Multivariate probabilty and statistics

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Date of Presentation

Overview

This presentation is ..

Random variables

Random variable is...

Random vectors

Random vector is...

$$\mathbf{z} = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix}$$

Probability density

(Function, Examples)

$$p_z(a) = \frac{p(z \text{ is in } [a, a+\nu])}{\nu}$$

$$p_{\mathbf{z}}(\mathbf{a}) = \frac{p(z_i \text{ is in } [a_i, a_i+\nu] \text{ for all } i)}{\nu^n}$$

Marginal and joint probabilities

(Definition, examples)
Marginal:

$$p_{z_1}(z_1) = \int p(z_1, z_2) dz_2$$

or more rigorosly:

$$p_{z_1}(\nu_1) = \int p(\nu_1, \nu_2) d\nu_2$$

Conditional probabilities

(continuous case, discrete case)

$$p(z_2|z_1 = a) = \frac{p_{\mathbf{z}}(a, z_2)}{\int p_{\mathbf{z}}(a, z_2) dz_2}$$

or more concisely:

$$p(z_2|z_1 = a) = \frac{p_{\mathbf{z}}(a, z_2)}{p_{z_1}(a)} = \text{e.t.c.}$$

for discrete case the integral is replaced by sum, that is

$$P_{z_1}(z_1) = \sum_{z_2} P_{\mathbf{z}}(z_1, z_2)$$

Independence

definition

$$p(z_2|z_1) = p(z_2) \text{for every } z_1 \text{ and } z_2$$

which implies that

$$\frac{p(z_1, z_2)}{p(z_1)} = p(z_2) \text{ or otherway around that } p(z_1, z_2) = p(z_1)p(z_2)$$

for all z_1 and z_2S

Expectation

(continuous, discrete (maybe otherway around would be more intuitive?))

$$E\{\mathbf{z}\} = \int p_{\mathbf{z}}(\mathbf{z})\mathbf{z}d\mathbf{z}$$

(write the components?)
its linear:

$$E\{a\mathbf{z} + b\mathbf{z}\} = aE\{\mathbf{z}\} + bE\{\mathbf{z}\}$$

similarily for any matrix $\mathbf{M}$

$$E\{\mathbf{Mz}\} = \mathbf{M}E\{\mathbf{z}\}$$

Variance, covariance and correlation

(dim = 1, covariance matrix)

$$var(z_1) = E\{z_1^2\} - (E\{z_1\})^2 = E\{(z_1 - E\{z_1\})^2\}$$

covariance and correlation

$$cov(z_1, z_2) = E\{z_1 z_2\} - E\{z_1\}E\{z_2\}$$

$$corr(z_1, z_2) = \frac{cov(z_1, z_2)}{\sqrt{var(z_1)var(z_2)}}$$

Covariance Matrix

$$\mathbf{C}(\mathbf{z}) = \begin{pmatrix} cov(z_1, z_1) & cov(z_1, z_2) \dots cov(z_1, z_n) \\ cov(z_2, z_1) & cov(z_2, z_2) \dots cov(z_2, z_n) \\ \vdots & \ddots \quad \vdots \\ cov(z_n, z_1) & cov(z_n, z_2) \dots cov(z_n, z_n) \end{pmatrix}$$

or more concisely:

$$\mathbf{C}(\mathbf{z}) = E\{\mathbf{z}\mathbf{z}^T\} - E\{\mathbf{z}\}E\{\mathbf{z}\}^T$$

Independence and covariance

For independent variables z_1 and z_2 the relation

$$E\{g_1(z_1)g_2(z_2)\} = E\{g_1(z_1)\}E\{g_2(z_2)\}$$

holds for any functions g_1 and g_2 (rigorously speaking: for any measurable functions), which implies that independant variables are uncorreleted (take $g_1(z) = g_2(z) = z$).

Bayesian inference

Baye's rule

$$p(\mathbf{s}|\mathbf{z}) = \frac{p(\mathbf{z}|\mathbf{s})p_{\mathbf{s}}(\mathbf{s})}{\int p(\mathbf{z}|\mathbf{s})p_{\mathbf{s}}(\mathbf{s})ds}$$

Or more rigorously WRITE HERE?

Non informative priors

definition

$$p(\mathbf{s}|\mathbf{z}) = \frac{p(\mathbf{z}|\mathbf{s})c}{\int p(\mathbf{z}|\mathbf{s})cds} = \frac{p(\mathbf{z}|\mathbf{s})}{\int p(\mathbf{z}|\mathbf{s})ds}$$

Bayesian inference as a incremental learning process

Explanation

Parameter estimation and likelihood

(statistical model, estimation, likelihood & log-likelihood)

Maximum likelihood and maximum posteriori

(definition, definition)

Template examples

Examples from the template on the next slides:

Tables and Figures

- Use `tabular` for basic tables — see Table 1, for example.
- You can upload a figure (JPEG, PNG or PDF) using the files menu.
- To include it in your document, use the `includegraphics` command (see the comment below in the source code).

Item	Quantity
Widgets	42
Gadgets	13

Table 1: An example table.

Readable Mathematics

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n} \sum_i^n X_i$$

denote their mean. Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.