10/3

CS229 ch3 Generalized linear model = $g(\mu) = \theta^T x$

$\downarrow$ link function

linear model (linear in θ)

ex $x \in \mathbb{R}^d$, $\mu = \theta_0 + \theta_1 x_1 + \theta_2 x_2 \cdots \theta_d x_d = \theta^T x$

$$\mu = \theta_0 x + \theta_1 x^2 + \theta_2 x^3 \ldots \theta_{d-1} x^d = [\theta_0\ \theta_1 \cdots \theta_d] \begin{bmatrix} x \\ x^2 \\ \vdots \\ x^d \end{bmatrix} = \theta^T T(x)$$

The exponential family

$$p(y;\varphi) = b(y)\, e^{\varphi^T y - a(\varphi)}$$

$$= b(y) \exp(\varphi^T y - a(\varphi))$$

Bernolli distribution (meon $= \phi$)

$$p(y) = \phi^y (1-\phi)^{1-y}$$

$$= e^{y \ln\phi + (1-y)\ln(1-\phi)}$$

$$= e^{y(\ln\phi - \ln(1-\phi)) + \ln(1-\phi)}$$

$$= e^{y \ln\frac{\phi}{1-\phi} + \ln(1-\phi)}$$

Bernolli

$b(y) = 1$, $y = \ln\frac{\phi}{1-\phi} \Rightarrow e^y = \frac{\phi}{1-\phi} \Rightarrow \phi = \frac{e^y}{1+e^y} \Rightarrow 1-\phi = \frac{1}{1+e^y}$

$a(y) = -\ln(1-\phi) = \ln(1+e^y)$

GLM : $h_\theta(x) = E(y|x)$

$$= \phi$$

$$= \frac{1}{1+e^{-\eta}} = \frac{1}{1+e^{-\theta^T x}}$$

Possion distribution (mean λ)

$$p(y:\lambda) = \frac{\lambda^y e^{-\lambda}}{y!} = \frac{1}{y!}\exp(y\ln\lambda - \lambda)$$

$$b(y) = \frac{1}{y!} \Rightarrow \eta = \ln\lambda \Rightarrow \lambda = e^{\eta}$$

$$GLM = E[y|x] = \lambda = e^{\eta} = e^{\theta^T x}$$

Normal distribution (mean μ) (var=1)

$$p(y:\mu) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(y-\mu)^2}{2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2} + \mu y - \frac{\mu^2}{2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}\right) \cdot e^{\mu y - \frac{\mu^2}{2}}$$

$$b(\eta) = \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} \qquad \eta = \mu \qquad a(\eta) = \frac{\mu^2}{2} = \frac{y^2}{2}$$

$$GLM = h_\theta(x) = E[y|x] = \mu = \eta = \theta^T x$$

The exponential family

$$p(y:\eta) = b(y)\exp(\eta^T T(y) - a(\eta))$$

Consider normal distribution

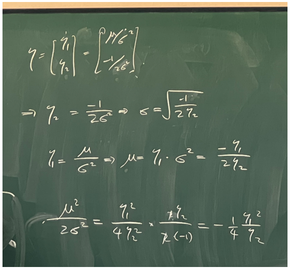
$$p(y;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(y-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}} e^{-\ln\sigma} e^{-\frac{y^2}{2\sigma^2} + \frac{\mu y}{2\sigma^2} - \frac{\mu^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{\left[\frac{\mu}{\sigma^2},\ \frac{-1}{2\sigma^2}\right]\left[\begin{matrix} y \\ y^2 \end{matrix}\right] - \frac{\mu^2}{2\sigma^2} - \ln\sigma}.$$

☆ normal distribution is an exponetial family with two paremeter

$b(\eta) = \frac{1}{\sqrt{2\pi}}$ $\quad a(\eta) = \frac{\mu^2}{2\sigma^2} + \ln\sigma = \frac{-\eta_1^2}{4\pi} - \frac{1}{2}\ln(-2\eta_2)$

$\eta = \begin{bmatrix} \frac{\mu}{\sigma^2} \\ -\frac{1}{2\sigma^2} \end{bmatrix} = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \quad T(y) = \begin{bmatrix} y \\ y^2 \end{bmatrix}$



1-2 週

1 週

2-3 week