MIS 381N
Stochastic Control and Optimization: Project 1

Problem Description

Bonds are a form of debt issued by governments and companies. You can purchase a bond for a given price, and it has a maturity date in the future at which point you will receive the face value of the bond. For example, you could purchase a bond for $98 (price) and at maturity the bond issuer will pay you $100 (face value). Coupon bonds are the bonds that offer a periodic payment during the life of the bond. Let's say you purchase a bond for $102 with an annual coupon payment of $5, a face value of $100 and maturity three years from now. Then you will pay $102 immediately to purchase the bond, receive $5 coupon payments at the end of years 1, 2 and finally $5+$100=$105 at the end of year 3. For additional information see http://guides.wsj.com/personal-finance/investing/what-is-a-bond/.

Dedication or cash-flow matching is a technique used to fund known liabilities in the future. The intent is to form a portfolio of assets whose cash inflows will exactly match cash outflows of the liabilities. The liabilities will be paid off, as they become due, without the need to sell or buy assets in the future. The portfolio is formed today and held until all liabilities are paid off. Dedicated portfolios usually only consist of risk-free non-callable bonds since future cash inflows need to be known when the portfolio is constructed. Dedicated portfolios eliminate the risk caused by change in interest rates. Corporates, municipalities, and pension funds routinely use such strategies. For example, corporates and municipalities sometimes want to fund liabilities stemming from projects that they might have initiated. **To simplify the problem, assume that one can buy a fraction of a bond.**

In this project, we will be writing and using an R program to construct a dedicated portfolio.

The Specifics

1. First, formulate the dedicated portfolio construction problem as a linear

program. Clearly list and describe the decision variables, the objective, and all the constraints. You can assume a face value = 100 for all bonds.

2. Use the following test case and solve the LP in R. Use lp() **instead** of solveLP(), because solveLP gives the wrong min.c and max.c.

Liability Schedule (L_t)

Year, t	1	2	3	4	5	6	7	8
Liability ($)	12,000	18,000	20,000	20,000	16,000	15,000	12,000	10,000

Bonds available right now (time t=0):

Bond	1	2	3	4	5	6	7	8	9	10
Price	102	99	101	98	98	104	100	101	102	94
Coupon	5	3.5	5	3.5	4	9	6	8	9	7
Maturity	1	2	2	3	4	5	5	6	7	8

The **rounded up** optimal solution to this test case is purchasing 63 of bond1, 126 of bond3, 152 of bond4, 157 of bond5, 124 of bond6, 125 of bond8, 105 of bond9, and 94 of bond10. **Your solution should be very close to this.**

3. Next we will write a function in R that can construct a portfolio for any set of liabilities and bonds. Starting from the R solution from the previous step, construct an R function that takes four-vector inputs P, C, M, L (**in that order**):

P is the vector containing the prices of $i = 1, \ldots, N$ bonds

C is the vector containing the coupon payments for the N bonds.

M is the vector containing the maturity (in years) for the N bonds.

L is the vector of non-negative liabilities for $i = 1, \ldots, T$ years.

And outputs the solution from your call to lp(). For example, if you assign Sol = lp(…), then you need to return Sol from your function. Name and save this function as dedicate_gZ.m where Z is your group number. For example, group 3's filename will be **dedicate_g3.R.** The function should take only four inputs. To test your function, use the test case from the previous step.

4. Finally, construct a dedicated portfolio using this liability stream and the current bond information from the Wall Street Journal (WSJ) Online U.S. Treasury Quotes (http://online.wsj.com/mdc/public/page/2_3020-treasury.html). It is worth to notice that the coupons are paid semi-annually. Make sure you note the date of your portfolio construction in your project report. Also, plot and interpret the sensitivity parameters that you obtain. You need to not only plot Liability date vs. the duals which is related to Liability constraints, but also interpret all parameters.

Date	Liability
6/30/17	9,000,000
12/31/17	9,000,000
6/30/18	10,000,000
12/31/18	10,000,000
6/30/19	6,000,000
12/31/19	6,000,000
6/30/20	9,000,000
12/31/20	9,000,000
6/30/21	10,000,000
12/31/21	10,000,000
6/30/22	5,000,000
12/31/22	3,000,000

The bonds considered here are slightly different from the bonds mentioned before in terms of coupon. The bonds here usually pay coupon semiannually. For example, a bond with an annual $5 coupon will pay $2.5 on both 6/30 and 12/31.

Submission Instructions

Please submit your report and dedicate_gZ.R file to Canvas.

A side note

It should be noted, however, that dedicated portfolios cost typically from 3% to 7% more in dollars terms than do "immunized" portfolios that are constructed based on matching present value, duration, and convexity of the assets and liabilities. The present value of the liability stream L_t for $t = 1, \dots, T$ is $P = \sum_{t=1}^{T} L_t/(1+r_t)^t$, where r_t denotes the risk-free rate in year t. Its duration is $D = (1/P)\sum_{t=1}^{T} tL_t/(1+r_t)^t$ and its convexity is $C = (1/P)\sum_{t=1}^{T} t(t+1)L_t/(1+r_t)^{t+2}$. Intuitively, duration is the average (discounted) time at which the liabilities occur, whereas convexity, a bit like variance, indicates how concentrated the cash flows are over time. For a portfolio that consists only of risk-free bonds, the present value P* of the portfolio future cash inflows can be computed using the same risk-free rate r_t (this would not be the case for a portfolio containing risky bonds). Similarly, for the duration D* and convexity C* of the portfolio future cash inflows, an "immunized" portfolio can be constructed based on matching P*=P, D*=D, and C*=C. Portfolios that are constructed by matching these three factors are immunized against parallel shifts in the yield curve, but there may still be a great deal of exposure and vulnerability to other types of shifts, and they need to be actively managed, which can be costly. By contrast, dedicated portfolios do not need to be managed after they are constructed.