Stochastic Control and Optimization
MIS 381N

This assignment is graded on Credit/No-Credit.

That is, if you complete the homework and it is acceptable, you will get credit. If you don't submit or if the submitted work is not acceptable, you will not get credit. Getting a credit is required to obtain a grade for the group project that follows.

Please write a report that solves the following problems. Make sure that your report includes the mathematical formations and the solutions of these optimization problems.

Problem 1:

Max is in a pie-eating contest that lasts 1 hour. Each torte that he eats takes 2 minutes. Each apple pie that he eats takes 3 minutes. He receives 4 points for each torte and 5 points for each pie. What should Max eat so as to get the most points? Solve the problem using the graphical method.

Next, let's see what happens if he would like to stick to his preference of eating at least as many pies as tortes. That is; the number of pies he eats should be greater than or equal to the number of tortes. By how many points does this constraint decrease Max's total points?

Problem 2:

A farmer in Iowa owns 450 acres of land. He is going to plant each acre with wheat or corn. Each acre planted with wheat (corn) yields $2,000 ($3,000) profit, requires three (two) workers, and requires two (four) tons of fertilizer. There are currently 1,000 workers and 1,200 tons of fertilizer available.

a) Formulate the optimization problem and solve the problem graphically.

b) Solve the problem in R and verify that the solutions are the same.

c) What happens to the decision variables and the total profit when the availability of fertilizer varies from 200 tons to 2200 tons in 100-ton increments? When does the farmer discontinue producing wheat? When does he stop producing corn? (Run a loop for different values of availability of fertilizer from 200 tons to 2200 tons).

Problem 3:

Star Oil Company is considering five different investment opportunities. Table 1 below gives the cash outflows and net present values in millions of dollars.

Star Oil has $40 million available for investment now (time 0); it estimates that one year from now (time 1) $20 million will be available for investment. Star Oil may purchase any fraction of each investment. In this case, the cash outflows and NPV are adjusted accordingly.

For example, if Star Oil purchases one-fifth of investment 3, then a cash outflow of 1/5 * 5 = $1 million would be required at time 0, and a cash outflow of 1/5 * 5 = $1 million would be required at time 1. The one-fifth share of investment 3 would yield an NPV of 1/5*16 = $3.2 million. Star Oil wants to maximize the NPV that can be

obtained by investing in investments 1-5. Formulate an LP that will help achieve this goal. Assume that any funds leftover at time 0 cannot be used at time 1.

Table 1: Cash Flows and Net Present Value for Investments in Capital Budgeting

	Investment ($)				
	1	2	3	4	5
Time 0 cash outflow	11	53	5	5	29
Time 1 cash outflow	3	6	5	1	34
NPV	13	16	16	14	39

Problem 4: (The Diet Problem)

The goal of the diet problem is to select a set of foods that will satisfy a set of daily nutritional requirement at minimum cost. Suppose there are three foods available, corn, milk, and bread. Besides, there are restrictions on the number of calories (between 2000 and 2250) and the amount of Vitamin A (between 5000 and 50,000). The table below shows, for each food, the cost per serving, the amount of Vitamin A per serving, and the number of calories per serving. Also, suppose that the maximum number of servings for each food is 10.

Food	Cost per serving	Vitamin A	Calories
Corn	$0.18	107	72
2% Milk	$0.23	500	121
Wheat Bread	$0.05	0	65

Paper and wood products companies need to define cutting schedules that will maximize the total wood yield of their forests over some planning period. Suppose that a firm with control of 2 forest units wants to identify the best cutting schedule over a planning horizon of 3 years. Forest unit 1 has a total acreage of 2 and unit 2 has a total of 3. The studies that the company has undertaken predict that each acre in unit 1(2) will have 1, 1.3, 1.4 (1, 1.2, 1.6) tons of woods available for harvesting in year 1, 2, 3 respectively. Based on its prediction of economic conditions, the company believes that it should harvest at least 1.2, 1.5, 2 tons of wood in year 1, 2, 3 separately. Due to the availability of equipment and personnel, the company can harvest at most 2, 2, 3 tons of wood in year 1, 2, 3. What is this company's best cutting strategy that maximizes the total weights of wood? Here discounting of the time value should not be considered.

Deliverables

You can either hand write or type your report, but make sure that you submit a PDF file. Please name your report as hw2_x.pdf (where x is your eid).
You don't need to submit your R code, we will decide complete or incomplete based on your report.