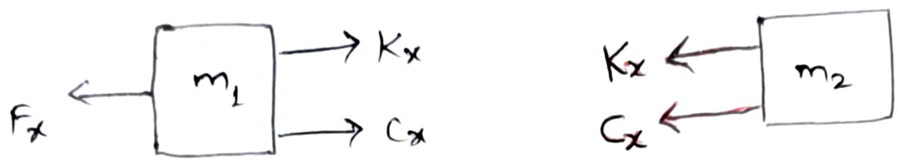
1(a)* Equations for Translation Motion:



$$\boxed{m_1\ddot{x}_1 = -K_x(x_1 - x_2) - C_x(\dot{x}_1 - \dot{x}_2)}$$

$$\boxed{m_2\ddot{x}_2 = K_x(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2)}$$

m_1 : mass of unsprung (kg)

m_2 : Sprung mass (kg)

x_1 : position of unsprung (m)

x_2 : position of sprung mass (m)

$\dot{x}_1$ = Velocity of unsprung mass (m/s)

$\ddot{x}_1$ = Acceleration of unsprung mass (ms^{-2})

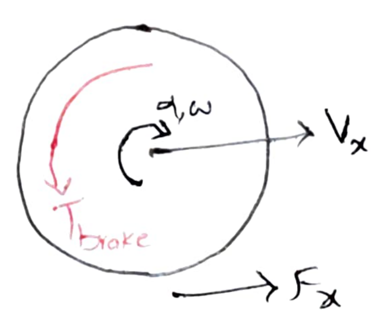
$\dot{x}_2$ = Velocity of sprung mass (m/s)

$\ddot{x}_2$ = Acceleration of sprung mass (m/s^2)

K_x = Spring stiffness (N/ms^{-1})

C_x = Damping coefficient (N/m)

1(b) Rotational Dynamics:



$$\boxed{I\alpha = R_{eff} \cdot F_x - T_{Brake}}$$

R_{eff} = Wheel Radius (m)

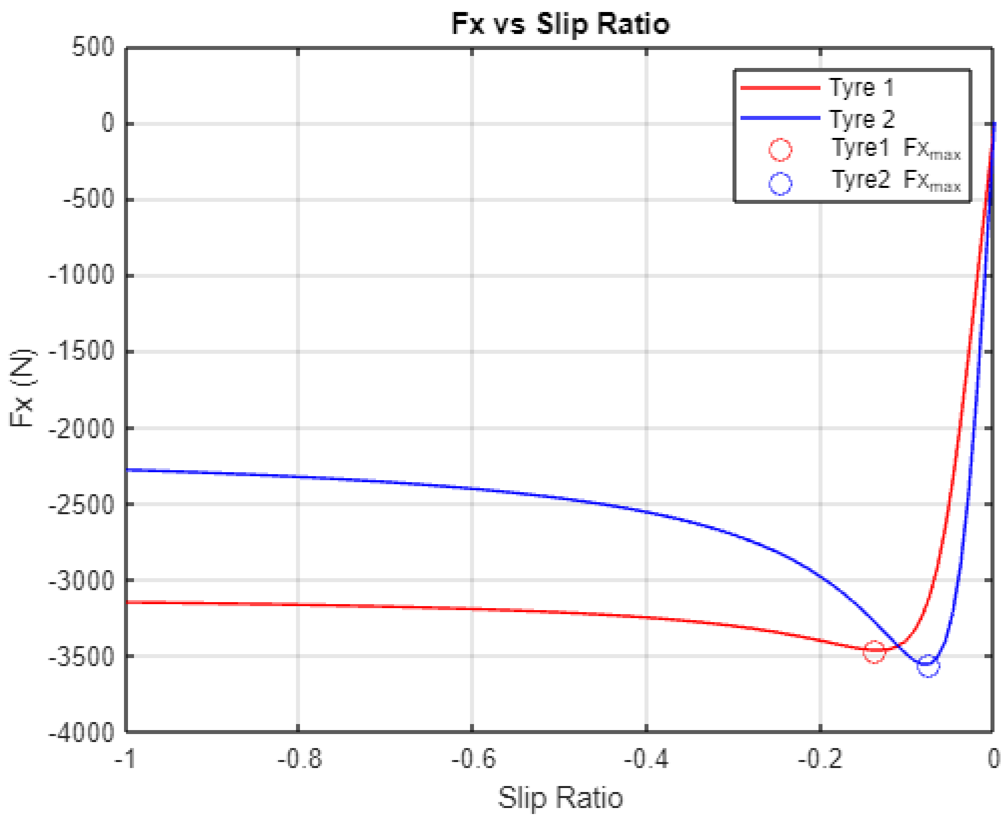
T_{Brake} = Brake Torque (NM)

α = Wheel Angular Acceleration

1 (b). Fx vs Slip ratio for Tire-1 and Tire-2

```
T1_var = load("F:\MS\Vechicle Dynamics\Assignment2\variables\Tire1_var.mat");
T2_Var = load("F:\MS\Vechicle Dynamics\Assignment2\variables\Tire2_var.mat");

% Plot Fx vs Slip Ratio
plot( T1_var.Tire1_var.slipRatio_T1.data, T1_var.Tire1_var.Fx_T1.Data, "r",
T2_Var.Tire2_var.slipRatio_T2.Data, T2_Var.Tire2_var.Fx_T2.Data, "b")
```

```
hold on;

% FInding Peak Braking Force for Tire 1 and Tire2
[max_Fx_T1, idx_max_T1] = min(T1_var.Tire1_var.Fx_T1.Data);
slip_ratio_at_max_Fx_T1 = T1_var.Tire1_var.slipRatio_T1.data(idx_max_T1);
plot(slip_ratio_at_max_Fx_T1, max_Fx_T1, 'ro', 'MarkerSize', 8);

[max_Fx_T2, idx_max_T2] = min(T2_Var.Tire2_var.Fx_T2.Data);
slip_ratio_at_max_Fx_T2 = T2_Var.Tire2_var.slipRatio_T2.Data(idx_max_T2);
plot(slip_ratio_at_max_Fx_T2, max_Fx_T2, 'bo', 'MarkerSize', 8);

xlabel('Slip Ratio')
ylabel('Fx (N)')
grid on
title('Fx vs Slip Ratio')
legend('Tyre 1','Tyre 2', 'Tyre1 Fx_{max}', 'Tyre2 Fx_{max}')
```




```
% Maximum Fx for Tire 1 and Corresponding Slip Ratio
[max_Fx_T1, idx_max_T1] = min(T1_var.Tire1_var.Fx_T1.Data);
slip_ratio_at_max_Fx_T1 = T1_var.Tire1_var.slipRatio_T1.data(idx_max_T1);

fprintf('Peak brake force for Tyre-1 is %f, corresponding slip ratio is %f',
max_Fx_T1, slip_ratio_at_max_Fx_T1);
```

```
Peak brake force for Tyre-1 is -3463.204541, corresponding slip ratio is -0.137701
```

```
% Maximum Fx for Tire 2 and Corresponding Slip Ratio
[max_Fx_T2, idx_max_T2] = min(T2_Var.Tire2_var.Fx_T2.Data);
slip_ratio_at_max_Fx_T2 = T2_Var.Tire2_var.slipRatio_T2.Data(idx_max_T2);

fprintf('Peak brake force for Tyre-2 is %f, corresponding slip ratio is %f',
max_Fx_T2, slip_ratio_at_max_Fx_T2);
```

```
Peak brake force for Tyre-2 is -3555.320852, corresponding slip ratio is -0.075040
```

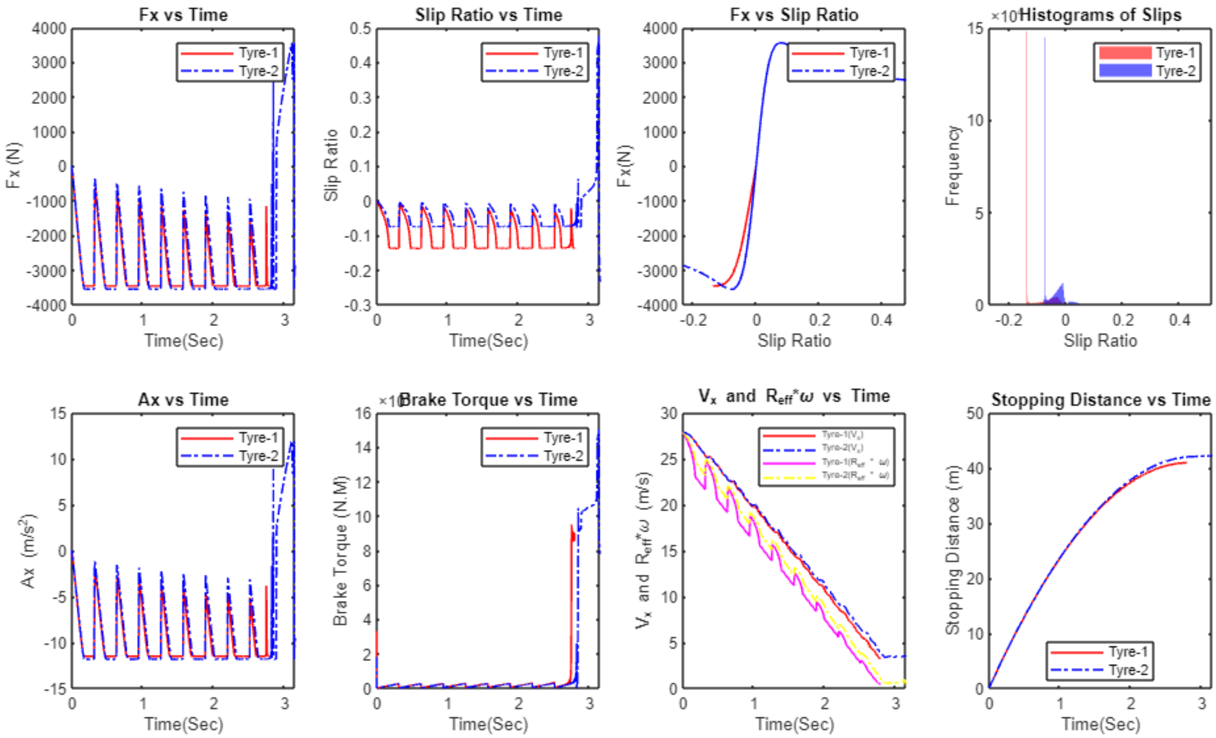
1 (C) A PID controller has been used to maintain the slip such that the Brake Torque will be adjusted.

1 (D)

```
clc;
clear all;
```

```
%Load all the data
T1_vars = load("F:\MS\Vechicle Dynamics\Assignment2\variables\Tire1_vars.mat");
T2_Vars = load("F:\MS\Vechicle Dynamics\Assignment2\variables\Tire2_vars.mat");

Fx_T1 = T1_vars.out.Fx_T2;
slip_T1 = T1_vars.out.slipRatio_T2;
ax_T1 = T1_vars.out.ax_T2;
BrakeTorque_T1 = T1_vars.out.BrakeTorque_T2;
Vx_T1 = T1_vars.out.Vx_T2;
Reffxw_T1 = T1_vars.out.Reffxw_T2;
Stop_dist_T1 = T1_vars.out.stopping_dist_T2;
Time_T1 = T1_vars.out.stopping_dist_T2.Time;

Fx_T2 = T2_Vars.out.Fx_T2;
slip_T2 = T2_Vars.out.slipRatio_T2;
ax_T2 = T2_Vars.out.ax_T2;
BrakeTorque_T2 = T2_Vars.out.BrakeTorque_T2;
Vx_T2 = T2_Vars.out.Vx_T2;
Reffxw_T2 = T2_Vars.out.Reffxw_T2;
Stop_dist_T2 = T2_Vars.out.stopping_dist_T2;
Time_T2 = T2_Vars.out.stopping_dist_T2.Time;

figure('Position', [100, 100, 800, 600]);
legend_font_size = 8;

% Fx vs Time
subplot(2,4, 1);
plot(Time_T1, Fx_T1.Data, 'r-', Time_T2, Fx_T2.Data, 'b-.')
xlabel('Time(Sec)')
ylabel('Fx (N)')
legend('Tyre-1', 'Tyre-2',  'FontSize', legend_font_size)
title('Fx vs Time')

% Slip vs Time
subplot(2,4, 2);
plot(Time_T1, slip_T1.Data, 'r-', Time_T2, slip_T2.Data, 'b-.')
xlabel('Time(Sec)')
ylabel('Slip Ratio')
legend('Tyre-1', 'Tyre-2',  'FontSize', legend_font_size)
title('Slip Ratio vs Time')

%Fx vs Slip Ratio
subplot(2,4, 3);
plot(slip_T1.Data, Fx_T1.Data, 'r-', slip_T2.Data, Fx_T2.Data,'b-.')
xlabel('Slip Ratio')
ylabel('Fx(N)')
legend('Tyre-1', 'Tyre-2',  'FontSize', legend_font_size)
title('Fx vs Slip Ratio')

%Histogram of Slip Ratio
```

```
subplot(2,4, 4);
histogram(slip_T1.Data, 'FaceColor', 'r','EdgeColor','none')
hold on
histogram(slip_T2.Data, 'FaceColor', 'b','EdgeColor','none')
xlabel('Slip Ratio')
ylabel('Frequency')
legend('Tyre-1', 'Tyre-2',  'FontSize', legend_font_size)
title('Histograms of Slips')

% ax vs Time
subplot(2,4, 5);
plot(Time_T1, ax_T1.Data, 'r-', Time_T2, ax_T2.Data, 'b-.')
xlabel('Time(Sec)')
ylabel('Ax (m/s^{2})')
legend('Tyre-1', 'Tyre-2',  'FontSize', legend_font_size)
title('Ax vs Time')

% Brake Torque vs Time
subplot(2,4, 6);
plot(Time_T1, BrakeTorque_T1.Data, 'r-', Time_T2, BrakeTorque_T2.Data, 'b-.')
xlabel('Time(Sec)')
ylabel('Brake Torque (N.M)')
legend('Tyre-1', 'Tyre-2',  'FontSize', legend_font_size)
title('Brake Torque vs Time')

% Vx and Reff.w vs Time
subplot(2,4, 7);
plot(Time_T1, Vx_T1.Data, 'r-', Time_T2, Vx_T2.Data, 'b-.', Time_T1,
Reffxw_T1.Data, 'm-', Time_T2, Reffxw_T2.Data, 'y-.')
xlabel('Time(Sec)')
ylabel('V_{x} and R_{eff}*\omega (m/s)')
legend('Tyre-1(V_{x})', 'Tyre-2(V_{x})', 'Tyre-1(R_{eff} * \omega)',
'Tyre-2(R_{eff} * \omega)',  'FontSize', 5)
title('V_{x} and R_{eff}*\omega vs Time')

% Stopping Distance vs Time
subplot(2,4, 8);
plot(Time_T1, Stop_dist_T1.Data, 'r-', Time_T2, Stop_dist_T2.Data, 'b-.')
xlabel('Time(Sec)')
ylabel('Stopping Distance (m)')
legend('Tyre-1', 'Tyre-2',  'FontSize', legend_font_size, 'Location', 'best')
title('Stopping Distance vs Time')

pos = get(gcf, 'Position');
set(gcf, 'Position',pos+[1000 -1000 1000 1000])
saveas(gcf, 'larger_subplots.png');
```




```
% Stopping Distance
final_dist_T1 = Stop_dist_T1.Data(end);
final_dist_T2 = Stop_dist_T2.Data(end);
fprintf('Stopping Distance for Tyre-1 is %f (m)\nStopping Distance for Tyre-2 is %f (m)\n', final_dist_T1, final_dist_T2);
```

```
Stopping Distance for Tyre-1 is 40.853819 (m)
Stopping Distance for Tyre-2 is 42.137842 (m)
```

```
% Stopping Time
final_time_T1 = Time_T1(end);
final_time_T2 = Time_T2(end);
fprintf('Stopping Time for Tyre-1 is %f (Sec)\nStopping Time for Tyre-2 is %f (Sec)\n', final_time_T1, final_time_T2);
```

```
Stopping Time for Tyre-1 is 2.795990 (Sec)
Stopping Time for Tyre-2 is 3.157000 (Sec)
```

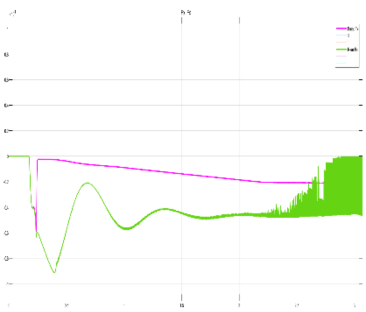
```
export("live_script.mlx")
```

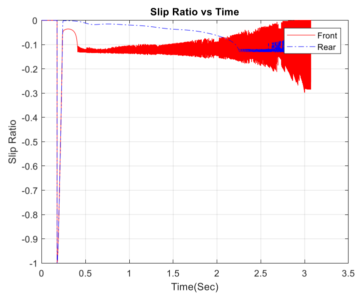
```
ans =
'F:\MS\Vechicle Dynamics\Assignment2\live_script.pdf'
```

2(Q)

An If-Else controller has been designed to maintain the slip at the desired value. The controller increases or Decreases the Brake Pressure to obtain the desired slip.

The below figure shows the plots of different parameters from the Half car Model simulation:

<table>
<tr>
<td>


Fx vs Time (Magenta – Front, Green Rear)
</td>
<td>


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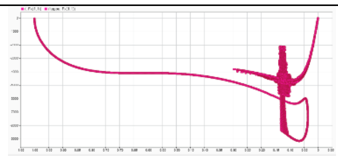
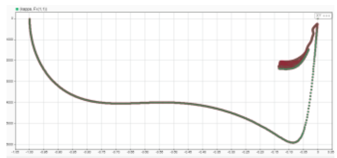
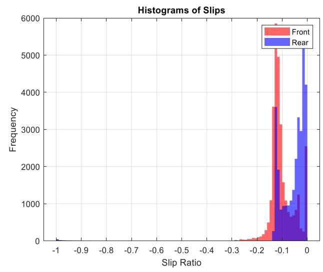


Fx vs Slip Ratio (Front)



Fx vs Slip Ratio (Rear)
</td>
<td>


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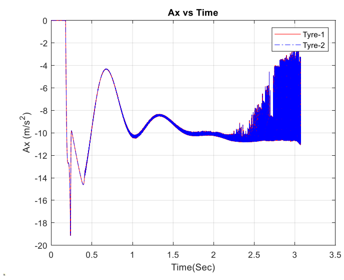

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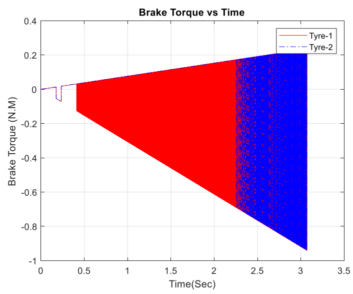

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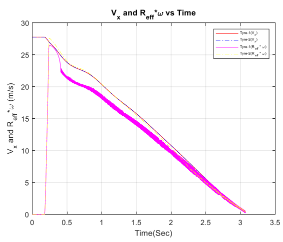

</td>
<td>
Stopping Distance : 45.783 m
Stopping Time : 3.07 sec
</td>
</tr>
</table>