

Problem Set 1

Topics \rightarrow Gamma Distribution,
chi-square Distribution

Q.1> The moment generating function
of a random variable X is

$$(1-2t)^{-6} \quad t < \frac{1}{2}$$

find $P(X < 5.23)$

Q.2> Suppose $X \sim \text{Gamma}\left(\frac{r}{2}, \beta\right)$ $r \in \mathbb{Z}_+, \beta > 0$
find the distribution of

$$Y = \frac{2X}{\beta}$$

Q.3> $M(t) = \frac{1}{(1-t)}$

Describe the distribution

Q.4>
$$f(x) = \begin{cases} \frac{1}{x^2} & 1 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

find the mean

Q.5> $X \sim \text{Gamma}(3, 4)$

Find

$$3.28 < P < 25.2$$

Remark: \rightarrow Gamma tables aren't
available, some other
tables are

Q.6) Additive property of Gamma

$$X_i \sim \text{Gamma}(\alpha_i, \beta) \quad [\text{All are independent}]$$

Define

$$Y = \sum X_i$$

Prove

$$Y \sim \text{Gamma}(\sum \alpha_i, \beta)$$

Hint \rightarrow Use m.g.f

Q.7) $X_i \sim \chi^2(r_i)$ [independent]

$$Y = \sum X_i$$

Prove $Y \sim \chi^2(\sum r_i)$

Results

\rightarrow Mean, Variance, pdf, mgf of Gamma & chi-square

- Inter-conversions
- Table Lookup
- Additive property