Multivariate probabilty and statistics

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Date of Presentation

Overview

This presentation is ..

Random variables

Discrete random variable is a variable which can have any value from some given range with some given probabilty. Example: outcome of throwing a dice. The outcome can have any value from 1 to 6, each with the probability 1/6.
Continuos random variable is a variable which can have any value from some given continuous range of values, with some given probabilities for the value been in any given interval. Example WRITE THIS!

Random vectors

Random vector is a n-tuple of random variables. Each of these can be either similar or different types of variables. More formaly:

$$\mathbf{z} = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix}$$

where $\mathbf{z}$ is the random vector and the random variables z_i are it's components.

Probability density

(Function, Examples)

$$p_z(a) = \frac{p(z \text{ is in } [a, a+\nu])}{\nu}$$

$$p_{\mathbf{z}}(\mathbf{a}) = \frac{p(z_i \text{ is in } [a_i, a_i+\nu] \text{ for all } i)}{\nu^n}$$

Marginal and joint probabilities

(Definition, examples)
Marginal:

$$p_{z_1}(z_1) = \int p(z_1, z_2) dz_2$$

or more rigorosly:

$$p_{z_1}(\nu_1) = \int p(\nu_1, \nu_2) d\nu_2$$

Conditional probabilities

(continuous case, discrete case)

$$p(z_2|z_1 = a) = \frac{p_{\mathbf{z}}(a, z_2)}{\int p_{\mathbf{z}}(a, z_2) dz_2}$$

or more concisely:

$$p(z_2|z_1 = a) = \frac{p_{\mathbf{z}}(a, z_2)}{p_{z_1}(a)} = \text{e.t.c.}$$

for discrete case the integral is replaced by sum, that is

$$P_{z_1}(z_1) = \sum_{z_2} P_{\mathbf{z}}(z_1, z_2)$$

Independence

definition

$$p(z_2|z_1) = p(z_2) \text{for every } z_1 \text{ and } z_2$$

which implies that

$$\frac{p(z_1, z_2)}{p(z_1)} = p(z_2) \text{ or otherway around that } p(z_1, z_2) = p(z_1)p(z_2)$$

for all z_1 and z_2S

Expectation

(continuous, discrete (maybe otherway around would be more intuitive?))

$$E\{\mathbf{z}\} = \int p_{\mathbf{z}}(\mathbf{z})\mathbf{z}d\mathbf{z}$$

(write the components?)
its linear:

$$E\{a\mathbf{z} + b\mathbf{z}\} = aE\{\mathbf{z}\} + bE\{\mathbf{z}\}$$

similarily for any matrix $\mathbf{M}$

$$E\{\mathbf{Mz}\} = \mathbf{M}E\{\mathbf{z}\}$$

Variance, covariance and correlation

(dim = 1, covariance matrix)

$$var(z_1) = E\{z_1^2\} - (E\{z_1\})^2 = E\{(z_1 - E\{z_1\})^2\}$$

covariance and correlation

$$cov(z_1, z_2) = E\{z_1 z_2\} - E\{z_1\}E\{z_2\}$$

$$corr(z_1, z_2) = \frac{cov(z_1, z_2)}{\sqrt{var(z_1)var(z_2)}}$$

Covariance Matrix

$$\mathbf{C}(\mathbf{z}) = \begin{pmatrix} cov(z_1, z_1) & cov(z_1, z_2) \dots cov(z_1, z_n) \\ cov(z_2, z_1) & cov(z_2, z_2) \dots cov(z_2, z_n) \\ \vdots & \ddots \quad \vdots \\ cov(z_n, z_1) & cov(z_n, z_2) \dots cov(z_n, z_n) \end{pmatrix}$$

or more concisely:

$$\mathbf{C}(\mathbf{z}) = E\{\mathbf{z}\mathbf{z}^T\} - E\{\mathbf{z}\}E\{\mathbf{z}\}^T$$

Independence and covariance

For independent variables z_1 and z_2 the relation

$$E\{g_1(z_1)g_2(z_2)\} = E\{g_1(z_1)\}E\{g_2(z_2)\}$$

holds for any functions g_1 and g_2 (rigorously speaking: for any measurable functions), which implies that independant variables are uncorreleted (take $g_1(z) = g_2(z) = z$).

Bayesian inference

Baye's rule

$$p(\mathbf{s}|\mathbf{z}) = \frac{p(\mathbf{z}|\mathbf{s})p_{\mathbf{s}}(\mathbf{s})}{\int p(\mathbf{z}|\mathbf{s})p_{\mathbf{s}}(\mathbf{s})ds}$$

Or more rigorously WRITE HERE?

Non informative priors

definition

$$p(\mathbf{s}|\mathbf{z}) = \frac{p(\mathbf{z}|\mathbf{s})c}{\int p(\mathbf{z}|\mathbf{s})c ds} = \frac{p(\mathbf{z}|\mathbf{s})}{\int p(\mathbf{z}|\mathbf{s}) ds}$$

Bayesian inference as a incremental learning process

Explanation

Parameter estimation and likelihood

(statistical model, estimation, likelihood & log-likelihood)

Maximum likelihood and maximum posteriori

(definition, definition)

Template examples

Examples from the template on the next slides:

Tables and Figures

- Use `tabular` for basic tables — see Table 1, for example.
- You can upload a figure (JPEG, PNG or PDF) using the files menu.
- To include it in your document, use the `includegraphics` command (see the comment below in the source code).

Item	Quantity
Widgets	42
Gadgets	13

Table 1: An example table.

Readable Mathematics

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n} \sum_{i}^{n} X_i$$

denote their mean. Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.