      # Initialization.
80      origin, goal = origin.reshape((2, 1)), goal.reshape((2, 1))
81      error = ((self.FFDtransform(origin) - goal) ** 2).sum()
82      p = origin
83      ite = 0
84
85      # Iterations.
86      while error > epsilon and ite < 1e3:
87          grad = self.compute_gradient(p[0, 0], p[1, 0], goal[0, 0], goal[1,
88          0])
89          hess = self.compute_hessian(p[0, 0], p[1, 0], goal[0, 0], goal[1,
90          0])
91          decrement = np.linalg.solve(hess, grad)
92          s = 1
93          p_new, error_new = np.zeros_like(p), error
94
95          # Use Armijo line search.
96          while error_new - error + alpha * lr * s * (grad * decrement).sum()
97              > 0:
98              p_new = p - lr * decrement * s
99              error_new = ((self.FFDtransform(p_new) - goal) ** 2).sum()
100             s *= 0.5
101             if np.linalg.norm(p - p_new) < 1e-2:
102                 break
103             p, error = p_new, error_new
104             ite += 1
105
106         return {'sol': p, 'error': error, 'ite': ite}

107     def FFDinvtransform(self, coord: np.float):
108         """
109
110         Based on given coordinate matrix, solve the original coordinate matrix
111         before FFD.
112
113         Input: transformed coordinate matrix of shape 2 * N;
114         Output: original matrix of shape 2 * N, mean square error.
115
116         """

```

```

107     # Initialization.
108     origin = 2 * coord - self.FFDtransform(coord)
109
110     # Compute the inverse.
111     inv = np.zeros_like(origin)
112     mse = 0
113     sum_iter = 0
114     stone = coord.shape[1] // 100
115     for i in range(coord.shape[1]):
116         # opt = self.optim_gd(origin[:, i], coord[:, i])
117         opt = self.optim_newton(origin[:, i], coord[:, i])
118         inv[:, i] = opt['sol'].flatten()
119         mse += opt['error']
120         sum_iter += opt['ite']
121         if i % stone == 0:
122             print('Computing inverse,\titerations: {0:7d},\t{1}% completed;
123             '.format(sum_iter, i / stone))
124     mse = np.sqrt(mse / coord.shape[1])
125     return {'inv': inv, 'mse': mse}

```

For evaluation, we use mean square error (MSE) to measure the accuracy of our solution. Suppose we get the inverse $\hat{X}' \approx \mathcal{T}^{-1}(X)$, then

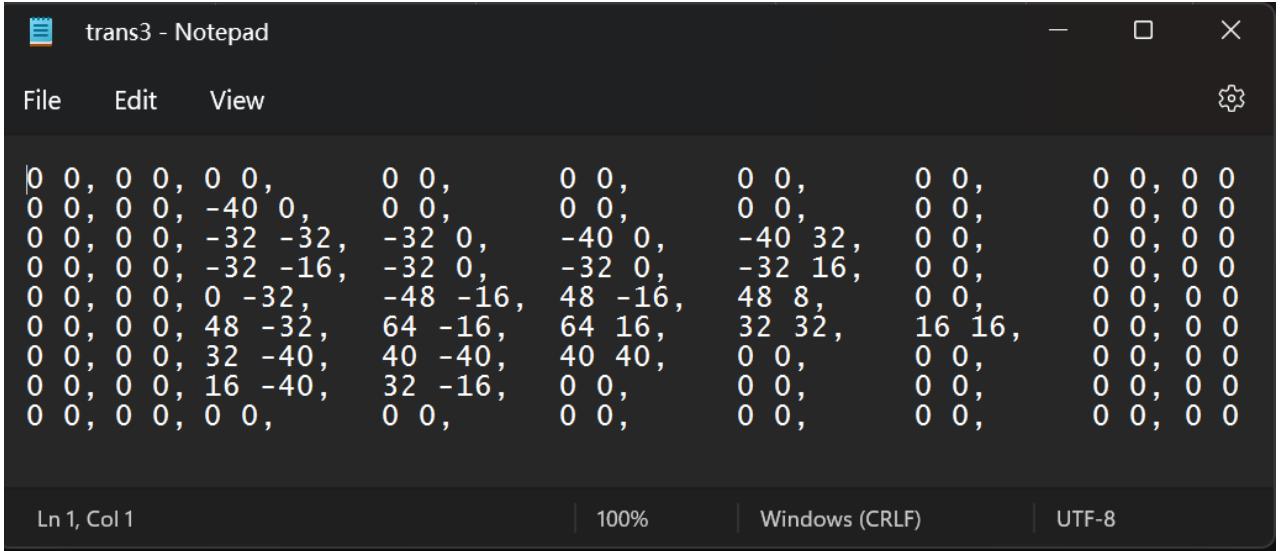
$$MSE = \frac{1}{\sqrt{N}} \|\mathcal{T}(\hat{X}') - X\|_F = \sqrt{\frac{1}{N} \sum_{i=1}^N \|\mathcal{T}(\hat{X}'_i) - X_i\|_2^2},$$

where N is the number of pixels in the image. MSE indicates the mean error of transformed pixels. It holds that $MSE \leq \sqrt{\epsilon}$.

3 Experiment

We tested our code on several images and different input data. In our experiment, we set the number of control points along one axis as $n_x = 9$, $n_y = 9$ (in total 81 points), with 8 intervals of length $l_x = 64$, $l_y = 64$. All our input images are resized to 513×513 .

Our input data includes images and offset of control points. The offset of control points is a $9 \times 9 \times 2$ tensor, referred as $[\phi_{ij}]_{i=0,1,\dots,8, j=0,1,\dots,8}$ with each $\phi_{ij} \in \mathbb{R}^2$. A preview for data is shown below.



The screenshot shows a Notepad window titled "trans3 - Notepad". The content of the file is a 9x9 grid of numerical values representing control point offsets. The values are mostly zero, with some non-zero entries like -40, -32, -16, 48, -32, -16, 40, 40, etc., appearing in specific positions. The Notepad interface includes a menu bar with File, Edit, View, and a status bar at the bottom showing "Ln 1, Col 1", "100%", "Windows (CRLF)", and "UTF-8".

0 0, 0 0, 0 0,	0 0,	0 0,	0 0,	0 0,	0 0, 0 0
0 0, 0 0, -40 0,	0 0,	0 0,	0 0,	0 0,	0 0, 0 0
0 0, 0 0, -32 -32,	-32 0,	-40 0,	-40 32,	0 0,	0 0, 0 0
0 0, 0 0, -32 -16,	-32 0,	-32 0,	-32 16,	0 0,	0 0, 0 0
0 0, 0 0, 0 -32,	-48 -16,	48 -16,	48 8,	0 0,	0 0, 0 0
0 0, 0 0, 48 -32,	64 -16,	64 16,	32 32,	16 16,	0 0, 0 0
0 0, 0 0, 32 -40,	40 -40,	40 40,	0 0,	0 0,	0 0, 0 0
0 0, 0 0, 16 -40,	32 -16,	0 0,	0 0,	0 0,	0 0, 0 0
0 0, 0 0, 0 0,	0 0,	0 0,	0 0,	0 0,	0 0, 0 0

The code to generate transformed image is shown below. It read in images and offset data, then compute the FFD inverse and restore the image by linear interpolation. We save the inverse FFD in our data folder so that redundant computation is not required next time.

```
1 def main():
2     parser = argparse.ArgumentParser(description='Transformation')
3     parser.add_argument('-i', '--img_path', type=str, default='images/cat.png',
4                         help='the path of source image.')
5     parser.add_argument('-o', '--offset_path', type=str,
6                         default='data/trans1.txt',
7                         help='the path of txt file which stores the offset
matrix of control points.')
8     args = parser.parse_args()
9
10    img = cv2.imread(args.img_path)
11    r, g, b = img[:, :, 2], img[:, :, 1], img[:, :, 0]
12    coord = generate_coord(r.shape)
13    npy_path = osp.splitext(args.offset_path)[0] + '_inv.npy'
14
15    if osp.exists(npy_path):
16        # If the data has been fitted, use the saved result directly.
```

```

17     ffdinv = np.load.npy_path, allow_pickle=True).item()
18
19 else:
20     transformer = FFD()
21     transformer.update_offset(args.offset_path)
22     ffdinv = transformer.FFDInvtransform(coord)
23     # Save the inverse transformation result.
24     np.save.npy_path, ffdinv)
25
26 print('Restore the image by interpolation.')
27 img_ffd_r = bilinear_interp(r, ffdinv['inv'])
28 img_ffd_g = bilinear_interp(g, ffdinv['inv'])
29 img_ffd_b = bilinear_interp(b, ffdinv['inv'])
30 img_ffd = np.zeros_like(img)
31 img_ffd[:, :, 2], img_ffd[:, :, 1], img_ffd[:, :, 0] = img_ffd_r, img_ffd_g
32 , img_ffd_b
33 print('mse: {}'.format(ffdinv['mse']))
34
35 # Visualization.
36 ...
37 tr = osp.splitext(args.offset_path)[0][-1]
38 cv2.imwrite(osp.splitext(args.img_path)[0] + f'_ffd_{tr}.png', img_ffd)

```

In experiment, we tested both gradient descent method and Newton's method. It turns out Newton's method needs much less iterations (often $10^4 \sim 10^6$) than gradient method (often $10^5 \sim 10^7$).

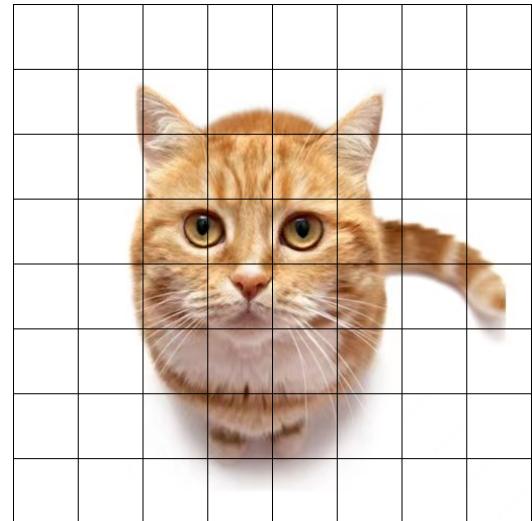
The visualization result is shown below.

From the result it's seen the effect of deformation is satisfactory. The topological structure within grid doesn't change much, and the original image is correctly transformed according to the deformation field defined by FFD (The grid points in right images are control points).

For quantitative evaluation, the MSE of transformed data are all less than 0.5, indicating the accuracy of our estimate of inverse.



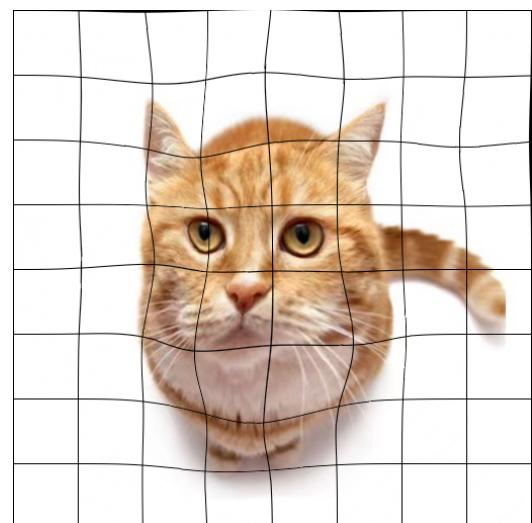
(1) Original image cat.png



(2) Original image cat.png with grids



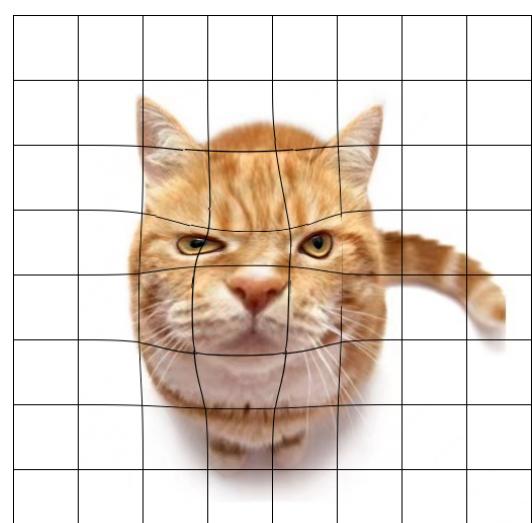
(3) FFD transformation result 1



(4) Result 1 with grids, $MSE = 0.1333$



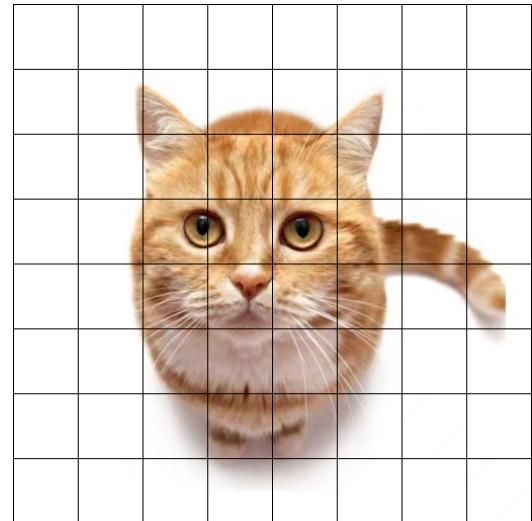
(5) FFD transformation result 2



(6) Result 2 with grids, $MSE = 0.1663$



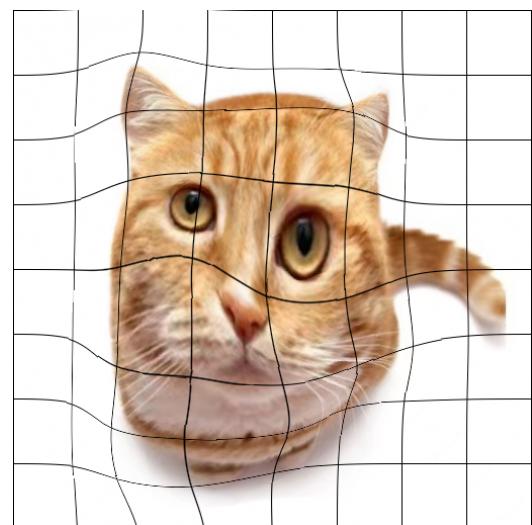
(7) Original image cat.png



(8) Original image cat.png with grids



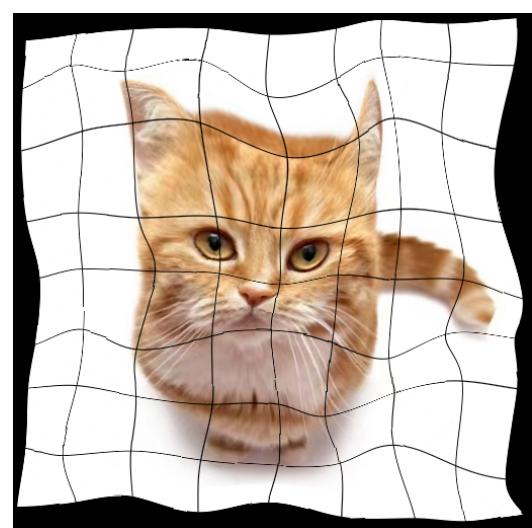
(9) FFD transformation result 3



(10) Result 3 with grids, $MSE = 0.3086$



(11) FFD transformation result 4



(12) Result 4 with grids, $MSE = 0.3743$

Additional Results



(13) deGaulle.png



(14) FFD transformation 1



(15) FFD transformation 2



(16) meisje_met_de_parel.png



(17) FFD transformation 3



(18) FFD transformation 4

4 Appendix

4.1 Libraries

We use the `numpy` library to support the large-scale computation of matrices which store the grey level information of images.

We use `opencv` library (`cv2`) to read the input images and save the output images.

We use `matplotlib` to show images before and after processing.

We also use `argparse` to read in the arguments for processing, which can be specified in the command line.

4.2 IDE

All our experiments are done on PyCharm Community Edition 2022.1.2.