MIS381N

Stochastic Control and Optimization: Homework 7

This assignment is graded on Credit/No-Credit.

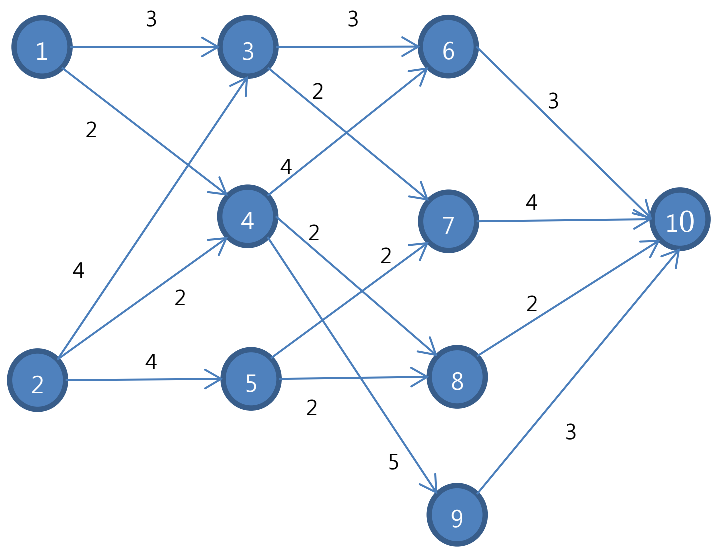
That is, if you complete the homework and it is acceptable, you will get credit. If you don't submit or if the submitted work is not acceptable, you will not get credit. Getting a credit is required to obtain a grade for the group project that follows.

Please write a report that solves the following problems.

For each problem, you need to define the value function, calculate the boundary conditions, and give the Bellman equation.

Problem 1:

In the network pictured below, find the shortest path from node 1 to node 10 and the shortest path from node 2 to node 10. Please use dynamic programming instead of network programming.



Problem 2:

Suppose that a new car costs $20,000 and that the annual operating cost and resale value of the car are as shown in Table below. For example, if you buy a new car and immediately sell it, you will only get $18,000 back. If you don't sell the new car, the operation cost for the 1st year is $600. If I have a new car now, determine a replacement policy that minimizes the net cost of owning and operating a car for the next six years. You don't need a car after the 6th year.

Please use dynamic programming instead of network programming.

Age of Car (Years)	Resale Value ($)	Operating Cost ($)	
0	18,000	600	(new)
1	14,000	1,000	(year 1)
2	12,000	1,600	(year 2)
3	8,000	2,400	(year 3)
4	6,000	3,200	(year 4)
5	4,000	4,400	(year 5)
6	2,000	6,000	(year 6)

Problem 3:

A sales representative lives in Bloomington and must be in Indianapolis on day 4. On each of the first 3 days, he can sell his wares in Indianapolis, Bloomington, or Chicago. From past experience, he believes that he can earn \$120 from spending a day in Indianapolis, \$160 from spending a day in Bloomington, and \$170 from spending a day in Chicago. Where should he spend the first three days and nights of the week to maximize his sales income less travel costs? Travel costs are shown in Table below.

From	To		
	Indianapolis	Bloomington	Chicago
Indianapolis	—	50	20
Bloomington	50	—	70
Chicago	20	70	—

Problem 4:

A company is considering how to invest \$6,000 and has three investments available. The goal is to maximize Net Present Value (NPV). Let d_j represent the amount invested (in thousands) in investment j. Then $r_j(d_j)$ is the net present value for the investment amount d_j in investment j using the following equations:

$$r_1(d_1) = 7d_1 + 2 \qquad (d_1 > 0)$$

$$r_2(d_2) = 3d_2 + 7 \qquad (d_2 > 0)$$

$$r_3(d_3) = 4d_3 + 5 \qquad (d_3 > 0)$$

$$r_1(0) = r_2(0) = r_3(0) = 0$$

Solve this problem to maximize NPV using dynamic programming.

Some Hints: Think of the stages as being the investments. At stage t=1, you make a decision to invest 0, 1, 2, 3, 4, 5, or 6 (in thousands) in investment 1. For example, if you invest $d_1 = 1$ (\$1,000) at t=1, then the NPV for that investment is $r_1(1) = 7 * 1 +$

$2 = 9$ and there would be \$5,000 remaining for the other investments. If $d_2 = 4$ (\$4,000) is invested in investment 2, the NPV for that investment is $r_2(5) = 3 * 4 + 7 = 19$ and there would be \$1,000 left over for investment 3. The optimal 3rd stage decision is $d_3 = 1$ for $r_3(1) = 4 * 1 + 5 = 9$.

To tackle this, start at the end and work backwards. At stage t=3 (i.e. investment 3), any remaining funds should be fully invested in investment 3 since $r_3(d_3)$ is non-negative and increasing in d_3. There are 7 possible states to consider at t=3, one for each potential balance remaining for investment. So, you should calculate $V(0,3), V(1,3), \dots, V(6,3)$ using the notation $V(s,t)$ to represent the optimal value at stage (investment) when the state (remaining balance for investment) is s:

$$V(s = 0, t = 3) = r_3(0) = 0$$

$$V(s = 1, t = 3) = r_3(1) = 4 * 1 + 5 = 9$$

$$\vdots$$

$$V(s = 6, t = 3) = r_3(6) = 4 * 6 + 5 = 29$$

At stage t=2, you should consider the optimal allocation to investment 2 when you have s funds remaining (depending on the stage 1 choices), and considering what the optimal value is in stage 3 after you invest d_2 in investment 2:

$$V(s, t = 2) = \max_{d_2}\{\, r_2(d_2) + V(s - d_2, t + 1 = 3)\,\}$$

At stage t=1, you will calculate the following for each s:

$$V(s, t = 1) = \max_{d_1}\{\, r_1(d_1) + V(s - d_1, t + 1 = 2)\,\}$$

Problem 5:

An electronics firm has a contract to deliver the following number of smart watches during the next three months; month 1, 200 smart watches; month 2, 300 smart watches; month 3, 300 smart watches. For each smart watch produced during months 1 and 2, a \$100 variable cost is incurred; for each smart watch produced during month 3, a $120 variable cost is incurred. The inventory cost is $15 for each smart watch in stock at the end of a month. The cost of setting up for production during a month is $2500. Smart watches made during a month may be used to meet demand for that month or any future month. Assume that production during each month must be a multiple of 100. Given that the initial inventory level is 0 units, use dynamic programming to determine an optimal production schedule.

Deliverables

You can either hand write or type your report, but make sure that you submit a PDF file. Please name your report as hw7_x.pdf(where x is your eid).

You do not need to submit your R code, we will decide complete or incomplete based on your report.