

Preface to the First Edition

"Basic Electrical Engineering" is a compulsory course in the first year for BE students of all disciplines of engineering. This text is meant for the use as introductory course in electrical engineering. It covers the syllable of "Basic Electrical Engineering" as prescribed for IOE students.

This book is organized into seven chapters. In first four chapters, we shall deal with circuit solving techniques in DC network whereas in remaining portions we shall discuss AC analysis of circuits. Throughout this text, the basic concepts involved in electrical engineering are emphasized step by step procedures for solving the problems are described well. The problems have been worked out in sufficient details so that the reader can follow the procedures easily. The detailed solutions of past questions of IOE examinations are presented chapter wise in this book. Additional questions chapter and also listed at the end of this book to help the readers strengthen their understanding of the course.

It is earnestly hoped that this text will facilitate students not only to score better in examinations but also to gain clear knowledge of the subject matter discussed. Every care has been taken to avoid misprints and errors. We would be grateful to readers for bringing into notice any of such errors or misprints they may come across while going through the book.

We would like to express our deep gratitude to Kathmandu Engineering college (KEC) Kalimati for providing us platform and exposure in the education field as lecturers. We are very thankful to the publishers of "G.L. Book House Pvt. Ltd.", Maitighar, Kathmandu for giving us this marvelous opportunity and bringing out this book in such a short time. We appreciate the sincere efforts of Yadu Kumar, Rekha Baral, Hemanta Shrestha and Bashu Kalauni for shaping this book in presentable form and being patient with us while developing the book.

Without great support of Department of Electrical Engineering, KEC it would not have been possible to write this text timely. Our seniors and colleagues have always been very responsive in providing necessary advice and without their generous support and inspiration our work would have lacked proper supervision. We express our warm thanks to them for their guidance. We would also like to thank our family, friends and students for their valuable support and companionship.

Lastly would like to dedicate the first edition of this text to all our students who have encouraged us to develop this book.
Best wishes to all our readers.

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1

GENERAL ELECTRIC SYSTEM

1.1 Electrical Current

Electric current may be defined as the time rate of net motion of electric charge across a cross sectional area in definite direction. At normal state, there is only random motion of free electrons in a metal which doesn't constitute an electric current because there is no net transfer of charge across a cross sectional area.

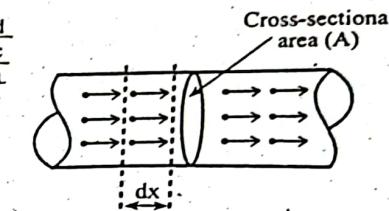


Fig. 1.1

Let,

n = number of free electrons

v = drift velocity of electrons (in meter per sec)

A = cross-sectional area of the conductor

dx = distance moved by electrons in a small time interval of dt

= vdt

e = charge on an electron

Volume swept in time dt = $dx \times A$

∴ Net number of electrons transfer across dx section in dt sec

$$= nAvdt$$

∴ Net charge transfer across dx section in dt sec is given by

$$dq = neAvdt$$

Now, Electric current, $i = \frac{dq}{dt} = \frac{neAvdt}{dt}$

$i = neAv$ (coulombs/sec or ampere),

Current density, $J = \frac{i}{A} = nev$ (A/m^2)

1.2 Electric Circuit

An electric circuit is a closed path composed of various components through which electric current completes its path.

Different constituent parts of an electrical system are

- (i) Source; provides energy to circuit such as battery, generators.
- (ii) Conductor; used to carry current such as wires, cables.
- (iii) Safety devices; for protection such as fuses, breakers.
- (iv) Controlling devices; for control such as switch.
- (v) Load; elements that utilizes electrical energy such as resistors.

1.3 Electromotive force and potential difference

Electromotive force (e.m.f) is the force that causes a current of electricity to flow. The potential difference (p.d) V , between two points in a circuit is the electrical pressure or voltage required to drive the current between them. The volt is unit of p.d and e.m.f.

E.m.f of device, say a battery, is a measure of the energy the battery gives to each coulomb of charge. Thus, if a battery supplies 4 joules of energy per coulomb, we say that it has an e.m.f of 4 volts.

The potential difference between two points in an electrical circuit is that difference in their electrical state which tends to cause flow of electric current between them. Here, the potential difference between point A and B is 2 volts, it means that each coulomb will give up an energy of 2 joules in moving from A to B.

1.4.1 Electrical units

Quantity	Unit	Symbol
Electric current	ampere	A
Power	watt	W, J/s
Quantity of electricity	coulomb	C, A/s
Potential, p.d, emf	volt	V
Electric field strength	volt per metre	V/m
Resistance	ohm	Ω , V/A
Capacitance	farad	F
Magnetic flux	weber	wb
Inductance	henry	H
Magnetic flux density	tesla	T

1.4.2 Prefix and symbol

Factor by which units is multiplied	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

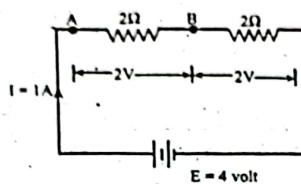


Fig. 1.2

1.5.1 Ohm's law

For a fixed metal conductor, the temperature and other conditions remaining constant, the current (I) through it is proportional to the potential difference (V) between its ends.

Mathematically,

$$\frac{V}{I} = \text{constant}$$

$$\text{or, } \frac{V}{I} = R$$

Where, R is the resistance of the conductor between the two points considered.

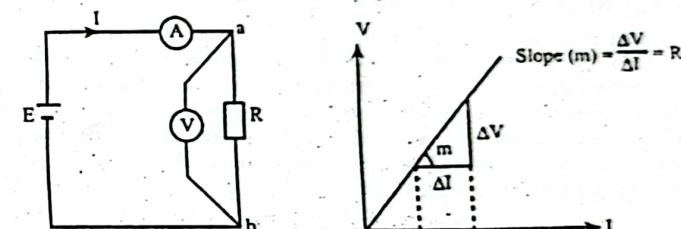


Fig. 1.3 Verification of Ohm's law

1.5.2 Ohmic and non-ohmic conductor

The conductor which exactly satisfy the ohm's law i.e. voltage/ current relationship is linear is called ohmic conductor or linear resistor. Example: Metals.

The conductor which doesn't exactly satisfy the ohm's law i.e. voltage/current relationship is not linear is called non-ohmic conductor or non linear resistor.

Example: Diode.

1.6 Resistors, resistivity

Resistance may be defined as, the property of material by virtue of which it opposes the flow of electrons through the material. The circuit element having this property is known as resistor.

The resistance of a conducting wire is found to be :

- (i) directly proportional to its length ($R \propto l$)
- (ii) inversely proportional to its area of cross sectional ($R \propto \frac{1}{a}$)
- (iii) depends upon the nature of the conducting material and
- (iv) depends upon temperature.

$$\text{i.e. } R \propto \frac{l}{a}$$

$$\therefore R = \rho \frac{l}{a}$$

Where, ρ (ohm) is a constant of material called specific resistance or resistivity of a material. If $l = 1\text{m}$ and $a = 1\text{m}^2$ then $\rho = R$. Hence, the resistivity of a material (ρ) is the resistance offered by 1m of its length and having a cross-section of 1m^2 .

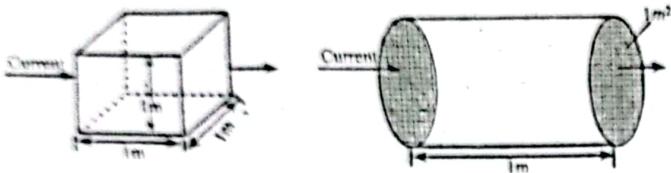


Fig. 1.4 Definition of resistivity of a material

The S.I unit of resistivity is ohm-metre.

1.7.1 Effect of temperature on resistance

The resistance of all pure metals and alloys increases with increase in temperature. Over the normal range of operating temperature the variation of resistance with temperature is linear. The temperature coefficient for pure metals and alloys is positive, whereas semi-conductors and insulating material have negative temperature coefficient. Thus, the resistance of semi-conductors and insulators decreases with increase in temperature. The temperature coefficient of metals decreases with increase in temperature.

1.7.2 Temperature coefficient of resistance

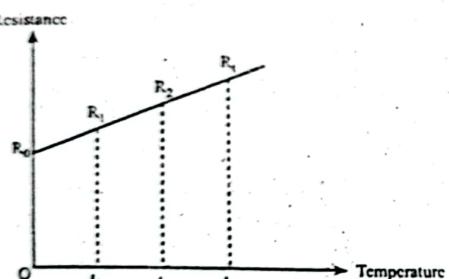


Fig. 1.5

If the resistance of any pure metal is plotted on a temperature base, the graph is as shown in figure.

Here, R_0 = resistance at 0°C

R_t = resistance at $t^\circ\text{C}$

Since, Change in resistance is directly proportional to the initial resistance and to the rise in temperature. So,

$$(R_t - R_0) \propto R_0 \quad (\text{i})$$

$$(R_t - R_0) \propto t \quad (\text{ii})$$

Combining equations (i) and (ii), we get

$$(R_t - R_0) \propto R_0 t$$

$$(R_t - R_0) = \alpha_t R_0 t \quad (\text{iii})$$

Where α_t is the temperature coefficient of resistance at 0°C whose value depends upon the nature of material and temperature.

From equation (iii),

$$R_t = R_0 + \alpha_t R_0 t$$

$$\therefore R_t = R_0 [1 + \alpha_t t]$$

Also,

From equation (iii)

$$\alpha_t = \frac{R_t - R_0}{R_0 t}$$

Temperature coefficient of resistance may be defined as the ratio of increase in resistance per degree of rise of temperature to the original resistance.

Note: If the resistance of a conductor is R_2 at $t_2^\circ\text{C}$ and R_1 at $t_1^\circ\text{C}$ ($t_1 < t_2$), then

$$R_2 = R_1 [1 + \alpha_t (t_2 - t_1)]$$

Example: Show that if α_t is the resistance temperature coefficient of a conductor at $t_1^\circ\text{C}$ then resistance temperature coefficient at $t_2^\circ\text{C}$ is given by $\frac{1}{\alpha_t} + (t_2 - t_1)$

Solution:

Let R_1, R_2 and R_3 be the resistances of a conductor at temperatures t_1, t_2 and $t_3^\circ\text{C}$ respectively.

Then,

$$R_2 = R_1 [1 + \alpha_t (t_2 - t_1)] \quad (\text{i})$$

$$R_3 = R_1 [1 + \alpha_t (t_3 - t_1)] \quad (\text{ii})$$

Also,

$$R_3 = R_2 [1 + \alpha_t (t_3 - t_2)]$$

$$\text{or, } \frac{R_1}{R_2} = [1 + \alpha_t (t_3 - t_2)] \quad (\text{iii})$$

Now, We divide equation (ii) by equation (i)

$$\frac{R_1}{R_2} = \frac{R_1 [1 + \alpha_t (t_3 - t_1)]}{R_1 [1 + \alpha_t (t_2 - t_1)]}$$

$$\text{or, } \frac{R_1}{R_2} = \frac{1 + \alpha_t (t_3 - t_1) + \alpha_t (t_1 - t_2) - \alpha_t (t_2 - t_1)}{1 + \alpha_t (t_2 - t_1)}$$

$$\text{or, } \frac{R_1}{R_2} = 1 + \frac{\alpha_t (t_3 - t_2)}{1 + \alpha_t (t_2 - t_1)} \quad (\text{iv})$$

Comparing equation (iii) and equation (iv), we get,

$$\text{or, } 1 + \alpha_2(t_1 - t_2) = 1 + \frac{\alpha_1(t_1 - t_2)}{1 + \alpha_1(t_2 - t_1)}$$

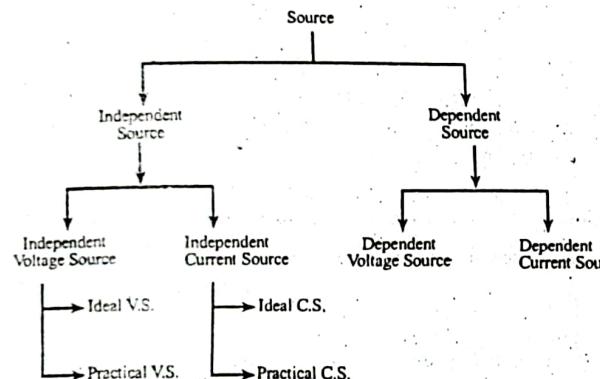
$$\text{or, } \alpha_2(t_1 - t_2) = \frac{\alpha_1(t_1 - t_2)}{1 + \alpha_1(t_2 - t_1)}$$

$$\text{or, } \alpha_2 = \frac{\alpha_1}{1 + \alpha_1(t_2 - t_1)}$$

$$\alpha_2 = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)}$$

1.8 Source

It is one of the constituent of an electrical system responsible for the flow of current in the circuit.



1.8.1 Ideal and practical voltage sources

An ideal voltage source is that which can give a constant terminal voltage across the load over infinite variation in load or load current. An ideal voltage source possess zero internal resistance or impedance.

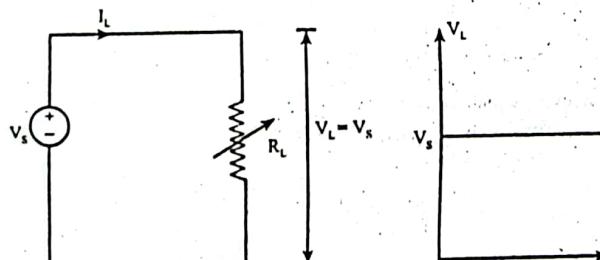


Fig. 1.6 (a) Ideal voltage source

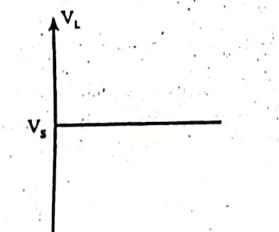


Fig. 1.6 (b) Volt - amp characteristics of ideal voltage source

Realization of such voltage source is not possible. An actual voltage source will have some internal resistance. Hence, the terminal voltage across load will decrease according to load current due to voltage drop in internal resistance. A real voltage source can be treated mathematically as an ideal voltage source in series with its internal resistance.

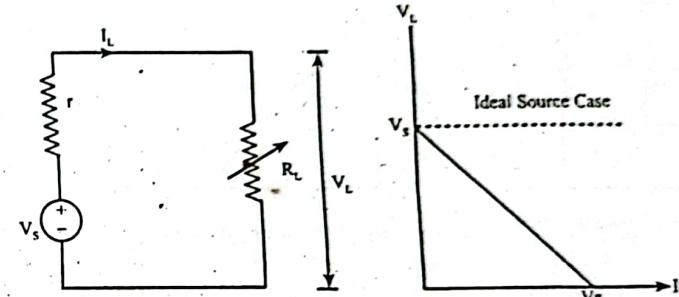


Fig. 1.7 (a) Real voltage source

Fig. 1.7 (b) Volt - amp characteristics of real voltage source

The relation between source voltage and terminal load voltage is given by,

$$V_L = V_s - I_L r$$

When $I_L = 0$ then $V_L = V_s$

$$\text{When } I_L = I_{sc} = \frac{V_s}{r} \text{ then } V_L = V_s - I_L r = V_s - \frac{V_s}{r} \times r = 0$$

1.8.2 Ideal and practical current sources

An ideal current source is that source which gives a fixed constant load current despite infinite variation in load or load voltage. An ideal current source posses infinite internal resistance or impedance. For an ideal source load current, $I_L = I_s$, whatever the load voltage may be.

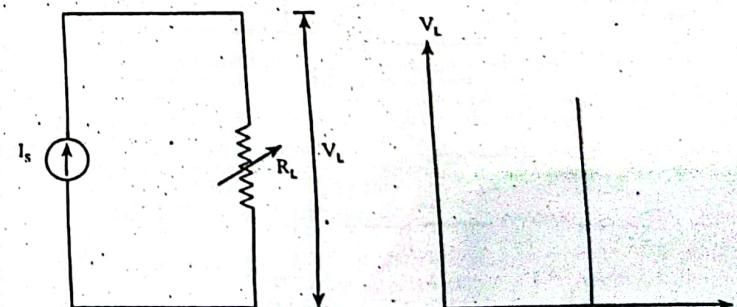


Fig. 1.8 (a) Ideal current source

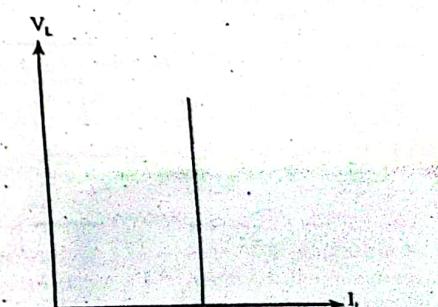


Fig. 1.8 (b) Volt-amp characteristics of ideal current source

For a practical current source, the load current is given by,

$$I_L = I_s - \frac{V_L}{R_s}$$

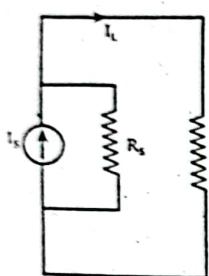


Fig. 1.9 (a) Practical current source

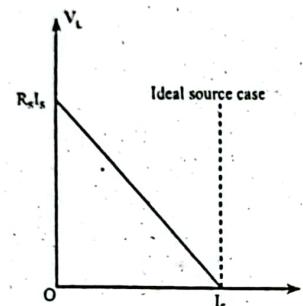


Fig. 1.9 (b) Volt-amp characteristics of practical current source

Here, the two extreme cases are,

when $R_L = 0, V_L = 0$ then $I_L = I_s$

when $R_L = \infty, I_L = 0$ then $V_L = I_s R_s$

1.9 Source transformation

Two sources are defined as being equivalent if they produce identical values of terminal voltage V_L and load current I_L connected to an identical load resistance R_L .

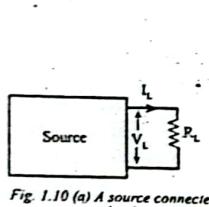


Fig. 1.10 (a) A source connected to load

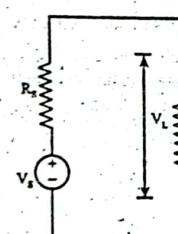


Fig. 1.10 (b) Voltage source representation

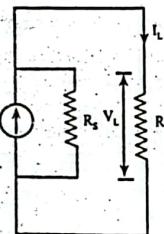


Fig. 1.10 (c) Current source representation

Example:

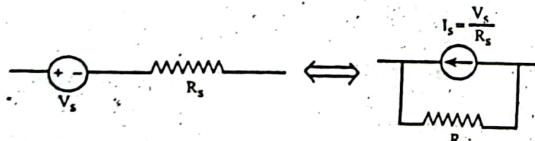


Fig. 1.11 Source transformation

Exam Solutions

1. An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5A is passed through the parallel combination, it is found that the current in the aluminium wire is 3A. The diameter of aluminium wire is 1mm. Determine the diameter of copper wire, the resistivity of copper is $0.017 \mu\Omega \cdot \text{m}$ and that of aluminium is $0.028 \mu\Omega \cdot \text{m}$. [2065 Kartik]

Solution:

Using subscript symbols 1 and 2 for aluminium (Al) and copper (Cu) respectively.

We have,

$$l_1 = 7.5 \text{ m}; a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (1 \times 10^{-3})^2 = 7.854 \times 10^{-7} \text{ m}^2$$

$$\rho_1 = 0.028 \mu\Omega \cdot \text{m}$$

$$I = 5 \text{ A}, I_1 = 3 \text{ A}, I_2 = I - I_1 = 2 \text{ A}$$

$$l_2 = 6 \text{ m}, \rho_2 = 0.017 \mu\Omega \cdot \text{m}$$

$$\therefore R_1 = \rho_1 \frac{l_1}{a_1}$$

$$= 0.028 \times 10^{-6} \times \frac{7.5}{7.854 \times 10^{-7}}$$

$$= 0.2673 \Omega$$

Voltage drop across the two ends of Al wire,

$$V_1 = I_1 R_1 = 3 \times 0.2673 = 0.8022 \text{ V}$$

$$R_2 = \rho_2 \frac{l_2}{a_2}$$

$$= \frac{0.017 \times 10^{-6} \times 6}{a_2} \Omega$$

Voltage drop through the two ends of Cu wire,

$$V_2 = I_2 R_2 = 2 \times \frac{0.017 \times 10^{-6} \times 6}{a_2}$$

$$= \frac{2.04 \times 10^{-7}}{a_2} \text{ V}$$

Since the wires are connected in parallel. So $V_1 = V_2$ i.e.,

$$0.8022 = \frac{2.04 \times 10^{-7}}{a_2}$$

$$\text{or, } a_2 = \frac{2.04 \times 10^{-7}}{0.8022}$$

$$\text{or, } \frac{\pi d_2^2}{4} = 2.54 \times 10^{-7}$$

$$\therefore d_2 = 0.569 \times 10^{-3} \text{ m}$$

Hence, diameter of copper wire is $0.569 \times 10^{-3} \text{ m}$ or 0.569 mm .

2. 1 km of wire having a diameter of 11.7 mm and of resistance 0.031 Ω is drawn so that its diameter becomes 5mm. What does its resistance become? [2066 Kartik]

Solution:

Using subscript symbols 1 and 2 for the wire before and after it is drawn.

$$R_1 = 0.031 \Omega$$

$$r_1 = \frac{1}{2} \times 11.7 = 5.85 \times 10^{-3} \text{ m}$$

$$r_2 = \frac{1}{2} \times 5 = 2.5 \times 10^{-3} \text{ m}$$

Now,

$$R = \rho \frac{l}{a} = \rho \frac{V}{a^2} = \rho \frac{V}{a^2} \quad (\text{Where, } V = \text{volume of wire})$$

Since, volume of wire is constant and the materials same, so:

$$R \propto \frac{1}{a^2}$$

$$\text{or, } \frac{R_2}{R_1} = \left(\frac{a_1}{a_2} \right)^2$$

$$\text{or, } R_2 = R_1 \left(\frac{a_1}{a_2} \right)^2$$

$$\text{or, } R_2 = R_1 \left(\frac{\pi r_1^2}{\pi r_2^2} \right)^2$$

$$\text{or, } R_2 = R_1 \left(\frac{r_1}{r_2} \right)^4$$

$$\text{or, } R_2 = 0.031 \times \left(\frac{5.85 \times 10^{-3}}{2.5 \times 10^{-3}} \right)^4$$

$$\therefore R_2 = 0.928 \Omega$$

3. A coil is connected across a constant dc source of 120V. It draws a current of 12 Amp at room temperature of 25°C. After 5 hours of operation, its temperature rises to 65°C and current reduces to 8 Amp. Calculate:

(i) Current when its temperature has increased to 80°C.

(ii) Temperature coefficient of resistance at 30°C. [2067 Mangsir]

Solution:

$$\text{Here, Resistance of coil at } 25^\circ\text{C}, R_{25} = \frac{120}{12} = 10 \Omega$$

$$\text{Resistance of coil at } 65^\circ\text{C}, R_{65} = \frac{120}{8} = 15 \Omega$$

$$\therefore R_{65} = R_{25} [1 + \alpha_{25} (65 - 25)]$$

$$\text{or, } 15 = 10 [1 + \alpha_{25} \times 40]$$

$$\therefore \alpha_{25} = 0.0125^\circ\text{C}$$

- (i) Resistance of coil at 80°C

$$R_{80} = R_{25} [1 + \alpha_{25} (80 - 25)]$$

$$= 10 [1 + 0.0125 \times 55]$$

$$= 16.875 \Omega$$

$$\text{Current at this temperature of } 80^\circ\text{C} = \frac{120}{16.875}$$

$$= 7.111 \text{ A}$$

- (ii). Temperature coefficient of resistance at 30°C,

$$\alpha_{30} = \frac{1}{\frac{1}{\alpha_{25}} + (30 - 25)} = \frac{1}{\frac{1}{0.0125} + 5}$$

$$\therefore \alpha_{30} = 0.01176^\circ\text{C}$$

4. The temperature rise of a machine field winding was determined by the measurement of the winding resistance. At 20°C, the field resistance was 150 Ω . After running the m/c for 6 hours at full load, the resistance was 175 Ω . The temperature coefficient of resistance of the copper winding is $4.3 \times 10^{-3}/\text{K}$ at 0°C. Determine the temperature rise of the m/c. [2068 Baishakh]

Solution:

$$\text{Resistance at } 20^\circ\text{C}, R_{20} = 150 \Omega$$

$$\text{Resistance at full load, } R_t = 175 \Omega$$

$$\text{Temperature coefficient of resistance at } 0^\circ\text{C}, \alpha_0 = 4.3 \times 10^{-3}/\text{K}$$

$$\text{Temperature rise, } t - 20 = ?$$

We know,

$$\alpha_{20} = \frac{1}{\frac{1}{\alpha_0} + (20 - 0)}$$

$$= \frac{1}{\frac{1}{4.3 \times 10^{-3}} + 20}$$

$$\therefore \alpha_{20} = 3.959 \times 10^{-3}/\text{K}$$

$$\text{Since, } R_t = R_{20} [1 + \alpha_{20} (t - 20)]$$

$$\text{or, } 175 = 150 [1 + 3.959 \times 10^{-3} \times (t - 20)]$$

$$\therefore t - 20 = 42.098^\circ\text{C}$$

Hence, temperature rise is 42.098°C.

5. The coil of a relay takes a current of 0.12A when it is at the room temperature of 15°C and connected across a 60V supply. If the minimum operating current of the relay is 0.1A, calculate the temperature above which the relay will fail to operate when connected to the same supply. Resistance temperature coefficient of the coil material is 0.0043 per °C at 6°C. [2068 Bhadra]

Solution:

$$\text{Resistance at } 15^\circ\text{C}, R_{15} = \frac{60}{0.12} = 500 \Omega$$

$$\text{Resistance at } t^\circ\text{C}, R_t = \frac{60}{0.1} = 600 \Omega$$

Temperature coefficient of the coil at 6°C , $\alpha_6 = 0.0043/\text{C}$ Temperature coefficient of the coil at 15°C ,

$$\alpha_{15} = \frac{1}{\frac{1}{\alpha_6} + (15 - 6)} = \frac{1}{0.0043 + (15 - 6)}$$

$$\therefore \alpha_{15} = 4.1397 \times 10^{-3}/\text{C}$$

Since,

$$R_t = R_{15}[1 + \alpha_{15}(t - 15)]$$

$$\text{or, } 600 = 500[1 + 4.1397 \times 10^{-3}(t - 15)]$$

$$\text{or, } t - 15 = 48.31^\circ\text{C}$$

$$\therefore t = 63.31^\circ\text{C}$$

Hence, for the temperature above 63.31°C , relay will fail to operate.

6. A 230V metal filament lamp has its filament 50cm long with cross sectional area of $3 \times 10^{-6} \text{ cm}^2$. Specific resistance of the filament metal at 20°C is $4 \times 10^{-8} \Omega \text{ cm}$. If the working temperature of the filament is 2000°C , find the wattage of the lamp. Temperature coefficient of resistance of the filament material at 20°C is 0.0055 per degree centigrade. [2009 Bhadra]

Solution:Resistance of filament at 20°C ,

$$R_{20} = \rho_{20} \frac{l}{A}$$

$$= 4 \times 10^{-8} \times 10^{-2} \times \frac{50 \times 10^{-2}}{3 \times 10^{-6} \times 10^{-2}} = 66.6667 \Omega$$

Temperature coefficient of resistance at 20°C , $\alpha_{20} = 0.0055/\text{C}$.

We know,

Resistance of filament at 2000°C ,

$$R_{2000} = R_{20} [1 + \alpha_{20}(2000 - 20)]$$

$$\text{or, } R_{2000} = 66.6667[1 + 0.0055 \times 1980]$$

$$\therefore R_{2000} = 792.6671 \Omega$$

Wattage of the lamp,

$$P = \frac{V^2}{R_{2000}} = \frac{230^2}{792.6671} = 66.7368 \text{ watt}$$

7. A lead wire and an iron wire are connected in parallel. Their specific resistances are in the ratio 49:24. The former carries 80% more current than the latter and the latter is 47% longer than the former. Determine the ratio of their cross-sectional areas. [2009 Ashad]

Solution:

Let suffix 1 represent lead and suffix 2 represent iron, we are given,

$$\frac{\rho_1}{\rho_2} = \frac{49}{24}$$

$$\text{if } i_1 = 1, i_2 = 1.8; \text{ if } i_1 = 1, i_2 = 1.47$$

Now,

$$R_1 = \rho_1 \frac{l_1}{A_1} \text{ and } R_2 = \rho_2 \frac{l_2}{A_2}$$

Since the two wires are in parallel,

$$i_1 = \frac{V}{R_1} \text{ and } i_2 = \frac{V}{R_2}$$

$$\therefore \frac{i_2}{i_1} = \frac{R_1}{R_2} = \frac{\rho_1 l_1}{\rho_2 l_2}$$

$$\therefore \frac{A_2}{A_1} = \frac{i_2}{i_1} \times \frac{\rho_2 l_2}{\rho_1 l_1} = \frac{1.8}{1.47} \times \frac{24}{49} = 0.4$$

8. The resistivity of a metal alloy is $50 \times 10^{-8} \Omega \text{ m}$. A sheet of material 15cm long, 6 cm wide and 0.014 cm thick. Calculate the resistance in the direction:

- (a) along the length and
- (b) along the thickness.

[2009 chaitra]

Solution:

- (a) along the length

Here,

$$l = 15 \text{ cm} = 0.15 \text{ m}$$

$$A = 6 \times 10^{-2} \times 0.014 \times 10^{-2} = 8.4 \times 10^{-6} \text{ m}^2$$

$$\therefore R = \rho \frac{l}{A} = 50 \times 10^{-8} \times \frac{0.15}{8.4 \times 10^{-6}} = 8.928 \times 10^{-3} \Omega$$

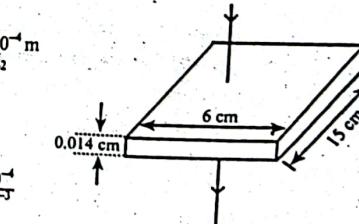
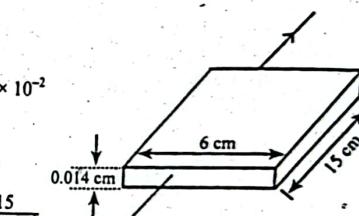
- (b) along the thickness

Here,

$$l = 0.014 \text{ cm} = 1.4 \times 10^{-4} \text{ m}$$

$$a = 6 \times 10^{-2} \times 15 \times 10^{-2} = 9 \times 10^{-3} \text{ m}^2$$

$$\therefore R = \rho \frac{l}{a} = 50 \times 10^{-8} \times \frac{1.4 \times 10^{-4}}{9 \times 10^{-3}} = 7.77 \times 10^{-9} \Omega$$



$$R_1 = R_{01}(1 + 70\alpha_1); R_2 = R_{02}(1 + 70\alpha_2)$$

Power ratio at 70°C is;

$$\frac{P_2}{P_1} = \frac{\left(\frac{V^2}{R_2}\right)}{\left(\frac{V^2}{R_1}\right)} = \frac{R_1}{R_2} = \frac{R_{01}(1 + 70\alpha_1)}{R_{02}(1 + 70\alpha_2)}$$

Using equation (i)

$$\begin{aligned} \frac{P_2}{P_1} &= \frac{(1 + 15\alpha_1)(1 + 70\alpha_1)}{(1 + 15\alpha_1)(1 + 70\alpha_2)} \\ &= \frac{(1 + 15 \times 0.005)(1 + 70 \times 0.004)}{(1 + 15 \times 0.004)(1 + 70 \times 0.005)} = 0.962 \end{aligned}$$

13. A coil has a resistance of 18 Ω when its mean temperature is 20°C and 20Ω when its mean temperature is 50°C. Find its mean temperature rise when its resistance is 21 Ω and the ambient temperature is 15°C. [2071 Bhadra]

Solution:

Here,

Resistance of coil at 20°C, $R_{20} = 18 \Omega$

Resistance of coil at 50°C, $R_{50} = 20 \Omega$

Ambient temperature, $t_{amb} = 15^\circ C$

Let, t be the mean temperature when the resistance is 21 Ω i.e. $R_t = 21 \Omega$
we know,

$$\begin{aligned} R_{20} &= R_{15}[1 + \alpha_{15}(20 - 15)] \\ \text{or, } 18 &= R_{15}[1 + 5\alpha_{15}] \quad \dots \text{(i)} \end{aligned}$$

Also,

$$\begin{aligned} R_{50} &= R_{15}[1 + \alpha_{15}(50 - 15)] \\ \text{or, } 20 &= R_{15}[1 + 35\alpha_{15}] \quad \dots \text{(ii)} \end{aligned}$$

dividing equation (i) by equation (ii)

we get,

$$\begin{aligned} \frac{18}{20} &= \frac{1 + 5\alpha_{15}}{1 + 35\alpha_{15}} \\ \text{or, } 0.9(1 + 35\alpha_{15}) &= 1 + 5\alpha_{15} \\ \text{or, } 0.9 + 31.5\alpha_{15} &= 1 + 5\alpha_{15} \\ \text{or, } 26.5\alpha_{15} &= 0.1 \\ \therefore \alpha_{15} &= \frac{1}{265} /^\circ C \end{aligned}$$

Now,

Putting the value of α_{15} in eq (i), we get,

$$R_{15} = \frac{18}{1 + 5 \times \frac{1}{265}}$$

= 17.67 Ω

Since, $R_t = R_{15}[1 + \alpha_{15}(t - 15)]$

$$\text{or, } 21 = 17.67[1 + \frac{1}{265}(t - 15)]$$

$$15 = 49.94^\circ C$$

temperature rise is 49.94°C

14. A coil connected to a constant dc supply of 100V drew a current of 13A at room temperature of 25°C. After sometime, its temperature increased to 70°C and current fell to 8.5 A. Find the current it will draw when its temperature will further rise to 80°C. Also find the temperature coefficient of resistance of the coil at 20°C. [2071 Magh]

Solution:

Here,

$$\text{Resistance of coil at } 25^\circ C, R_{25} = \frac{100}{13} = 7.692 \Omega$$

$$\text{Resistance of coil at } 70^\circ C, R_{70} = \frac{100}{8.5} = 11.765 \Omega$$

we know,

$$R_{25} = R_0[1 + \alpha_0(25 - 0)] \quad \dots \text{(i)}$$

$$R_{70} = R_0[1 + \alpha_0(70 - 0)] \quad \dots \text{(ii)}$$

where, R_0 = resistance of coil at 0°C

α_0 = temperature coefficient of resistance at 0°C.

dividing equation (i) by equation (ii), we get

$$\frac{R_{25}}{R_{70}} = \frac{1 + 25\alpha_0}{1 + 70\alpha_0}$$

$$\text{or, } \frac{7.692}{11.765} = \frac{1 + 25\alpha_0}{1 + 70\alpha_0}$$

$$\text{or, } 0.654(1 + 70\alpha_0) = 1 + 25\alpha_0$$

$$\text{or, } 0.654 + 45.766\alpha_0 = 1 + 25\alpha_0$$

$$\therefore \alpha_0 = 0.0167 /^\circ C$$

Using equation (i)

$$R_0 = \frac{R_{25}}{1 + 25\alpha_0} = \frac{7.692}{1 + 25 \times 0.0167} = 5.426 \Omega$$

Resistance of coil at 80°C

$$\begin{aligned} R_{80} &= R_0[1 + \alpha_0(80 - 0)] \\ &= 5.426[1 + 0.0167 \times 80] \\ &= 12.675 \Omega \end{aligned}$$

Temperature coefficient of resistance of coil at 20°C,

$$\alpha_{20} = \frac{1}{R_0 + (20 - 0)} = \frac{1}{\frac{1}{0.0167} + 20} = 0.0125 /^\circ C$$

15. The field winding of d.c. motor connected across 230V supply takes 1.15A at room temperature of 20°C. After working for some hours the current falls to 0.26A, the supply voltage remaining constant. Calculate the final working temperature of field winding. Resistance temperature coefficient of copper at 20°C is $\frac{1}{254.5}$. [2071 chaitra]

Solution,

Here,

$$\text{Resistance at } 20^\circ C, R_{20} = \frac{230}{1.15} = 200 \Omega$$

$$\text{Resistance at } t^\circ C, R_t = \frac{230}{0.26} = 884.615 \Omega$$

Additional Problems

1. A rectangular metal strip has the dimensions $x = 10 \text{ cm}$, $y = 0.5 \text{ cm}$, $z = 0.2 \text{ cm}$. Determine the ratio of the resistances R_x , R_y and R_z between the respective pair of opposite faces.

Solution:

$$l_x = 10 \text{ cm}; a_x = 0.5 \text{ cm} \times 0.2 \text{ cm} = 0.1 \text{ cm}^2$$

$$l_y = 0.5 \text{ cm}; a_y = 10 \text{ cm} \times 0.2 \text{ cm} = 2 \text{ cm}^2$$

$$l_z = 0.2 \text{ cm}; a_z = 10 \text{ cm} \times 0.5 \text{ cm} = 5 \text{ cm}^2$$

$$R_x = \rho \frac{l_x}{a_x} = \rho \frac{10}{0.1} = 100 \rho$$

$$R_y = \rho \frac{l_y}{a_y} = \rho \frac{0.5}{2} = 0.25 \rho$$

$$R_z = \rho \frac{l_z}{a_z} = \rho \frac{0.2}{5} = 0.04 \rho$$

$$\therefore R_x : R_y : R_z = 100 : 0.25 : 0.04 = 10000 : 25 : 4$$

2. A semicircular ring of copper has an inner radius of 80mm, radial thickness 40 mm and axial thickness 60 mm. Calculate the resistance of the ring at 50°C between its two end faces. Resistivity of copper at 20°C is $1.724 \times 10^{-8} \Omega\text{m}$. Resistance temperature coefficient of copper at 0°C is $0.0043/\text{ }^\circ\text{C}$.

Solution:

The semicircular ring is shown in figure

$$\text{Mean radius of the ring} = \frac{80 + 120}{2}$$

$$= 100 \text{ mm} = 10 \text{ cm.}$$

$$\text{Mean length of the ring} = 10\pi \text{ cm} = 31.416 \text{ cm.}$$

$$\text{Area of cross section of the ring} = 6 \times 4 = 24 \text{ cm}^2$$

$$\alpha_0 = 0.0043/\text{ }^\circ\text{C}$$

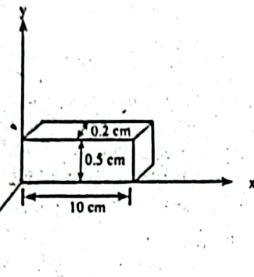
$$\alpha_{20} = \frac{1}{\frac{1}{0.0043} + (20 - 0)} = 0.00396/\text{ }^\circ\text{C}$$

$$\rho_{20} = 1.724 \times 10^{-8} \Omega\text{m}$$

$$\therefore \rho_{50} = \rho_{20} [(1 + \alpha_{20}(50 - 20))] = 1.724 \times 10^{-8} [1 + 0.00396 \times 30] \\ = 1.929 \times 10^{-8} \Omega\text{m.}$$

$$R_{50} = \rho_{50} \times \frac{l}{a} = 1.929 \times 10^{-8} \times \frac{31.416 \times 10^{-2}}{24 \times 10^{-4}} = 2.525 \times 10^{-6} \Omega$$

3. A specimen of copper wire has a specific resistance of $1.7 \times 10^{-8} \Omega\text{m}$ at 0°C and has temperature coefficient of $1/254.5$ per degree celsius at 20°C . Find the specific resistance and temperature coefficient at 70°C .



Solution:

$$\rho_0 = 1.7 \times 10^{-8} \Omega\text{m}$$

$$\alpha_{20} = \frac{1}{254.5}/\text{ }^\circ\text{C}$$

$$\alpha_0 = \frac{1}{\frac{1}{254.5} + (0 - 20)} = \frac{2}{469}/\text{ }^\circ\text{C}$$

$$\rho_{70} = \rho_0 [1 + \alpha_0 (70 - 0)] = 1.7 \times 10^{-8} \left[1 + \frac{2}{469} \times 70 \right] = 2.2074 \times 10^{-8} \Omega\text{m}$$

\therefore Temperature coefficient at 70°C ,

$$\alpha_{70} = \frac{1}{\frac{1}{\alpha_{20}} + (70 - 20)} = \frac{1}{\frac{1}{254.5} + 50} = \frac{2}{609}/\text{ }^\circ\text{C} = \frac{1}{304.5}/\text{ }^\circ\text{C}$$

4. A 60W, 240V incandescent filament lamp is switched on at 20°C . The operating temperature of the filament is 2000°C . Determine the current taken by the lamp at the instant of switching on. The temperature coefficient of resistance of filament material is $0.0045/\text{ }^\circ\text{C}$.

Solution:

Let R_1 and R_2 be the resistance of the filament of the lamp at 20°C and 2000°C respectively.

Here, Temperature coefficient at 20°C , $\alpha_{20} = 0.0045/\text{ }^\circ\text{C}$

$$P = \frac{V^2}{R}$$

$$\therefore R = \frac{V^2}{P}$$

$$\text{Then, } R_{2000} = \frac{(240)^2}{60} = 960 \Omega$$

$$\therefore R_{2000} = R_{20} [1 + \alpha_{20} (2000 - 20)]$$

$$\text{or, } 960 = R_{20} [1 + 0.0045 \times 1980]$$

$$\therefore R_{20} = 96.9 \Omega$$

Therefore, current taken by the lamp at the instant of

$$\text{switching on} = \frac{V}{R_{20} \cdot 96.9} = 2.48 \text{ A}$$

5. At 30°C , two coils connected in series having resistance of 800Ω and 400Ω respectively and temperature coefficient of resistance coil 1 and coil 2 are,

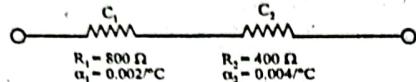
$$\alpha_{C1} = 0.002/\text{ }^\circ\text{C at } 30^\circ\text{C}$$

$$\alpha_{C2} = 0.004/\text{ }^\circ\text{C at } 30^\circ\text{C}$$

(i) Find the resistance of combination at 60°C .

(ii) Effective temperature coefficient of resistance of combination at 30°C .

Solution:



For coil 1;

$$R_{60} = R_{30} [1 + \alpha_{30} (60 - 30)] = 800 [1 + 0.002 \times 30] = 848 \Omega$$

For coil 2;

$$R_{60} = R_{30} [1 + \alpha_{30} (60 - 30)] = 400 [1 + 0.004 \times 30] = 448 \Omega$$

(i) Resistance of combination at 60°C ,

$$R_{60 (\text{comb})} = 848 \Omega + 448 \Omega = 1296 \Omega$$

$$(ii) R_{30 (\text{comb})} = 800 + 400 = 1200 \Omega$$

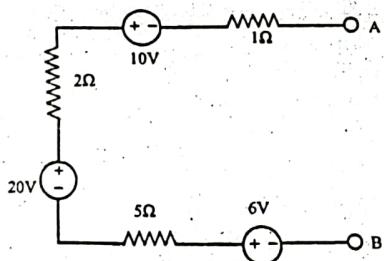
$$\therefore R_{60 (\text{comb})} = R_{30 (\text{comb})} [1 + \alpha_{30 (\text{comb})} (60 - 30)]$$

$$\text{or, } 1296 = 1200 [1 + \alpha_{30 (\text{comb})} \times 30]$$

$$\therefore \alpha_{30 (\text{comb})} = 2.67 \times 10^{-3}/^\circ\text{C}.$$

Hence, temperature coefficient of resistance of combination at 30°C is $2.67 \times 10^{-3}/^\circ\text{C}$.

6. Find the equivalent current source for the circuit shown in figure.



Solution:

The equivalent voltage source is obtained as follows:

$$V_{eq} = 6 + 20 - 10 = 16 \text{ V}$$

$$R_{eq} = 5 + 2 + 1 = 8 \Omega$$

Fig (a) represents the single equivalent voltage source. The equivalent current source can be obtained as,

$$I_{eq} = \frac{16}{8} = 2 \text{ A}; R = 8 \Omega$$

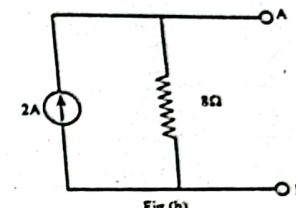
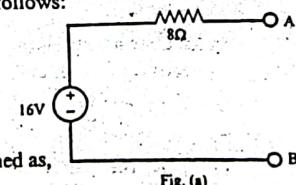
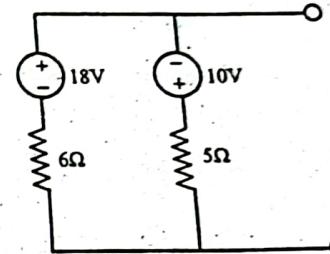


Fig (b) represents the required equivalent current source.

7. Obtain a single current source for the network shown in figure.

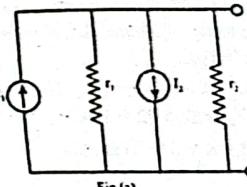


Solution:

Let us first convert the voltage sources to equivalent current source [fig (a)].

$$\text{Here, } I_1 = \frac{18}{6} = 3 \text{ A}; r_1 = 6 \Omega$$

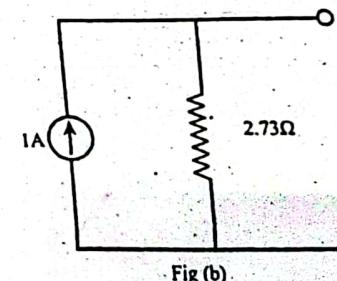
$$I_2 = \frac{10}{5} = 2 \text{ A}; r_2 = 5 \Omega$$



The equivalent current source is obtained as,

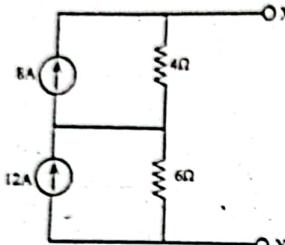
$$I_{eq} = I_1 - I_2 = 1 \text{ A}$$

$$R = \frac{I_1 I_2}{r_1 + r_2} = \frac{6 \times 5}{6 + 5} = 2.73 \Omega$$



Hence, fig(b) represents the equivalent current source.

8. Convert the following circuit into a single voltage source.



Solution:

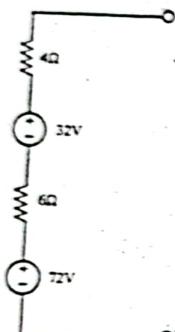


Fig. (a)

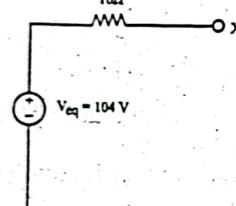


Fig. (b)

Let us first convert the multiple current sources to equivalent voltage sources as shown in fig (a). Next, these voltage sources are transformed into single voltage source fig(b).

Where,

$$V_{eq} = 32 + 72 = 104 \text{ V}$$

$$\text{and } R = 4 + 6 = 10 \Omega$$



2

DC CIRCUITS

2.1 DC Circuit

A dc circuit is an electric circuit that consists of any combination of constant voltage sources, constant current sources and resistors. Direct current is the unidirectional flow of electric charge.

2.2 Series Circuits

When the resistors are connected end to end so that they form only one path for the flow of current then resistors are said to be connected in series and such circuits are known as series circuits.

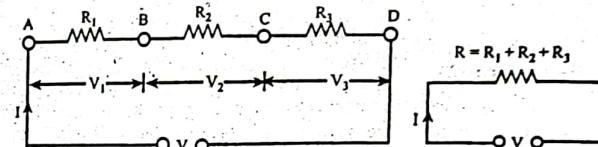


Fig. 2.1 (a): Resistors in Series

Fig. 2.1 (b): Equivalent resistance

In figure 2.1 (a), A and D are the free ends of three resistors AB, BC and CD connected in series. Let R_1 , R_2 and R_3 be the respective resistance, R = resistance of combination, V = total p.d across the resistors and I = current strength. Then,

$$V = IR \quad \dots \dots \dots \text{(i)}$$

But, V = sum of the individual p.d across R_1 , R_2 and R_3

$$\therefore V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \quad \dots \dots \dots \text{(ii)}$$

Using equations (i) and (ii), we get

$$IR = IR_1 + IR_2 + IR_3$$

$$\therefore R = R_1 + R_2 + R_3 \quad \text{Also, } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

i.e. If a number of resistors are connected in series, then combined resistance of the system equals the sum of the individual resistances.

2.2.1 Voltage divider rule

$$V_1 = IR_1 = \frac{V}{R_1 + R_2} \times R_1 = \frac{V}{G_1 + G_2} \times G_1$$

(Where, $G = \frac{1}{R}$ = conductance)

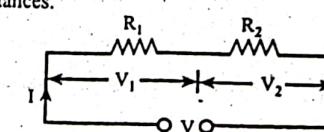


Fig. 2.2 Two resistors in series

$$V_2 = IR_2 = \frac{V}{R_1 + R_2} \times R_2 = \frac{V}{G_1 + G_2} \times G_2$$

Also,

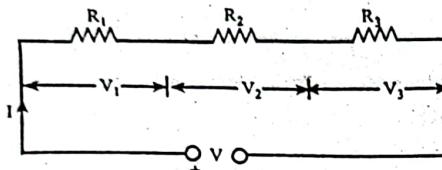


Fig. 2.3 Three resistors in series

$$V_1 = IR_1 = \frac{V}{R_1 + R_2 + R_3} \times R_1 = \frac{V}{G_1 G_2 + G_2 G_3 + G_3 G_1} \times G_2 G_3$$

$$V_2 = IR_2 = \frac{V}{R_1 + R_2 + R_3} \times R_2 = \frac{V}{G_1 G_2 + G_2 G_3 + G_3 G_1} \times G_3 G_1$$

$$V_3 = IR_3 = \frac{V}{R_1 + R_2 + R_3} \times R_3 = \frac{V}{G_1 G_2 + G_2 G_3 + G_3 G_1} \times G_1 G_2$$

2.3 Parallel Circuits

When a number of resistors are connected in such a way that one end of each of them is joined to a common point and the other ends being joined to another common point then resistors are said to be connected in parallel and such circuits are known as parallel circuits.

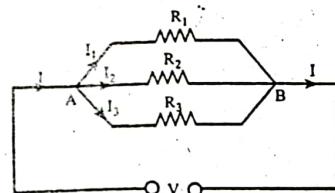


Fig. 2.4 (a) Resistors in parallel

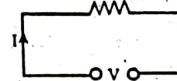


Fig. 2.4 (b) Equivalent resistance

In fig 2.4 (a), three resistors of resistance R_1 , R_2 and R_3 are connected between the common points A and B. Same p.d (V) exists between the ends of each resistors but the amount of current passing through each is different, depending upon their resistances. Suppose the main current I is divided into I_1 , I_2 and I_3 through resistors R_1 , R_2 and R_3 . If R is the combined resistance between A and B, then

$$I = \frac{V}{R} \dots\dots\dots (i)$$

Since the main current (I), which enters the combination, must also come out as such,

$$\text{so, } I = I_1 + I_2 + I_3$$

$$\text{or, } I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \dots\dots\dots (ii)$$

Using equations (i) and (ii), we get

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{Also, } G = G_1 + G_2 + G_3$$

i.e. If a number of conductors are connected in parallel, the reciprocal of the combined resistance is equal to the sum of the reciprocal of the individual resistances.

2.3.1 Current divider rule

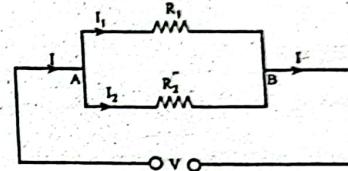


Fig. 2.5 Two resistors in parallel

$$I_1 = \frac{V}{R_1} = \frac{IR}{R_1} \quad [R \text{ is the equivalent resistance of combination}]$$

$$\text{or, } I_1 = \frac{I \times \left(\frac{1}{R_1}\right)}{\frac{1}{R}}$$

$$\text{or, } I_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} \times I$$

$$\text{or, } I_1 = \frac{I}{R_1 + R_2} \times R_2 = \frac{1}{G_1 + G_2} \times G_1$$

$$\text{Similarly, } I_2 = \frac{I}{G_1 + G_2} \times G_2$$

Also,

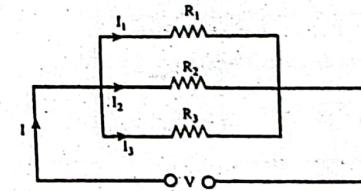


Fig. 2.6 Three resistors in parallel

$$I_1 = \frac{V}{R_1} = \frac{IR}{R_1} \quad [R \text{ is the equivalent resistance of combination}]$$

$$\text{or, } I_1 = \frac{I \times \left(\frac{1}{R_1}\right)}{\frac{1}{R}}$$

$$\text{or, } I_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \times I$$

$$\therefore I_1 = \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \times I = \frac{G_1}{G_1 + G_2 + G_3} \times I$$

$$\text{Similarly, } I_2 = \frac{G_2}{G_1 + G_2 + G_3} \times I$$

$$I_3 = \frac{G_3}{G_1 + G_2 + G_3} \times I$$

2.4 Circuit containing series and parallel connections

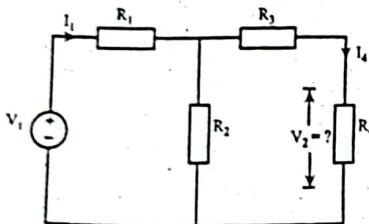


Fig. 2.7 Resistors in series and parallel

The total resistance seen from the source is

$$R = \frac{V_1}{I_1} = R_1 + R_2 \parallel (R_3 + R_4)$$

$$\text{or, } \frac{V_1}{I_1} = R_1 + \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$$

$$\text{or, } \frac{V_1}{I_1} = \frac{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$$

$$\begin{aligned} \text{Therefore, } I_1 &= \frac{(R_2 + R_3 + R_4)V_1}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)} \\ &= \frac{(R_2 + R_3 + R_4)V_1}{R_1 R_2 + R_3 R_1 + R_1 R_4 + R_2 R_3 + R_2 R_4} \end{aligned}$$

Using the current division formula, we can find,

$$I_4 = \frac{I_1}{R_2 + R_3 + R_4} \times R_2$$

Finally,

Since, $V_2 = I_4 R_4$

$$\text{We have, } V_2 = \frac{R_2 R_4 I_1}{R_2 + R_3 + R_4}$$

2.4.1 Summary of series and parallel circuit

Circuit	Potential difference	Current	Resistance
Series	Each load uses a portion of the total p.d. supplied by the battery.	The current is the same through the series circuit.	The current decreases when more resistors are added because resistance increases.
Parallel	Each load uses all the potential difference supplied by the battery.	The current divides into different paths.	Adding resistors in parallel decreases the total resistance of the circuit.

2.5 Kirchhoff's law

In simple circuits, we can carry out the analysis of current, voltage and resistance simply with the help of Ohm's law. However, in complex circuit or network, the calculations can be done with the help of Kirchhoff's laws, which are stated as follows:

(i) Kirchhoff's current law

(ii) Kirchhoff's voltage law

(iii) Kirchhoff's current law

This law is also called as Kirchhoff's first law or point law or junction rule. According to this law, at any node of a several circuit elements, the sum of currents entering the node must equal the sum of currents leaving it. This law is based on conservation of charge.

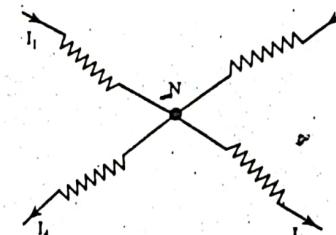


Fig. 2.8

In this figure, the currents directed towards node N are $I_1, I_2, -I_3$ and $-I_4$.

So, $I_1 + I_2 + (-I_3) + (-I_4) = 0$

$$\sum_{k=1}^n I_k = 0$$

Where n is the total number of branches with current flowing towards or away from the node.

(ii) Kirchhoff's voltage law

This law is also called Kirchhoff's second law or Kirchhoff's loop (or mesh) rule. According to this law, in any closed electrical circuit or mesh, the algebraic sum of all the electromotive forces (e.m.fs) and voltage drops in resistors is equal to zero. This law is based on conservation of energy.

In any closed circuit or mesh,
Algebraic sum of e.m.fs + Algebraic
sum of voltage drops = 0
Applying KVL in above mesh;
 $E + (-V_1) + (-V_2) + (-V_3) = 0$
 $\therefore E = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$

Sign rules for KVL

- Give positive sign to all rise in voltage and negative to voltage drops. Thus, if we move from negative (-ve) terminal of a battery to positive (+ve) terminal, a positive sign should be given, since there is a rise in voltage. On the other hand, if we go from positive (+ve) terminal to negative (-ve) terminal, a negative sign should be given, since there is a drop in voltage.

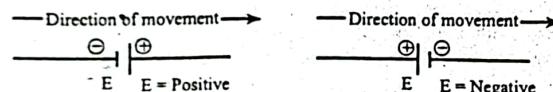


Fig. 2.10

- For a conductor of resistance R, if the direction of current is same/opposite as the direction of movement, then voltage ($= IR$) should be taken negative/positive, since current flows from higher voltage to lower one and hence there is a voltage drop/voltage rise while crossing the conducting element. This is illustrated below;

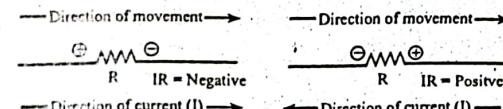


Fig. 2.11

2.6 Power and energy in dc circuit

As a charge carrier q move around a circuit and drops an amount of potential V in time t, it loses an amount of potential energy qV . The power or the rate at which it loses energy is $\frac{qV}{t}$. Since the current I is equal to $\frac{q}{t}$, the power can be expressed as,

$$P = \frac{qV}{t} = VI \text{ joules/sec or watts}$$

We can combine the equations for power and Ohm's law to get expressions for power in terms of resistance.

$$P = I^2R = \frac{V^2}{R} \text{ joules/sec or watts}$$

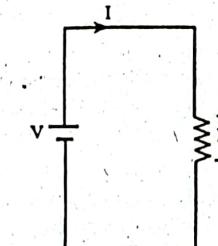


Fig. 2.12

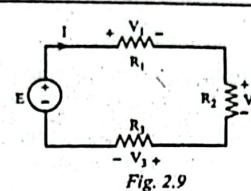


Fig. 2.9

As current flows through a resistor, the resistor heats up. The heat in joules is given by,

$$H = I^2Rt = Pt \text{ joules}$$

Where t is the time in seconds. In other words, a resistor heats up more when there is a high current running through a strong resistor over a long stretch of time.

2.7 Open Circuit and Short Circuit

2.7.1 Open Circuit

Two points are said to be open circuited if there is no direct connection between them. It represents a break in the continuity of the circuit. Due to this break,

- Resistance R between two points is infinite.
- There is no flow of current between two points.

Example:

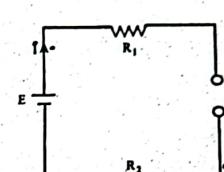


Fig. 2.13 (a) Open in series circuit

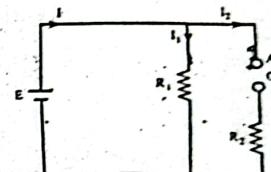


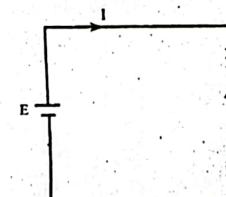
Fig. 2.13 (a) Open in parallel circuit

2.7.2 Short Circuit

When two points are connected together by a thick metallic wire (resistance less wire), they are said to be short circuited. Due to this,

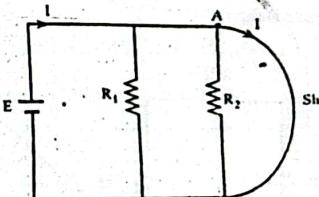
- Resistance R between points is zero.
- No voltage can exist across it, $V = IR = 0$
- Current through it is very large (theoretically infinite) called short circuited current.

Example:



$$I = \frac{E}{R_1}$$

Fig. 2.14 (a) Short in series circuit



$$I = \infty; \text{excessive current}$$

Fig. 2.14 (b) Short in parallel circuit

Exam Solutions

1. It is proposed to work in series two lamps at their rated power of 100W, 200V and 40W, 220V on a 440V supply by connecting a resistor R in parallel with 40 watt lamp. Find the value of R if total power drawn from the source is 400 watt. [2064 Poush]

Solution:

Considering lamp resistance constant

$$\text{Resistance of first lamp, } R_1 = \frac{200^2}{100} = 400 \Omega$$

$$\text{Resistance of second lamp, } R_2 = \frac{220^2}{40} = 1210 \Omega$$

Since, Power drawn from source is 400 watt

$$\text{So, } P = \frac{V^2}{R_{eq}}$$

$$\text{or, } 400 = \frac{440^2}{R_{eq}}$$

$$\text{or, } R_{eq} = \frac{440^2}{400}$$

$$\text{or, } 400 + (1210 \parallel R) = \frac{440^2}{400}$$

$$\text{or, } 400 + \frac{1210 \times R}{1210 + R} = \frac{440^2}{400}$$

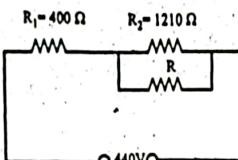
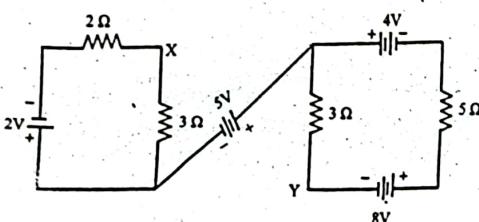
$$\text{or, } 400 + \frac{1210 \times R}{1210 + R} = 484$$

$$\text{or, } \frac{1210 \times R}{1210 + R} = 84$$

$$\text{or, } 1210 R = 101640 + 84 R$$

$$\therefore R = 90.27 \Omega$$

2. What is the difference of potential between X and Y in the network shown in figure below. [2067 Mangsir]



Solution:

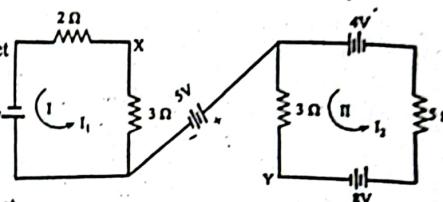
Let I_1 and I_2 be the mesh current of mesh I and mesh II respectively.

Applying KVL on mesh I, we get

$$2 - 3I_1 - 2I_2 = 0$$

$$\text{or, } 2 - 5I_1 = 0$$

$$\therefore I_1 = \frac{2}{5} \text{ A}$$



Applying KVL on mesh II, we get

$$8 - 5I_2 + 4 - 3I_2 = 0$$

$$\text{or, } 12 - 8I_2 = 0$$

$$\therefore I_2 = \frac{3}{2} \text{ A.}$$

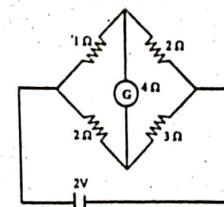
Now, $V_{XY} = V_X - V_Y$

$= 3I_2 - 5 - 3I_1$ [Write KVL equation; move from Y to X]

$$= 3 \times \frac{3}{2} - 5 - 3 \times \frac{2}{5} = -1.7 \text{ volt.}$$

Which indicates that Y is at higher potential with respect to X.

3. Calculate the current through the galvanometer in the bridge circuit as shown in figure given below using Kirchhoff's laws. [2067 Mangsir]



Solution:

Let the current distribution in the network be as shown in figure

Applying KVL to mesh ABCA, we get

$$-I_1 - 4I_3 + 2I_2 = 0 \quad \dots \dots \dots (i)$$

Also,

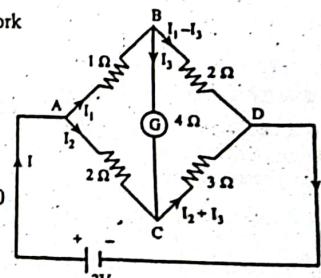
Applying KVL to mesh BCDB, we get

$$-4I_3 - 3(I_2 + I_3) + 2(I_1 - I_2) = 0$$

$$\text{or, } -4I_3 - 3I_2 - 3I_3 + 2I_1 - 2I_2 = 0$$

$$\text{or, } 2I_1 - 3I_2 - 9I_3 = 0 \quad \dots \dots \dots (ii)$$

and,



Applying KVL to mesh ACDA, we get,

$$-2I_2 - 3(I_2 + I_3) + 2 = 0$$

$$\text{or, } -2I_2 - 3I_2 - 3I_3 + 2 = 0$$

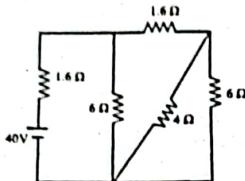
or, $-5I_2 - 3I_3 = -2 \dots \dots \dots \text{(iii)}$
solving equations (i), (ii) and (iii), we get,

$$I_1 = \frac{15}{22} \text{ A}, I_2 = \frac{17}{44} \text{ A}, I_3 = \frac{1}{44} \text{ A}$$

Hence, The current through the galvanometer is $\frac{1}{44} \text{ A}$

4. Find the current through 4Ω resistance.

[2068 Bhadra]



Solution:

Equivalent resistance of the given circuit

$$\begin{aligned} R_{eq} &= [(6 \parallel 4) + 1.6] \parallel 6 + 1.6 \\ &= \left[\frac{6 \times 4}{6+4} + 1.6 \right] \parallel 6 + 1.6 \\ &= 4 \parallel 6 + 1.6 = \frac{4 \times 6}{4+6} + 1.6 = 4\Omega \end{aligned}$$

$$\therefore \text{current, } I = \frac{V}{R_{eq}} = \frac{40}{4} = 10 \text{ A}$$

Using current division rule

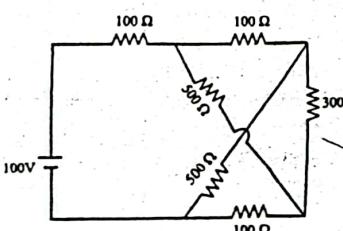
$$I_2 = \frac{1}{6 + \left(1.6 + \frac{4 \times 6}{4+6} \right)} \times 6 = \frac{10}{6+4} \times 6 = 6 \text{ A}$$

$$\therefore I_3 = \frac{I_2}{4+6} \times 6 = \frac{6}{10} \times 6 = 3.6 \text{ A}$$

Hence, the current through 4Ω resistance is 3.6 A.

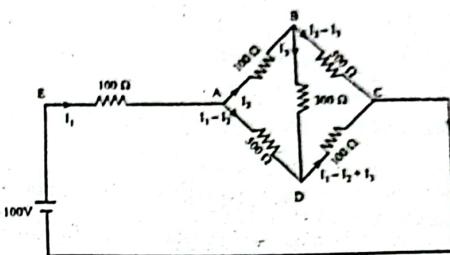
5. Determine the current supplied by the battery in the circuit shown in figure below.

[2068 Bhadra]



Solution:

Redrawing the circuit,



Using Kirchhoff's laws to find the current supplied by the battery we consider current distribution in the network be as shown in figure:

Applying KVL on mesh EADCE, we get

$$100 - 100 I_1 - 500 (I_1 - I_2) - 100 (I_1 - I_2 + I_3) = 0$$

$$\text{or, } -700 I_1 + 600 I_2 - 100 I_3 = -100 \dots \dots \text{(i)}$$

Applying KVL on mesh ABDA, we get,

$$-100 I_2 - 300 I_3 + 500 (I_1 - I_2) = 0$$

$$\text{or, } 500 I_1 - 600 I_2 - 300 I_3 = 0 \dots \dots \text{(ii)}$$

Applying KVL on mesh BCDB, we get,

$$-500 (I_2 - I_3) + 100 (I_1 - I_2 + I_3) + 300 I_3 = 0$$

$$\text{or, } 100 I_1 - 600 I_2 + 900 I_3 = 0 \dots \dots \text{(iii)}$$

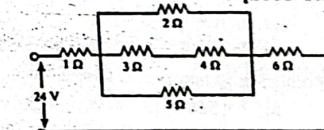
solving equations (i), (ii) and (iii), we get

$$I_1 = \frac{3}{10} \text{ A}, I_2 = \frac{1}{5} \text{ A}, I_3 = \frac{1}{10} \text{ A}$$

Hence, the current supplied by the battery in the circuit is $\frac{3}{10} \text{ A}$.

6. Find the equivalent resistance in the figure shown and power dissipated in the 5Ω resistor.

[2068 Chaitra]



Solution:

Equivalent resistance of the circuit,

$$R = 1 + (2 \parallel 7 \parallel 5) + 6$$

$$= 1 + \left\{ \left(\frac{2 \times 7}{2+7} \right) \parallel 5 \right\} + 6 = 1 + \left(\frac{14}{9} \parallel 5 \right) + 6$$

$$= 1 + \frac{70}{59} + 6 = 8.1864 \Omega$$

$$\text{Total current, } I = \frac{24}{8.1864} = 2.9317 \text{ A}$$

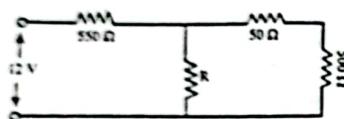
Now, Using current division rule,
Current flowing through 5Ω resistor,

$$I_{10} = \frac{G_{50}}{G_{50} + G_{50} + G_{50}} \times I$$

$$= \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{5} + \frac{1}{5}} \times 2.9317 = 0.6957 \text{ A}$$

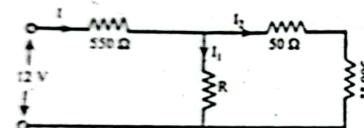
Power dissipated in 5Ω resistor = $I_{10}^2 R$
 $= (0.6957)^2 \times 5 = 2.4197 \text{ watt}$

7. Determine the value of unknown resistor 'R' in the circuit below if the voltage drop across 500Ω resistor is 2.5 volts. [2068 Shrawan]



Solution:

We have,
Voltage drop across 500Ω resistor = 2.5 volt



So, Current flowing through 500Ω resistor.

$$I_2 = \frac{2.5}{500}$$

$$\therefore I_2 = 0.005 \text{ A.}$$

Voltage drop across $R = \text{voltage drop across } (50 + 500) \Omega$
 $\text{or, } I_1 R = 0.005 \times (50 + 500)$
 $\text{or, } I_1 R = 2.75 \dots \text{(i)}$

Voltage drop across 550Ω resistor = $12 - I_2 (50 + 500)$
 $\text{or, } I \times 550 = 12 - 0.005 \times (50 + 500)$
 $\text{or, } I \times 550 = 9.25$
 $\therefore I = 0.0168 \text{ A}$

Simply by KCL,

$$I = I_1 + I_2$$

$$\text{or, } 0.0168 = I_1 + 0.005$$

$$\therefore I_1 = 0.0168 - 0.005 = 0.0118 \text{ A}$$

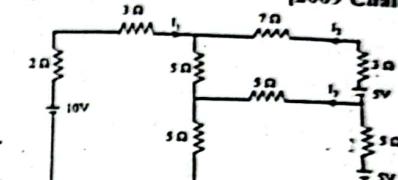
We know, Voltage drop across $R = I_1 (50 + 500)$

$$\text{or, } I_1 \times R = 0.005 \times 550$$

$$\text{or, } 0.0118 \times R = 2.75$$

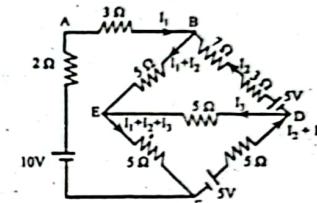
$$\therefore R = 233.0508 \Omega$$

8. Find the currents I_1 , I_2 , I_3 using Kirchhoff's law and also find the power output of each voltage source of figure below. [2069 Chaitra]

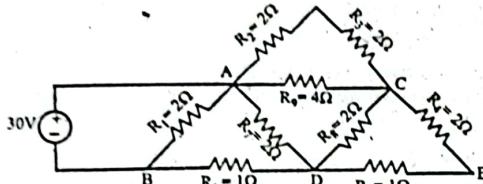


Solution:

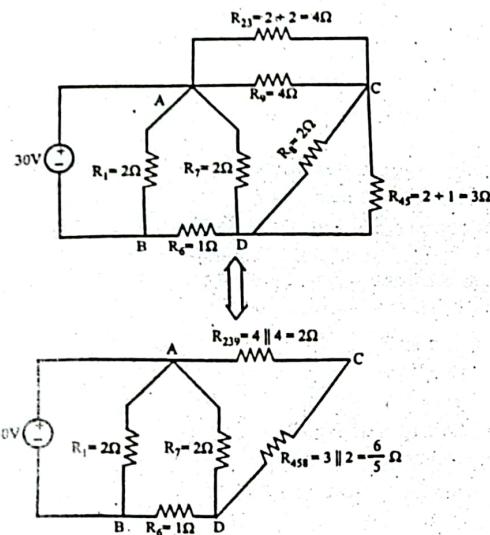
Redrawing the given circuit,



9. Using series-parallel combination of resistances find the current delivered by the source in the following circuit. [2069 Ashad]



Solution:

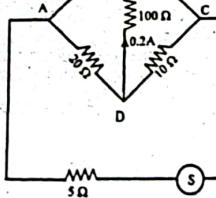


∴ Equivalent resistance of combination is,

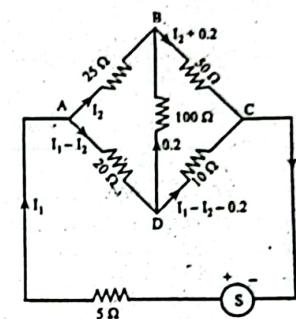
$$R_{eq} = \left[\left(2 + \frac{6}{5} \right) \| 2 + 1 \right] \| 2 = \left[\left(\frac{16}{5} \| 2 \right) + 1 \right] \| 2 = \frac{29}{13} \| 2 = \frac{58}{55} \Omega$$

$$\text{Hence, current delivered by the source} = \frac{30}{58} = 28.45 \text{ A}$$

10. Using Kirchhoff's laws determine the magnitude of source current and polarity of the source S, If the current flowing through branch BD is 0.2A from D to B in the circuit shown below. [2069 Ashad]



Solution:



Let the polarity of S be as shown. Then,

Applying KVL to mesh ABDA, we get

$$-25I_2 + 0.2 \times 100 + 20(I_1 - I_2) = 0$$

$$\text{or, } -25I_2 + 20 + 20I_1 - 20I_2 = 0$$

Applying KVL to mesh BCDB, we get

$$-50(I_2 + 0.2) + 10(I_1 - I_2 - 0.2) - 0.2 \times 100 = 0$$

$$\text{or, } -50I_2 - 10 + 10I_1 - 10I_2 - 2 - 20 = 0$$

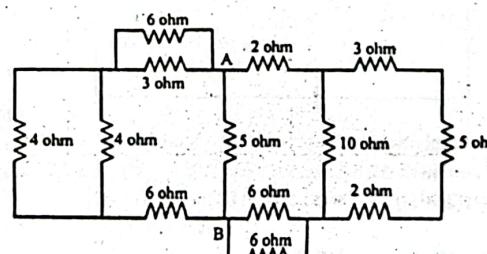
solving equations (i) and (ii), we get

$$I_1 = -3.52 \text{ A}, I_2 = -1.12 \text{ A}$$

The magnitude of current I_1 indicates that polarity of source S is reverse of the shown one.

11. Find the equivalent resistance across the terminals A and B, R_{AB}

[2070 Ashad]



Current reading in the ammeter,

$$I_2 = \frac{1}{11 + (15 + 13)} \times 11$$

[∴ Using current divider rule]

$$\text{or, } I_2 = \frac{2.5}{39} \times 11$$

$$\therefore I_2 = 0.7051 \text{ A}$$

Now,

Emf of the battery,
 $E = I \times (\text{equivalent resistance of given circuit})$

$$\text{or, } E = 2.5 \times [(15 + 13)/11] + (8 + 11)$$

$$\text{or, } E = 2.5 \times [(28/11) + (8 + 11)]$$

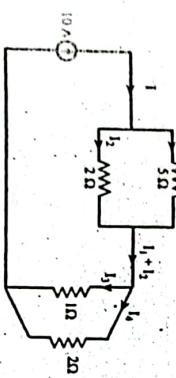
$$\text{or, } E = 2.5 \times \left[\frac{28 \times 11}{28 + 11} + 19 \right]$$

$$\text{or, } E = 2.5 \times 26.897$$

$$\therefore E = 67.2425 \text{ Volt}$$

Additional Problems

1. Find I_1, I_2, I_3 and I_4 in given figure:



Solution:

Using the current division method,

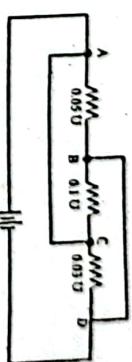
$$I_1 = \frac{2}{5+2} \times 1 = \frac{2}{7} \times 1 = 2.857 \text{ A}$$

$$I_2 = \frac{5}{5+2} \times 1 = \frac{5}{7} \times 1 = 7.143 \text{ A}$$

$$I_3 = \frac{2}{1+2} \times 1 = \frac{2}{3} \times 1 = 6.67 \text{ A}$$

$$I_4 = \frac{1}{1+2} \times 1 = \frac{1}{3} \times 10 = 3.33 \text{ A}$$

2. For the circuit shown below, calculate the power consumed by the 0.1Ω resistor.



Solution:
 The given circuit can be redrawn as,

Equivalent conductance,

$$G = G_1 + G_2 + G_3 = 0.05 + 0.1 + 0.03 = 0.18 \text{ U (or mho or siemens)}$$

Since, $I = GV$

$$\text{So, } I = 0.18 \times 24$$

$$\therefore I = 4.32 \text{ A}$$

Now, current flowing through 0.1Ω

$$I_2 = \frac{G_2}{G_1 + G_2 + G_3} \times I = \frac{0.1}{0.05 + 0.1 + 0.03} \times 4.32 = 2.4 \text{ A}$$

∴ Power consumed by the 0.1Ω resistor
 $= V I_2 = 24 \times 2.4 = 57.6 \text{ watt}$

3. Find V such that 100 C charge is injected to the 50V source in 1 min in the circuit.

Solution:

In order to inject the 100 C charge to the 50V source, the current in the loop must be anti-clockwise. 50V

$$\text{But, } I = \frac{Q}{t} = \frac{100}{60} = 1.67 \text{ A} \quad [\because Q = 100 \text{ C}; t = 1 \text{ min} = 60 \text{ sec}]$$

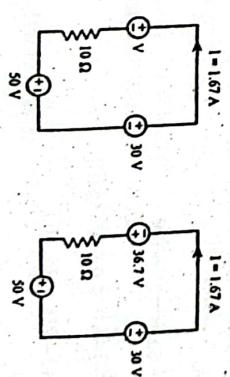
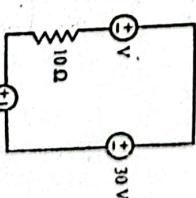
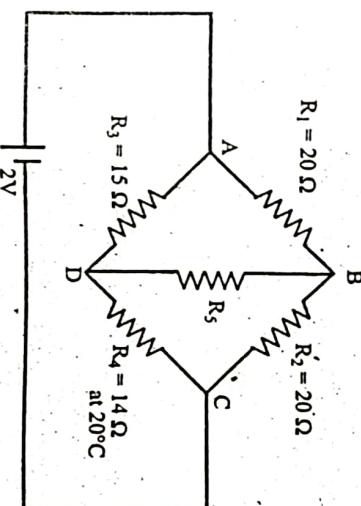


Fig. (b)

Applying KVL in loop [Fig. (a)]
 $V - 30 + 50 + 10I = 0$

12. In the circuit shown in the figure, resistors R_1 , R_2 , R_3 have negligible temperature coefficient of resistance. If resistor R_4 has a temperature coefficient of resistance of 0.004 per $^{\circ}\text{C}$, and a resistance of 14 Ohms at 20°C , what rise in temperature is required to reduce the current in resistor R_4 to zero?



Solution:

For no current in R_3 , the wheatstone bridge is balanced.

$$\therefore \frac{20}{15} = \frac{20}{R_4}$$

$$\therefore R_4 = 15\Omega$$

Let, t be temperature at which 14Ω resistance becomes 15Ω

$$\therefore \frac{R_4}{R_{20}} = \frac{15}{14}$$

$$= \frac{R_0[1 + \alpha_0 t]}{R_0[1 + \alpha_0 \times 20]}$$

$$= \frac{1 + t \times 0.004}{1 + 20 \times 0.004}$$

$$\text{or, } \frac{15}{14} = \frac{1 + 0.04t}{1.08}$$

$$\text{or, } 1 + 0.004t = 1.08 \times \frac{15}{14}$$

$$\therefore t = \frac{0.15714}{0.004} \\ = 39.29^{\circ}\text{C}$$



NETWORK THEOREMS

3.1 NETWORK TERMINOLOGY

While discussing network theorems and techniques, one often comes across the following terms.

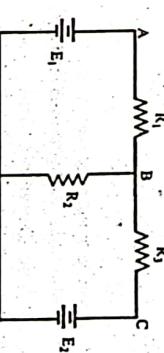


Fig. 3.1

i) Active element

An active element is one which supplies electrical energy to the circuit. Thus in figure, E_1 and E_2 are the active elements because they supply energy to the circuit.

ii) Passive element

A passive element is one which receives electrical energy and then either converts it into heat (resistance) or stores in an electric field (capacitance) or magnetic field (inductance). Thus in figure, there are three passive elements namely R_1 , R_2 and R_3 .

iii) Node

A node of a network is an equipotential surface at which two or more circuit elements are joined. Here A, B, C and D are nodes.

iv) Junction

A junction is that point in a network where three or more circuit elements are joined. In figure, there are only two junction points, B and D.

v) Branch

A branch is that part of a network which lies between two junction points. Referring to figure, there are a total of three branches; BAD, BCD and BD.

vi) Loop

A loop is any closed path of a network. Thus in figure ABDA, BCDB and ABCDA are the loops.

vii) Mesh

A mesh is the most elementary form of a loop and cannot be further divided into other loops. In figure, both loops ABDA and BCDB are meshes because they cannot be further divided into other loops.

viii) **Unilateral circuit**

A unilateral circuit is one whose characteristics are not same in either direction.

For example: diode, transistor.

ix) **Bilateral circuit**

A bilateral circuit is one whose characteristics are same in either direction. For example: resistor, transmission line.

3.2.1 Nodal analysis

This method of circuit analysis is based on seeking the solution of a network in terms of voltage of nodes measured with respect to reference node. A reference node is usually the point in network called ground. Nodal analysis is the application of KCL.

Steps for nodal analysis

Find the possible number of junction.

- Select one node as the ground reference. The choice doesn't affect the result and is just a matter of convention. Choosing the node with most connections can simplify the analysis.

- Assign a variable for each node whose voltage is unknown. If the voltage is already known, it is not necessary to assign a variable.

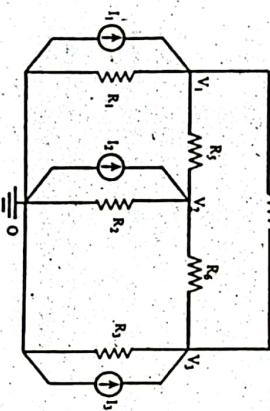
- For each unknown voltage, form an equation based on Kirchhoff's current law.

- If there are voltage sources between two unknown junctions, join the two nodes as supernode. The currents of the two nodes are combined in a single equation and a new equation for the voltage is formed.

- Solve the system of simultaneous equations for each unknown voltage.

On the basis of source present, the circuit may be classified as follows and can be solved accordingly by Nodal analysis.

D) Circuit containing only current source

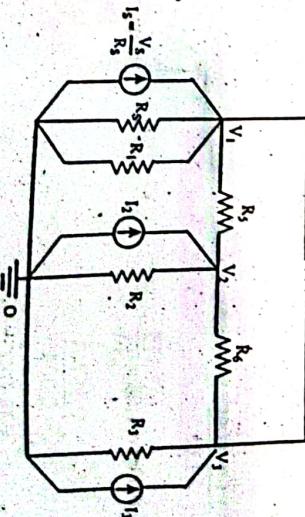


Applying KCL at node 1,

$$\frac{V_1 - 0}{R_1} + \frac{V_1 - V_2}{R_3} + \frac{V_1 - V_3}{R_4} = I_1$$

or,

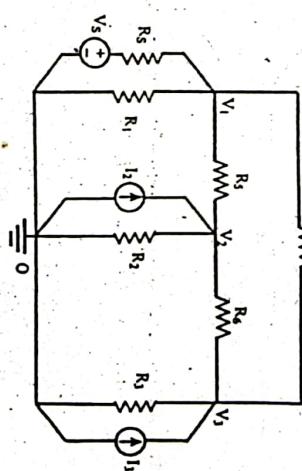
$$\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right)V_1 + \left(-\frac{1}{R_3}\right)V_2 + \left(\frac{1}{R_4}\right)V_3 = I_1 \dots\dots\dots (i)$$



Applying KCL at node 1,

$$\frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_3} + \frac{V_1 - 0}{R_4} = 0 \dots\dots\dots (i)$$

As voltage source is transformable into current source,



Applying KCL at node 2,

$$\frac{V_2 - 0}{R_1} + \frac{V_2 - V_1}{R_3} + \frac{V_2 - V_3}{R_4} = I_2$$

or,

$$\left(-\frac{1}{R_3}\right)V_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)V_2 + \left(-\frac{1}{R_4}\right)V_3 = I_2 \dots\dots\dots (ii)$$

or,

$$G_{21}V_1 + G_{22}V_2 + G_{23}V_3 = I_2 \dots\dots\dots (iiia)$$

Applying KCL at node 3,

$$\frac{V_3 - 0}{R_4} + \frac{V_3 - V_2}{R_6} + \frac{V_3 - V_1}{R_5} = I_3 \dots\dots\dots (iii)$$

or,

$$\left(-\frac{1}{R_5}\right)V_1 + \left(-\frac{1}{R_6}\right)V_2 + \left(\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}\right)V_3 = I_3 \dots\dots\dots (iii)$$

or,

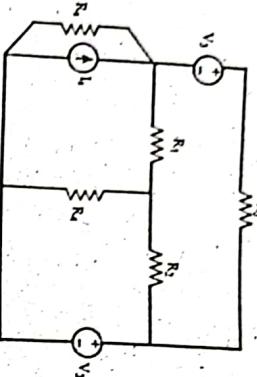
$$G_{31}V_1 + G_{32}V_2 + G_{33}V_3 = I_3 \dots\dots\dots (iiia)$$

Solving equations (i), (ii) and (iii), we can find node voltages V_1 , V_2 and V_3 .

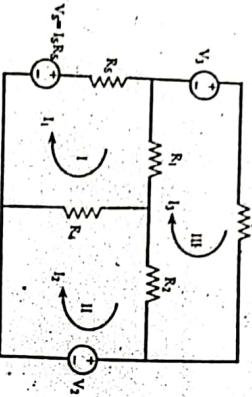
II) Circuit containing voltage source in addition to current source.

a) Voltage source transformable into current source:

- i) Current source present in the common branch of any two loops.



Transforming the current source into voltage source.



Applying KVL to the loop I, we get

$$I_s R_s - I_1 R_1 - R_1 (I_1 - I_3) - R_4 (I_1 - I_2) = 0$$

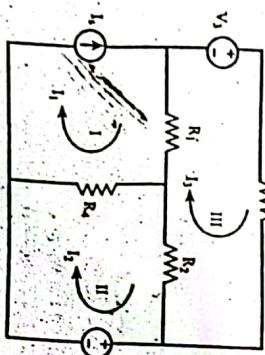
$$\text{or, } (R_s + R_1 + R_4) I_1 + (-R_1) I_2 + (-R_4) I_3 = I_s R_s \dots \dots \dots \text{(i)}$$

$$R_{1r} I_1 + R_{2r} I_2 + R_{3r} I_3 = I_s R_s \dots \dots \dots \text{(ia)}$$

Equations (ii) and (iii) are same as above

b) Current source not transformable into voltage source

- i) Current source present in the perimeter of any individual loop.



As we are interested to know the loop current and we can see that

current source is present in the perimeter of loop I,

So, directly we write

$$I_1 = I_s \dots \dots \dots \text{(i)}$$

Equations (ii) and (iii) are same as above.

If the current source occurs as a common element between the two meshes they one of the way is to form a supermesh from two meshes that contain current source in their common branch and then write KVL equation for supermesh.

$$\text{Here, } I_3 - I_1 = I_s \dots \dots \dots \text{(i)}$$

Applying KVL for supermesh I and III, we get

$$V_3 - I_3 R_3 - R_2 (I_3 - I_2) - R_4 (I_1 - I_2) + V_1 = 0$$

$$\text{or, } I_3 R_3 + R_2 (I_3 - I_2) + R_4 (I_1 - I_2) = V_1 + V_3$$

Equation (ii) is same as above.

3.3 Star-delta and delta-star transformation

In many circuit applications, we encounter components connected together in one of two ways to form a three-terminal network; the 'Delta' or Δ (also known as the pi or π) configuration and the 'Star' or Y (also known as the T) configuration:

3.3.1 Delta – Star transformation

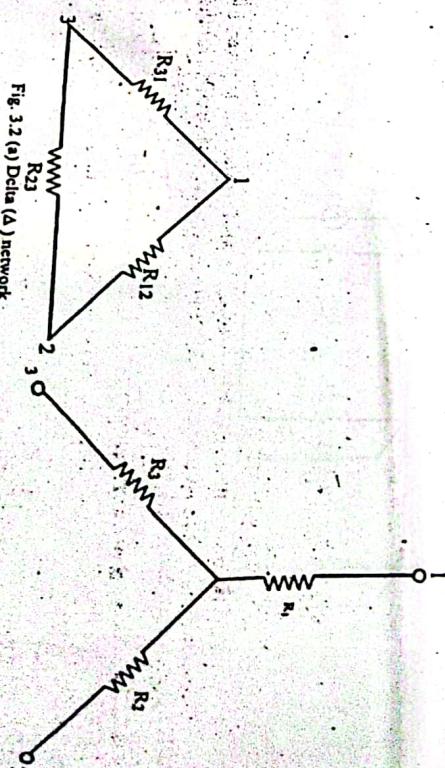


Fig. 3.2 (a) Delta (Δ) network

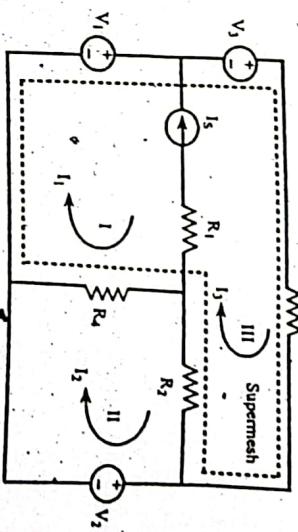


Fig. 3.2 (b) Star (Y) network

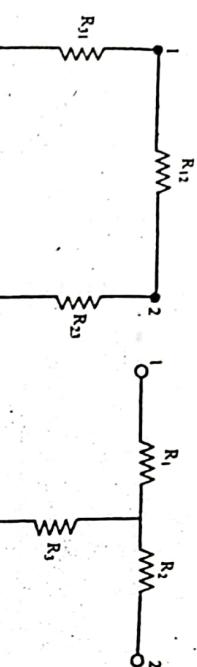


Fig. 3.3 (a) Pi (N) network

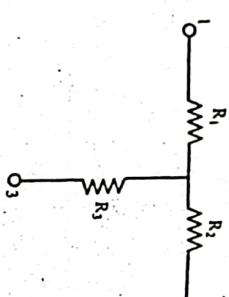


Fig. 3.3 (b) Tee (T) network

Consider the two circuits shown in the figure 3.2 (a) and figure 3.2 (b). They will be equivalent if the resistance measured between any two terminals 1, 2 and 3 is the same in the two cases.

$$[R_{12}]_Y = [R_{12}]_L \quad \text{(i)}$$

$$\text{or, } R_1 + R_2 = \frac{R_{12}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{21}} \quad \text{(ii)}$$

$$\text{Similarly, } R_2 + R_3 = \frac{R_{12}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \quad \text{(iii)}$$

$$\text{and, } R_3 + R_1 = \frac{R_{12}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{(iv)}$$

Adding equations (i), (iii) and (iv), we get

$$R_1 + R_2 + R_3 + R_1 = \frac{R_{12}(R_{21} + R_{31}) + R_{21}(R_{31} + R_{12}) + R_{31}(R_{12} + R_{21})}{R_{12} + R_{23} + R_{31}}$$

$$\text{or, } 2(R_1 + R_2 + R_3) = \frac{2(R_{12}R_{21} + R_{21}R_{31} + R_{31}R_{12})}{R_{12} + R_{23} + R_{31}}$$

$$\text{or, } R_1 + R_2 + R_3 = \frac{R_{12}R_{21} + R_{21}R_{31} + R_{31}R_{12}}{R_{12} + R_{23} + R_{31}} \quad \text{(v)}$$

Subtracting equations (ii), (iii), (iv) from (v), we get

$$R_1 + R_2 + R_3 - (R_1 + R_2) = \frac{R_{12}R_{21} + R_{21}R_{31} + R_{31}R_{12} - R_{12}R_{21} - R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\therefore R_3 = \frac{R_{21}R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{(vi)}$$

$$\text{Similarly, } R_2 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{(vii)}$$

$$R_1 = \frac{R_{12}R_{21}}{R_{12} + R_{23} + R_{31}} \quad \text{(viii)}$$

From above it may be noted that resistance of each arm of the star is given by the product of the resistance of the two delta sides that meet at its end divided by the sum of the three delta resistances.

3.3.2 Star - delta transformation

Now, Multiplying equations (viii) and (vii), (vi) and (viii) and then adding them, we get,

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12}R_{21}R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{(ix)}$$

$$\begin{aligned} \text{Dividing equation (ix) by equation (vii), we get} \\ R_{12} &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = R_1 + R_2 + \frac{R_1 R_2}{R_3} \quad \text{(x)} \\ \text{Similarly, } R_{23} &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = R_2 + R_3 + \frac{R_2 R_3}{R_1} \quad \text{(xi)} \\ R_{31} &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = R_1 + R_3 + \frac{R_3 R_1}{R_2} \quad \text{(xii)} \end{aligned}$$

Hence, the equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistance.

3.4 Superposition theorem

If a number of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of the currents that would be produced in it when each source acts alone replacing all other independent sources by their internal resistances.

Steps for solving a network using the principle of superposition

- Take only one independent source of voltage/current and deactivate the other independent voltage/ current sources (for voltage sources, remove the source and short circuit the respective circuit terminals and for current sources, just delete the source keeping the respective circuit terminals open). Obtain branch currents.
- Repeat the above step for each of the independent sources.

The total current in any branch of the circuit is the algebraic sum of currents due to each source. When finding total current in any branch, it is necessary to take into account the directions of the currents caused by each individual source, currents flowing in the same direction being additive, currents flowing in opposite direction being subtractive.

Explanation



Fig. 3.4 (a)

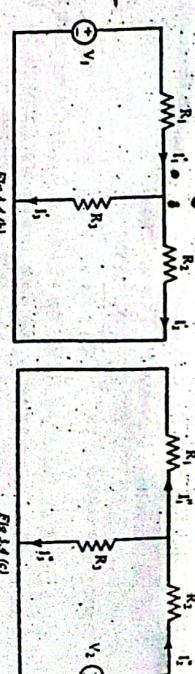


Fig. 3.4 (b)

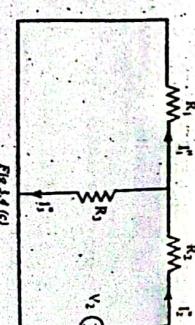


Fig. 3.4 (c)

In figure 3.4 (a), to apply Superposition theorem, let us first take the source V_1 alone at first replacing V_2 by short circuit [Fig. 3.4 (b)]

$$\text{Here, } I_1' = \frac{V_1}{R_2 R_3 + R_1}$$

$$I_2' = \frac{R_1}{R_1 + R_3} + R_2$$

$$I_3' = I_1 R_2 + R_3$$

Next, replacing V_1 by short circuit, let the circuit be energized by V_2 only [Fig. 3.4c]

$$\text{Here, } I_2' = \frac{V_2}{R_1 R_3 + R_2}$$

$$I_1' = I_2' \frac{R_3}{R_1 + R_3} \text{ and } I_3' = I_1' - I_2'$$

As per Superposition theorem,

$$I_3 = I_1 + I_2$$

$$I_2 = I_2' - I_1'$$

$$I_1 = I_1' - I_2'$$

It may be noted that during application of Superposition, the direction of currents calculated for each source should be taken care of.

3.5 Thevenin's theorem

A linear two terminal network can be replaced by an equivalent circuit composed of a voltage source V_{th} in series with a resistance R_{th} . The voltage V_{th} is the voltage across the open circuited terminals and R_{th} is the equivalent resistance of the network as seen from the terminals with all independent source suppressed.

To understand the application of this theorem, consider the circuit shown in figure 3.5 (a). It is desired to find the current through the resistance R_L . The steps are:

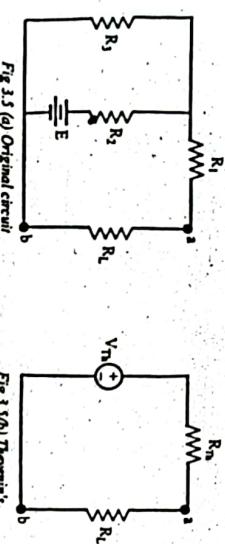


Fig. 3.5(a) Original circuit

3.6 Norton's theorem

Any two terminals of a network containing linear passive and active elements may be replaced by an equivalent current source I_N in parallel with a resistance R_N , where I_N is the current flowing through a short circuit placed across the terminals and R_N is the equivalent resistance of the network as seen from the two terminals with all independent sources suppressed.

To understand the application of this theorem consider the circuit shown in figure 3.6 (a). It is desired to find the current through the resistance R_L . The steps are:

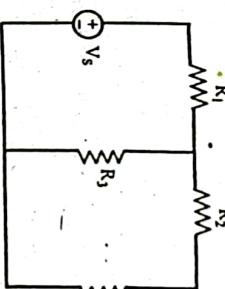


Fig. 3.6 (a) Original circuit

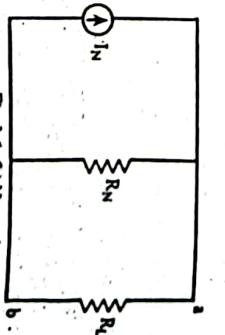


Fig. 3.6 (b) Norton's equivalent circuit

- i) Short circuit the pair of terminals at which Norton's equivalent is to be determined.

- ii) Find the current flowing through the short circuited path. This is I_N .

- iii) Redraw the circuit with all the voltage sources short circuited and all the current sources open circuited. Determine the resistance of the network as seen from the terminals ab. This is R_N (R_N is same as R_{th}).

- iv) The Norton's equivalent circuit for terminals ab is shown in figure 3.6 (b). The current through resistance R_L is;

$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

3.7 Maximum Power transfer theorem

For loads connected directly to a dc voltage supply, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source i.e., $R_L = R_s$.

Consider the circuit of figure 3.7 where a battery having source resistance R_s is supplying by a load resistance R_L . The power delivered to the load is

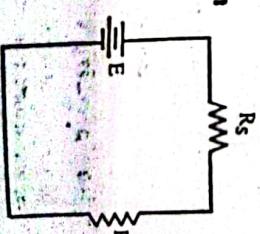


Fig. 3.7

- Remove the resistance R_L thus creating an open circuit at terminals ab.
- Find out the voltage between ab. This voltage is V_{th} .
- In the given circuit, $V_{th} = E - \frac{ER_s}{R_s + R_L}$
- Short circuit the battery and find the resistance R_N of the network as seen from terminals ab. Actually, voltage source are removed by internal resistance and current source if present is open circuited.

$$P = I^2 R_L = \frac{E^2 R_L}{(R_L + R_s)^2} \quad \dots \dots \dots (1)$$

When R_L is zero, P is zero. When R_L is infinite, P is again zero. For some intermediate value of R_L , the power delivered is maximum. This value of R_L can be found by putting $\frac{dP}{dR_L}$ equal to zero.

$$\frac{dP}{dR_L} = 0$$

$$\text{or, } \frac{d}{dR_L} \left[\frac{E^2 R_L}{(R_L + R_S)^2} \right] = 0$$

$$\text{or, } (R_L + R_S)^2 E^2 - E^2 R_L \times 2(R_L + R_S) = 0$$

$$\text{or, } (R_S + R_L)^2 - 2R_L (R_S + R_L) = 0$$

$$\text{or, } E^2 (R_S + R_L) (R_S + R_L - 2R_L) = 0$$

$$\text{or, } E^2 (R_S + R_L) (R_S - R_L) = 0$$

$$\text{or, } E^2 (R_S^2 - R_L^2) = 0$$

$$\text{Since, } E \neq 0$$

$$R_S = R_L$$

Thus, the power delivered to the load is maximum when the load resistance equals the source resistance.

Now,

If $R_L = R_S$ then from equation (i)

Power delivered to load;

$$P = I^2 R_L = \frac{E^2 R_L}{4R_L} = \frac{E^2}{4}$$

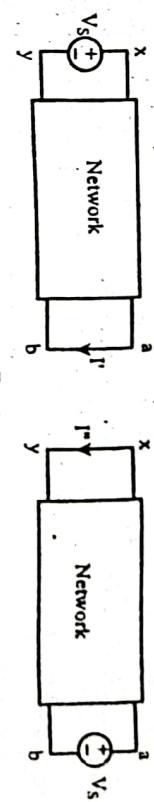


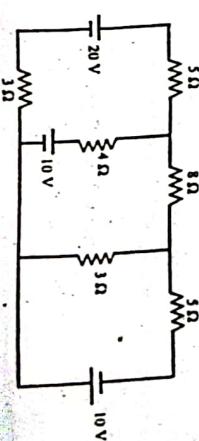
Fig. 3.8

It may be observed that the currents obtained in step-2 and step-4 are identical to each other for the validation of reciprocity theorem.

Exam Solutions

(Nodal and Mesh Analysis)

1. Use nodal analysis method to find the current through 8Ω resistor in the circuit shown below. [2004 Shrawan]



Solution:

3.8 Reciprocity theorem

In a linear bilateral network having one independent source and no dependent sources, an important relation exists between a source voltage in one branch and current in some other branch.

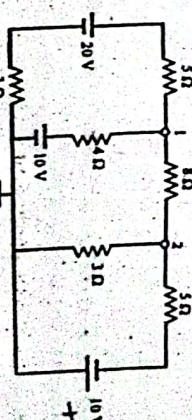
The reciprocity theorem states that, "If a source of emf E , located at one point in a network composed of linear bilateral circuit elements, produces a current I at a point in the network, the same source of emf E acting at the second point will produce current at the first point."

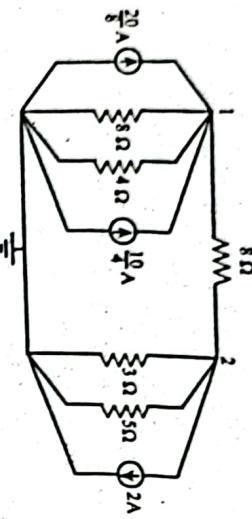
Reciprocity theorem is also applicable to network containing a single current source. In this case the theorem states that, "If a current source I_1 located at one point in the network, the same source of current I acting at second point will produce the same voltage V at the first point."

Steps for Solving a network using Reciprocity theorem

- The branches between which reciprocity is to be established are to be selected first.
- The current in the branch is obtained using conventional network analysis.
- The voltage source is interchanged between the branches concerned.
- The current in the branch where the voltage source was existing earlier is calculated.

Transforming voltage source into current source,





Let O be the reference node and the voltage of nodes 1 and 2 be V_1 and V_2 respectively.

Applying KCL at node 1;

$$\frac{20}{8} + \frac{10}{4} = \frac{V_1 - 0}{8} + \frac{V_1 - V_2}{4} + \frac{V_1 - V_2}{8}$$

$$\text{or, } 5 = \frac{V_1}{8} + \frac{V_1}{4} + \frac{V_1 - V_2}{8}$$

$$\text{or, } 5 = \frac{1}{2}V_1 - \frac{1}{8}V_2 \quad \dots\dots\dots (i)$$

Applying KCL at node 2;

$$0 = \frac{V_2 - V_1}{8} + \frac{V_2 - 0}{3} + \frac{V_2 - 0}{5} + 2$$

$$\text{or, } 0 = -\frac{V_1}{8} + \frac{V_2}{8} + \frac{V_2}{3} + \frac{V_2}{5} + 2$$

$$\text{or, } -2 = -\frac{V_1}{8} + \frac{79}{120}V_2 \quad \dots\dots\dots (ii)$$

Solving equations (i) & (ii) we get

$$V_1 = 9.7010 \text{ volt}$$

$$V_2 = -1.1960 \text{ volt}$$

The negative voltage at node 2 indicates that node 2 is at lower potential with respect to reference node.

Hence,

Current flowing through 8Ω resistor

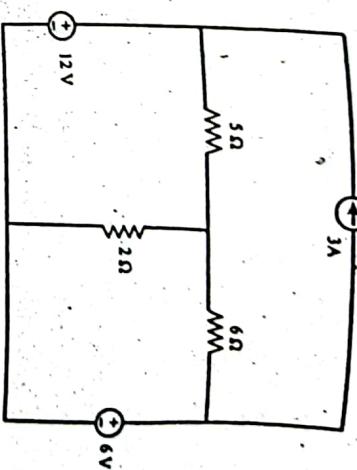
$$\begin{aligned} &= \frac{V_1 - V_2}{8} \\ &= \frac{9.7010 - (-1.1960)}{8} \\ &= 1.3621 \text{ A.} \end{aligned}$$

2. Use mesh current analysis method to calculate node voltages and current through resistors in the following network.

[2003 Kartik]

The negative sign indicates that the current flows in the direction opposite our assumptions.

Solution:



The three loop currents are shown in figure. For these loops, applying Kirchhoff's voltage law,

Here, $I_3 = -3A$.

Loop I:

$$12 - 5(I_1 - I_3) - 2(I_1 - I_2) = 0$$

$$\text{or, } 12 - 5I_1 + 5I_3 - 2I_1 + 2I_2 = 0$$

$$\text{or, } 12 - 5I_1 - 15 - 2I_1 + 2I_2 = 0 [\because I_3 = -3A]$$

$$\text{or, } -7I_1 + 2I_2 = 3 \quad \dots\dots\dots (i)$$

Loop II:

$$-6 - 2(I_2 - I_1) - 6(I_2 - I_3) = 0$$

$$\text{or, } -6 - 2I_2 + 2I_1 - 6I_2 + 6I_3 = 0$$

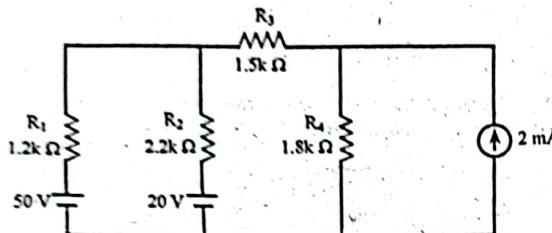
$$\text{or, } 2I_1 - 8I_2 - 18 = 6 [\because I_3 = -3A]$$

$$\text{or, } 2I_1 - 8I_2 = 24 \quad \dots\dots\dots (ii)$$

Solving (i) & (ii), we get

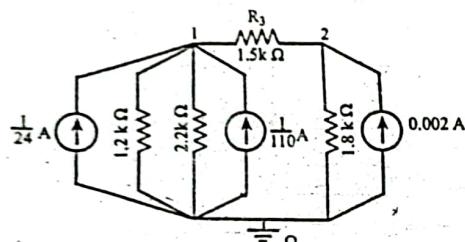
$$\begin{aligned} I_1 &= -1.3846A \\ I_2 &= -3.3462A \end{aligned}$$

5. Using nodal analysis determine the current that flows through resistor R_3 .
[2066 Magh]



Solution:

Converting possible voltage source into current source.



Let 0 be the reference node and the voltage of nodes 1 and 2 be V_1 and V_2 respectively.

Applying KCL at node 1, we get

$$\frac{1}{24} + \frac{1}{110} = \frac{V_1 - 0}{1.2 \times 1000} + \frac{V_1 - 0}{2.2 \times 1000} + \frac{V_1 - V_2}{1.5 \times 1000}$$

$$\text{or, } 0.0508 = \frac{43}{22000} V_1 - \frac{1}{1500} V_2 \quad \dots \text{(i)}$$

Applying KCL at node 2, we get

$$0.002 = \frac{V_2 - 0}{1.8 \times 1000} + \frac{V_2 - V_1}{1.5 \times 1000}$$

$$\text{or, } 0.002 = -\frac{1}{1500} V_1 + \frac{11}{9000} V_2 \quad \dots \text{(ii)}$$

Solving equations (i) and (ii), we get

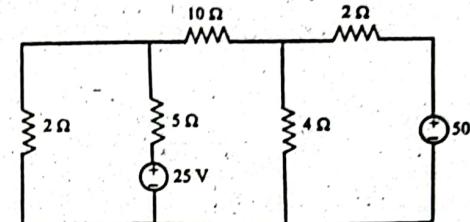
$$V_1 = 32.6171 \text{ V}$$

$$V_2 = 19.4275 \text{ V}$$

∴ Current flowing through R_3 resistor

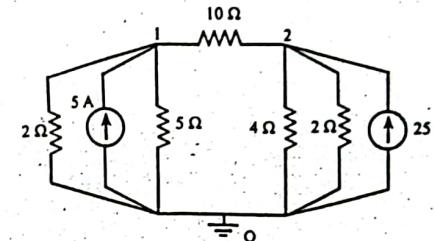
$$= \frac{V_1 - V_2}{R_3} = \frac{32.6171 - 19.4275}{1.5 \times 1000} = 8.79 \times 10^{-3} \text{ A}$$

6. Use nodal method to find the current through 10Ω resistor for circuit shown below.
[2071 Shravan, 2068 Balshakh, 2067 Ashad]



Solution:

Converting the possible voltage source into current source.



Let 0 be the reference node and the voltage of node 1 and 2 be V_1 and V_2 respectively.

Applying KCL at node 1, we get

$$5 = \frac{V_1 - 0}{2} + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{10}$$

$$\text{or, } 5 = \frac{4}{5} V_1 - \frac{1}{10} V_2 \quad \dots \text{(i)}$$

Applying KCL at node 2, we get

$$25 = \frac{V_2 - 0}{4} + \frac{V_2 - 0}{2} + \frac{V_2 - V_1}{10}$$

$$\text{or, } 25 = -\frac{1}{10} V_1 + \frac{17}{20} V_2 \quad \dots \text{(ii)}$$

Solving equations (i) and (ii), we get

$$V_1 = 10.075 \text{ V}$$

$$V_2 = 30.597 \text{ V}$$

$$\therefore \text{Current through } 10\Omega \text{ resistor} = \frac{V_1 - V_2}{10}$$

$$= \frac{30.597 - 10.075}{10}$$

$$= 2.052 \text{ A}$$

Now,

Current flowing through $R_1 = 2\Omega$ resistor = $10 - 2.2737 = 7.7263$ A

Current flowing through $R_2 = 4\Omega$ resistor = $2.2737 - 1.1921 = 1.0816$ A

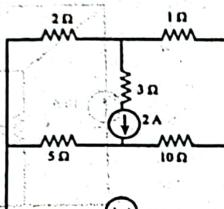
Current flowing through $R_3 = 3\Omega$ resistor = $2.2737 + 1.4349 = 3.7086$ A

Current flowing through $R_4 = 5\Omega$ resistor = $1.1921 + 1.4349 = 2.6270$ A

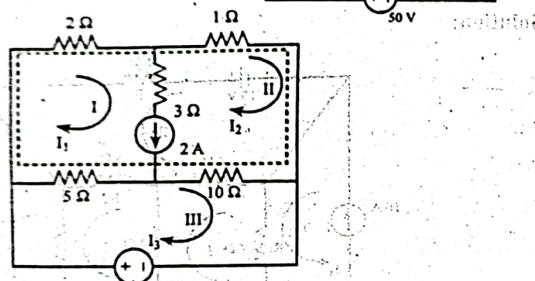
Current flowing through $R_5 = 4\Omega$ resistor = $7.5 - 1.4349 = 6.0651$ A

Current flowing through $R_6 = 1\Omega$ resistor = 1.1921 A

11. Determine the current in the 5Ω resistor in the network shown below, using loop formulation method. [2066 Kartik]



Solution:



Consider loop currents be as shown in figure.

Here,

Mesh I and mesh II are super mesh.

so,

$$I_1 - I_2 = 2 \quad \text{(i)}$$

Applying KVL on super mesh, we get

$$-2I_1 - I_2 - 10(I_2 - I_3) - 5(I_1 - I_3) = 0$$

$$\text{or, } -2I_1 - I_2 - 10I_2 + 10I_3 - 5I_1 + 5I_3 = 0$$

$$\text{or, } -7I_1 - 11I_2 + 15I_3 = 0 \quad \text{(ii)}$$

Applying KVL on mesh III, we get

$$50 - 5(I_3 - I_1) - 10(I_3 - I_2) = 0$$

$$\text{or, } 50 - 5I_3 + 5I_1 - 10I_3 + 10I_2 = 0$$

$$\text{or, } 5I_1 + 10I_2 - 15I_3 = -50 \quad \text{(iii)}$$

Solving equations (i), (ii) and (iii) we get

$$I_1 = 17.3333 \text{ A}, I_2 = 15.3333 \text{ A}, I_3 = 19.3333 \text{ A}$$

So,

The current flowing in 5Ω resistor

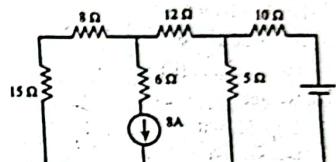
$$= I_3 - I_1$$

$$= 19.3333 - 17.3333$$

$$= 2 \text{ A}$$

12. Calculate the current through 15Ω resistor in figure given below:

[2066 magh]



Solución:

Consider loop currents be as shown in figure. Here mesh I and mesh II are supermeshes.

$$\text{Here, } I_1 - I_2 = 8 \quad \text{(i)}$$

Applying KVL on supermesh, we get

$$-15I_1 - 8I_1 - 12I_2 - 5(I_2 - I_3) = 0$$

$$\text{or, } -23I_1 - 12I_2 - 5I_3 = 0$$

$$\text{or, } -23I_1 - 17I_2 + 5I_3 = 0 \quad \text{(ii)}$$

Applying KVL in mesh III we get

$$-10I_3 - 48 - 2I_3 - 5(I_3 - I_2) = 0$$

$$\text{or, } -10I_3 - 48 - 2I_3 - 5I_3 + 5I_2 = 0$$

$$\text{or, } 5I_2 - 17I_3 = 48 \quad \text{(iii)}$$

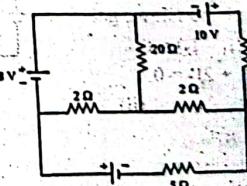
Solving (i), (ii) and (iii) we get

$$I_1 = 2.858 \text{ A}, I_2 = -5.142 \text{ A}, I_3 = -4.3359 \text{ A}$$

Current flowing through 15Ω resistor = 2.858 A

13. Determine current in 5Ω resistor by mesh analysis in figure below.

[2068 Chaitra]



Solution:

Consider loop currents be as shown in figure.

Applying KVL on mesh I, we get

$$8 - 20(I_1 - I_2) - 2(I_1 - I_3) = 0$$

$$\text{or, } 8 - 20I_1 + 20I_2 - 2I_1 + 2I_3 = 0$$

$$\text{or, } -22I_1 + 20I_2 + 2I_3 = -8 \quad \dots \dots \dots (i)$$

Applying KVL on mesh II, we get

$$10 - 3I_2 - 2(I_2 - I_3) - 20(I_2 - I_1) = 0$$

$$\text{or, } 10 - 3I_2 - 2I_2 + 2I_3 - 20I_2 + 20I_1 = 0$$

$$\text{or, } 20I_1 - 25I_2 + 2I_3 = -10 \quad \dots \dots \dots (ii)$$

Applying KVL on mesh III, we get

$$12 - 2(I_1 - I_3) - 2(I_3 - I_2) - 5I_3 = 0$$

$$\text{or, } 12 - 2I_1 + 2I_3 - 2I_3 + 2I_2 - 5I_3 = 0$$

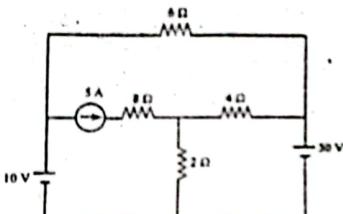
$$\text{or, } 2I_1 + 2I_2 - 9I_3 = -12 \quad \dots \dots \dots (iii)$$

Solving equations (i), (ii) and (iii) we get

$$I_1 = 4.679 \text{ A}, I_2 = 4.411 \text{ A}, I_3 = 3.353 \text{ A}$$

Hence, Current in 5Ω resistor is 3.353 A

14. Find all branch current using mesh analysis method in the following circuit. [2070 Bharda]

**Solution:**

The three mesh currents are shown in figure. Here mesh I and mesh III are supermesh

$$\text{So, } I_1 - I_3 = 5 \text{ A} \quad \dots \dots \dots (i)$$

Applying KVL on supermesh I and III, we get

$$10 - 6I_1 - 4(I_1 - I_2) - 2(I_1 - I_3) = 0$$

$$\text{or, } 10 - 6I_1 - 4I_3 + 4I_2 - 2I_1 + 2I_2 = 0$$

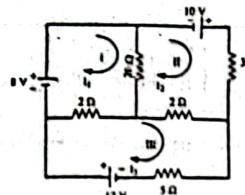
$$\text{or, } -2I_1 + 6I_2 - 10I_3 = -10 \quad \dots \dots \dots (ii)$$

Applying KVL on mesh II, we get

$$-30 - 2(I_2 - I_1) - 4(I_2 - I_3) = 0$$

$$-30 - 2I_2 + 2I_1 - 4I_2 + 4I_3 = 0$$

$$I_1 + 4I_3 = 30 \quad \dots \dots \dots (iii)$$



Solving equations (i), (ii) and (iii), we get

$$I_1 = \frac{5}{3} \text{ A} = 1.667 \text{ A}$$

$$I_2 = -\frac{20}{3} \text{ A} = -6.667 \text{ A}$$

$$I_3 = -\frac{10}{3} \text{ A} = -3.333 \text{ A}$$

Negative current indicates that our direction of assumption of current flow is opposite to its actual flow.

Now, Current in a branch containing 6Ω resistor = 3.333 A

Current in a branch containing 8Ω resistor = 5 A

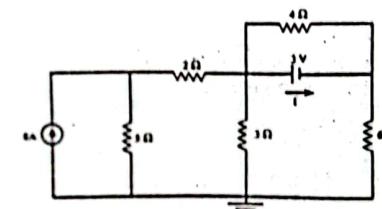
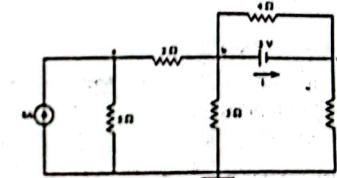
Current in a branch containing 4Ω resistor = $6.667 - 3.333 = 3.334$ A

Current in a branch containing 2Ω resistor = $1.667 + 6.667 = 8.334$ A

Current in a branch containing 10 V battery = 1.667 A

Current in a branch containing 30 V battery = 6.667 A

15. Find the value of 'I' through the voltage source using Nodal analysis. [2071 Magh]

**Solution:**

Let 'd' be the reference node and the voltage of node a, b and c be V_a , V_b and V_c respectively.

Here, node b and node c are super node.

$$\text{So, } V_b - V_c = 3$$

Also,

$$0 = \frac{V_b - V_a}{2} + \frac{V_b - 0}{3} + \frac{V_c - 0}{6}$$

$$0 = \frac{V_b}{2} - \frac{V_a}{2} + \frac{V_b}{3} + \frac{V_c}{6}$$

$$\text{or, } \frac{1}{2}V_a - \frac{5}{6}V_b - \frac{1}{6}V_c = 0 \quad \dots \dots \dots \text{(ii)}$$

Applying KCL at node a, we get

$$8 = \frac{V_a - V_b}{2} + \frac{V_a - V_c}{5}$$

$$\text{or, } 8 = \frac{V_a}{2} - \frac{V_b}{2} + \frac{V_a}{5}$$

$$\text{or, } 8 = \frac{7}{10}V_a - \frac{1}{2}V_b \quad \dots \dots \dots \text{(iii)}$$

Solving equations (i), (ii) and (iii), we get

$$V_a = \frac{55}{3}V$$

$$V_b = \frac{29}{3}V$$

$$V_c = \frac{20}{3}V$$

We are now clear that with reference to node d, node a is at higher potential compared to node b and node c. Also, node b is at higher potential compared to node c.

Now,

applying KCL at node b, in the above circuit.

(Incoming current)_{nodeb} = (Outgoing current)_{nodeb}

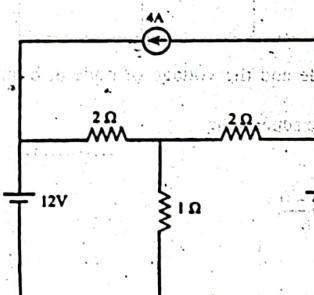
$$\frac{V_a - V_b}{2} = \frac{V_b - V_c}{4} + \frac{V_b - 0}{3} + I$$

$$\text{or, } \frac{55 - 29}{3 \times 2} = \frac{29 - 20}{3 \times 4} + \frac{29}{3 \times 3} + I$$

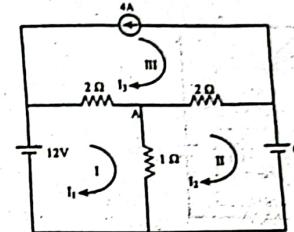
$$\text{or, } \frac{13}{3} = \frac{3}{4} + \frac{29}{9} + I$$

$$\therefore I = \frac{13}{36} = 0.3611 A$$

16. Calculate the current flowing through 1 ohm resistor for network shown below using loop current method. [2071 chaitra]



Solution:



The three loop currents are shown in figure. For these loops, we apply Kirchhoff's Voltage Law.

Here, $I_3 = -4A$

Loop I :

$$12 - 2(I_1 - I_3) - 1(I_1 - I_2) = 0$$

$$\text{or, } 12 - 2I_1 + 2I_3 - I_1 + I_2 = 0$$

$$\text{or, } -3I_1 + I_2 + 2I_3 = -12$$

$$\text{or, } -3I_1 + I_2 + 2(-4) = -12$$

$$\text{or, } -3I_1 + I_2 = -4$$

.....(i)

Loop II :

$$-6 - 1(I_2 - I_1) - 2(I_2 - I_3) = 0$$

$$\text{or, } -6 - I_2 + I_1 - 2I_2 + 2I_3 = 0$$

$$\text{or, } I_1 - 3I_2 + 2I_3 = 6$$

$$\text{or, } I_1 - 3I_2 + 2(-4) = 6$$

$$\text{or, } I_1 - 3I_2 = 14$$

.....(ii)

Solving equations (i) and (ii), we get

$$I_1 = -0.25A,$$

$I_2 = -4.75A$, Negative sign indicates that the current flows in the direction opposite to our assumptions.

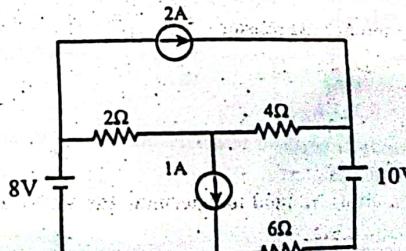
Now, the current flowing through

$$1\Omega \text{ resistor} = I_2 - I_1$$

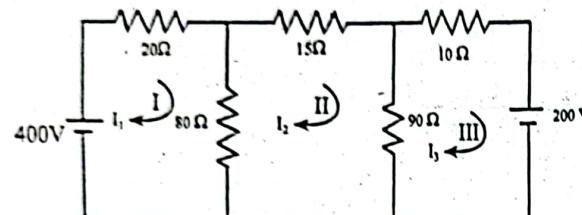
$$= 4.75 - 0.25$$

$$= 4.5A \quad (\text{top to bottom}) (A \text{ to } B)$$

17. Obtain the currents flowing through the 2 ohm resistor of the network shown below using mesh analysis. [2072 Ahswin]



Solution:



Consider loop currents be as shown in figure.

Applying KVL on mesh I, we get

$$400 - 20I_1 - 80(I_1 - I_2) = 0$$

$$\text{or, } -100I_1 + 80I_2 = -400 \quad \dots\dots\dots (i)$$

Applying KVL on mesh II, we get

$$-15I_2 - 90(I_2 - I_3) - 80(I_2 - I_1) = 0$$

$$\text{or, } -15I_2 - 90I_2 + 90I_3 - 80I_2 + 80I_1 = 0$$

$$\text{or, } 80I_1 - 185I_2 + 90I_3 = 0 \quad \dots\dots\dots (ii)$$

Applying KVL on mesh III, we get

$$-10I_3 - 200 - 90(I_3 - I_2) = 0$$

$$\text{or, } 90I_2 - 100I_3 = 200 \quad \dots\dots\dots (iii)$$

Solving equations (i), (ii) and (iii), we get

$$I_1 = 6.8A, I_2 = 3.5A, I_3 = 1.15A$$

\therefore The power dissipation in 15Ω resistor $= I_2^2 \times 15$

$$= 3.5^2 \times 15$$

$$= 183.75W$$

Additional questions (Nodal and mesh analysis)

1. In the circuit configuration of figure shown below, determine the number of

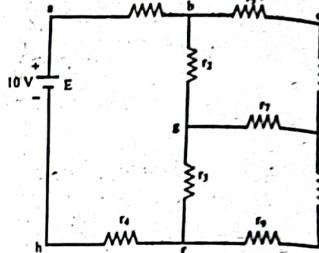
i. circuit elements

ii. nodes

iii. Junction points

iv. branches

v. meshes



Solution:

i. No. of circuit elements = 10 [9 resistors + 1 voltage source]

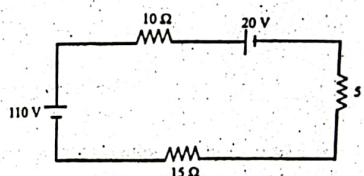
ii. No. of nodes = 8 [a, b, c, d, e, f, g, h]

iii. No. of junction points = 4 [b, d, g, f]

iv. No. of branches = 6 [r2, r3, r7, (r1 + r4), (r9 + r8), (r5 + r6)]

v. No. of meshes = 3 [abghfa, bcdgb, defgd]

2. Find the current that flow and p.d. across each resistor in the circuit given below



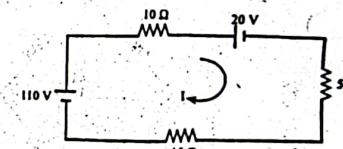
Solution:

Applying KVL to mesh, we get

$$110 - 10I - 20 - 5I - 15I = 0$$

$$\text{or, } -30I + 90 = 0$$

$$\therefore I = 3A$$



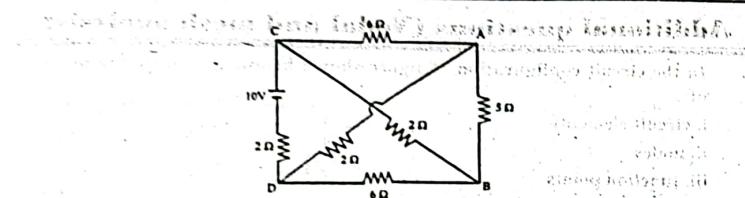
Current through each resistor, $I = 3A$

p.d. across 10Ω resistor $= 10 \times 3 = 30V$

p.d. across 5Ω resistor $= 5 \times 3 = 15V$

p.d. across 15Ω resistor $= 15 \times 3 = 45V$

3. Calculate the voltage across AB in the circuit shown below and indicate the polarity of voltage.



Solution:

The circuit can be redrawn as;

Applying KVL to mesh I, we get

$$10 - 2(I_1 - I_2) - 6(I_1 - I_3) - 2I_1 = 0$$

$$\text{or, } 10 - 2I_1 + 2I_2 - 6I_1 + 6I_3 - 2I_1 = 0$$

$$\text{or, } -5I_1 + I_2 + 3I_3 = -5 \quad \dots \dots \dots (i)$$

Applying KVL to mesh II, we get

$$-6I_2 - 5(I_2 - I_3) - 2(I_2 - I_1) = 0$$

$$\text{or, } -6I_2 - 5I_2 + 5I_3 + 2I_1 = 0$$

$$\text{or, } 2I_1 - 13I_2 + 5I_3 = 0 \quad \dots \dots \dots (ii)$$

Applying KVL to mesh III, we get,

$$-2I_3 - 6(I_3 - I_1) - 5(I_3 - I_2) = 0$$

$$\text{or, } -2I_3 - 6I_3 + 6I_1 - 5I_3 + 5I_2 = 0$$

$$\text{or, } 6I_1 + 5I_2 - 13I_3 = 0 \quad \dots \dots \dots (iii)$$

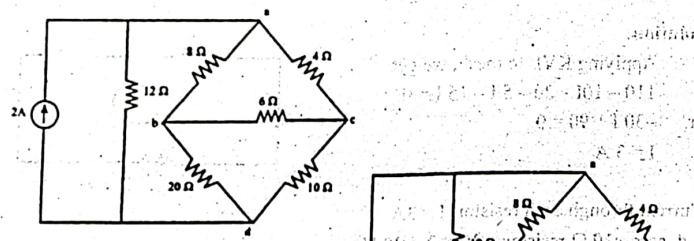
Solving equations (i), (ii) and (iii), we get

$$I_1 = 1.8 \text{ A}, I_2 = 0.7 \text{ A}, I_3 = 1.1 \text{ A}$$

$$\therefore \text{Current in branch AB, } (I_3 - I_2) = 1.1 - 0.7 = 0.4 \text{ A (B to A)}$$

$$\therefore \text{Voltage across AB, } V_{BA} = V_B - V_A = 0.4 \times 5 = 2 \text{ V}$$

4. Use nodal analysis method to find the voltage drop across 10Ω resistor of the circuit shown below:



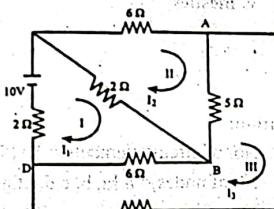
Solution:

The circuit can be redrawn as,

Let d be the reference node

and the voltage of node a, b, c

and d be V_a, V_b, V_c and V_d respectively, then $V_d = 0$



Applying KCL at node a, we get

$$\frac{V_a - V_b}{8} + \frac{V_a - V_c}{4} + \frac{V_a - V_d}{12} = 2$$

$$\text{or, } \left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4} \right) V_a - \frac{V_b}{8} - \frac{V_c}{4} = 2$$

$$\text{or, } \frac{11}{24} V_a - \frac{V_b}{8} - \frac{V_c}{4} = 2 \quad \dots \dots \dots (i)$$

Applying KCL at node b, we get

$$\frac{V_b - V_a}{8} + \frac{V_b - V_c}{6} + \frac{V_b - V_d}{20} = 0$$

$$\text{or, } -\frac{1}{8} V_a + \left(\frac{1}{8} + \frac{1}{6} + \frac{1}{20} \right) V_b - \frac{V_c}{6} = 0$$

$$\text{or, } -\frac{1}{8} V_a + \frac{41}{120} V_b - \frac{V_c}{6} = 0 \quad \dots \dots \dots (ii)$$

Applying KCL at node c, we get

$$\frac{V_c - V_a}{4} + \frac{V_c - V_b}{6} + \frac{V_c - V_d}{10} = 0$$

$$\text{or, } -\frac{1}{4} V_a - \frac{1}{6} V_b + \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{10} \right) V_c = 0$$

$$\text{or, } -\frac{1}{4} V_a - \frac{1}{6} V_b + \frac{31}{60} V_c = 0 \quad \dots \dots \dots (iii)$$

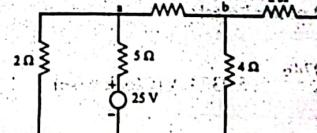
Solving equations (i), (ii) and (iii) we get

$$V_a = 10.5 \text{ V}, V_b = 7.5 \text{ V}, V_c = 7.5 \text{ V}$$

Hence, Voltage drop across 10Ω resistor = $V_c = 7.5 \text{ V}$.

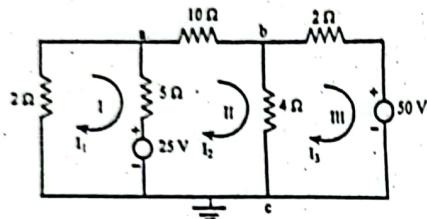
5. Use the node voltage method to solve the mesh currents in the network

shown in below:



Solution:

Let, c be the reference node and the voltage of node a and b be V_a and V_b respectively.



Applying KCL at node a;

$$\frac{V_a - V_c}{2} + \frac{V_a - 25}{5} + \frac{V_a - V_b}{10} = 0$$

$$\text{or, } \frac{V_a - 0}{2} + \frac{V_a - 25}{5} + \frac{V_a - V_b}{10} = 0$$

$$\text{or, } \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right) V_a - \frac{1}{10} V_b = \frac{25}{5}$$

$$\text{or, } \frac{4}{5} V_a - \frac{1}{10} V_b = 5 \quad \dots \dots \dots \text{(i)}$$

Applying KCL at node b;

$$\frac{V_b - V_c}{4} + \frac{V_b - V_a}{10} + \frac{V_b - 50}{2} = 0$$

$$\text{or, } -\frac{1}{10} V_a + \left(\frac{1}{4} + \frac{1}{10} + \frac{1}{2}\right) V_b - \frac{1}{4} V_c = \frac{50}{2}$$

$$\text{or, } -\frac{1}{10} V_a + \frac{17}{20} V_b = 25 \quad \dots \dots \dots \text{(ii)}$$

Solving (i) and (ii), we get

$$V_a = 10.0746 \text{ V}, V_b = 30.5970 \text{ V}$$

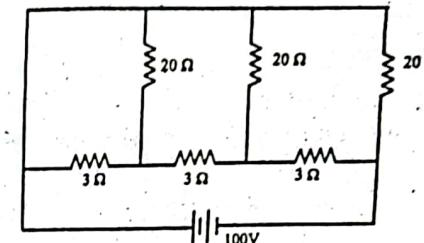
Mesh current I_1 can be determined by finding the current in resistor 2Ω of first mesh. Also mesh current I_2 can be determined by finding the current in resistor 10Ω of second mesh and finally mesh current I_3 can be determined by finding the current in resistor 2Ω of third mesh.

$$I_1 = I_{2\Omega} = \frac{V_a - 0}{2} = \frac{10.0746}{2} = 5.0373 \text{ A (Clock wise)}$$

$$I_2 = I_{10\Omega} = \frac{V_b - V_a}{10} = \frac{30.5970 - 10.0746}{10} = 2.0522 \text{ A (clockwise)}$$

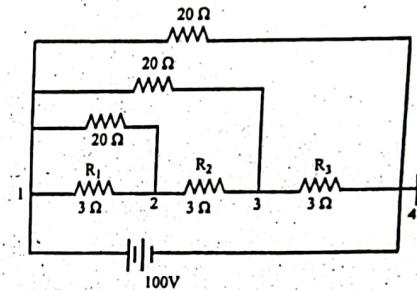
$$I_3 = I_{2\Omega} = \frac{V_b - 50 - 0}{2} = \frac{30.5970 - 50}{2} = -9.7015 \text{ A (counter clockwise)}$$

6. Determine the voltage across the 3Ω resistors in the circuit shown.



Solution:

Redrawing the figure and solving by using nodal analysis.



Consider node 4 as the reference node and the voltage of nodes 1, 2 and 3 be V_1 , V_2 and V_3 respectively then $V_1 = 100 \text{ V}$

Applying KCL at node 2;

$$\frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{20} + \frac{V_2 - 0}{3} = 0$$

$$\text{or, } \left(\frac{1}{3} + \frac{1}{20} + \frac{1}{3}\right) V_2 - \frac{1}{3} V_3 = \frac{100}{3} + \frac{100}{20} \quad [\because V_1 = 100 \text{ V}]$$

$$\text{or, } \frac{43}{60} V_2 - \frac{1}{3} V_3 = \frac{115}{3} \quad \dots \dots \dots \text{(i)}$$

Applying KCL at node 3;

$$\frac{V_3 - V_2}{20} + \frac{V_3 - V_1}{3} + \frac{V_3 - 0}{3} = 0$$

$$\text{or, } -\frac{1}{3} V_2 + \left(\frac{1}{20} + \frac{1}{3} + \frac{1}{3}\right) V_3 = \frac{100}{20} \quad [\because V_1 = 100 \text{ V}]$$

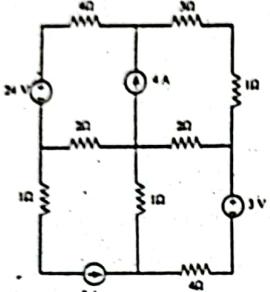
$$\text{or, } -\frac{1}{3} V_2 + \frac{43}{60} V_3 = 5 \quad \dots \dots \dots \text{(ii)}$$

Solving equations (i) and (ii), we get

$$V_2 = 72.3948 \text{ V}$$

$$V_3 = 40.6487 \text{ V}$$

9. Determine the mesh currents in the given circuit using mesh analysis.



Solution:

Consider loop currents be as shown in figure $I_3 = 2A$

Here, Mesh I and mesh II are supermesh

so,

$$I_1 - I_2 = 4A \quad \dots \dots \dots (i)$$

Applying KVL on super mesh

$$-24 - 2(I_1 - I_3) - 2(I_2 - I_4) - 4I_2 - 4I_1 = 0$$

$$\text{or, } -24 - 2I_1 + 2I_3 - 2I_2 + 2I_4 - 4I_2 - 4I_1 = 0$$

$$\text{or, } -6I_1 - 6I_2 + 2I_4 = 20 \quad \dots \dots \dots (ii) \quad [\because I_3 = 2A]$$

Using equation (i), equation (ii) can be written as,

$$\text{or, } -6(4 + I_2) - 6I_2 + 2I_4 = 20$$

$$\text{or, } -24 - 6I_2 - 6I_2 + 2I_4 = 20$$

$$\text{or, } -6I_2 + I_4 = 22 \quad \dots \dots \dots (iii)$$

Applying KVL on mesh IV, we get

$$-4I_4 + 3 - 2(I_4 - I_2) - 1(I_4 - I_3) = 0$$

$$\text{or, } -4I_4 + 3 - 2I_4 + 2I_2 - I_4 + I_3 = 0$$

$$\text{or, } 2I_2 - 7I_4 = -5 \quad \dots \dots \dots (iv) \quad [\because I_3 = 2A]$$

Solving equation (iii) and equation (iv) we get

$$I_2 = -3.725 A, I_4 = -0.35 A$$

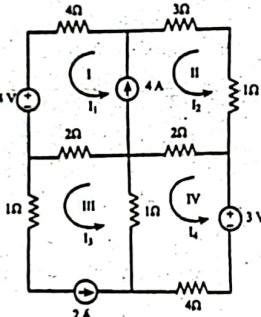
$$\therefore I_1 = 4 + I_2 = 4 + (-3.725) = 0.275 A$$

The negative current indicates that the direction of current flow is opposite to our assumed direction.



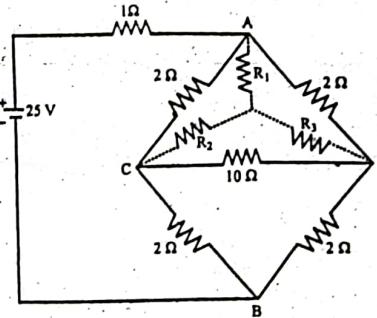
Star – delta and delta – star transformation Exam solutions

1. Using Delta - star transformation determine the current drawn from the supply for the network given below. [2064 Poush]



Solution:

The network given in figure can be redrawn as,



The three delta connected resistances between nodes A, C and D can be converted into equivalent star configuration. The values are,

$$R_1 = \frac{2 \times 2}{2 + 10 + 2} = \frac{2}{7} \Omega$$

$$R_2 = \frac{10 \times 2}{2 + 10 + 2} = \frac{10}{7} \Omega$$

$$R_3 = \frac{2 \times 10}{2 + 10 + 2} = \frac{10}{7} \Omega$$

After this transformation, the circuit is shown in figure below.

$$\text{Total resistance} = 1 + \frac{2}{7} + \left(\frac{10}{7} + 2 \right) \parallel \left(\frac{10}{7} + 2 \right)$$

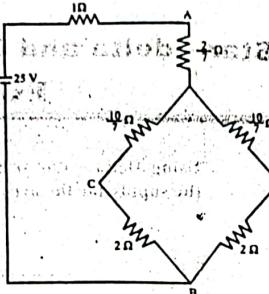
$$= 1 + \frac{2}{7} + \left(\frac{24}{7} \parallel \frac{24}{7} \right)$$

$$\text{short circuit resistance} = 1 + \frac{2}{7} + \frac{12}{7}$$

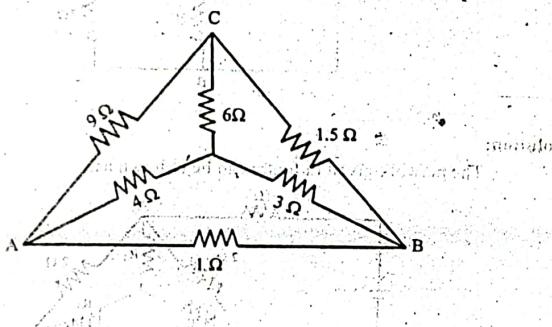
$$\text{parallel total} = 3 \Omega$$

$$\therefore \text{Current drawn from the supply} = \frac{25}{3}$$

$$= 8.333 \text{ A.}$$



2. In the network shown below, using star/delta transformation, calculate the network resistance between A and B. [2066 Kartik]



Solution:

Here, the three star connected resistances between nodes A, B and C can be converted into equivalent delta configuration. The values are,

$$R_{AB} = \frac{4 \times 3 + 3 \times 6 + 6 \times 4}{6} = 9 \Omega$$

$$R_{BC} = \frac{4 \times 3 + 3 \times 6 + 6 \times 4}{4} = 13.5 \Omega$$

$$R_{CA} = \frac{4 \times 3 + 3 \times 6 + 6 \times 4}{3} = 18 \Omega$$

After this transformation, the circuit is shown in the figure below.

Now,

Equivalent resistance between A and B is,

$$R_{AB} = [(9 \parallel 18) + (1.5 \parallel 13.5)] \parallel [9 \parallel 1]$$

$$= \left[\frac{9 \times 18}{9 + 18} + \frac{1.5 \times 13.5}{1.5 + 13.5} \right] \parallel \left[\frac{9 \times 1}{9 + 1} \right]$$

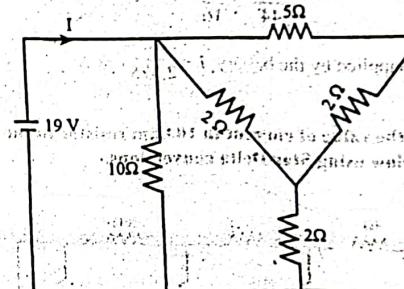
$$= \frac{147}{20} \parallel \frac{9}{10}$$

$$= \frac{147}{20} \times \frac{9}{10}$$

$$= \frac{147}{20} + \frac{9}{10}$$

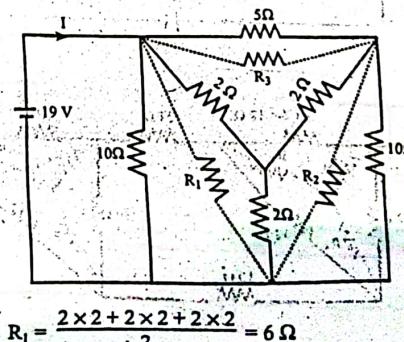
$$= 0.8018 \Omega$$

3. Find the current I as shown in figure below using star - delta transformation. [2068 Chaitra]



Solution:

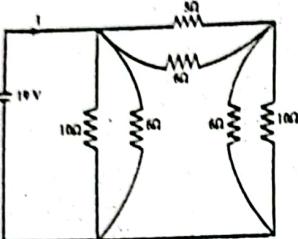
Three resistances 2Ω , 2Ω and 2Ω are star connected. Transforming them into delta with ends at the same points.



$$R_1 = \frac{2 \times 2 + 2 \times 2 + 2 \times 2}{2} = 6 \Omega$$

Similarly, $R_2 = 6 \Omega$, $R_3 = 6 \Omega$

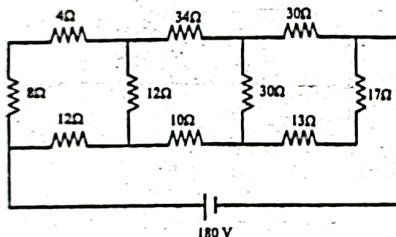
After this transformation, the circuit is shown in the figure below.



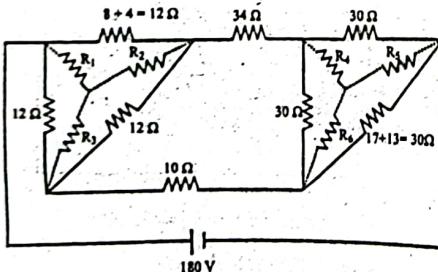
$$\begin{aligned}\text{Total resistance} &= [(10 \parallel 6) + (5 \parallel 6)] \parallel (10 \parallel 16) \\&= \left[\frac{10 \times 6}{10+6} + \frac{5 \times 6}{5+6} \right] \parallel \left[\frac{10 \times 6}{10+6} \right] \\&= \frac{285}{44} \parallel \frac{60}{16} = \frac{\frac{285}{44} \times \frac{60}{16}}{\frac{285}{44} + \frac{60}{16}} = 2.375 \Omega\end{aligned}$$

$$\therefore \text{Current supplied by the battery, } I = \frac{19}{2.375} = 8 \text{ A}$$

4. Determine the value of current in 10 ohm resistor in the network shown in figure below using Star/Delta conversions. [2069 Bhadra]



Solution:



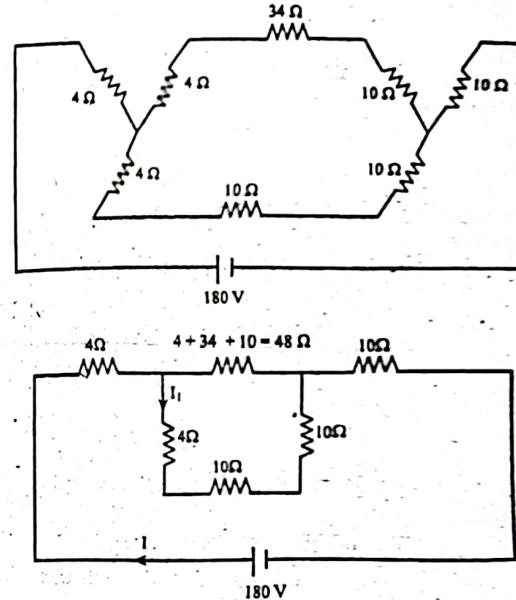
Three resistances at the left in the circuit (12Ω , 12Ω , 12Ω) and three resistances at the right in the circuit (30Ω , 30Ω , 30Ω) are delta connected. So, transforming them into star.

We get,

$$R_1 = R_2 = R_3 = \frac{12 \times 12}{12 + 12 + 12} = 4 \Omega$$

$$R_4 = R_5 = R_6 = \frac{30 \times 30}{30 + 30 + 30} = 10 \Omega$$

After this transformation, the circuit is shown in the figure below:



Equivalent resistance of the circuit,

$$\begin{aligned}
 \text{Req.} &= 4 + [48 \parallel (4 + 10 + 10)] + 10 \\
 &= 4 + [48 \parallel 24] + 10 \\
 &= 4 + \frac{48 \times 24}{48 + 24} + 10 \\
 &= 30 \Omega \\
 \therefore \text{Current, } I &= \frac{180}{30} = 6 \text{ A}
 \end{aligned}$$

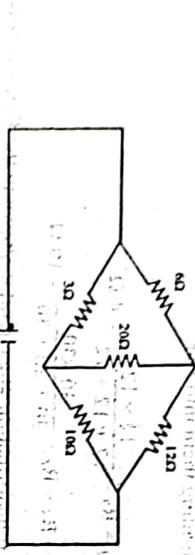
Now,

Current through $24\ \Omega$ ($4 + 10 + 10 = 24\ \Omega$) branch.

$$I_1 = \frac{1}{24+48} \times 48 = \frac{6}{72} \times 48 = 4 \text{ A}$$

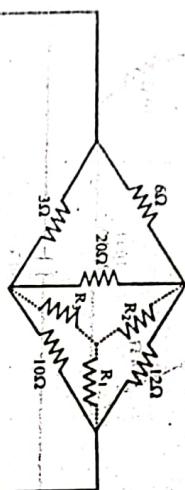
Hence, the current through $10\ \Omega$ resistor (which is a part of series branch of 24 ohm) is also 4A .

5. Determine the current in 20Ω resistor of the network shown in figure below using Star - Delta transformation. [2009 IIT-JEE Q. 10] [2010 IIT-JEE Q. 10] [2010 CBSE JEE Main]



Solution:

Resistances 12Ω , 10Ω and 20Ω are delta connected. Transforming them into star with ends at the same points.



After this transformation, the circuit is shown in figure below.

$$R_1 = \frac{12 \times 10}{12 + 10 + 20} = 2.857 \Omega$$

$$R_2 = \frac{20 \times 12}{12 + 10 + 20} = 5.714 \Omega$$

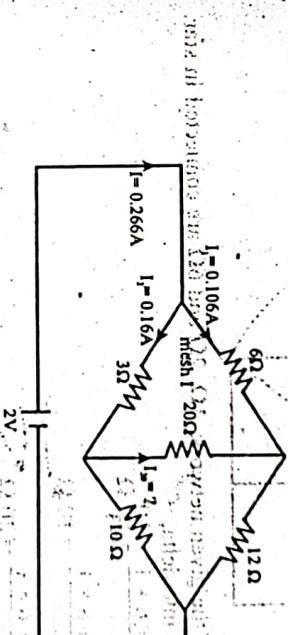
$$R_3 = \frac{20 \times 10}{12 + 10 + 20} = 4.762 \Omega$$

Using current division rule,

$$\begin{aligned} I_1 &= \frac{1}{(6 + 5.714) + (3 + 4.762) \times (3 + 4.762)} \\ &= \frac{0.266}{19.476} \times 7.762 \\ &= 0.106 \text{ A} \end{aligned}$$

(using KCL)

Now, to find the current in 20Ω resistor,



Applying KVL in mesh I considering branch current method, we get

$$-6I_1 + 20I_{20} + 3I_2 = 0$$

Or,

$$-6 \times 0.106 + 20 \times I_{20} + 3 \times 0.106 = 0$$

Or,

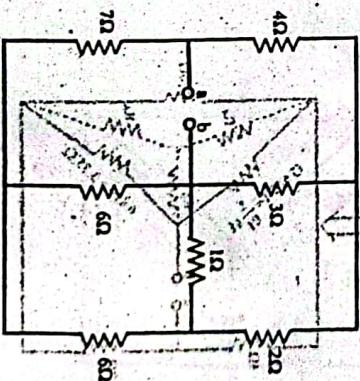
$$I_{20} = \frac{0.156}{20} = 0.0078 \text{ A}$$

$$I = 0.266 \text{ A}$$

$$I_1 = 0.16 \text{ A}$$

$$I_{20} = 0.0078 \text{ A}$$

6. Using star - delta transformation, find the equivalent resistance between terminals 'a' and 'b'.

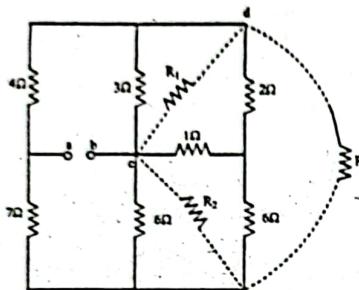


Here,

$$\text{Current, } I = [(6 + 5.714) \parallel (3 + 4.762)] + 2.857$$

$$\begin{aligned} &= \frac{2}{[11.714 \parallel 7.762] + 2.857} \cdot \frac{1}{\frac{1}{11.714} + \frac{1}{7.762} + \frac{1}{2.857}} \\ &= \frac{2}{4.669 + 2.857} = 0.266 \text{ A} \end{aligned}$$

Solution:



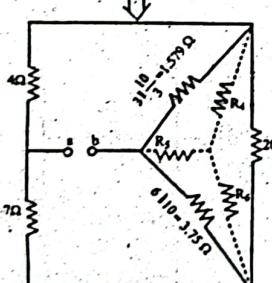
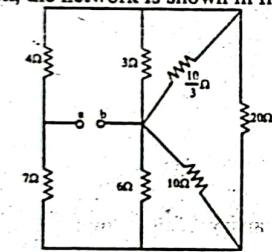
In the right side of the given network, 1Ω , 2Ω and 6Ω are connected in star. Transforming them into delta,

$$R_1 = \frac{1 \times 2 + 2 \times 6 + 6 \times 1}{6} = \frac{10}{3} \Omega$$

$$R_2 = \frac{1 \times 2 + 2 \times 6 + 6 \times 1}{2} = 10 \Omega$$

$$R_3 = \frac{1 \times 2 + 2 \times 6 + 6 \times 1}{1} = 20 \Omega$$

After this transformation, the network is shown in figure below,



Considering delta to star transformation;

We get

$$R_4 = \frac{1.579 \times 20}{1.579 + 20 + 3.75}$$

$$= 1.247 \Omega$$

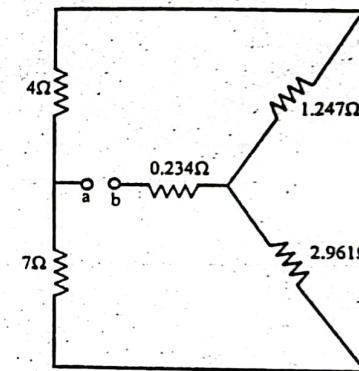
$$R_5 = \frac{3.75 \times 1.579}{1.579 + 20 + 3.75}$$

$$= 0.234 \Omega$$

$$R_6 = \frac{20 \times 3.75}{1.579 + 20 + 3.75}$$

$$= 2.961 \Omega$$

After transformation, the circuit is shown in figure below,



Hence,

the equivalent resistance between terminals 'a' and 'b',

$$R_{ab} = [(4 + 1.247) // (7 + 2.961)] + 0.234$$

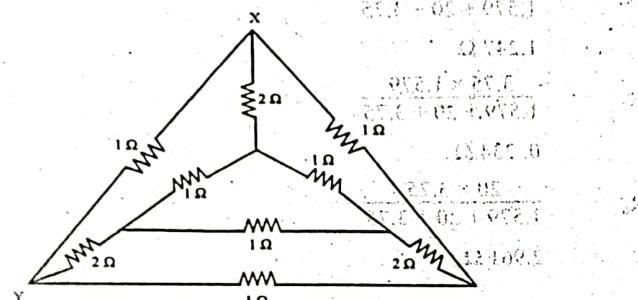
$$= [5.247 // 9.96] + 0.23$$

$$= 3.437 + 0.234$$

$$= 3.671 \Omega$$

Additional Questions

1. Determine the resistance between the point X and Y for the network given below.



Solution:

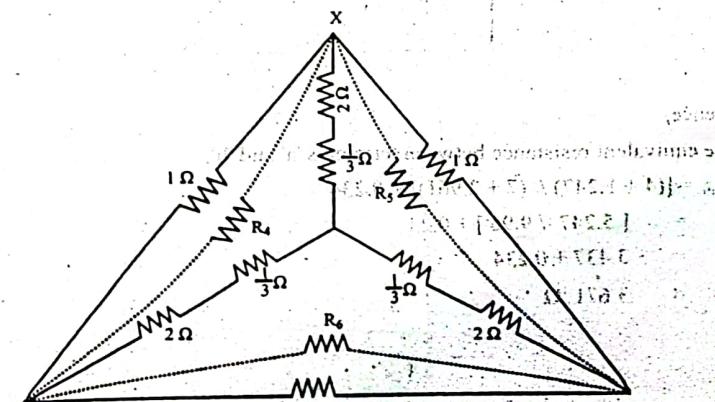
Inner three resistances 1Ω , 1Ω and 1Ω are delta connected. Transforming them into star with ends at the same points.

$$R_1 = \frac{1 \times 1}{1 + 1 + 1} = \frac{1}{3}\Omega$$

Similarly,

$$R_2 = \frac{1}{3}\Omega, R_3 = \frac{1}{3}\Omega$$

After this transformation, the circuit is shown in the figure below,



Converting inner star connected resistances

$2 + \frac{1}{3} = \frac{7}{3}\Omega, \frac{7}{3}\Omega$ and $\frac{7}{3}\Omega$ into delta connection.

$$R_4 = \frac{\frac{7}{3} \times \frac{7}{3} + \frac{7}{3} \times \frac{7}{3} + \frac{7}{3} \times \frac{7}{3}}{\frac{7}{3}} = 7\Omega$$

Similarly,

$$R_5 = 7\Omega, R_6 = 7\Omega$$

Now,

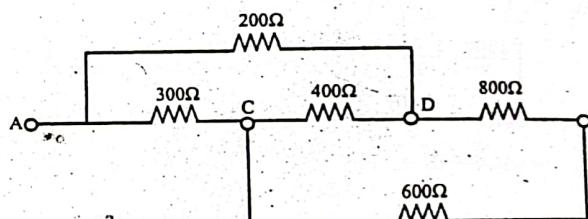
The resistance between the points X and are Y is,

$$R_{xy} = [(R_5 \parallel 1) + (R_6 \parallel 1)] \parallel [R_4 \parallel 1]$$

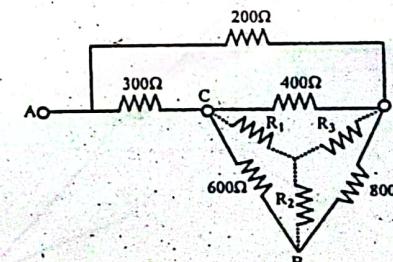
$$= [(7 \parallel 1) + (7 \parallel 1)] \parallel [7 \parallel 1] = \left[\frac{7}{8} + \frac{7}{8} \right] \parallel \left[\frac{7}{8} \right] = \frac{7}{4} \parallel \frac{7}{8}$$

$$\therefore R_{xy} = \frac{7}{12}\Omega$$

2. Using Star/delta and delta/ star conversion, find the resistance between the terminals A and B.



Solution:



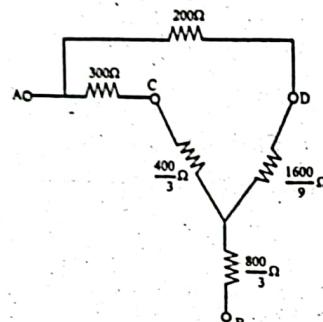
The three delta connected resistances between nodes B, C and D can be connected into equivalent star configuration. The values are,

$$R_1 = \frac{600 \times 400}{600 + 400 + 800} = \frac{400}{3}\Omega$$

$$R_2 = \frac{600 \times 800}{600 + 400 + 800} = \frac{800}{3} \Omega$$

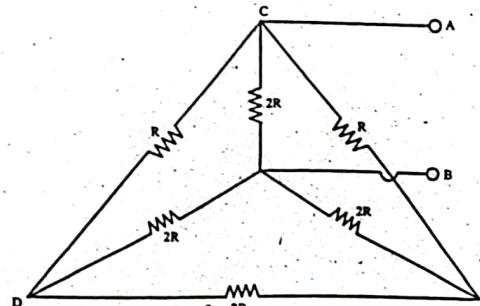
$$R_3 = \frac{400 \times 800}{600 + 400 + 800} = \frac{1600}{9} \Omega$$

After this transformation the circuit is shown in the figure below.

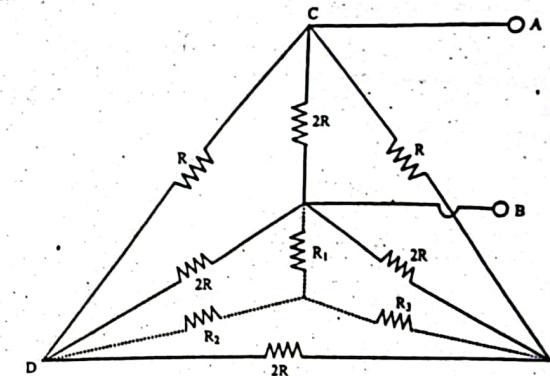


$$\begin{aligned} \therefore R_{AB} &= \left[\left(300 + \frac{400}{3} \right) \parallel \left(200 + \frac{1600}{9} \right) \right] + \frac{800}{3} \\ &= \left[\frac{1300}{3} \parallel \frac{3400}{9} \right] + \frac{800}{3} \\ &= 201.826 + \frac{800}{3} \\ &= 468.493 \Omega \end{aligned}$$

3. Making use of star/ delta transformation, determine the resistance between terminals A and B.



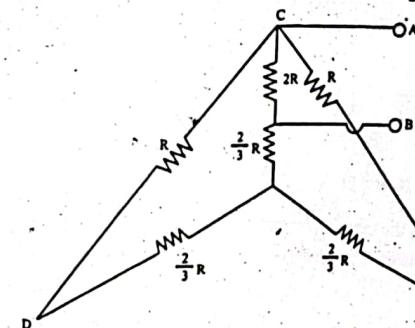
Solution:



The three delta connected resistances between nodes B, D and E can be converted into equivalent star configuration. The values are,

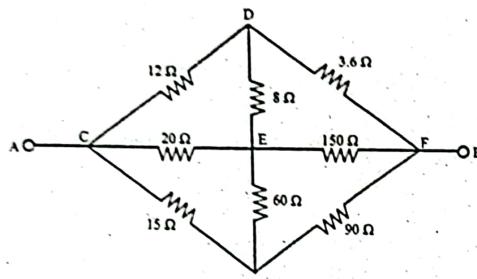
$$R_1 = R_2 = R_3 = \frac{2R \times 2R}{2R + 2R + 2R} = \frac{4R^2}{6R} = \frac{2}{3}R$$

After this transformation the circuit is shown in figure below.

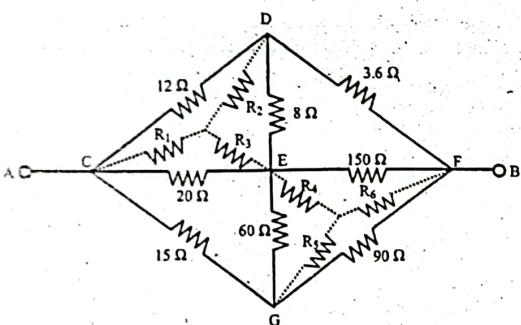


$$\begin{aligned} R_{AB} &= \left[\left\{ \left(R + \frac{2}{3}R \right) \parallel \left(R + \frac{2}{3}R \right) \right\} + \frac{2}{3}R \right] \parallel [2R] \\ &= \left[\left(\frac{5}{3}R \parallel \frac{5}{3}R \right) + \frac{2}{3}R \right] \parallel 2R \\ &= \left(\frac{5}{6}R + \frac{2}{3}R \right) \parallel 2R \\ &= \frac{3}{2}R \parallel 2R = \frac{\frac{3}{2}R \times 2R}{\frac{3}{2}R + 2R} = \frac{3R}{\frac{7}{2}} = \frac{6}{7}R \end{aligned}$$

4. Compute the resistance measured between terminals A and B of the circuit shown in figure below.



Solution:



Considering delta to star transformation between nodes C, D, E and E, F, G.

$$R_1 = \frac{12 \times 20}{12 + 20 + 8} = 6 \Omega$$

$$R_2 = \frac{12 \times 8}{12 + 20 + 8} = 2.4 \Omega$$

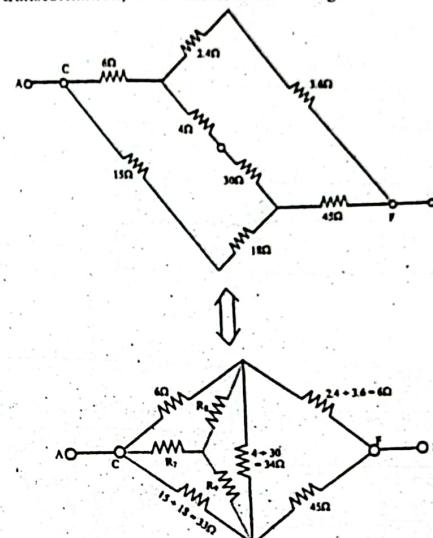
$$R_3 = \frac{20 \times 8}{12 + 20 + 8} = 4 \Omega$$

$$R_4 = \frac{60 \times 150}{60 + 150 + 90} = 30 \Omega$$

$$R_5 = \frac{60 \times 90}{60 + 150 + 90} = 18 \Omega$$

$$R_6 = \frac{150 \times 90}{60 + 150 + 90} = 45 \Omega$$

After transformation, the circuit is shown in figure below.



Considering delta to star transformation;

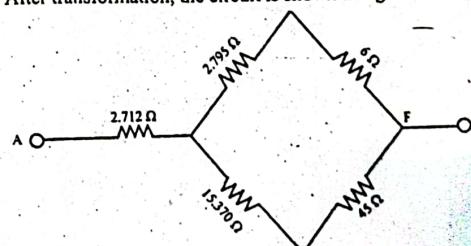
We get

$$R_8 = \frac{6 \times 34}{6 + 34 + 33} = 2.795 \Omega$$

$$R_7 = \frac{6 \times 33}{6 + 33 + 34} = 2.712 \Omega$$

$$R_9 = \frac{33 \times 34}{6 + 33 + 34} = 15.370 \Omega$$

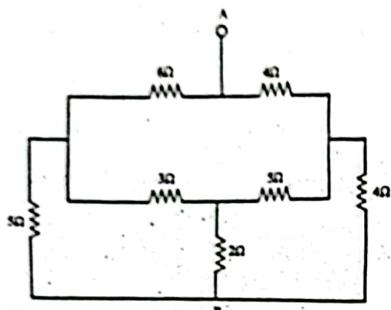
After transformation, the circuit is shown in figure below,



Hence, Resistance between terminals A and B,

$$\begin{aligned} R_{AB} &= 2.712 + [(6 + 2.795) \parallel (15.370 + 45)] \\ &= 2.712 + [8.795 \parallel 60.370] \\ &= 2.712 + 7.677 = 10.389 \Omega \end{aligned}$$

5. Find the equivalent resistance between terminal A and B using delta to star transformation.



Solution:

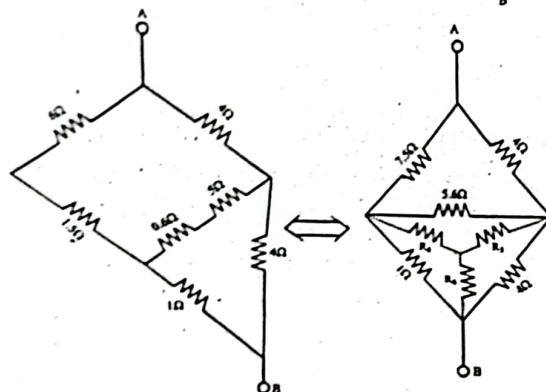
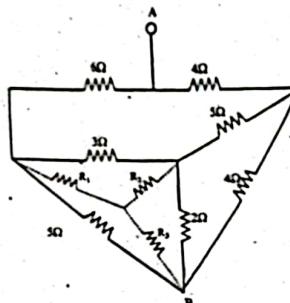
Considering delta to star transformation,

$$R_1 = \frac{5 \times 3}{5 + 3 + 2} = 1.5 \Omega$$

$$R_2 = \frac{3 \times 2}{5 + 3 + 2} = 0.6 \Omega$$

$$R_3 = \frac{5 \times 2}{5 + 3 + 2} = 1 \Omega$$

After transformation;



Again,

Considering delta to star transformation,

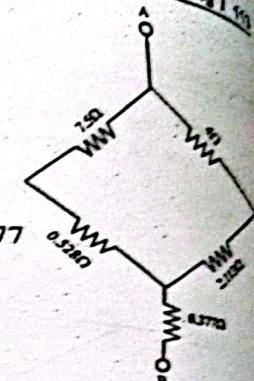
$$R_4 = \frac{1 \times 5.6}{1 + 5.6 + 4} = 0.528 \Omega$$

$$R_5 = \frac{5.6 \times 4}{1 + 5.6 + 4} = 2.113 \Omega$$

$$R_6 = \frac{1 \times 4}{1 + 5.6 + 4} = 0.377 \Omega$$

After transformation,

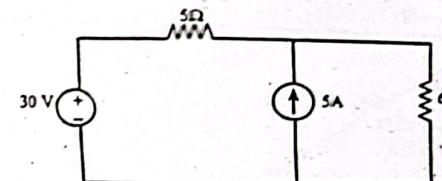
$$R_{AB} = [(7.5 + 0.528) \parallel (4 + 2.113)] + 0.377 \\ = (8.028 \parallel 6.113) + 0.377 = 3.847 \Omega$$



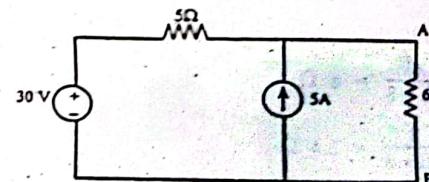
Superposition theorem

Exam solutions:

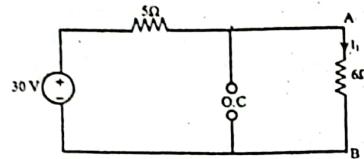
1. Use Superposition theorem to calculate current through the 6Ω resistor in the following network. [2063 Kartik]



Solution:



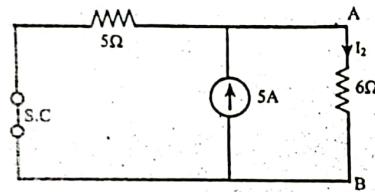
Here are two sources in the given circuit. We shall determine the current through the 6Ω resistor due to each source acting alone.
Considering 30 V source only;



Here, The 5 A current source is open-circuited then by Ohm's law

$$I_1 = \frac{30}{(5+6)} = 2.727 \text{ A (A to B)}$$

Considering 5 A source only;



The 30V voltage source is short circuited as shown in figure. The 5A current source is supplying current to 5Ω and 6Ω resistors in parallel.

By current division rule,

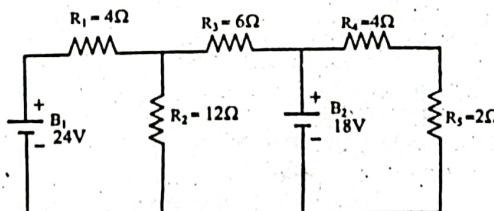
$$I_2 = \frac{5}{5+6} \times 5 = 2.273 \text{ A (A to B)}$$

Now,

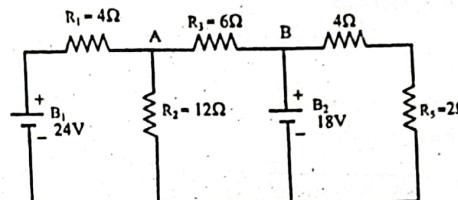
From the principle of Superposition,
Current through 6 Ω resistor is,

$$\begin{aligned} I &= I_1 + I_2 \\ &= 2.727 + 2.273 = 5 \text{ A (A to B)} \end{aligned}$$

2. Using Superposition theorem, find the current in resistor R_3 in the circuit shown in figure below. [2065 Kartik]

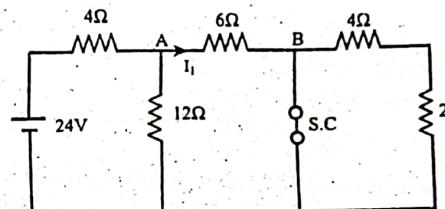


Solution:



Here are two sources in the given circuit. We shall determine the current through the R_3 resistor due to each source acting alone.

Considering $B_1 = 24\text{V}$ source only;



$$\text{Total resistance} = (6\parallel 12) + 4$$

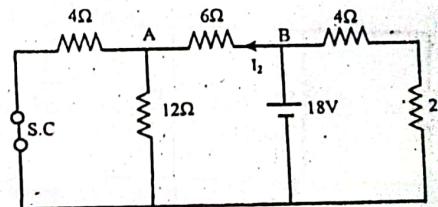
$$= \frac{6 \times 12}{6+12} + 4 = 8 \Omega$$

$$\text{Current supplied by battery} = \frac{24}{8} = 3 \text{ A}$$

Using current division rule,

$$\begin{aligned} \text{Current through } 6 \Omega \text{ resistor, } I_1 &= \frac{3}{6+12} \times 12 \\ &= 2 \text{ A (A to B)} \end{aligned}$$

Considering $B_2 = 18\text{V}$ source only;



$$\begin{aligned} \text{Total resistance} &= [(4\parallel 12) + 6] \parallel [(4+2)] \\ &= \left[\frac{4 \times 12}{4+12} + 6 \right] \parallel [6] \\ &= 9 \parallel 6 \end{aligned}$$

$$\frac{9 \times 6}{9+6} = 3.6 \Omega$$

Current supplied by the battery

$$= \frac{18}{3.6} = 5 \text{ A}$$

Using current division rule,

$$I_2 = \frac{5}{[(4 \parallel 12) + 6] + (4 + 2)} \times (4 + 2)$$

$$= \frac{5}{3+6+6} \times 6$$

$$= 2 \text{ A (B to A)}$$

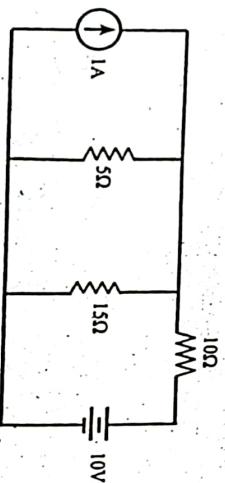
Now,

From principle of Superposition,

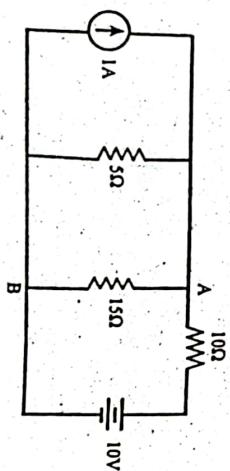
$$\begin{aligned} \text{Current through } 6 \Omega \text{ resistor} &= I_1 - I_2 \\ &= 0 \text{ A} \end{aligned}$$

3.

Calculate the current in the 15Ω resistor in the network shown in figure below using Superposition theorem. [2007 Ashad]

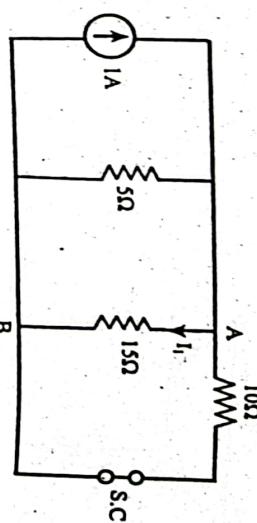


Solution:



Here are two sources in the given circuit. We shall determine the current through the 15Ω resistor due to each source acting alone.

Considering 1 A current source only;

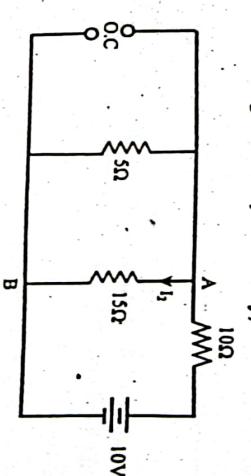


Here, 5Ω , 15Ω and 10Ω resistors are connected in parallel.

Using current division rule,

$$\begin{aligned} \text{Current in the } 15 \Omega \text{ resistor, } I_1 &= \frac{1}{\frac{1}{5} + \frac{1}{15} + \frac{1}{10}} \times 1 \\ &= \frac{2}{11} \text{ A (A to B)} \end{aligned}$$

Considering 10V battery source only,



$$\text{Total resistance} = (5 \parallel 15) + 10 = \frac{5 \times 15}{5+15} + 10$$

$$= \frac{55}{4} \Omega = 13.75 \Omega$$

Current supplied by the battery

$$= \frac{10}{13.75} = \frac{8}{11} \text{ A}$$

Current through 15Ω resistor,

$$= \frac{8}{5+15} \times 5$$

$= \frac{2}{11} \text{ A (A to B)}$

Now,

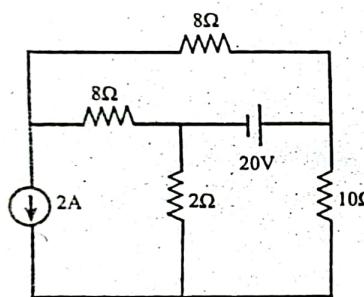
From the principle of Superposition,

Current through 15Ω resistor = $I_1 + I_2$

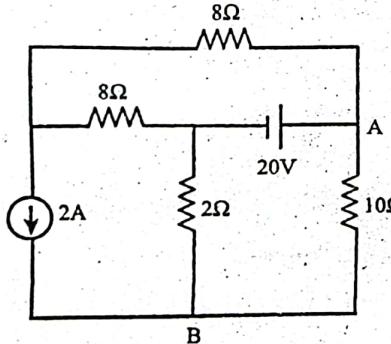
$$= \frac{2}{11} + \frac{2}{11}$$

$$= \frac{4}{11} \text{ A (A to B)}$$

4. Use Superposition theorem to find the current flowing through the 10Ω resistor shown in the figure. [2068 Baishakhi]

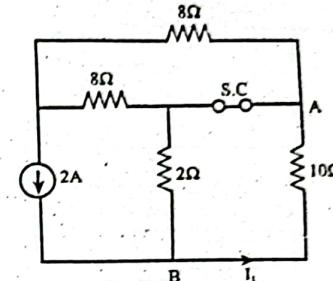


Solution:



Here are two sources in the given circuit. We shall determine the current through the 10Ω resistor due to each source acting alone.

Considering 2A current source only;

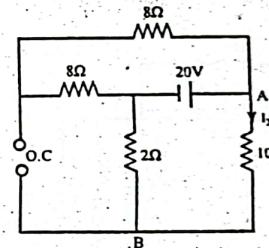


Using current division rule,

$$\text{Current flowing through } 10\Omega \text{ resistor, } I_1 = \frac{2}{2+10} \times 2$$

$$= \frac{4}{12} = \frac{1}{3} \text{ A (B to A)}$$

Considering 20V battery source only;



Current flowing through 10Ω resistors,

$$I_2 = \frac{20}{2+10}$$

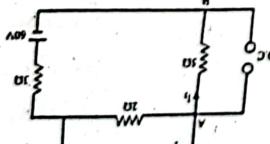
$$= \frac{5}{3} \text{ A (A to B)}$$

Now, from the principle of Superposition,

$$\begin{aligned} \text{Current through the } 10\Omega \text{ resistor} &= I_2 - I_1 \\ &= \frac{5}{3} - \frac{1}{3} = \frac{4}{3} \\ &= 1.333 \text{ A (A to B)} \end{aligned}$$

5. Apply Superposition theorem to the circuit shown below to find the voltage drop V across the 5Ω resistor. [2068 Bhadra]

Considering 60V source only:



Current through 5Ω resistor, by Ohm's law

$$I_3 = \frac{5+2+3}{60} = 6 \text{ A (A to B)}$$

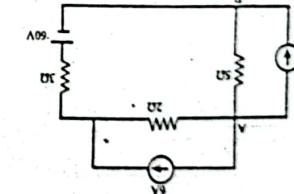
Now, from the principle of Superposition, we get

$$\therefore \text{Voltage across } 5\Omega \text{ resistors, } V = V_{AB} = 3.8 \times 5 = 19 \text{ V}$$

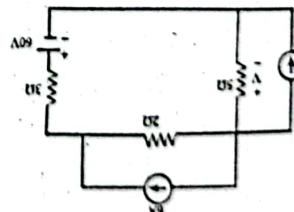
6. Find the current in 0.5Ω resistor in the network shown, by using superposition theorem. [2009 Asstt]
-

Solution:

Here are three sources in the given circuit. We shall determine the current through the 5Ω resistor due to each source acting alone.



Solution:



120 / Network Theorems

7. Here are two sources in the given circuit. We shall determine the current through $R_L = 0.5\Omega$ resistor due to each source acting alone.
-

Considering 15V source only:

Here are two sources in the given circuit. We shall determine the current through $R_L = 0.5\Omega$ resistor due to each source acting alone.

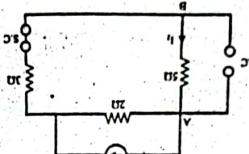
Using current division rule,

$$I_1 = \frac{5 + (3+2) \times (3+2)}{2} = 1 \text{ A (B to A)}$$

Considering 6 A current source only:

$$I_2 = \frac{6}{2 + (5+3) \times 2} = \frac{6}{16} = \frac{3}{8} \text{ A (B to A)}$$

Using current division rule,



$$I_1 = \frac{5 + (3+2) \times (3+2)}{2} = 1 \text{ A (B to A)}$$

Considering 6 A current source only:

$$I_2 = \frac{6}{2 + (5+3) \times 2} = \frac{6}{16} = \frac{3}{8} \text{ A (B to A)}$$

Using current division rule,

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$$I_2 = \frac{6}{2 + (5+3) \times 2} = \frac{6}{16} = \frac{3}{8} \text{ A (B to A)}$$

Using current division rule,

$$I_1 = \frac{5 + (3+2) \times (3+2)}{2} = 1 \text{ A (B to A)}$$

Considering 2 A current source only:

$$I_1 = \frac{5 + (3+2) \times (3+2)}{2} = 1 \text{ A (B to A)}$$

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Considering 2 A current source only:

$$I_1 = \frac{5 + (3+2) \times (3+2)}{2} = 1 \text{ A (B to A)}$$

Considering 2 A current source only:

$$= \frac{7}{13} + 1$$

$$= \frac{20}{13} \Omega$$

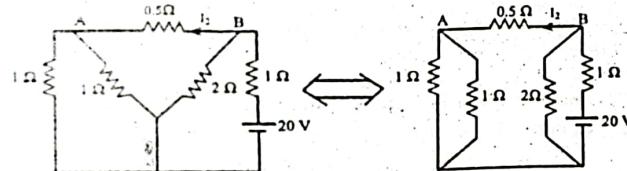
Current supplied by the battery = $\frac{15}{\left(\frac{20}{13}\right)}$
 $= 9.75 \text{ A.}$

Current flowing through 0.5Ω resistor,

$$I_1 = \frac{9.75}{1 + \left(0.5 + \frac{2}{3}\right)} \times 1$$

$$= 4.5 \text{ A. (A to B)}$$

Considering 20 V source only;



$$\begin{aligned} \text{Total resistance} &= [(1 \parallel 1) + 0.5] \parallel 2 + [1] \\ &= (0.5 + 0.5) \parallel 2 + [1] \\ &= 1 \parallel 2 + [1] \\ &= \frac{2}{3} + 1 = \frac{5}{3} \Omega \end{aligned}$$

Current supplied by the battery $= \frac{20}{\left(\frac{5}{3}\right)}$
 $= 12 \text{ A}$

Current flowing through 0.5Ω resistor

$$I_2 = \frac{12}{[(1 \parallel 1) + 0.5] + 2} \times 2$$

$$\therefore I_2 = 8 \text{ A} \quad (\text{B to A})$$

Now, from the principle of Superposition,

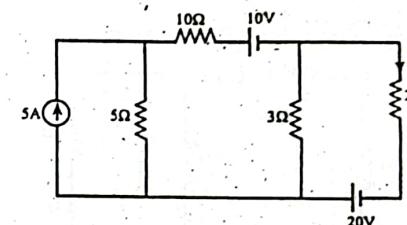
We get,

Current through 0.5Ω resistor

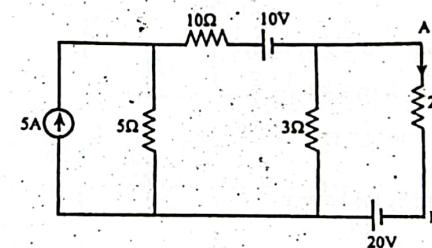
$$\begin{aligned} &= I_2 - I_1 \\ &= 8 - 4.5 \\ &= 3.5 \text{ (B to A)} \end{aligned}$$

7.

Use Superposition theorem to find the current I_L through 2Ω resistor in figure below. [2070 Ashad]

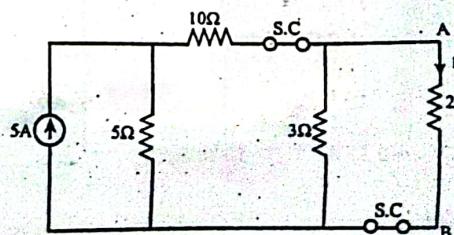


Solution:



Here are three sources in the given circuit. We shall determine the current through the 2Ω resistor due to each source acting alone.

Considering 5A current source only;



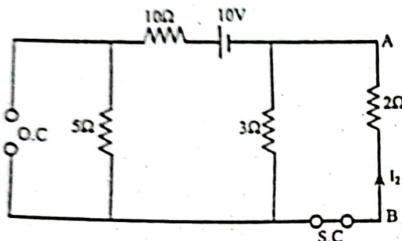
Current flowing through 10Ω resistor

$$= \frac{5}{5 + \left(10 + \frac{3 \times 2}{3+2}\right)} \times 5 \\ = 1.543 \text{ A}$$

Current flowing through 2Ω resistor,

$$I_2 = \frac{1.543}{3+2} \times 3 \\ = 0.926 \text{ A (A to B)}$$

Considering $10V$ source only;



$$\text{Total resistance} = 10 + 5 + (3 \parallel 2)$$

$$= 10 + 5 + \frac{3 \times 2}{3+2} \\ = 16.2 \Omega$$

Current supplied by the battery

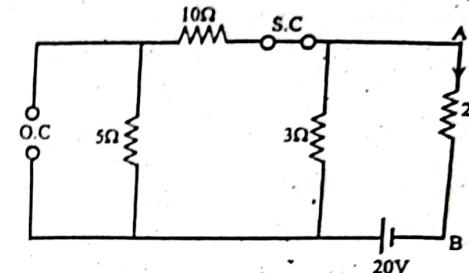
$$= \frac{10}{16.2} \\ = 0.617 \text{ A}$$

Using current division rule,

Current flowing through 2Ω resistor,

$$= \frac{0.617}{3+2} \times 3 \\ = 0.37 \text{ A (B to A)}$$

Considering $20V$ source only;



$$\text{Total resistance} = [(5+10) \parallel 3] + 2 \\ = [15 \parallel 3] + 2 \\ = \frac{15 \times 3}{15+3} + 2 \\ = 4.5 \Omega.$$

Current flowing through 2Ω resistor

$$I_1 = \frac{20}{4.5} \\ = 4.444 \text{ A (A to B)}$$

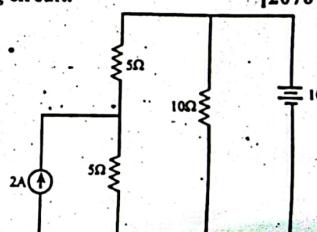
Now,

From the principle of Superposition, we get,

Current flowing through 2Ω resistor,

$$I_L = I_1 + I_2 - I_3 \\ = 0.926 + 4.444 - 0.37 \\ = 5 \text{ A (A to B)}$$

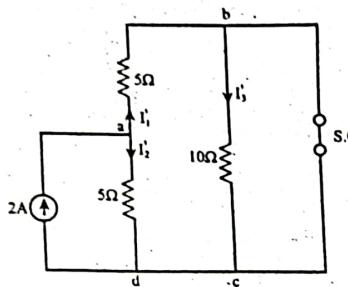
8. Using Superposition theorem, determine currents in all the resistors of the following circuit. [2070 Bhadra]



Solution:

We have two sources in the given circuit. We shall determine the current through the resistors due to each source acting alone.

Considering 2A source only;



Here,

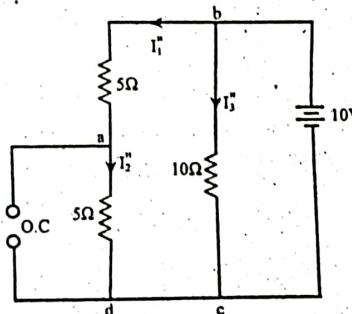
$$I_3 = 0 \quad (\text{current flows through short circuit path})$$

Using current division rule,

$$I_1' = \frac{2}{5+5} \times 5 \\ = 1 \text{ A} \quad (\text{a to b})$$

$$I_2' = \frac{2}{5+5} \times 5 \\ = 1 \text{ A} \quad (\text{a to d})$$

Considering 10V source only;



$$\text{Here, } I_3 = \frac{V_{bc}}{10} = \frac{10}{10} = 1 \text{ A} \quad (\text{b to c})$$

$$I_1'' = \frac{V_{bd}}{5+5} = \frac{10}{10} = 1 \text{ A} \quad (\text{b to a})$$

$$\text{Also, } I_2'' = 1 \text{ A} \quad (\text{a to d})$$

Now,

From principle of Superposition,

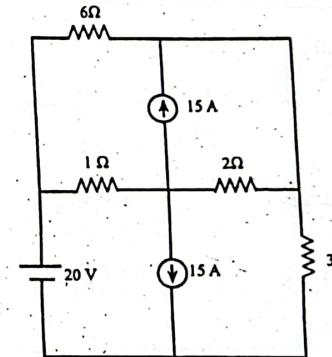
$$I_1 = I_1' - I_1'' = 1 - 1 = 0$$

$$I_2 = I_2' + I_2'' = 1 + 1 = 2 \text{ A} \quad (\text{a to d})$$

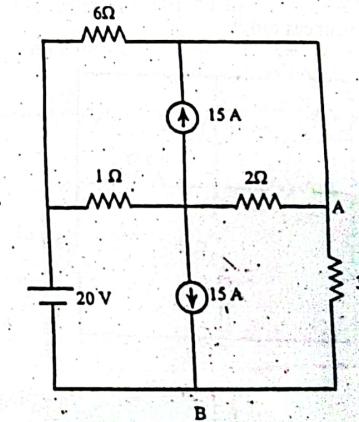
$$I_3 = I_3'' + I_3' = 1 - 0 = 1 \text{ A} \quad (\text{b to c})$$

Hence, current through 5 Ω resistor (at top) is 0A, current through 5 Ω resistor (at bottom) is 2A and current through 10 Ω resistor is 1 A.

9. Calculate the voltage drop across 3 Ω resistor using Superposition theorem in the circuit given below.
[2071 Bhadra]

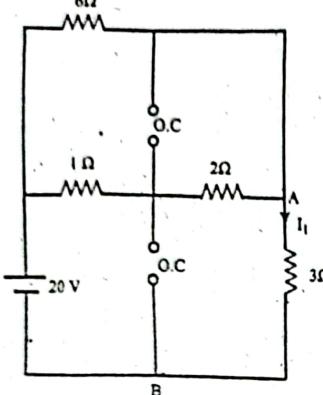


Solution:



Here are three sources in the given circuit. We shall determine the voltage drop across 3 Ω resistor due to each source acting alone.

Considering 20V voltage source only :



$$\begin{aligned}\text{Total resistance} &= [6 // (1 + 2)] + 3 \\ &= [6 // 3] + 3 \\ &= \frac{6 \times 3}{6 + 3} + 3 \\ &= 5 \Omega\end{aligned}$$

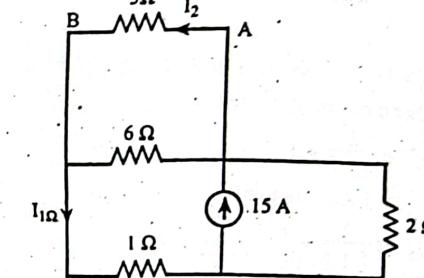
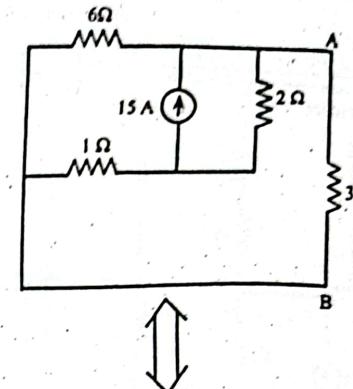
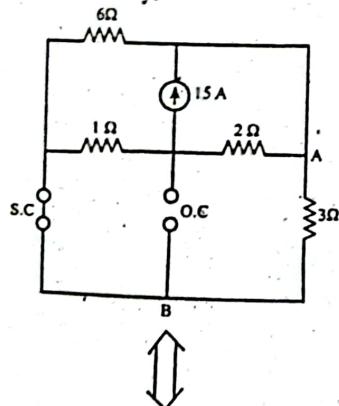
Current flowing through 3Ω resistor,

$$\begin{aligned}I_1 &= \frac{20}{5} \\ &= 4 \text{ A (A to B)}\end{aligned}$$

Voltage drop across 3Ω resistor,

$$\begin{aligned}V_{AB} &= 4 \times 3 \\ &= 12 \text{ V}\end{aligned}$$

Considering 15A current sources only:



Using current division rule,

$$\begin{aligned}I_{1B} &= \frac{15}{[(3 // 6) + 1] + 2} \times 2 \\ &= \frac{15}{5} \times 2 \\ &= 6 \text{ A}\end{aligned}$$

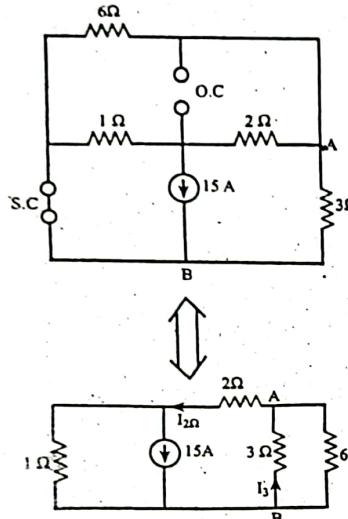
Again, Using current division rule,

$$\begin{aligned}I_2 &= \frac{6}{3 + 6} \times 6 \\ &= 4 \text{ A (A to B)}\end{aligned}$$

Voltage drop across 3Ω resistor,

$$\begin{aligned}V_{AB}'' &= 4 \times 3 \\ &= 12 \text{ V}\end{aligned}$$

Considering 15A current source only :



Using current division rule,

$$\begin{aligned} I_{1\Omega} &= \frac{15}{1 + [(3//6) + 2]} \times 1 \\ &= \frac{15}{5} \times 1 = 3 \text{ A} \end{aligned}$$

Again, using current division rule,

$$\begin{aligned} I_3 &= \frac{3}{3+6} \times 6 \\ &= 2 \text{ A (B to A)} \end{aligned}$$

Voltage drop across 3 Ω resistor,

$$V_{BA}''' = 2 \times 3 = 6 \text{ V}$$

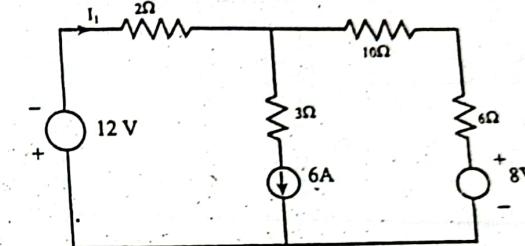
Now,

From principle of Superposition,

Voltage drop across 3 Ω resistor,

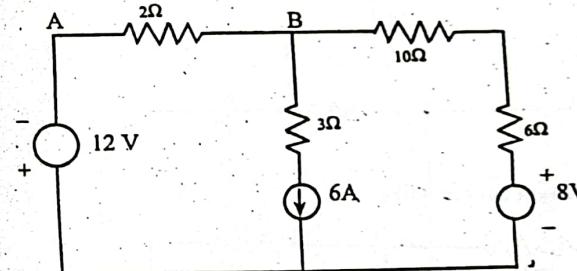
$$\begin{aligned} V_{3\Omega} &= V_{AB} \\ &= V_{AB}'' + V_{AB}''' - V_{BA}''' \\ &= 12 + 12 - 6 \\ &= 18 \text{ V} \end{aligned}$$

10. Determine the current I_1 in the circuit shown below using Super position theorem. [2072 Ashwin]



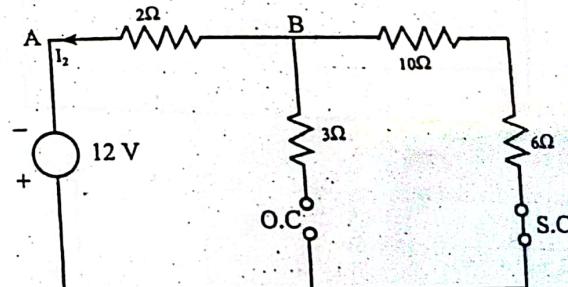
Solution:

Here, I_1 is the current flowing through 2Ω resistor.



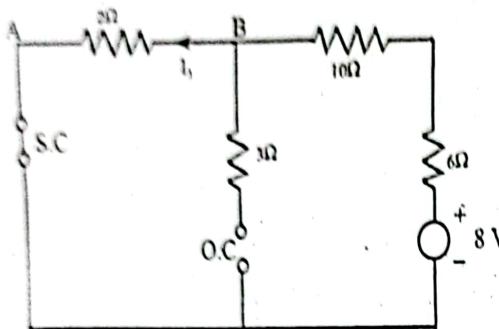
Here, are three sources in the given circuit. We shall determine the current through the 2Ω resistor due to each source acting alone.

Considering 12V source only;



$$\text{Here, } I_2 = \frac{12}{(6+10+12)} = \frac{2}{3} \text{ A (B to A)}$$

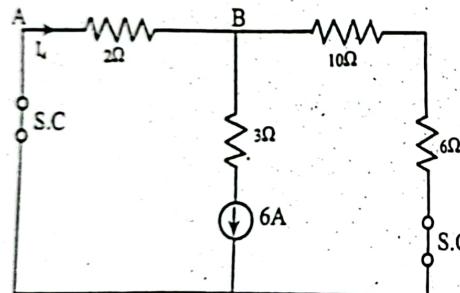
Considering 8V source only;



Here,

$$I_1 = \frac{8}{(2 + 10 + 6)} \\ = \frac{4}{9} \text{ A (B to A)}$$

Considering 6A current source only;



Using current divider rule,

$$I_2 = \frac{6}{2 + (10 + 6)} \times (10 + 6) \\ = \frac{16}{3} \text{ A (A to B)}$$

Now,

From the principle of superposition, we get

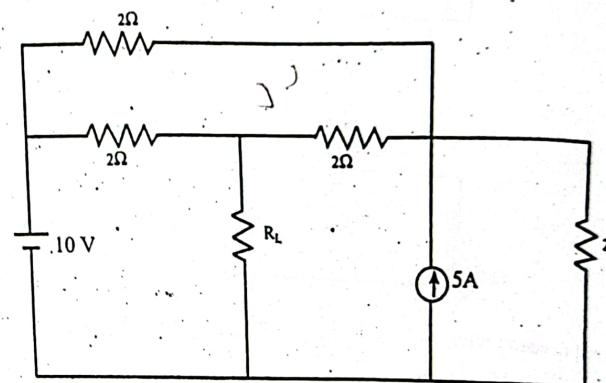
Current through 2Ω resistor,

$$I_1 = \frac{16}{3} - \left(\frac{2}{3} + \frac{4}{9} \right)$$

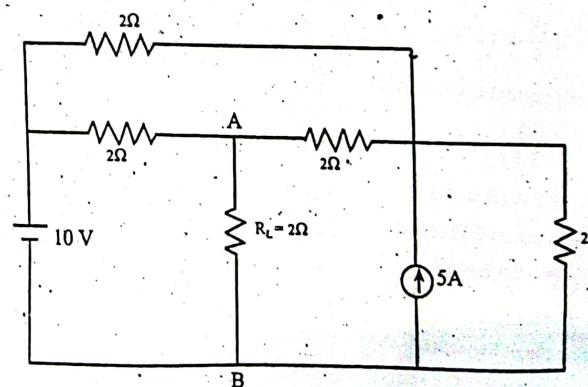
$$= \frac{38}{9} \text{ A}$$

$$\therefore I_1 = 4.222 \text{ A (A to B)}$$

11. Find current on load resistor R_L , if its resistance is 2Ω, using Superposition theorem. [2072 Chaitra]

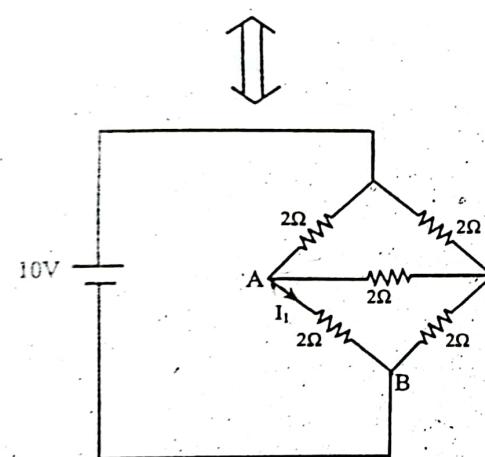
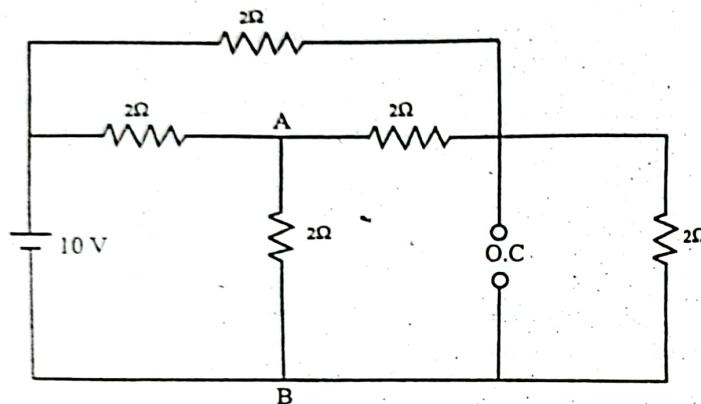


Solution:



Here are two sources in the given circuit. We shall determine the current through $R_L = 2\Omega$ resistor due to each source acting alone.

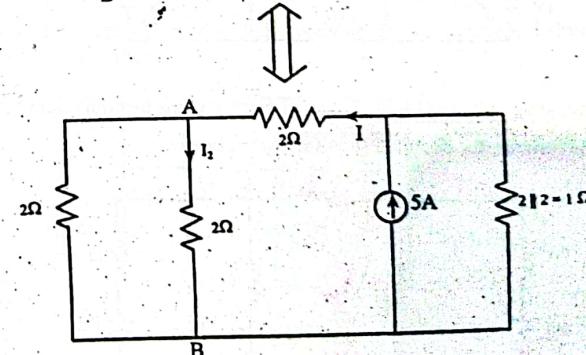
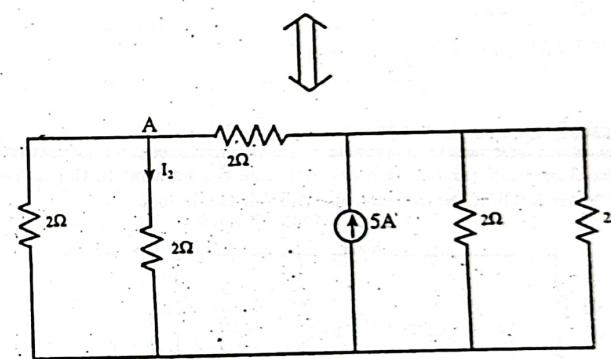
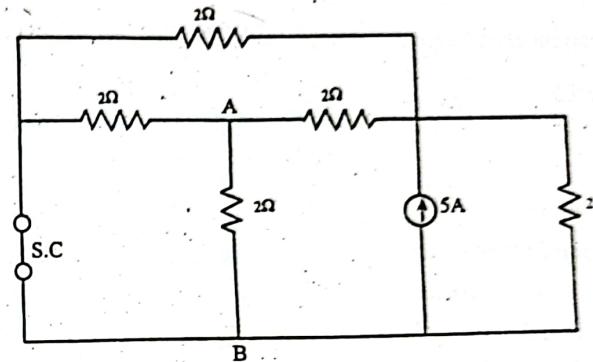
Considering 10 V source only;



By observing the given circuit, we can conclude that it is a balanced bridge.

$$\begin{aligned} I_1 &= \frac{10}{2+2} \\ &= \frac{10}{4} \\ &= 2.5\text{A (A to B)} \end{aligned}$$

Considering 5A current source only;



Using current divider rule,

$$I = \frac{5}{(2 \parallel 2) + 1} \times 1$$

$$= \frac{5}{1+2+1} \times 1 \\ = 1.25 \text{ A}$$

Again, using current divider rule,

$$I_2 = \frac{1}{2+2} \times 2$$

$$= \frac{1.25}{2+2} \times 2$$

$$= 0.625 \text{ A (A to B)}$$

Now, from the principle of superposition, we get

Current through $R_L = 2\Omega$ resistor,

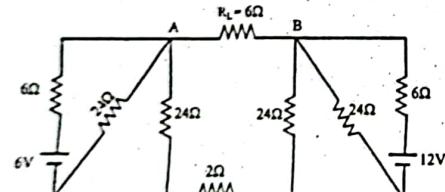
$$= I_1 + I_2$$

$$= 2.5 + 0.625$$

$$= 3.125 \text{ A (A to B)}$$

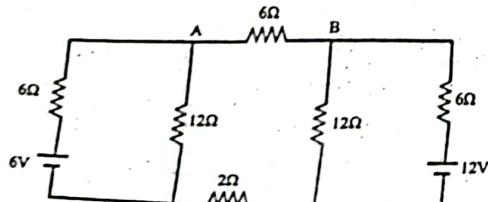
Additional questions

1. Use the Superposition theorem to determine the current in the current in the branch AB of the network shown in figure below.



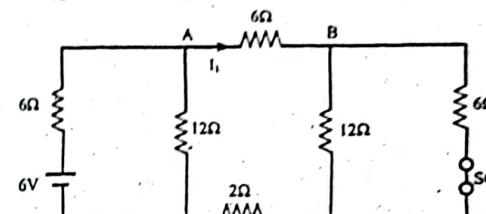
Solution:

Converting both $24\Omega \parallel 24\Omega$ parallel combinations by equivalent $\frac{24 \times 24}{48} = 12\Omega$ resistor, the circuit becomes as shown below.



Here are two sources in the given circuit. We shall determine the current through the 6Ω resistor due to each source acting alone.

Considering 6V source only;

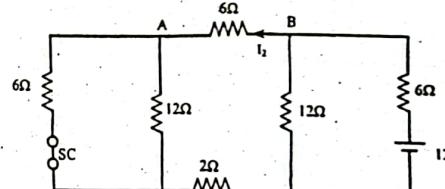


$$\text{Total resistance} = [(12 \parallel 6) + 2 + 6] \parallel 12 + [6] \\ = [(4 + 2 + 6) \parallel 12] + [6] \\ = 6 + 6 = 12\Omega$$

$$\text{Current supplied by the battery} = \frac{6}{12} = 0.5 \text{ A}$$

$$\text{Current through } 6\Omega \text{ resistor, } I_1 = \frac{0.5}{12 + (6 + 4 + 2)} \times 12 \\ = 0.25 \text{ A (A to B)}$$

Considering 12 V source only;



$$\text{Total resistance} = [(6 \parallel 12) + 6 + 2] \parallel 12 + 6 \\ = [(4 + 6 + 2) \parallel 12] + 6 \\ = 6 + 6 = 12\Omega$$

$$\text{Current supplied by the battery} = \frac{12}{12} = 1 \text{ A}$$

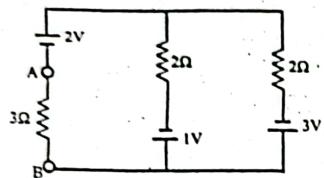
$$\text{Current through } 6\Omega \text{ resistor, } I_2 = \frac{1}{12 + [6 + 2 + (6 \parallel 12)]} \times 12 \\ = 0.5 \text{ A (B to A)}$$

Now,

From the principle of Superposition,
Current through the 6Ω resistor is,

$$I = I_2 - I_1 \\ = 0.5 - 0.25 \\ = 0.25 \text{ A (B to A)}$$

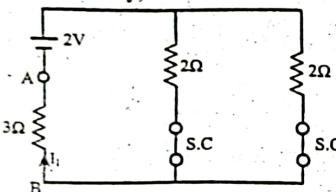
Find the current through 3 ohms resistor of the circuit shown below by Superposition theorem.



Solution:

There are three sources in the given circuit. We shall determine the current through the 3Ω resistor due to each source acting alone.

Considering 2V source only;

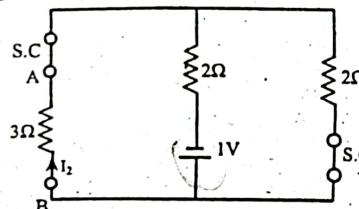


$$\begin{aligned}\text{Total resistance} &= (2 \parallel 2) + 3 \\ &= 1 + 3 \\ &= 4 \Omega\end{aligned}$$

$$\text{Current supplied by the battery} = \frac{2}{4} = 0.5 \text{ A}$$

Current through 3Ω resistor, $I_1 = 0.5 \text{ A}$ (B to A)

Considering 1V source only;



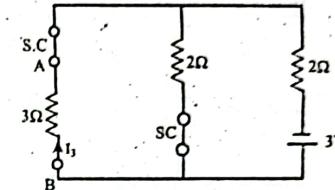
$$\begin{aligned}\text{Total resistance} &= (3 \parallel 2) + 2 \\ &= \frac{6}{5} + 2 \\ &= 3.2 \Omega\end{aligned}$$

$$\begin{aligned}\text{Current supplied by the battery} &= \frac{1}{3.2} \\ &= 0.313 \text{ A}\end{aligned}$$

Current through 3Ω resistor,

$$I_2 = \frac{0.313}{3+2} \times 2 = 0.125 \text{ A (B to A)}$$

Considering 3V source only;



$$\text{Total resistance} = (3 \parallel 2) + 2$$

$$\begin{aligned}&= \frac{3 \times 2}{3+2} + 2 \\ &= 3.2 \Omega\end{aligned}$$

$$\text{Current supplied by the battery} = \frac{3}{3.2} = 0.938 \text{ A}$$

$$\begin{aligned}\text{Current through } 3\Omega \text{ resistor} &= \frac{0.938}{3+2} \times 2 \\ &= 0.375 \text{ A (B to A)}\end{aligned}$$

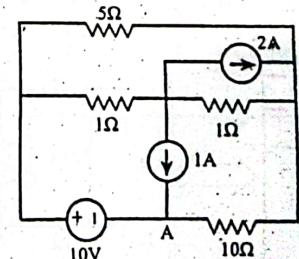
Now,

From the principle of Superposition,

Current through the 3Ω resistor

$$\begin{aligned}&= I_1 + I_2 + I_3 \\ &= 0.5 + 0.125 + 0.375 \\ &= 1 \text{ A (B to A)}\end{aligned}$$

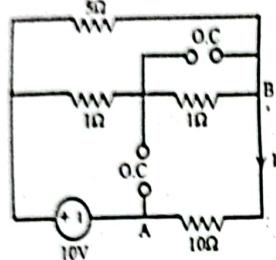
3. Find the current in the 10Ω resistor in the circuit below using Superposition theorem:



Solution:

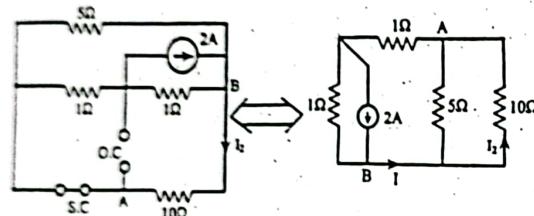
We have three sources in the given circuit. We shall determine the current through the 10Ω resistor due to each source acting alone.

Considering 10V source only;



$$I_1 = \frac{10}{5 + (1 + 1)} = \frac{10}{7} = 0.875 \text{ A (B to A)}$$

Considering 2A current source only;

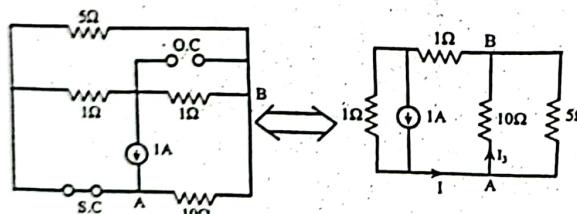


Using current division rule,

$$I_1 = \frac{2}{1 + [1 + (5 \parallel 10)]} = \frac{2}{1 + \left[1 + \frac{10}{3}\right]} = 0.375 \text{ A}$$

$$\therefore I_2 = \frac{1}{5 + 10} \times 5 = \frac{0.375}{15} \times 5 = 0.125 \text{ A (B to A)}$$

Considering 1A current source only;



Using current division rule,

$$I_1 = \frac{1}{1 + [1 + (10 \parallel 5)]} \times 1$$

$$= \frac{1}{1 + \left[1 + \frac{50}{15}\right]}$$

$$= 0.1875 \text{ A}$$

$$I_3 = \frac{1}{10 + 5} \times 5 = \frac{0.1875}{15} \times 5 = 0.0625 \text{ A (B to A)}$$

Now,

From principle of Superposition,

Current flowing through 10Ω resistor,

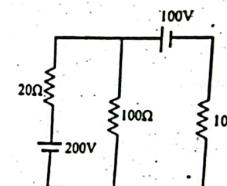
$$= I_1 + I_2 - I_3$$

$$= 0.875 + 0.125 - 0.0625$$

$$= 0.9375 \text{ A (B to A)}$$

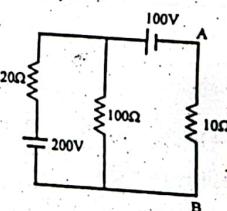
Thevenin's theorem**Exam solutions**

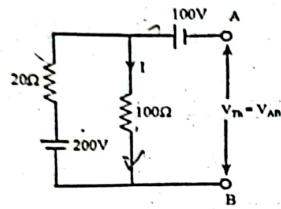
1. Use Thevenin's theorem to calculate current through 10Ω resistor in the following network. [2064 Shrawan]



Solution:

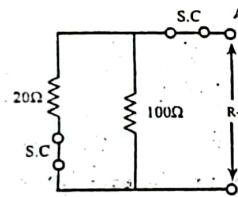
Here the given network is,



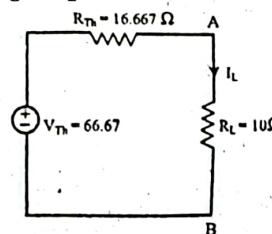
To find V_{Th} :

$$\begin{aligned} V_{Th} &= V_{AB} = V_A - V_B \\ &= 100 \times \frac{200}{20+100} - 100 \quad [\text{write KVL equation; move from B to A}] \\ &= 66.67 \text{ V} \end{aligned}$$

This indicates that A is at a higher potential with respect to B.

To find R_{Th} :

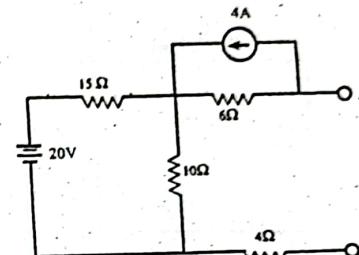
$$\begin{aligned} R_{Th} &= R_{AB} = 20 \parallel 100 \\ &= \frac{20 \times 100}{20+100} \\ &= 16.667 \Omega \end{aligned}$$

To find current I_L in $R_L = 10 \Omega$ 

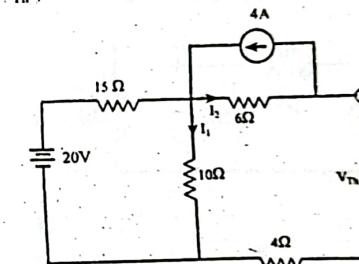
By Ohm's law,

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{66.67}{16.667 + 10} = 2.5 \text{ A (A to B)}$$

2. Find the Thevenin's equivalent circuit for terminal pair AB of the network shown in figure given below. [2067 Mangsir]

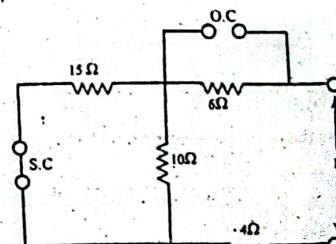


Solution:

To find V_{Th} :

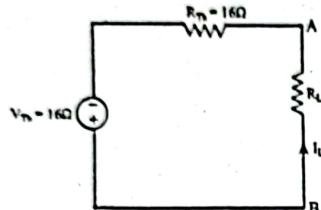
$$\begin{aligned} V_{Th} &= V_{AB} \\ &= V_A - V_B \\ &= 10 \times I_1 - 6 \times I_2 \quad [\text{Write KVL equation; move from B to A}] \\ &= 10 \times \frac{20}{15+10} - 6 \times 4 = -16 \text{ V} \end{aligned}$$

Which indicates that B is at higher potential with respect to A.

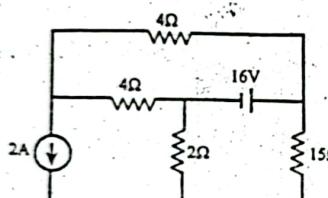
To find R_{Th} :

$$\begin{aligned} R_{Th} &= R_{AB} = (15 \parallel 10) + 6 + 4 \\ &= \frac{15 \times 10}{15+10} + 6 + 4 = 16 \Omega \end{aligned}$$

Thevenin's equivalent circuit

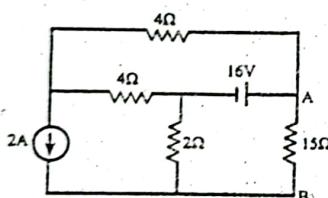


3. Use Thevenin's theorem to find the current flowing through 15Ω resistor of the network of figure below. [2009 Bhadra]

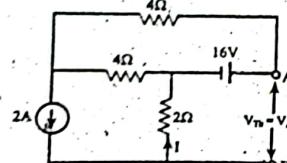


Solution:

Here, the given network is,



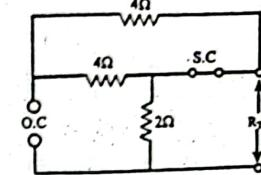
To find V_{Th} :



$$\begin{aligned} V_{Th} &= V_{AB} = V_A - V_B \\ &= -2 \times I + 16 \quad [\text{Write KVL equation; move from B to A}] \\ &= -2 \times 2 + 16 = 12 \text{ V} \end{aligned}$$

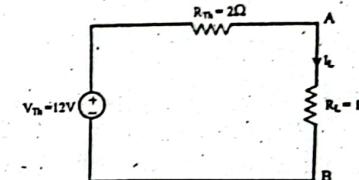
Which indicates A is at higher potential with respect to B.

To find R_{Th} :



$$R_{Th} = 2 \Omega$$

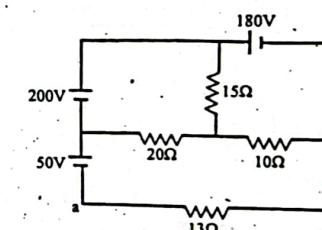
To find current I_L in $R_L = 15\Omega$



By Ohm's law,

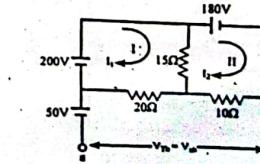
$$\begin{aligned} I_L &= \frac{V_{Th}}{R_{Th} + R_L} = \frac{12}{2 + 15} \\ &= 0.706 \text{ A (A to B)} \end{aligned}$$

4. Applying Thevenin's theorem, calculate the magnitude and direction of current in the 13Ω resistor in the circuit shown in the following figure. [2009 Ashad]



Solution:

To find V_{Th} :



Applying KVL on mesh I and mesh II, we get
Mesh I:

$$200 - 15(I_1 - I_2) - 20I_1 = 0$$

Or, $-35I_1 + 15I_2 = -200 \quad \dots \text{(i)}$

Mesh II :

$$-180 - 10I_2 - 15(I_2 - I_1) = 0$$

Or, $15I_1 - 25I_2 = 180 \quad \dots \text{(ii)}$

Solving equations (i) and (ii), we get

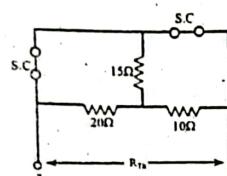
$$I_1 = 3.538 \text{ A}$$

$$I_2 = -5.077 \text{ A}$$

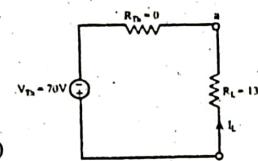
Negative current indicates that our direction of assumption of current flow is opposite to the actual flow.

$$\begin{aligned} V_{Th} &= V_{ab} = V_a - V_b \\ &= 10I_2 - 20I_1 - 50 \quad [\text{Write KVL equation; move from b to a}] \\ &= 10 \times 5.077 - 20 \times 3.538 - 50 = -70 \text{ V} \end{aligned}$$

Which indicates b is at higher potential with respect to a.

To find R_{Th} ;

$$R_{Th} = 0$$

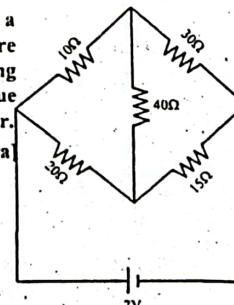
To find current I_L in $R_L = 13\Omega$ 

By Ohm's law,

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{70}{0 + 13} = 5.385 \text{ A} \quad (\text{b to a})$$

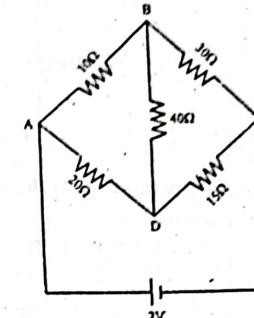
5. The resistance of the various arms of a Wheatstone bridge are shown in figure below. The battery has an emf of 2V. Using Thevenin's theorem, determine the value and direction of current in the 40Ω resistor.

[2070 Bhadral]



Solution:

Here, the given circuit is,

To find V_{Th} ;

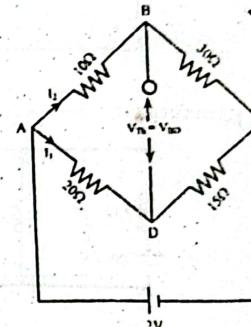
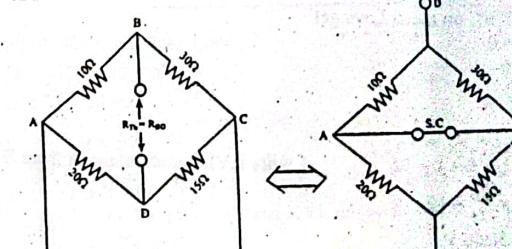
Here,

$$\begin{aligned} V_{AB} &= V_A - V_B \\ &= I_2 \times 10 \\ &= \frac{2}{10+30} \times 10 \\ &= 0.5 \text{ V} \end{aligned}$$

$$\begin{aligned} V_{AD} &= V_A - V_D \\ &= I_1 \times 20 \\ &= \frac{2}{20+15} \times 20 \\ &= 1.143 \text{ V} \end{aligned}$$

Now,

$$\begin{aligned} V_{Th} &= V_{BD} \\ &= V_{AD} - V_{AB} \\ &= 1.143 - 0.5 \\ &= 0.643 \text{ V} \end{aligned}$$

To find R_{Th} ;

$$\begin{aligned} R_{Th} &= R_{BD} = (10 \parallel 30) + (20 \parallel 15) \\ &= \frac{10 \times 30}{10 + 30} + \frac{20 \times 15}{20 + 15} = 7.5 + 8.571 = 16.071 \Omega \end{aligned}$$

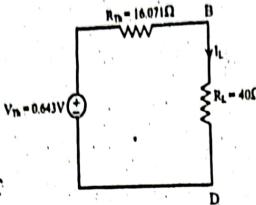
To find current I_L in $R_L = 40\Omega$:

By Ohm's law,

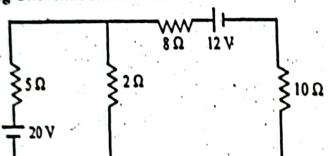
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{0.643}{16.071 + 40}$$

$$= 0.0115 \text{ A (B to D)}$$

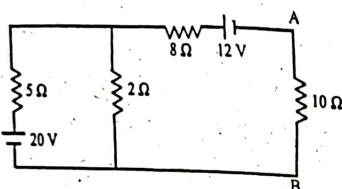


6. For the circuit shown in figure below, calculate the current in the 10 ohm resistance using Thevenin's theorem. [2071-Shrawan]

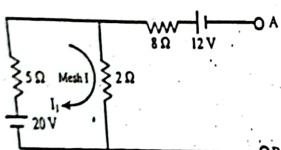


Solution :-

Here, the given circuit is,



To find V_{Th} :



Applying KVL on mesh I, we get

$$20 - 5I_1 - 2I_1 = 0$$

$$\text{or, } 7I_1 = 20$$

$$\therefore I_1 = \frac{20}{7} \text{ A}$$

$$\therefore V_{Th} = V_{AB} = V_A - V_B$$

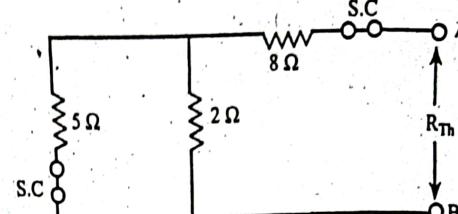
$$= 2I_1 - 8 \times 0 - 12$$

$$= 2 \times \frac{20}{7} - 12$$

$$= -6.286 \text{ V}$$

Which indicates B is at higher potential with respect to A.

To find R_{Th} :

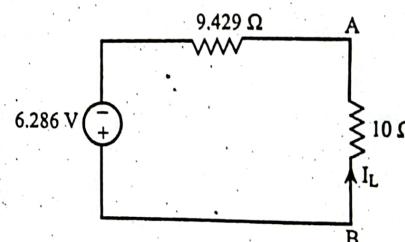


$$R_{Th} = (5/2) + 8$$

$$= \frac{5 \times 2}{5+2} + 8$$

$$= 9.429 \Omega$$

To find current I_L through $R_L = 10\Omega$:



By Ohm's law,

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{6.286}{9.429 + 10}$$

$$= 0.3235 \text{ A (B to A)}$$

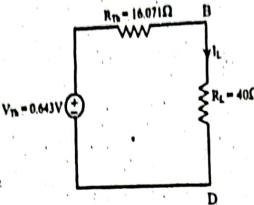
To find current I_L in $R_L = 40\Omega$:

By Ohm's law,

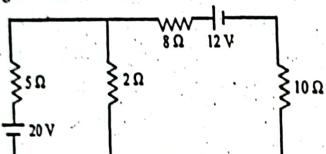
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{0.643}{16.071 + 40}$$

$$= 0.0115 \text{ A (B to D)}$$

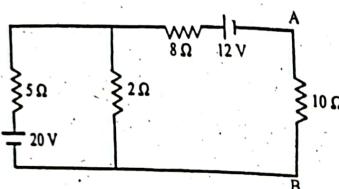
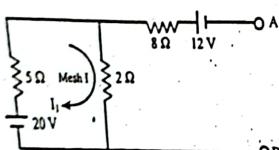


6. For the circuit shown in figure below, calculate the current in the 10Ω resistance using Thevenin's theorem. [2071-Shrawan]



Solution :-

Here, the given circuit is,

To find V_{Th} :

Applying KVL on mesh I, we get

$$20 - 5I_1 - 2I_1 = 0$$

$$\text{or, } 7I_1 = 20$$

$$\therefore I_1 = \frac{20}{7} \text{ A}$$

$$\therefore V_{Th} = V_{AB} = V_A - V_B$$

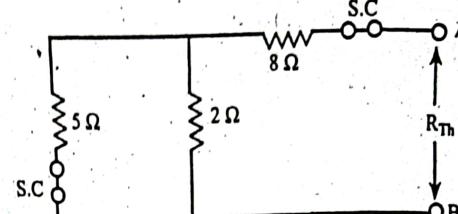
[write KVL equation; move from B to A]

$$= 2I_1 - 8 \times 0 - 12$$

$$= 2 \times \frac{20}{7} - 12$$

$$= -6.286 \text{ V}$$

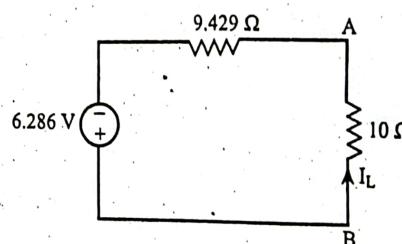
Which indicates B is at higher potential with respect to A.

To find R_{Th} :

$$R_{Th} = (5/2) + 8$$

$$= \frac{5 \times 2}{5+2} + 8$$

$$= 9.429 \Omega$$

To find current I_L through $R_L = 10\Omega$:

By Ohm's law,

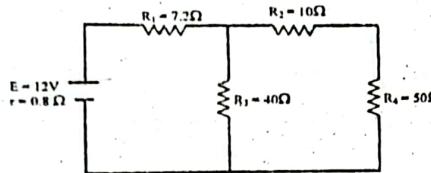
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{6.286}{9.429 + 10}$$

$$= 0.3235 \text{ A (B to A)}$$

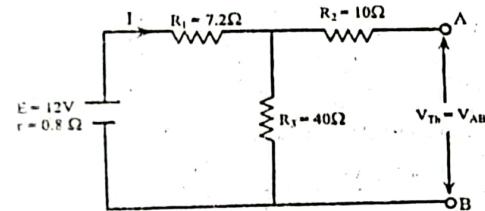
Additional questions

1. Determine the current through 50Ω resistance in the circuit shown below using Thevenin's theorem.



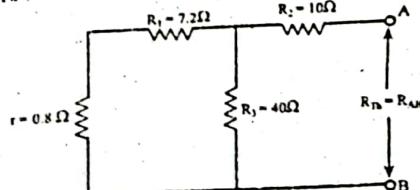
Solution:

To find V_{Th} :



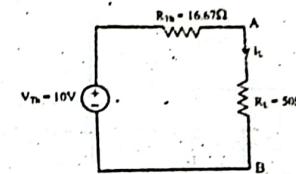
$$\begin{aligned} V_{Th} &= V_{AB} = IR_3 \\ &= \left(\frac{E}{r + R_1 + R_3} \right) \times R_3 \\ &= \left(\frac{12}{0.8 + 7.2 + 40} \right) \times 40 \\ &= 10V \end{aligned}$$

To find R_{Th} :



$$\begin{aligned} R_{Th} &= [(R_1 + r) \parallel R_3] + R_2 \\ &= [(7.2 + 0.8) \parallel 40] + 10 \\ &= (8 \parallel 40) + 10 \\ &= \frac{8 \times 40}{8 + 40} + 10 \\ &= 16.67\Omega \end{aligned}$$

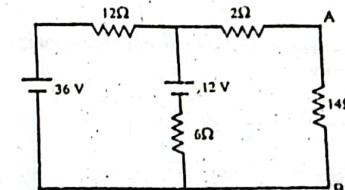
To find current I_L in $R_L = 50\Omega$:



By Ohm's law,

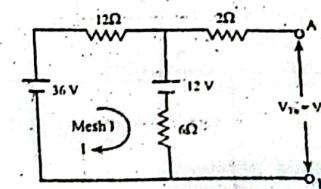
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{10}{16.67 + 50} = 0.15A \text{ (A to B)}$$

2. Using Thevenin's theorem, calculate the potential difference across terminals A and B of the given circuit.



Solution:

To find V_{Th} :



Applying KVL on Mesh I, we get

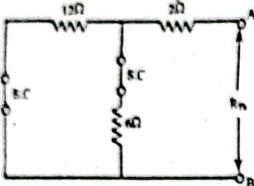
$$36 - 12I - 12 - 6I = 0$$

$$\text{or, } -18I + 24 = 0$$

$$I = \frac{4}{3}A$$

$$\begin{aligned} V_{Th} &= V_{AB} = V_A - V_B \\ &= 6I + 12 \quad [\text{Write KVL equation; move from B to A}] \\ &= 6 \times \frac{4}{3} + 12 = \frac{24}{3} + 12 = 20V \end{aligned}$$

Which indicates A is at higher potential with respect to B.

To find R_{Th} :

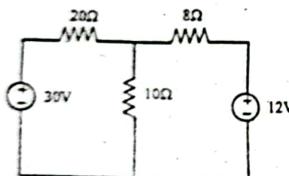
$$R_{Th} = (12 \parallel 6) + 2 = \frac{12 \times 6}{12+6} + 2 = 6 \Omega$$

To find p.d across 14 Ω resistance.

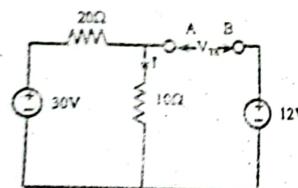
Using voltage divider rule,

$$\begin{aligned} \text{P.d across } 14 \Omega \text{ resistance} &= \frac{V_{Th}}{R_{Th} + R_L} \times R_L \\ &= \frac{20}{6+14} \times 14 = 14 \text{ V} \end{aligned}$$

3. Using Thevenin's theorem, calculate the current through the 8 Ω resistor of the circuit given below.

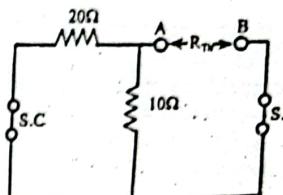


Solution:

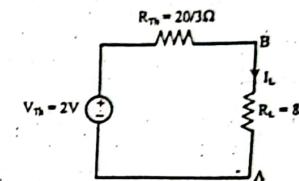
To find V_{Th} :

$$\begin{aligned} V_{Th} &= V_{AB} = V_A - V_B \\ &= -12 + 10 \times I \quad [\text{Write KVL equation; move from B to A}] \\ &= -12 + 10 \times \frac{30}{(20+10)} \\ &= -2 \text{ V} \end{aligned}$$

It means that point A is at lower potential with respect to B, or point B is at a higher potential than point A.

To find R_{Th} :

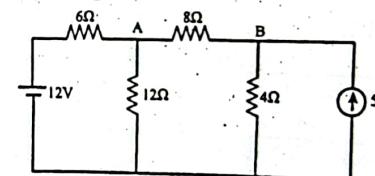
$$R_{Th} = 20 \parallel 10 = \frac{20 \times 10}{20+10} = \frac{20}{3} \Omega$$

To find current I_L in $R_L = 8\Omega$:

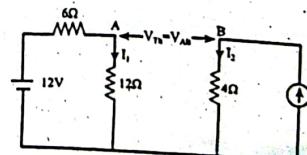
By Ohm's law,

$$\begin{aligned} I_L &= \frac{V_{Th}}{R_{Th} + R_L} = \frac{2}{\frac{20}{3} + 8} \\ &= \frac{3}{22} = 0.136 \text{ A (B to A)} \end{aligned}$$

4. Using Thevenin's theorem, calculate the current flowing through the 8Ω resistor in the circuit given below.



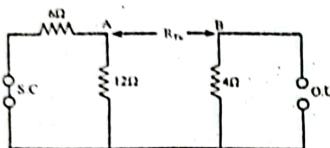
Solution:

To find V_{Th} :

$$\begin{aligned} V_{Th} &= V_{AB} = V_A - V_B \\ &= -4I_2 + 12I_1 \quad [\text{Write KVL equation; move from B to A}] \\ &= -4 \times 5 + 12 \times \frac{12}{12+16} \\ &= -12 \text{ V} \end{aligned}$$

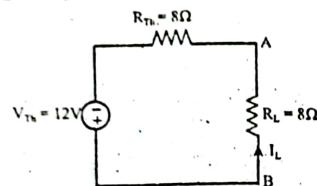
Which indicates that B is at higher potential with respect to A.

To find R_{Th} :



$$R_{Th} = (6 \parallel 12) + 4 = \frac{6 \times 12}{6 + 12} + 4 = 8\Omega$$

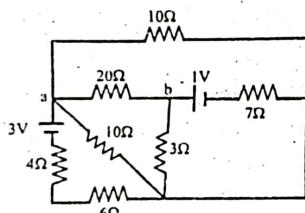
To find current I_L in $R_L = 8\Omega$:



By Ohm's law

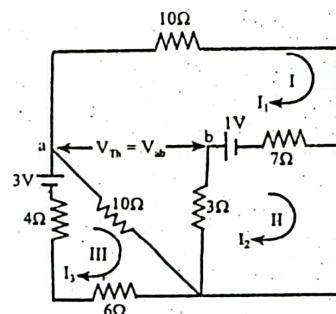
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{12}{8 + 8} = 0.75 \text{ A} \quad (\text{B to A})$$

5. By the use of Thevenin's theorem, find the value of the current in 20 ohms resistor.



Solution:

To find V_{Th} :



Applying KVL to mesh I, we get

$$-10I_1 - 7(I_1 - I_2) + 1 - 3(I_2 - I_3) - 10(I_3 - I_1) = 0$$

$$\text{or, } -30I_1 + 10I_2 + 10I_3 = -1 \quad \dots\dots\dots(1)$$

Applying KVL to mesh II, we get

$$-3(I_2 - I_1) - 1 - 7(I_2 - I_1) = 0$$

$$\text{or, } 10I_1 - 10I_2 = 1 \quad \dots\dots\dots(2)$$

Applying KVL to mesh III, we get

$$-(6 + 4)I_3 + 3 - 10(I_3 - I_1) = 0$$

$$\text{or, } 10I_1 - 20I_3 = -3 \quad \dots\dots\dots(3)$$

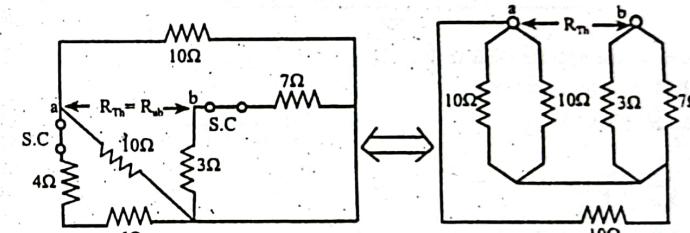
Solving equations (1), (2) and (3) we get

$$I_1 = \frac{1}{10} \text{ A}, I_2 = 0, I_3 = \frac{1}{5} \text{ A}$$

$$V_{Th} = V_{ab} = V_a - V_b \\ = -3(I_1 - I_2) + 10(I_3 - I_1) \quad [\text{write KVL equation; move from B to A}]$$

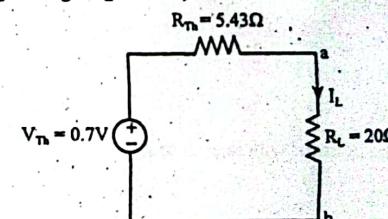
$$= -3 \times \left(\frac{1}{10} - 0\right) + 10 \left(\frac{1}{5} - \frac{1}{10}\right) \\ = -\frac{3}{10} + 10 \times \frac{1}{10} = 0.7 \text{ V}$$

To find R_{Th} :



$$R_{Th} = [(10 \parallel 10) \parallel 10] + [3 \parallel 7] \\ = \frac{10}{3} + 2.1 \\ = 5.43 \Omega$$

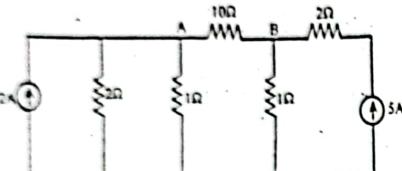
To find current I_L through $R_L = 20 \Omega$:



By Ohm's law,

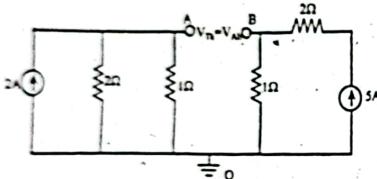
$$\begin{aligned} I_L &= \frac{V_{Th}}{R_{Th} + R_L} \\ &= \frac{0.7}{3.43 + 20} \\ &= 0.028 \text{ A (A to B)} \end{aligned}$$

6. What is the power loss in the 10Ω resistor? Use Thevenin's theorem.



Solution:

To find V_{Th} :



Here, we use nodal analysis to find V_{Th}

Applying KCL at node A, we get

$$\frac{V_A - 0}{2} + \frac{V_A - 0}{1} = 2$$

$$\text{or, } V_A \left(\frac{1}{2} + 1 \right) = 2$$

$$\therefore V_A = \frac{4}{3} \text{ V}$$

Also,

Applying KCL at node B, we get

$$\frac{V_B - 0}{1} = 5$$

$$\therefore V_B = 5 \text{ V}$$

$$V_{Th} = V_{AB} = V_A - V_B$$

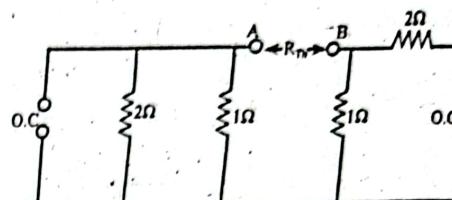
$$= \frac{4}{3} - 5$$

$$= -3.667 \text{ V}$$

Which indicates B is at higher potential with respect to A.

$$\therefore V_{Th} = 3.667 \text{ V}$$

To find R_{Th} :

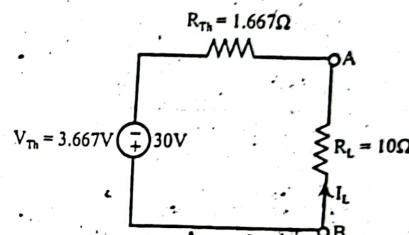


$$R_{Th} = R_{AB} = (2 \parallel 1) + 1$$

$$= \frac{2 \times 1}{2+1} + 1$$

$$= 1.667 \Omega$$

To find current I_L through $R_L = 10\Omega$:



By Ohm's law

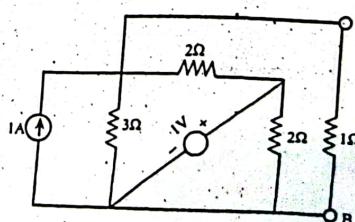
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{3.667}{1.667 + 10}$$

$$= 0.314 \text{ A (B to A)}$$

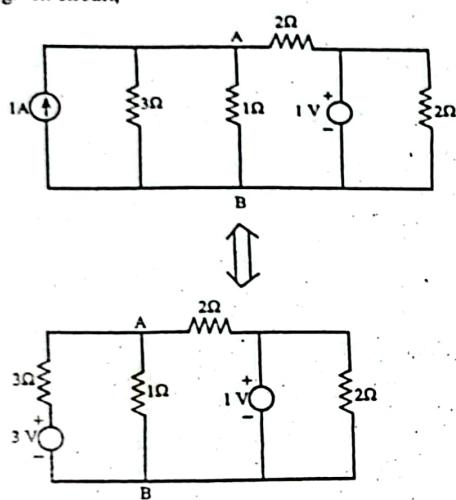
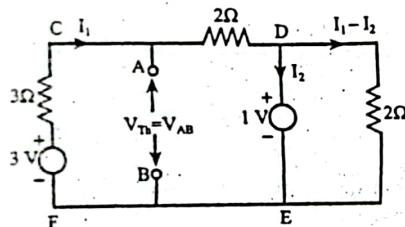
$$\begin{aligned} \text{Power loss in } 10\Omega \text{ resistor} &= (0.314)^2 \times 10 \\ &= 0.986 \text{ W} \end{aligned}$$

7. Determine the current in the 1Ω resistor across AB of network shown below using Thevenin's theorem.



Solution:

Redrawing of the given circuit,

To find V_{Th} :

Applying KVL to mesh CDEFC, we get

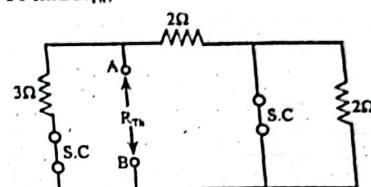
$$3 - 3I_1 - 2I_2 - 1 = 0$$

$$\text{or, } 2 - 5I_1 = 0$$

$$\therefore I_1 = \frac{2}{5} \text{ A}$$

$$\begin{aligned} \therefore V_{Th} &= V_{AB} = V_A - V_B \\ &= 3 - 3I_1 \quad [\text{Write KVL equation; move from B to A}] \\ &= 3 - 3 \times \frac{2}{5} = 1.8 \text{ V} \end{aligned}$$

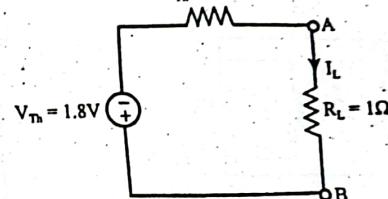
which indicates A is at higher potential with respect to B.

To find R_{Th} :

$$\begin{aligned} R_{Th} &= 3 \parallel 2 = \frac{3 \times 2}{3 + 2} \\ &= 1.2 \Omega \end{aligned}$$

To find current I_L through $R_L = 1\Omega$:

$$R_{Th} = 1.2 \Omega$$



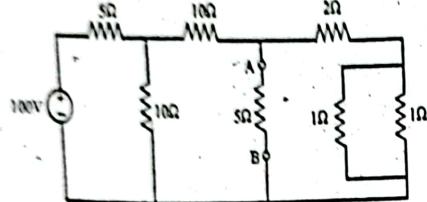
By Ohm's law

$$\begin{aligned} I_L &= \frac{V_{Th}}{R_{Th} + R_L} \\ &= \frac{1.8}{1.2 + 1} \\ &= 0.818 \text{ A (A to B)} \end{aligned}$$

Norton's Theorem

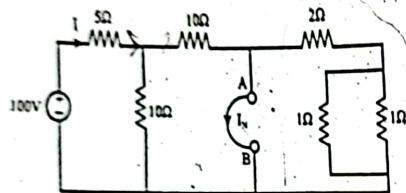
1. Determine the current flowing through the 5Ω resistor connected between AB in the circuit shown below using Norton's theorem.

[2066 Kartik]



Solution:

To find I_N :



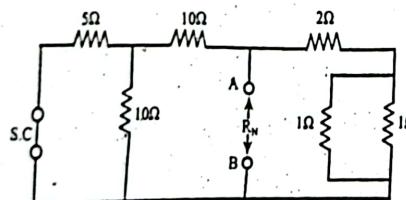
Here,

Current supplied by the battery

$$I = \frac{100}{(10 \parallel 10) + 5} = \frac{100}{\frac{10 \times 10}{10+10} + 5} = 10 \text{ A}$$

$$\text{Short circuit current, } I_N = \frac{I}{10+10} \times 10 = \frac{10}{10+10} \times 10 = 5 \text{ A (A to B)}$$

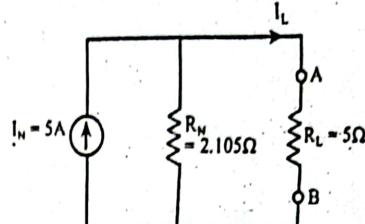
To find R_N :



Resistance of the network seen from the terminals A and B

$$\begin{aligned} R_N &= R_{AB} \\ &= [(5 \parallel 10) + 10] \parallel [2 + (1 \parallel 1)] \\ &= \left(\frac{50}{15} + 10\right) \parallel \left(2 + \frac{1}{2}\right) = \frac{40}{3} \parallel \frac{5}{2} = \frac{\frac{40}{3} \times \frac{5}{2}}{\frac{40}{3} + \frac{5}{2}} = \frac{40}{19} = 2.105 \Omega \end{aligned}$$

To find current I_L in $R_L = 5\Omega$:



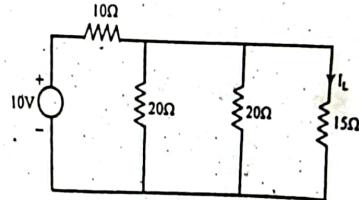
Using current division rule,

Current through 5Ω resistor,

$$I_L = \frac{5}{2.105 + 5} \times 2.105 = 1.48 \text{ A (A to B)}$$

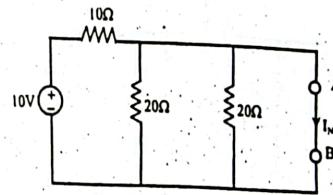
2. Determine the current I_L through 15Ω resistor in the network by Norton's theorem.

[2067 Ashadhi]



Solution:

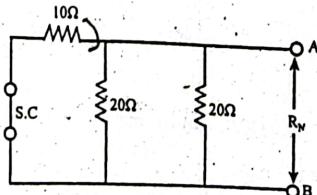
To find I_N :



With 15Ω resistor removed and terminals AB short circuited,

$$\text{Short circuit current, } I_N = \frac{10}{10} = 1 \text{ A (A to B)}$$

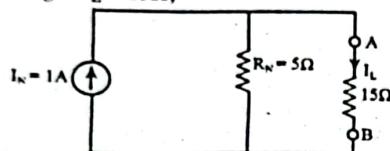
To find R_N :



Resistance of the network seen from the terminals A and B is:

$$\begin{aligned} R_N &= 10 \parallel 20 \parallel 20 \\ &= \frac{10 \times 20 \times 20}{10 \times 20 + 20 \times 20 + 20 \times 10} = \frac{4000}{800} = 5 \Omega \end{aligned}$$

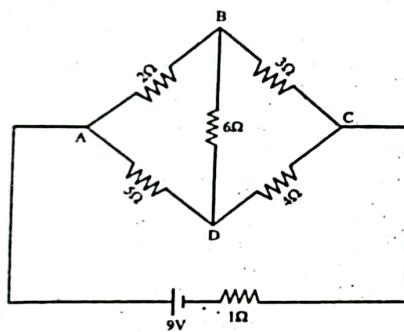
To find current I_L through $R_L = 15\Omega$:



Using current division rule,

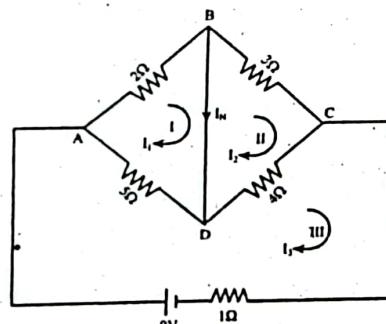
$$\text{Current through } 15\Omega \text{ resistor} = \frac{1}{5+15} \times 5 = \frac{1}{4} = 0.25 \text{ A (A to B or top to bottom)}$$

3. Calculate the current in the 6Ω resistor in the network shown below using Norton's theorem. [2067 Mangsir]



Solution:

To find I_N :



Using mesh analysis, applying KVL on mesh I, mesh II and mesh III

Mesh I:

$$-2I_1 - 5(I_1 - I_3) = 0$$

$$\text{or, } -2I_1 - 5I_1 + 5I_3 = 0$$

$$\text{or, } -7I_1 + 5I_3 = 0 \dots \text{(i)}$$

Mesh II:

$$-3I_2 - 4(I_2 - I_1) = 0$$

$$\text{or, } -7I_2 + 4I_1 = 0 \dots \text{(ii)}$$

Mesh III:

$$9 - 5(I_3 - I_1) - 4(I_3 - I_2) - I_3 = 0$$

$$\text{or, } 9 - 5I_3 + 5I_1 - 4I_3 + 4I_2 - I_3 = 0$$

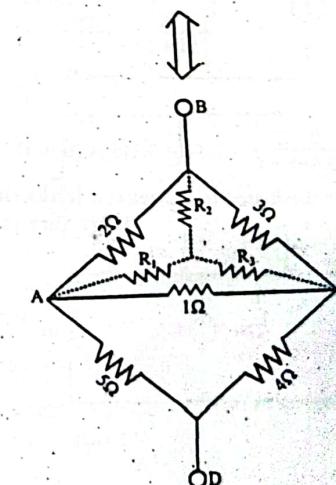
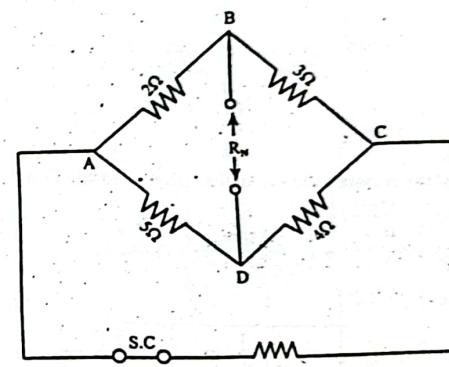
$$\text{or, } 5I_1 + 4I_2 - 10I_3 = -9 \dots \text{(iii)}$$

Solving equations (i), (ii) and (iii), we get

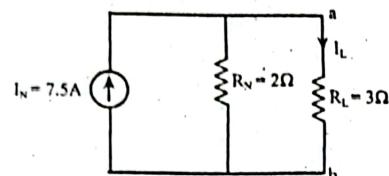
$$I_1 = 1.552 \text{ A}, I_2 = 1.241 \text{ A}, I_3 = 2.172 \text{ A}$$

$$\therefore I_N = I_1 - I_2 = 1.552 - 1.241 = 0.311 \text{ A (B to D)}$$

To find R_N :



To find current I_L in $R_L = 3\Omega$



Using current division rule,

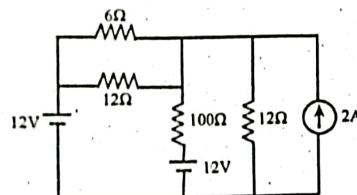
$$\text{Current in } 3\Omega \text{ resistor, } I_L = \frac{7.5}{2+3} \times 2$$

$$\therefore I_L = 3 \text{ A (a to b)}$$

Hence,

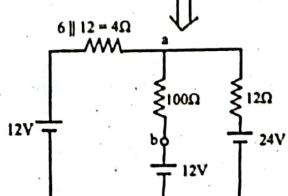
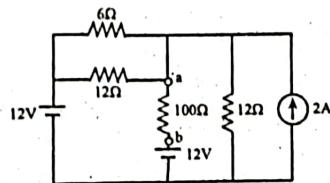
the power dissipated in 3Ω resistor in the circuit,
 $= I_L^2 \times 3 = 3^2 \times 3 = 9 \times 3 = 27 \text{ W}$

5. Use Norton's theorem to find the current through 100Ω resistor of the circuit below. [2070 Magh]

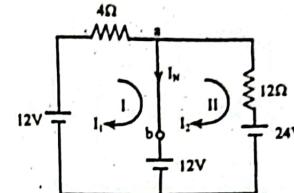


Solution:

The given circuit is;



To find I_{N1}



Applying KVL to mesh I, we get

$$12 - 4I_1 - 12 = 0$$

$$\therefore I_1 = 0$$

Applying KVL to mesh II, we get

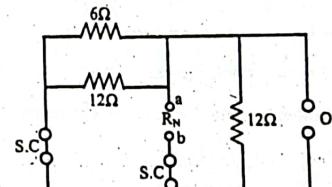
$$-12 I_2 - 24 + 12 = 0$$

$$\therefore I_2 = -1 \text{ A}$$

Negative current indicates that the direction of current is opposite to our assumed direction.

$$\therefore I_N = I_2 = 1 \text{ A (a to b)}$$

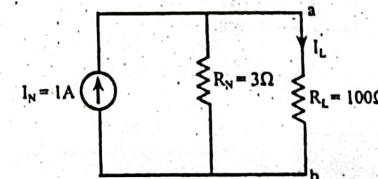
To find R_N :



Resistance of the network seen from the terminals a and b is,

$$R_N = R_{ab} = 6 \parallel 12 \parallel 12 = \frac{6 \times 12 \times 12}{6 \times 12 + 12 \times 12 + 12 \times 6} = 3\Omega$$

To find current I_L through $R_L = 100\Omega$:



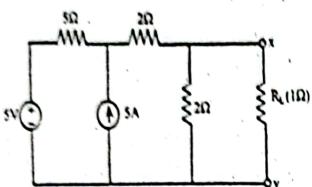
using current division rule,

$$\text{Current through } 100\Omega \text{ resistor, } I_L = \frac{1}{3+100} \times 3 = 0.02912 \text{ A (a to b)}$$

6. Write down the steps to calculate Norton's equivalent resistance in the circuit with a suitable example. [2070 Ashad]

Solution:

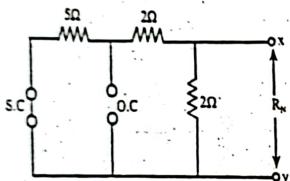
Suppose we have a circuit.



Here, we want to find Norton's equivalent resistance across terminals xy. For this,

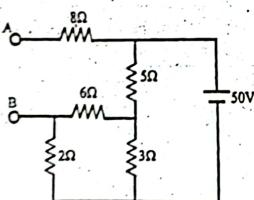
- We remove load resistance (R_L), thus creating an open circuit at terminals xy.
- We redraw the circuit with all the voltage sources short circuited and all current sources open circuited.
- Determining the resistance of the network as seen from the terminals xy as R_N (Norton's equivalent resistance).

To find R_N in above circuit;



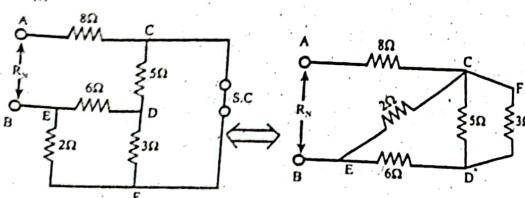
$$R_N = (5 + 2) \parallel 2 = 7 \parallel 2 = \frac{7 \times 2}{7+2} = 1.556 \Omega$$

- Find the Norton's equivalent resistance between the terminals A and B in the given circuit. [2070 Bhadra]



Solution:

To find R_N :



Norton's equivalent resistance

$$\begin{aligned} R_N &= R_{AB} = [[(5 \parallel 3) + 6] \parallel 2] + 8 \\ &= \left[\left(\frac{15}{8} + 6 \right) \parallel 2 \right] + 8 \\ &= [7.875 \parallel 2] + 8 \\ &= \frac{126}{79} + 8 \\ &= 9.595 \Omega \end{aligned}$$

- Thevenin's theorem and Norton's theorem are dual of each other. Justify the statement with suitable example. [2070 Ashad]

Ans:

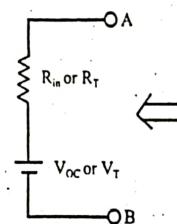


Fig: (a)

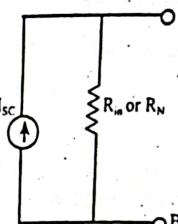


Fig: (b)

Thevenin's equivalent can be converted into its Norton's equivalent and vice-versa. Thevenin's equivalent is shown in fig (a). Norton's current source equals the current I_{SC} which flows through a short across terminals A and B.

Hence,

$$I_{SC} = \frac{V_{oc}}{R_{in}}$$

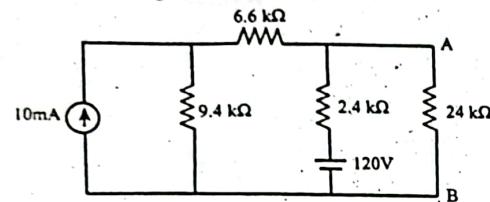
Likewise a Norton's circuit can be converted into its Thevenin's equivalent. The Thevenin's equivalent source V_{oc} or V_T is the voltage on open circuit and is given as

$$V_{oc} \text{ or } V_T = I_{SC} R_{in}$$

Hence, Thevenin's theorem and Norton's theorem are dual of each other.

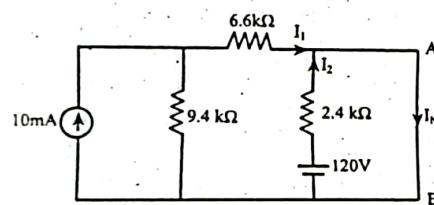
Additional Questions:

1. For the network shown below derive Norton's equivalent circuit and find the current through $24\text{ k}\Omega$ resistance.



Solution:

To find I_N :

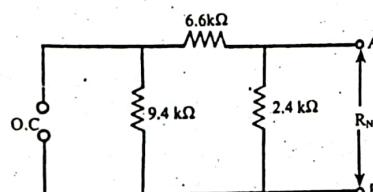


I_N = Current due to 10 mA source + Current due to 120 V source

$$= I_1 + I_2$$

$$= \frac{10}{9.4 + 6.6} \times 9.4 + \frac{120}{2.4} = 55.875 \text{ mA (A to B)}$$

To find R_N :

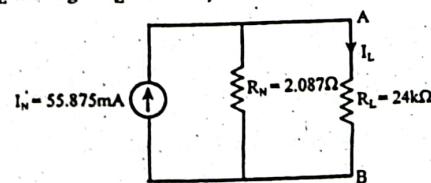


Resistance of the network seen from the terminals A and B is;

$$R_N = R_{AB} = (6.6 + 9.4) \parallel 2.4$$

$$= 16 \parallel 2.4 = 2.087 \Omega$$

To find current I_L through $R_L = 24\text{ k}\Omega$:

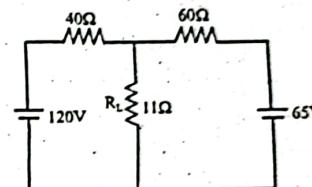


Using current division rule,

Current through $24\text{ k}\Omega$ resistor,

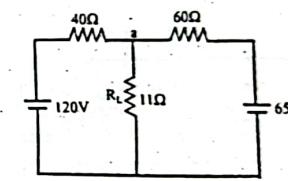
$$I_L = \frac{55.875}{2.087 + 24} \times 2.087 = 4.47 \text{ mA}$$

2. Find the current through R_L in the circuit below using Norton's theorem.

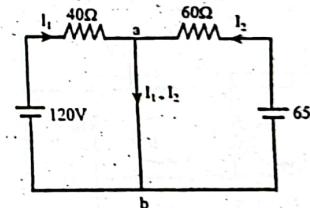


Solution:

The given circuit is;



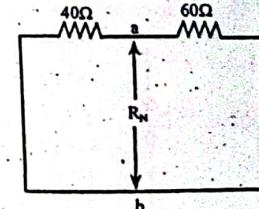
To find I_N :



$$I_N = I_1 + I_2$$

$$= \frac{120}{40} + \frac{65}{60} = 4.083 \text{ A (a to b)}$$

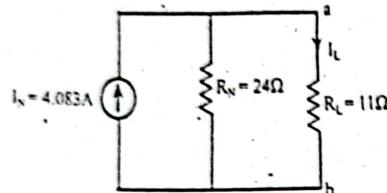
To find R_N :



Resistance of the network seen from the terminals a and b is;

$$R_N = R_{ab} = 40 \parallel 60 \\ = \frac{40 \times 60}{40 + 60} = 24 \Omega$$

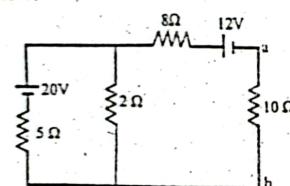
To find Current I_L through $R_L = 11\Omega$;



Using current division rule,

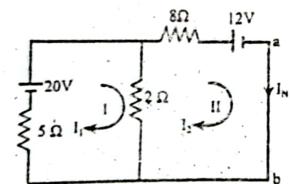
$$I_L = \frac{4.083}{24 + 11} \times 24 = 2.8 \text{ A (a to b)}$$

3. Use Norton's theorem to find the current in 10Ω resistance in the circuit given below:



Solution:

To find I_N ;



Applying KVL on mesh I, we get

$$20 - 2(I_1 - I_2) - 5I_1 = 0$$

or, $-2I_1 + 2I_2 - 5I_1 = -20$

or, $-7I_1 + 2I_2 = -20 \dots\dots\dots (i)$

Applying KVL on mesh II, we get

$$-8I_2 - 12 - 2(I_2 - I_1) = 0$$

or, $-8I_2 - 12 - 2I_2 + 2I_1 = 0$

or, $2I_1 - 10I_2 = 12 \dots\dots\dots (ii)$

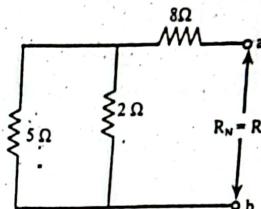
Solving equations (i) and (ii), we get

$$I_1 = \frac{8}{3} \text{ A}, I_2 = -\frac{2}{3} \text{ A}$$

Negative current indicates that our direction of assumption of flow of current is opposite to actual flow of current.

$$\therefore I_2 = I_N = \frac{2}{3} \text{ A (b to a)}$$

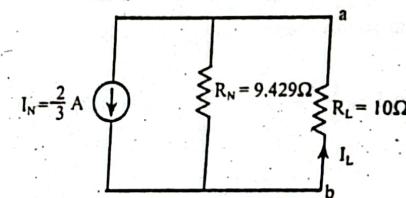
To find R_N ;



Resistance of the network seen from the terminals a and b is;

$$R_N = R_{ab} = (5 \parallel 2) + 8 \\ = \frac{5 \times 2}{5 + 2} + 8 = 9.429 \Omega$$

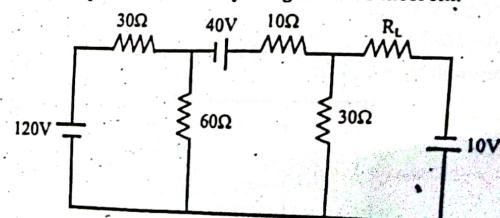
To find Current I_L through $R_L = 10\Omega$;



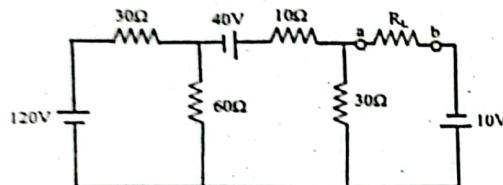
Using current division rule,

$$I_L = \frac{2}{3} \times \frac{9.429}{9.429 + 10} = 0.324 \text{ A (b to a)}$$

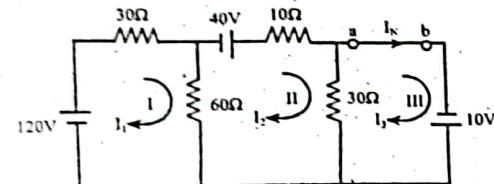
4. The two terminal battery resistor network shown below is to be connected to the load resistor (R_L) of 24 ohms. Determine the load current and power delivered by using Norton's theorem.



Solution:



To find I_N :



Applying KVL to mesh I, we get

$$120 - 30I_1 - 60(I_1 - I_2) = 0$$

or, $-90I_1 + 60I_2 = -120 \dots\dots\dots (i)$

Applying KVL to mesh II, we get

$$40 - 10I_2 - 30(I_2 - I_3) - 60(I_2 - I_1) = 0$$

or, $60I_1 - 100I_2 + 30I_3 = -40 \dots\dots\dots (ii)$

Applying KVL to mesh III, we get

$$10 - 30(I_3 - I_2) = 0$$

or, $30I_2 - 30I_3 = -10 \dots\dots\dots (iii)$

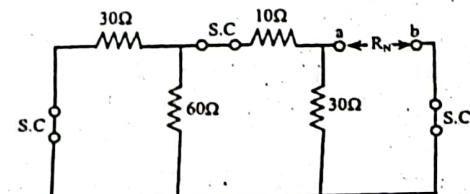
Solving equations (i), (ii) and (iii), we get

$$I_1 = 4.22 \text{ A}, I_2 = 4.33 \text{ A}, I_3 = 4.67 \text{ A}$$

\therefore Norton's current or short circuit current is,

$$I_N = I_3 = 4.67 \text{ A} \text{ (a to b)}$$

To find R_N :



Resistance of the network seen from the terminals a and b is;

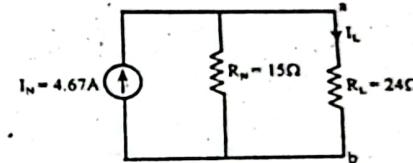
$$R_N = R_{ab} = [(30 \parallel 60) + 10] \parallel 30$$

$$= [20 + 10] \parallel 30$$

$$= 30 \parallel 30$$

$$= 15 \Omega$$

To find current I_L through $R_L = 24 \Omega$:



Using current division rule,

$$I_L = \frac{I_N}{R_N + R_L} \times R_L$$

$$= \frac{4.67}{15 + 24} \times 15$$

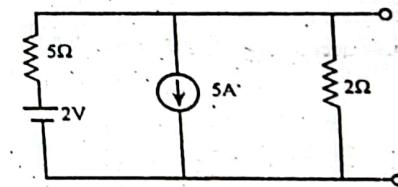
$$= 1.796 \text{ A}$$

Power delivered, $P = I_L^2 R_L$

$$= (1.796)^2 \times 24$$

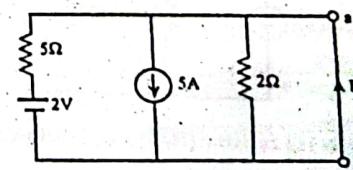
$$= 77.415 \text{ W}$$

5. Find the Norton's equivalent circuit across a – b for the network shown in figure below. Use Superposition theorem to find the short circuit current at network terminals.

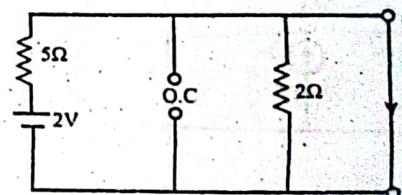


Solution:

To find I_N :

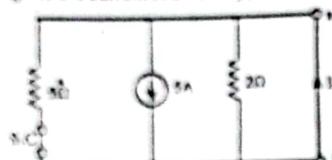


Firstly taking the 2 V source only,



$$I_1 = \frac{V}{R} = \frac{10}{5} = 2 \text{ A} \quad (\text{x to y})$$

Secondly, taking the 5A current source only,



$$I_2 = 5 \text{ A} \quad (\text{b to x})$$

Now,

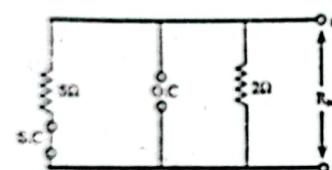
From principle of Superposition, we get

$$I_{N1} = I_1 + I_2$$

$$= 2 + 5 = 7 \text{ A}$$

$$= 4.6 \text{ A} \quad (\text{b to x})$$

To find R_N :

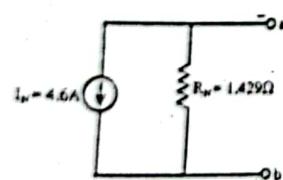


Resistance of the network seen from the terminals a and b is;

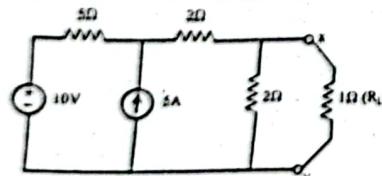
$$R_N = R_{ab} = 5 \parallel 2$$

$$= \frac{5 \times 2}{5 + 2} = 1.429 \Omega$$

Norton's equivalent circuit;

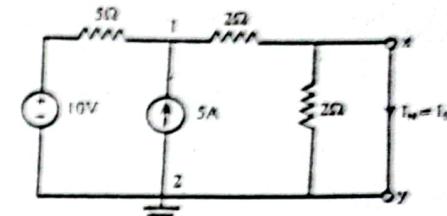


6. Find the power loss in 1Ω resistor (R_L) using Norton's theorem.



Solution:

To find I_N :



Assuming node 1 with potential V and node 2 as the reference node.

Then,

Applying KCL at node 1, we get

$$\frac{V - 10 - 0}{5} + \frac{V - 0}{2} = 5$$

$$\text{or, } \frac{V - 10}{5} + \frac{V}{2} = 5$$

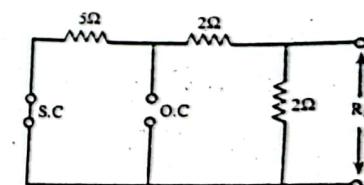
$$\text{or, } \frac{V}{5} + \frac{V}{2} = 5 + \frac{10}{5}$$

$$\text{or, } \left(\frac{1}{5} + \frac{1}{2} \right) V = 7$$

$$\therefore V = 10V$$

$$\therefore I_N = \frac{V - 0}{2} = \frac{10}{2} = 5 \text{ A} \quad (\text{x to y})$$

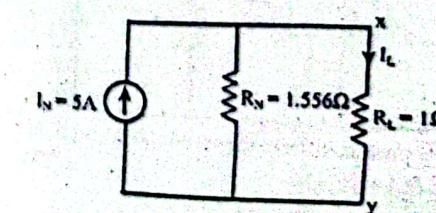
To find R_N :



Resistance of the network seen from the terminals x and y is;

$$R_N = R_{xy} = (5 + 2) \parallel 2 = 7 \parallel 2 = 1.556 \Omega$$

To find current I_L through $R_L = 1 \Omega$:

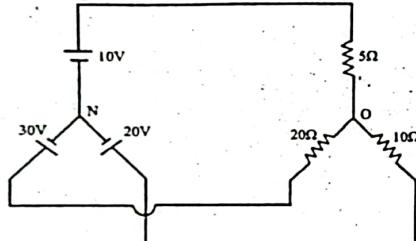


Using current division rule,

$$I_L = \frac{5}{1.556 + 1} \times 1.556 = 3.043 \text{ A (x to y)}$$

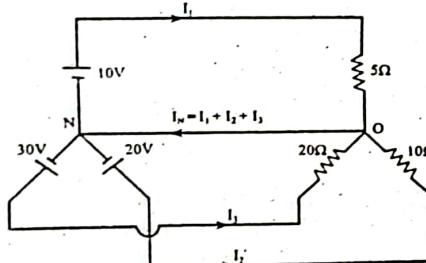
∴ Power loss in 1Ω resistor = $(3.043)^2 \times 1 = 9.26 \text{ W}$

7. Using Norton's theorem, find current which would flow in a 25Ω resistor connected between points N and O in figure below:



Solution:

To find I_N :

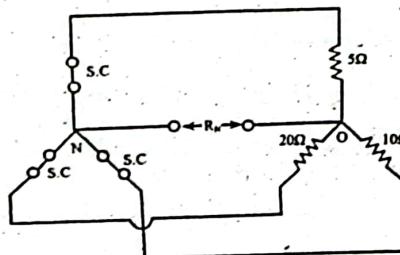


Total current in short-circuit across ON is equal to the sum of currents driven by different batteries through their respective resistances.

$$I_N = I_{SC} = I_1 + I_2 + I_3$$

$$= \frac{10}{5} + \frac{20}{10} + \frac{30}{20} = 5.5 \text{ A}$$

To find R_N :

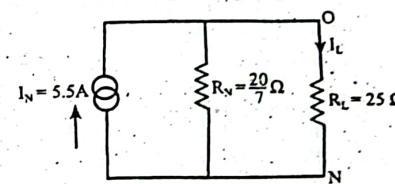


Resistance of the network seen from the terminals N and O is;

$$R_N = R_{NO} = 5 \parallel 20 \parallel 10$$

$$= \frac{5 \times 20 \times 10}{5 \times 20 + 20 \times 10 + 10 \times 5} = \frac{20}{7} \Omega$$

To find current I_L through $R_L = 25 \Omega$:



Using current division rule,

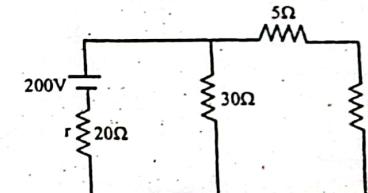
$$I_L = \frac{I_N}{R_N + R_L} \times R_N = \frac{5.5}{\frac{20}{7} + 25} \times \frac{20}{7} = 0.564 \text{ A (O to N)}$$

Maximum Power Transfer Theorem

Exam solutions

1. In the network shown below find the resistance R so that maximum power is transferred by 200 V source of internal resistance 20Ω .

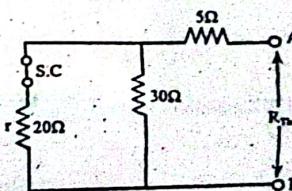
[2003 Kartik]



Solution:

In order to determine maximum power transfer, we determine the Thevenin's equivalent network.

To find R_{TH} :



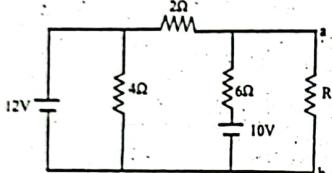
$$R_{Th} = (20 \parallel 30) + 5$$

$$= \frac{20 \times 30}{20 + 30} + 5 = 17 \Omega$$

Hence, the value of resistance R must be 17Ω for the maximum power to be drawn by it following maximum power transfer theorem ($R = R_{Th} = 17 \Omega$).

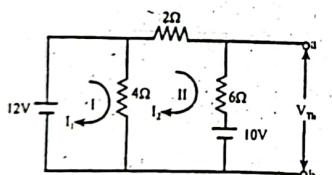
2. Calculate the value of R to receive maximum power and hence the maximum power received by it for the network given in figure.

[2064 Poush]

**Solution:**

Here,

The open circuit voltage V_{OC} (also called Thevenin's voltage V_{Th}) which appears across terminals a and b is calculated and also the resistance of the circuit as looked into the network (also called Thevenin's resistance R_{Th}) from the points a and b is calculated.

To find V_{Th} :

Applying KVL on mesh I, we get

$$12 - 4(I_1 - I_2) = 0$$

$$\text{or, } -4I_1 + 4I_2 = -12 \quad \dots \dots \text{(i)}$$

Applying KVL on mesh II, we get

$$-2I_2 - 6I_2 - 10 - 4(I_2 - I_1) = 0$$

$$\text{or, } -8I_2 - 10 - 4I_2 + 4I_1 = 0$$

$$\text{or, } 4I_1 - 12I_2 = 10 \quad \dots \dots \text{(ii)}$$

Solving equations (i) and (ii), we get

$$I_1 = 3.25 \text{ A}, I_2 = 0.25 \text{ A}$$

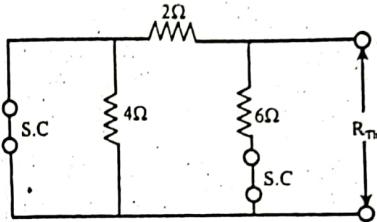
Open circuit voltage

$$V_{OC} = V_{Th} = V_{ab}$$

$$= V_a - V_b$$

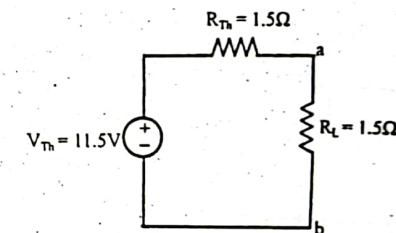
$$= 10 + 6 \times I_2 \quad [\text{Write KVL equation; move from b to a}]$$

$$= 10 + 6 \times 0.25 = 11.5 \text{ V}$$

To find R_{Th} :

$$R_{Th} = 2 \parallel 6 = \frac{2 \times 6}{2 + 6} = 1.5 \Omega$$

For maximum power transfer, R_L should be equal to R_{Th} i.e. 1.5Ω ; whole circuit upto ab can now be replaced by a single source of e.m.f. and resistance as,



$$\text{Maximum power drawn by load} = \frac{V_{Th}^2}{4R_L}$$

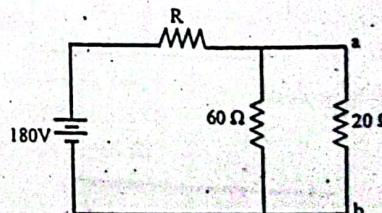
$$= \frac{11.5^2}{4 \times 1.5}$$

$$= 22.042 \text{ W}$$

3. Consider the circuit shown in the figure: Determine

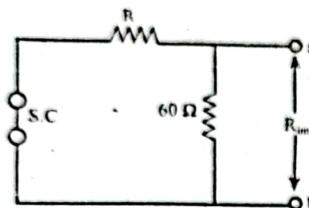
- The value of R so that the load of 20Ω should draw maximum power.
- The value of maximum power drawn by the load.

[2065 Krt]

**Solution:**

Here, load given is of 20Ω . From maximum power transfer theory, maximum power will be delivered to the load when the load resistance is equal to internal resistance of the source ($R_{int} = R_{Th}$). So, $R_{int} = 20 \Omega$.

i. To find R_{in} :



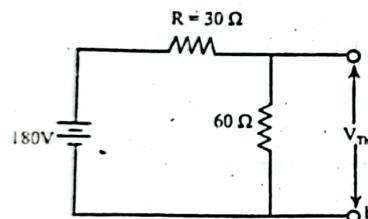
$$R_{\text{in}} = R \parallel 60 = \frac{60R}{R+60}$$

$$\text{or, } 20 = \frac{60R}{R+60}$$

$$\text{or, } 20R + 1200 = 60R$$

$$\therefore R = 30\Omega$$

ii. We find open circuit voltage V_{oc} (also called the Thevenin's voltage V_{th}) across terminals a and b.



$$V_{\text{th}} = V_{ab} = \frac{180}{30+60} \times 60$$

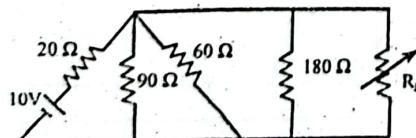
$$V_{\text{th}} = 120 \text{ V}$$

Maximum power drawn by the load,

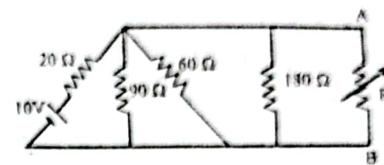
$$\begin{aligned} P_{\text{max}} &= \frac{V_{\text{th}}^2}{4R_L} \\ &= \frac{120^2}{4 \times 20} \\ &= 180 \text{ watt} \end{aligned}$$

4. For the circuit shown below, what should be the value of R_L to get maximum power? What is the maximum power delivered to the load?

[2066 kartlk]



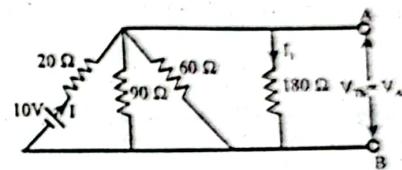
Solution:



Here,

The open circuit voltage V_{oc} (also called Thevenin's voltage V_{th}) which appears across terminals A and B is calculated and also the resistance of the circuit as looked into the network (also called Thevenin's resistance R_{th}) from the points A and B is calculated.

To find V_{th} :

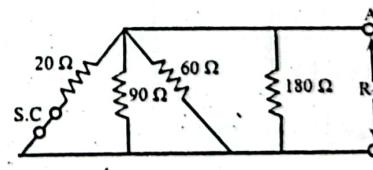


$$\begin{aligned} \text{Current delivered by battery, } I &= \frac{10}{20 + (90 \parallel 60 \parallel 180)} \\ &= \frac{10}{20 + \frac{90 \times 60 \times 180}{(90 \times 60 + 60 \times 180 + 180 \times 90)}} = \frac{10}{20 + 30} = \frac{1}{5} \text{ A} \end{aligned}$$

$$\therefore V_{\text{th}} = V_{AB}$$

$$\begin{aligned} &= \text{Source emf} - \text{voltage drop in } 20\Omega \text{ resistor} \\ &= 10 - 1 \times 20 = 10 - \frac{1}{5} \times 20 = 6 \text{ V} \end{aligned}$$

To find R_{th} :



$$\begin{aligned} R_{\text{th}} &= R_{AB} = 20 \parallel 90 \parallel 60 \parallel 180 \\ &= \left(\frac{20 \times 90}{20+90} \right) \parallel \left(\frac{60 \times 180}{60+180} \right) \\ &= \frac{180}{11} \parallel 45 = 12 \Omega \end{aligned}$$

As per the maximum power transfer theorem,

$$R_{\text{th}} = R_L = 12 \Omega$$

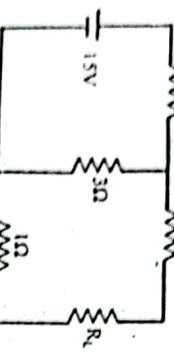
Then,

Amount of maximum power delivered to the load is given by,

$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{6^2}{4 \times 12} = 0.75 \text{ W}$$

5. Find the value of R_L such that maximum power will be transferred to R_L .

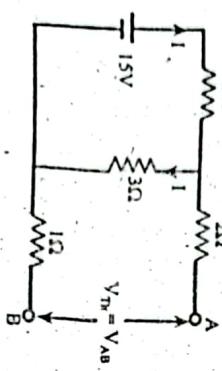
Find the value of the maximum power.



Solution:

In order to determine the maximum power transfer, we determine the Thevenin's equivalent network.

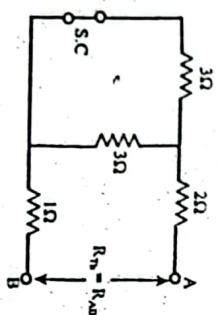
To find V_{Th} :



Using voltage divider rule,

$$V_{Th} = V_{AB} = 1 \times 3 = \frac{15}{3+2} \times 3 = 7.5 \text{ V}$$

To find R_{Th} :



$$\begin{aligned} R_{Th} &= R_{AB} \\ &= (3 \parallel 3) + 2 + 1 \\ &= \frac{3 \times 3}{3+3} + 3 = 4.5 \Omega \end{aligned}$$

As per the maximum power transfer theorem,

$$R_{Th} = R_L = 4.5 \Omega$$

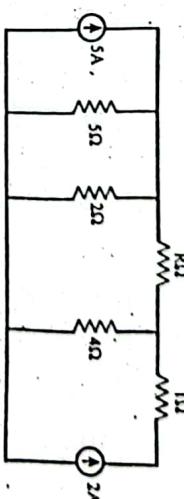
Then,

Amount of maximum power delivered to the load is given by,

$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{7.5^2}{4 \times 4.5} = 3.125 \text{ watt}$$

6. Find the value of R such that maximum power transfer takes place from the current sources to the load R in figure below. Obtain the amount of power transfer.

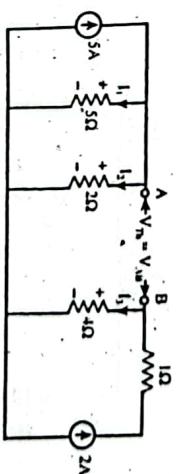
[2068 Bhadra]



Solution:

In order to determine the maximum power transfer, we determine the Thevenin's equivalent circuit.

To find V_{Th} :



Using current division rule,

$$I_B = \frac{5}{5+2} \times 5 = 3.571 \text{ A}$$

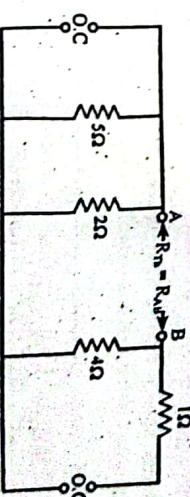
$I_3 = 2 \text{ A}$

Then,

$$\begin{aligned} V_{Th} &= V_{AB} = V_A - V_B \\ &= -4I_3 + 2I_2 \quad [\text{Write KVL equation; move from B to A}] \\ &= -4 \times 2 + 2 \times 3.571 = -0.858 \text{ V} \end{aligned}$$

Which indicates B is at higher potential with respect to A.

To find R_{Th} :



$$\begin{aligned} R_{Th} &= R_{AB} = (5 \parallel 2) + 4 \\ &= \frac{5 \times 2}{5+2} + 4 = 5.429 \Omega \end{aligned}$$

As per maximum power transfer theorem,

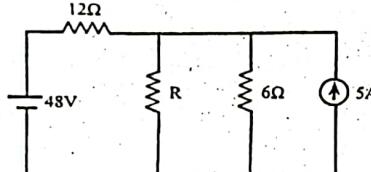
$$R = R_{Th} = 5.429 \Omega$$

Then,

Amount of maximum power transfer is given by,

$$\begin{aligned} P_{max} &= \frac{V_{Th}^2}{4R_{Th}} \\ &= \frac{0.858^2}{4 \times 5.429} = 0.034 \text{ watt} \end{aligned}$$

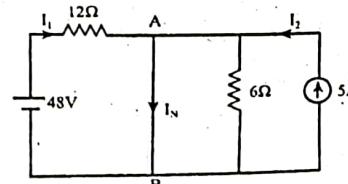
7. Use Norton's theorem to calculate the value of R that will absorb maximum power from the circuit shown in the figure below. Also calculate the maximum power drawn by it. [2069 Chaitra]



Solution:

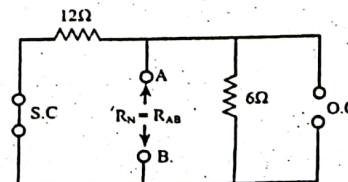
In order to determine the maximum power drawn by R , we first determine Norton's equivalent network.

To find I_N :



$$I_N = I_1 + I_2 = \frac{48}{12} + 5 = 9 \text{ A} \quad (\text{A to B})$$

To find R_N :



$$\begin{aligned} R_N &= R_{AB} = 12 \parallel 6 \\ &= \frac{12 \times 6}{12+6} = 4 \Omega \end{aligned}$$

As per maximum power transfer theorem,

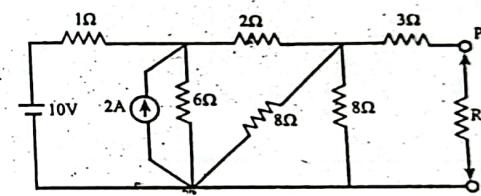
$$R_N = R = 4 \Omega$$

Then,

Maximum power drawn by R ;

$$\begin{aligned} P_{max} &= \left(\frac{I_N}{2}\right)^2 \times R \\ &= \frac{I_N^2}{4} \times R = \left(\frac{9}{2}\right)^2 \times 4 = 81 \text{ watt} \end{aligned}$$

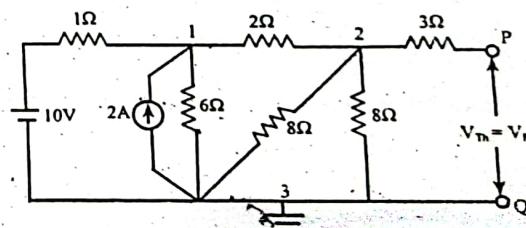
8. Using maximum power transfer theorem, find the value of R_L connected between terminals P and Q so that maximum power is developed across R_L . Find the value of maximum power also. [2068 Magh]



Solution:

In order to determine the maximum power transfer, we determine the Thevenin's equivalent circuit.

To find V_{Th} :



In order to find V_{Th} , here we use nodal analysis.

For this, let node 3 be the reference node. The voltage at node 1 and node 2 be V_1 and V_2 respectively.

Then, Applying KCL at node 1;

$$\frac{V_1 - 10 - 0}{1} + \frac{V_1 - 0}{6} + \frac{V_1 - V_2}{2} = 2$$

$$\text{or, } V_1 - 10 + \frac{V_1}{6} + \frac{V_1}{2} - \frac{V_2}{2} = 2$$

$$\text{or, } \frac{5}{3}V_1 - \frac{V_2}{2} = 12 \quad \dots \dots \dots \text{(i)}$$

Applying KCL at node 2;

$$\frac{V_2 - 0}{8} + \frac{V_2 - 0}{8} + \frac{V_2 - V_1}{2} = 0$$

$$\text{or, } \frac{V_2}{8} + \frac{V_2}{8} + \frac{V_2}{2} - \frac{V_1}{2} = 0$$

$$\text{or, } -\frac{1}{2}V_1 + \frac{3}{4}V_2 = 0 \quad \dots \dots \dots \text{(ii)}$$

Solving equations (i) and (ii), we get

$$V_1 = 9V, V_2 = 6V$$

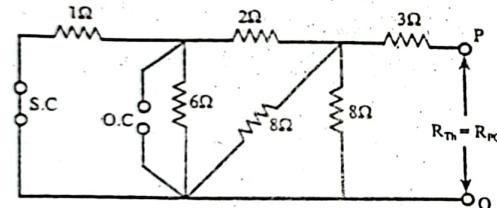
$$V_{Th} = V_{PQ}$$

= Voltage drop in 8Ω resistor

$$= V_2$$

$$= 6V$$

To find R_{Th} :



$$R_{Th} = R_{PQ}$$

$$= [(1 \parallel 6) + 2] \parallel [8 \parallel 8] + 3$$

$$= \left[\left(\frac{6}{7} + 2 \right) \parallel [8 \parallel 8] \right] + 3 = \left[\frac{20}{7} \parallel [8 \parallel 8] \right] + 3 = \frac{\frac{20}{7} \times 8 \times 8}{\frac{20}{7} \times 8 + 8 \times 8 + 8 \times \frac{20}{7}} + 3$$

$$= \frac{5}{3} + 3 = 4.67 \Omega$$

As per maximum power transfer theorem,

$$R_{Th} = R_L = 4.67 \Omega$$

Then,

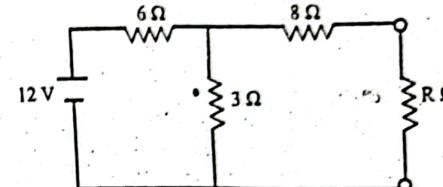
Maximum power drawn by R_L :

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

$$= \frac{6^2}{4 \times 4.67}$$

$$= 0.93 \text{ watt}$$

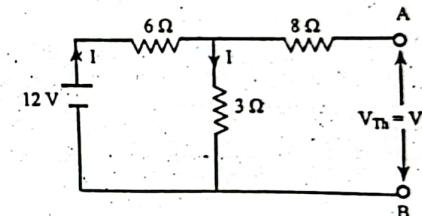
9. Determine the value of R for maximum power to R and calculate the power delivered under this condition. [2071 Bhadral]



Solution:

In order to determine maximum power transfer, we determine the Thevenin's equivalent network.

To find V_{Th} :

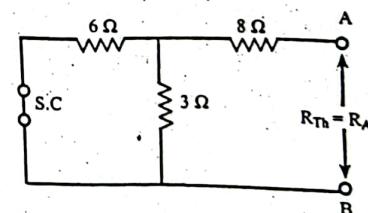


Using voltage divider rule,

$$V_{Th} = V_{AB} = 1 \times 3$$

$$= \frac{12}{6+3} \times 3 = 4V$$

To find R_{Th} :



$$R_{Th} = R_{AB} = (6 \parallel 3) + 8$$

$$= \frac{6 \times 3}{6+3} + 8$$

$$= 10 \Omega$$

As per the maximum power transfer theorem,

$$R_{Th} = R_L = 10 \Omega$$

Then,

Amount of maximum power delivered to the load is given by,

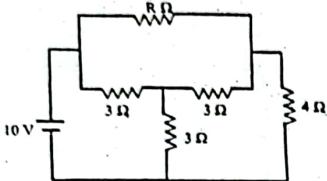
$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{4^2}{4 \times 10}$$

$$= 0.4 \text{ watt}$$

Hence, the value of R is 10Ω

For maximum power and power delivered under this condition is 0.4 watt.

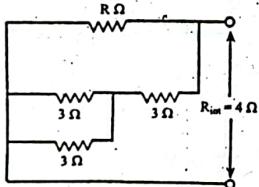
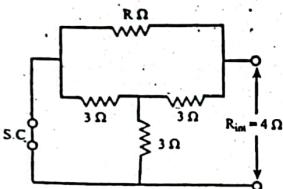
10. Determine the value of R in the given network such that 4Ω resistor consumes maximum power.
[2071 magh]



Solution :-

Maximum power will be delivered to the load of 4Ω resistance when the load resistance is equal to the internal resistance of the source ($R_{int} = R_{Th}$). So, $R_{int} = 4 \Omega$

To Find R_{int} :



We know,

$$R_{int} = [(3//3) + 3] // R$$

$$\text{or, } 4 = 4.5 // R$$

$$\text{or, } 4 = \frac{4.5R}{4.5 + R}$$

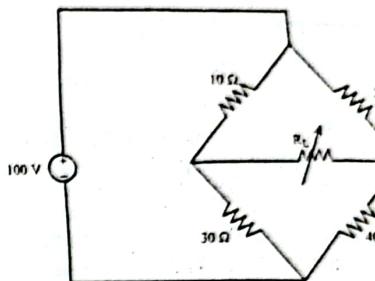
$$\text{or, } 4(4.5 + R) = 4.5R$$

$$\text{or, } 18 + 4R = 4.5R$$

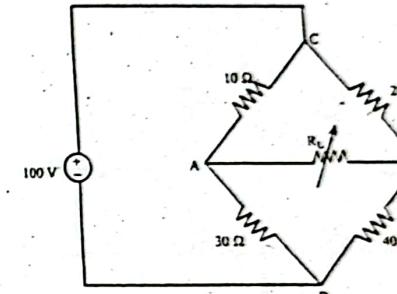
$$\text{or, } 0.5R = 18$$

$$\therefore R = 36 \Omega$$

11. Determine the value of load resistance R_L to receive maximum power from the source. Also, find the maximum power delivered to the load in the circuit shown in figure below.
[2071 chaitra]

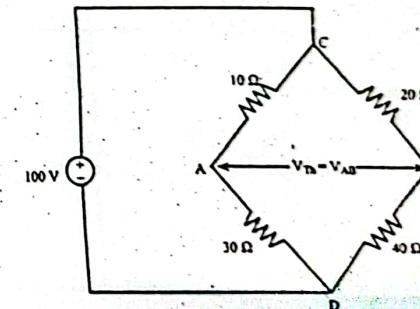


Solution:



Here, the open circuit voltage V_{oc} (also called Thevenin's voltage V_{Th}) which appears across terminals A and B is calculated and also the resistance of the circuit as looked into the network (also called Thevenin's resistance R_{Th}) from the points A and B is calculated.

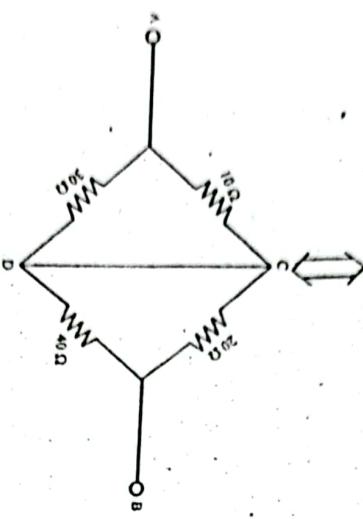
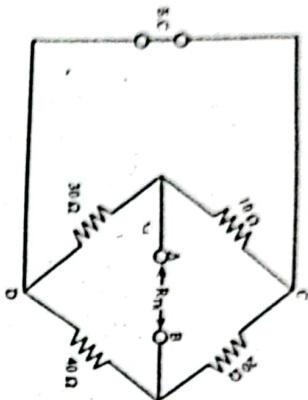
To find V_{Th} :



$$\begin{aligned} V_{Th} &= V_{AB} = V_{CA} - V_{CB} \\ &= \frac{100}{10+30} \times 10 - \frac{100}{20+40} \times 20 \\ &= 25 - \frac{100}{3} = -8.33 \text{ V} \end{aligned}$$

The negative sign indicates that R_L is at higher potential with respect to A.
To find R_{Th} .

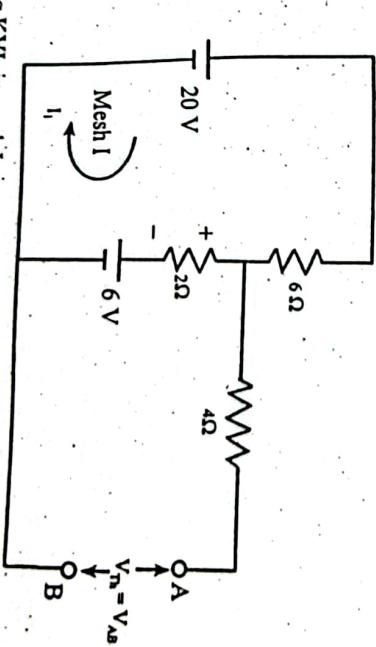
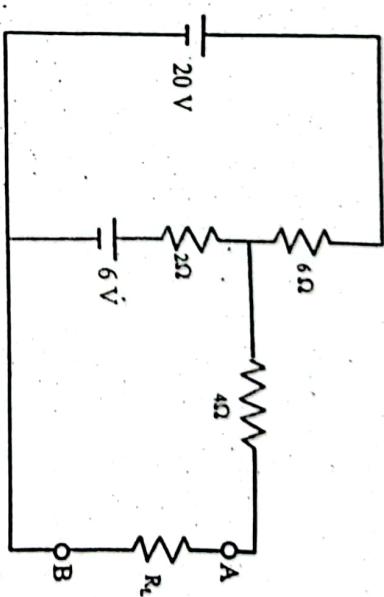
12. Find Thevenin's equivalent circuit across AB for the following figure below and find the value R_L to obtain maximum power in R_L . Also determine the maximum power. [2072 Ashwin]



Solution:

In order to find the maximum power, we determine Thevenin's equivalent circuit.

To find V_{Th} :



Using KVL in mesh I, we get;

$$20 - 6I_1 - 2I_1 - 6 = 0$$

$$\text{or, } -8I_1 + 14 = 0$$

$$\text{or, } I_1 = \frac{14}{8}$$

$$\therefore I_1 = 1.75 \text{ A}$$

$$\begin{aligned} \therefore R_{Th} &= R_{AB} = (10 // 30) + (20 // 40) \\ &= \frac{10 \times 30}{10 + 30} + \frac{20 \times 40}{20 + 40} \\ &= 7.5 \Omega + 13.33 \\ &= 20.83 \Omega \end{aligned}$$

As per maximum power transfer theorem,

$$R_{Th} = R_{AB} = R_L = 20.83 \Omega$$

Then,

$$P_{Max} = R_{Th}^2 / 4R_{Th}$$

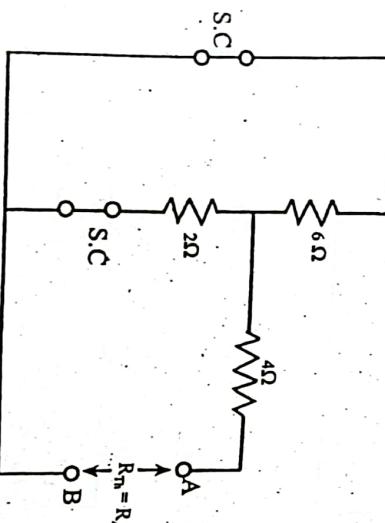
$$\begin{aligned} P_{Max} &= \frac{V_{Th}^2}{4R_{Th}} \\ &= \frac{8.33^2}{4 \times 20.83} \\ &= 0.883 \text{ watt} \end{aligned}$$

Then,

$$V_{Th} = V_{AB} = V_A - V_B$$

$$\begin{aligned} &= 6 + 2I_1 \quad [\text{Write KVL equation; move from B to A}] \\ &= 6 + 2 \times 1.75 \\ &= 9.5 \text{ V} \end{aligned}$$

To find R_{Th} :



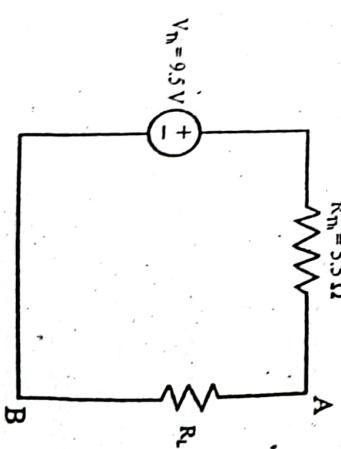
$$R_{Th} = R_{AB} = (6/2) + 4$$

$$= \frac{6 \times 2}{5+2} + 4$$

$$= \frac{12}{8} + 4$$

$$= 5.5 \Omega$$

Thevenin's equivalent circuit:



$$V_{Th} = V_{AB} = V_A - V_B$$

$$= 6 I_2 - 4 \times 0 - 4 \quad [\text{Write KVL equation; move from B to A}]$$

$$= 6 \times \frac{8}{(4+6)} - 4 \quad [\because I_2 = \frac{8}{4+6} = \frac{8}{10} \text{ A}]$$

Now,

As per maximum power transfer theorem,

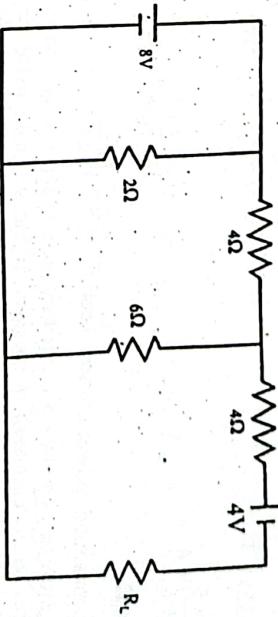
$$R_L = R_{Th} = 5.5 \Omega$$

Then,

Maximum power drawn by R_L :

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{9.5^2}{4 \times 5.5} = 4.102 \text{ watt}$$

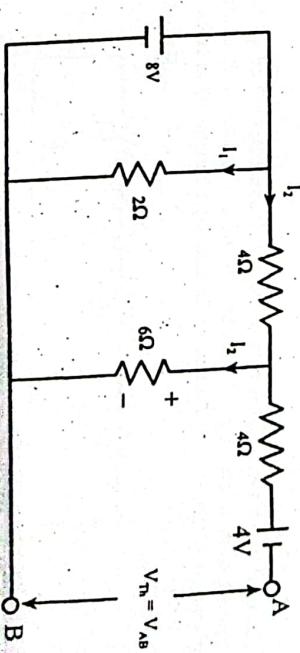
13. Find the value of R_L for which the maximum power is transferred in the load resistance R_L . Also, find the maximum power that can be transferred to the load resistance R_L . [2072 Chairra]



Solution:

In order to determine maximum power transfer, we determine the Thevenin's equivalent network.

To find V_{Th} :



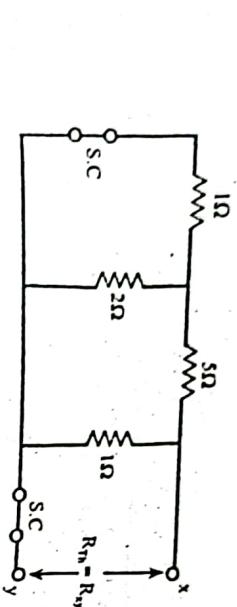
$$V_{Th} = V_{AB} = V_A - V_B$$

$$= 6 I_2 - 4 \times 0 - 4$$

$$= 6 I_2 - 4$$

$$= 6 \times \frac{8}{(4+6)} - 4$$

To find R_{Th} :



$$R_{Th} = R_{xy}$$

$$= [(1 \parallel 2) + 5] \parallel 1 = \left[\frac{2}{3} + 5 \right] \parallel 1 = \frac{17}{3} \parallel 1 = 0.85 \Omega$$

Now,

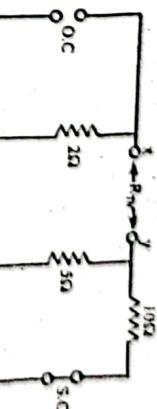
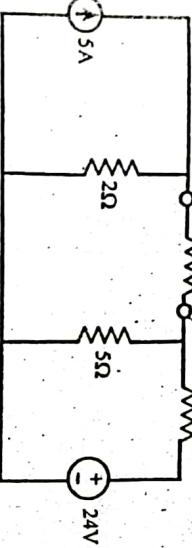
For maximum power, $P_{L(\max)}$,

$$R = R_{Th} = 0.85 \Omega$$

$$P_{L(\max \text{ power})} = \frac{V_{Th}^2}{4R_{Th}}$$

$$= \frac{6.4^2}{4 \times 0.85} = 12.04 \text{ W}$$

2. What should be the value of R such that maximum power transfer can take place from the rest of the network to R in figure below. Obtain the amount of this power.



$$R_{Th} = R_{xy}$$

$$= (5 \parallel 10) + 2 = \frac{5 \times 10}{5+10} + 2 = \frac{16}{3} \Omega = 5.33 \Omega$$

As per maximum power transfer theorem,

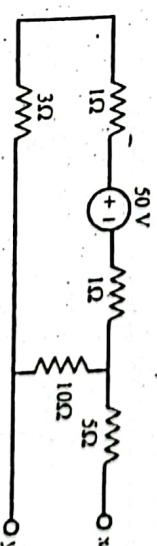
$$R = R_{Th} = \frac{16}{3} \Omega = 5.33 \Omega$$

And,

$$P_{L(\max \text{ power})} = \frac{V_{Th}^2}{4R_{Th}}$$

$$= \frac{2^2}{4 \times 5.33} = 0.188 \text{ watt}$$

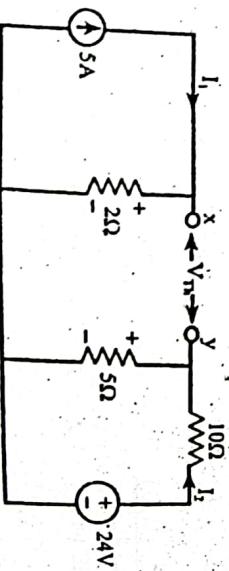
3. What resistance should be connected across $x - y$ in the circuit shown in figure below such that maximum power is developed across this load resistance? What is the amount of this maximum power?



Solution:

In order to determine the maximum power transfer, we determine the Thevenin's equivalent circuit.

To find V_{Th} :



$$V_{Th} = V_{xy} = V_x - V_y$$

$$= -5I_1 + 2I_1$$

[Write KVL equation; move from y to x].

$$= -5 \times \left(\frac{24}{5+10} \right) + 2 \times 5 = 2 \text{ volt}$$

$$= \frac{50}{15} + 5 = 8.333 \Omega$$

$$R_{int} = [(1 + 1 + 3) \parallel 10] + 5$$

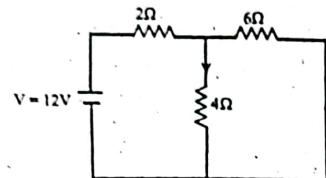
$$= [5 \parallel 10] + 5$$

$$= \frac{5 \times 10}{5+10} + 5$$

Reciprocity Theorem

Exam Solutions

1. Verify the reciprocity theorem in the network given below. [2067 Mangir]



Solution:

In order to verify the reciprocity theorem, the network is drawn as;

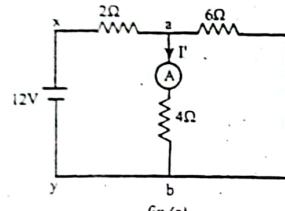


fig (a)

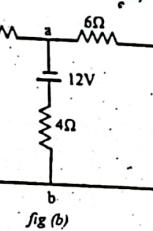


fig (b)

Considering fig (a),

$$\text{Equivalent resistance, } R_{eq} = (4 \parallel 6) + 2 = \frac{4 \times 6}{4+6} + 2 = 4.4 \Omega$$

$$\text{Current supplied by battery} = \frac{12}{4.4} = \frac{30}{11} A$$

$$\text{Ammeter current, } I' = \frac{30}{4+6} \times 6 = \frac{18}{11} A$$

Now, Considering fig (b),

$$\text{Equivalent resistance, } R_{eq} = (2 \parallel 6) + 4 = \frac{2 \times 6}{2+6} + 4 = 5.5 \Omega$$

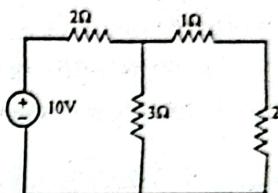
$$\text{Current supplied by battery} = \frac{12}{5.5} = \frac{24}{11} A$$

$$\text{Ammeter current, } I'' = \frac{24}{2+6} \times 6 = \frac{18}{11} A$$

Hence, we observed that when the source was in branch xy as in fig (a), the ab branch current was $\frac{18}{11} A$ and when the source was in branch ab as in fig (b), the xy branch current was $\frac{18}{11} A$. This proves the reciprocity theorem.

Additional Questions

1. Show the application of reciprocity theorem in the network given below.



Solution:

Let the circuit be drawn as;

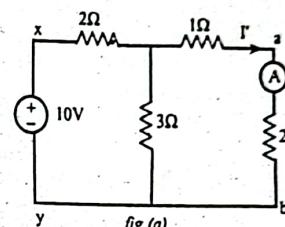


fig (a)

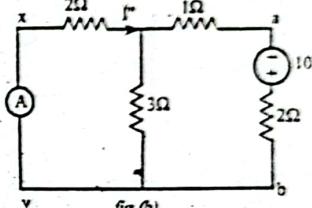


fig (b)

Considering fig (a),

Equivalent resistance,

$$\begin{aligned} R_{eq} &= [(1+2) \parallel 3] + 2 \\ &= (3 \parallel 3) + 2 \\ &= 3.5 \Omega \end{aligned}$$

$$\text{Current supplied by battery} = \frac{10}{3.5} = \frac{20}{7} A$$

$$\text{Ammeter current, } I' = \frac{20}{3+(1+2)} \times 3 = \frac{10}{7} A$$

Now,

Considering fig (b),

$$\text{Equivalent resistance, } R_{eq} = (2 \parallel 3) + (1+2)$$

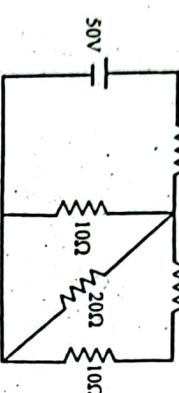
$$\begin{aligned} &= \frac{2 \times 3}{2+3} + 3 \\ &= 4.2 \Omega \end{aligned}$$

$$\text{Current supplied by battery} = \frac{10}{4.2} = \frac{50}{21} \text{ A}$$

$$\text{Ammeter current, } I'' = \frac{50}{3+2} \times 3 = \frac{10}{7} \text{ A}$$

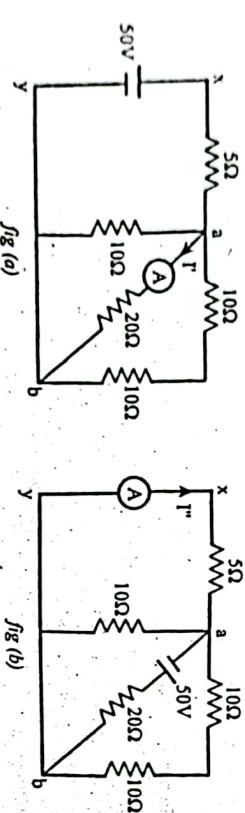
Hence, we observed that when the source was in branch xy as in fig (a), the ab branch current was $\frac{10}{7}$ A and when the source was in branch ab as in fig (b), the xy branch current was $\frac{10}{7}$ A. This proves the reciprocity theorem.

2. Show the validity of reciprocity theorem in the circuit given below:



Solution:

Let the circuit be drawn as;



Considering fig (a),

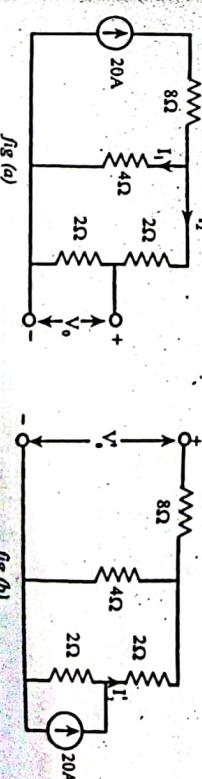
$$\text{Equivalent resistance, } R_{eq} = (10 \parallel 20 \parallel 20) + 5$$

$$= 5 + 5$$

$$= 10 \Omega$$

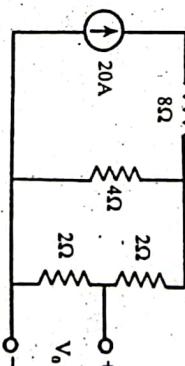
$$\text{Current supplied by battery} = \frac{50}{10} = 5 \text{ A}$$

$$= 5 \text{ A}$$



Solution:

Let the circuit be drawn as;



3. Verify the reciprocity theorem for the circuit shown in figure.

$$\text{Ammeter current, } I'' = \frac{1}{\frac{1}{5} + \frac{1}{10} + \frac{1}{20}} \times 2.187$$

$$= 1.25 \text{ A}$$

Hence, we observed that when the source was in branch xy as in fig (a), the ab branch current is 1.25 A and when the source was in branch ab as in fig (b), the xy branch current becomes 1.25 A. This proves the reciprocity theorem.

$$\text{Ammeter current, } I'' = \frac{\frac{1}{20}}{\frac{1}{10} + \frac{1}{20} + \frac{1}{20}} \times 5 = 1.25 \text{ A}$$

Now,

Considering fig (b),

$$\begin{aligned} \text{Equivalent resistance, } R_{eq} &= (5 \parallel 10 \parallel 20) + 20 \\ &= 2.86 + 20 \end{aligned}$$

$$\begin{aligned} \text{Current supplied by battery} &= \frac{50}{22.86} \\ &= 2.187 \text{ A} \end{aligned}$$

Considering fig (a),

By current division rule

$$I_2 = 20 \times \frac{4}{4 + (2 + 2)} \\ = 10 \text{ A}$$

$$\text{Therefore, } V_0 = 2 \times 10 \\ = 20 \text{ V}$$

Considering fig (b),

By current division rule,

$$I_2' = 20 \times \frac{2}{2 + (4 + 2)} \\ = 5 \text{ A}$$

$$\text{Therefore, } V_0' = 5 \times 4 \\ = 20 \text{ V}$$

From the above, it can be seen that $V_0 = V_0'$. Hence, the reciprocity theorem is verified.



4

INDUCTANCE AND CAPACITANCE IN ELECTRIC CIRCUITS

4.1 Capacitor

A capacitor is circuit element capable of storing energy in an electrostatic field. In its elementary form, it consists of two insulated parallel metallic plates with air or any other insulating material separating the plates.

4.2 Capacitance

Capacitance is a measure of ability of a capacitor to store an electric charge when a potential difference is applied between the two plates. It is denoted by 'C' and its unit is farad (F).

A capacitor has a capacitance of one farad when 1 volt charges it with 1 coulomb of electricity.

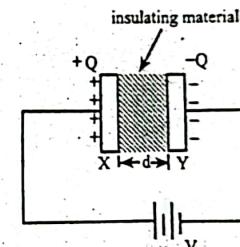


Fig. 4.1

It has been found experimentally that charge Q stored in a capacitor is directly proportional to the p.d. (V) across the plates.

$$\text{i.e. } Q \propto V$$

$$\text{or, } \frac{Q}{V} = \text{constant} = C$$

The constant of proportionality C is called Capacitance of the capacitor.

Electric field strength (E) between two plates,

$$E = \frac{V}{d} \quad \text{(i) [where d is distance between two parallel plates]}$$

If the charge on the plates X and Y is of Q coulombs,

$$\text{Electric flux density, } D = \frac{Q}{A} \quad \text{(ii) [where A is overlapping area of parallel plates]}$$

Dividing equation (ii) by equation (i), we get

Inductance is the property of a circuit element that opposes the rise or fall of current through it by inducing emf across the circuit element. The circuit element having this property is known as inductor.

4.6 Self inductance

Self inductance is usually just called inductance symbolized by L . Self inductance is a measure of a coil's ability to establish an induced voltage as a result of a change in its current. The induced voltage always opposes the change in current which is basically a statement of Lenz's law. The unit of inductance is henry (H).

Let us consider an inductor with copper wire wound on an iron ring as shown, Self induced emf,

$$e = -N \frac{d\phi}{dt} \quad \dots \dots \text{(i)}$$

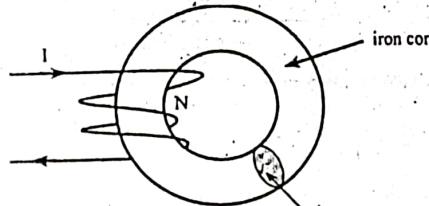


Fig. 4.5

Flux is defined as the ratio of magnetomotive force (mmf) to the reluctance.

$$\text{i.e. } \phi = \frac{\text{MMF(magnetomotive force)}}{\text{R}(reluctance)} \quad \dots \dots \text{(ii)}$$

We know,

$$\text{MMF} = NI$$

Since,

$$R \propto \frac{1}{A}, R \propto \frac{1}{A}$$

$$\text{or, } R \propto \frac{l}{A}$$

$$\therefore R = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$$

Where,

$\mu = \mu_0 \mu_r$ = permeability of medium

$$\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}$$

Then,

Equation (ii) can be written as,

$$\phi = \frac{NI}{l} \quad \dots \dots \text{(iii)}$$

Now, substituting the value of ϕ from equation (iii) in equation (i), we get,

$$\begin{aligned} e &= -N \frac{d}{dt} \left[\frac{NI}{l} \right] \\ &= -N \frac{N}{l} \times \frac{dI}{dt} \\ &= -N^2 \frac{\mu_0 \mu_r A}{l} \times \frac{dI}{dt} \\ \therefore e &= -L \frac{dI}{dt} \quad \dots \dots \text{(iv)} \end{aligned}$$

Where,

$$L = \frac{N^2 \mu_0 \mu_r A}{l} = \frac{N\phi}{I} \text{ is coefficient of self inductance.}$$

A coil is said to have a self inductance of one henry if a current of 1A, when following through it, produces flux linkage of 1 wb-turn in it.

4.7 Energy Stored in an inductor

When the current in a circuit, of a coil of inductance L henry, increases from zero to its maximum steady value of I amperes, work has to be done against the opposing induced emf. Let i and e are the respective value of the current and induced emf after a time t seconds. Then the work done in establishing the steady state value of current is given by,

$$\begin{aligned} W &= \int_0^t i e dt = \int_0^t i \left(L \frac{di}{dt} \right) dt = \int_0^t L i di \\ \therefore E &= W = \frac{1}{2} L I^2 \text{ joules} \end{aligned}$$

This is the expression for energy stored in an inductor.

4.8 Magnetic coupling

Two coils are said to be magnetically coupled, if either full or part of the magnetic flux produced by one links with that of the other. Let L_1 = self inductance of coil '1'; L_2 = self inductance of coil '2' and M = mutual inductances of two coils. Then,

$$M = k \sqrt{L_1 L_2}$$

Where k is coefficient of coupling. If the full flux produced by coil '1' links with the flux produced by coil '2' then $k = 1$ and $M = \sqrt{L_1 L_2}$.

If there is no common flux between the two coils, then they are said to be magnetically isolated.

4.9.1 Inductors in series

i) Series - aiding

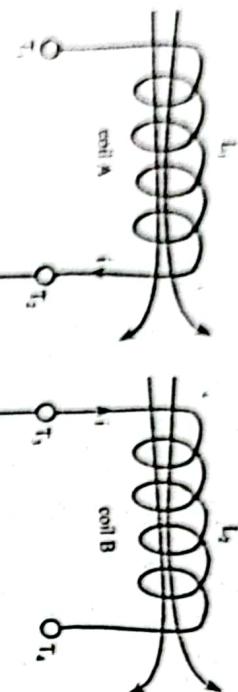


Fig. 4.6 Series aiding connection

Let the two coils be so joined in series that their fluxes (or m.m.f.s) are additive i.e. in the same direction.

Let,

M = coefficient of mutual inductance

L_1 = coefficient of self inductance of first coil

L_2 = coefficient of self inductance of second coil

$$\text{Then, Self induced emf in A} = e_1^i = -L_1 \frac{di}{dt}$$

Mutually induced emf in A due to change of current in B is $e_1^m = -M \frac{di}{dt}$

Total emf induced in coil A;

$$e_1 = e_1^i + e_1^m$$

$$= -(L_1 + M) \frac{di}{dt}$$

Similarly,

Total emf induced in coil B;

$$e_2 = -(L_2 + M) \frac{di}{dt}$$

So, the total induced emf in the circuit is given as,

$$\begin{aligned} e &= e_1 + e_2 \\ &= -(L_1 + M) \frac{di}{dt} - (L_2 + M) \frac{di}{dt} \\ &= -\frac{di}{dt} (L_1 + L_2 + 2M) \quad \dots \dots \dots \text{(i)} \end{aligned}$$

If L is the equivalent inductance then total induced emf in that single coil would have been

$$e = -L \frac{di}{dt} \quad \dots \dots \dots \text{(ii)}$$

Equating equations (i) and (ii), we get

$$L = L_1 + L_2 + 2M$$

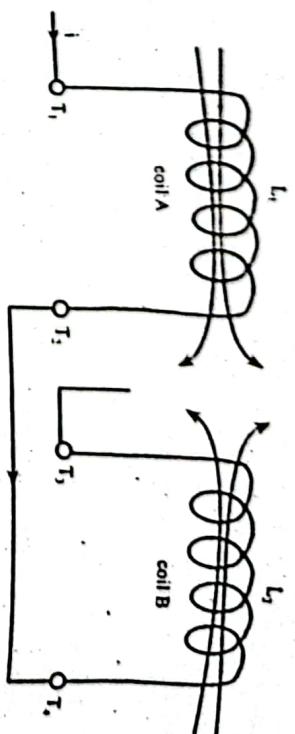


Fig. 4.7 Series opposing connection

For series opposing connections, mutually induced emf opposes the self induced emf.

As before,

Total emf induced in coil A;

$$e_1 = -(L_1 - M) \frac{di}{dt}$$

Similarly,

Total emf induced in coil B;

$$e_2 = -(L_2 - M) \frac{di}{dt}$$

So, the total induced emf in the circuit is given as;

$$e = e_1 + e_2 = -(L_1 + L_2 - 2M) \frac{di}{dt} \quad \dots \dots \dots \text{(iii)}$$

If L is the equivalent inductance then total induced emf in that single coil would have been;

$$e = -L \frac{di}{dt} \quad \dots \dots \dots \text{(iv)}$$

From equations (iii) and (iv), we get

$$\therefore L = L_1 + L_2 - 2M$$

In general

$$L = L_1 + L_2 + 2M \quad (\text{if m.m.f.s are additive})$$

$$L = L_1 + L_2 - 2M \quad (\text{if m.m.f.s are subtractive})$$

4.9.2 Inductors in parallel

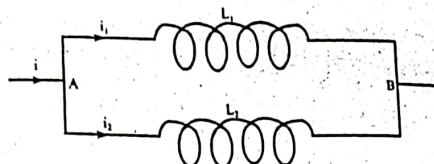


Fig. 4.8

Let the two coils be joined in parallel that the mutual field assists the separate fields.

Let,

M = coefficient of mutual inductance

L_1 = coefficient of self inductance of first coil

L_2 = coefficient of self inductance of second coil

Here,

At node A, by KCL

$$i = i_1 + i_2$$

$$\text{or, } \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad \dots \dots \dots \text{(i)}$$

In each coils, both self and mutual induced emf are produced. Since the coils are in parallel, these emfs are equal.

$$\text{i.e. } e_1^L + e_1^M = e_2^L + e_2^M$$

$$\text{or, } -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = -L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

$$\text{or, } (L_2 - M) \frac{di_2}{dt} = (L_1 - M) \frac{di_1}{dt}$$

$$\text{or, } \frac{di_1}{dt} = \frac{(L_2 - M)}{(L_1 - M)} \frac{di_2}{dt} \quad \dots \dots \dots \text{(ii)}$$

From equations (i) and (ii)

$$\frac{di}{dt} = \frac{L_2 - M}{L_1 - M} \frac{di_2}{dt} + \frac{di_2}{dt}$$

$$\frac{di}{dt} = \left[\left(\frac{L_2 - M}{L_1 - M} \right) + 1 \right] \frac{di_2}{dt} \quad \dots \dots \dots \text{(iii)}$$

If L is the equivalent inductance of the combination then induced emf

$$e = -L \frac{di}{dt}$$

= induced emf in parallel combination

= induced emf in any one coil

$$\text{i.e. } -L \frac{di}{dt} = -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$\text{or, } \frac{di}{dt} = \frac{L_1}{L} \frac{di_1}{dt} + \frac{M}{L} \frac{di_2}{dt} \quad \dots \dots \dots \text{(iv)}$$

$$\text{or, } \frac{di}{dt} = \frac{1}{L} \left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right)$$

Substituting the value of $\frac{di_1}{dt}$ from equation (ii) in equation (iv), we get

$$\frac{di}{dt} = \frac{1}{L} \left[L_1 \frac{(L_2 - M)}{(L_1 - M)} + M \right] \frac{di_2}{dt} \quad \dots \dots \dots \text{(v)}$$

From equations (iii) and (v), we have

$$\frac{L_2 - M}{L_1 - M} + 1 = \frac{1}{L} \left[L_1 \frac{(L_2 - M)}{(L_1 - M)} + M \right]$$

$$\text{or, } \frac{L_2 - M + L_1 - M}{L_1 - M} = \frac{1}{L} \left[\frac{L_1 L_2 - L_1 M + L_1 M - M^2}{L_1 - M} \right]$$

$$\therefore L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \quad [\text{When mutual field assists the separate fields}]$$

Similarly,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \quad [\text{When the two fields oppose each other}]$$

Exam Solutions

1. A capacitor of $2 \mu\text{F}$ is charged to ~~600 V~~. Find the energy stored in the capacitor and calculate power if the capacitor is discharged uniformly to 400 V in 3 seconds. [2064 Shrawan]

Solution:

Here

Capacitance, $C = 2 \mu\text{F}$

Voltage to which the capacitor has been

charged, $V_1 = 600 \text{ V}$

Voltage to which the capacitor has been

discharged, $V_2 = 400 \text{ V}$

Energy stored in the capacitor, before

discharge,

$$W_1 = \frac{1}{2} CV_1^2$$

$$= \frac{N^2 A \mu_0 \mu_r I}{1 \times l}$$

$$= \frac{N^2 A \mu_0 \mu_r I}{l}$$

$$= \frac{1200^2 \times 80 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1000}{0.4}$$

$$= 36.19 \text{ H}$$

Energy stored in the capacitor, after discharge

$$W_2 = \frac{1}{2} CV_2^2$$

$$= \frac{1}{2} \times 2 \times 10^{-6} \times 400^2$$

$$= 0.16 \text{ J}$$

Energy removed from the capacitor

$$= W_1 - W_2$$

$$= 0.36 - 0.16$$

$$= 0.2 \text{ J}$$

Duration of discharge, $t = 3$ seconds

$$\text{Power, } P = \frac{\text{Energy removed in J or } W \text{--s}}{t \text{ in seconds}}$$

$$= \frac{0.2}{3}$$

$$= \frac{1}{15} \text{ watt}$$

$$\text{Hence, } l = 50 \text{ cm} = 0.5 \text{ m}, A = 60 \text{ mm}^2, L = 70 \text{ mH}$$

$$\mu_0 = 4\pi \times 10^{-7}, \mu_r = 1400, N = ?$$

[2066 Magh]

3. An inductor is to be made with a copper wire wound on an iron core having mean length of 50 cm with a cross sectional area of 60 mm^2 . If the required value of inductance is 70 mH, calculate the number of turns required. It is given that the relative permeability of the core is 1400.

Solution:

2.

- A solenoid of 1200 turns has iron core of cross-section area 80 cm^2 and mean length 0.4m. Find the self inductance of the coil and calculate induced emf if a current of 0.2 A is switched OFF in 0.01 sec. (Relative permeability of iron is 1000)

[2063 Kartik]

Solution:

Here,

- $I = 0.4 \text{ m}$, $N = 1200$ turns, $A = 80 \text{ cm}^2$, $\mu_0 = 4\pi \times 10^{-7}$, $\mu_r = 1000$

Self-inductance, $L = \frac{N\Phi}{I}$

$$\frac{N}{I} \times \frac{NI}{\lambda \mu_0 \mu_r}$$

4. An air cored coil is 2.5 cm long and has an average cross-sectional area of 2 cm^2 . Determine the number of turns if the coil has an inductance of 100 μH .

[2068 Chaitra]

Solution:

Here,

- $L = 100 \mu\text{H} = 100 \times 10^{-6} \text{ H}$, $I = 2.5 \text{ cm} = 0.025 \text{ m}$
- $A = 2 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$, $\mu_0 = 4\pi \times 10^{-7}$, $\mu_r = 1$

Since,

$$L = \frac{N^2 A \mu_0 I}{l}$$

$$\text{or, } C_A = \frac{80}{120} C_B \quad (\text{i})$$

Now,

$$100 \times 10^{-6} = \frac{N^2 \times 2 \times 10^{-4} \times 4 \times 10^{-7} \times 1}{0.025}$$

or,

$$N = 99.74 \approx 100 \text{ turns}$$

5. Two coils of inductance of 4 and 6 henry are connected in parallel. If their mutual inductance is 3 henry, calculate the equivalent inductance of the combination if mutual inductance

- i) Supports the self inductance, and

- ii) Oppose the self inductance

Solution:

$$\text{i. } L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$$= \frac{4 \times 6 - 3^2}{4 + 6 - 2 \times 3} = 3.75 \text{ H}$$

$$\text{ii. } L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

$$= \frac{4 \times 6 - 3^2}{4 + 6 + 2 \times 3} = 0.9375 \text{ H}$$

[2068 Surawal]

$$C_B' = C_B + 3$$

New charges on C_A and C_B' will be equal.

$$\text{i.e. } 140 C_A = (200 - 140)(C_B + 3)$$

$$\text{or, } 140 C_A = 60(C_B + 3)$$

$$\text{or, } 140 \times \frac{80}{120} C_B = 60(C_B + 3)$$

$$\text{or, } \frac{100}{3} C_B = 180$$

$$\therefore C_B = 5.4 \mu\text{F}$$

From equation (i)

$$C_A = \frac{80}{120} \times 5.4 \\ = 3.6 \mu\text{F}$$

6. Two capacitors, A and B are connected in series across a 200 V dc supply. The p.d. across A is 120 V. This p.d. is increased to 140 V, when a $3\mu\text{F}$ capacitor is connected in parallel with B. Calculate the capacitances of A and B.[2069 Bhadrab]

Solution:

- Let the capacitances of the capacitors A and B be C_A and C_B respectively and potential difference across them be V_A and V_B respectively.

Solution:

6. Calculate the inductance that must be connected in parallel with a 100 mH inductor to give a total inductance of 70 mH. Assume no mutual inductance between the two.

[2070 Ashad]

- Let L_1 and L_2 be the inductances of inductors 1 and 2 connected in parallel. Also, L_{eq} be the equivalent inductance of the parallel connection. Since, no mutual inductance is between the two

so, $M = 0$

Since,

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\text{or, } \frac{1}{70} = \frac{1}{100} + \frac{1}{L_2}$$

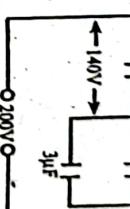
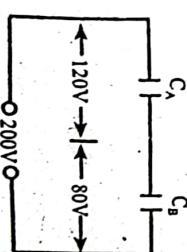
$$\therefore L_2 = 233.33 \text{ mH}$$

$$V_B = 200 - V_A = 200 - 120 = 80 \text{ V}$$

For series connection, charge is same

$$\text{i.e. } Q = C_A V_A = C_B V_B$$

$$\text{or, } C_A \times 120 = C_B \times 80$$



7. A capacitor with capacitance of $2\mu F$ is connected in series with another capacitor whose capacitance is C_r . If the equivalent capacitance of the combination is $1.5 \mu F$ calculate the value of C_r . What would be the equivalent capacitance if they were connected in parallel? (2011 Maharashtra Board)

Solution:

Let C_1 and C_2 be the capacitance of capacitor 1 and capacitor 2 respectively. Then,

By question,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \quad (\text{in series})$$

$$\text{or, } C_{eq} = \frac{2C_r}{2+C_r}$$

$$\text{or, } 1.5 = \frac{2C_r}{2+C_r}$$

$$\text{or, } 3 + 1.5 C_r = 2 C_r$$

$$\therefore C_r = 6 F$$

Now,

$$C_{eq} = C_1 + C_2 \quad (\text{in parallel})$$

$$= 2 + 6$$

$$= 8 \mu F$$

Hence,

The value of C_r is $6 \mu F$ and the equivalent capacitance is $8 \mu F$ when capacitors are connected in parallel.

8. Three capacitors A, B and C have capacitances 10, 50, and $25 \mu F$ respectively. Calculate.
- Charge on each when connected in parallel to a 250V supply
 - Total capacitance and
 - P.d across each when connected in series [2012 kartik]

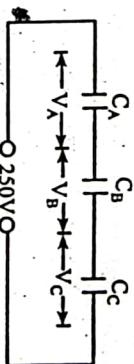
Solution:

Here,

Capacitance of capacitor A, $C_A = 10 \mu F$

Capacitance of capacitor B, $C_B = 50 \mu F$

Capacitance of capacitor C, $C_C = 25 \mu F$



We know,

$$\frac{1}{C_T} = \frac{1}{C_A} + \frac{1}{C_B} + \frac{1}{C_C}$$

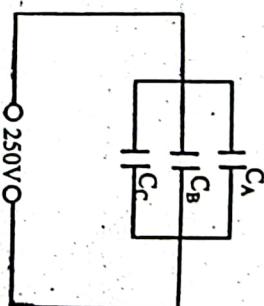
Where, C_T is the equivalent capacitance in series.

$$\text{or, } \frac{1}{C_T} = \frac{1}{10} + \frac{1}{50} + \frac{1}{25}$$

$$\therefore C_T = 6.25 \mu F$$

Charge across the capacitors will be same.

$$Q_T = Q = Q_A = Q_B = Q_C = C_T V$$



$$\text{Charge on capacitor A, } Q_A = C_A V$$

$$= 10 \times 10^{-6} \times 250$$

$$= 2.5 \times 10^{-3} C$$

$$\text{Charge on capacitor B, } Q_B = C_B V$$

$$= 50 \times 10^{-6} \times 250$$

$$= 0.0125 C$$

$$\text{Charge on capacitor C, } Q_C = C_C V$$

$$= 25 \times 10^{-6} \times 250$$

$$= 6.25 \times 10^{-3} C$$

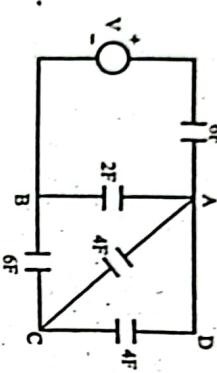
$$\text{ii) Total capacitance, } C_T = C_A + C_B + C_C$$

$$= (10 + 50 + 25) \mu F$$

$$= 85 \mu F$$

iii)

3. Determine the equivalent capacitance of the combination shown in the figure.



Solution:

Between A and C, there are two 4 F capacitors in parallel, so

$$C_{AC} = 4 + 4 = 8 \text{ F}$$

C_{AB} = Equivalent of 8 F in series with 6 F

$$C_{AB} = \frac{8 \times 6}{8+6} = \frac{24}{7} \text{ F}$$

$$C_{AB} = \frac{24}{7} + 2 = \frac{38}{7} \text{ F}$$

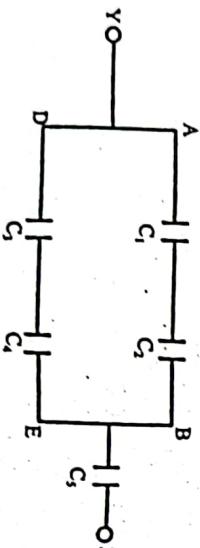
Now,

Equivalent capacitance of combination

$$= \text{Equivalent of } 6 \text{ F in series with } \frac{38}{7} \text{ F}$$

$$\therefore C_{eq} = \frac{\frac{38}{7} \times 6}{\frac{38}{7} + 6} = 2.85 \text{ F}$$

4. Find the equivalent capacitance of the following network.



Solution:

$$C_p = C_{AB} + C_{DE}$$

$$= \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}$$

$$C_{eq} = C_{pq}$$

$$= \frac{C_p C_3}{C_p + C_3} \cdot \frac{\left(\frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4} \right) C_3}{\left(\frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4} \right) + C_3}$$

5. Two coils have self inductances of 6.8 mH and 4.5 mH respectively. When they are connected in series, the total inductances in series aiding and series opposing connections are 19.6 mH and 3mH respectively. Find mutual inductance and coefficient of coupling.

Solution:

Here,

$$L_{\text{additive}} = 19.6 \text{ mH}$$

$$L_{\text{subtractive}} = 3 \text{ mH}$$

$$L_1 = 6.8 \text{ mH}$$

$$L_2 = 4.5 \text{ mH}$$

$$L_{\text{additive}} = L_1 + L_2 + 2M$$

$$19.6 = 6.8 + 4.5 + 2M$$

or,

$$M = 4.15 \text{ mH}$$

Also

$$M = k \sqrt{L_1 L_2}$$

$$\text{or, } k = \frac{M}{\sqrt{L_1 L_2}}$$

$$= \frac{4.15}{\sqrt{6.8 \times 4.5}}$$

$$\therefore k = 0.7502$$

6. The coefficient of coupling between two coils is 0.85. Coil 1 has 250 turns. When the current in coil 1 is 2A, the total flux of this coil is 3×10^{-3} weber. When I_1 is changed from 2A to zero linearly in 2 milliseconds, the voltage induced in coil 2 is 63.75V. Find L_1 , L_2 , M and N_2 .

Solution:

$$\begin{aligned} L_1 &= \frac{N_1 \Phi_1}{I_1} = \frac{250 \times 3 \times 10^{-4}}{2} \\ &= 37.5 \times 10^{-3} \text{ H} \end{aligned}$$

$$v_2 = M \frac{di_1}{dt}$$

$$\text{or, } 63.75 = M \times \frac{2}{2 \times 10^{-3}}$$

$$M = 63.75 \times 10^{-3} \text{ H}$$

$$\text{Also, } M = k \sqrt{L_1 L_2}$$

$$\text{or, } 63.75 \times 10^{-3} = 0.85 \sqrt{37.5 \times 10^{-3} \times L_2}$$

$$\therefore L_2 = 150 \times 10^{-3} \text{ H}$$

Since, self inductance $\propto N^2$

$$\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$$

$$\text{or, } \frac{37.5 \times 10^{-3}}{150 \times 10^{-3}} = \frac{250^2}{N_2^2}$$

$$\therefore N_2 = 500 \text{ turns}$$



ALTERNATING QUANTITIES

BASIC TERM & CONCEPTS

An alternating quantity (voltage or current) is one which changes continuously in magnitude and alternates in direction at regular intervals of time.

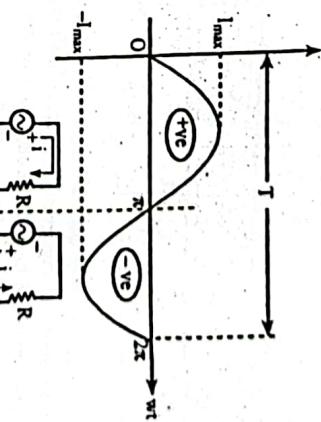


Fig. 5.1

Waveform – The shape of the curve of the voltage or current when plotted against time as base is called waveform. Fig. 5.1 shows the waveform of an alternating current varying sinusoidally.

[Sinusoid refers to the signal that has form of the sine or cosine function.]

Time period – The time taken by the alternating quantities (voltage/current) to complete one cycle is called time period and is denoted by T.

Frequency – The number of cycles completed in one second is called frequency and is denoted by f.

$$f = \frac{1}{T}$$

Unit → hertz (Hz) or cycles/second

Amplitude – The maximum value which an alternating quantity attains during one complete cycle is called its amplitude.

In fig. 5.1, amplitude is denoted by I_{\max} .

Phase – The phase of an alternating quantity is defined as the fraction of time period that it has elapsed since its waveform has passed through zero position of the reference line.



In fig. 5.2,

$$\text{Phase of } A = \frac{T}{8}$$

$$\text{Phase of } B = \frac{T}{4}$$

$$\text{Phase of } C = \frac{T}{2}$$

Phase difference

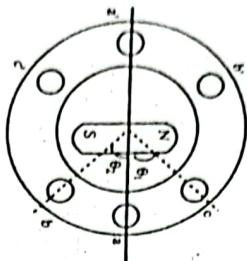


Fig. 5.3

Consider a generator having three coils $a - a'$, $b - b'$ & $c - c'$. According to Faraday's law of electromagnetic induction emf will induce in all three coils namely e_a , e_b and e_c . But, the instant at which they attend maximum value is different and so is the time at which they pass through zero of reference line.

At time when e_a passes through zero, e_b has already passed through zero i.e. e_b is θ_1 angle ahead of e_a . Similarly, e_c is θ_2 angle behind e_a .

Therefore, the phase difference between e_a and e_b is θ_1 and e_c is leading e_b by a phase angle of θ_2 . Similarly, the phase difference between e_a and e_c is θ_2 and e_b is lagging e_c by θ_2 phase angle.

Hence,

$$e_a = E_m \sin \omega t$$

$$e_b = E_m \sin (\omega t + \theta_1)$$

$$e_c = E_m \sin (\omega t + \theta_2)$$

Instantaneous value of alternating quantity

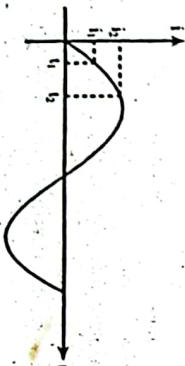


Fig. 5.5

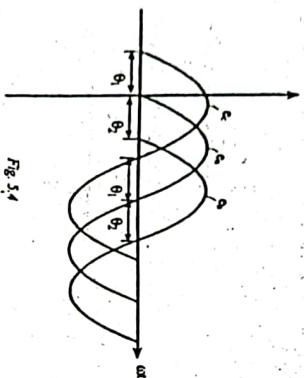


Fig. 5.6

The value of the alternating quantities at any particular instant of time, is called instantaneous value of alternating quantity.

- Instantaneous value of current at time $t_1 = i_1$
- Instantaneous value of current at time $t_2 = i_2$

Average value of alternating quantity

The average value of an alternating current is equal to the value of direct current which transfers across any circuit the same amount of charge during a given time as is transferred by the alternating current for the same time in the same circuit.

Mathematically integral form,

$$\therefore I_{\text{avg}} = \frac{-1}{T} \int_i dt$$

[However, for symmetrical waveform, average value for a complete cycle is zero]

Symmetrical waveforms are those sinusoidal or non-sinusoidal wave forms whose two half cycles (positive and negative) are exactly similar.

Root-mean square (rms) Effective/ Virtual value of alternating quantity

The rms value of an alternating current or voltage is equal to that direct current or voltage which when flows or applied to a given resistance for a given time produces the same amount of heat as when the alternating current or voltage is flowing or applied to the same resistance for the same time.

In integral form,

$$I_{\text{rms}} = \sqrt{\frac{1}{T} \int i^2 dt}$$

(Note : rms value of a waveform is greater than the average value of the particular waveform)

Form factor

$$\text{Form factor} = \frac{\text{Rms value of alternating quantity}}{\text{Average value of alternating quantity}}$$

Peak factor

$$\text{Peak factor} = \frac{\text{Peak value of alternating quantity}}{\text{Rms value of alternating quantity}}$$

Phasor Diagram

An alternating quantities (voltage/current) are represented by straight lines having definite direction and length. Such lines are called phasors and diagrams in which phasors represent currents, voltages and their phase difference are known as phasor diagrams. The phasor diagrams can be drawn either to represent maximum or effective values.

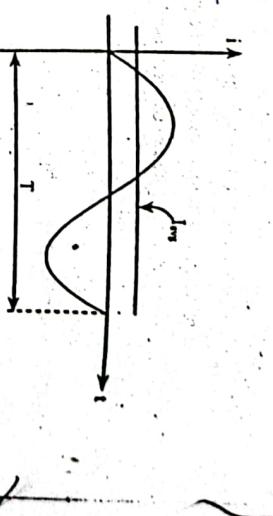


Fig. 5.7

Some of common conventions are-

1. Counter - clockwise direction of rotation of phasors is usually taken as positive direction of rotation of phasors i.e. a phasor rotated in counter - clockwise direction from a given phasor is said to lead the given phasor while a phasor rotated in clockwise direction is said to lag the given phasor.
2. For series circuit, the current phasor is usually taken as reference phasor.
For parallel circuit, voltage phasor is usually taken as reference phasor.

Some illustration of phasor diagrams.

$$e_1 = E_m \sin \omega t$$

$$e_2 = E_m \sin (\omega t - \theta_2)$$

$$e_c = E_m \sin (\omega t + \theta_1)$$

Phasor diagram

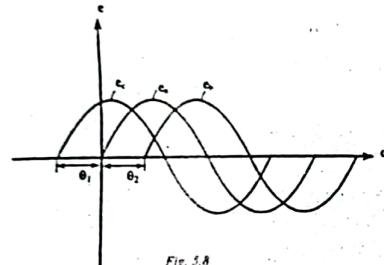


Fig. 5.8

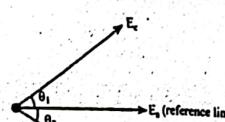


Fig. 5.9

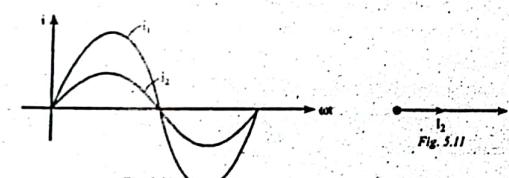


Fig. 5.10

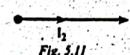


Fig. 5.11

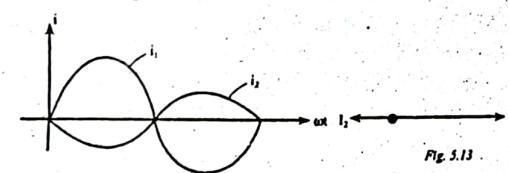


Fig. 5.12

Fig. 5.13

Note:

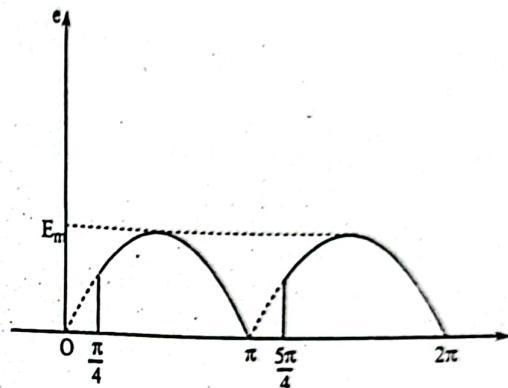
$$\begin{aligned} \sin n\pi &= 0 \\ \cos n\pi &= (-1)^n \\ \cos 2n\pi &= 1 \end{aligned}$$

Steps to find out rms and average values

1. Find the time period of the waveform i.e. T
2. Find the required equation of the waveform for a complete cycle.
3. Find the rms and average values using the equations.

Exam Solutions

1. Calculate the average and rms value of full wave rectified sine wave as shown below. [2072 Chaitra]



Solution:

Here,

$$e = 0 \quad [0 < \theta < \pi/4]$$

$$e = E_m \sin \theta \quad [\pi/4 < \theta < \pi]$$

Now,

$$\begin{aligned} V_{\text{average}} &= \frac{1}{\pi} \left[\int_0^{\pi/4} 0 d\theta + \int_{\pi/4}^{\pi} E_m \sin \theta d\theta \right] \\ &= \frac{1}{\pi} \left[\int_{\pi/4}^{\pi} E_m \sin \theta d\theta \right] \\ &= \frac{E_m}{\pi} [-\cos \theta]_{\pi/4}^{\pi} \\ &= -\frac{E_m}{\pi} [\cos \pi - \cos \pi/4] \\ &= -\frac{E_m}{\pi} \left[-1 - \frac{1}{\sqrt{2}} \right] \\ &= 0.5433 E_m \end{aligned}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{\pi} \left[\int_0^{\pi/4} 0^2 d\theta + \int_{\pi/4}^{\pi} (E_m \sin \theta)^2 d\theta \right]}$$

$$\text{or, } V_{\text{rms}}^2 = \frac{1}{\pi} \left[\int_0^\pi (E_m \sin \theta)^2 d\theta \right]$$

$$\text{or, } V_{\text{rms}}^2 = \frac{E_m^2}{\pi} \int_0^\pi \sin^2 \theta d\theta$$

$$\text{or, } V_{\text{rms}}^2 = \frac{E_m^2}{\pi} \int_0^\pi \frac{1 - \cos 2\theta}{2} d\theta$$

$$\text{or, } V_{\text{rms}}^2 = \frac{E_m^2}{\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^\pi$$

$$\text{or, } V_{\text{rms}}^2 = \frac{E_m^2}{2\pi} \left[\left(\pi - \frac{\sin 2\pi}{2} \right) - \left(\frac{\pi}{4} - \frac{\sin \frac{\pi}{2}}{2} \right) \right]$$

$$\text{or, } V_{\text{rms}}^2 = \frac{E_m^2}{2\pi} \left[\left(\pi - 0 \right) - \left(\frac{\pi}{4} - \frac{1}{2} \right) \right]$$

$$\text{or, } V_{\text{rms}}^2 = \frac{E_m^2}{2\pi} \left[\pi - \frac{\pi}{4} + \frac{1}{2} \right]$$

$$\therefore V_{\text{rms}} = 0.6742 E_m$$

2.

Calculate the rms value of current of the following triangular wave form. [2071 Chaitra]

Solution:

The waveform completes one cycle

from 0 to 2π .

Hence, time period of the waveform
is 2π as denoted in fig.

Required equation

For $0 \leq \omega t < \pi$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\text{or, } y - 0 = \frac{18 - 0}{\pi - 0} (x - 0)$$

$$\text{or, } y = \frac{18x}{\pi}$$

$$\therefore i = \frac{18}{\pi} (\omega t)$$

$$\text{For } \pi < \omega t < 2\pi$$

$$\text{or, } y - 18 = \frac{0 - 18}{2\pi - \pi} (x - \pi)$$

$$\text{or, } y = 18 - \frac{18}{\pi} (x - \pi)$$

$$\text{or, } y = 18 - \frac{18}{\pi} x + 18$$

$$\text{or, } y = 36 - \frac{18}{\pi} x$$

[$\pi < \omega t < 2\pi$]

Rms value of current,

$$I_{\text{rms}} = \sqrt{\frac{1}{T_0} \int_0^T i^2 d(\omega t)}$$

$$I_{\text{rms}}^2 = \frac{1}{2\pi} \left[\int_0^\pi \left\{ \frac{18}{\pi} (\omega t) \right\}^2 d(\omega t) + \int_\pi^{2\pi} \left\{ 36 - \frac{18}{\pi} (\omega t) \right\}^2 d(\omega t) \right]$$

$$= \frac{1}{2\pi} \left[\int_0^\pi \frac{324}{\pi^2} (\omega t)^2 d(\omega t) + \int_\pi^{2\pi} \left\{ (36)^2 - 2 \times 36 \times \frac{18}{\pi} (\omega t) + \frac{324}{\pi^2} (\omega t)^2 \right\} d(\omega t) \right]$$

$$= \frac{1}{2\pi} \left[\frac{324}{\pi^2} \left| \frac{3}{3} \right| + 1296 (\omega t) \Big|_\pi^{2\pi} - \frac{1296}{\pi} \left| \frac{(\omega t)^2}{2} \right|_\pi^{2\pi} + \frac{324}{\pi^2} \left| \frac{(\omega t)^3}{3} \right|_\pi^{2\pi} \right]$$

$$= \frac{1}{2\pi} \left[\frac{324}{\pi^2} \frac{\pi^3}{3} + 1296\pi - \frac{1296}{\pi} \frac{3\pi^2}{2} + \frac{324}{\pi^2} \frac{7\pi^3}{3} \right]$$

$$= \frac{1}{2\pi} \left[\frac{324\pi}{3} + 1296\pi - \frac{1296 \times 3}{2} \pi + \frac{324 \times 7\pi}{3} \right]$$

$$= \frac{1}{2\pi} \times 216\pi$$

$$\text{or, } I_{\text{rms}} = \sqrt{108}$$

$$\text{or, } I_{\text{rms}} = 10.39 \text{ A}$$

The rms value of current of the above waveform is 10.39 A.

3. An alternating current of frequency 50 Hz has a maximum value of 120 A. Write down the equation for its instantaneous value. Find also the instantaneous value after 1/360 sec and the time taken to reach 96 A for the first time. [2071 Magh]

Solution:

Given,

Frequency (f) = 50 HzMaximum value of current (I_m) = 120 A

Equation for its instantaneous value

$$i = I_m \sin (\omega t)$$

$$= 120 \sin (2\pi f t)$$

$$= 120 \sin (2\pi \times 50 t)$$

$$= 120 \sin (100\pi t)$$

$$i = 120 \sin(100\pi t)$$

∴ Instantaneous value after $1/360$ sec;

$$i = 120 \sin(100\pi \times \frac{1}{360})$$

$$= 120 \sin\left(100 \times \frac{\pi}{360} \times 180^\circ\right)$$

$$= 120 \sin 50^\circ = 91.92 \text{ A}$$

∴ The time taken to reach 96A for the first time,

$$i = 120 \sin(100\pi t)$$

$$\text{or, } 96 = 120 \sin(100\pi t)$$

$$\text{or, } \frac{96}{120} = \sin(100\pi t)$$

$$\text{or, } \sin^{-1}\left(\frac{96}{120}\right) = 100\pi t$$

$$\text{or, } 0.927 = 100\pi t$$

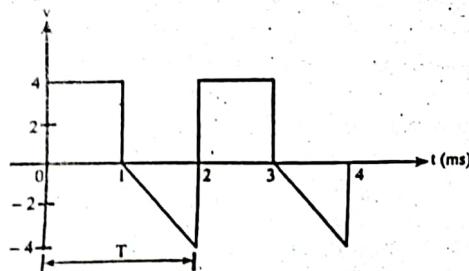
$$\text{or, } t = 2.95 \times 10^{-3}$$

$$\therefore t = 2.95 \text{ millisecond}$$

4. Define cycle, time period, angular velocity, frequency, and average and rms value of an alternating quantity. [2071 Shawan]

[Please refer to the theory]

5. Calculate the Rms value and Average value of the voltage wave given below and hence compute the form factor. [2071 Bhadra]



Solution:

Time period (T) = 2 ms

Required equation:

For $0 < t < 1$

$$\therefore v = 4$$

For $1 < t < 2$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\text{or, } y - 0 = \frac{-4 - 0}{2 - 1} (x - 1)$$

$$\text{or, } y = -4x + 4$$

$$\text{or, } y = 4 - 4x$$

$$v = 4 - 4t$$

Rms value of voltage

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$V_{\text{rms}}^2 = \frac{1}{2} \left[\int_0^1 4^2 dt + \int_1^2 (4 - 4t)^2 dt \right]$$

$$V_{\text{rms}}^2 = \frac{1}{2} \left[16t \Big|_0^1 + \int_1^2 (16 - 32t + 16t^2) dt \right]$$

$$\text{or, } V_{\text{rms}}^2 = \frac{1}{2} \left[16 + 16t \Big|_0^2 - 32 \frac{t^2}{2} \Big|_0^2 + \frac{16t^3}{3} \Big|_0^2 \right]$$

$$\text{or, } V_{\text{rms}}^2 = \frac{1}{2} \left[16 + 16 - 48 + \frac{112}{3} \right]$$

$$\text{or, } V_{\text{rms}}^2 = \frac{1}{2} \times \frac{64}{3}$$

$$\text{or, } V_{\text{rms}}^2 = \frac{32}{3}$$

$$\therefore V_{\text{rms}} = \sqrt{\frac{32}{3}} = 3.266 \text{ volt}$$

Average value of voltage,

$$V_{\text{avg}} = \frac{1}{T} \int_0^T v dt$$

$$= \frac{1}{2} \left[\int_0^1 4 dt + \int_1^2 (4 - 4t) dt \right]$$

$$= \frac{1}{2} \left[4t \Big|_0^1 + 4t \Big|_1^2 - \frac{4t^2}{2} \Big|_1^2 \right]$$

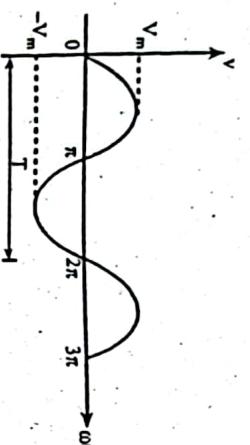
$$= \frac{1}{2} [4 + 4 - 6]$$

$$= \frac{1}{2} \times 2$$

$$= 1 \text{ volt}$$

$$\text{Form factor} = \frac{V_{\text{rms}}}{V_{\text{avg}}} = \frac{3.266}{1} = 3.266$$

6. Find the rms and average values of the given figure.



Solution:

The waveform completes one cycle from 0 to 2π .

Hence, time period of the waveform is 2π as denoted in fig.

Required equations:

$$V = V_m \sin \omega t \quad \text{for } [0 \leq \omega t \leq 2\pi]$$

$$\therefore V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^{2\pi} V^2 d\omega t} \quad \text{for } [0 \leq \omega t \leq 2\pi]$$

$$\text{or, } V_{\text{rms}}^2 = \frac{1}{2\pi} \int_0^{2\pi} (V_m \sin \omega t)^2 d\omega t$$

$$= \frac{1}{2\pi} \int_0^{2\pi} V_m^2 \sin^2 \omega t d\omega t$$

$$= \frac{1}{2\pi} V_m^2 \int_0^{2\pi} \frac{1 - \cos 2\omega t}{2} d\omega t$$

$$= \frac{V_m^2}{2\pi} \times \frac{1}{2} \left[\omega t \left|_0^{2\pi} \right. - \frac{\sin 2\omega t}{2} \left|_0^{2\pi} \right. \right]$$

$$= \frac{V_m^2}{2\pi} \times \left[2\pi - 0 - \frac{\sin 4\pi - \sin 0}{2} \right]$$

$$= \frac{V_m^2}{4\pi} \times 2\pi = \frac{V_m^2}{2}$$

$$\therefore V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

Similarly,

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} \quad \text{for sinusoidal current waveform}$$

Average value in complete cycle,

$$V_{\text{average}} = \frac{1}{T} \int_0^T V d\omega t$$

Thus, for symmetrical waveforms, average value in half cycle

Average value in half cycle

$$V_{\text{average}} = \frac{1}{\pi} \int_0^\pi V d\omega t$$

$$= \frac{1}{\pi} \int_0^\pi V_m \sin \omega t d\omega t$$

$$= \frac{1}{\pi} \times V_m [-\cos \omega t]_0^\pi$$

$$= \frac{V_m}{\pi} [-\cos \pi + \cos 0]$$

$$= \frac{V_m}{\pi} [1 + 1] = \frac{2V_m}{\pi}$$

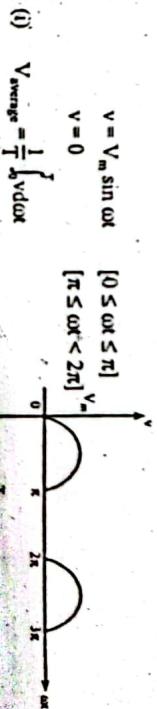
$$\therefore V_{\text{average}} = \frac{2V_m}{\pi}$$

Similarly,

$$I_{\text{average}} = \frac{2I_m}{\pi}$$

7. Calculate the (i) average value and (ii) rms value of voltage waveform shown in fig.

[2065 Kartik]



$$V = V_m \sin \omega t \quad [0 \leq \omega t \leq \pi]$$

$$V = 0 \quad [\pi \leq \omega t < 2\pi]$$

$$(i) \quad V_{\text{average}} = \frac{1}{T} \int_0^T V d\omega t$$

$$= \frac{1}{2\pi} \left[\int_0^{\pi/2} V_m \sin \omega t d\omega t + \int_{\pi/2}^{\pi} 0 d\omega t \right] - \frac{1}{2\pi} V_m [-\cos \omega t]_0^\pi$$

$$= \frac{V_m}{2\pi} [-\cos \pi + \cos 0] - \frac{V_m}{2\pi} [1 + 1] = \frac{V_m}{2\pi} \times 2$$

$$V_{\text{average}} = \frac{1}{T} \int_0^T v d\omega t$$

$$= \frac{1}{2\pi} \int_0^{\pi/2} V_m \sin \omega t d\omega t$$

$$= \frac{1}{2\pi} \times V_m [-\cos \omega t]_0^{\pi/2}$$

$$= \frac{V_m}{2\pi} [-\cos 2\pi + \cos 0]$$

$$= \frac{V_m}{2\pi} [-1 + 1] \quad [\because \cos 2\pi = (-1)^2 = 1]$$

$$= 0$$

$$\therefore V_{\text{avg}} = \frac{V_m}{\pi} \int_0^T v dt$$

$$(ii) V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$\text{or, } V_{\text{rms}} = \frac{1}{2\pi} \left[\int_0^\pi (V_m \sin \omega t)^2 dt + \int_0^\pi 0^2 dt \right]$$

$$\text{or, } V_{\text{rms}}^2 = \frac{1}{2\pi} V_m^2 \int_0^\pi \frac{1 - \cos 2\omega t}{2} dt$$

$$= \frac{1}{4\pi} V_m^2 \left[\omega t \Big|_0^\pi - \frac{\sin 2\omega t}{2} \Big|_0^\pi \right] = \frac{1}{4\pi} V_m^2 \times \pi = \frac{V_m^2}{4}$$

$$\therefore V_{\text{rms}} = \frac{V_m}{2}$$

5. Find the (i) average value (ii) rms value and (iii) form factor of the full wave rectified sine wave shown in fig.



Solution:

$$v = V_m \sin \omega t \quad [0 \leq \omega t \leq \pi]$$

$$(i) V_{\text{average}} = \frac{1}{T} \int_0^T v dt$$

$$= \frac{1}{\pi} \int_0^\pi V_m \sin \omega t dt$$

$$= \frac{1}{\pi} V_m [-\cos \omega t]_0^\pi = \frac{V_m}{\pi} [-\cos \pi + \cos 0] = \frac{V_m}{\pi} [1 + 1]$$

$$\therefore V_{\text{avg}} = \frac{2V_m}{\pi}$$

$$(ii) V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$\text{or, } V_{\text{rms}}^2 = \frac{1}{T} \int_0^T V_m^2 \sin^2 \omega t dt$$

$$= \frac{V_m^2}{\pi} \int_0^\pi \frac{1 - \cos 2\omega t}{2} dt$$

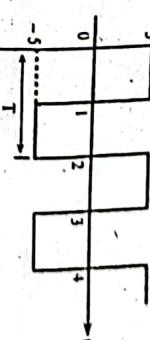
$$(iii) \text{Form factor} = \frac{V_{\text{rms}}}{V_{\text{avg}}} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = \frac{\frac{\pi}{2} V_m}{2V_m} = \frac{\pi}{4\sqrt{2}} = 1.11$$

9. Calculate the average and rms values for the figure shown below:

$$v(0) = 5 \quad [0 \leq t < 1]$$

$$v(0) = -5 \quad [1 < t < 2]$$

[2004 Poush]



Solution:

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$\text{or, } V_{\text{rms}}^2 = \frac{1}{2} \left[\int_0^1 (5)^2 dt + \int_1^2 (-5)^2 dt \right]$$

$$= \frac{1}{2} \left[25t \Big|_0^1 + 25t \Big|_1^2 \right]$$

$$= \frac{1}{2} [25 \times 1 + 25 \times 1] = \frac{50}{2} = 25$$

$$\therefore V_{\text{rms}} = \sqrt{25} = 5V$$

Average value in complete cycle

$$V_{\text{average}} = \frac{1}{T} \int_0^T v dt = \frac{1}{2} \left[\int_0^1 5dt + \int_1^2 (-5)dt \right]$$

$$= \frac{1}{2} \left[5t \Big|_0^1 + (-5)t \Big|_1^2 \right] = \frac{1}{2} [5 - 5] = 0$$

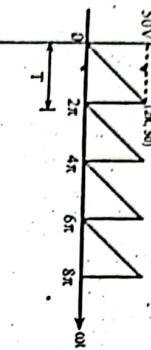
Since, symmetrical waveform's average value in complete cycle is zero.

Average value in half cycle

$$V_{\text{average}} = \frac{1}{2} \int_0^T v dt$$

$$= \frac{5}{2} \left[t - \frac{5}{2}(1-t) \right] \Big|_0^1 = 2.5V$$

10. Calculate the average, rms value, form factor and peak factor of the saw tooth waveform as shown in fig. [2068 Chaitta]



Solution:

We know

The equation of line is $y = mx$

i.e. $v = m(\omega t)$

$$\therefore \text{In fig. Slope (m)} = \frac{50}{2\pi}$$

OR Using two point formula;

$$x_1 = 0 \quad y_1 = 0$$

$$x_2 = 2\pi \quad y_2 = 50$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$(y - 0) = \frac{50 - 0}{2\pi - 0} (x - 0)$$

$$y = \frac{50}{2\pi} x$$

$\therefore v = \frac{50}{2\pi} (\omega t) \quad [0 \leq \omega t \leq 2\pi]$

Now,

$$V_{\text{average}} = \frac{1}{T} \int_0^T v dt$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{50}{2\pi} (\omega t) dt = \frac{1}{2\pi} \times \frac{50}{2\pi} \left[\frac{(\omega t)^2}{2} \right]_0^{2\pi}$$

$$= \frac{50}{(2\pi)^2} \times \frac{(2\pi)^2}{2} = 25V$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$V_{\text{rms}}^2 = \frac{1}{2\pi} \int_0^T \left(\frac{50}{2\pi} \omega t \right)^2 dt$$

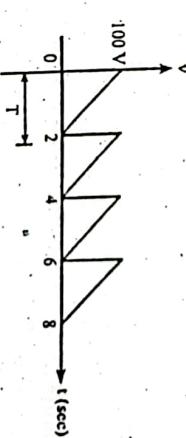
$$= \frac{1}{2\pi} \times \frac{(50)^2}{(2\pi)^2} \frac{(\omega t)^3}{3} \Big|_0^T = \frac{(50)^2}{(2\pi)^3} \times \frac{(2\pi)^3}{3} = \frac{(50)^2}{3}$$

$$\therefore V_{\text{rms}} = \frac{50}{\sqrt{3}} = 28.8675V$$

$$\text{Form factor} = \frac{V_{\text{peak}}}{V_{\text{average}}} = \frac{28.8675}{25} = 1.15$$

$$\text{Peak factor} = \frac{V_{\text{peak}}}{V_{\text{rms}}} = \frac{V_{\text{peak}}}{28.8675} = 1.73$$

11. Find the average value, rms value, form factor and peak factor of the voltage waveform given below. [2070 Asad]



Solution:

Using two point formula,

$$(0, 100) \Rightarrow x_1 = 0 \quad y_1 = 100$$

$$(2, 0) \Rightarrow x_2 = 2 \quad y_2 = 0$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 100 = \frac{0 - 100}{2 - 0} (x - 0)$$

$$y = 100 - \frac{100}{2} x$$

$$\therefore v = 100 - 50t \quad [0 \leq t \leq 2]$$

$$V_{\text{average}} = \frac{1}{T} \int_0^T v dt$$

$$= \frac{1}{2} \int_0^2 (100 - 50t) dt = \frac{1}{2} \left[100t - 50 \frac{t^2}{2} \right]_0^2$$

$$= \frac{1}{2} \left[100 \times 2 - 50 \times \frac{4}{2} \right] = \frac{1}{2} [200 - 100]$$

$$\therefore V_{\text{average}} = 50V$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$\text{or, } V_{\text{rms}}^2 = \frac{1}{2} \int_0^T (100 - 50t)^2 dt$$

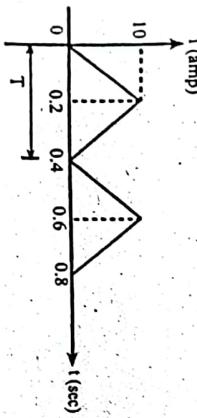
$$\begin{aligned} &= \frac{1}{2} \int_0^T [(100)^2 - 2 \times 100 \times 50t + (50t)^2] dt \\ &= \frac{1}{2} \left[(100)^2 t \Big|_0^{0.2} - 10000 \frac{t^2}{2} \Big|_0^{0.2} + (50)^2 \frac{t^3}{3} \Big|_0^{0.2} \right] \\ &= \frac{1}{2} \left[(100)^2 \times 2 - (100)^2 \times \frac{4}{2} + (50)^2 \times \frac{8}{3} \right] = \frac{1}{2} (50)^2 \times \frac{8}{3} = (50)^2 \times \frac{4}{3} \\ \therefore V_{\text{rms}} &= 50 \times \sqrt{\frac{2}{3}} = 57.735 \end{aligned}$$

$$\text{Form factor} = \frac{V_{\text{peak}}}{V_{\text{average}}} = \frac{57.735}{50} = 1.154$$

$$\text{Peak factor} = \frac{V_{\text{peak}}}{V_{\text{rms}}} = \frac{100}{57.735} = 1.732$$

12. Find the average and rms values of the triangular waveform of current shown in fig below. Also calculate the form factor and peak amplitude factor for the triangular wave form

[2063 Karlik]



Solution:

For $0 \leq t < 0.2$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\text{or, } y - 0 = \frac{10 - 0}{0.2 - 0} (x - 0)$$

$$\text{or, } y = \frac{10}{0.2} x$$

$$i = 50t \quad [0 \leq t < 0.2]$$

For $0.2 < t < 0.4$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\text{or, } y - 10 = \frac{0 - 10}{0.4 - 0.2} (x - 0.2)$$

$$\text{or, } y = 10 - \frac{10}{0.2} (x - 0.2)$$

$$\text{or, } y = 10 - 50x + 10$$

$$\text{or, } y = 20 - 50x$$

$$i = 20 - 50t \quad [0.2 < t < 0.4]$$

$$I_{\text{average}} = \frac{1}{0.4} \left[\int_{0.2}^{0.4} 50t dt + \int_{0.4}^{0.6} (20 - 50t) dt \right]$$

$$\begin{aligned} &= \frac{1}{0.4} \left[50 \frac{t^2}{2} \Big|_0^{0.4} + 20t \Big|_0^{0.6} - 50 \frac{t^2}{2} \Big|_0^{0.6} \right] \\ &= \frac{1}{0.4} \left[50 \times \frac{(0.2)^2}{2} + 20 \times (0.2) - 50 \times \frac{(0.4)^2}{2} + 50 \times \frac{(0.2)^2}{2} \right] = \frac{1}{0.4} \times 2 = 5A \end{aligned}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$$

$$\begin{aligned} I_{\text{rms}}^2 &= \frac{1}{0.4} \left[\int_0^{0.2} (50t)^2 dt + \int_{0.2}^{0.4} (20 - 50t)^2 dt \right] \\ &= \frac{1}{0.4} \left[(50)^2 \frac{t^3}{3} \Big|_0^{0.2} + \frac{(20 - 50t)^2}{2} \Big|_{0.2}^{0.4} \right] \\ &= \frac{1}{0.4} \left[(50)^2 \times \frac{(0.2)^3}{3} - \frac{(10)^3}{3(50)} \right] \end{aligned}$$

OR Expanding the square term & integrating individually.

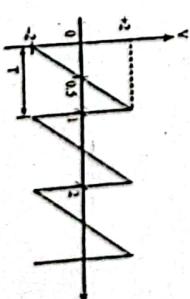
$$\therefore I_{\text{rms}} = \sqrt{\frac{100}{3}} = 5.7735A$$

$$\text{Form factor} = \frac{I_{\text{peak}}}{I_{\text{average}}} = \frac{5.7735}{5} = 1.154$$

$$\text{Peak amplitude factor} = \frac{I_{\text{peak}}}{I_{\text{rms}}} = \frac{10}{5.7735} = 1.732$$

13. Find the rms and average values of the waveform given in figure below.

[2070 Magh]



Solution:
Using two-point formula for equation throughout 0 to 1

i.e. (0, -2) and (1, 2)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\text{or, } y - (-2) = \frac{2 - (-2)}{1 - 0} (x - 0)$$

$$\text{or, } y + 2 = \frac{2 + 2}{1} (x - 0)$$

$$\text{or, } y = 4x - 2$$

$$\therefore v = 4t - 2 \quad [0 \leq t < 1]$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T (4t - 2)^2 dt}$$

$$\text{or, } V_{\text{rms}}^2 = \int_0^T [(4t)^2 - 2 \times 4t \times 2 + 2^2] dt$$

$$= (4t)^2 \left[\frac{t^2}{2} \right]_0^T - 16t \left[\frac{t^2}{2} \right]_0^T + 4t \left[t \right]_0^T = \frac{16}{3}T - \frac{16}{2}T + 4 = \frac{4}{3}T$$

$$\therefore V_{\text{rms}} = 1.154 \text{ V}$$

Average value in complete cycle

$$V_{\text{average}} = \frac{1}{T} \int_0^T (4t - 2) dt = 4 \left[\frac{t^2}{2} \right]_0^T - 2t \Big|_0^T = \frac{4}{2}T - 2T = 0$$

Average value in half cycle

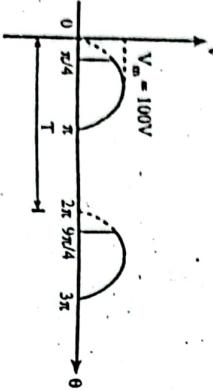
$$V_{\text{average}} = \frac{1}{0.5} \int_0^{0.5} (4t - 2) dt = \frac{1}{0.5} \left[4 \left(\frac{0.5t^2}{2} \right) - 2t \right]_0^{0.5} = \frac{1}{0.5} \left[4 \left(\frac{0.5 \cdot 0.25}{2} \right) - 2(0.5) \right]$$

$$\therefore V_{\text{average}} = -1 \text{ V}$$

i.e. - ve sign indicate the Average value for lower half cycle, hence lies below the reference line.

14. Determine the average and rms values of voltage for sinusoidal voltage waveform as shown in figure below.

[2067 Mansir]



Solution:

$$v = 0 \quad [0 \leq \theta < \frac{\pi}{4}]$$

$$v = 100 \sin \theta \quad \left[\frac{\pi}{4} < \theta < \pi \right]$$

$$v = 0 \quad [\pi < \theta < 2\pi]$$

$$V_{\text{average}} = \frac{1}{2\pi} \int_{\pi/4}^{\pi} 100 \sin \theta d\theta$$

$$= \frac{1}{2\pi} 100 \left[-\cos \theta \right]_{\pi/4}^{\pi}$$

$$= \frac{1}{2\pi} 100 \left[-\cos \pi + \cos \frac{\pi}{4} \right]$$

$$= \frac{1}{2\pi} 100 \left[1 + \frac{1}{\sqrt{2}} \right] = \frac{1}{2\pi} \times 100 \left(\frac{\sqrt{2} + 1}{\sqrt{2}} \right) = 27.169 \text{ V}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_{\pi/4}^{\pi} (100 \sin \theta)^2 d\theta}$$

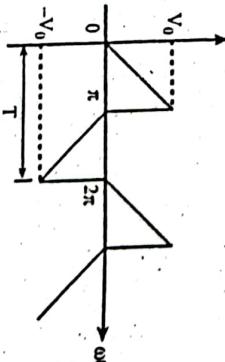
$$\text{or, } V_{\text{rms}}^2 = \frac{1}{2\pi} (100)^2 \int_{\pi/4}^{\pi} \frac{1 - \cos 2\theta}{2} d\theta$$

$$= \frac{1}{2\pi} (100)^2 \left[\frac{1}{2} \left[\theta \right]_{\pi/4}^{\pi} - \frac{\sin 2\theta}{2} \right]$$

$$= \frac{1}{4\pi} (100)^2 \left[\left(\pi - \frac{\pi}{4} \right) - \frac{\sin 2\pi - \sin 2\frac{\pi}{4}}{2} \right] = \frac{1}{4\pi} (100)^2 \left[\pi - \frac{\pi}{4} + \frac{\sin \frac{\pi}{2}}{2} \right] = \frac{1}{4\pi} (100)^2 \left[\frac{3\pi}{4} + \frac{1}{2} \right] = 2272.88$$

$$\therefore V_{\text{rms}} = 47.67 \text{ V}$$

15. Determine the average and rms values of voltage for voltage waveform as shown in figure below.



For $0 \leq \omega t < \pi$

$(0, 0), (\pi, V_0)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 0 = \frac{V_0 - 0}{\pi - 0} (x - 0)$$

$$\text{or, } y = \frac{V_0}{\pi} x$$

$$\therefore v = \frac{V_0}{\pi} (\omega t) \quad [0 \leq \omega t < \pi]$$

For $\pi < \omega t < 2\pi$

($\infty, 0$) & ($2\pi, -V_0$)

$$y - 0 = \frac{-V_0 - 0}{2\pi - \pi} (\omega t - \pi)$$

$$\text{or, } y = \frac{-V_0}{\pi} (\omega t - \pi)$$

$$\text{or, } y = V_0 - \frac{V_0}{\pi} \omega t$$

$$\therefore v = V_0 - \frac{V_0}{\pi} (\omega t) \quad [\pi < \omega t < 2\pi]$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$\begin{aligned} \text{or, } V_{rms}^2 &= \frac{1}{2\pi} \left[\int_0^\pi \left(\frac{V_0 \omega t}{\pi} \right)^2 dt + \int_\pi^{2\pi} \left(V_0 - \frac{V_0}{\pi} \omega t \right)^2 dt \right] \\ &= \frac{1}{2\pi} \left[\left(\frac{V_0}{\pi} \right)^2 \times \frac{(\omega t)^3}{3} \Big|_0^\pi + \int_\pi^{2\pi} \left\{ V_0^2 - 2 \times V_0 \frac{V_0}{\pi} (\omega t) + \left(\frac{V_0}{\pi} \right)^2 (\omega t)^2 \right\} dt \right] \\ &= \frac{1}{2\pi} \left[\left(\frac{V_0}{\pi} \right)^2 \times \frac{\pi^3}{3} + V_0^2 (\omega t) \Big|_\pi^{2\pi} - \frac{2V_0^2 (\omega t)^2}{\pi} \Big|_\pi^{2\pi} + \left(\frac{V_0}{\pi} \right)^2 \frac{(\omega t)^3}{3} \Big|_\pi^{2\pi} \right] \\ &= \frac{1}{2\pi} \left[\left(\frac{V_0}{\pi} \right)^2 \frac{\pi^3}{3} + V_0^2 \pi - \frac{2V_0^2 (2\pi)^2}{\pi} + \frac{2V_0^2 \pi^2}{\pi} + \left(\frac{V_0}{\pi} \right)^2 \frac{(2\pi)^3}{3} - \left(\frac{V_0}{\pi} \right)^2 \frac{(\pi)^3}{3} \right] \\ &= \frac{1}{2\pi} \left[\frac{V_0^2 \pi^2}{3} + V_0^2 \pi - 4V_0^2 \pi + V_0^2 \pi + \frac{8V_0^2 \pi}{3} - \frac{V_0^2 \pi}{3} \right] = \frac{1}{2\pi} \times \frac{2}{3} V_0^2 \pi = \frac{V_0^2}{3} \end{aligned}$$

$$\therefore V_{rms} = \frac{V_0}{\sqrt{3}}$$

Average value in complete cycle:

$$\begin{aligned} V_{average} &= \frac{1}{T} \int_0^T v dt \\ &= \frac{1}{2\pi} \left[\int_0^\pi \frac{V_0}{\pi} (\omega t) dt + \int_\pi^{2\pi} \left(V_0 - \frac{V_0}{\pi} (\omega t) \right) dt \right] \\ &= \frac{1}{2\pi} \left[\frac{V_0}{\pi} \frac{(\omega t)^2}{2} \Big|_0^\pi + V_0 (\omega t) \Big|_\pi^{2\pi} - \frac{V_0}{\pi} \frac{(\omega t)^2}{2} \Big|_\pi^{2\pi} \right] \\ &= \frac{1}{2\pi} \left[\frac{V_0 \pi^2}{2} + V_0 \pi - \frac{V_0 (2\pi)^2}{2} + \frac{V_0 (\pi)^2}{2} \right] \\ &= \frac{1}{2\pi} \left[\frac{V_0 \pi}{2} + V_0 \pi - 2V_0 \pi + \frac{1}{2} V_0 \pi \right] = 0 \end{aligned}$$

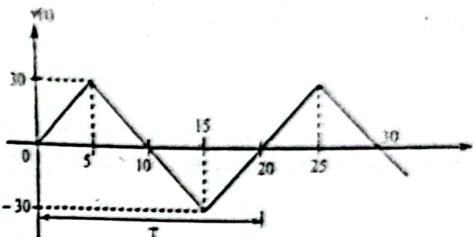
Average value in half cycle:

$$V_{average} = \frac{1}{\pi} \int_0^\pi \frac{V_0}{\pi} (\omega t) dt = \frac{1}{\pi} \times \frac{V_0 (\omega t)^2}{2} \Big|_0^\pi = \frac{1}{\pi} \times \frac{V_0}{\pi} \times \frac{\pi^2}{2}$$

$$\therefore V_{average} = \frac{V_0}{2}$$

Additional Questions

1. Determine the average and rms values of voltage for current waveform shown in figure below.



Solution:

Since the waveform is symmetrical, average value in a complete cycle is zero.

Therefore, average and rms values is calculated in half-cycle.

For $0 \leq t < 5$

$$y = mx$$

$$\text{or, } v = \frac{30}{5} t$$

$$\therefore v = 6t \quad [0 \leq t < 5]$$

For $5 < t < 10$:

$(5, 30) \& (10, 0)$

$$y - 30 = \frac{0 - 30}{10 - 5} (x - 5)$$

$$\text{or, } y = 30 - 6x + 30$$

$$\therefore v = 60 - 6t \quad [5 < t < 10]$$

$$V_{rms} = \sqrt{\frac{1}{T/2} \int_0^{T/2} v^2 dt}$$

$$\begin{aligned} \text{or, } V_{rms}^2 &= \frac{1}{10} \left[\int_0^5 (6t)^2 dt + \int_5^{10} (60 - 6t)^2 dt \right] \\ &= \frac{1}{10} \left[36 \frac{t^3}{3} \Big|_0^5 + \int_5^{10} [(60)^2 - 2 \times 60 \times 6t + (6t)^2] dt \right] \\ &= \frac{1}{10} \left[36 \times \frac{5^3}{3} + (60)^2 \Big|_5^{10} - 720 \frac{t^2}{2} \Big|_5^{10} + (6t)^2 \frac{t}{3} \Big|_5^{10} \right] \\ &= \frac{1}{10} \left[36 \times \frac{125}{3} + (60)^2 (10 - 5) - 720 \frac{(10)^2}{2} + 720 \frac{(5)^2}{2} + (5)^2 \frac{(10)^2}{3} - (5)^2 \frac{(5)^2}{3} \right] \\ &= \frac{1}{10} (1500 + 1800 - 36000 + 9000 + 12000 - 1500) = 300 \end{aligned}$$

$$\therefore V_{rms} = 17.32 \text{ V}$$

$$V_{\text{average}} = \frac{1}{T/2} \int_0^{T/2} v dt$$

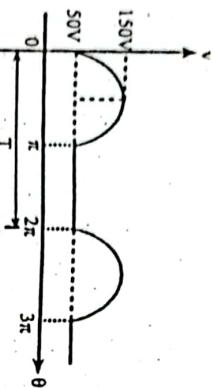
$$= \frac{1}{10} \left[\int_0^6 dt + \int_6^{10} (60 - 6t) dt \right]$$

$$= \frac{1}{10} \left[6 \left[\frac{t^2}{2} \right]_0^6 + 60t \Big|_6^{10} - 6 \left[\frac{t^2}{2} \right]_6^{10} \right]$$

$$= \frac{1}{10} \left[6 \times \frac{(5)^2}{2} + 60 \times 5 - 6 \times \frac{(10)^2}{2} + 6 \times \frac{(5)^2}{2} \right]$$

$$= 15 \text{ V}$$

2. A transmission line carries a dc voltage of 50V and half-wave rectified sinusoidal voltage as shown in figure. Calculate (i) average value (ii) rms value.



$$v = 50 + 100 \sin \theta \quad \text{for } [0 \leq \theta < \pi]$$

$$[\text{Note: } V_m \sin \theta \Rightarrow V_m = 150 - 50 = 100]$$

$$v = 50 \quad \text{for } [\pi < \theta < 2\pi]$$

$$V_{\text{average}} = \frac{1}{2\pi} \left[\int_0^\pi (50 + 100 \sin \theta) d\theta + \int_\pi^{2\pi} 50 d\theta \right]$$

$$= \frac{1}{2\pi} \left[50\theta \Big|_0^\pi + 100 (-\cos \theta) \Big|_0^\pi + 50\theta \Big|_\pi^{2\pi} \right]$$

$$= \frac{1}{2\pi} [50\pi + 100 (-\cos \pi + \cos 0) + 50\pi]$$

$$= \frac{1}{2\pi} [100\pi + 100 (1 + 1)] = 81.83\text{V}$$

$$V^2_{\text{average}} = \frac{1}{2\pi} \left[\int_0^\pi (50 + 100 \sin \theta)^2 d\theta + \int_\pi^{2\pi} (50)^2 d\theta \right]$$

$$= \frac{1}{2\pi} \left[\int_0^\pi [(50)^2 + 10000 \sin \theta + (100)^2 \sin^2 \theta] d\theta + (50)^2 \theta \Big|_\pi^{2\pi} \right]$$

$$= \frac{1}{2\pi} \left[(50)^2 \theta \Big|_0^\pi + 10000 (-\cos \theta) \Big|_0^\pi + (100)^2 \int_0^\pi \frac{1 - \cos 2\theta}{2} d\theta + (50)^2 \pi \right]$$

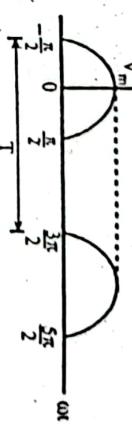
$$= \frac{1}{2\pi} \left[(50)^2 \pi + 10000 \times 2 + (100)^2 \left\{ \frac{1}{2} \theta \Big|_0^\pi - \frac{1}{2} \frac{\sin 2\theta}{2} \Big|_0^\pi \right\} + (50)^2 \pi \right]$$

$$\therefore V_{\text{average}} = \frac{V_m}{2}$$

$$= \frac{1}{2\pi} \left[(50)^2 2\pi + 10000 \times 2 + \frac{(100)^2}{2} \pi \right]$$

$$\therefore V_{\text{rms}} = 90.46 \text{ V}$$

3. Determine the average and rms values of voltage for sinusoidal voltage waveform as shown in figure below.



Solution:

$$v = V_m \cos \omega t \quad \left[-\frac{\pi}{2} < \omega t < \frac{\pi}{2} \right]$$

$$v = 0 \quad \left[\frac{\pi}{2} < \omega t < \frac{3\pi}{2} \right]$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2 d\omega t}$$

$$V^2_{\text{rms}} = \frac{1}{\frac{3\pi}{2} - (-\frac{\pi}{2})} \left[\int_{-\pi/2}^{\pi/2} (V_m \cos \omega t)^2 d\omega t \right]$$

$$= \frac{1}{2\pi} V_m^2 \int_{-\pi/2}^{\pi/2} \frac{1 + \cos 2\omega t}{2} d\omega t$$

$$= \frac{1}{2\pi} V_m^2 \frac{1}{2} \left[\omega t \Big|_{-\pi/2}^{\pi/2} + \frac{\sin 2\omega t}{2} \Big|_{-\pi/2}^{\pi/2} \right]$$

$$= \frac{1}{2\pi} V_m^2 \frac{1}{2} \left[\frac{\pi}{2} + \frac{\pi}{2} + \frac{\sin \pi - \sin(-\pi)}{2} \right] = \frac{V_m^2}{4\pi} [\pi] = \frac{V_m^2}{4}$$

$$\therefore V_{\text{rms}} = \frac{V_m}{2}$$

$$V_{\text{average}} = \frac{1}{T} \int_0^T v d\omega t$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} V_m \cos \omega t d\omega t$$

$$= \frac{1}{2\pi} V_m \sin \omega t \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi} V_m \left[\sin \frac{\pi}{2} - \sin \left(-\frac{\pi}{2} \right) \right] = \frac{V_m}{2\pi} [1 - (-1)]$$

$$\therefore V_{\text{average}} = \frac{V_m}{\pi}$$

15.1 Alternating Quantities

4. Determine the average and rms values of current for current waveform as shown in figure below.

Solution:

$$i(t) = \begin{cases} 5t & ; 0 < t < 2 \\ -10 & ; 2 < t < 4 \end{cases}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$$

$$\text{or, } I_{rms}^2 = \frac{1}{4} \left[\int_0^2 (5t)^2 dt + \int_2^4 (-10)^2 dt \right]$$

$$= \frac{1}{4} \left[25 \frac{t^3}{3} \Big|_0^2 + 100t \Big|_2^4 \right] = \frac{1}{4} \left[25 \left(\frac{2^3}{3} \right) + 100 (4-2) \right]$$

$$= \frac{1}{4} \left[25 \times \frac{8}{3} + 100 \times 2 \right] = \frac{200}{3}$$

$$\therefore I_{rms} = 8.165 \text{ A}$$

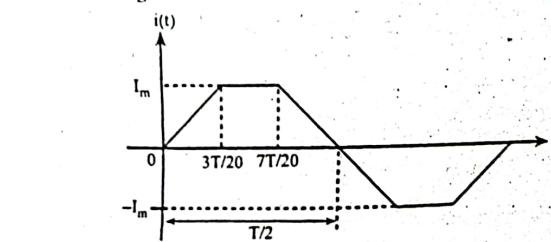
$$\text{Now, } I_{average} = \frac{1}{4} \left[\int_0^2 5t dt + \int_2^4 (-10) dt \right]$$

$$= \frac{1}{4} \left[5 \frac{t^2}{2} \Big|_0^2 + (-10)t \Big|_2^4 \right] = \frac{1}{4} \left[5 \left(\frac{2^2}{2} \right) + (-10)(4-2) \right]$$

$$= \frac{1}{4} [10 - 20] = -\frac{5}{2} = -2.5 \text{ A}$$

-ve sign indicates the average value lies below the reference line.

5. Determine the average and rms values of current for current waveform as shown in figure below.



$$\text{For } 0 < t < \frac{3T}{20}$$

$$y = mx$$

$$\text{or, } i = \frac{I_m}{\frac{3T}{20}} t$$

$$\therefore i = \frac{20}{3T} I_m t$$

$$\text{For } \frac{3T}{20} < t < \frac{7T}{20}$$

$$y = I_m$$

$$\text{For } \frac{7T}{20} < t < \frac{T}{2}$$

$$\left(\frac{7T}{20}, I_m \right) \& \left(\frac{T}{2}, 0 \right)$$

$$y - I_m = \frac{0 - I_m}{\frac{T}{2} - \frac{7T}{20}} \left(x - \frac{7T}{20} \right)$$

$$\text{or, } y = I_m - \frac{I_m}{\frac{3T}{20}} \left(x - \frac{7T}{20} \right)$$

$$\text{or, } y = I_m - \frac{20}{3T} x + \frac{7}{3} I_m$$

$$\text{or, } y = \frac{10}{3} I_m - \frac{20}{3T} I_m x$$

$$\therefore i = \frac{10}{3} I_m - \frac{20}{3T} I_m t$$

$$i(t) = \begin{cases} \frac{20}{3T} I_m t & ; 0 < t < \frac{3T}{20} \\ I_m & ; \frac{3T}{20} < t < \frac{7T}{20} \\ \frac{10}{3} I_m - \frac{20}{3T} I_m t & ; \frac{7T}{20} < t < \frac{T}{2} \end{cases}$$

$$I_{rms}^2 = \frac{1}{T} \left\{ \int_0^{\frac{3T}{20}} \left(\frac{20}{3T} I_m t \right)^2 dt + \int_{\frac{3T}{20}}^{\frac{7T}{20}} (I_m)^2 dt + \int_{\frac{7T}{20}}^{\frac{T}{2}} \left(\frac{10}{3} I_m - \frac{20}{3T} I_m t \right)^2 dt \right\}$$

$$= \frac{2}{T} \left[\left(\frac{20}{3T} \right)^2 I_m^2 \frac{t^3}{3} \Big|_0^{\frac{3T}{20}} + I_m^2 t \Big|_{\frac{3T}{20}}^{\frac{7T}{20}} + \int_{\frac{7T}{20}}^{\frac{T}{2}} \left(\frac{10}{3} I_m - \frac{20}{3T} I_m t \right)^2 dt \right]$$

$$= \frac{2}{T} \left[\left(\frac{20}{3T} \right)^2 I_m^2 \left(\frac{3T}{20} \right)^3 + I_m^2 \left(\frac{4T}{20} \right) + \left(\frac{10}{3} \right)^2 I_m^2 \Big|_{\frac{7T}{20}}^{\frac{T}{2}} - \frac{400}{9T} I_m^2 t^2 \Big|_{\frac{7T}{20}}^{\frac{T}{2}} + \left(\frac{20}{3T} \right)^2 I_m^2 \frac{t^3}{3} \Big|_{\frac{7T}{20}}^{\frac{T}{2}} \right]$$

$$= \frac{2}{T} \left[\frac{3T}{20} I_m^2 \frac{1}{3} + I_m^2 \frac{T}{5} + \left(\frac{10}{3} \right)^2 I_m^2 \frac{3T}{20} - \frac{400}{9T} I_m^2 \frac{T^2}{2} + \left(\frac{20}{3T} \right)^2 I_m^2 \frac{657T^3}{24000} \right]$$

$$= \frac{2}{7} \left[l_m^2 \left(\frac{1}{20} + l_m^2 \left(\frac{1}{5} + l_m^2 \left(\frac{1}{3} + l_m^2 \left(\frac{5T}{6} - \frac{17}{6} l_m^2 T + \frac{73}{60} l_m^2 T^2 \right) \right) \right) \right) \right]$$

$$+ \frac{2 l_m^2 T}{7} \left[\frac{1}{20} + \frac{1}{5} + \frac{5}{3} - \frac{17}{6} + \frac{73}{60} \right] = 2 l_m^2 \times \frac{3}{10}$$

$$\therefore I_{m0} = \sqrt{\frac{3}{5} l_m}$$

$$= 0.7425 l_m$$

$$I_{\text{average}} = \frac{1}{2} \left[\int_0^{T/2} \frac{20}{3l_m} t dt + \int_{T/2}^{3T/2} l_m t dt + \int_{3T/2}^{5T/2} \left(\frac{10}{3} l_m - \frac{20}{3} \frac{t^2}{l_m} \right) dt \right]$$

$$= \frac{2}{3} \left[\frac{20}{3l_m} \frac{t^2}{2} \right]_0^{T/2} + l_m t \left[\frac{10}{3} t - \frac{20}{3} \frac{t^3}{l_m} \right]_{T/2}^{5T/2} - \frac{20}{3} \frac{l_m^2}{2} \left[\frac{t^2}{2} \right]_{T/2}^{5T/2}$$

$$= \frac{2}{3} \left[\frac{20}{3l_m} \frac{1}{2} \left(\frac{5T}{20} \right)^2 + l_m \left(\frac{4T}{5} + \frac{10}{3} l_m \frac{3T}{20} - \frac{20}{3} \frac{l_m^2}{2} \frac{5T^2}{800} \right) \right]$$

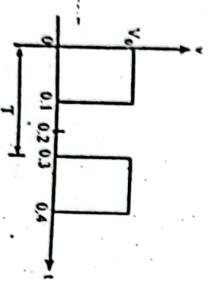
$$= \frac{2}{3} \left[\frac{3T}{4l_m} + \frac{T}{5l_m} + \frac{T}{2l_m} - \frac{17}{40} T l_m \right]$$

$$= \frac{2}{3} \times T l_m \left[\frac{3}{40} + \frac{1}{5} + \frac{1}{20} - \frac{17}{40} \right]$$

$$= 2 l_m \times \frac{7}{20}$$

$$= \frac{7}{10} l_m$$

6. Calculate the average and rms values and form factor and peak factor of the following figure.



$$V = \begin{cases} V_0 & ; 0 < t < 0.1 \\ 0 & ; 0.1 < t < 0.3 \end{cases}$$

$$V_{m0}^2 = \frac{1}{T} \int_0^T V^2 dt = \frac{1}{T} V_0^2 \cdot 0.1 = \frac{1}{T} V_0^2 \cdot \left[-\frac{1}{0.3} V_0^2 \times 0.1 \right] = \frac{V_0^2}{3}$$

$$\therefore V_{m0} = \frac{V_0}{\sqrt{3}}$$

$$V_{\text{average}} = \frac{1}{0.3} \int_0^1 V_0 dt$$

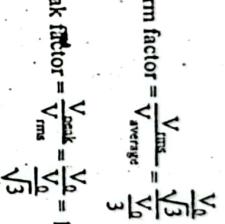
$$= \frac{1}{0.3} V_0 \cdot t \Big|_0^1 = \frac{1}{0.3} V_0 \times 0.1$$

$$\therefore V_{\text{average}} = \frac{V_0}{3}$$

$$\therefore \text{Form factor} = \frac{V_{\text{peak}}}{V_{\text{average}}} = \frac{\sqrt{3}}{0.1} = 1.73$$

$$\therefore \text{Peak factor} = \frac{V_{\text{peak}}}{V_{\text{rms}}} = \frac{V_0}{\sqrt{3}} = 1.73$$

7. A full wave - rectified sinusoidal voltage is clipped at $\frac{1}{\sqrt{2}}$ of its max value. Find its rms and average value.



$$V = \begin{cases} V_m \sin \omega t & ; 0 < \omega t < \frac{\pi}{4} \\ \frac{V_m}{\sqrt{2}} & ; \frac{\pi}{4} < \omega t < \frac{3\pi}{4} \\ V_m \sin \omega t & ; \frac{3\pi}{4} < \omega t < \pi \end{cases}$$

$$V_{\text{average}} = \frac{1}{T} \int_0^T v dt$$

$$= \frac{1}{\pi} \left[\int_0^{\pi/4} V_m \sin \omega t dt + \int_{\pi/4}^{3\pi/4} \frac{V_m}{\sqrt{2}} \omega t dt + \int_{3\pi/4}^{\pi} V_m \sin \omega t dt \right]$$

$$= \frac{1}{\pi} \left[V_m \left[-\cos \omega t \right]_0^{\pi/4} + \frac{V_m}{\sqrt{2}} \omega t \Big|_{\pi/4}^{3\pi/4} + V_m \left[-\cos \omega t \right]_{3\pi/4}^{\pi} \right]$$

$$= \frac{1}{\pi} \left[V_m \left[-\cos \frac{\pi}{4} + \cos 0 \right] + \frac{V_m}{\sqrt{2}} \left(\frac{3\pi}{4} - \frac{\pi}{4} \right) + V_m \left[-\cos \pi + \cos \frac{3\pi}{4} \right] \right]$$

$$= \frac{1}{\pi} \left[V_m \left(-\frac{1}{\sqrt{2}} + 1 \right) + \frac{V_m}{\sqrt{2}} \frac{\pi}{2} + V_m \left(1 + \left(-\frac{1}{\sqrt{2}} \right) \right) \right] = 0.54 V_m$$

$$\begin{aligned}
 V_{\text{rms}} &= \sqrt{\frac{1}{T} \int_0^T v^2 dt} \\
 V_{\text{rms}}^2 &= \frac{1}{\pi} \left[\int_0^{\pi/4} V_m^2 \sin^2 \omega t dt + \int_{\pi/4}^{\pi} \left(\frac{V_m}{\sqrt{2}} \right)^2 dt + \int_{\pi/4}^{\pi} V_m^2 \sin^2 \omega t dt \right] \\
 &= \frac{1}{\pi} V_m^2 \left[\int_0^{\pi/4} \frac{1 - \cos 2\omega t}{2} dt + \int_{\pi/4}^{\pi} \frac{1}{2} dt + \int_{\pi/4}^{\pi} \frac{1 - \cos 2\omega t}{2} dt \right] \\
 &= \frac{1}{\pi} V_m^2 \left[\omega \left[\frac{1}{2} - \frac{\sin 2\omega t}{2} \right] \Big|_0^{\pi/4} + \omega t \Big|_{\pi/4}^{\pi} + \omega \left[\frac{1}{2} - \frac{\sin 2\omega t}{2} \right] \Big|_{\pi/4}^{\pi} \right] \\
 &= \frac{1}{\pi} V_m^2 \left[\frac{\pi}{4} - \frac{\sin \frac{\pi}{2} - \sin 0}{2} + \frac{\pi}{2} + \frac{\pi}{4} - \frac{\sin 2\pi - \sin \frac{3\pi}{2}}{2} \right] \\
 &= \frac{1}{\pi} V_m^2 \left[\pi - \frac{1}{2} - \frac{1}{2} \right] = \frac{1}{\pi} \times \frac{V_m^2}{2} [\pi - 1] \\
 &= 0.3408 V_m^2 \\
 V_{\text{rms}} &= 0.584 V_m
 \end{aligned}$$

Adding two alternating quantities Exam Solutions

1. Two currents i_1 and i_2 are given as $i_1 = 10 \sin \left(314t + \frac{\pi}{14} \right)$ A and $i_2 = 8 \sin \left(314t - \frac{\pi}{3} \right)$ A. Find (i) $i_1 + i_2$ and (ii) $i_1 - i_2$

Write answer in sinusoidal form. Also draw phasor diagram.

[2070 Bhadra]

Solution:

$$\begin{aligned}
 i_1 &= 10 \sin \left(314t + \frac{\pi}{14} \right) \text{ A} \\
 i_2 &= 8 \sin \left(314t - \frac{\pi}{3} \right) \text{ A} \\
 i_1 + i_2 &= 10 \sin \left(314t + \frac{\pi}{14} \right) + 8 \sin \left(314t - \frac{\pi}{3} \right) \\
 &= 10 \sin 314t \cos \frac{\pi}{14} + 10 \cos 314t \sin \frac{\pi}{14} + 8 \sin 314t \cos \frac{\pi}{3} \\
 &\quad - 8 \cos 314t \sin \frac{\pi}{3}
 \end{aligned}$$

$$\begin{aligned}
 &= 9.75 \sin 314t + 2.22 \cos 314t + 4 \sin 314t - 6.93 \cos 314t \\
 &= 13.75 \sin 314t - 4.71 \cos 314t \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 i_1 &= I_m \sin (\omega t + \alpha) \\
 &= I_m \sin \omega t \cos \alpha + I_m \cos \omega t \sin \alpha \quad (2)
 \end{aligned}$$

Comparing (1) and (2)

$$I_m \cos \alpha = 13.75 \quad (3)$$

$$I_m \sin \alpha = -4.71 \quad (4)$$

Squaring and adding

$$I_m^2 = (13.75)^2 + (-4.71)^2$$

$$\therefore I_m = 14.53 \text{ A}$$

Dividing (4) by (3)

$$\frac{I_m \sin \alpha}{I_m \cos \alpha} = \frac{-4.71}{13.75}$$

$$\therefore \alpha = -18.91^\circ$$

$$\therefore i_1 + i_2 = 14.53 \sin (314t - 18.91^\circ) \text{ A}$$

In phasor notation,

$$\tilde{i}_1 + \tilde{i}_2 = 14.53 \angle -18.91^\circ \text{ A}$$

Again,

$$\begin{aligned}
 i_1 - i_2 &= 10 \sin \left(314t + \frac{\pi}{14} \right) - 8 \sin \left(314t - \frac{\pi}{3} \right) \\
 &= 10 \sin 314t \cos \frac{\pi}{14} + 10 \cos 314t \sin \frac{\pi}{14} - 8 \sin 314t \cos \frac{\pi}{3} \\
 &\quad + 8 \cos 314t \sin \frac{\pi}{3} \\
 &= 9.75 \sin 314t + 2.22 \cos 314t - 4 \sin 314t + 6.93 \cos 314t \\
 &= 5.75 \sin 314t + 9.15 \cos 314t \quad (5) \\
 i_1 - i_2 &= I_m \sin (314t + \beta) \\
 &= I_m \sin 314t \cos \beta + I_m \cos 314t \sin \beta \quad (6)
 \end{aligned}$$

Comparing (5) and (6)

$$I_m \cos \beta = 5.75 \quad (7)$$

$$I_m \sin \beta = 9.15 \quad (8)$$

Squaring and adding (3) and (4)

$$I_m^2 = (5.75)^2 + (9.15)^2$$

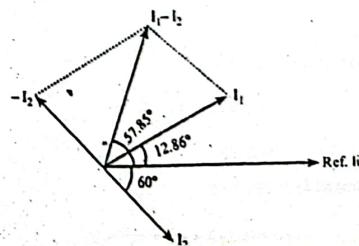
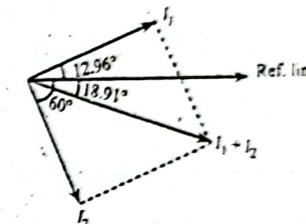
$$I_m = 10.81 \text{ A}$$

Dividing (8) by (7)

$$\frac{I_m \sin \beta}{I_m \cos \beta} = \frac{9.15}{5.75}$$

$$\therefore \beta = 57.85^\circ$$

$$\therefore i_1 - i_2 = 10.81 \sin (314t + 57.85^\circ)$$



$$\tilde{i}_1 - \tilde{i}_2 = 10.81 \angle 57.85^\circ A$$

Alternate Method

Resolving the phasors \tilde{i}_1 and \tilde{i}_2 into x - component and y - component

$$|i_x| = i_1 \cos 12.86^\circ = 0.975 i_1$$

$$|i_y| = i_1 \sin 12.86^\circ = 0.222 i_1$$

$$|i_x| = i_2 \cos 60^\circ = \frac{1}{2} i_2$$

$$|i_y| = i_2 \sin 60^\circ = \frac{\sqrt{3}}{2} i_2$$

$$x - \text{component } i_x = i_{x1} + i_{x2} = 0.975 i_1 + \frac{1}{2} i_2 = 0.975 \times 10 + \frac{1}{2} \times 8 = 13.75$$

$$y - \text{component } i_y = i_{y1} - i_{y2}$$

$$= 0.222 \times 10 - \frac{\sqrt{3}}{2} \times 8 = -4.708$$

Now, Magnitude of current = $\sqrt{i_x^2 + i_y^2}$

$$= \sqrt{(13.75)^2 + (-4.708)^2}$$

$$= 14.53 A$$

$$\begin{aligned} \text{Phase of current} &= \tan^{-1} \left(\frac{y - \text{comp}}{x - \text{comp}} \right) \\ &= \tan^{-1} \left(\frac{-4.708}{13.76} \right) = -18.91^\circ \end{aligned}$$

$$i_1 + i_2 = 14.53 \sin(-18.91^\circ) A$$

In phasor notation,

$$\tilde{i}_1 + \tilde{i}_2 = 14.53 \angle -18.91^\circ A$$

For $i_1 - i_2$:

$$x - \text{component } i_x = i_{x1} - i_{x2}$$

$$= 0.975 \times 10 - \frac{1}{2} \times 8$$

$$= 5.75$$

$$y - \text{component } i_y = i_{y1} + i_{y2}$$

$$= 0.222 \times 10 + \frac{\sqrt{3}}{2} \times 8$$

$$= 9.148$$

$$\text{Now, mag. of current} = \sqrt{5.75^2 + 9.148^2}$$

$$= 10.81 A$$

$$\begin{aligned} \text{Phase of current} &= \tan^{-1} \left(\frac{9.148}{5.75} \right) = 57.85^\circ \\ i_1 - i_2 &= 10.81 \sin(314t + 57.85^\circ) A \end{aligned}$$

In phasor notation,

$$\tilde{i}_1 - \tilde{i}_2 = 10.81 \angle 57.85^\circ A$$

2. A sinusoidal voltage is applied to three parallel branches yielding branch currents, $i_1 = 14.14 \sin(\omega t - 45^\circ)$, $i_2 = 28.3 \cos(\omega t - 60^\circ)$ and $i_3 = 7.07 \sin(\omega t + 60^\circ)$ (i) Find complete time expression for the source current (ii) Draw the phasor diagram in terms of effective values. Use the voltage as reference. [2009 Chaitra]

Solution:

$$\text{Given, } i_1 = 14.14 \sin(\omega t - 45^\circ)$$

$$i_2 = 28.3 \cos(\omega t - 60^\circ)$$

$$= 28.3 \sin(90^\circ + \omega t - 60^\circ) = 28.3 \sin(\omega t + 30^\circ)$$

$$i_3 = 7.07 \sin(\omega t + 60^\circ)$$

(i)

Let the source current be i ,

$$\begin{aligned} i &= i_1 + i_2 + i_3 = 14.14 \sin(\omega t - 45^\circ) + 28.3 \sin(\omega t + 30^\circ) \\ &\quad + 7.07 \sin(\omega t + 60^\circ) \end{aligned}$$

$$= 14.14 (\sin \omega t \cos 45^\circ - \sin 45^\circ \cos \omega t) + 28.3 (\sin \omega t \cos 30^\circ$$

$$+ \cos \omega t \sin 30^\circ) + 7.07 (\sin \omega t \cos 60^\circ + \sin 60^\circ \cos \omega t)$$

$$= 9.999 \sin \omega t - 9.99 \cos \omega t + 24.51 \sin \omega t + 14.15 \cos \omega t$$

$$+ 3.53 \sin \omega t + 6.12 \cos \omega t$$

$$\therefore i = 38.03 \sin \omega t + 10.28 \cos \omega t \quad \dots \dots \dots (i)$$

Now,

$$= I_m \sin \omega t \cos \alpha + I_m \cos \omega t \sin \alpha \quad \dots \dots \dots (ii)$$

Comparing (i) and (ii)

$$I_m \cos \alpha = 38.03 \quad \dots \dots \dots (iii)$$

$$I_m \sin \alpha = 10.28 \quad \dots \dots \dots (iv)$$

Squaring & adding

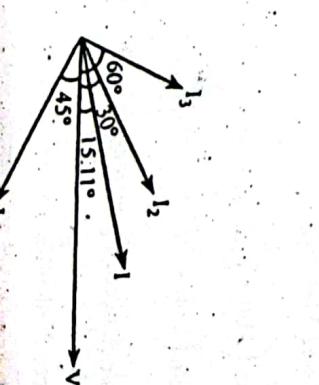
$$I_m^2 = (38.03)^2 + (10.28)^2$$

$$\therefore I_m = 39.40 A$$

Dividing (iv) by (iii)

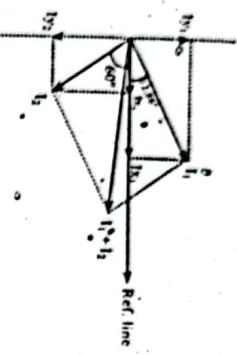
$$\frac{I_m \sin \alpha}{I_m \cos \alpha} = \frac{10.28}{38.03}$$

$$\text{or, } \alpha = \tan^{-1} \left(\frac{10.28}{38.03} \right)$$



Now, mag. of current = $\sqrt{5.75^2 + 9.148^2}$

= 10.81 A



$$\therefore \alpha = 15.11^\circ$$

Hence, $\tilde{I} = 39.40 \angle 15.11^\circ \text{A}$ (in phasor notation)
 $i(t) = 39.40 \sin(\omega t + 15.11^\circ)$ (in time domain)

Drawing phasor diagram,

Alternately; Resolving into x - component and y - component

$$i_1 = 14.14 \sin(\omega t - 45^\circ)$$

$$i_2 = 28.3 \cos(\omega t - 60^\circ)$$

$$= 28.3 \sin(90^\circ + \omega t - 60^\circ) = 28.3 \sin(\omega t + 30^\circ)$$

$$i_3 = 7.07 \sin(\omega t + 60^\circ)$$

Resultant x - component

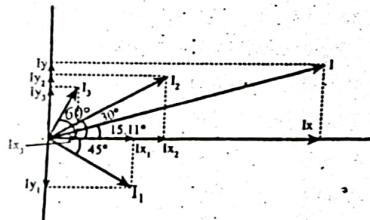
$$Ix = Ix_1 + Ix_2 + Ix_3$$

$$= 14.14 \cos 45^\circ + 28.3 \cos 30^\circ + 7.07 \cos 60^\circ = 38.04$$

Resultant y - component

$$Iy = -Iy_1 + Iy_2 + Iy_3$$

$$= -14.14 \sin 45^\circ + 28.3 \sin 30^\circ + 7.07 \sin 60^\circ = 10.27$$



$$\text{Magnitude of total current} = \sqrt{(38.04)^2 + (10.27)^2} = 39.402 \text{A}$$

$$\text{Phase of total current} = \tan^{-1}(10.27/38.04) = 15.11^\circ$$

$$\therefore i(t) = 39.402 \sin(\omega t + 15.11^\circ) \text{A}$$

3. Describe phasor representation and addition of two sinusoids

$$i_3 = i_1 + i_2, \text{ Illustrate}$$

(i) Position of the phasors for $t = 0$

(ii) Sinusoidal waveform for increasing time.

[2069 Bhadra]

Solution:

Let us consider two sinusoidal currents whose equation is given as

$$i_1 = I_m_1 \sin \omega t \text{ and}$$

$$i_2 = I_m_2 \sin(\omega t + \theta_2)$$

Addition of these sinusoidal currents is given by

$$i_3 = i_1 + i_2$$

$$= I_m_1 \sin \omega t + I_m_2 \sin(\omega t + \theta_2)$$

$$= I_m_1 \sin \omega t + I_m_2 \sin \omega \cos \theta_2 + I_m_2 \cos \omega \sin \theta_2$$

$$i_3 = (I_m_1 + I_m_2 \cos \theta_2) \sin \omega t + (I_m_2 \sin \theta_2) \cos \omega t \quad \dots \dots \text{(i)}$$

The addition of two sinusoids of the same frequency results in another sinusoid. Hence i_3 can be represented as;

$$i_3 = I_m_3 \sin(\omega t + \theta_3)$$

$$= I_m_3 \sin \omega \cos \theta_3 + I_m_3 \sin \theta_3 \cos \omega$$

$$i_3 = (I_m_3 \cos \theta_3) \sin \omega t + (I_m_3 \sin \theta_3) \cos \omega t \quad \dots \dots \text{(ii)}$$

From (i) and (ii) we get

$$I_m_3 \cos \theta_3 = I_m_1 + I_m_2 \cos \theta_2 \quad \dots \dots \text{(iii)}$$

$$I_m_3 \sin \theta_3 = I_m_2 \sin \theta_2 \quad \dots \dots \text{(iv)}$$

From (iii) and (iv) equations we can determine the amplitude I_m_3 and phase angle θ_3 .

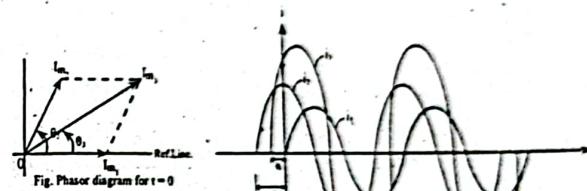


Fig. Phasor diagram for $t = 0$

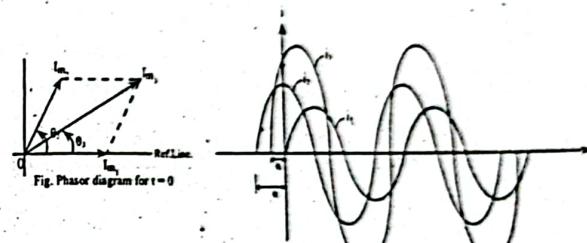


Fig. Sinusoidal waveform for increasing time

4. In a parallel ac circuit consisting of two branches, the branch current are as follows: $i_1(t) = 10 \sin\left(314t - \frac{\pi}{12}\right)$ and $i_2(t) = 5 \sin\left(314t - \frac{\pi}{3}\right)$. Find the expression of total instantaneous current being drawn from the supply by these two branches. [2062 Bhadra]

Solution:

$$\text{Given, } i_1(t) = 10 \sin\left(314t - \frac{\pi}{12}\right)$$

$$i_2(t) = 5 \sin\left(314t - \frac{\pi}{3}\right)$$

Total instanteneous current drawn from supply

$$i(t) = i_1(t) + i_2(t)$$

$$= 10 \sin\left(314t - \frac{\pi}{12}\right) + 5 \sin\left(314t - \frac{\pi}{3}\right)$$

$$= 10 \sin 314t \cos \frac{\pi}{12} - 10 \cos 314t \sin \frac{\pi}{12} + 5 \sin 314t \cos \frac{\pi}{3}$$

$$- 5 \cos 314t \sin \frac{\pi}{3}$$

$$= 9.66 \sin 314t - 2.59 \cos 314t + 2.5 \sin 314t - 4.33 \cos 314t$$

$$= 12.16 \sin 314t - 6.92 \cos 314t$$

The average power in a complete cycle is given by

$$\begin{aligned}
 P_{\text{average}} &= \int_0^{\pi} \frac{P}{\pi} dt \\
 &= \frac{1}{\pi} \int_0^{\pi} V_m I_m \sin^2 \omega t dt \\
 &= \frac{V_m I_m}{\pi} \int_0^{\pi} \frac{1 - \cos 2\omega t}{2} dt \\
 &= \frac{V_m I_m}{2\pi} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_0^{\pi} \\
 &= \frac{V_m I_m}{2\pi} (\pi - 0) \\
 &= \frac{V_m I_m}{2} \\
 &= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \\
 &= V_{\text{rms}} I_{\text{rms}}
 \end{aligned}$$

$$P_{\text{average}} = V_{\text{rms}} I_{\text{rms}}$$

Hence, power in resistive circuit is always positive though fluctuating. This means that voltage source constantly delivers power to the circuit and the circuit consumes it.

AC through pure inductor

Consider an AC circuit with a pure inductance 'L' excited by an ac sinusoidal voltage described by the equation

$$v = V_m \sin \omega t \quad \dots \dots \dots (1)$$

Let 'i' be the instantaneous value of current through the inductance which is also time varying in nature.

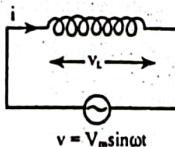
Hence, according to Faraday's law of electromagnetic induction, emf is induced across the inductance given by equation,

$$v_L = L \frac{di}{dt} \quad \dots \dots \dots (2)$$

By KVL, $v = v_L$

$$\text{or, } V_m \sin \omega t = L \frac{di}{dt}$$

$$\text{or, } di = \frac{V_m}{L} \sin \omega t dt$$



Integrating both sides,

$$\int di = \int \frac{V_m}{L} \sin \omega t dt$$

$$\Rightarrow i = \frac{V_m}{L} \left(\frac{-\cos \omega t}{\omega} \right)$$

$$\Rightarrow i = \frac{V_m}{\omega L} (-\cos \omega t)$$

$$\Rightarrow i = \frac{V_m}{\omega L} \sin (\omega t - 90^\circ)$$

$$\Rightarrow i = \frac{V_m}{X_L} \sin (\omega t - 90^\circ)$$

Where, $X_L = \omega L = 2\pi f L$

X_L is known as inductive reactance of L.

Thus, X_L opposes flow of current (functions same as resistor). Its unit is Ohm (Ω).

$$\Rightarrow i = I_m \sin (\omega t - 90^\circ) \text{ where, } I_m = \frac{V_m}{X_L}$$

Here, 'i' is also ac in nature and lags by 90° with respect to v.

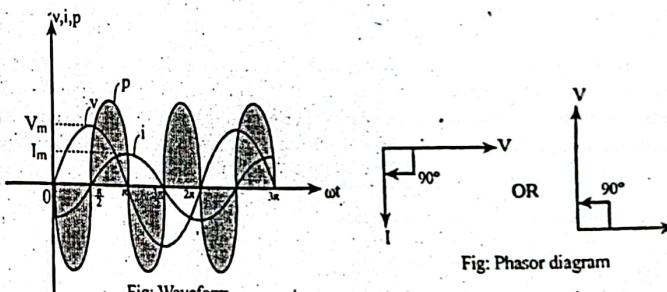


Fig: Waveform

Fig: Phasor diagram

Hence, it is concluded that in purely inductive coil, voltage and current are in different phase; current lags behind applied voltage by $\frac{\pi}{2}$.

Now, Instantaneous power $p = v \times i$

$$= V_m \sin \omega t \times I_m \sin (\omega t - 90^\circ)$$

$$= V_m \sin \omega t \times -I_m \cos \omega t = \frac{-V_m I_m}{2} \sin 2\omega t$$

The power waveform may be plotted by multiplying at every instant the values of voltage and current obtained from their waveforms.

Average power for complete cycle is

$$\begin{aligned} P_{\text{average}} &= \frac{1}{\pi} \int_0^{\pi} p \, d\omega t \\ &= \frac{1}{\pi} \int_0^{\pi} \left[-\frac{V_m I_m}{2} \sin 2\omega t \right] d\omega t \\ &= -\frac{V_m I_m}{2\pi} \left[\frac{-\cos 2\omega t}{2} \right]_0^{\pi} \\ &= -\frac{V_m I_m}{2\pi} \left[\frac{-1+1}{2} \right] \\ &= 0 \end{aligned}$$

Hence, purely inductive circuit does not consume power and thus produces no useful work in an electric circuit. However, it draws a 90° lagging current, which is utilized in establishing magnetic field.

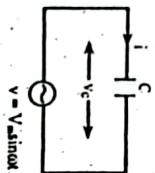
Explanation from waveform

The power curve is a sine wave of twice the frequency of the current or voltage wave; i.e. power curve completes two cycles when current or voltage wave completes one cycle.

From cycle $\frac{\pi}{2} < \omega t < \pi$ the power curve is above horizontal axis hence p is positive. So the circuit draws energy from the source. The energy is stored as the magnetic field in the inductor. From cycle $\pi < \omega t < \frac{3\pi}{2}$, the power curve is below horizontal axis hence p is negative so, previously stored energy is returned to the source.

Now, as the energy stored in positive cycle is equal to the energy stored in negative cycle, the total energy dissipated during every cycle is zero. Thus average power (real power) over complete cycle in a purely inductive circuit is zero.

AC through pure capacitor



Consider an AC circuit with a capacitor 'C' excited by an ac sinusoidal voltage source, given by $v = V_m \sin \omega t$ (1)

Let ' v_c ' be the instantaneous voltage across the capacitor.

$$v_c = \frac{q}{C} \quad \dots \dots \dots (2)$$

Where, q = charge on the capacitor at any instant

$$\text{By KVIL, } v = v_c$$

$$\text{or, } V_m \sin \omega t = \frac{q}{C}$$

$$\text{or, } q = CV_m \sin \omega t$$

Differentiating both sides w.r.t. time

$$\begin{aligned} \frac{dq}{dt} &= \frac{d}{dt}(V_m C \sin \omega t) \\ \Rightarrow i &= V_m C \omega \cos \omega t \\ \Rightarrow i &= \omega V_m C \cos \omega t \\ \Rightarrow i &= \frac{V_m}{\omega C} \sin(\omega t + 90^\circ) \\ \Rightarrow i &= \frac{V_m}{X_C} \sin(\omega t + 90^\circ) \end{aligned}$$

$$\text{Where, } X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

X_C is capacitive reactance of C .

Thus, X_C opposes the flow of current (functions same as resistor). Its unit is ohm (Ω).

$$\therefore i = I_m \sin(\omega t + 90^\circ) \quad \text{where, } I_m = \frac{V_m}{X_C}$$

Here, 'i' is also ac in nature and leads by 90° w.r.t to v .

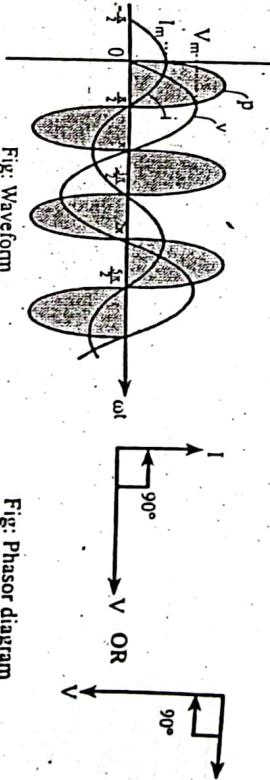
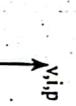


Fig: Phasor diagram

Hence, in purely capacitive circuit, current leads applied voltage by $\frac{\pi}{2}$.

$$\text{Instantaneous power } p = v \times i$$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t + 90^\circ)$$

Average power over a complete cycle is

$$\begin{aligned} P_{\text{average}} &= \frac{1}{\pi} \int_0^{\pi} p \, d\omega t \\ &= \frac{1}{\pi} \int_0^{\pi} \left[\frac{V_m I_m}{2} \sin 2\omega t \cos \omega t \right] d\omega t = \frac{V_m I_m}{2\pi} \left[\frac{\cos 2\omega t}{2} \right]_0^{\pi} \\ &= \frac{V_m I_m}{2\pi} \left[\frac{-1+1}{2} \right] = 0 \end{aligned}$$

Hence, pure capacitor does not consume real power and produces no useful work in the circuit. However, it draws a 90° leading current, which is used in establishing the electric field.

Explanation from waveform

The power curve is a sine wave of twice the frequency of the voltage or current wave. During first quarter cycle the power curve is positive and the circuit draws energy from source and capacitor is charged. During second quarter cycle the power curve is negative, the energy stored in the capacitor is returned to the source i.e. the capacitor is discharged. Thus, the total active energy during each cycle of the current is zero. Thus, in purely capacitive circuit the active power over a complete cycle is zero.

Impedance and Admittance

Resistive circuit	Purely inductive circuit	Purely capacitive circuit
$i = \frac{V}{R} \sin \omega t$	$i = \frac{V}{X_L} \sin(\omega t - 90^\circ)$	$i = \frac{V}{X_C} \sin(\omega t + 90^\circ)$
In phasor form,	In phasor form,	In phasor form,
$\tilde{i} = \frac{V \angle 0^\circ}{R}$	$\tilde{i} = \frac{V \angle -90^\circ}{X_L}$	$\tilde{i} = \frac{V \angle 90^\circ}{X_C}$
$R = \frac{V \angle 0^\circ}{\tilde{i}}$	$\tilde{i} = \frac{V \angle 0^\circ}{X_L \angle 90^\circ} = \frac{V \angle 0^\circ}{jX_L}$	$\tilde{i} = \frac{V \angle 0^\circ}{X_C \angle -90^\circ} = \frac{V \angle 0^\circ}{-jX_C}$
$R = \frac{\tilde{V}}{\tilde{i}}$	$jX_L = \frac{V \angle 0^\circ}{\tilde{i}}$	$\tilde{i} = \frac{1 \times V \angle 0^\circ}{X_C \angle -90^\circ} [j^2 = -1]$
$\therefore Z = R$	$jX_L = \frac{\tilde{V}}{\tilde{i}}$	$\frac{X_C}{j} = \frac{V \angle 0^\circ}{\tilde{i}}$
$\therefore Z = jX_L$	$-jX_C = \frac{\tilde{V}}{\tilde{i}}$	$G + jB = \frac{1}{R \pm jX} = \frac{1}{R \pm jX} \times \frac{R \mp jX}{R \mp jX}$
		$= \frac{R \mp jX}{R^2 + X^2}$
		Thus, $G = \frac{R}{R^2 + X^2}; B = \mp \frac{X}{R^2 + X^2}$

$$\text{Thus, } Z = \frac{\tilde{V}}{\tilde{i}}$$

Where, Z is a frequency-dependent quantity known as impedance.

The impedance (Z) of a circuit is the ratio of the phasor voltage (V) to the phasor current (I), measured in ohms (Ω).

As a complex quantity, Z can be expressed in rectangular form as

$$Z = R + jX$$

Where,

$$R = \text{Re}(Z)$$
 is the resistance

$$X = \text{Imag}(Z)$$
 is the reactance

If X is positive $Z = R + jX$

$Z \rightarrow$ inductive impedance

$X \rightarrow$ inductive reactance

Current lags Voltage

If X is negative $Z = R - jX$

$X \rightarrow$ Capacitive impedance

Current leads Voltage

In polar form $Z = |Z| \angle \theta$

$$\text{and, } R = |Z| \cos \theta \\ X = |Z| \sin \theta$$

For Inductive circuit:

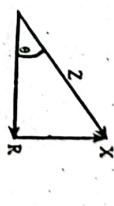


Fig: Impedance diagram
For Capacitive circuit:



Admittance (Y) of a circuit is the reciprocal of impedance (Z), measured in siemens (S) or mho (Ω).

$$Y = \frac{1}{Z} = \frac{1}{R + jX}$$

Where,
 $G = \text{Re}(Y)$ is the conductance
 $B = \text{Imag}(Y)$ is the susceptance

$$G + jB = \frac{1}{R \pm jX} = \frac{1}{R \pm jX} \times \frac{R \mp jX}{R \mp jX}$$

Thus, $G = \frac{R}{R^2 + X^2}; B = \mp \frac{X}{R^2 + X^2}$

Consider an ac circuit with a resistance connected in series with an inductance and excited by ac voltage.

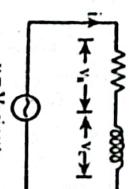
Now,
Voltage drop across resistance

$$\tilde{V}_R = \tilde{i}R \quad [\text{no phase difference between } v \text{ and } i]$$

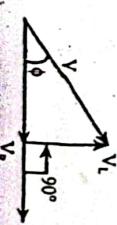
$\tilde{V}_L = \tilde{i}(jX_L)$ [i lags v_L by 90°]

$$\text{By KVL, } \tilde{V} = \tilde{V}_R + \tilde{V}_L \dots \dots \dots (i)$$

Now, drawing phasor diagram from equation (i)



$$v = V_{\text{sin} \omega t}$$



$$\text{From diagram } V = \sqrt{V_R^2 + V_L^2}$$

$$\text{or, } V = \sqrt{I^2 R^2 + I^2 X_L^2}$$

$$\text{or, } V = I \sqrt{R^2 + X_L^2}$$

$$\text{or, } \frac{V}{I} = \sqrt{\frac{R^2 + X_L^2}{I^2}}$$

$$\text{or, } \frac{V}{I} = |Z|$$

Where, $|Z| = \sqrt{R^2 + X_L^2}$ → impedance of RL circuit.

From phasor diagram, it shows that current lags voltage by a phase angle ϕ which is less than 90° .

$$\text{Now, } \tan \phi = \frac{V_L}{V_R} = \frac{X_L}{R}$$

$$\phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\text{Also, } \tilde{I} = \frac{\tilde{V}}{Z} \quad (\text{in phasor})$$

$$\tilde{I} = \frac{V \angle 0^\circ}{|Z| \angle \phi} = \frac{V}{|Z|} \angle (-\phi)$$

$$\therefore \tilde{I} = I \angle -\phi \quad \dots \dots \dots (2) \text{ where } I = \frac{V}{|Z|}$$

Thus in an inductive circuit, current lags applied voltage by an angle ϕ .

If the instantaneous voltage is represented by

$$v = V_m \sin \omega t$$

then instantaneous value of current is

$$i = I_m \sin(\omega t - \phi) \text{ where, } I_m = \frac{V_m}{|Z|}$$

Now, Instantaneous power $p = v \times i$

$$= V_m I_m \times \frac{1}{2} [2 \sin \omega t \sin(\omega t - \phi)]$$

$$= \frac{V_m I_m}{2} [\cos \phi - \cos(2\omega t - \phi)]$$

$$= \frac{1}{2} V_m I_m \cos \phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$$

The power waveform can be plotted by multiplying the values of current and voltage at each instant obtained from their waveforms.

V.v.i.p

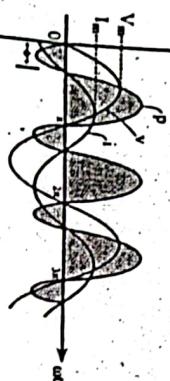


Fig: Waveform

This power consists of two parts

1. Constant part $\rightarrow \frac{1}{2} V_m I_m \cos \phi$
2. Pulsating component $\rightarrow \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$

This component has frequency twice of voltage or current.
The average power of this pulsating part is zero.

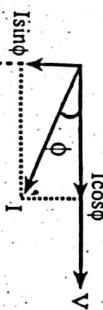
Hence, only constant part $\frac{1}{2} V_m I_m \cos \phi$ contributes power to the circuit.

$$\therefore P_{\text{average}} = \frac{1}{2} V_m I_m \cos \phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi$$

[Note: V_m, I refers to rms values of voltage & current respectively]

Power in RL series circuit

Drawing phasor diagram of voltage and current in RL circuit;



Here, current 'I' has two components.

- i. $I \cos \phi \rightarrow$ component of 'I' in phase with 'V'
- ii. $I \sin \phi \rightarrow$ component of 'I' in quadrature to 'V'

Accordingly, two types of power can be defined:

1. Active Power (P): It is the product of rms value of the voltage in the circuit and in phase component of rms value of current. Its unit is watt.



Active power, $P = V \times \text{component of } I \text{ in phase with } V'$

$$P = VI \cos \phi$$

$$P = IZ \times I \frac{R}{Z}$$

$$\therefore P = I^2 R \text{ where, } \frac{R}{Z} = \cos \phi$$

Thus, active power is only consumed in resistance. It is the actual power which is responsible for production of heat. Thus, the power which is actually consumed or utilized in an ac circuit is called true or real or active power of the circuit.

- Reactive power (Q): It is the product of rms value of voltage and quadrature components of rms value of current. Its unit is VAR (volt - ampere reactive). Reactive power, $Q = V \times \text{component of } I \text{ in direction perpendicular to } V'$



$$\therefore \phi = \tan^{-1} \frac{X_C}{R}$$

$$= IZ \times 1 \cdot \frac{R}{Z} = I^2 R$$

$$\text{Also, } \tilde{I} = \frac{\tilde{V}}{Z} = \frac{V}{|Z|} e^{j\phi}$$

$$\therefore \tilde{I} = I e^{j\phi} \quad \text{where, } I = \frac{V}{|Z|}$$

Thus in a capacitive circuit, current leads voltage by an angle ϕ .

The instantaneous voltage and current are described as:

$$v = V_m \sin \omega t$$

$$i = I_m \sin (\omega t + \phi) \quad \text{where } I_m = \frac{V_m}{Z}$$

Instantaneous power P

$$\begin{aligned} &= v \times i \\ &= V_m \sin \omega t \times I_m \sin (\omega t + \phi) \\ &= \frac{V_m I_m}{2} [2 \sin \omega t \sin (\omega t + \phi)] \\ &= \frac{V_m I_m}{2} [\cos \phi - \cos (2\omega t + \phi)] \\ &= \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos (2\omega t + \phi) \end{aligned}$$

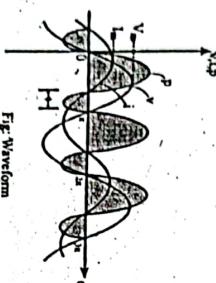


Fig: Wavetom

Power in $R C$ circuit

$$I \sin \phi$$

$$\begin{matrix} I \cos \phi \\ \downarrow \\ V \end{matrix}$$

Fig: Phasor diagram

Here, current has two components

$\Rightarrow I \cos \phi$ ~ component in phase with voltage

$\Rightarrow I \sin \phi$ ~ component perpendicular with voltage

Accordingly two powers can be defined

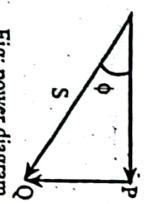


Fig: power diagram

$$\begin{aligned} \text{i)} \quad \text{If } R = X_C \text{ then } \phi = \tan^{-1} \left(\frac{X_C}{R} \right) = 45^\circ, \text{ p.f.} = 0.707 \\ \therefore P = Q \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad \text{If } R > X_C \text{ then } \phi = \tan^{-1} \left(\frac{X_C}{R} \right) < 45^\circ, \text{ p.f.} > 0.707 \\ \therefore P > Q \end{aligned}$$

$$\begin{aligned} \text{iii)} \quad \text{If } R < X_C \text{ then } \phi = \tan^{-1} \left(\frac{X_C}{R} \right) > 45^\circ, \text{ p.f.} < 0.707 \\ \therefore P < Q \end{aligned}$$

$$\begin{aligned} \text{iv)} \quad \text{If } X_C = 0, \text{ then } \phi = \tan^{-1} \left(\frac{X_C}{R} \right) = 0, \text{ p.f.} = 1 \text{ (max)} \\ \therefore Q = 0 \quad [\text{Resistive circuit}] \end{aligned}$$

$$\begin{aligned} \text{v)} \quad \text{If } R = 0, \text{ then } \phi = \tan^{-1} \left(\frac{X_C}{R} \right) = 90^\circ, \text{ p.f.} = 0 \text{ (min)} \\ \therefore P = 0 \quad [\text{Capacitive circuit}] \end{aligned}$$

A.C through RLC series circuit

Here, Inductive reactance (X_L) = $\omega L = 2\pi fL$

$$\text{Capacitive reactance } (X_C) = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

Let,

$$V_R = \text{Voltage drop across R} \quad [v_R \text{ is in phase with } i]$$

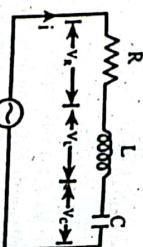
$$V_L = \text{voltage across inductor} \quad [i \text{ lags } v \text{ by } 90^\circ]$$

$$V_C = \text{voltage across capacitor} \quad [i \text{ leads } v \text{ by } 90^\circ]$$

$$\text{By KVL, } \tilde{V} = \tilde{V}_R + \tilde{V}_L + \tilde{V}_C$$

$$= \tilde{I}R + \tilde{I}jX_L + \tilde{I}(-jX_C)$$

$$\begin{aligned} \tilde{V} &= \tilde{I}(R + jX_L - jX_C) \quad \text{.....(i)} \\ \text{Case i: If } X_L > X_C \text{ i.e., } V_L > V_C \text{ i.e., the circuit will be inductive} \\ \tilde{V} &= \tilde{I}[R + j(X_L - X_C)] \\ \text{or, } \tilde{V} &= \tilde{I}Z \end{aligned}$$



Where, $Z = R + j(X_L - X_C)$

= impedance of the circuit

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}, \quad \phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$$\text{Also, } \tilde{I} = \frac{\tilde{V}}{Z}$$

$$= \frac{V \angle 0^\circ}{Z \angle \phi}$$

$$= \frac{V}{|Z|} \angle -\phi$$

$$\boxed{\tilde{I} = I \angle -\phi}$$

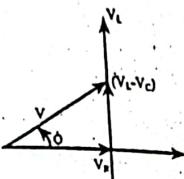


Fig: Phasor diagram

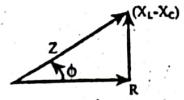


Fig: Impedance triangle

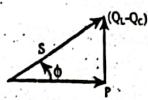


Fig: Power diagram

$\Rightarrow i$ lags v by ϕ angle and P.F. will be lagging

Thus, if $X_L > X_C$ circuit will be inductive

Case II: If $X_L < X_C$, $V_L < V_C$ i.e. circuit will be capacitive

$$\tilde{V} = \tilde{I} [R - j(X_C - X_L)]$$

$$\text{or, } \tilde{V} = \tilde{I} Z$$

Where, $Z = R - j(X_C - X_L)$

= impedance of the circuit

$$|Z| = \sqrt{R^2 + (X_C - X_L)^2}, \quad \phi = \tan^{-1} \left[\frac{-(X_C - X_L)}{R} \right]$$

$$\text{Also, } \tilde{I} = \frac{\tilde{V}}{Z} = \frac{V \angle 0^\circ}{|Z| \angle -\phi} = \frac{V}{|Z|} \angle \phi$$

$$\therefore \tilde{I} = I \angle \phi \quad \text{where } I = \frac{V}{|Z|}$$

$\Rightarrow i$ leads v by ϕ angle & p.f. will be leading.

Thus, if $X_C > X_L$ the circuit will be capacitive.

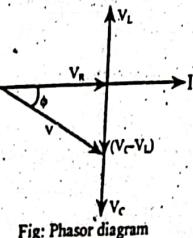


Fig: Phasor diagram

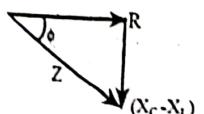


Fig: Impedance triangle

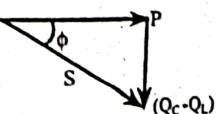
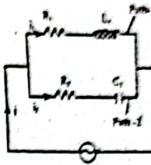


Fig: Phasor diagram

AC parallel circuit



Let, V = Rms value of applied voltage

I_1 = Rms value of current through path - 1

I_2 = Rms value of current through path - 2

$$\text{Now, } \tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{\tilde{V}}{R_1 + jX_L} = \frac{V \angle 0^\circ}{|Z_1| \angle \phi_1} = I_1 \angle -\phi_1$$

$$\text{Where, } Z_1 = \sqrt{R_1^2 + (X_L)^2}, \quad \phi_1 = \tan^{-1} \left(\frac{X_L}{R_1} \right) \Rightarrow i_1 \text{ lags } v \text{ by } \phi_1$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{\tilde{V}}{R_2 - jX_C} = \frac{V \angle 0^\circ}{|Z_2| \angle -\phi_2} = I_2 \angle \phi_2$$

$$\text{Where, } Z_2 = \sqrt{R_2^2 + X_C^2}, \quad \phi_2 = \tan^{-1} \left(\frac{-X_C}{R_2} \right) \Rightarrow i_2 \text{ leads } v \text{ by } \phi_2$$

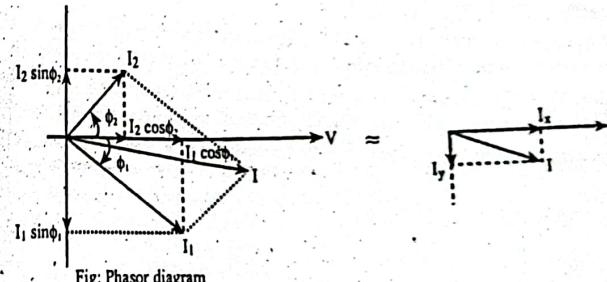


Fig: Phasor diagram

$I_1 \cos \phi_1$ = Active component of I_1

$I_1 \sin \phi_1$ = Reactive component of I_1

$I_2 \cos \phi_2$ = Active component of I_2

$I_2 \sin \phi_2$ = Reactive component of I_2

Net active component $I_x = I_1 \cos \phi_1 + I_2 \cos \phi_2$

Net reactive component $I_y = I_1 \sin \phi_1 - I_2 \sin \phi_2$

$$\therefore I = \sqrt{I_x^2 + I_y^2}$$

$$\therefore \phi = \tan^{-1} \left(\frac{I_y}{I_x} \right)$$

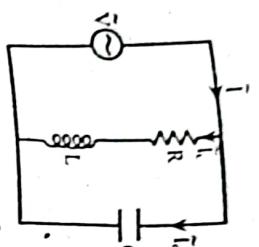


Fig.Circuit diagram 2

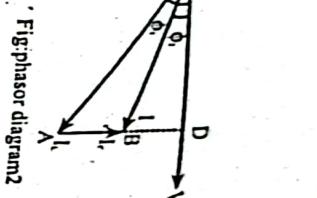


Fig.phasor diagram2

From phasor diagram.
 $\phi_2 < \phi_1$

$$\cos\phi_2 > \cos\phi_1$$

Power factor of circuit 2 > power factor of circuit 1.

1. Thus, by connecting capacitor power factor is improved from $\cos\phi_1$ to $\cos\phi_2$.

In Δ OBD of phasor diagram 2,

$$\cos\phi_2 = \frac{OD}{OB} \Rightarrow OD = OB \cos\phi_2 = I \cos\phi_2 \dots (1)$$

In Δ OAD of phasor diagram 1,

$$\cos\phi_1 = \frac{OD}{OA} \Rightarrow OD = OA \cos\phi_1$$

$$= I_L \cos\phi_1 \dots (2)$$

From (1) & (2)

$$I_L \cos\phi_1 = I \cos\phi_2 \dots (3)$$

Since, $\cos\phi_2 > \cos\phi_1$

2. Thus the new current drawn from the supply is less than the load current I_L .

Multiplying V on both sides of (3)

$$V I_L \cos\phi_1 = V I \cos\phi_2$$

$$\therefore P_1 = P_2$$

3. Thus the power taken from the supply remains same.

Calculation for rating of capacitor to improve power factor.
 Let us draw phasor diagram for the above circuit.

Here,

P = Active power drawn from the supply.

Q_1 = Reactive power taken by load.

Q_2 = Reactive power taken from the supply.

Q_C = leading reactive power drawn by the capacitor from the supply.

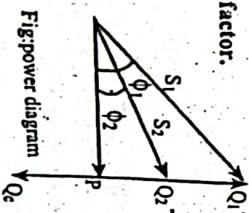


Fig.power diagram

$$\begin{aligned} \text{From power diagram,} \\ \tan\phi_1 &= \frac{Q_1}{P} \\ \text{or, } Q_1 &= P \tan\phi_1 \dots (1) \\ \tan\phi_2 &= \frac{Q_2}{P} \\ \text{or, } Q_2 &= P \tan\phi_2 \dots (2) \end{aligned}$$

Now,

$$\begin{aligned} Q_C &= Q_1 - Q_2 \\ &= P \tan\phi_1 - P \tan\phi_2 \\ (\because Q_C &= P(\tan\phi_1 - \tan\phi_2)) \text{ VAR} \end{aligned}$$

Thus, the above equation gives the VAR rating of the capacitors to improve the power factor from $\cos\phi_1$ to $\cos\phi_2$.
 The value of capacitance can be obtained
 Since,

$$Q_C = V I_C \sin 90^\circ$$

$$= V I_C$$

$$= V^2 \times \omega C$$

$$\therefore Q_C = \frac{V}{X_C} = \frac{V}{\frac{1}{\omega C}} = \omega C V$$

$$\text{Also, } I_C = \frac{V}{X_C} = \frac{V}{\frac{1}{\omega C}} = \omega C V$$

$$\therefore Q_C = \frac{\omega C}{\omega V^2} F, \text{ where } V \text{ is rms value of supply voltage.}$$

Thus, the above equation gives the capacitance value in Farad for power factor correction.

- Step - by - step calculation procedure for series AC circuits.**
- Determine the impedance in rectangular form

$$Z = (R + j(X_L - X_C)) \Omega$$

$$\text{Where: } X_L = \omega L, \omega L = 2\pi fL, \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}, \Omega$$

- Express the given supply voltage [i.e. as reference phasor] in phasor form.

Given, $v = V_m \sin\omega t$

$$3. \text{ Determine circuit current by Ohm's law}$$

$$\tilde{v} = V \angle 0^\circ \text{ where, } V = \frac{V_m}{\sqrt{2}}, V \rightarrow \text{rms voltage}$$

$$\tilde{i} = \frac{\tilde{v}}{Z} = \frac{V \angle 0^\circ}{R + j(X_L - X_C)} = \frac{V \angle 0^\circ}{Z \angle \phi} = \frac{V}{Z} \angle -\phi A$$

To express \tilde{i} in equation from

$$i(t) = I_m \sin(\omega t - \phi)$$

$$\text{Where, } I_m = \sqrt{2} \times I = \sqrt{2} \times \frac{V}{Z}$$

Exam Solutions

4. Determine phase difference between supply voltage and circuit current i.e. $\phi = 0^\circ - (-\phi) = \phi$
5. Determine the power factor $\cos \phi$. Specify whether the power factor is lagging or leading. If current lags voltage the power factor is lagging. If current leads voltage the power factor is leading.
6. Determine power in the circuit.

Active power (P) $= VI \cos \phi$ watt

Reactive power (Q) $= VI \sin \phi$ VAR

Apparent power (S) $= VI$ VA

Alternatively,

$$S = \tilde{V} \tilde{I}^* \quad \text{Where } \tilde{I}^* \text{ is complex conjugate of } \tilde{I}$$

$$= P + jQ$$

$$\therefore P = \operatorname{Re}[\tilde{V} \tilde{I}^*] \text{ watt}$$

$$Q = \operatorname{Imag}[\tilde{V} \tilde{I}^*] \text{ VAR}$$

Note: Lagging current gives a positive Q i.e. reactive power is consumed by the circuit.

Leading current gives a negative Q i.e. reactive power is supplied by the circuit.

7. Determine the voltage drop at each element.

$$\tilde{V}_R = \tilde{I} R$$

$$\tilde{V}_L = \tilde{I} (jX_L)$$

$$\tilde{V}_C = \tilde{I} (-jX_C)$$

8. Draw the phasor diagram

Step - by - step calculation procedure for single - phase AC parallel circuits.

1. Solve each branch by the procedure given for single - phase series circuits.
2. Determine the current through each branch.
3. Add the branch current to determine the total supply current \tilde{I} .
4. Determine the phase difference ϕ between \tilde{V} & \tilde{I} and determine the overall power factor $\cos \phi$.
5. Determine total impedance Z by Ohm's law

$$Z = \frac{\tilde{V}}{\tilde{I}}$$

$$\text{Alternatively, } \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

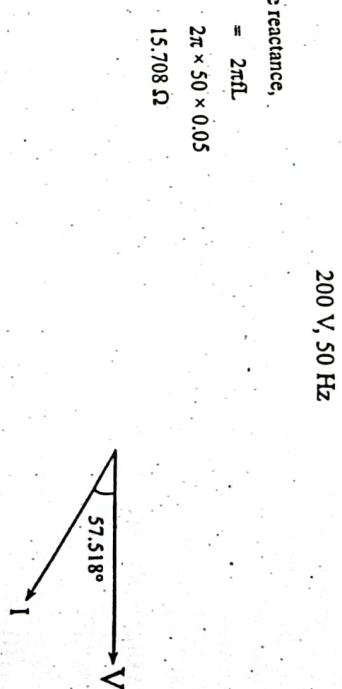
$$\therefore Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Here,



Inductive reactance,

$$\begin{aligned} X_L &= 2\pi f L \\ &= 2\pi \times 50 \times 0.05 \\ &= 15.708 \Omega \end{aligned}$$



Phasor Diagram

Impedance,

$$\begin{aligned} Z &= R + jX_L \\ &= 10 + j 15.708 \Omega \end{aligned}$$

$$\text{Current, } I = \tilde{I} = \frac{\tilde{V}}{Z}$$

$$\begin{aligned} &\frac{200}{10 + j 15.708} \angle 0^\circ \\ &= 10.741 \angle -57.518^\circ \end{aligned}$$

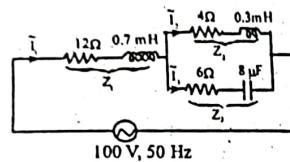
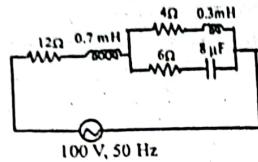
$$\text{Power factor, } \cos \phi = \cos 57.518^\circ = 0.537 \text{ (lag)}$$

$$\begin{aligned} \text{Power consumed, } P &= I^2 R = (10.741)^2 \times 10 = 1153.691 \text{ watt} \\ 2. \quad \text{Determine the total impedance, power factor and current in each branch of the circuit below. Also draw the phasor diagram showing all the currents.} & [2072 \text{ Ashwin}] \end{aligned}$$

1. A coil has an inductance of $0.05H$ and a resistance of 10Ω . It is connected to a sinusoidal $200V, 50Hz$ supply. Calculate the impedance, current, power factor and power consumed. [2072 Ashwin]

Solution:

Solution:



Here,

$$\begin{aligned} Z_1 &= R_1 + jX_{L_1} \\ &= 12 + j2\pi f L_1 \\ &= 12 + j2\pi \times 50 \times 0.7 \times 10^{-3} \\ &= 12 + j0.2199 \Omega \end{aligned}$$

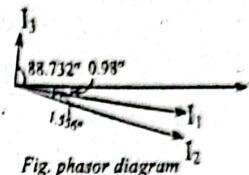
$$\begin{aligned} Z_2 &= R_2 + jX_{L_2} \\ &= 4 + j2\pi f L_2 \\ &= 4 + j2\pi \times 50 \times 0.3 \times 10^{-3} \\ &= 4 + j0.094 \Omega \end{aligned}$$

$$\begin{aligned} Z_3 &= R_3 - jX_{C_3} \\ &= 6 - j\frac{1}{2\pi f C_3} \\ &= 6 - j\frac{1}{2\pi \times 50 \times 8 \times 10^{-6}} \\ &= 6 - j397.887 \Omega \end{aligned}$$

Total impedance,

$$\begin{aligned} Z &= Z_1 + (Z_2 \parallel Z_3) \\ &= (12 + j0.2199) + \frac{Z_2 Z_3}{Z_2 + Z_3} \\ &= (12 + j0.2199) + \frac{(4 + j0.094)(6 - j397.887)}{(4 + j0.094) + (6 - j397.887)} \\ &= 16 + j0.2737 \Omega \end{aligned}$$

$$\text{Current, } \tilde{I}_1 = \frac{\tilde{V}}{Z} = \frac{100 \angle 0}{16 + j0.2737} = 6.249 \angle -0.98^\circ$$



$$\begin{aligned} \text{Power factor, } \cos \phi &= \cos(0.98^\circ) \\ &= 0.999 \text{ (lag)} \end{aligned}$$

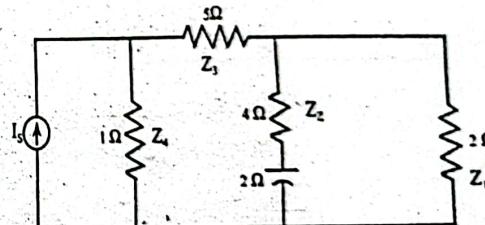
$$\text{Current in branch 2, } \tilde{I}_2 = \frac{\tilde{I}_1}{Z_1 + Z_3} \times Z_3$$

$$\begin{aligned} &= \frac{(6.249 \angle -0.98^\circ)}{(4 + j0.094) + (6 - j397.887)} \times (6 - j397.887) \\ &= 6.249 \angle -1.556^\circ \text{ A} \end{aligned}$$

$$\text{Current in branch 3, } \tilde{I}_3 = \tilde{I}_1 - \tilde{I}_2$$

$$\begin{aligned} &= (6.279 \angle -0.98^\circ) - (6.249 \angle -1.556^\circ) \\ &= 0.0623 \angle 88.732^\circ \text{ A} \end{aligned}$$

3. In the given circuit, find the current through the inductor, what is the equivalent impedance? (2072 Kartik)



Solution:

Here,

$$Z_1 = 2 \Omega$$

$$Z_2 = 4 - j2 \Omega$$

$$Z_3 = j5 \Omega$$

$$Z_4 = 1 \Omega$$

Current through the inductor,

$$I_{\text{inductor}} = \frac{I_s}{Z_4 + \{Z_1 + (Z_2 \parallel Z_3)\}} \times Z_4 \quad [\because \text{using current divider rule}]$$

$$\frac{I_s}{1 + \left\{ j5 + \frac{2(4-j2)}{2+(4-j2)} \right\}} \times 1 \\ = (0.186 \angle -63.435^\circ) I_s$$

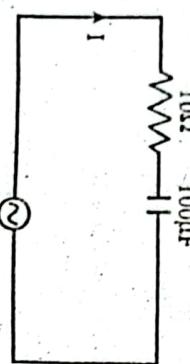
Equivalent impedance,

$$Z = [(Z_1 \parallel Z_2) + Z_3] \parallel Z_4 \\ = \left[\frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3 \right] \parallel Z_4 \\ = \left[\frac{2 \times (4-j2)}{2+(4-j2)} + j5 \right] \parallel 1 \\ = (1.4 + 4.8j) \parallel 1 \\ = \frac{(1.4 + 4.8j) \times 1}{(1.4 + 4.8j) + 1} = 0.9167 + j0.1667 \Omega$$

$$= 0.9317 \angle 10.305^\circ \Omega$$

4. A 10Ω resistor is connected in series with a $100\mu F$ capacitor to a $230V, 50$ Hz supply. Find (i) the impedance (ii) current (iii) power factor (iv) phase angle (v) voltage across the resistor and the capacitor. [2072 Magh]

Solution:



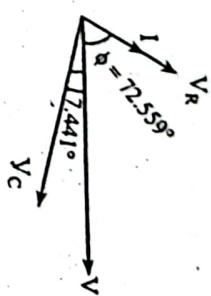
$230 V, 50$ Hz

Here,

Impedance, $Z = R - jX_c$

$$= 10 - j \frac{1}{2\pi f C} \\ = 10 - j \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} \\ = 10 - j 31.831 \Omega$$

$$\text{Current, } I = \frac{\tilde{V}}{Z} = \frac{230 \angle 0^\circ}{10 - j 31.831} = 6.893 \angle 72.559^\circ \text{ A}$$



$$\text{Power factor, } \cos \phi = \cos (72.559^\circ) \\ = 0.2997^\circ \text{ (lead)}$$

$$\text{Phase angle, } \phi = 72.559^\circ$$

$$\text{Voltage across the resistor, } \tilde{V}_R = \tilde{I}R = (6.893 \angle 72.559^\circ) \times 10 \\ = 68.93 \angle 72.559^\circ \text{ V}$$

Voltage across the capacitance,

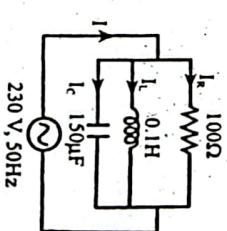
$$\tilde{V}_C = \tilde{I}(-jX_c) \\ = (6.893 \angle 72.559^\circ) \times (-j31.831) \\ = 219.411 \angle -17.441^\circ$$

5. Three elements, a resistance of 100Ω , an inductance of 0.1 H and a capacitance of $150\mu F$ are connected in parallel to a $230V, 50$ Hz supply. Calculate the:

- (i) Current in each element
- (ii) Supply current
- (iii) Phase angle between the supply voltage and the supply current with the help of a phasor diagram. (2072 Magh)

Solution:

$230 V, 50$ Hz



$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.416 \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}} = 21.221 \Omega$$

$\tilde{V} = 230 \angle 0^\circ \text{ V}$

$$i_R = \frac{\tilde{V}}{R} = \frac{230 \angle 0}{100} = 2.3 \angle 0^\circ A$$

$$\tilde{i}_L = \frac{\tilde{V}}{jX_L} = \frac{230 \angle 0}{j31.416} = 7.3211 \angle -90^\circ A$$

$$\tilde{i}_C = \frac{\tilde{V}}{-jX_C} = \frac{230 \angle 0}{-j21.221} = 10.838 \angle 90^\circ A$$

- i) Total current drawn from the supply

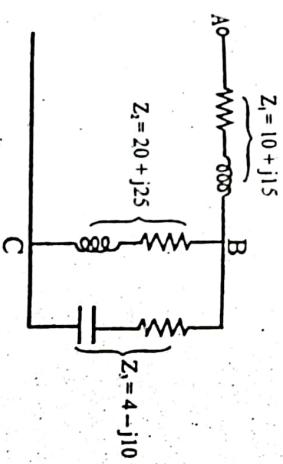
$$\begin{aligned} \tilde{I} &= \tilde{i}_R + \tilde{i}_L + \tilde{i}_C \\ &= (2.3 \angle 0^\circ) + (7.3211 \angle -90^\circ) + (10.838 \angle 90^\circ) \\ &= 4.2022 \angle 56.8159^\circ \end{aligned}$$

iii)



Phase angle between the supply voltage and the supply current = 56.8159°

- 6) In the circuit shown in figure below, determine the equivalent impedance that appears across the terminals A-C. [2072 Magnt]



Here,

$$Z_1 = R_1 + jX_1 = 30 + j2\pi \times 50 \times 75 \times 10^{-9} = 30 + j23.562 \Omega$$

$$\begin{aligned} Z_t &= R_t - jX_C = 20 - j\frac{1}{2\pi \times 50 \times 100 \times 10^{-9}} \\ &= 20 - j31.831 \Omega \end{aligned}$$

$$\begin{aligned} i) \quad \tilde{I}_1 &= \frac{\tilde{V}}{Z_1} = \frac{240 \angle 0}{30 + j23.562} = 6.292 \angle -38.146^\circ A \\ \tilde{I}_2 &= \frac{\tilde{V}}{Z_t} = \frac{240 \angle 0}{20 - j31.831} = 6.384 \angle 57.858^\circ A \end{aligned}$$

Solution:

Equivalent impedance,

$$Z_{AC} = (Z_1 \parallel Z_2) + Z_1 = \frac{Z_2 Z_1}{Z_2 + Z_1} + Z_1$$

$$= \frac{(20 + j23.562)(4 - j31.831)}{(20 + j23.562) + (4 - j31.831)} + (10 + j5)$$

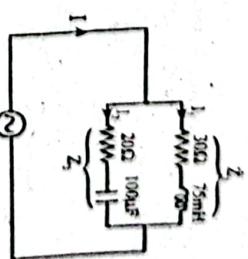
$$= 18.014 + j5.8339 \Omega$$

7.

- A circuit consisting of a resistance of 30Ω in series with an inductance of $75mH$ is connected in parallel with a circuit consisting of a resistance of 20Ω in series with a capacitance of $10mF$. If the parallel combination is connected to a $240 V, 50 Hz$ single phase supply, calculate

- The current in each branch
- The total current and power factor and
- Power consumed. Also draw a neat phasor diagram

Solution:



(2072 Chaitra)

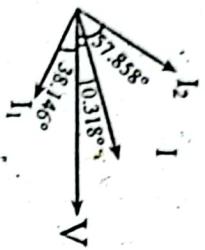
$$= 8.482 \angle 10.318^\circ \text{ A}$$

Power factor, $\cos\phi = \cos(10.318^\circ) = 0.984$ (lead)

- iii) Total power consumed, $P = V I \cos\phi$

$$= 240 \times 8.482 \times 0.984$$

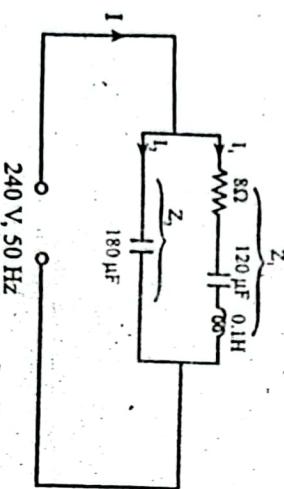
= 2003.109 watt



8. For a series path with a resistance of 8Ω , capacitor of $120\mu\text{F}$ and an inductance of 0.1 H , a capacitor of $180\mu\text{F}$ is kept in parallel. Then the combination is fed by $240\text{ V}, 50\text{ Hz}, 1 - \phi$ supply. Calculate branch currents, total current from active power and reactive power consumed by the circuit. Also show phasor diagram.

[2072 Chaitra]

Solution:



$$\begin{aligned} I_1 &= \frac{\tilde{V}}{Z_1} = \frac{240 \angle 0^\circ}{8 + j4.8901} = 25.5967 \angle -31.4359^\circ \text{ A} \\ \tilde{I}_1 &= \frac{\tilde{V}}{Z_1} = \frac{240 \angle 0^\circ}{-17.6839j} = 13.5717 \angle 90^\circ \text{ A} \\ \text{Total current from the supply} \\ \tilde{I} &= \tilde{I}_1 + \tilde{I}_2 \\ &= (25.5967 \angle -31.4359^\circ) + (13.5717 \angle 90^\circ) \\ &= 21.8409 \angle 0.5821^\circ \text{ A} \end{aligned}$$

Power factor of whole circuit,

$$\cos\phi = \cos(0.5821^\circ)$$

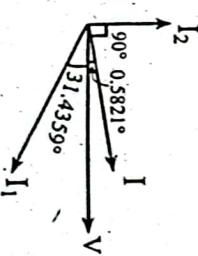
= 0.9999 (lead)

Active power consumed,

$$\begin{aligned} Q &= VI \sin\phi = 240 \times 21.8409 \times \sqrt{1 - (\cos^2\phi)} \\ &= 240 \times 21.8409 \times \sqrt{1 - (\cos 0.5821^\circ)^2} \\ &= 53.2536 \text{ VAR (negative VAR)} \\ &= 5.24 \text{ kW} \end{aligned}$$

Reactive power,

$$\begin{aligned} Q &= VI \sin\phi = 240 \times 21.8409 \times \sqrt{1 - (\cos^2\phi)} \\ &= 240 \times 21.8409 \times \sqrt{1 - (\cos 0.5821^\circ)^2} \\ &= 53.2536 \text{ VAR (negative VAR)} \end{aligned}$$



Here,

$$Z_1 = R_1 + j(X_{L1} - X_{C1})$$

$$\begin{aligned} &= 8 + j \left(2\pi \times 50 \times 0.1 - \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} \right) \\ &= 8 + j 4.8901 \Omega \end{aligned}$$

$$Z_2 = -jX_{C2} = \frac{-j}{2\pi \times 50 \times 180 \times 10^{-6}}$$

9. Derive the equation for instantaneous current flowing through a pure capacitor when excited by AC sinusoidal voltage, $V = V_m \sin\omega t$. Draw the waveform of voltage and current and phase or diagram of the circuit. Show analytically and graphically that it does not consume real power.

[2071 Chaitra]

10. A series R-L-C circuit having $R = 100\Omega$, $L = 0.12\text{ H}$ and $C = 28.27\mu\text{F}$ is fed from a $100\text{ V}, 50\text{ Hz}$ supply. Find the current flowing, active power, reactive power, power factor, rms values of voltage across each elements. Also draw phase diagram.

[2071 Chaitra]

Solution:

Given, 100V, 50 Hz, 1φ Supply

$$\begin{aligned} \text{Now, } X_L &= 2\pi fL \\ &= 2\pi \times 50 \times 0.12 \\ &= 37.7 \Omega \\ X_C &= \frac{1}{2\pi fC} \\ &= \frac{1}{2\pi \times 50 \times 28.27 \times 10^{-6}} \\ &= 112.6 \Omega \end{aligned}$$

Total impedance of the circuit

$$\begin{aligned} Z &= R + jX_L - jX_C \\ &= 100 + j37.7 - j112.6 \\ &= 100 - j74.9 \Omega \end{aligned}$$

$$\bar{V} = 100 \angle 0^\circ V$$

$$\therefore \bar{V} = \bar{I}Z$$

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{100 \angle 0^\circ}{100 - j74.9} = 0.8 \angle 36.83^\circ A$$

$$\text{Phase difference } (\phi) = 0^\circ - 36.83^\circ = -36.83^\circ$$

$$\text{Active Power (P)} = VI \cos \phi$$

$$\begin{aligned} &= 100 \times 0.8 \times \cos(-36.83^\circ) \\ &= 64.03 \text{ watt} \end{aligned}$$

$$\text{Reactive power (Q)} = VI \sin \phi$$

$$\begin{aligned} &= 100 \times 0.8 \times \sin(-36.83^\circ) \\ &= -47.95 \text{ VAR} \end{aligned}$$

(Negative sign indicates that the circuit supplies reactive power)

$$\begin{aligned} \text{Power factor} &= \text{Cos}\phi = \text{Cos}(-36.83^\circ) \\ &= 0.8 \text{ (lead)} \end{aligned}$$

Voltage across resistor

$$\begin{aligned} \bar{V}_R &= \bar{I}R \\ &= (0.8 \angle 36.83^\circ) \times (100) \end{aligned}$$

$$= 80 \angle 36.83^\circ V$$

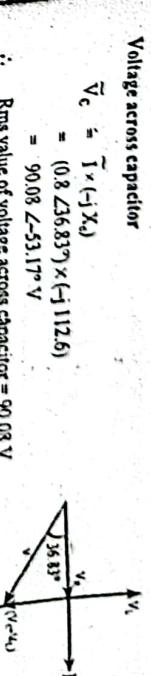
\therefore Rms value of voltage across resistor = 80 V

Voltage across inductor

$$\begin{aligned} \bar{V}_L &= \bar{I} \times jX_L \\ &= (0.8 \angle 36.83^\circ) \times j37.7 \end{aligned}$$

$$= 30.16 \angle 126.83^\circ V$$

\therefore Rms value of voltage across inductor = 30.16 V



11. A coil is connected in series with a resistance of 30Ω across of 240V, 50 Hz power supply. The reading of a voltmeter across coil is 180V and across a resistor is 130V. Calculate resistance and reactance of coil. Also find power factor of whole circuit.

$$\begin{aligned} I &= \frac{\text{Voltage across resistor (V_R)}}{\text{Resistance}} \\ &= \frac{130}{30} = 4.33 \text{ A} \end{aligned}$$

Solution:

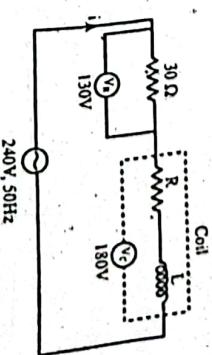
Given,

$$\text{Voltage across coil (V_L)} = 180V$$

$$\text{Voltage across resistor (V_R)} = 130V$$

$$240V, 50Hz \text{ power supply.}$$

Total current of the circuit



The circuit is capacitive.

Rms value of voltage across capacitor = 90.03 V

Since $X_C > X_L$, $V_C > V_L$.

Drawing phasor diagram

The circuit is capacitive.

$$\begin{aligned} \text{Impedance of the coil, } Z_C &= \frac{\text{Voltage across coil}}{\text{total current of the circuit}} \\ &= \frac{180}{4.33} \\ &= 41.57 \Omega \end{aligned}$$

Where, $V_{C_R} \rightarrow$ Voltage across resistance R of coil.

$$\begin{aligned} C &= \frac{Q_C}{\omega V^2} \\ &= \frac{26.57}{2\pi \times 50 \times (25)^2} \\ &= 1.353 \times 10^{-4} \text{ F} \\ &= 135.3 \mu\text{F} \end{aligned}$$

The value of required capacitance is $135.3 \mu\text{F}$.

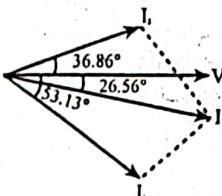
14. Explain the operation of purely capacitive circuit excited by a sinusoidal source and hence prove that average power consumed by such circuit is zero. Draw necessary waveforms. [2071 Bhadra]
[Please refer to the theory]
15. For the circuit given below, calculate the current I . Draw the phasor diagram of the circuit. [2071 Bhadra]

Solution:

$$\begin{aligned} \tilde{V} &= 100 \angle 0^\circ \\ Z_1 &= 3 + j4 \Omega \\ Z_2 &= 8 - j6 \Omega \\ \tilde{I}_1 &= \frac{\tilde{V}}{Z_1} = \frac{100 \angle 0^\circ}{3+j4} = 20 \angle -53.13^\circ \text{ A} \\ \tilde{I}_2 &= \frac{\tilde{V}}{Z_2} = \frac{100 \angle 0^\circ}{8-j6} = 10 \angle 36.86^\circ \text{ A} \end{aligned}$$

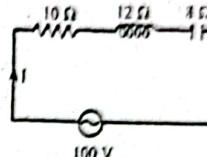
$$\begin{aligned} \therefore \tilde{I} &= \tilde{I}_1 + \tilde{I}_2 \\ &= (20 \angle -53.13^\circ) + (10 \angle 36.86^\circ) \\ &= 22.36 \angle -26.56^\circ \text{ A} \end{aligned}$$

Drawing phasor diagram



16. Derive the equation for the instantaneous current when A.C. voltage is supplied to a series R-L circuit. Draw phasor diagrams and analyze power in the circuit. [2071 Bhadra]
[Please refer to the theory]

17. An electric circuit is being supplied by an a.c. source of 100 Vrms. The circuit has a resistance of 10Ω , inductor of 12Ω reactance and capacitance of 8Ω reactance connected in series. Compute the active power and power factor of the circuit. [2071 Bhadra]



Solution:

$$\begin{aligned} \tilde{V} &= 100 \angle 0^\circ \\ Z &= R + jX_L - jX_C \\ &= 10 + j12 - j8 \\ &= 10 + j4 \end{aligned}$$

$$\tilde{V} = \tilde{I} Z$$

$$\begin{aligned} \tilde{I} &= \frac{\tilde{V}}{Z} \\ &= \frac{100 \angle 0^\circ}{10 + j4} \\ &= 9.28 \angle -21.8^\circ \text{ A} \end{aligned}$$

$$\begin{aligned} \text{Phase difference } (\phi) &= 0^\circ - (-21.8^\circ) \\ &= 21.8^\circ \end{aligned}$$

$$\begin{aligned} \text{Active power } (P) &= VI \cos \phi \\ &= 100 \times 9.28 \times \cos 21.8^\circ \\ &= 861.63 \text{ Watt.} \end{aligned}$$

$$\begin{aligned} \text{Power factor } (pf) &= \cos \phi \\ &= \cos 21.8^\circ \\ &= 0.928 \text{ (lagging)} \end{aligned}$$

18. Derive the equation for instantaneous current flowing through a pure capacitor when excited by AC sinusoidal voltage $v = V_m \sin \omega t$. Draw the wave form of voltage and current and phasor diagram of the circuit. Show analytically and graphically that it does not consume real power.

[2070 Chaitra, 2071 Chaitra]

[Please refer to the theory]

19. A coil takes 1.3 kVA and 1.2 kVAR when connected to a 240V, 50 Hz sinusoidal supply. Calculate (i) Power dissipated (ii) current and (iii) inductance of the coil. [2070 Chaitra]

Solution:

$$\begin{aligned} \text{Given, } 240V, 50\text{Hz supply} \\ \text{Reactive power } (Q) &= 1.2 \text{ kVAR} \\ \text{Apparent power } (S) &\perp 1.3 \text{ kVA} \\ \text{(i) Power dissipated } (P) & \\ P^2 &= S^2 - Q^2 \end{aligned}$$

$$\begin{aligned} P &= \sqrt{S^2 - Q^2} \\ &= \sqrt{(1.3)^2 - (1.2)^2} \\ &= 0.5 \text{ kW} \\ &= 500 \text{ W} \end{aligned}$$

- (ii) Current (I)
Power - diagram

$$\begin{aligned} \tan\phi &= \frac{Q}{P} \\ \tan\phi &= \frac{1.2}{0.5} \\ \therefore \phi &= 67.38^\circ \end{aligned}$$

$$P = V I \cos\phi$$

$$\text{or, } 500 = 240 \times 1 \times \cos 67.38^\circ$$

$$\therefore I = \frac{500}{240 \times \cos 67.38^\circ} = 5.4166 \text{ A}$$

- (iii) Inductance of the coil (L)

$$\tilde{V} = \tilde{I}Z$$

$$240 \angle 0^\circ = (5.4166 \angle -67.38^\circ)Z$$

$$\therefore Z = \frac{240 \angle 0^\circ}{5.4166 \angle -67.38^\circ} = 44.308 \angle 67.38^\circ$$

Comparing with $Z = R + jX_L$

$$\therefore X_L = 40.8998$$

$$\text{or, } 2\pi fL = 40.8998$$

$$\text{or, } L = \frac{40.8998}{2\pi \times 50}$$

$$\therefore L = 0.13018 \text{ H}$$

20. A circuit consisting of a resistance of 30Ω in series with an inductance of 75mH is connected in parallel with a circuit consisting of a resistance of 20Ω in series with a capacitance of $100\mu\text{F}$. If the parallel combination is connected to a 240V , 50Hz , single phase supply. Calculate (i) The total current (ii) Power factor (iii) Active and reactive power. Also draw a neat phasor diagram. [2070 Chaitra]

Solution:

Given,

$240\text{V}, 50\text{Hz}, 1\phi$ supply

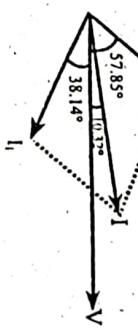
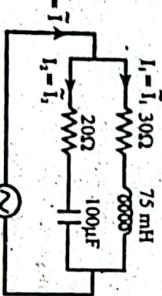
Now,

$$X_L = 2\pi fL$$

$$= 2\pi \times 50 \times 75 \times 10^{-3}$$

$$= 23.56\Omega$$

$$\therefore Z_L = R_L + jX_L$$



21. Define power factor, active, reactive and apparent power in ac circuit. Also draw the phasor diagram. [2070 Magh]
[Please refer to the theory]
22. A voltage $e(t) = 100 \sin 314t$ is applied across series circuit consisting of 10Ω resistance, 0.0318 H inductance and a capacitor of $63.6 \mu\text{F}$. Calculate expression for $i(t)$, phase difference between voltage and current, power factor, apparent power and active power. [2070 Magh]

Solution:

Given,

$240\text{V}, 50\text{Hz}, 1\phi$ supply

$$X_L = 2\pi fL$$

$$= 2\pi \times 50 \times 75 \times 10^{-3}$$

$$= 23.56\Omega$$

$$\therefore Z_L = R_L + jX_L$$

$$R = 10\Omega$$

$$X_L = 2\pi fL$$

$$\begin{aligned} &= 30 + j23.56\Omega \\ X_C &= \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} \\ &= 31.83\Omega. \\ \therefore Z_L &= R_L - jX_C = 20 - j31.83\Omega \end{aligned}$$

Now,
Total impedance (Z) = $Z_L \parallel Z_L$

$$\text{So, } \tilde{I}_1 = \frac{\tilde{V}}{Z_L} = \frac{240 \angle 0^\circ}{30 + j23.56} = 6.29 \angle -38.14^\circ \text{ A}$$

$$\begin{aligned} \tilde{I}_1 &= \frac{\tilde{V}}{Z_L} = \frac{240 \angle 0^\circ}{20 - j31.83} = 6.38 \angle 57.85^\circ \text{ A} \\ \text{Hence,} \end{aligned}$$

$$\begin{aligned} \text{(i) Total current } (\tilde{I}) &= \tilde{I}_1 + \tilde{I}_2 \\ &= (6.29 \angle -38.14^\circ) + (6.38 \angle 57.85^\circ) \\ &= 8.478 \angle 10.32^\circ \text{ A} \end{aligned}$$

$$\begin{aligned} \text{(ii) Power factor} \\ \cos\phi &= \cos 10.32^\circ = 0.9838 \text{ (lead)} \\ \text{(iii) Active power (P)} &= V I \cos\phi \\ \text{Reactive power (Q)} &= V I \sin\phi \\ &= 240 \times 8.478 \times \sin(10.32^\circ) = 364.597 \text{ VAR} \end{aligned}$$

$$\begin{aligned} &= \omega L \\ &= 314 \times 0.0318 \\ &= 9.9852 \Omega \end{aligned}$$

$$X_C = \frac{1}{\omega C}$$

$$= \frac{1}{314 \times 63.6 \times 10^{-6}}$$

$$= 50.074 \Omega$$

$$\begin{aligned} Z &= R + jX_L - jX_C \\ &= 10 + j9.9852 - j50.074 \\ &= 10 - j40.088 \Omega \end{aligned}$$

$$e(t) = 100 \sin 314t$$

Comparing with $e(t) = E_m \sin(\omega t + \phi)$

$$\therefore \omega = 314 \quad \phi = 0^\circ$$

$$E_m = 100$$

$$\therefore E_{rms} = \frac{E_m}{\sqrt{2}}$$

$$= \frac{100}{\sqrt{2}}$$

$$= 70.71 \text{ V}$$

$$\therefore \tilde{E} = 70.71 \angle 0^\circ$$

$$\therefore \tilde{E} = \tilde{I}Z$$

$$\begin{aligned} \tilde{I} &= \frac{\tilde{E}}{Z} \\ &= \frac{70.71 \angle 0^\circ}{10 - j40.088} \end{aligned}$$

$$= 1.71 \angle 75.99^\circ$$

$$= 1.71 \angle 76^\circ \text{ A}$$

$$\therefore I_{rms} = 1.71 \text{ A}$$

$$I_m = \sqrt{2} I_{rms} = \sqrt{2} \times 1.71 = 2.42 \text{ A}$$

$$\therefore i(t) = 2.42 \sin(314t + 76^\circ) \text{ A}$$

Phase difference between voltage and current

$$\phi = 0^\circ - 76^\circ = -76^\circ$$

Power factor = $\cos \phi = \cos(-76^\circ) = 0.2419$ (lead)

$$\begin{aligned} \text{Apparent power (S)} &= \tilde{V} \tilde{I}^* \\ &= (70.71 \angle 0^\circ) \times (1.71 \angle -76^\circ) \\ &= 120.91 \angle -76^\circ \text{ VA} \\ &= (29.25 - j17.32) \text{ VA} \end{aligned}$$

$$\therefore \text{Apparent power (S)} = 120.91 \text{ VA}$$

Comparing with $S = P + jQ$

$$\text{Active power (P)} = 29.25 \text{ W}$$

Reactive power (Q) = -117.33 VAR (negative sign indicates the circuit supplies reactive power)

23. Two impedances $Z_1 = (10 + j5)$ and $Z_2 = (8 + j6)$ are joined in parallel across a voltage of $V = 200 + j0$. Calculate magnitudes and phases of circuit current and branch currents. Draw phasor diagram. [2070 Bhadra]

Given, $\tilde{V} = 200 + j0$

$$= 200 \angle 0^\circ$$

$$Z_1 = 10 + j5 \Omega$$

$$\text{Now, } \tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{200 \angle 0^\circ}{10 + j5}$$

$$= 17.88 \angle -26.56^\circ \text{ A}$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{200 \angle 0^\circ}{8 + j6}$$

$$= 20 \angle -36.87^\circ \text{ A}$$

$$\tilde{I} = \tilde{I}_1 + \tilde{I}_2$$

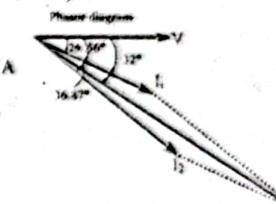
$$= (17.88 \angle -26.56^\circ) + (20 \angle -36.87^\circ)$$

$$= 37.73 \angle -32^\circ \text{ A}$$

Circuit current (\tilde{I}) = $37.73 \angle -32^\circ \text{ A}$

Branch current $\tilde{I}_1 = 17.88 \angle -26.56^\circ \text{ A}$

$$\tilde{I}_2 = 20 \angle -36.87^\circ \text{ A}$$



24. An inductive load of 4kW at a lagging power factor of 0.8 is connected across a $220\text{V}, 50\text{Hz}$ supply. Calculate the value of the capacitance to be connected in parallel with the load to bring the resultant power factor to 0.95 lagging. [2070 Bhadra]

Solution:

Given, $220\text{V}, 50\text{Hz}$ supply

$$\text{Power (P)} = 4 \text{ kW}$$

$$\text{p.f.}_1 = 0.8 \text{ (lag)}$$

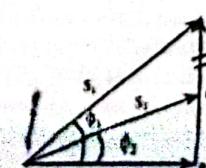
$$\text{p.f.}_2 = 0.95 \text{ (lag)}$$

$$\begin{aligned} \text{Now, } \text{p.f.}_1 &= 0.8 & \text{p.f.}_2 &= 0.95 \\ \cos \phi_1 &= 0.8 & \cos \phi_2 &= 0.95 \\ \therefore \phi_1 &= 36.87^\circ & \therefore \phi_2 &= 18.19^\circ \end{aligned}$$

Drawing power diagram

From figure,

$$\begin{aligned} Q_1 &= P \tan \phi_1 \\ &= 4000 \times \tan 36.87^\circ \end{aligned}$$



$$\begin{aligned} &= 3000.01 \text{ VAR} \\ Q_2 &= P \tan \phi_2 \\ &= 4000 \times \tan 18.19^\circ \\ &= 1314.359 \text{ VAR} \end{aligned}$$

$$\begin{aligned} \text{Reactive Power supplied } (Q_C) &= Q_1 - Q_2 \\ &= 3000.01 - 1314.359 \\ &= 1685.65 \text{ VAR} \\ Q_C &= \omega CV^2 \end{aligned}$$

$$\therefore C = \frac{Q_C}{\omega V^2} = \frac{1685.65}{2 \times \pi \times 50(220)^2} = 110.86 \mu\text{F}$$

25. Required value of capacitance (C) = $110.86 \mu\text{F}$
 Two impedances ($3 - j4$) and $(8 + j6)$ are connected in parallel across an ac voltage source. If the total current drawn from the source is 25A , find the total active power consumed by the impedances. [2070 Ashad]

Solution:

Given,

$$Z_1 = 3 - j4$$

$$Z_2 = 8 + j6$$

Total current drawn (I) = 25A

Let, $\tilde{I} = 25 \angle 0^\circ \text{A}$

$$\text{Total impedance } Z = Z_1 \parallel Z_2 = (3 - j4) \parallel (8 + j6) = 4 - j2 \Omega$$

$$\tilde{V} = \tilde{I}Z$$

$$= (25 \angle 0^\circ) \times (4 - j2)$$

Now, phase difference between voltage and current

$$\phi = -26.565^\circ - 0^\circ$$

$$\begin{aligned} \text{Active power consumed (P)} &= V I \cos \phi \\ &= 111.803 \times 25 \cos(-26.565) \\ &= 2449.9 = 2500 \text{ watts} \end{aligned}$$

26. An industrial load consists of the following:

- i) A load of 200 kVA @ 0.8 power factor lagging
 - ii) A load of 50 kW @ unity power factor.
 - iii) A load of 48 kW @ 0.6 power factor leading
- Calculate the total kW , total kVAR , total kVA and the overall power factor.

[2070 Ashad]

Solution:
 Load 1: Since p.f lagging (inductive load)

$$\begin{aligned} S_1 &= 200 \text{ kVA} \\ \text{p.f}_1 &= 0.8 (\text{lag}) \end{aligned}$$

$$\begin{aligned} \cos \phi_1 &= 0.8 \\ \therefore \phi_1 &= 36.87^\circ \end{aligned}$$

$$\begin{aligned} \therefore P_1 &= S_1 \cos \phi_1 = 200 \times \sin 36.87^\circ = 160 \text{ kW} \\ \therefore Q_1 &= S_1 \sin \phi_1 = 200 \times \cos 36.87^\circ = 120 \text{ kVAR} \end{aligned}$$

Load 2: Since p.f. = 1 (Resistive load)

$$\begin{aligned} \text{p.f}_2 &= 1 \Rightarrow \phi_2 = 0^\circ \\ P_2 &= 0 \end{aligned}$$



$$\therefore S_2 = \sqrt{P_2^2 + Q_2^2} = \sqrt{P_2^2} = P_2 = 50 \text{ kVA}$$

Load 3: Since p.f. leading (Capacitive load)

$$\begin{aligned} P_3 &= 48 \text{ kW} \\ \text{p.f}_3 &= 0.6 \\ \phi_3 &= 53.13^\circ \end{aligned}$$

$$\therefore Q_3 = P_3 \tan \phi_3 = 48 \tan 53.13^\circ = 64 \text{ kVAR}$$

$$\therefore S_3 = \sqrt{P_3^2 + Q_3^2} = \sqrt{48^2 + 64^2} = 80 \text{ kVA}$$

$$\therefore \text{Total kW}(P) = P_1 + P_2 + P_3$$

$$= 160 + 50 + 48$$

$$= 258 \text{ kW}$$

$$\therefore \text{Total kVAR}(Q) = Q_1 + Q_2 - Q_3 \text{ [phasor sum]}$$

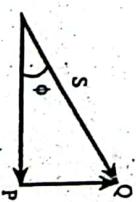
$$= 120 + 0 - 64$$

$$= 56 \text{ kVAR}$$

$$\therefore \text{Total kVA}(S) = \sqrt{P^2 + Q^2} = 264 \text{ kVA}$$

From fig.

$$\cos \phi = \frac{P}{S} = \frac{258}{264} = 0.977$$



∴ Total power factor (P.F) = 0.977 (lag)

27. A 100 kW load at 0.8 lagging power factor is being supplied by a $220\text{V}, 50 \text{ Hz}$ source. Calculate the reactive power drawn from the source. If a capacitor, connected parallel to the load improves its power factor to 0.9.

Find the capacitance of the capacitor. Also calculate the current drawn from the source before and after connecting the capacitor. [2070 Ashad]

Solution:

Given,

220 V, 50 Hz source.

Power (P) = 100 kW

p.f₁ = 0.8 (lag)

p.f₂ = 0.9 (lag)

cos φ₁ = 0.8

$$\phi_1 = 36.87^\circ$$



From figure,

$$Q_1 = P \tan \phi_1$$

$$= 100 \times \tan 36.87^\circ$$

$$= 75 \text{ kVAR}$$

∴ Reactive power drawn from the source = 75 kVAR

$$P = VI_1 \cos \phi_1$$

$$100 \times 10^3 = 220 \times I_1 \times 0.8$$

$$\therefore I_1 = 568.18 \text{ A}$$

Current drawn from the source before connecting the capacitor = 568.18 A

$$P \cdot f_2 = 0.9$$

$$\cos \phi_2 = 0.9$$

$$\therefore \phi_2 = 25.84^\circ$$

∴ Q₂ = P tan φ₂ = 100 × tan 25.84° = 48.428 kVAR

Reactive power compensated by the capacitor

$$\therefore Q_C = Q_1 - Q_2 = 75 - 48.428 = 26.572 \text{ kVAR}$$

$$\therefore Q_C = \omega C V^2$$

$$\therefore C = \frac{Q_C}{\omega V^2} = \frac{26.572 \times 10^3}{2\pi \times 50 \times (220)^2}$$

$$= 1747.547 \mu\text{F}$$

∴ Capacitance of the capacitor = 1747.547 μF

Now, P = VI₂ cos φ₂

$$100 \times 10^3 = 220 \times I_2 \times 0.9$$

$$\therefore I_2 = 505.05 \text{ A}$$

∴ Current drawn from the source after connecting the capacitor I₁ = 505.05 A

Thus, this shows power factor improvement causes less current to be drawn from the circuit for same amount of active power consumption.

28. For the parallel circuit shown below, calculate

- (i) Rms value for current, power factors and active power of path 1.
- (ii) Rms value of current, power factor and reactive power of path 2.
- (iii) Rms value of current and power factor of the whole circuit.

Solution:

Given,

$$v = 325 \sin 377t$$

Comparing with $v = V_m \sin(\omega t + \phi)$

$$V_m = 325 \quad \omega = 377 \quad \phi = 0^\circ$$

$$V_m = \frac{v}{\sqrt{2}} = \frac{325}{\sqrt{2}} = 229.3 \angle 0^\circ \text{ V}$$

$$\therefore \tilde{V} = 229.3 \angle 0^\circ \text{ V}$$

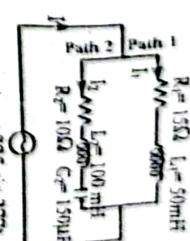
$$Z_1 = R_1 + jX_{L_1} = 15 + j377 \times 50 \times 10^{-3}$$

$$= 15 + j18.85 \Omega$$

$$Z_2 = R_2 + jX_{C_2} = 10 + j18.85 \Omega$$

$$= R_2 + j(X_{L_1} - X_C) = 10 + j(377 \times 100 \times 10^{-3} - \frac{1}{377 \times 150 \times 10^{-6}})$$

$$= 10 + j(37.7 - 17.68) = 10 + j20 \Omega$$



$$v = 325 \sin 377t$$

(i) For Path 1

$$\tilde{V}_1 = \tilde{I}_1 Z_1$$

$$\tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{229.3 \angle 0^\circ}{15 + j18.85} = 9.54 \angle -51.49^\circ \text{ A}$$

Rms value of current of path 1 = 9.54 A

$$\therefore p.f_1 = \cos \phi_1 = \cos(0^\circ - (-51.49^\circ))$$

$$= \cos 51.49^\circ$$

$$= 0.622 \text{ (lag)}$$

$$\begin{aligned} &= 229.8 \times 9.54 \times 0.622 \\ &= 1363.605 \text{ watts} \end{aligned}$$

(ii) For path 2

$$\tilde{V} = \tilde{I}_2 Z_2$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{229.8 \angle 0^\circ}{10 + j 20} = 10.28 \angle -63.43^\circ \text{ A}$$

∴ Rms value of current of path 2 = 10.28 A

$$\therefore \text{p.f.} = \cos \phi_2 = \cos (0^\circ - (-63.43^\circ)) = \cos 63.43^\circ = 0.447 \text{ (lag)}$$

$$\begin{aligned} \text{Reactive power (Q}_2\text{)} &= VI_2 \sin \phi_2 \\ &= 229.8 \times 10.28 \sin 63.43^\circ \\ &= 2112.85 \text{ VAR} \end{aligned}$$

(iii) Since parallel circuit,

$$\begin{aligned} \tilde{I} &= \tilde{I}_1 + \tilde{I}_2 = (9.54 \angle -51.49^\circ) + (10.28 \angle -63.43^\circ) \\ &= 19.71 \angle -57.68^\circ \text{ A} \end{aligned}$$

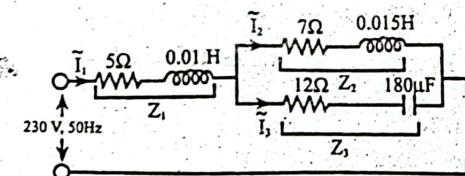
Rms value of current of the whole circuit = 19.71 A

$$\begin{aligned} \therefore \text{p.f.} = \cos \phi &= \cos(0^\circ - (-57.68^\circ)) \\ &= \cos 57.68^\circ \\ &= 0.534 \text{ (lag)} \end{aligned}$$

29. In a network shown in figure below, determine:

- Total impedance
- Total current
- The current in each branch
- The overall power factor.
- Volt amperes, active power & reactive power.

[2069 Chaitra]



Solution:

Given,

230V, 50Hz supply

$$\begin{aligned} Z_1 &= R_1 + jX_{L_1} \\ &= 5 + j 2\pi f L_1 \end{aligned}$$

$$= 5 + j 2\pi \times 50 \times 0.01$$

$$= 5 + j 3.14 \Omega$$

$$Z_1 = R_1 + jX_{L_1} = 7 + j2\pi \times 50 \times 0.015 = 7 + j 4.71 \Omega$$

$$Z_3 = R_3 - jX_c = 12 - j \frac{1}{2\pi \times 50 \times 180 \times 10^{-6}} = 12 - j 17.68 \Omega$$

Now,

$$\begin{aligned} \text{(i) } Z &= Z_1 + (Z_2 \parallel Z_3) \\ &= Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \\ &= (5 + j 3.14) + \frac{(7 + j 4.71)(12 - j 17.68)}{(7 + j 4.71) + (12 - j 17.68)} \\ &= (5 + j 3.14) + (7.65 + j 1.68) \\ &= (12.65 + j 4.82) \Omega \end{aligned}$$

$$\therefore \text{Total impedance (Z)} = 12.65 + j 4.82 \Omega$$

Now,

$$\text{(ii) } \tilde{V} = 230 \angle 0^\circ$$

$$\tilde{I}_1 = \frac{\tilde{V}}{Z} = \frac{230 \angle 0^\circ}{12.65 + j 4.82} = 16.99 \angle -20.86^\circ \text{ A} = 17 \angle -20.86^\circ \text{ A}$$

$$\therefore \text{Total current} = 17 \angle -20.86^\circ \text{ A}$$

(iii) Using current division rule,

$$\begin{aligned} \tilde{I}_2 &= \frac{\frac{1}{Z_2}}{\frac{1}{Z_2} + \frac{1}{Z_3}} \times \tilde{I}_1 \\ &= \frac{\frac{1}{(7 + j 4.71)}}{\frac{1}{(7 + j 4.71)} + \frac{1}{(12 - j 17.68)}} \times (17 \angle -20.86) \\ &= 15.79 \angle -42.37^\circ \text{ A} \end{aligned}$$

Using KCL,

$$\begin{aligned} \tilde{I}_3 &= \tilde{I}_1 - \tilde{I}_2 \\ &= (17 \angle -20.86^\circ) - (15.79 \angle -42.37^\circ) \\ &= 6.23 \angle 47.39^\circ \end{aligned}$$

(iv) Phase difference between voltage and current

$$\begin{aligned} \phi &= 0^\circ - (-20.86^\circ) \\ &= 20.86^\circ \end{aligned}$$

$$\text{P.f.} = \cos \phi = \cos 20.86^\circ = 0.934 \text{ (lag)}$$

Current lags voltage in phase angle

(v) Volt amperes = Apparent power (S) = VI = 230 × 17 = 3910 VA

$$\text{Active power (P)} = VI \cos \phi = 230 \times 17 \times \cos 20.86^\circ = 3653.71 \text{ watts}$$

$$\text{Reactive power (Q)} = VI \sin \phi = 230 \times 17 \times \sin 20.86 = 1392.29 \text{ VAR}$$

30. What is power factor in ac circuit? Explain the disadvantages of low power factor.
[please refer to the theory]
31. When a voltage $v = 10 \sin(500t - 60^\circ)$ V is applied to a series ac circuit, the current is $i = 6 \sin(500t - 10^\circ)$. Find (i) Power factor (ii) Circuit parameters (iii) Apparent, active and reactive power. (iv) Also indicate whether the circuit is capacitive or inductive and why? [2069 Ashd]

Solution:

Given,

$$v = 10 \sin(500t - 60^\circ) \text{V}$$

Comparing with $v = V_m \sin(\omega t + \phi)$

$$V_m = 10$$

$$\omega = 500$$

$$\phi = -60^\circ$$

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ V}$$

$$\therefore \tilde{V} = 7.07 \angle -60^\circ \text{V}$$

$$i = 6 \sin(500t - 10^\circ) \text{A}$$

Comparing with $i = I_m \sin(500t + \phi)$

$$\therefore I_m = 6 \quad \phi = -10^\circ$$

$$\therefore I_{rms} = \frac{6}{\sqrt{2}} = 4.24 \text{ A}$$

$$\therefore \tilde{I} = 4.24 \angle -10^\circ \text{A}$$

Phase difference between voltage & current $\phi = -60^\circ - (-10^\circ) = -50^\circ$

$$(i) \quad \therefore p.f. = \cos\phi = \cos(-50^\circ) = 0.6427 \text{ (lead)}$$

Lead as current leads voltage in phase angle

(ii) Circuit parameters

$$\therefore \tilde{V} = \tilde{I}Z$$

$$\therefore Z = \frac{\tilde{V}}{\tilde{I}} = \frac{7.07 \angle -60^\circ}{4.24 \angle -10^\circ}$$

$$= 1.07 - j 1.27 \Omega$$

Comparing with $Z = R - jX_C$

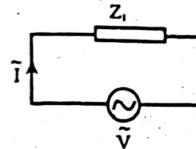
$$\therefore R = 1.07 \Omega$$

$$\therefore X_C = 1.27 \Omega$$

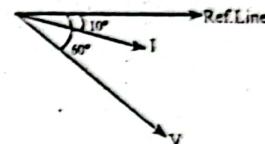
$$\text{Now, } X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{500 \times 1.27} = 1574.8 \mu\text{F}$$

\therefore Circuit parameters Resistance (R) = 1.07Ω

Capacitance (C) = $1574.8 \mu\text{F}$

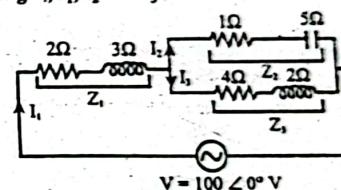


- (iii) Apparent power (S) = $VI = 7.07 \times 4.24 = 29.976 \text{ VA}$
Active power (P) = $VI \cos\phi = 7.07 \times 4.24 \times \cos(-50^\circ) = 19.27 \text{ W}$
Reactive power (Q) = $VI \sin\phi = 7.07 \times 4.24 \times \sin(-50^\circ) = -22.96 \text{ VAR}$
(-ve sign indicates reactive power is being supplied by the capacitive circuit)
- (iv) The circuit is capacitive as current leads voltage by 50° phase angle and also impedance $Z = R - jX_C$ which indicates capacitive circuit. The reactive power is being supplied by the circuit which also shows the circuit to be capacitive.



32. Following figure shows a series parallel circuit. Find:

- (i) Total impedance (ii) current drawn from the circuit (iii) Voltage across the parallel branches (iv) current flowing through each parallel branch (v) power factor (vi) Active, reactive and apparent power. Draw phasor diagram showing V , I_1 , I_2 and I_3 . [2069 Ashad]



Solution:

Given,

$$\tilde{V} = 100 \angle 0^\circ \text{V}$$

$$Z_1 = 2 + j3 \Omega$$

$$Z_2 = 1 - j5 \Omega$$

$$Z_3 = 4 + j2 \Omega$$

$$(i) \quad Z = Z_1 + (Z_2 \parallel Z_3) = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= (2 + j3) + \frac{(1 - j5)(4 + j2)}{(1 - j5) + (4 + j2)}$$

$$= (2 + j3) + (3.64 - j 1.41) = 5.64 + j 1.58 \Omega$$

\therefore Total impedance of the circuit (Z) = $5.64 + j 1.58 \Omega = 5.86 \angle 15.71^\circ \Omega$

$$(ii) \quad \tilde{V} = \tilde{I}_1 Z$$

$$\tilde{I}_1 = \frac{\tilde{V}}{Z}$$

$$= \frac{100 \angle 0^\circ}{5.64 + j 1.58}$$

$$= 17.05 \angle -15.71^\circ \text{A}$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(12 + j16)(10 - j20)}{(12 + j16) + (10 - j20)} = 20 \Omega$$

Total impedance of the circuit = 20Ω

$$(ii) \tilde{V} = \tilde{I}Z$$

$$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{200 \angle 53.8^\circ}{20} = 10 \angle 53.8^\circ A$$

$$(iii) \tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{200 \angle 53.8^\circ}{12 + j16} = 10 \angle 0.669^\circ A$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{200 \angle 53.8^\circ}{10 - j20} = 8.94 \angle 117.234^\circ A$$

(iv) Phase difference between voltage and current

$$\phi = 53.8^\circ - 53.8^\circ = 0$$

$$P.F. = \cos \phi = \cos 0^\circ = 1$$

\therefore Power factor of the circuit = unity (1)

(v) Active power (P)

$$= VI \cos \phi = 200 \times 10 \times \cos 0^\circ = 2000 \text{ Watts}$$

Reactive power (Q)

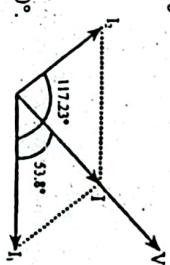
$$= 0$$

Apparent power (S)

$$= 200 \times 10$$

$$= 2000 \text{ VA}$$

Taking I_1 as reference phasor as its phase angle is nearly 0° .



36. Two coils A and B are connected in series across a 240V, 50 Hz supply. The resistance of A is 5Ω and the inductance of B is 0.015 H . If the input from the supply is 3 kW and 2 kVAR , find the inductance of A and the resistance of B. Calculate the voltage across each coil. [2008 Bhabra]

Solution:

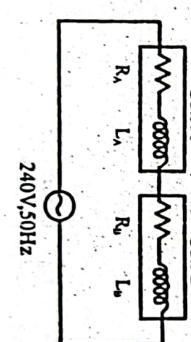
Given,

$$R_A = 5\Omega$$

$$L_B = 0.015 \text{ H}$$

Active power (P) = 3 kW

Reactive power (Q) = 2 kVAR



$$\text{Comparing (i) & (ii)}$$

$$R_A + R_B = 13.295 \quad (\text{iii})$$

$$X_{L_A} + X_{L_B} = 8.863 \quad (\text{iv})$$

$$\text{Now, } \tilde{V} = \tilde{I}Z$$

$$\therefore \tilde{I} = 15.02 \angle -33.69^\circ A$$

$$\text{Hence, } \tilde{V} = \tilde{I}Z$$

$$= \frac{\tilde{V}}{I}$$

$$= \frac{240 \angle 0^\circ}{15.02 \angle -33.69^\circ}$$

$$= 15.02 \angle -33.69^\circ$$

Since inductive load lags v by angle ϕ

Hence, $\tilde{V} = \tilde{I}Z$

$$Z = \frac{\tilde{V}}{I}$$

$$= 15.02 \angle -33.69^\circ$$

$$\text{Also, Total impedance } Z = Z_A + Z_B$$

$$= R_A + jX_{L_A} + R_B + jX_{L_B}$$

$$\therefore Z = (R_A + R_B) + j(X_{L_A} + X_{L_B}) \quad (\text{v})$$

$$3 \times 10^3 = 240I \cos \phi \quad (\text{i})$$

$$\text{Also, } Q = VI \sin \phi$$

$$2 \times 10^3 = 240I \sin \phi \quad (\text{ii})$$

Dividing (ii) by (i)

$$\frac{2 \times 10^3}{3 \times 10^3} = \frac{240I \sin \phi}{240I \cos \phi}$$

$$\therefore \frac{2}{3} = \tan \phi$$

$$\therefore \phi = \tan^{-1}\left(\frac{2}{3}\right)$$

$$= 33.69^\circ$$

$$\text{Now, } P = VI \cos \phi$$

$$3 \times 10^3 = 240I \cos 33.69^\circ$$

$$\therefore I = 15.02 A$$

Now, $\tilde{V} = 240 \angle 0^\circ V$

Comparing (i) & (ii)

$$R_A + R_B = 13.295 \quad (\text{iii})$$

$$X_{L_A} + X_{L_B} = 8.863 \quad (\text{iv})$$

$$\begin{aligned} R_A &=? \\ L_B &=? \\ R_B &=? \\ V_A &=? \quad V_B=? \end{aligned}$$

We know,

$$P = VI \cos \phi$$

From (iv)

$$X_{L_A} + X_{L_B} = 8.863$$

$$\text{or, } \omega L_A + \omega L_B = 8.863$$

$$\text{or, } \omega(L_A + L_B) = 8.863$$

$$\text{or, } 2\pi \times 50 (L_A + 0.015) = 8.863$$

$$\text{or, } L_A + 0.015 = 0.0282$$

$$\therefore L_A = 0.013 \text{ H}$$

Voltage across coil A

$$\tilde{V}_A$$

$$= \tilde{I}Z_A$$

$$= (15.02 \angle -33.69^\circ) \times (R_A + jX_{L_A})$$

$$= (15.02 \angle -33.69^\circ) \times (5 + j2\pi \times 50 \times L_A)$$

$$= (15.02 \angle -33.69^\circ) \times (5 + j4.08)$$

$$= 96.93 \angle 5.52^\circ \text{ V}$$

Voltage across coil B

$$\tilde{V}_B$$

$$= \tilde{I}Z_B$$

$$= (15.02 \angle -33.69^\circ) \times (R_B + jX_{L_B})$$

$$= (15.02 \angle -33.69^\circ) \times (8.295 + j2\pi \times 50 \times 0.015)$$

$$= 143.29 \angle -4.09^\circ \text{ V}$$

37. Two impedances consists of (resistance of 15Ω and series connected inductance of 0.04 H) and (resistance of 10Ω , inductance of 0.1 H and a capacitance of $100 \mu\text{F}$, all in series) are connected in series and are connected to a $230 \text{ V}, 50 \text{ Hz}$ ac source. Find: (i) current drawn (ii) voltage across each impedance (iii) total power factor and (iv) draw the phasor diagram.

[2068 Bhadra]

Solution:

$$Z_1 = R_1 + jX_{L_1}$$

$$= 15 + j\omega L_1$$

$$= 15 + j2\pi \times 50 \times 0.04$$

$$= 15 + j12.56 \Omega$$

$$Z_2 = R_2 + jX_{L_2} - jX_C$$

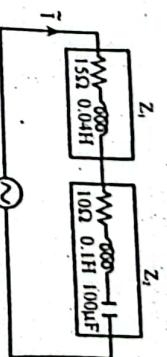
$$= R_2 + j\omega L_2 - j\frac{1}{\omega C}$$

$$= 10 + j2\pi \times 50 \times 0.1 - j\frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$= 10 + j31.41 - j31.83$$

$$= 10 - j0.42 \Omega$$

Total impedance



$$Z = Z_1 + Z_2$$

$$= (15 + j12.56) + (10 - j0.42)$$

$$= (25 + j12.14)\Omega$$

$$(i) \quad \tilde{I} = \frac{\tilde{V}}{Z} = \frac{230 \angle 0^\circ}{25 + j12.14} = 8.275 \angle -25.9^\circ \text{ A}$$

$$(ii) \quad \text{Voltage across } Z_1 (\tilde{V}_{Z_1}) = \tilde{I}Z_1$$

$$= (8.275 \angle -25.9^\circ)(15 + j12.56)$$

$$= 161.91 \angle 14.04^\circ \text{ V}$$

$$\text{Voltage across } Z_2 (\tilde{V}_{Z_2}) = \tilde{I}Z_2$$

$$= (8.275 \angle -25.9^\circ)(10 - j0.42)$$

$$= 82.82 \angle -28.30^\circ \text{ V}$$

$$(iii) \quad \text{Total p.f.} = \cos \phi$$

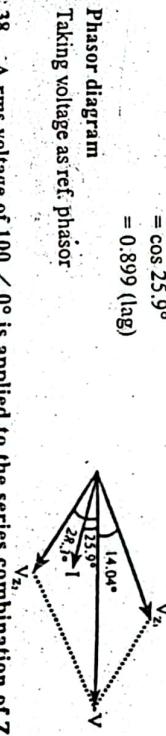
$$= \cos(0 - (-25.9))$$

$$= \cos 25.9^\circ$$

$$= 0.899 \text{ (lag)}$$

Phasor diagram

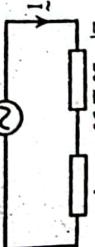
Taking voltage as ref. phasor



38. A rms voltage of $100 \angle 0^\circ$ is applied to the series combination of Z_1 and Z_2 where $Z_1 = 20 \angle 30^\circ$. The effective voltage drop across Z_2 is known to be $40 \angle -30^\circ$. Find the reactive component of Z_2 .

Solution:

$$Z_1 = 20 \angle 30^\circ \quad Z_2$$



$$\tilde{V} = 100 \angle 0^\circ$$

Given, $\tilde{V}_{z_2} = 40 \angle -30^\circ \text{ V}$

By KVL

$$\tilde{V} = \tilde{V}_{z_1} + \tilde{V}_{z_2}$$

or, $100 \angle 0^\circ = \tilde{V}_1 + 40 \angle -30^\circ$

$$\therefore \tilde{V}_1 = (100 \angle 0^\circ) - (40 \angle -30^\circ)$$

$$= 68.351 \angle 17.014^\circ V$$

$$\tilde{V}_{z_1} = \tilde{I} Z_1$$

$$\therefore \tilde{I} = \frac{\tilde{V}_{z_1}}{Z_1} = \frac{68.351 \angle 17.014^\circ}{20 \angle 30^\circ} = 3.418 \angle -12.98^\circ A$$

$$\tilde{V}_{z_2} = \tilde{I} Z_2$$

$$\therefore Z_2 = \frac{\tilde{V}_{z_2}}{\tilde{I}} = \frac{40 \angle -30^\circ}{3.418 \angle -12.98^\circ} = 11.192 - j 3.424 \Omega$$

\therefore The reactive component of Z_2 is -3.424Ω .

39. A series circuit consists of resistance equal to 4Ω and inductance of $0.01H$. The applied voltage is $283 \sin(300t + 90^\circ) V$. Calculate the followings:

- Power factor
- Expression for $i(t)$
- The power dissipated in the circuit
- Voltage drop across each elements and

(v) Draw a phasor diagram [2068 Baisakh, 2071 Shawal]

Solution:

Given,

$$v = 283 \sin(300t + 90^\circ)$$

$$V_{rms} = \frac{283}{\sqrt{2}} = 200.11 V$$

$$\tilde{V} = 200.11 \angle 90^\circ V$$

Impedance of the circuit (Z) = $R + jX_L$

$$= R + j\omega L$$

$$= 4 + j300 \times 0.01 = 4 + j3 \Omega$$

$$\text{Now, } \tilde{I} = \frac{\tilde{V}}{Z} = \frac{200.11 \angle 90^\circ}{4 + j3} = 40.02 \angle 53.13^\circ A$$

$$\therefore I_m = \sqrt{2} I_{rms} = \sqrt{2} \times 40.02 = 56.6 A$$

$$\therefore i(t) = 56.6 \sin(300t + 53.13^\circ) A$$

$$P.f = \cos\phi = \cos[90^\circ - 53.13^\circ]$$

$$= \cos 36.87^\circ = 0.8 (\text{lag})$$

$$\text{Power dissipated in the circuit (P)} = V I \cos\phi$$

$$= 200.11 \times 40.02 \times 0.8$$

$$= 6406.71 \text{ watts}$$

Voltage drop across resistor $\tilde{V}_R = \tilde{I} R$

$$= (40.02 \angle 53.13^\circ) \times 4$$

$$= 160.08 \angle 53.13^\circ V$$

$$\begin{aligned} \text{Voltage drop across inductor } \tilde{V}_L &= \tilde{I} (jX_L) \\ &= (40.02 \angle 53.13^\circ) (j3) \\ &= 120.06 \angle 143.13^\circ V \end{aligned}$$

Phasor diagram



40. Define cycle, time period, angular velocity, frequency, average and rms value of an alternating quantity. [Please refer to the theory]
41. In a purely inductive circuit when excited by a sinusoidal voltage, show mathematically and graphically, that the current lags the applied voltage by 90° and also show the average power consumed in the inductor is zero. [2068 Baisakh]

[Please refer to the theory]

42. An emf $e_0 = 141.4 \sin(377t + 30^\circ)$ is impressed on the impedance coil having a resistance of 4Ω and an inductive reactance of 1.25Ω measured at 25 Hz . What is the equation of the current? Also find the equation for the resistive drop e_R and inductive drop e_L . [2067 Mangir]

[2068 Baisakh]

Solution:

Given,

$$e_0 = 141.4 \sin(377t + 30^\circ)$$

$$\omega = 377$$

$$2\pi f = 377$$

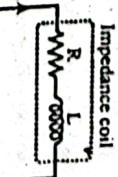
$$\therefore f = 60 \text{ Hz}$$

$$At 25 \text{ Hz}, X_L = 1.25 \Omega$$

$$At 60 \text{ Hz}, X_L = \frac{1.25}{25} \times 60$$

$$= 3\Omega$$

$$\therefore Z = R + jX_L = 4 + j3\Omega$$



$$E_{rms} = \frac{e_0}{\sqrt{2}} = 99.98 V = 100V$$

$$\therefore E_0 = 100 \angle 30^\circ V$$

$$\tilde{I} = \frac{\tilde{E}_0}{Z} = \frac{100 \angle 30^\circ}{4 + j3}$$

$$= 20 \angle -6.86^\circ A$$

$$I_m = 20\sqrt{2} = 28.28 A$$

$$\therefore i(t) = 28.28 \sin(377t - 6.86^\circ) A$$

$$E_R = \tilde{I} R$$

$$= (20 \angle -6.86^\circ)^2 4$$

$$= 80 \angle -6.86^\circ V$$

$$\begin{aligned}c_R(t) &= 80 \times \sqrt{2} \sin(377t - 6.86^\circ) \\&= 113.14 \sin(377t - 6.86^\circ) \text{ V}\end{aligned}$$

$$\bar{E}_L = \tilde{I}(jX_L)$$

$$\begin{aligned}&= 60 \angle 83.14^\circ \text{ V} \\&= (20 \angle -6.86^\circ)(j3)\end{aligned}$$

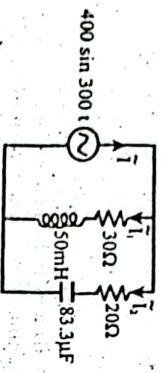
$$c_L(t) = 60 \times \sqrt{2} \sin(377t + 83.14^\circ)$$

$$= 84.85 \sin(377t + 83.14^\circ) \text{ V}$$

43. For the circuit below, calculate

- Magnitude and phase angles of current in each of the branches.
- Active, reactive, apparent power and power factor of the circuit, and
- Draw the vector diagram indicating branch currents and supply voltage.

[2067 Ashadh]



Solution:

Given,

$$\begin{aligned}v &= 400 \sin 300t \\V &= \frac{400}{\sqrt{2}} \angle 0^\circ = 282.84 \angle 0^\circ \text{ V}\end{aligned}$$

$$Z_1 = R_1 + jX_L = 30 + j300 \times 50 \times 10^{-3} = 30 + j15 \Omega$$

$$Z_2 = R_2 - jX_C = 20 - j \frac{1}{\omega C} = 20 - j \frac{1}{300 \times 83.3 \times 10^{-6}} = 20 - j40 \Omega$$

$$(i) \quad \tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{282.84 \angle 0^\circ}{30 + j15} = 8.43 \angle -26.56^\circ \text{ A}$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{282.84 \angle 0^\circ}{20 - j40} = 6.32 \angle 63.43^\circ \text{ A}$$

By KCL;

$$\begin{aligned}\tilde{I} &= \tilde{I}_1 + \tilde{I}_2 = (8.43 \angle -26.56^\circ) + (6.32 \angle 63.43^\circ) \\&= 10.54 \angle 10.32^\circ \text{ A}\end{aligned}$$

Phase difference between voltage and current
 $\phi = 0^\circ - 10.32^\circ = -10.32^\circ$

(ii) Active power of the circuit (P) = $VI \cos \phi$

$$\begin{aligned}&= 282.84 \times 10.54 \cos(-10.32^\circ) \\&= 2932.9 \text{ watts}\end{aligned}$$

Reactive power of the circuit (Q) = $VI \sin \phi$

$$\begin{aligned}&= 282.84 \times 10.54 \sin(-10.32^\circ) \\&= -534.05 \text{ VAR}\end{aligned}$$

(-ve sign indicates reactive power is being supplied by the circuit)

$$\begin{aligned}\text{Apparent power of the circuit } (S) &= \sqrt{P^2 + Q^2} = 2981.12 \text{ VA} \\ \text{Power factor of the circuit } P.F. &= \cos \phi = \cos(-10.32^\circ) = 0.9838 \text{ (lead)}\end{aligned}$$



44. A circuit consists of the following in parallel

- a resistor of 50Ω
- an inductance of 2H
- a capacitor of $10\mu\text{F}$

In this circuit a $200\text{V}, 50\text{Hz}$ source is applied calculate

- the total current drawn from the supply

- Complex power

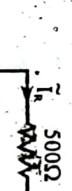
- Active power

- Reactive power

- Power factor of the circuit

Solution:

Given,



[2065 Kartik]

$$\begin{aligned}X_L &= 2\pi f L = 2\pi \times 50 \times 2 = 628.32\Omega \\X_C &= \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318.31\Omega\end{aligned}$$

$$(A) \quad \tilde{I}_R = \frac{\tilde{V}}{R} = \frac{200 \angle 0^\circ}{500} = 0.4 \angle 0^\circ \text{ A}$$

$$\tilde{I}_L = \frac{\tilde{V}}{jX_L} = \frac{200 \angle 0^\circ}{j628.32} = 0.318 \angle -90^\circ \text{ A}$$

$$\tilde{I}_C = \frac{\tilde{V}}{-jX_C} = \frac{200 \angle 0^\circ}{-j318.31} = 0.628 \angle 90^\circ \text{ A}$$

Total current drawn from the supply

$$\begin{aligned}\tilde{I} &= \tilde{V}_R + \tilde{V}_L + \tilde{V}_C = (0.4 \angle 0^\circ) + (0.318 \angle -90^\circ) + (0.628 \angle 90^\circ) \\ &= 0.506 \angle 37.77^\circ A\end{aligned}$$

(B) Complex power is defined as

$$\begin{aligned}S &= \tilde{V} \tilde{I}^* \\ &= (200 \angle 0^\circ) \times (0.506 \angle -37.77^\circ) \\ &= 79.99 - j 61.98 \\ &= 101.2 \angle -37.77^\circ \text{ VA}\end{aligned}$$

[Note: $I = 0.506 \angle 37.77^\circ A$

$$I^* = 0.506 \angle -37.77^\circ A]$$

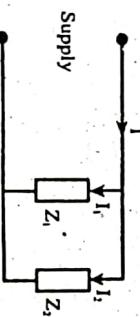
(C) Comparing $S = P + jQ$ with $S = 79.99 - j 61.98$

(D) Active power (P) = 79.99 watts
(-ve sign indicates capacitive circuit)

(E) Reactive power (Q) = -61.98 VAR

Two impedances in the circuit shown in figure below are $Z_1 = (1k + j2.7k)\Omega$ and $Z_2 = (790 - j1.6k)\Omega$. The total current taken from the supply is 15 mA.

Calculate the two branch currents. [2006 Magh]



Solution:

$$\text{Given, } \tilde{V} = 15 \text{ mA} \angle 0^\circ = 15 \times 10^{-3} \angle 0^\circ A$$

$$Z_1 = 1000 + j 2.7 \times 1000$$

$$= 1000 + j 2700 \Omega$$

$$Z_2 = 790 - j 1.6 \times 1000$$

$$= 790 - j 1600 \Omega$$

Total impedance $Z := Z_1 \parallel Z_2$

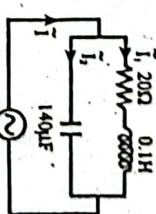
$$\begin{aligned}Z &= \frac{Z_1 Z_2}{Z_1 + Z_2} \\ &= \frac{(1000 + j 2700)(790 - j 1600)}{(1000 + j 2700) + (790 - j 1600)} \\ &= 2205.02 - j 1057.28 \Omega\end{aligned}$$

$$\therefore \tilde{V} = \tilde{I}Z = (15 \times 10^{-3} \angle 0^\circ)(2205.02 - j 1057.28) = 36.68 \angle -25.62^\circ V$$

$$\therefore \tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{36.68 \angle -25.62^\circ}{1000 + j 2700} = 0.0127 \angle -95.29^\circ = 12.74 \angle -95.29^\circ \text{ mA}$$

$$\therefore \tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{36.68 \angle -25.62^\circ}{790 - j 1600} = 0.0205 \angle 38.102^\circ = 20.55 \angle 38.102 \text{ mA}$$

46. A coil having resistance of 20Ω and inductance of $0.1H$, connected in parallel with a $140 \mu F$ capacitor is supplied by $100 V$, 60 Hz sinusoidal source. Find active power, reactive power, apparent power and power factor of the circuit and draw the phasor diagram. [2005 Shrawan]



Solution:

$$\text{Given, } \tilde{V} = 100 \angle 0^\circ V$$

$$Z_1 = R + j X_L = 20 + j 2\pi f L = 20 + j 2\pi \times 60 \times 0.1 = 20 + j 37.699 \Omega$$

$$Z_2 = -j X_C = -j \frac{1}{\omega C} = -j \frac{1}{2\pi f C}$$

$$= -j \frac{1}{2\pi \times 60 \times 140 \times 10^{-6}} = -j 18.95 \Omega$$

$$\text{Now, } \tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{100 \angle 0^\circ}{20 + j 37.699} = 2.34 \angle -62.05^\circ A$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{100 \angle 0^\circ}{-j 18.95} = 5.28 \angle 90^\circ A$$

$$\therefore \tilde{I} = \tilde{I}_1 + \tilde{I}_2 = (2.34 \angle -62.05) + (5.28 \angle 90^\circ)$$

$$= 3.39 \angle 71.15^\circ A$$

Power factor = p.f. = $\cos \phi$

$$\begin{aligned}&= \cos(0 - 71.15^\circ) \\ &= \cos(-71.15^\circ) \\ &= 0.323 (\text{lead})\end{aligned}$$

$$\text{Active power (P)} = VI \cos \phi$$

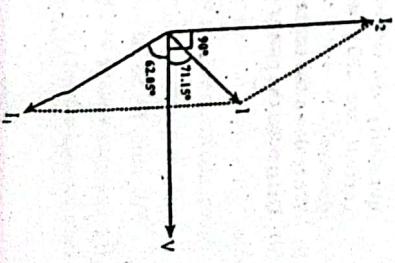
$$= 100 \times 3.39 \times 0.323 = 109.5 \text{ W}$$

$$\text{Reactive power (Q)} = VI \sin \phi$$

$$= 100 \times 3.39 \times \sin(-71.15^\circ)$$

$$= -320.82 \text{ VAR}$$

$$\text{Apparent power (S)} = VI$$



$$\begin{aligned} &= 100 \times 3.39 \\ &= 339 \text{ VA} \end{aligned}$$

47. In a R - L - C series circuit has 20Ω resistor, 30 mH inductor and $100\mu\text{F}$ capacitor. The supply voltage is given as $v(t) = 100\sqrt{2} \sin 1000t$. Calculate the phase and magnitude of current and voltages across all the elements in the circuit. [2062 Bhadra]

Solution:

$$\text{Given, } v(t) = 100\sqrt{2} \sin 1000t$$

$$\omega = 1000$$

$$V_m = 100\sqrt{2}$$

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}} = 100\text{V}$$

$$\therefore \tilde{V} = 100 \angle 0^\circ \text{ V}$$

$$\text{Impedance } Z = R + jX_L - jX_C$$

$$X_L = \omega L = 1000 \times 30 \times 10^{-3} = 30\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000 \times 100 \times 10^{-6}} = 10\Omega$$

$$\therefore Z = R + jX_L - jX_C = 20 + j30 - j10 = 20 + j20\Omega$$

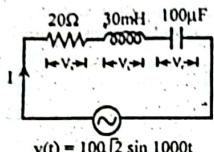
as $Z = R + jX_L$ hence the circuit is inductive,
also, ($X_L > X_C$)

$$\therefore \tilde{I} = \frac{\tilde{V}}{Z} = \frac{100 \angle 0^\circ}{20 + j20} = 3.53 \angle -45^\circ \text{ A}$$

$$\therefore \tilde{V}_R = \tilde{I}R = (3.53 \angle -45^\circ) \times 20 = 70.6 \angle -45^\circ \text{ V}$$

$$\therefore \tilde{V}_L = \tilde{I}(jX_L) = (3.53 \angle -45^\circ) \times (j30) = 105.9 \angle 45^\circ \text{ V}$$

$$\therefore \tilde{V}_C = \tilde{I}(-jX_C) = (3.53 \angle -45^\circ) \times (-j10) = 35.3 \angle -135^\circ \text{ V}$$



Additional Problems

- The maximum values of the alternating voltage and current are 400V and 20A respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of voltage and current are 283V and 10 A respectively at $t = 0$ both increasing positively.
 - Write down the expressions for voltage and current at time t .
 - Determine the power consumed in the circuit.

Solution,

$$\text{Given, maximum voltage } V_{max} = 400\text{V}$$

$$\text{maximum current } I_{max} = 20\text{A}$$

$$\text{Frequency } f = 50\text{ Hz}$$

$$\text{Instantaneous voltage } v = 283\text{V at } t = 0$$

$$\text{Instantaneous current } i = 10\text{ A at } t = 0$$

Let ϕ be the phase difference w.r.t the point corresponding to $t = 0$.

We have expression of voltage as

$$v = V_{max} \sin(\omega t + \phi)$$

$$\text{or, } 283 = 400 \sin(\omega t + \phi)$$

$$\text{or, } 283 = 400 \sin \phi$$

$$\therefore \phi = \sin^{-1}\left(\frac{283}{400}\right) = 45.03^\circ$$

So, expression of voltage at time t

$$v = 400 \sin(\omega t + \phi)$$

$$= 400 \sin(2\pi ft + \phi)$$

$$= 400 \sin(2\pi \times 50 t + 45.03^\circ)$$

$$\therefore v = 400 \sin(314 t + 45.03^\circ)$$

Again, expression for current

$$i = I_{max} \sin(\omega t + \phi)$$

$$\text{or, } 10 = 20 \sin(\omega t + \phi)$$

$$\text{or, } 10 = 20 \sin \phi$$

$$\therefore \phi' = \sin^{-1}\left(\frac{10}{20}\right) = 30^\circ$$

So, expression of current at time t

$$i = I_{max} \sin(2\pi ft + \phi')$$

$$= 20 \sin(2\pi \times 50 t + 30^\circ)$$

$$\therefore i = 20 \sin(314 t + 30^\circ)$$

Power consumed in the circuit,

$$P = V_{rms} I_{rms} \cos \phi$$

$$\therefore V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{400}{\sqrt{2}} = 282.84 \text{ V}$$

$$\therefore I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.14 \text{ A}$$

$$\begin{aligned} \phi &= \text{phase difference between voltage and current} \\ &= 45.03^\circ - 30^\circ = 15.03^\circ \end{aligned}$$

$$P = 282.84 \times 14.14 \times \cos 15.03^\circ = 3862.54 \text{ W}$$

- A circuit of 20Ω resistance in series with capacitance of $200\mu\text{F}$, connected across 50 Hz supply. The current through the circuit is $10.8 \sin 314 t \text{ A}$. Determine the voltage across each component and across the circuit.

Solution:

Let v be the voltage across the circuit.

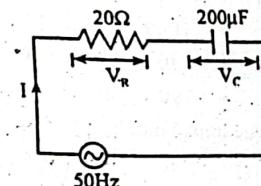
$$i = 10.8 \sin 314t$$

$$i = \frac{10.8}{\sqrt{2}} \angle 0^\circ = 7.64 \angle 0^\circ \text{ A}$$

$$\begin{aligned} X_C &= \frac{1}{\omega C} = \frac{1}{2\pi f \times C} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} \\ &= 15.91 \Omega \end{aligned}$$

impedance of the circuit

$$Z = R - jX_C = 20 - j15.91 \Omega$$



$$\tilde{V} = \tilde{I} Z = (7.64 \angle 0^\circ) \times (20 - j15.91) = 195.25 \angle -38.502^\circ V$$

Now, Voltage across resistance

$$\tilde{V}_R = \tilde{I} R = (7.64 \angle 0^\circ) \times (20) = 152.8 \angle 0^\circ V$$

Voltage across capacitance

$$\tilde{V}_C = \tilde{I} (-jX_C) = (7.64 \angle 0^\circ) \times (-j15.91) = 121.55 \angle -90^\circ V$$

3. A series R - C circuit takes a power of 7,000 W when connected to 200V, 50 Hz supply. The voltage across the resistor is 130 V. Calculate (i) the resistance and impedance (ii) Also write equations for v(t) and i(t).

Solution:

Given, Power (P) = 7,000 W

Voltage (V) = 200 V

Voltage across resistor (V_R) = 130 V

From phasor diagram,

$$\cos\phi = \frac{V_R}{V} = \frac{130}{200} = 0.65 \text{ (leading)}$$

$$\therefore \phi = \cos^{-1}(0.65) = 49.46^\circ$$

We know, $P = VI \cos\phi$

$$7,000 = 200 \times I \times 0.65$$

$$\therefore I = \frac{7000}{200 \times 0.65} = 53.84 A$$

$$\text{Resistance } R = \frac{V_R}{I} = \frac{130}{53.84} = 2.41 \Omega$$

$$\text{Impedance } Z = \frac{V}{I} = \frac{200}{53.84} = 3.71 \Omega$$

Capacitive reactance

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(3.71)^2 - (2.41)^2} = 2.82 \Omega$$

$$\text{We know, } X_C = \frac{1}{\omega C}$$

$$\text{or, } 2.82 = \frac{1}{2\pi \times 50 \times C}$$

$$\text{or, } C = \frac{1}{2\pi \times 50 \times 2.82}$$

$$\therefore \text{Capacitance } (C) = 1128.75 \mu F$$

Equation for voltage,

$$v(t) = V_{max} \sin(\omega t + \phi) = 200 \times \sqrt{2} \sin(2\pi \times 50t + 0^\circ)$$

[Taking voltage as reference thus $\phi = 0$ at time t]

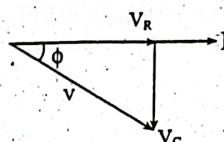
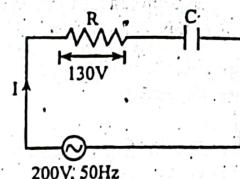
$$= 200 \times \sqrt{2} \sin 314t$$

$$\therefore v(t) = 282.84 \sin 314t$$

Equation for current,

$$i(t) = I_{max} \times \sin(\omega t + \phi) = 53.84 \times \sqrt{2} \sin(2\pi \times 50t + 49.46^\circ)$$

$$= 76.14 \sin(314t + 49.46^\circ)$$



4. An alternating current of 1.5 A flows in a circuit when applied voltage is 300V. The power consumed is 225 W. Find resistance and reactance of the circuit.

Solution:

Current (I) = 1.5 A

Voltage (V) = 300 V

Power consumed (P) = 225 W

$P = VI \cos\phi$

$$\text{or, } 225 = 300 \times 1.5 \times \cos\phi$$

$$\text{or, } \cos\phi = \frac{225}{300 \times 1.5} = 0.5$$

$$\therefore \phi = \cos^{-1}(0.5) = 60^\circ$$

Impedance of the circuit

$$Z = \frac{V}{I} = \frac{300}{1.5} = 200 \Omega$$

$$\text{Resistance } (R) = Z \cos\phi = 200 \times 0.5 = 100 \Omega$$

$$\text{Reactance } (X) = \sqrt{Z^2 - R^2} = \sqrt{(200)^2 - (100)^2} = 173.2 \Omega$$

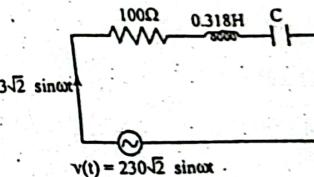
5. A series RLC circuit consists of a 100Ω resistor, an inductor of 0.318H and capacitor of unknown value. When the circuit is energised by $230\sqrt{2} \sin \omega t$ volt supply, the current was found to be $i = 2.3\sqrt{2} \sin \omega t$ amperes. Find.

(i) the value of the capacitance

(ii) the voltage across the inductor

(iii) the total power consumed, assume $\omega = 314.5$

Solution:



$$v(t) = 230\sqrt{2} \sin \omega t$$

$$\tilde{V} = \frac{230\sqrt{2}}{\sqrt{2}} \angle 0^\circ = 230 \angle 0^\circ V$$

$$i(t) = 2.3\sqrt{2} \sin \omega t$$

$$\tilde{I} = \frac{2.3\sqrt{2}}{\sqrt{2}} \angle 0^\circ = 2.30 \angle 0^\circ A$$

Impedance of the circuit

$$Z = \frac{\tilde{V}}{\tilde{I}} = \frac{230 \angle 0^\circ}{2.3 \angle 0^\circ} = 100 \Omega$$

We have, Resistance (R) = 100Ω

$$\text{Inductive reactance } (X_L) = \omega L = 314.5 \times 0.318 = 100.01 \Omega = 100 \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{or, } 100 = \sqrt{(100)^2 + (100 - X_c)^2}$$

$$\text{or, } (100)^2 = (100)^2 + (100 - X_c)^2$$

$$\therefore X_c = 100 \Omega$$

$$\text{Capacitive reactance } X_c = \frac{1}{\omega C}$$

$$\text{or, } C = \frac{1}{\omega X_c} = \frac{1}{314.5 \times 100}$$

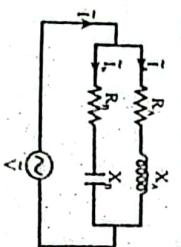
$$\therefore \text{Capacitance } (C) = 31.8 \mu F$$

Voltage across inductor.

$$\tilde{V}_L = I j X_L = (2.3 \angle 0^\circ) \times (j100) = 230 \angle 90^\circ V$$

$$\text{Power consumed } (P) = VI \cos \phi = 230 \times 2.3 \times \cos 0^\circ = 529 W$$

6. The currents in each branch of a two branched parallel circuit are given by expressions $i_a = 7.07 \sin(314t - \frac{\pi}{4})$ and $i_b = 21.2 \sin(314t + \frac{\pi}{3})$. The supply voltage is given by $v = 354 \sin 314 t$. Calculate supply current and the ohmic values of components assuming two pure components in each branch. State whether the reactive components are inductive or capacitive.



Solution:

Given,

$$v = 354 \sin 314t$$

$$\tilde{V} = \frac{354}{\sqrt{2}} \angle 0^\circ = 250.3 \angle 0^\circ V$$

$$i_a = 7.07 \sin(314t - \frac{\pi}{4})$$

$$\tilde{i}_a = \frac{7.07}{\sqrt{2}} \angle -\frac{\pi}{4} = 4.99 \angle -45^\circ$$

7. The circuits with impedances $Z_1 = (10 + j15) \Omega$ and $Z_2 = (6 - j8) \Omega$ are connected in parallel. If the supply current is 20A, what is the power dissipated in each branch.

Solution:

Given,

$$Z_1 = 10 + j15 \Omega$$

$$Z_2 = 6 - j8 \Omega$$

Let,

$$i_b = 21.2 \sin(314t + \frac{\pi}{3})$$

$$\tilde{i}_b = \frac{21.2}{\sqrt{2}} \angle \frac{\pi}{3} = 14.99 \angle 60^\circ = 15 \angle 60^\circ A$$

Supply current,

$$\tilde{I} = \tilde{I}_a + \tilde{I}_b = (5 \angle -45^\circ) + (15 \angle 60^\circ) = 14.53 \angle 40.59^\circ A$$

$$\text{Expression for supply current,}$$

$$i = 14.53 \times \sqrt{2} \sin(314t + 40.59^\circ) = 20.54 \sin(314t + 40.59^\circ) A$$

Impedance of branch A,

$$Z_A = \frac{\tilde{V}}{\tilde{I}_a} = \frac{250.3 \angle 0^\circ}{5 \angle -45^\circ} = 35.39 + j35.39 \Omega$$

$$\text{Since impedance is in form } R + jX;$$

$$\text{Resistance of branch A}$$

$$R_A = 35.39 \Omega$$

$$\text{Reactance of branch A,}$$

$$X_A = 35.39 \Omega$$

Thus, above reactance is inductive in nature as $Z = R + jX_L$.

Impedance of branch B,

$$Z_B = \frac{\tilde{V}}{\tilde{I}_b} = \frac{250.3 \angle 0^\circ}{15 \angle 60^\circ} = 8.34 - j14.45 \Omega$$

$$\text{Since impedance is in form } R - jX;$$

$$\text{Resistance of branch B}$$

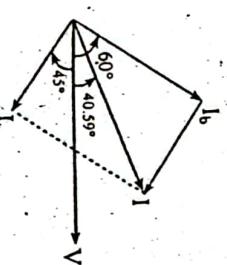
$$R_B = 8.34 \Omega$$

$$\text{Reactance of branch B}$$

$$X_B = 14.45 \Omega$$

Thus, above reactance is capacitive in nature as $Z = R - jX_C$.

Phasor diagram,



- Given,
- $Z_1 = 10 + j15 \Omega$
- $Z_2 = 6 - j8 \Omega$
- Let,
- $\tilde{I} = 20 \angle 0^\circ A$
- Total impedance of the circuit

$$Z = Z_1 \parallel Z_2 = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

$$= \frac{(10 + j15) \times (6 - j8)}{(10 + j15) + (6 - j8)} = 9.67 - j3.61 \Omega$$

Now, $\tilde{V} = \tilde{I}Z$
 $= (20\angle 0^\circ) \times (9.67 - j3.61) = 206.44 \angle -20.47^\circ V$

Branch currents,

$$\tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{206.44 \angle -20.47^\circ}{10 + j15} = 11.45 \angle -76.78^\circ A$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{206.44 \angle -20.47^\circ}{6 - j8} = 20.64 \angle 32.66^\circ A$$

Power dissipated in branch 1,

$$P_1 = V I_1 \cos \phi_1$$

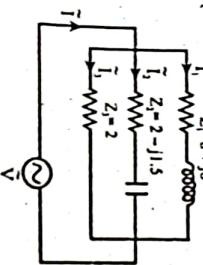
$$= 206.44 \times 11.45 \times \cos(-20.47^\circ - (-76.78^\circ)) = 1311.16 W$$

Power dissipated in branch 2,

$$P_2 = V I_2 \cos \phi_2$$

$$= 206.44 \times 20.64 \times \cos(-20.47^\circ - 32.66^\circ) = 2556.56 W$$

8. Three impedances $Z_1 = (8 + j6)\Omega$, $Z_2 = (2 - j1.5)\Omega$ and $Z_3 = 2\Omega$ are connected in parallel across a 50 Hz supply. If the current through Z_1 is $(3 + j4) A$, calculate the current through the other impedances and also the power absorbed by this parallel circuit.



Given, $I_1 = 3 + j4 A = 5\angle 53.13^\circ A$

$$Z_1 = 8 + j6 \Omega$$

$$Z_2 = 2 - j1.5 \Omega$$

$$Z_3 = 2 \Omega$$

Now,

$$\tilde{V} = \tilde{I}_1 Z_1 = (3 + j4) \times (8 + j6) = j50 = 50\angle 90^\circ V$$

Current through Z_2 ,

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{50\angle 90^\circ}{2 - j1.5} = (-12 + j16) = 20 \angle 126.87^\circ A$$

Current through Z_3

$$\tilde{I}_3 = \frac{\tilde{V}}{Z_3} = \frac{50\angle 90^\circ}{2} = j25 = 25\angle 90^\circ A$$

Supply current

$$\tilde{I} = \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 \\ = (3 + j4) + (-12 + j16) + (j25) = -9 + j45 = 45.89 \angle 101.31^\circ A$$

Power absorbed by the circuit,

$$P = VI \cos \phi = 50 \times 45.89 \times \cos(90^\circ - 101.31^\circ) = 2249.94 W$$

9. In the circuit shown in figure calculate

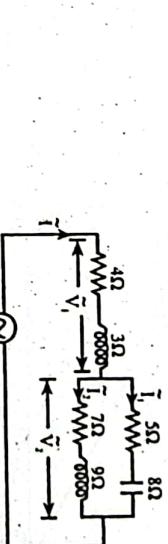
(i) the impedance of the entire circuit

(ii) total current

(iii) overall power factor

(iv) current in each parallel branch

Also draw the phasor diagram showing applied voltage and various currents.



Solution:

$$\tilde{V} = 220 \angle 0^\circ V$$

Impedance of the entire circuit,

$$Z = (4 + j3) + [(5 - j8) \parallel (7 + j9)]$$

$$= (4 + j3) + \left[\frac{(5 - j8) \times (7 + j9)}{(5 - j8) + (7 + j9)} \right]$$

$$= (4 + j3) + (8.78 - j1.65) = 12.78 + j1.13 \Omega$$

Total current

$$\tilde{I} = \frac{\tilde{V}}{Z}$$

$$= \frac{220 \angle 0^\circ}{12.78 + j1.13} = 17.12 \angle -6.03^\circ A$$

Power factor,

$$pf = \cos \phi = \cos(0^\circ - (-6.03^\circ)) = 0.994 \text{ (lag)}$$

Voltage drop across $(4 + j3) \Omega$,

$$\begin{aligned} \tilde{V}_1 &= \tilde{I}(4 + j3) \\ &= (17.12 \angle -6.03^\circ) \times (4 + j3) = 85.6 \angle 30.84^\circ V \end{aligned}$$

Voltage drop across parallel branch.

$$\tilde{V}_i = \tilde{V} - \tilde{V}_1 = (220 \angle 0^\circ) - (85.6 \angle 30.84^\circ)$$

$$= 152.93 \angle -16.67^\circ V$$

Current in branch $Z_1 = (5 - j8) \Omega$

$$\tilde{I}_1 = \frac{\tilde{V}_i}{Z_1} = \frac{152.93 \angle -16.67^\circ}{(5 - j8)}$$

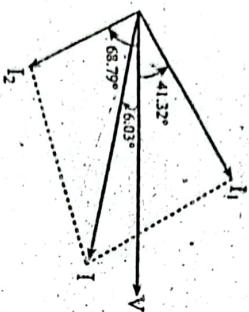
$$= 16.21 \angle 41.32^\circ A$$

Current in branch $Z_2 = (7 + j9) \Omega$

$$\tilde{I}_2 = \frac{\tilde{V}_i}{Z_2} = \frac{152.93 \angle -16.67^\circ}{(7 + j9)}$$

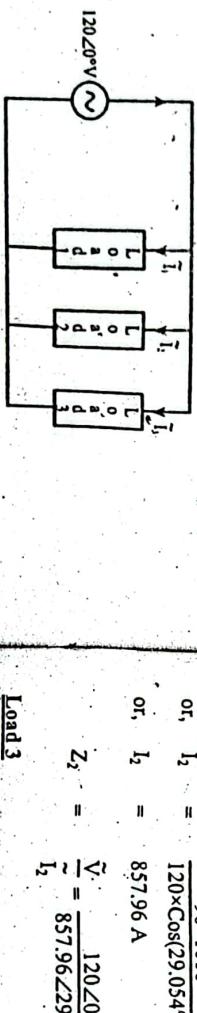
$$= 13.41 \angle -68.79^\circ A$$

Phasor diagram



10. Three loads are connected in parallel to a $120 \angle 0^\circ$ Vrms source. Load 1 absorbs 60 k VAR at $\text{pf} = 0.85$ lagging, Load 2 absorbs 90 kW and 50 k VAR leading, and load 3 absorbs 100 kW at $\text{pf} = 1$.
- Find the equivalent impedance.
 - Calculate the power factor of the parallel combination.
 - Determine the current supplied by the source.

Solution:



Given, Load 1 absorbs 60 k VAR at $\text{pf} = 0.85$ lagging

Load 2 absorbs 90 kW and 50 k VAR leading

Load 3 absorbs 100 kW at $\text{pf} = 1$

Load 1:

$$Q_1 = 60 \text{ k VAR}, \text{pf}_1 = 0.85 (\text{lag})$$

$$\text{Cos}\phi_1 = 0.85$$

$$\therefore \phi_1 = \text{Cos}^{-1}(0.85) = 31.79^\circ$$

$$Q_1 = V_1 \text{ Sin}\phi_1$$

$$\text{or, } 60 \text{ kVAR} = 120 \times I_1 \times \text{Sin}(31.79^\circ)$$

$$\text{or, } I_1 = \frac{60 \times 1000}{120 \times \text{Sin}(31.79^\circ)}$$

$$\therefore I_1 = 949.11 A$$

$$Z_1 = \frac{\tilde{V}}{I} = \frac{120 \angle 0^\circ}{949.11 \angle -31.79^\circ} = 0.107 + j0.066 \Omega$$

Load 2

$$P_2 = 90 \text{ kW}, Q_2 = 50 \text{ k VAR leading}$$

$$P_2 = V I_2 \text{ Cos}\phi_2$$

$$\text{or, } 90 \times 1000 = 120 \times I_2 \times \text{Cos}\phi_2 \quad \dots \dots \dots \text{(i)}$$

$$Q_2 = V I_2 \text{ Sin}\phi_2$$

$$\text{or, } 50 \times 1000 = 120 \times I_2 \times \text{Sin}\phi_2 \quad \dots \dots \dots \text{(ii)}$$

Dividing (ii) by (i).

$$\frac{50 \times 1000}{90 \times 1000} = \frac{120 \times I_2 \times \text{Sin}\phi_2}{120 \times I_2 \times \text{Cos}\phi_2}$$

$$\text{or, } \frac{5}{9} = \tan\phi_2$$

$$\text{or, } \phi_2 = \tan^{-1}(5/9)$$

$$\therefore \phi_2 = 29.054^\circ$$

$$P_2 = V I_2 \text{ Cos}\phi_2$$

$$\text{or, } 90 \times 1000 = 120 \times I_2 \times \text{Cos}(29.054^\circ)$$

$$\text{or, } I_2 = \frac{90 \times 1000}{120 \times \text{Cos}(29.054^\circ)}$$

$$\text{or, } I_2 = 857.96 A$$

$$\text{Load 3}$$

$$Z_3 = \frac{\tilde{V}}{I_3} = \frac{120 \angle 0^\circ}{857.96 \angle 29.054^\circ} = 0.122 - j0.067 \Omega$$

$$\begin{aligned}P_3 &= 100 \text{ kW} \\ \text{or, } V I_3 \cos \phi_3 &= 100 \times 1000 \\ \text{or, } 120 \times I_3 \times 1 &= 100 \times 1000 \\ \text{or, } I_3 &= \frac{100 \times 1000}{120} \\ \therefore I_3 &= 833.33 \text{ A}\end{aligned}$$

$$\begin{aligned}Z_3 &= \frac{\tilde{V}}{I_3} = \frac{120 \angle 0^\circ}{833.33 \angle 0^\circ} \\ &= 0.144 \Omega.\end{aligned}$$

a) Total impedance = ?

$$\begin{aligned}\text{total supply current } \tilde{I} &= \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 \\ &= (949.11 \angle -31.79^\circ) + (857.96 \angle 29.054^\circ) + (833.33 \angle 0^\circ) \\ &= 2391.51 \angle -1.997^\circ \text{ A}\end{aligned}$$

$$\text{total impedance } Z = \frac{\tilde{V}}{\tilde{I}}$$

$$\begin{aligned}&= \frac{120 \angle 0^\circ}{2391.51 \angle -1.997^\circ} \\ &= 0.05014 + j1.7485 \times 10^{-3} \Omega \\ &= 50.14 + j1.7485 \text{ m}\Omega\end{aligned}$$

(b) Power factor of parallel combination

$$\begin{aligned}\text{pf} &= \cos \phi \\ &= \cos(0^\circ - (-1.997^\circ)) \\ &= 0.9994 \text{ (lagging)}\end{aligned}$$

(c) Current supplied by source

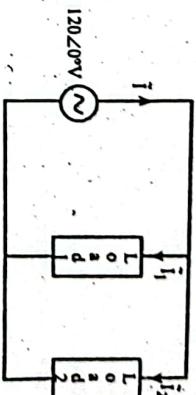
$$\begin{aligned}\tilde{I} &= 2391.51 \angle -1.997^\circ \text{ A} \\ &= 2.392 \angle -1.997^\circ \text{ kA}\end{aligned}$$

11. Two loads connected in parallel draw a total of 2.4 kW at 0.8 pf lagging from a 120-V rms, 60-Hz line. One load absorbs 1.5 kW at a 0.707 pf lagging. Determine:

(a) The pf of the second load.

- (b) The parallel element required to correct the pf to 0.9 lagging for the two loads.

Solution:



Given,

120 Vrms, 60 Hz line
Total power P = 2.4 kW at pf = 0.8 lagging

$$\text{Load } 1 P_1 = 1.5 \text{ kW}, \text{pf}_1 = 0.707 \text{ lagging}$$

$$\text{pf} = \cos \phi$$

$$\text{or, } \phi = \cos^{-1}(0.8)$$

$$\therefore \phi = 36.86^\circ$$

$$P = V I \cos \phi$$

$$\text{or, } 2.4 \times 1000 = 120 \times I \times \cos(36.86^\circ)$$

$$\text{or, } I = \frac{120 \times 1000}{2.4 \times \cos(36.86^\circ)}$$

$$\therefore I = 24.99 \approx 25 \text{ A}$$

$$\therefore \tilde{I} = 25 \angle -36.86^\circ \text{ A}$$

$$\text{pf}_1 = 0.707$$

$$\text{or, } \cos \phi_1 = 0.707$$

$$\text{or, } \phi_1 = \cos^{-1}(0.707)$$

$$\therefore \phi_1 = 45.01^\circ$$

$$\begin{aligned}P_1 &= V I_1 \cos \phi_1 \\ \text{or, } 1.5 \times 1000 &= 120 \times I_1 \times \cos(45.01^\circ)\end{aligned}$$

$$\text{or, } I_1 = \frac{1.5 \times 1000}{120 \times \cos(45.01^\circ)}$$

$$\therefore I_1 = 17.68 \text{ A}$$

$$\therefore \tilde{I}_1 = 17.68 \angle -45.01^\circ \text{ A}$$

Using KCL,

$$\tilde{I} = \tilde{I}_1 + \tilde{I}_2$$

$$\text{or, } \tilde{I}_2 = \tilde{I} - \tilde{I}_1$$

$$= (25\angle-36.86^\circ) - (17.68\angle-45.01^\circ)$$

$$\therefore \tilde{I}_2 = 7.92\angle-18.37^\circ \text{ A}$$

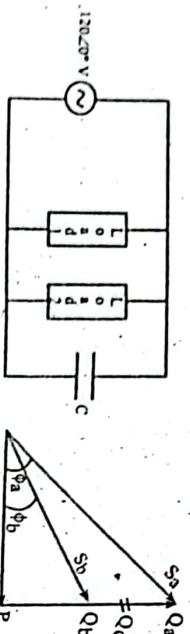
$$\text{pf}_2 = \cos\phi_2 = \cos(0^\circ - (-18.37^\circ))$$

$$= 0.949 \text{ (lagging)}$$

The pf of the second load = 0.949 (lagging)

Now,

To correct the pf to 0.9 lagging for the two loads capacitor must be connected.



Reactive power supplied by capacitor

$$Q_c = Q_a - Q_b$$

$$= P \tan \phi_a - P \tan \phi_b$$

$$= P (\tan \phi_a - \tan \phi_b)$$

$$[\phi_a = 36.86^\circ \text{ & pf} = 0.9]$$

$$\phi_b = \cos^{-1}(0.9) = 25.84^\circ]$$

$$Q_2 = 2.4 \times 1000 (\tan(36.86^\circ) - \tan(25.84^\circ))$$

$$= 637.08 \text{ VAR.}$$

We know,

$$C = \frac{Q_2}{\omega V^2} = \frac{637.08}{2\pi \times f \times V^2}$$

$$= \frac{637.08}{2\pi \times 60 \times (120)^2} = 1.1735 \times 10^{-4} \text{ F}$$

$$= 117.35 \mu\text{F}$$

The required capacitance is 177.35 μF .



7

THREE PHASE CIRCUIT ANALYSIS

Generation of a Three Phase Supply

When three identical coils are placed with their axes at 120° apart from each other in the presence of uniformly rotating magnetic field, a sinusoidal voltage is generated across each coil according to Faraday's law of electromagnetism. This is the working principle of a three-phase generator. It has three sets of coils RR', YY' and BB' kept 120° apart from each other mounted on a rotor. When the rotor (magnet) is rotated in clockwise direction with a constant angular velocity of ω rad/s, the flux linkage associated with the coil changes with respect to time.

Hence, according to Faraday's law of electromagnetism, emf will induce in all three coils.

Since, rotor is rotating with constant angular velocity ω , the generated voltages have same frequency. Also, since the coils are identical, the generated voltages have the same magnitudes, but there is a phase difference of 120° between these voltages.

The generated voltage (emfs) in the coils are given by

$$e_R = E_m \sin \omega t$$

$$e_Y = E_m \sin (\omega t - 120^\circ)$$

$$e_B = E_m \sin (\omega t - 240^\circ)$$

$$= E_m \sin (\omega t + 120^\circ) \quad [\text{Note: phase angle of } -240^\circ \text{ is same as } +120^\circ]$$

In polar form

$$\tilde{E}_R = E \angle 0^\circ$$

$$\tilde{E}_Y = E \angle -120^\circ$$

$$\tilde{E}_B = E \angle -240^\circ = E \angle +120^\circ$$

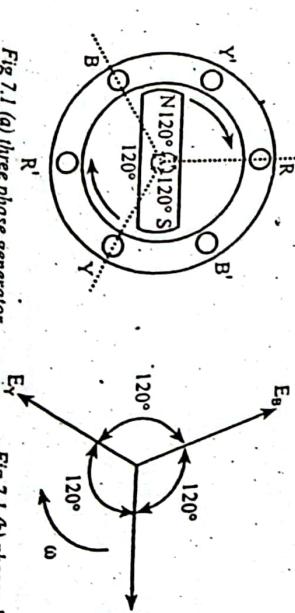


Fig 7.1 (a) three phase generator

Fig 7.1 (b) phasor diagram

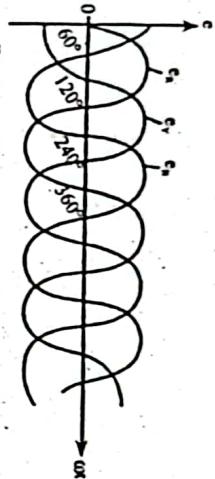


Fig 7.1 (c) Waveform of emfs.

Here, E_R leads E_V by 120° and E_V leads E_B by 120° . Also, emfs attain their maximum value in order E_{RR} , E_{VR} and E_{BR} .

Phase sequence /Phase order: The sequence in which the emf induced in the three - phase attain their peaks is known as phase sequence. For the arrangement shown in figure in which the coils are rotating in clockwise direction, the phase sequence is RYB. If the rotor is rotated in anticlockwise direction, the emfs attain their peaks in order E_{RB} , E_{VR} , E_{RR} . In this case phase sequence is BYR or RBY. Thus phase sequence determines direction of rotation.

Advantages of three phase system over single phase system.

- (1) A three - phase machine has a smaller size than a single - phase machine of the same power output.
- (2) The conductor material required to transmit a given power at a given voltage over a given distance by a three - phase system is less than that by an equivalent single phase system.
- (3) In single phase system the power delivered is pulsating and becomes zero at certain intervals. In three phase system power delivered to the load is constant though power of one phase may be negative.

Methods of Connections of Three - Phase system

There are two methods of interconnecting the three phases. They are called star and delta connections.

1. Star or Wye (Y) Connection

If three similar ends are connected together then they form star connected 3ϕ system. The free ends are connected to the external circuit through three conductors called lines. The point N is called neutral point where the three similar ends are connected and wire brought out from neutral point is called neutral wire. The three line conductors and a neutral wire provide a three-phase, four-wire supply. The neutral point is connected to ground.

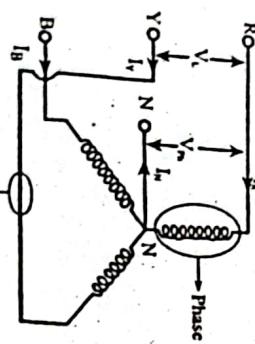
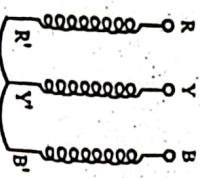
Fig 7.2 (a) 3ϕ 4 wire star connected system

Fig 7.2 (b) Star connection

Phase voltage— It is the voltage across a phase coil. In other words, it is the voltage between any line to neutral. It is denoted by V_{ph} or V_p .

In star connection,

Phase Current — I_R , I_Y , I_B

Line current — It is the current through phase coil. It is denoted by I_{ph} or I_p .

Line Voltage— V_{RN} , V_{YN} , V_{BN}

Line voltage— It is the voltage between two lines. It is denoted by V_L .

In Y connection,

Line current — I_R , I_Y , I_B

Line current — I_{ph} , I_p

Phase Current — I_R , I_Y , I_B

Phase Current — I_{ph} , I_p

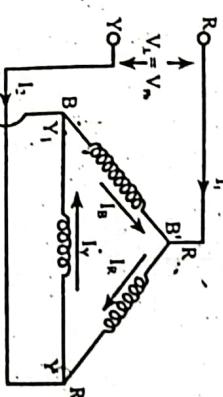
Phase Voltage— V_{ph} , V_{RN} , V_{YN} , V_{BN}

Line voltage— V_L

$$\boxed{I_L = I_{ph} \\ V_L \neq V_{ph}}$$

2. Delta or Mesh (Δ) Connection.

If the dissimilar ends are connected in such a way that they form a loop, the system is said to be 3ϕ delta connected system.

In Δ system,

Phase voltages — V_{RN} , V_{YN} , V_{BN}

Phase currents — I_R , I_Y , I_B

Line voltages — V_{RY} , V_{YB} , V_{BR}

Line current — I_L , I_1 , I_2

In Δ connection,

$$\boxed{V_L = V_{ph} \\ I_L \neq I_{ph}}$$

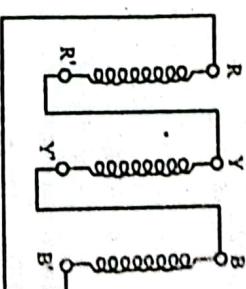


Fig 7.3 (b) Delta connection

In Δ system,

- Phase voltages — V_{RY} , V_{YB} , V_{BR}
- Phase currents — I_R , I_Y , I_B
- Line voltages — V_{RN} , V_{YN} , V_{BN}

In Δ connection,

$$\boxed{\begin{array}{l} V_L = V_{ph} \\ I_L \neq I_{ph} \end{array}}$$

Balanced System

A 3ϕ balanced system is one in which

- all phase voltages are equal in magnitude and displaced from one another by 120° .
- all phase currents are equal in magnitude and displaced from one another by 120° .

A 3ϕ balanced load is that in which the loads connected across three phases are identical.

Voltage, Current and Power relation in 3ϕ Y-connected balanced system

From figure,

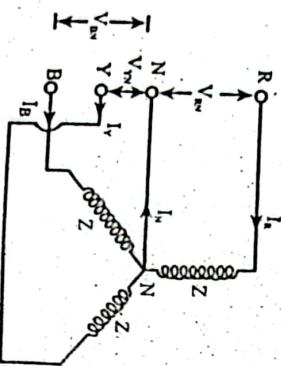


Fig 7.4 (a) 3ϕ Y-connected balanced system

$$\begin{aligned} I_L &= I_{ph} \\ I_R &= I_Y = I_B = I_L \end{aligned}$$

Let the phase sequence be RYB

$$\tilde{V}_{RN} = V_{ph} < 0^\circ$$

$$\tilde{V}_{YN} = V_{ph} < -120^\circ$$

$$\tilde{V}_{BN} = V_{ph} < 120^\circ$$

Therefore,

$$\tilde{I}_R = \frac{\tilde{V}_{RN}}{Z} = \frac{V_{ph} < 0^\circ}{Z} = \frac{V_{ph}}{Z} < -\phi$$

$$\tilde{I}_Y = \frac{\tilde{V}_{YN}}{Z} = \frac{V_{ph} < -120^\circ}{Z} = \frac{V_{ph}}{Z} < -120^\circ - \phi$$

$$\tilde{I}_B = \frac{\tilde{V}_{BN}}{Z} = \frac{V_{ph} < 120^\circ}{Z} = \frac{V_{ph}}{Z} < 120^\circ - \phi$$

Line current $- I_1, I_2, I_3$

displaced by 120° phase angle from each other.

It is seen that the magnitude of each line current is $\frac{V_{ph}}{Z}$ and line current are displaced by 120° phase angle from each other.

$$\text{KCL at point } N, \tilde{I}_N = \tilde{I}_R + \tilde{I}_Y + \tilde{I}_B = \frac{V_{ph}}{Z} < (-\phi) - \frac{V_{ph}}{Z} < (-120^\circ - \phi) + \frac{V_{ph}}{Z} < (120^\circ - \phi) = 0$$

Hence, phasor sum of line currents must be zero.

Now drawing the phasor diagram.

Also, in a balanced three phase star-connected load the current in the neutral wire is zero.

Note: Steps to draw phasor diagram

- First draw phasor voltage & phasor current.
- Then draw line quantities.

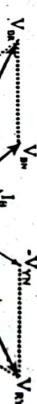


Fig 7.4 (b) Phasor diagram of 3ϕ Y-connected balanced system

From figure,

$$V_{RY} = V_{RN} + V_{NY} = V_{RN} - V_{YN}$$

From figure,

$$V_{RY} = V_{RN} + V_{NY}$$

or,

$$V_{RY} = V_{RN} - V_{YN}$$

or,

$$V_{RY} = \sqrt{(V_{RN})^2 + (V_{YN})^2 + 2V_{RN}V_{YN} \cos 60^\circ}$$

or,

$$V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}\left(\frac{1}{2}\right)}$$

or,

$$V_L = \sqrt{3}V_{ph}$$

$\therefore V_L = \sqrt{3}V_{ph}$

In star connected balanced system

$$\boxed{V_L = \sqrt{3}V_{ph}}$$

Active power (P) = $P_R + P_Y + P_B$

$$= V_{RN} I_R \cos \phi + V_{YN} I_Y \cos \phi + V_{BN} I_B \cos \phi$$

$$= V_{ph} I_{ph} \cos \phi + V_{ph} I_{ph} \cos \phi + V_{ph} I_{ph} \cos \phi$$

$$\begin{aligned} \therefore P &= 3 V_m I_m \cos \phi \\ &= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos \phi \\ &= \sqrt{3} V_L I_L \cos \phi \end{aligned}$$

 $\therefore P = 3 V_m I_m \cos \phi$

$$\begin{aligned} \text{Reactive power (Q)} &= Q_R + Q_Y + Q_B = V_{RN} I_R \sin \phi + V_{VN} I_V \sin \phi + V_{BN} I_B \sin \phi \\ \therefore Q &= 3 V_m I_m \sin \phi = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \sin \phi = \sqrt{3} V_L I_L \sin \phi \end{aligned}$$

$$\begin{aligned} \therefore Q &= 3 V_m I_m \sin \phi = \sqrt{3} V_L I_L \sin \phi \\ \text{Thus, in Y connection, we conclude that} \\ \text{(i) } I_L &= I_m \\ V_L &= \sqrt{3} V_m \end{aligned}$$

$\therefore V_L = \sqrt{3} V_m$

(ii) Line voltages are 120° apart and also are the phase voltages.

- (iii) Line voltage leads the corresponding phase voltage by 30° .
 (iv) The phase difference between line currents and corresponding line voltages is $(30^\circ + \phi)$ with current lagging.

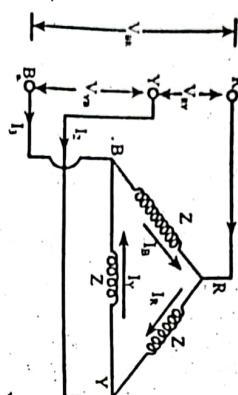
Voltage, Current and Power relation in 3φ Δ - connected system

Fig 7.5 (a) Delta connected balanced system

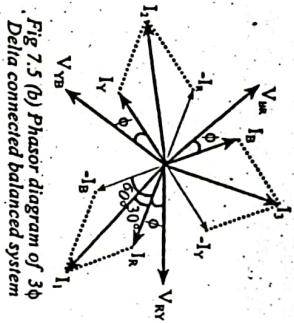


Fig 7.5 (b) Phasor diagram of 3φ Delta connected balanced system

Form figure,

$V_L = V_m$

Let the phase sequence by RYB

Now, drawing the phasor diagram

Note:

- First draw phase quantities.
- Then draw line quantities.

Applying KCL at node R

$$\begin{aligned} I_1 + I_B &= I_R \\ I_1 &= I_R - I_B \end{aligned}$$

$$I_R = I_V - I_B$$

$$I_V = I_B - I_Y$$

$$I_Y = \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \cos 60^\circ}$$

$$I_Y = \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \left(\frac{1}{2}\right)}$$

$$\text{or, } I_L = \sqrt{3} I_m$$

$$\begin{aligned} \text{In } \Delta-\text{connected system: } I_L &= \sqrt{3} I_m \\ \text{Active power (P)} &= P_R + P_Y + P_B \\ &= V_{RY} I_R \cos \phi + V_{VN} I_V \cos \phi + V_{BN} I_B \cos \phi \\ &= 3 V_m I_m \cos \phi = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos \phi \\ &= \sqrt{3} V_L I_L \cos \phi \end{aligned}$$

$$\begin{aligned} \therefore P &= 3 V_m I_m \cos \phi = \sqrt{3} V_L I_L \cos \phi \\ \text{Similarly, Reactive power (Q)} &= Q_R + Q_Y + Q_B = V_{RN} I_R \sin \phi + V_{VN} I_V \sin \phi + V_{BN} I_B \sin \phi \\ \text{Thus in } \Delta-\text{connected system, we conclude that} \\ \text{(i) } V_L &= V_m \\ I_L &= \sqrt{3} I_m \end{aligned}$$

- (ii) Line currents are 120° apart and also are the phase currents.
 (iii) Line currents are 30° behind the respective phase currents.
 (iv) The phase difference between line currents and corresponding line voltages is $(30^\circ + \phi)$ with current lagging.

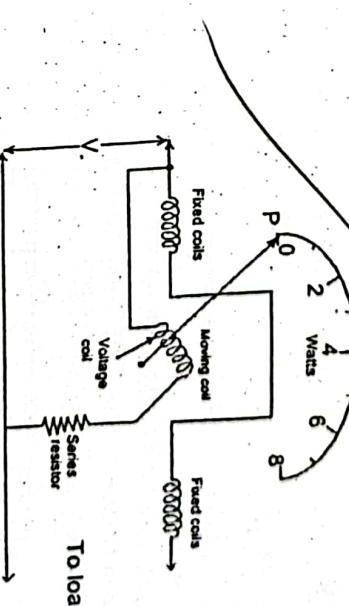
Dynamometer Wattmeter

Fig 7.6 (a) Dynamometer Wattmeter

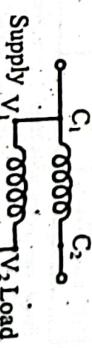


Fig 7.6 (b) Schematic diagram of Dynamometer Wattmeter

A dynamometer wattmeter is a moving coil instrument in which the magnetic field is produced not by a permanent magnet but by fixed coils. The fixed coils are usually arranged in two equal sections F and F placed together and parallel to each other.

The deflecting torque produced in the moving coil is proportional to the product of the magnetic flux density produced by the fixed coil and the current passing through the moving coil.

- $T_d \propto B \times I_2$
- But flux produced by the fixed coil is proportional to the current flowing through fixed coil.
- $B \propto I_1$

$\therefore T_d \propto I_1 \times I_2$

For dc circuit,

$$\begin{aligned} T_d &\propto I_1 I_2 & \text{But } I_2 &\propto V \\ \therefore T_d &\propto V I_1 \\ \therefore T_d &\propto \text{Power} \end{aligned}$$

For ac circuit

$$\begin{aligned} T_d &\propto \text{average value of } V \times i \\ T_d &\propto \frac{1}{2\pi} \int_0^{2\pi} v \times i \, d\omega \end{aligned}$$

$$T_d \propto \frac{V_m I_m}{2\pi} \int_0^{2\pi} \sin \omega t \sin(\omega t - \phi) \, d\omega$$

$$T_d \propto \frac{V_m I_m}{2\pi} \left[\frac{\cos \phi - \cos(2\omega t - \phi)}{2} \right]_0^{2\pi}$$

$$T_d \propto \frac{V_m I_m}{4\pi} \left[\cos \phi \times \omega t - \frac{\sin(2\omega t - \phi)}{2} \right]_0^{2\pi}$$

$$T_d \propto \frac{V_m I_m}{4\pi} \cos \phi \times 2\pi$$

$$T_d \propto \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos \phi$$

$$T_d \propto V_{rms} \times I_{rms} \cos \phi$$

$\therefore T_d \propto \text{Active power}$

Hence, deflection is proportional to active power in ac circuit.

Power Measurement in 1φ system by wattmeter

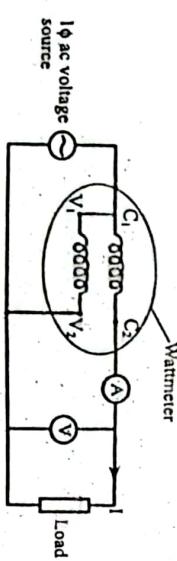


Fig 7.7 Power measurement in single phase circuit

Wattmeter reading

$$W = VI \cos \phi$$

where,

$\cos \phi$ = Power factor of the load

$$\text{Now, } \cos \phi = \frac{W}{VI} \text{ can be calculated}$$

Then, reactive power can be calculated as

$$Q = VI \sin \phi$$

Power measurement in 3φ system by two wattmeter method.

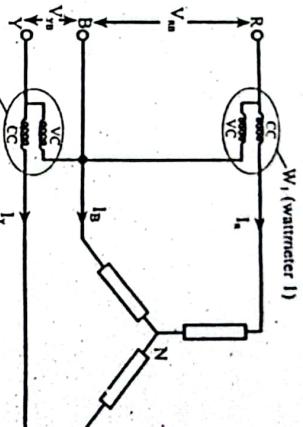


Fig 7.8 (a) Power measurement in 3 phase system

The total power in 3φ balanced as well as unbalanced load can be measured by using two-wattmeter method.

As shown in figure, the current coil of W_1 measures I_A , while its potential coil measures line voltage V_{AB} . Similarly, the current coil of W_2 measures I_B , while its potential coil measures line voltage V_{BC} .

Let an inductive load having power factor $\cos \phi$ is connected to each phase, then the phase current lags the corresponding phase voltage by an angle ϕ as shown in the phasor.

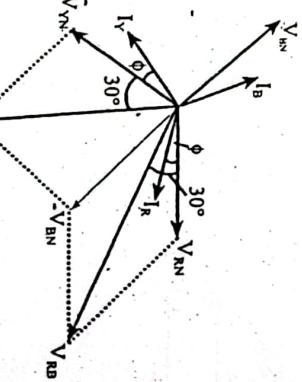


Fig 7.8 (b) Phasor diagram

For W_1 ,

$$V_{RB} = V_{RN} + V_{NB} = V_{RN} - V_{BN}$$

For W_2 ,

$$V_{VB} = V_{VR} + V_{NB} = V_{VR} - V_{BN}$$

From phasor diagram,

I_A leads V_{AB} by an angle $(30^\circ - \phi)$

Thus, the reading of wattmeter W_1 $= V_{RN} I_A \cos(30^\circ - \phi)$

Similarly,

I_Y lags V_{VB} by angle $(30^\circ + \phi)$

Thus, the reading of wattmeter $W_1 = V_{\text{VL}} I_{\text{V}} \cos (30^\circ + \phi)$

Now, Total active power of the circuit:

$$W = W_1 + W_2$$

$$= V_{\text{VL}} I_{\text{V}} \cos (30^\circ - \phi) + V_{\text{VL}} I_{\text{V}} \cos (30^\circ + \phi)$$

$$= V_{\text{VL}} [2 \cos 30^\circ \cos \phi]$$

$$= V_{\text{VL}} \times 2 \times \frac{\sqrt{3}}{2} \cos \phi = \sqrt{3} V_{\text{VL}} I_{\text{V}} \cos \phi$$

$$\therefore W = W_1 + W_2 = \sqrt{3} V_{\text{VL}} I_{\text{V}} \cos \phi$$

i.e. Total active power of 3φ circuit

Similarly,

$$W_1 = V_{\text{VL}} I_{\text{V}} \cos (30^\circ - \phi) - V_{\text{VL}} I_{\text{V}} \cos (30^\circ + \phi)$$

$$= V_{\text{VL}} I_{\text{V}} [\cos (30^\circ - \phi) - \cos (30^\circ + \phi)]$$

$$= V_{\text{VL}} I_{\text{V}} \times 2 \times \frac{1}{2} \sin 30^\circ \sin \phi$$

$$= V_{\text{VL}} I_{\text{V}} \sin \phi = V_{\text{VL}} I_{\text{V}} \sin \phi$$

or, $\sqrt{3} (W_1 - W_2) = \sqrt{3} V_{\text{VL}} I_{\text{V}} \sin \phi$

$$\therefore Q = \sqrt{3} (W_1 - W_2) = \sqrt{3} V_{\text{VL}} I_{\text{V}} \sin \phi$$

i.e. Total Reactive power of 3φ circuit

From (1) and (2),

Dividing (ii) by (i)

$$\frac{\sqrt{3} (W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3} V_{\text{VL}} I_{\text{V}} \sin \phi}{\sqrt{3} V_{\text{VL}} I_{\text{V}} \cos \phi}$$

or, $\tan \phi = \frac{\sqrt{3} (W_1 - W_2)}{(W_1 + W_2)}$

$$\therefore \phi = \tan^{-1} \left[\frac{\sqrt{3} (W_1 - W_2)}{(W_1 + W_2)} \right]$$

i.e. Power factor angle of the circuit

Effect of power factor on Wattmeter Reading

$$W_1 = V_{\text{VL}} I_{\text{V}} \cos (30^\circ - \phi)$$

$$W_2 = V_{\text{VL}} I_{\text{V}} \cos (30^\circ + \phi)$$

ϕ = P.f angle of the load

Case 1: When $\phi = 0^\circ$; $\cos \phi = 1$ (unity p.f.)

$$W_1 = V_{\text{VL}} I_{\text{V}} \cos 30^\circ$$

$$W_2 = V_{\text{VL}} I_{\text{V}} \cos 30^\circ$$

$\therefore W_1 = W_2$ = equal reading on both wattmeters

$$\Rightarrow \frac{W_2}{W_1} = 1$$

This is the case for resistive load

Case 2: When $\phi = 60^\circ$; $\cos 60^\circ = 0.5$ (lagging)

$$W_1 = V_{\text{VL}} I_{\text{V}} \cos (30^\circ - 60^\circ) = \frac{\sqrt{3}}{2} V_{\text{VL}} I_{\text{V}}$$

$$W_2 = V_{\text{VL}} I_{\text{V}} \cos (30^\circ + 60^\circ) = 0$$

$$\therefore W = W_1 + W_2 = \frac{\sqrt{3}}{2} V_{\text{VL}} I_{\text{V}}$$

Case 3: When $60^\circ < \phi < 90^\circ \Rightarrow 0.5 > \cos \phi > 0$

W_1 reading is positive.

W_2 reading is negative.
To obtain reading on W_2 , the connection NC/CC (Voltage Coil or Current coil) must be reversed and thus reading obtained will be taken as - ve reading

$$\Rightarrow \frac{W_2}{W_1} = (-\text{ve value})$$

Case 4: When $\phi = 90^\circ$; $\cos 90^\circ = 0$

$$W_1 = V_{\text{VL}} I_{\text{V}} \cos (30^\circ - 90^\circ)$$

$$= V_{\text{VL}} I_{\text{V}} \sin 30^\circ$$

$$W_2 = V_{\text{VL}} I_{\text{V}} \cos (30^\circ + 90^\circ)$$

$$= -V_{\text{VL}} I_{\text{V}} \sin 30^\circ$$

$$\therefore W_1 + W_2 = 0$$

$\therefore W_1$ and W_2 readings are equal and opposite.

$$\Rightarrow \frac{W_2}{W_1} = -1$$

This is the case for purely Inductive or Capacitive load.

The above analysis is plotted on curve known as watt - ratio curve.

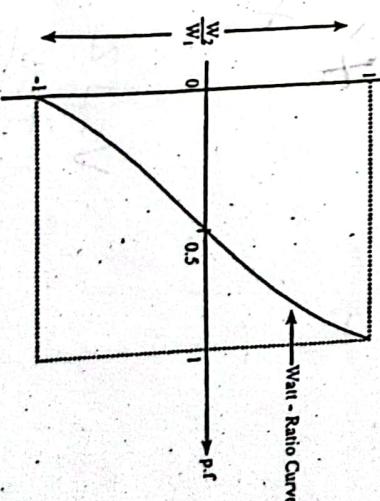


Fig 7.9 Watt Ratio Curve

Hence, if pf of load is less than 0.5 then it shows that one of the wattmeter shows negative reading.

Exam Solutions

1. A three - phase load consists of three similar inductive coils each of resistance 8Ω and inductance 20 mH . The supply voltage is 100V , 50 Hz . Calculate the line current and total power consumed by the load in star connection. Take phase sequence RYB.

Solution:

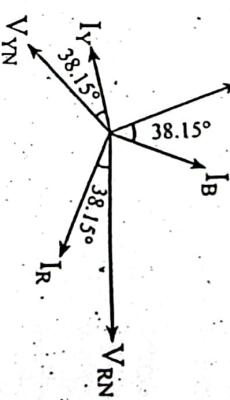
Given,

Star connected load.

$I_{\text{line}} = I_{\text{phase}}$ current

i.e. $I_L = I_P$

$$V_{\text{phase}} = \frac{100}{\sqrt{3}} = 57.74 \text{ V}$$



As the given load is balanced 3ϕ load, we know

$$\text{Total power consumed } (P) = \sqrt{3} V_L I_L \cos \phi$$

2. A three phase star connected system with line voltage 400 V is connected to three loads: $25 \angle 0^\circ$, $11 \angle -20^\circ$ and $15 \angle 10^\circ$ (also connected in star). Find the line currents, total power and current in the neutral of the system.

[2072 Kartik]

Solution:

Given, Line voltage, $V_L = 400\text{V}$

$$\text{Phase voltage, } V_P = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

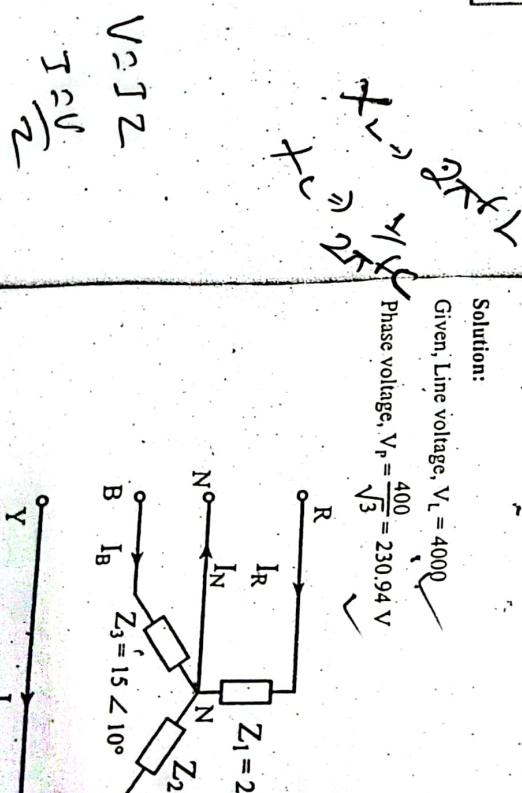
As the phase sequence is RYB

$$\tilde{V}_{RN} = 57.74 \angle 0^\circ \text{ V}$$

$$\begin{aligned} \tilde{V}_{VN} &= 57.74 \angle -120^\circ \text{ V} \\ \tilde{V}_{BN} &= 57.74 \angle +120^\circ \text{ V} \end{aligned}$$

Now, line currents:

$$\tilde{I}_R = \frac{\tilde{V}_{RN}}{Z} = \frac{57.74 \angle 0^\circ}{8 + j 6.283}$$



In Y-connected system,

Line current = Phase current

$$\text{i.e. } I_L = I_P$$

$$Z_1 = 25 \angle 0^\circ \Omega, Z_2$$

$$Z_L = 11 \angle -20^\circ \Omega$$

$$Z_3 = 15 \angle 10^\circ \Omega$$

Line currents

$$\tilde{I}_R = \frac{\tilde{V}_{RN}}{Z_1} = \frac{230.94 \angle 0^\circ}{25 \angle 0^\circ}$$

$$= 9.2376 \angle 0^\circ \text{ A}$$

$$\tilde{I}_Y = \frac{\tilde{V}_{RN}}{Z_2} = \frac{230.94 \angle -120^\circ}{11 \angle -20^\circ}$$

$$= 20.99 \angle -100^\circ \text{ A}$$

$$\tilde{I}_B = \frac{\tilde{V}_{RN}}{Z_3} = \frac{230.94 \angle 120^\circ}{15 \angle 10^\circ}$$

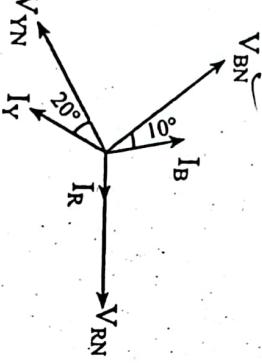
$$= 15.396 \angle 110^\circ \text{ A}$$

Current through neutral wire,

$$\tilde{I}_N = \tilde{I}_R + \tilde{I}_Y + \tilde{I}_B$$

$$= (9.2376 \angle 0^\circ) + (20.99 \angle -100^\circ) + (15.396 \angle 110^\circ)$$

$$= 6.212 \angle -86.98^\circ \text{ A}$$



Total active power

$$P_T = V_{RN} I_R \cos 0^\circ + V_{YN} I_Y \cos 20^\circ + V_{BN} I_B \cos 10^\circ$$

$$= 230.94 \times 9.2376 \times 1 + 230.94 \times 20.99 \times 0.940$$

$$+ 230.94 \times 15.396 \times 0.985$$

$$= 10.19 \text{ kW}$$

Total reactive power,

$$Q_T = V_{RN} I_R \sin 0^\circ + V_{YN} I_Y \sin (-20^\circ) + V_{BN} I_B \sin 10^\circ$$

$$= 230.94 \times 9.2376 \times 0 - 230.94 \times 20.99 \times 0.34$$

$$+ 230.94 \times 15.396 \times 0.174$$

$$= 1.03 \text{ kVAR}$$

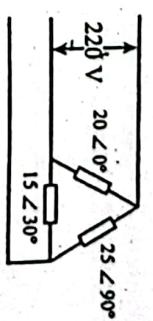
Total apparent power,

$$S_T = \sqrt{P_T^2 + Q_T^2}$$

$$= \sqrt{10.19^2 + (-1.03)^2}$$

$$= 10.24 \text{ kVA}$$

3. For the 3-phase delta connected circuit below. Determine the line currents and total active, reactive and apparent power. [2072 Marks]



Solution:

Redrawing the given circuit,

Let the phase sequence be RYB
In Δ -connected circuit
 $V_{\text{Line}} = V_{\text{Phase}} = 220 \text{ V}$

$$\text{i.e. } V_L = V_P = 220 \text{ V}$$

$$\tilde{I}_Y = \frac{\tilde{V}_{YB}}{Z_{YB}} = \frac{220 \angle -120^\circ}{15 \angle 90^\circ} = 14.67 \angle -150^\circ$$

$$\tilde{I}_B = \frac{\tilde{V}_{BR}}{Z_{BR}} = \frac{220 \angle +120^\circ}{15 \angle 30^\circ} = 11 \angle 120^\circ$$

Line currents,

$$\tilde{I}_L = \tilde{I}_R - \tilde{I}_B = (8.8 \angle -90^\circ) - (11 \angle 120^\circ)$$

$$= 19.13 \angle -73.29^\circ \text{ A}$$

$$\tilde{I}_T = \tilde{I}_Y - \tilde{I}_B = (14.67 \angle -150^\circ) - (8.8 \angle 90^\circ) = 12.79 \angle 173.42^\circ \text{ A}$$

$$\begin{aligned}\tilde{I}_j &= \tilde{I}_B - \tilde{I}_Y = (11 \angle 120^\circ) - (14.67 \angle -150^\circ) \\ &= 18.34 \angle 66.88^\circ\end{aligned}$$

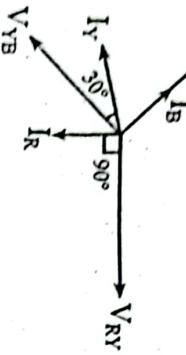
Hence,

Line voltage,

$V_L = 415 \text{ V}$

Phase voltage

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$



Total active power,

$$\begin{aligned}P &= V_{AY} I_A \cos 90^\circ + V_{BY} I_B \cos 30^\circ + V_{RY} I_R \cos 0^\circ \\ &= 220 \times 8.8 \times 0 + 220 \times 14.67 \times 0.5 + 220 \times 11 \times 1 \\ &= 5227.838 \text{ W}\end{aligned}$$

Total reactive power,

$$\begin{aligned}Q &= V_{AY} I_A \sin 90^\circ + V_{BY} I_B \sin 30^\circ + V_{RY} I_R \sin 0^\circ \\ &= 220 \times 8.8 \times 1 + 220 \times 14.67 \times 0.5 + 220 \times 11 \times 0 \\ &= 3549.7 \text{ VAR}\end{aligned}$$

Total apparent power,

$$\begin{aligned}S &= \sqrt{P^2 + Q^2} \\ &= \sqrt{5227.838^2 + 3549.7^2} \\ &= 6319.07 \text{ VA}\end{aligned}$$

4. For the circuit shown in figure below, calculate the current through the neutral and the total power consumed in the load.

[2007 Chaira]

Let, the phase sequence be RBY.
Then,

$$\tilde{I}_R = \frac{239.6 \angle 0^\circ}{10} = 23.96 \angle 0^\circ \text{ A}$$

$$\tilde{I}_B = \frac{239.6 \angle -120^\circ}{20} = 11.98 \angle -120^\circ \text{ A}$$

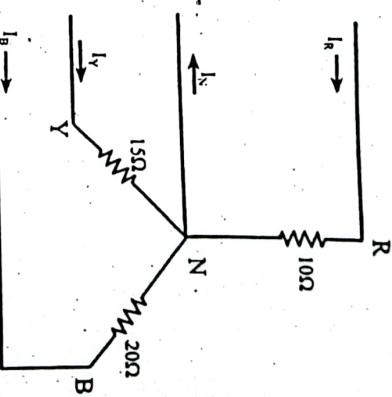
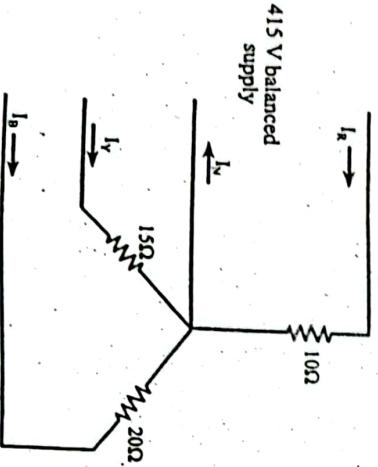
$$\tilde{I}_Y = \frac{239.6 \angle +120^\circ}{15} = 15.97 \angle +120^\circ \text{ A}$$

\therefore Current through the neutral,

$$\begin{aligned}\tilde{I}_N &= \tilde{I}_R + \tilde{I}_B + \tilde{I}_Y \\ &= (23.96 \angle 0^\circ) + (11.98 \angle -120^\circ) + (15.97 \angle 120^\circ) \\ &= 10.566 \angle 19.09^\circ \text{ A}\end{aligned}$$

Now,

$$\begin{aligned}\text{Total power consumed, } \dot{P} &= 23.96^2 \times 10 + 11.98^2 \times 20 + 15.97^2 \times 15 \\ &= 12.437 \text{ kW}\end{aligned}$$



5. Each phase of a 3-Phase, delta-connected load consists of an impedance, $Z = 20\angle 60^\circ$ ohm. The line voltage is 440 V at 50 Hz. Compute the power consumed by each phase impedance and the total power. What will be the reading of the two wattmeters connected?

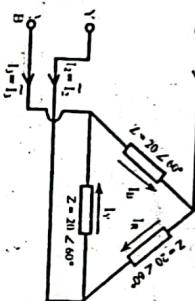
Solution:

Given,

$$\text{Line voltage } (V_L) = 440 \text{ V}$$

Frequency (f) = 50 Hz

From figure as Δ -connected,



Line voltage = phase voltage

$$V_L = V_{ph} = 440 \text{ V}$$

Phase Voltages

$$\bar{V}_{RY} = 440\angle 0^\circ \text{ V}$$

$$\bar{V}_{YB} = 440\angle -120^\circ \text{ V}$$

$$\bar{V}_{BR} = 440\angle 120^\circ \text{ V}$$

Phase Currents

$$\bar{I}_R = \frac{\bar{V}_{RY}}{Z} = \frac{440\angle 0^\circ}{20\angle 60^\circ} = 22\angle -60^\circ \text{ A}$$

$$\bar{I}_Y = \frac{\bar{V}_{YB}}{Z} = \frac{440\angle -120^\circ}{20\angle 60^\circ} = 22\angle 180^\circ \text{ A}$$

$$\bar{I}_B = \frac{\bar{V}_{BR}}{Z} = \frac{440\angle 120^\circ}{20\angle 60^\circ} = 22\angle 60^\circ \text{ A}$$

OR,

$$\bar{I}_Y = 22\angle (-60^\circ - 120^\circ) = 22\angle -180^\circ = 22\angle 180^\circ \text{ A}$$

$$\bar{I}_B = 22\angle (-60^\circ + 120^\circ) = 22\angle 60^\circ \text{ A}$$

Power Consumed by each phase impedance.

$$\therefore \text{Active power per phase } (P_R) = V_{ph} I_{ph} \cos \phi$$

$$\begin{aligned} \text{Phase difference } \phi &= 0^\circ - (-60^\circ) \\ &= 60^\circ \end{aligned}$$

$$\begin{aligned} \therefore P_R &= 440 \times 22 \times \cos 60^\circ \\ &= 4840 \text{ W} \end{aligned}$$

Power consumed by each phase impedance = 4840 W

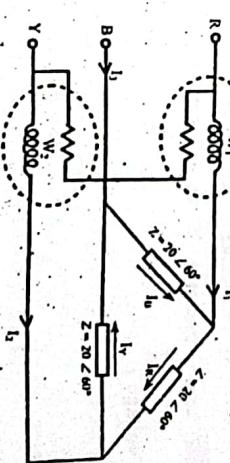
$$\therefore \text{Total Active power } (P) = 3 P_R = 3 \times 4840 = 14520 \text{ W}$$

OR,

$$\begin{aligned} \text{Total Reactive Power } (Q) &= 3 V_{ph} I_{ph} \sin \phi \\ &= 3 \times 440 \times 22 \times \sin 60^\circ \\ &= 25149.37 \text{ VAR} \end{aligned}$$

$$\begin{aligned} \text{Apparent Power } (S) &= \sqrt{P^2 + Q^2} \\ &= \sqrt{(14520)^2 + (25149.37)^2} = 29039.99 \approx 29040 \text{ VA} \end{aligned}$$

Now, To find the reading of the two wattmeters connected.



Let, W_1 and W_2 be the readings of the two wattmeters connected.

Total Active Power (P) = $W_1 + W_2$:

$$\text{Or, } 14520 = W_1 + W_2$$

$$\text{Total Reactive Power } (Q) = \sqrt{3} (W_1 - W_2)$$

$$\text{Or, } 25149.37 = \sqrt{3} (W_1 - W_2)$$

$$\text{Or, } \frac{25149.37}{\sqrt{3}} = W_1 - W_2 \quad \dots \dots \dots \text{(ii)}$$

Adding equations (i) and (ii), we get

$$14520 + \frac{25149.37}{\sqrt{3}} = 2W_1$$

$$\text{or, } 29039.99 = 2W_1$$

$$\therefore W_1 = 14519.99 \text{ W} = 14520 \text{ W}$$

Using equation (i)

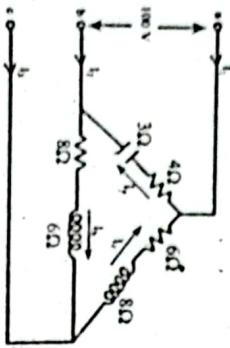
$$W_1 + W_2 = 14520$$

$$W_2 = 14520 - 14520 = 0$$

$$\text{The readings of two wattmeters connected are } 14520 \text{ W and } 0.$$

6. Discuss the advantages of three phase ac system over single phase ac system. For the given unbalanced delta connected load, find the phase currents, line currents and total power consumed by the load when phase sequence is abc. Construct the phasor diagram of currents and voltages in the load.

[2011 Magh]



Solution:

[Please refer to the theory for first part of the question]

For Δ-connected system

Line voltage = Phase voltage

$$V_L = V_{ph} = 100 \text{ V}$$

Phase Voltages

$$\bar{V}_{ab} = 100\angle 0^\circ \text{ V}$$

$$\bar{V}_{bc} = 100\angle -120^\circ \text{ V}$$

$$\bar{V}_{ca} = 100\angle 120^\circ \text{ V}$$

$$\begin{aligned} Z_a &= 4 - j3 \\ Z_b &= 8 + j6 \\ Z_c &= 6 + j8 \end{aligned}$$

Phase Currents

$$\bar{I}_a = \frac{\bar{V}_{ab}}{Z_a} = \frac{100\angle 0^\circ}{4 - j3} = 20\angle 36.86^\circ \text{ A}$$

$$\bar{I}_b = \frac{\bar{V}_{bc}}{Z_b} = \frac{100\angle -120^\circ}{8 + j6} = 10\angle -156.86^\circ \text{ A}$$

Total Power consumed by load

$$\begin{aligned} (P) &= P_a + P_b + P_c \\ &= 1600.21 + 800.10 + 599.86 = 3000.17 \text{ W} \end{aligned}$$

$$\begin{aligned} \bar{I}_c &= \frac{\bar{V}_{ca}}{Z_c} = \frac{100\angle 120^\circ}{6 + j8} = 10\angle 66.86^\circ \text{ A} \\ \bar{I}_1 + \bar{I}_2 &= \bar{I}_a \\ \therefore \bar{I}_2 &= \bar{I}_a - \bar{I}_1 \\ &= (10\angle -156.86^\circ) - (20\angle 36.86^\circ) \\ &= 29.81\angle -147.70^\circ \text{ A} \\ \bar{I}_3 + \bar{I}_b &= \bar{I}_c \\ \bar{I}_3 &= \bar{I}_c - \bar{I}_b \\ &= (10\angle 66.86^\circ) - (10\angle -156.86^\circ) \\ &= 18.56\angle 45^\circ \text{ A} \end{aligned}$$

Total Power consumed by load (P) = ?

Active Power in Phase a (P_a) = $V_{ph} I_{ph} \cos \phi$

$$\begin{aligned} &= 100 \times 20 \times \cos(0^\circ - 36.86^\circ) \\ &= 1600.21 \text{ W} \end{aligned}$$

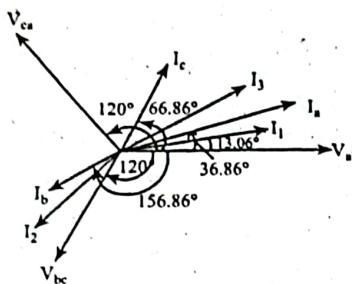
Active Power in Phase b (P_b)

$$\begin{aligned} &= 100 \times 10 \times \cos(-120^\circ - (-156.86^\circ)) \\ &= 800.10 \text{ W} \end{aligned}$$

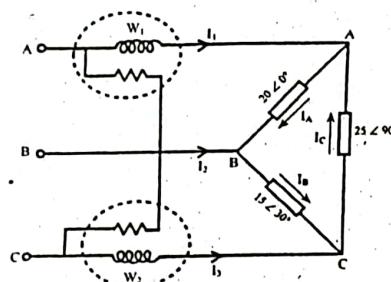
Active Power in Phase c (P_c)

$$\begin{aligned} &= 100 \times 10 \times \cos(120^\circ - 66.86^\circ) \\ &= 599.86 \text{ W} \end{aligned}$$

Phasor diagram



7. The supply system is 230 V, 3-Phase, 50 Hz. Determine the reading of wattmeters W_1 and W_2 . Phase sequence is AB-BC-CA. [2071 Bhadra]



Solution:

Given, 230V, 3φ, 50 Hz Supply

As Δ-connected system,

$$V_L = V_{ph} = 230 \text{ V}$$

Phase Voltages

$$\bar{V}_{AB} = 230 \angle 0^\circ \text{ V}$$

$$\bar{V}_{BC} = 230 \angle -120^\circ \text{ V}$$

$$\bar{V}_{CA} = 230 \angle 120^\circ \text{ V}$$

$$Z_A = 20 \angle 0^\circ$$

$$Z_B = 15 \angle 30^\circ$$

$$Z_C = 25 \angle 90^\circ$$

Phase Currents:

$$\bar{I}_A = \frac{\bar{V}_{AB}}{Z_A} = \frac{230 \angle 0^\circ}{20 \angle 0^\circ} = 11.5 \angle 0^\circ \text{ A}$$

$$\bar{I}_B = \frac{\bar{V}_{BC}}{Z_B} = \frac{230 \angle -120^\circ}{15 \angle 30^\circ} = 15.33 \angle -150^\circ \text{ A}$$

$$\bar{I}_C = \frac{\bar{V}_{CA}}{Z_C} = \frac{230 \angle 120^\circ}{25 \angle 90^\circ} = 9.2 \angle 30^\circ \text{ A}$$

Line Currents:

KCL at node A,

$$\bar{I}_1 + \bar{I}_C = \bar{I}_A$$

$$\therefore \bar{I}_1 = \bar{I}_A - \bar{I}_C = (11.5 \angle 0^\circ) - (9.2 \angle 30^\circ) = 5.79 \angle -52.47^\circ \text{ A}$$

KCL at node C,

$$\bar{I}_3 + \bar{I}_B = \bar{I}_C$$

$$\therefore \bar{I}_3 = \bar{I}_C - \bar{I}_B = (9.2 \angle 30^\circ) - (15.33 \angle -150^\circ) = 24.53 \angle 30^\circ \text{ A}$$

As shown in figure, the current coil of W_1 measures I_1 and potential coil of W_1 measures V_{AB} .

∴ Reading of Wattmeter

$$W_1 = V_{AB} \times I_1 \times \cos \phi_1$$

Where ϕ_1 is phase difference between V_{AB} & I_1 .

$$\therefore W_1 = 230 \times 5.79 \times \cos(0^\circ - (-52.47)) = 811.24 \text{ W}$$

Similarly, the current coil of W_2 measures I_3 and its potential coil measures V_{CB} .

$$\bar{V}_{CB} = -\bar{V}_{BC} = -(230 \angle -120^\circ) = 230 \angle 60^\circ \text{ V}$$

∴ Reading of wattmeter

$$W_2 = V_{CB} \times I_3 \times \cos \phi_2$$

Where ϕ_2 is phase difference between V_{CB} & I_3 .

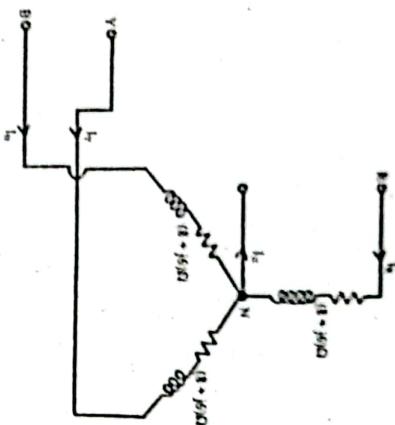
$$\therefore W_2 = 230 \times 24.53 \times \cos(60^\circ - 30^\circ) = 4886.02 \text{ W}$$

∴ The readings of two wattmeter W_1 and W_2 are 811.24 W and 4886.02 W respectively.

8. Calculate the amount of current through the neutral of a balanced 3-phase star connected circuit. Also verify with the phasor diagram.

Let a balanced 3-phase star connected load of $(8+j6) \Omega$ phase is connected to 3φ, 230 V, 50 Hz supply.

Phasor diagram



Solution:

From figure Y-connected load

Line current = phase current

 $I_L = I_P$

$$V_P = \frac{230}{\sqrt{3}} = 132.8 \text{ V}$$

Impedance per phase (Z) = $(8 + j6) \Omega$

$$\tilde{V}_{BN} = 132.8 \angle 0^\circ \text{ V}$$

$$\tilde{V}_{AN} = 132.8 \angle -120^\circ \text{ V}$$

$$\tilde{V}_{BA} = 132.8 \angle 120^\circ \text{ V}$$

$$\tilde{I}_P = \frac{\tilde{V}_{BN}}{Z} = \frac{132.8 \angle 0^\circ}{8+j6}$$

$$= 13.28 \angle -36.87^\circ \text{ A}$$

$$\tilde{I}_Y = \frac{\tilde{V}_{AN}}{Z} = \frac{132.8 \angle -120^\circ}{8+j6}$$

$$= 13.28 \angle -156.87^\circ \text{ A}$$

$$\tilde{I}_B = \frac{\tilde{V}_{BN}}{Z} = \frac{132.8 \angle 120^\circ}{8+j6} = 13.28 \angle 83.13^\circ \text{ A.}$$

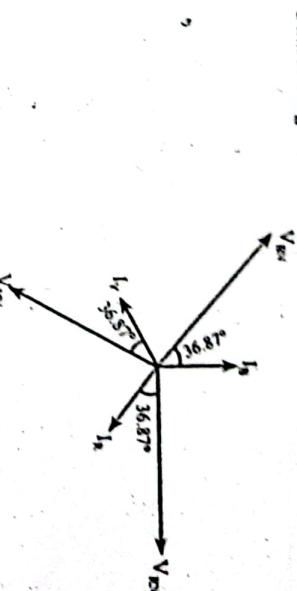
Applying KCL at N,

$$\begin{aligned} \tilde{I}_N &= \tilde{I}_R + \tilde{I}_Y + \tilde{I}_B \\ &= (13.28 \angle -36.87^\circ) + (13.28 \angle -156.87^\circ) + (13.28 \angle 83.13^\circ) = 0 \end{aligned}$$

$$\therefore \tilde{I}_N = 0$$

Thus the current through neutral of a balanced 3 phase star connected circuit is zero.

Phasor diagram

From phasor diagram, it is clear that the phase currents have equal magnitude and are 120° apart from each other thus, their algebraic sum is zero.

9. A 415 V, 3 Phase, 50 Hz induction motor takes 50 kW power from supply mains at 0.72 power factor lagging. A bank of capacitors is connected in delta across the line to improve the overall power factor to 0.9 lagging.

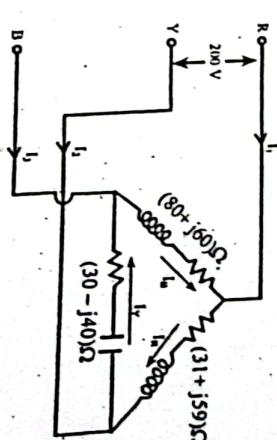
[2071 Shawan]

Solution:

[Please refer to solution of 2068 Baisakhi]

10. Three loads $(31+j59)\Omega$, $(30-j40)\Omega$ and $(80+j60)\Omega$ are connected in delta to a 3-phase, 200 V supply. Find the phase currents, line currents and total power absorbed.

[2071 Shawan]



Solution:

Let the phase sequence be RYB.

From figure Δ -connected system,

Line voltage = Phase voltage

$$\therefore V_L = V_P = 220 \text{ V}$$

Phase voltage:

$$\tilde{V}_{RY} = 200 \angle 0^\circ \text{ V}$$

$$\tilde{V}_{YA} = 200 \angle -120^\circ \text{ V}$$

$$\tilde{V}_{BR} = 200 \angle 120^\circ \text{ V}$$

$$Z_R = 31 + j59\Omega$$

$$Z_Y = 30 - j40\Omega$$

$$Z_B = 80 + j60\Omega$$

Phase Currents

$$\bar{I}_R = \frac{\bar{V}_{RY}}{Z_R} = \frac{200 \angle 0^\circ}{31 + j59} = 3 \angle -62.28^\circ A$$

$$\bar{I}_Y = \frac{\bar{V}_{YB}}{Z_Y} = \frac{200 \angle -120^\circ}{30 - j40} = 4 \angle -66.87^\circ A$$

$$\bar{I}_B = \frac{\bar{V}_{BR}}{Z_B} = \frac{200 \angle 120^\circ}{80 + j60} = 2 \angle 83.13^\circ A$$

Line Currents:

$$\bar{I}_1 + \bar{I}_2 = \bar{I}_R$$

$$\therefore \bar{I}_1 = \bar{I}_R - \bar{I}_B$$

$$= (3 \angle -62.28^\circ) - (2 \angle 83.13^\circ)$$

$$= 4.78 \angle -76.01^\circ A$$

$$\bar{I}_2 + \bar{I}_3 = \bar{I}_Y$$

$$\therefore \bar{I}_2 = \bar{I}_Y - \bar{I}_R = (4 \angle -66.87^\circ) - (3 \angle -62.28^\circ) = 1.037 \angle -80.24^\circ A$$

$$\bar{I}_3 + \bar{I}_Y = \bar{I}_B$$

$$\therefore \bar{I}_3 = \bar{I}_B - \bar{I}_Y$$

$$= (2 \angle 83.13^\circ) - (4 \angle -66.87^\circ) = 5.82 \angle 103.23^\circ A$$

Active Power in phase R

$$P_R = V_{RY} \times I_R \times \cos \phi_R$$

$$= 200 \times 3 \times \cos(0^\circ - (-62.28)) = 279.09 W$$

Active Power in phase Y

$$P_Y = V_{YB} \times I_Y \times \cos \phi_Y$$

$$= 200 \times 4 \times \cos(-120^\circ - (-66.87)) = 480 W$$

Active Power in phase B

$$P_B = V_{BR} \times I_B \times \cos \phi_B$$

$$= 200 \times 2 \times \cos(120^\circ - 83.13) = 320 W$$

$$\therefore \text{Total power absorbed } (P) = P_R + P_Y + P_B$$

$$= 279.09 + 480 + 320 = 1079.09 W$$

11. What are the two ways of connecting a 3-phase system? Draw their phasor diagrams and write down the relationship between phase and line voltages and phase and line current for these system.

[2070 Chaitra]

[Please refer to the theory]

12. A 220V, 3-phase voltage is applied to a balanced delta connected 3-phase load of phase impedance $(15 + j20)\Omega$ calculate;

(i) The phase voltages

(ii) The phasor current in each line

(iii) The power consumed per phase

(iv) Draw the phasor diagram
(v) What is the phasor sum of three line currents? Why does it have this value? [2070 Chaitra]

Solution:

Given, Line voltage (V_L) = 220V

[Remember the quantities given in the question is always assumed to be line quantities unless stated to be the other quantities]

From figure as Δ -connected system

Line voltage = phase voltage

$$\therefore V_L = V_{ph} = 220 V$$

Since balanced load condition

All phase impedances are equal i.e. $Z = (15 + j20)\Omega$

[It is convenient to do the calculation in terms of phase and then use to find the required solution]

The phase voltages

$$\bar{V}_{RY} = 220 \angle 0^\circ V$$

$$\bar{V}_{YB} = 220 \angle -120^\circ V$$

$$\bar{V}_{BR} = 220 \angle 120^\circ V$$

All phase voltages are 120° apart

(ii) Phasor current in each line

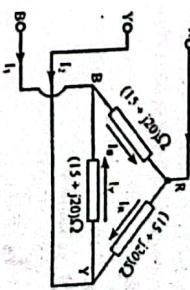
$$\tilde{I}_R = \frac{\bar{V}_{RY}}{Z} = \frac{220 \angle 0^\circ}{15 + j20} = 8.8 \angle -53.13^\circ A$$

Now, in balanced Δ -connected system all line currents and phase currents are 120° apart. Using this fact, we can write

$\tilde{I}_Y = 8.8 \angle (-53.13^\circ - 120^\circ)$ [Magnitude is same as it is balanced system]
= $8.8 \angle -173.13^\circ A$

$$\tilde{I}_B = 8.8 \angle (-53.13^\circ + 120^\circ)$$

$$= 8.8 \angle 66.87^\circ A$$



OR $\tilde{I}_V = \frac{\tilde{V}_{YB}}{Z} = \frac{220 \angle -120^\circ}{15 + j20} = 8.8 \angle -173.13^\circ A$

$$\tilde{I}_B = \frac{\tilde{V}_{YB}}{Z} = \frac{220 \angle 120^\circ}{15 + j20} = 8.8 \angle 66.87^\circ A$$

Line currents

KCL at node R,

$$\tilde{I}_1 + \tilde{I}_B = \tilde{I}_R$$

$$\tilde{I}_1 = \tilde{I}_R - \tilde{I}_B$$

$$= (8.8 \angle -53.13^\circ) - (8.8 \angle 66.87^\circ) = 15.24 \angle -83.13^\circ A$$

Similarly,

KCL at node Y,

$$\tilde{I}_2 + \tilde{I}_A = \tilde{I}_Y$$

$$\tilde{I}_2 = \tilde{I}_Y - \tilde{I}_A$$

$$= (8.8 \angle -173.13^\circ) - (8.8 \angle -53.13^\circ)$$

$$= 15.24 \angle 156.87^\circ A$$

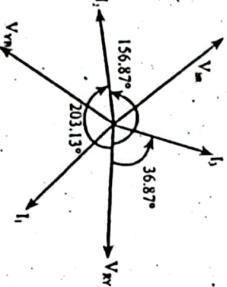
KCL at node B,

$$\tilde{I}_3 + \tilde{I}_V = \tilde{I}_B$$

$$\tilde{I}_3 = \tilde{I}_B - \tilde{I}_V$$

$$= (8.8 \angle 66.87^\circ) - (8.8 \angle -173.13^\circ) = 15.24 \angle 36.87^\circ A$$

OR



Solution:

Given,

$$V_L = 440 V$$

For Δ - connected load

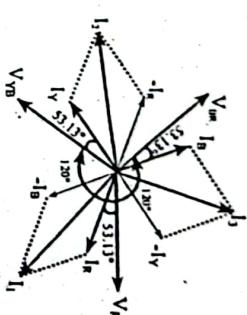
$$V_L = V_{ph} = 440 V$$

$$The phase difference \phi = 0^\circ - (-53.13^\circ) = 53.13^\circ$$

$$\therefore P_R = 220 \times 8.8 \times \cos 53.13^\circ = 1161.6027 \text{ watts}$$

$$\text{Power consumed per phase} = 1161.6027 \text{ watts}$$

(iv) Phasor diagram



(v) Phasor sum of three line currents

$$= \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3$$

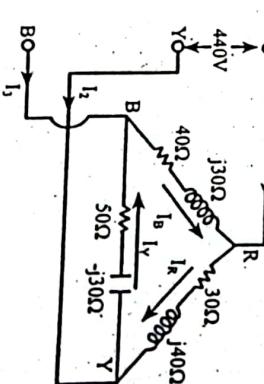
$$= (15.24 \angle -83.13^\circ) + (15.24 \angle 156.87^\circ) + (15.24 \angle 36.87^\circ)$$

$$= 0$$

The phasor sum of three line currents is zero because the line currents are equal in magnitude as in a balanced system and have a phase difference of 120° amongst themselves.

13. Explain two - wattmeter method for the measurement of power in a balanced three phase load. [2070 Chairita]
[Please refer to the theory]

14. For the delta connected load, find the phase currents, line currents, power (active, reactive and apparent) in each phases. Also determine the total active power consumed.



$$\tilde{V}_{RY} = 440 \angle 0^\circ V$$

$$-\tilde{V}_{YB} = 440 \angle -120^\circ V$$

$$\tilde{V}_{BR} = 440 \angle 120^\circ V$$

Phase Currents

$$\tilde{I}_R = \frac{\tilde{V}_{RY}}{Z_1} = \frac{440 \angle 0^\circ}{30 + j40} = 8.8 \angle -53.13^\circ A$$

$$\tilde{I}_Y = \frac{\tilde{V}_{YB}}{Z_2} = \frac{440 \angle -120^\circ}{50 - j30} = 7.54 \angle -89.03^\circ A$$

$$\tilde{I}_B = \frac{\tilde{V}_{BR}}{Z_3} = \frac{440 \angle 120^\circ}{40 + j30} = 8.8 \angle 83.13^\circ A$$

Line currents:

At node R, $\tilde{I}_1 + \tilde{I}_B = \tilde{I}_R$

$$\tilde{I}_1 = \tilde{I}_R - \tilde{I}_B = (8.8 \angle -53.13^\circ) - (8.8 \angle 83.13^\circ) = 16.33 \angle -75^\circ A$$

At node Y, $\tilde{I}_2 + \tilde{I}_R = \tilde{I}_Y$

$$\begin{aligned}\tilde{I}_2 &= \tilde{I}_Y - \tilde{I}_R = (8.8 \angle 83.13^\circ) - (8.8 \angle -53.13^\circ) \\ &= 5.176 \angle -174.47^\circ A\end{aligned}$$

At node B, $\tilde{I}_3 + \tilde{I}_Y = \tilde{I}_B$

$$\tilde{I}_3 = \tilde{I}_B - \tilde{I}_Y = (8.8 \angle 83.13^\circ) - (7.54 \angle -89.03^\circ) = 16.301 \angle 86.75^\circ A$$

Powers in each phases

Power in R phase

$$\phi_R = 0^\circ - (-53.13^\circ) = 53.13^\circ$$

$$\text{Active power } (P_R) = V_{RY} I_R \cos \phi_R$$

$$= 440 \times 8.8 \times \cos(53.13^\circ)$$

$$= 2323.205 W$$

Reactive power (Q_R) = $V_{RY} I_R \sin \phi_R$

$$= 440 \times 8.8 \times \sin(53.13^\circ)$$

$$= 3097.595 VAr$$

$$\text{Apparent power } (S_R) = V_{RY} I_R = 440 \times 8.8 = 3872 VA$$

Power in Y phase

$$\phi_Y = -120^\circ - (-89.03^\circ) = -30.97^\circ$$

$$P_Y = V_{YB} I_Y \cos \phi_Y = 440 \times 7.54 \times \cos(-30.97^\circ)$$

$$= 2844.63 W$$

$$Q_Y = V_{YB} I_Y \sin \phi_Y = 440 \times 7.54 \times \sin(-30.97^\circ) = -1707.2 VAr$$

$$S_Y = V_{YB} I_Y = 440 \times 7.54 = 3317.6 VA$$

Power in B phase

$$\phi_B = 120^\circ - 83.13^\circ = 36.87^\circ$$

$$P_B = 440 \times 8.8 \cos(36.87^\circ) = 3097.595 W$$

$$Q_B = 440 \times 8.8 \sin(36.87^\circ) = 2323.205 VAR$$

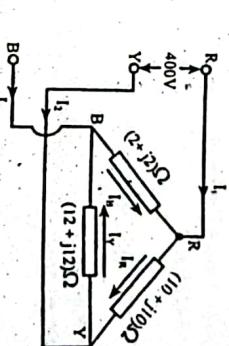
$$S_B = 440 \times 8.8 = 3872 VA$$

$$\text{Total active power consumed } (P) = P_R + P_Y + P_B$$

$$= 2323.205 + 2844.63 + 3097.595 = 8265.43 W$$

15. Three impedances of $(10 + j10)\Omega$, $(12 + j12)\Omega$ and $(2 + j2)\Omega$ are connected in delta to a 3-phase system with line voltage 400 V. Calculate all the phase currents, line currents, active powers, reactive powers and apparent power.

[2070 Bhadra]



Given, $V_L = 400 V$

In Δ -connected system, $V_L = V_{ph} = 400 V$

$$\tilde{V}_{RY} = 400 \angle 0^\circ V$$

$$\tilde{V}_{YB} = 400 \angle -120^\circ V$$

$$\tilde{V}_{BR} = 400 \angle 120^\circ V$$

$$Z_1 = 10 + j10 \Omega$$

$$Z_2 = 12 + j12 \Omega$$

$$Z_3 = 2 + j2 \Omega$$

Phase Currents

$$\tilde{I}_R = \frac{\tilde{V}_{RY}}{Z_1} = \frac{400 \angle 0^\circ}{10 + j10} = 28.28 \angle -45^\circ A$$

$$\tilde{I}_Y = \frac{\tilde{V}_{YB}}{Z_2} = \frac{400 \angle -120^\circ}{12 + j12} = 23.57 \angle -165^\circ A$$

$$\tilde{I}_B = \frac{\tilde{V}_{BR}}{Z_3} = \frac{400 \angle 120^\circ}{2 + j2} = 141.42 \angle 75^\circ A$$

Line Currents

$$\tilde{I}_1 = \tilde{I}_R - \tilde{I}_B = (28.28 \angle -45^\circ) - (141.42 \angle 75^\circ)$$

$$= 157.47 \angle -96.05^\circ A$$

$$\tilde{I}_2 = \tilde{I}_Y - \tilde{I}_R = (23.57^\circ - 165^\circ) - (23.57^\circ - 165^\circ) = 154.56^\circ \angle 67.41^\circ A$$

$$= 44.96 \angle 161.997^\circ A = 44.96 \angle 162^\circ A$$

Power in R phase

$$\Phi_R = 0 - (-45^\circ) = 45^\circ$$

Active power (P_R) = $V_{RN} I_R \cos \Phi_R = 400 \times 28.28 \times \cos 45^\circ = 7998.79 W$

Reactive power (Q_R) = $V_{RN} I_R \sin \Phi_R = 400 \times 28.28 \times \sin 45^\circ = 7998.79 VAr$

Apparent power (S_R) = $V_{RN} I_R = 400 \times 28.28 = 11312 VA$

Power in Y phase

$$\Phi_Y = -120^\circ - (-165^\circ) = 45^\circ$$

$P_Y = V_{RN} I_Y \cos \Phi_Y = 400 \times 23.57 \times \cos 45^\circ = 6666.602 W$

$Q_Y = V_{RN} I_Y \sin \Phi_Y = 400 \times 23.57 \times \sin 45^\circ = 6666.602 VAr$

$S_Y = V_{RN} I_Y = 400 \times 23.57 = 9428 VA$

Powers in B Phase

$$\Phi_B = 120^\circ - 75^\circ = 45^\circ$$

$P_B = 400 \times 141.42 \times \cos 45^\circ = 39999.61 W$

$Q_B = 400 \times 141.42 \times \sin 45^\circ = 39999.61 VAr$

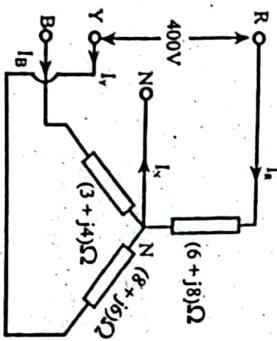
$S_B = 400 \times 141.42 = 56568 VA$

16. With the help of necessary phasor diagram and circuit diagram, explain the two wattmeter method of Active Power Measurement in three Phase AC system? What is the Variation of wattmeter readings with load Power Factor? [2009 Ashad]

[Please refer to the theory]

17. Three impedances of $(6+j8)\Omega$, $(8+j6)\Omega$, $(3+j4)\Omega$ are connected in star to a 3-phase, 4-wire system for which the line voltage is 400 V. Find the line currents, and active and reactive and apparent power per phase. Also find the current through neutral wire.

[2009 Ashad]



$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 V$$

In Y-connected system,

Line current = phase current

$$I_L = I_{ph}$$

$$Z_1 = (6+j8)\Omega \quad Z_2 = (8+j6)\Omega \quad Z_3' = (3+j4)\Omega$$

$$\tilde{V}_{RN} = 230.94 \angle 0^\circ V$$

$$\tilde{V}_{VN} = 230.94 \angle -120^\circ V$$

$$\tilde{V}_{BN} = 230.94 \angle 120^\circ V$$

$$\tilde{I}_R = \frac{\tilde{V}_{RN}}{Z_1} = \frac{230.94 \angle 0^\circ}{(6+j8)} = 23.094 \angle -53.13^\circ A$$

$$\tilde{I}_Y = \frac{\tilde{V}_{VN}}{Z_2} = \frac{230.94 \angle -120^\circ}{(8+j6)} = 23.094 \angle -156.87^\circ A$$

$$\tilde{I}_B = \frac{\tilde{V}_{BN}}{Z_3'} = \frac{230.94 \angle 120^\circ}{(3+j4)} = 46.188 \angle 66.87^\circ A$$

Current through neutral wire $\tilde{I}_N = \tilde{I}_R + \tilde{I}_Y + \tilde{I}_B = 18.403 \angle 54.21^\circ A$

Power in R phase

$$\Phi_R = 0^\circ - (-53.13^\circ) = 53.13^\circ$$

Active power (P_R) = $V_{RN} I_R \cos \Phi_R = 230.94 \times 23.094 \times \cos 53.13^\circ = 3200 W$

Reactive power (Q_R) = $V_{RN} I_R \sin \Phi_R = 230.94 \times 23.094 \times \sin 53.13^\circ = 4266.65 VAr$

Apparent power (S_R) = $V_{RN} I_R = 230.94 \times 23.094 = 5333.33 VA$

Power in Y Phase

$$\Phi_Y = -120^\circ - (-156.87^\circ) = 36.87^\circ$$

$P_Y = 230.94 \times 23.094 \times \cos 36.87^\circ = 4266.65 W$

$Q_Y = 230.94 \times 23.094 \times \sin 36.87^\circ = 3200 VAr$

$S_Y = 230.94 \times 23.094 = 5333.33 VA$

Power in B phase

$$\Phi_B = 120^\circ - (66.87^\circ) = 53.13^\circ$$

$P_B = 230.94 \times 46.188 \times \cos 53.13^\circ = 6400 W$

$Q_B = 230.94 \times 46.188 \times \sin 53.13^\circ = 8533.31 VAr$

$S_B = 230.94 \times 46.188 = 10666.66 VA$

Solution,
Given,

$$V_L = 400 V$$

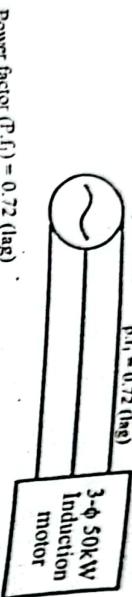
18. A three phase induction motor takes 50 kW at 415V, 50Hz and a power factor of 0.72 lagging. Determine the kVAR rating of capacitor bank to improve the power factor to 0.9 lagging. What capacitance per phase is required if the capacitor bank is connected in star connection? What is the advantage of power factor correction from the source point of view of motor itself?

[2009 Chaitra]

Solution.

Given, 415 V, 50Hz, $P_f_1 = 0.72$ (lag)

$$415V, 50\text{ Hz} \quad P_f_1 = 0.72 \text{ (lag)}$$



$$\text{Power factor } (P.f.) = 0.72 \text{ (lag)}$$

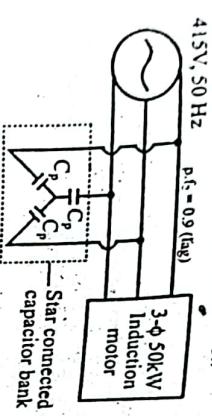
$$\Rightarrow \phi_1 = \cos^{-1}(0.72) = 43.94^\circ$$

kVAR rating of capacitor bank = ?

$$P.f. = 0.9 \text{ (lag)}$$

$$\Rightarrow \phi_1 = \cos^{-1}(0.9) = 25.84^\circ$$

Now, given that the capacitor bank is connected to star connection



$$\text{Power per phase } (P_p) = \frac{50}{3} \text{ kW} = 16.67 \text{ kW}$$

Now, Reactive power supplied when $P.f_1 = 0.72$ per phase

$$Q_{1p} = P_p \tan \phi_1$$

Reactive power supplied when $P.f_1 = 0.9$ per phase

$$Q_{2p} = P_p \tan \phi_2$$

From figure

$$\text{kVAR rating per phase } Q_{CP} = Q_{1p} - Q_{2p}$$

$$= P_p \tan \phi_1 - P_p \tan \phi_2$$

$$= P_p (\tan \phi_1 - \tan \phi_2)$$

$$= 16.67 (\tan 43.94^\circ - \tan 25.84^\circ)$$

$$= 7.99 \text{ kVAR} \approx 8 \text{ kVAR}$$

\therefore Required kVAR rating of capacitor bank

$$Q_C = 3 \times Q_{CP} = 3 \times 8 = 24 \text{ kVAR}$$

We know, $Q_{CP} = \omega V_{ph}^2 C_p$

$$C_p = \frac{Q_{CP}}{\omega V_{ph}^2}$$

Now, since star - connection

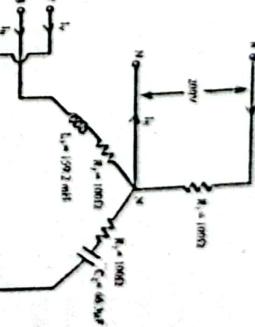
$$V_{ph} = \frac{V}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$

$$C_p = \frac{8 \times 10^4}{27050 \times (219.6)^2} = 4.435 \times 10^{-6} \text{ F} = 443.57 \mu\text{F}$$

\therefore Required capacitance per phase = 443.57 μF

19. In a 3 - phase, 4 wire Wye connected system the phase voltage $V_{ph} = 200\text{V}$ and its frequency is 60 Hz. The load impedance components are $R_L = 100\Omega$, $R_s = 100\Omega$, $C_s = 66.3 \mu\text{F}$, $R_d = 100\Omega$, $L_d = 159.2 \text{ mH}$. Calculate the three line currents and the neutral current.

[2009 Chairra]



Solution.
Given, 3φ, 4 wire Y - connected system.

$$V_{ph} = 200 \text{ V}, 60 \text{ Hz}$$

$$Z_1 = 100 \Omega$$

$$Z_2 = 100 - j \frac{1}{2 \times \pi \times 60 \times 66.3 \times 10^{-6}}$$

$$= 100 - j 40 \Omega$$

$$Z_3 = 100 + j 2\pi \times 60 \times 159.2 \times 10^{-3} = 100 + j 600$$

In Y - connected system

$$\therefore \tilde{I}_R = \frac{\tilde{V}_{RN}}{Z_2} = \frac{200 \angle 0^\circ}{100} = 2 \angle 0^\circ \text{ A}$$

$$\therefore \tilde{I}_Y = \frac{\tilde{V}_{RN}}{Z_3} = \frac{200 \angle -120^\circ}{100 - j 40} = 1.85 \angle -98.2^\circ \text{ A}$$

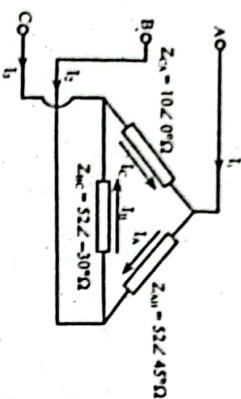
$$\therefore \tilde{I}_B = \frac{\tilde{V}_{RN}}{Z_1} = \frac{200 \angle 120^\circ}{100 + j 600} = 1.71 \angle 89.04^\circ \text{ A}$$

Neutral current $\tilde{I}_N = \tilde{I}_R + \tilde{I}_Y + \tilde{I}_B = 2 \angle 0^\circ + 1.85 \angle -98.2^\circ + 1.71 \angle 89.04^\circ$

$$= 1.77 \angle -3.93^\circ \text{ A}$$

20. A delta connected load of $Z_{AB} = 52 \angle 45^\circ \Omega$, $Z_{AC} = 52 \angle -30^\circ \Omega$ and $Z_{CB} = 10 \angle 0^\circ \Omega$ are connected to a 380 V, 3 phase AC source. Find the magnitude of the line currents and total power absorbed by loads, when phase sequence is ABC.

[2008 Chairra]



Solution:

380V, 3φ ac source

$$Z_{AB} = 52 \angle 45^\circ \Omega$$

$$Z_{AC} = 52 \angle -30^\circ \Omega$$

$$Z_{BC} = 10 \angle 0^\circ \Omega$$

In Δ -connected system

$$V_L = V_{Ph} = 380 \text{ V}$$

$\tilde{V}_{AB} = 380 \angle 0^\circ \text{ V}$

$\tilde{V}_{BC} = 380 \angle -120^\circ \text{ V}$

$\tilde{V}_{CA} = 380 \angle 120^\circ \text{ V}$

Phase Currents

$$\tilde{I}_A = \frac{\tilde{V}_{AB}}{Z_{AB}} = \frac{380 \angle 0^\circ}{52 \angle 45^\circ} = 7.3 \angle -45^\circ \text{ A}$$

$$\tilde{I}_B = \frac{\tilde{V}_{BC}}{Z_{BC}} = \frac{380 \angle -120^\circ}{52 \angle -30^\circ} = 7.3 \angle -90^\circ \text{ A}$$

$$\tilde{I}_C = \frac{\tilde{V}_{CA}}{Z_{CA}} = \frac{380 \angle 120^\circ}{10 \angle 0^\circ} = 3.8 \angle 120^\circ \text{ A}$$

Line Currents

$$\tilde{I}_1 = \tilde{I}_A - \tilde{I}_C = (7.3 \angle -45^\circ) - (3.8 \angle 120^\circ) = 11.01 \angle -50.12^\circ \text{ A}$$

$$\tilde{I}_2 = \tilde{I}_B - \tilde{I}_A = (7.3 \angle -90^\circ) - (7.3 \angle -45^\circ) = 5.58 \angle -157.5^\circ \text{ A}$$

$$\tilde{I}_3 = \tilde{I}_C - \tilde{I}_B = (3.8 \angle 120^\circ) - (7.3 \angle -90^\circ) = 10.76 \angle 100.17^\circ \text{ A}$$

Active power in A phase (P_A)

$$= V_{AB} I_A \cos \phi_A$$

$$= 380 \times 7.3 \cos [0^\circ - (-45^\circ)]$$

$$= 380 \times 7.3 \cos 45^\circ = 1961.514 \text{ W}$$

Active power in B phase (P_B)

$$= V_{BC} I_B \cos \phi_B$$

$$= 380 \times 7.3 \cos [-120^\circ - (-90^\circ)]$$

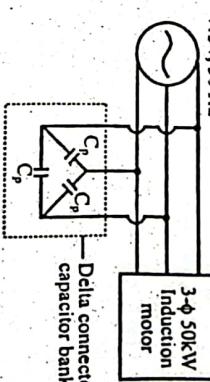
$$= 2402.35 \text{ W}$$

$$\begin{aligned} \text{Active power in C phase } (P_C) &= V_{CA} I_C \cos \phi_C \\ &= 380 \times 3.8 \cos [120^\circ - 120^\circ] = 1444 \text{ W} \\ \text{Total power absorbed by loads} &= P_A + P_B + P_C \\ &= 1961.514 + 2402.35 + 1444 \\ &= 5807.864 \text{ W} \end{aligned}$$

21. What are the advantages of three phase AC system over single phase ac system?
[Please refer to the theory]

22. A 415V, 3 phase, 50 Hz induction motor takes 50 kW power from supply mains at 0.72 power factor lagging. Capacitors are connected in delta across the line to improve the overall power factor. Calculate the capacitance per phase in order to raise the power factor to 0.9 lagging.

[2008 Baisakhi]



Solution

$$\text{Now, Power per phase } (P_p) = \frac{50}{3} \text{ kW} = 16.67 \text{ kW}$$

$$P.f_l = 0.72 \text{ (lag)}$$

$$\Rightarrow \phi_1 = \cos^{-1}(0.72) = 43.94^\circ$$

$$P.f_2 = 0.9 \text{ (lag)}$$

$$\Rightarrow \phi_2 = \cos^{-1}(0.9) = 25.84^\circ$$

From figure,

$$Q_{1P} = P_p \tan \phi_1 = 16.67 \times \tan 43.94^\circ = 16.06 \text{ kVAR}$$

$$Q_{2P} = P_p \tan \phi_2 = 16.67 \times \tan 25.84^\circ = 8.07 \text{ kVAR}$$

$$Q_{CP} = Q_{1P} - Q_{2P} = 16.06 - 8.07 \approx 8 \text{ kVAR}$$

Since Δ -connected capacitor bank

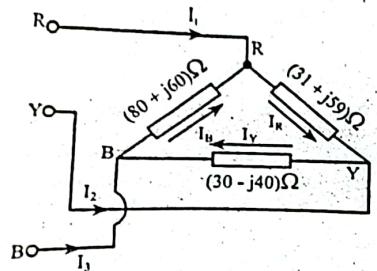
$$V_L = V_{Ph} = 415 \text{ V}$$

$$Q_{CP} = \omega V_{Ph}^2 C_p$$

$$C_p = \frac{Q_{CP}}{\omega V_{Ph}^2} = \frac{8 \times 10^3}{2\pi \times 50 \times (415)^2} = 147.86 \mu\text{F}$$

: Required capacitance per phase = 147.86 μF

23. Three phase loads $(31 + j 59)\Omega$, $(30 - j 40)\Omega$ and $(80 + j 60)\Omega$ are connected in delta to a 3 phase, 200V supply. Find the phase currents. Line currents and total power absorbed. [2068 Baisakh]



Solution:

In Δ -connected system
 $V_L = V_{ph} = 200V$

Phase Currents

$$\tilde{I}_R = \frac{220 \angle 0^\circ}{31 + j59} = 3.3 \angle -62.28^\circ A$$

$$\tilde{I}_Y = \frac{220 \angle -120^\circ}{30 - j40} = 4.4 \angle -66.87^\circ A$$

$$\tilde{I}_B = \frac{220 \angle 120^\circ}{80 + j60} = 2.2 \angle 83.13^\circ A$$

Line currents

$$\tilde{I}_1 = \tilde{I}_R - \tilde{I}_B = (3.3 \angle -62.28^\circ) - (2.2 \angle 83.13^\circ) = 5.26 \angle -76.01^\circ A$$

$$\tilde{I}_2 = \tilde{I}_Y - \tilde{I}_R = (4.4 \angle -66.87^\circ) - (3.3 \angle -62.28^\circ) = 1.14 \angle -80.24^\circ A$$

$$\tilde{I}_3 = \tilde{I}_B - \tilde{I}_Y = (2.2 \angle 83.13^\circ) - (4.4 \angle -66.87^\circ) = 6.4 \angle 103.23^\circ A$$

$$P_R = V_{RY} I_R \cos \phi_R = 200 \times 3.3 \times \cos(0^\circ - (-62.28^\circ)) = 306.997 W = 307 W$$

$$P_Y = V_{YB} I_Y \cos \phi_Y = 200 \times 4.4 \times \cos(-120^\circ - (-66.87^\circ)) = 528 W$$

$$P_B = V_{BR} I_B \cos \phi_B = 200 \times 2.2 \times \cos(120^\circ - 83.13^\circ) = 351.9995 W \approx 352 W$$

$$\text{Total power absorbed (P)} = P_R + P_Y + P_B = 307 + 528 + 352 = 1187 W$$

24. A balanced star connected load with impedance $(10 + j3)\Omega$ per phase is fed from a balanced 3 phase 400 volt supply. Calculate:

i) The phase voltages

ii) The line currents

iii) The power absorbed and

iv) Draw the phasor diagram. [2067 Mangsir]

Solution: $Z = (10 + j3)\Omega$

$V_L = 400V$

(i) Phase voltages

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 V$

$\tilde{V}_{RN} = 230.94 \angle 0^\circ V$

$\tilde{V}_{YN} = 230.94 \angle -120^\circ V$

$\tilde{V}_{BN} = 230.94 \angle 120^\circ V$

(ii) Line currents

In Y-connected system,

Line currents = phase currents

$\tilde{I}_R = \frac{\tilde{V}_{RN}}{Z} = \frac{230.94 \angle 0^\circ}{(10 + j3)} = 20.65 \angle -26.56^\circ A$

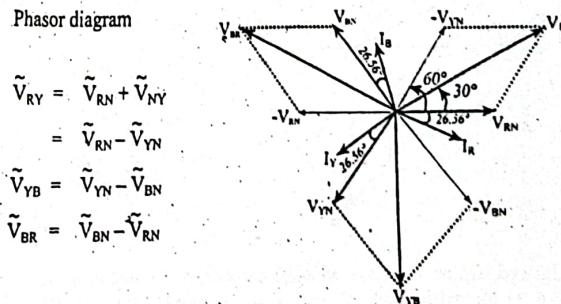
$\tilde{I}_Y = 20.65 \angle (-26.56^\circ - 120^\circ) = 20.65 \angle -146.56^\circ A$

$\tilde{I}_B = 20.65 \angle (-26.56^\circ + 120^\circ) = 20.65 \angle 93.44^\circ A$

(iii) Power absorbed (P) = $3V_{ph} I_{ph} \cos \phi$
 $= 3 \times 230.94 \times 20.65 \times \cos(26.56^\circ) = 12796.895 \text{ watts}$

OR $P = \sqrt{3} V_L I_L \cos \phi$
 $= \sqrt{3} \times 400 \times 20.65 \times \cos(26.56^\circ)$
 $= 12796.9 \text{ watts}$

(iv) Phasor diagram



25. A balanced star connected load of $(8 + j 6)\Omega$ per phase is connected to a three-phase, 50Hz, 380 V supply. Find the line current, power factor, active power, reactive power and total volt-amps. Also draw the phasor diagram. [2065 Kartik]

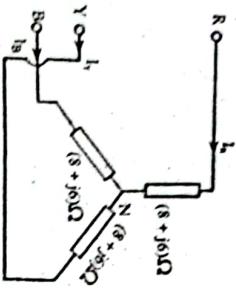
Solution:

$V_L = 380 V$

$$V_m = \frac{380}{\sqrt{3}} V = 219.39 V$$

$$Z = (8 + j6)\Omega$$

Phase current



In Y - connected system, line current = phase current

$$\tilde{I}_B = \frac{\tilde{V}_{BN}}{Z} = \frac{219.39 \angle 0^\circ}{8 + j6} = 21.94 \angle -36.87^\circ A$$

(i) Line current (I_L) = 21.94 A(ii) P.f. = $\cos \phi = \cos (0 - (-36.87)) = \cos 36.87 = 0.8$ (lag)(iii) Active power (P) = $\sqrt{3} V_L I_L \cos \phi$

$$= \sqrt{3} \times 380 \times 21.94 \times 0.8 \\ = 11552.36 W$$

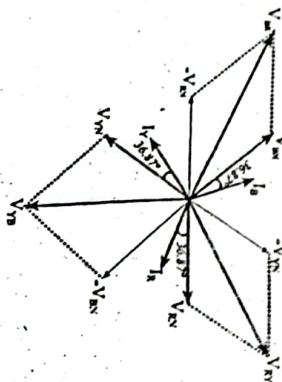
(v) Reactive power (Q) = $\sqrt{3} V_L I_L \sin \phi$

$$= \sqrt{3} \times 380 \times 21.94 \times \sin (36.87^\circ) \\ = 8664.293 \text{ VAR}$$

(v) Total volt-amp (S)

$$= \sqrt{3} V_L I_L \\ = \sqrt{3} \times 380 \times 21.94$$

$$= 14440.45 \text{ VA}$$



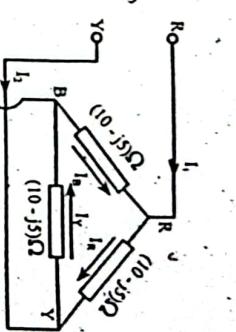
26.

A 380 V balanced three - phase voltage source is supplying a delta connected load bank with each of the loads equal to $10 - j5$ Ohm. Determine the phase current, line current and magnitude of voltage across each of the loads. Also calculate total active (real) power consumption of the load and power factor. Construct the phasor diagram of currents and voltages in the circuit.

[2002 Bhadral]

In Δ -connected system
 $V_L = 380 V$
 $V_{ph} = 380 V$
 $Z = (10 - j5)\Omega$

Phase current



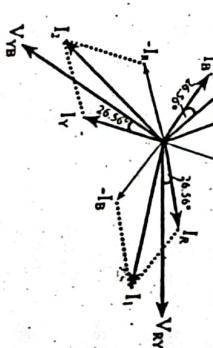
Magnitude of voltage across each of the loads = 380 V

Total active power consumption of the load

$$P = \sqrt{3} V_L I_L \cos \phi \\ = \sqrt{3} \times 380 \times 58.89 \cos (0^\circ - 26.56^\circ) \\ = \sqrt{3} \times 380 \times 58.89 \cos (-26.56^\circ) \\ = 34669.68 W$$

Power factor ($P.f.$) = $\cos (0^\circ - 26.56^\circ)$

$$= 0.894 \text{ (lead) [as current leads the voltage]}$$

[As Δ connected $V_L = V_{ph}$]

27.

The power input to a motor is measured by two wattmeters, which indicate 40 kW and 50 kW respectively. If the power factor of the motor be changed to 0.8 leading, determine the reading of two wattmeters. The total input power remains the same. Draw vector diagram for the second condition of load.

[2006 Mugh]

Given,
 Total input power = $40 \text{ kW} + 50 \text{ kW}$
 = 90 kW

Total power factor (P.f) = 0.8 (lead)

Let W_1 and W_2 be the required wattmeter readings.

As total input power remains the same

$$W_1 + W_2 = 90 \text{ kW} \quad \dots \dots \dots \text{(i)}$$

We know,

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi \quad \dots \dots \dots \text{(ii)}$$

$$W_1 - W_2 = V_L I_L \sin \phi \quad \dots \dots \dots \text{(iii)}$$

Dividing (iii) by (ii)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi}$$

$$\text{or, } \frac{W_1 - W_2}{W_1 + W_2} = \frac{\tan \phi}{\sqrt{3}} \quad \dots \dots \dots \text{(iv)}$$

We have,

$$\text{P.f} = 0.8$$

$$\cos \phi = 0.8$$

$$\therefore \phi = 36.87^\circ$$

Putting in equation (iv)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\tan 36.87^\circ}{\sqrt{3}}$$

$$\Rightarrow W_1 - W_2 = 0.43 (W_1 + W_2)$$

$$\Rightarrow W_1 - W_2 - 0.43 W_1 - 0.43 W_2 = 0$$

$$\Rightarrow 0.57 W_1 - 1.43 W_2 = 0 \quad \dots \dots \dots \text{(v)}$$

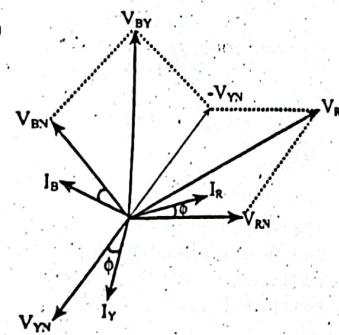
Solving (i) & (v)

$$\therefore W_1 = 64.35 \text{ kW}$$

$$\therefore W_2 = 25.65 \text{ kW}$$

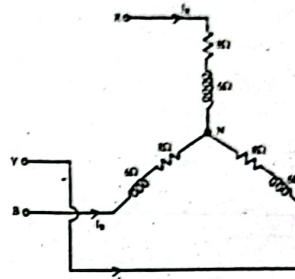
The required readings of two wattmeters are 64.35 kW and 25.65 kW

Considering star-connected load



Additional Problems

1. A star-connected three phase load has a resistance of 8Ω and an inductive reactance of 6Ω in each phase. It is fed from a $400V$, $3-\phi$ balanced supply. Determine the line current power factor, active and reactive power. Draw phasor diagram showing phase and line voltages and currents. If power measurements is made using two wattmeter method, what will be the readings of both wattmeters?



Solution:

Given,

$400 \text{ V}, 3-\phi$ balanced supply

$$Z = 8 + j6 \Omega$$

For star-connected system,

line current = phase current

$$I_L = I_{ph}$$

$$V_L = 400 \text{ V}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

Line currents

$$\tilde{I}_R = \frac{\tilde{V}_{RN}}{Z} = \frac{230.94 \angle 0^\circ}{8 + j6} = 23.094 \angle -36.86^\circ \text{ A}$$

$$\tilde{I}_Y = 23.094 \angle (-36.86^\circ - 120^\circ) = 23.094 \angle -156.86^\circ \text{ A}$$

$$\tilde{I}_B = 23.094 \angle (-36.86^\circ + 120^\circ) = 23.094 \angle 83.14^\circ \text{ A}$$

$$\begin{aligned} \text{Power factor (P.f)} &= \cos \phi \\ &= \cos (0^\circ - (-36.86^\circ)) \\ &= 0.8 \text{ (lagging)} \end{aligned}$$

$$\text{Active power (P)} = 3V_{ph} I_{ph} \cos \phi$$

$$= 3 \times 230.94 \times 23.094 \times 0.8$$

$$= 12,799.99 \text{ W}$$

$$\text{Reactive power } (Q) = 3 V_{ph} I_{ph} \sin \phi$$

$$= 3 \times 230.94 \times 23.094 \times \sin(36.86^\circ)$$

$$= 9297.78 \text{ VAR}$$

$$V_{ph} = 230.94 \text{ V}$$

$$I_{ph} = 23.094 \text{ A}$$

$$\sin 36.86^\circ = 0.600$$

$$\therefore \text{Reactive power } (Q) = 3 \times 230.94 \times 23.094 \times 0.600$$

$$= 13,800 \text{ VAR}$$

$$V_{ph} = 230.94 \text{ V}$$

$$I_{ph} = 23.094 \text{ A}$$

$$\cos 36.86^\circ = 0.8$$

$$\therefore \text{Active power } (P) = 3 \times 230.94 \times 23.094 \times 0.8$$

$$= 12,799.99 \text{ W}$$

$$V_{ph} = 230.94 \text{ V}$$

$$I_{ph} = 23.094 \text{ A}$$

$$\tan 36.86^\circ = 0.75$$

$$\therefore \tan \phi = -0.75$$

$$\therefore \phi = -36.869^\circ$$

$$\therefore \text{power factor (PF)} = \cos \phi$$

$$= \cos(-36.869^\circ)$$

$$= 0.8 \text{ (lead)}$$

We have,

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$\text{or, } 2762 = \sqrt{3} \times 416 \times I_L \times 0.8$$

$$\text{or, } I_L = \frac{2762}{\sqrt{3} \times 416 \times 0.8}$$

$$\therefore \text{line current (I}_L\text{)} = 4.79 \text{ A}$$

Now, Impedance of the circuit per phase

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{V_L \sqrt{3}}{I_L}$$

$$= \frac{\sqrt{3}}{4.79}$$

$$= 50.14 \Omega$$

Reactance per phase,

$$X_{ph} = Z_{ph} \sin \phi$$

$$= 50.14 \times \sin(36.869^\circ)$$

$$= 30.08 \Omega$$

$$X_{ph} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$\text{or, } C = \frac{1}{2\pi f X_{ph}} = \frac{1}{2\pi \times 50 \times 30.08} = 105.82 \mu\text{F}$$

\therefore Capacitance of each

capacitor = 105.82 μF

3. The power input to a synchronous motor is measured by two wattmeters both of which indicate 50 kW. If the pf of the motor be changed to 0.866 leading determine the readings of the wattmeters, the total input power remaining the same.

Solution:

Let the readings of the wattmeters for second condition be W_1 and W_2 .

Total input power of load = $2 \times 50 \text{ kW}$

$$W_1 + W_2 = 100 \text{ kW} \dots (1)$$

$$\text{pf} = 0.866 \text{ (leading)}$$

$$W_1 = 782 \text{ W}$$

$$W_2 = 1980 \text{ W}$$

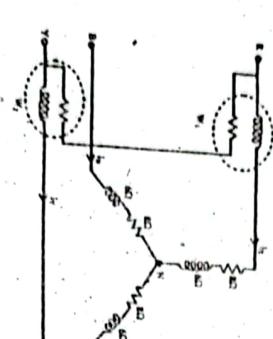
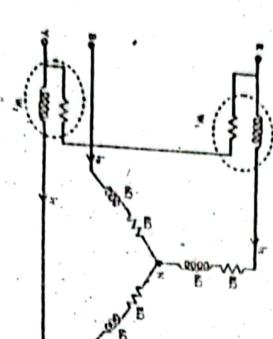
$$\text{Total active power } P = W_1 + W_2 = 782 + 1980 = 2762 \text{ W}$$

Given,

3 ϕ , 416 V, 50Hz supply

$W_1 = 782 \text{ W}$

$W_2 = 1980 \text{ W}$



Voltmeter readings.

$$W_1 = V_{RB} I_R \cos(30^\circ - \phi)$$

$$= 400 \times 23.094 \cos(30^\circ - 36.86^\circ)$$

$$= 9171.47 \text{ W}$$

$$W_2 = V_{RB} I_Y \cos(30^\circ + \phi)$$

$$= 400 \times 23.094 \cos(30^\circ + 36.86^\circ) = 3630.18 \text{ W}$$

2. A star-connected balanced load is supplied from a 3 ϕ balanced supply with a line voltage of 416 V at a frequency of 50 Hz. Each phase of the load consists of a resistance and a capacitor joined in series and the readings of two wattmeters connected to measure the load power supplied are 782 W and 1,980 W, both positive. Calculate

- (i) power factor of the circuit
(ii) Line current
(iii) the capacitance of each capacitor

$$\cos\phi = 0.866$$

$$\phi = \cos^{-1}(0.866) = 30^\circ \text{ (lead)}$$

We have,

$$\tan\phi = \sqrt{3} \frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{or, } \tan 30^\circ = \sqrt{3} \frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{or, } 0.577 = \sqrt{3} \frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{or, } 0.577 = \sqrt{3} \frac{(W_1 - W_2)}{100}$$

$$\text{or, } W_1 - W_2 = \frac{100 \times 0.577}{\sqrt{3}} \text{ kW}$$

$$\therefore W_1 - W_2 = 33.313 \text{ kW} \quad \dots \dots \dots (2)$$

From eqns (1) and (2)

$$W_1 = 66.656 \text{ kW}$$

$$W_2 = 33.343 \text{ kW}$$

4. Two wattmeter method is used to measure the power taken by a 3- ϕ inductive motor on no-load. The wattmeter readings are 375W and -50W. Calculate.

- pf of the motor at no load,
- Phase difference of voltage and current in two wattmeters.
- Reactive power taken by the load.

Solution:

$$\text{Given, } W_1 = 375 \text{ W}$$

$$W_2 = -50 \text{ W}$$

i) We have,

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{or, } \tan\phi = \frac{\sqrt{3}(375 - (-50))}{(375 + (-50))}$$

$$\text{or, } \tan\phi = 2.2649$$

$$\text{or, } \phi = \tan^{-1}(2.2649)$$

$$\therefore \phi = 66.18^\circ$$

$$\text{Power factor (pf)} = \cos\phi = \cos(66.18^\circ) = 0.403$$

ii) Phase angle difference in W_1 Wattmeter

$$= 30^\circ - \phi$$

$$= 30^\circ - 66.18^\circ$$

$$= -36.18^\circ$$

Phase angle difference in W_2 Wattmeter

$$= 30^\circ + \phi = 30^\circ + 66.18^\circ = 96.18^\circ$$

iii) Reactive Power (Q)

$$= \sqrt{3}(W_1 - W_2) = \sqrt{3}(375 - (-50))$$

$$= 736.12 \text{ VAR}$$

5. A 3 ϕ motor load has pf of 0.397 lagging. Two wattmeter connected to measure power so that the input is 30 kW. Find reading on each wattmeters.

Solution:

Let W_1 and W_2 be readings on two wattmeters.

$$\text{pf} = 0.397$$

$$\cos\phi = 0.397$$

$$\therefore \phi = \cos^{-1}(0.397) = 66.61^\circ$$

Given,

Total input power = 30 kW

$$\therefore W_1 + W_2 = 30 \text{ kW} \quad \dots \dots \dots (i)$$

$$\tan\phi = \sqrt{3} \frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{Or, } \tan(66.61^\circ) = \sqrt{3} \frac{(W_1 - W_2)}{30 \text{ kW}}$$

$$\text{Or, } \frac{2.31 \times 30 \text{ kW}}{\sqrt{3}} = (W_1 - W_2)$$

$$\therefore W_1 - W_2 = 40.1 \text{ kW} \quad \dots \dots \dots (ii)$$

Solving (i) & (ii) We get,

$$W_1 = 35.01 \text{ kW}$$

$$W_2 = -5.01 \text{ kW}$$

6. A three-phase source delivers 4.8 kVA to a wye-connected load with a phase voltage of 208 V and a power factor of 0.9 lagging. Calculate the source line current and the source line voltage.

Solution:

Apparent power delivered (S) = 4.8 kVA

Phase voltage (V_{ph}) = 208V

Power factor pf = 0.9 (lagging)

$$\text{pf} = \cos\phi = 0.9$$

$$\therefore \phi = \cos^{-1}(0.9) = 25.84^\circ$$

Total Active Power (P) = $S \cos\phi$

$$= 4.8 \times 1000 \times \cos(25.84^\circ)$$

$$= 4320.07 \text{ W}$$

Total Reactive Power (Q) = $S \sin\phi$

$$= 4.8 \times 1000 \times \sin(25.84^\circ)$$

$$= 2092.12 \text{ VAR}$$

Total active Power (P) = $3V_{ph}I_{ph}\cos\phi$

$$\text{or, } 4320.07 = 3 \times 208 \times I_{ph} \times \cos(25.84^\circ)$$

$$= 4320.07$$

$$\text{or, } I_{ph} = \frac{3 \times 208 \times \cos(25.84^\circ)}{3 \times 208}$$

$$\therefore I_{ph} = 7.69 \text{ A}$$

Since load is wye-connected,

Phase current = line current

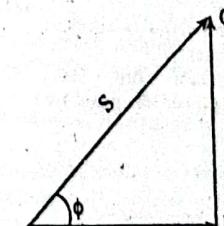
$$I_{ph} = I_L = 7.69 \text{ A}$$

The Source line current is 7.69 A

Line voltage = $\sqrt{3} \times \text{Phase voltage}$

$$= \sqrt{3} \times V_{ph} = \sqrt{3} \times 208 = 360.3 \text{ V}$$

The source line voltage is 360.3V.



7. Each phase load consists of a 20Ω resistor and a 10Ω inductive reactance. With a line voltage of $220V$ rms, calculate the average power taken by the load if
 (a) the three-phase loads are delta-connected.
 (b) the loads are wye-connected.

Solution:

$$Z = 20 + j10 \Omega$$

Line voltage $V_L = 220V$ rms

(a) Loads are delta-connected,

In Δ -connection,

Line voltage = Phase voltage

$$V_L = V_{ph} = 220V$$

$$\tilde{V}_R = 220 \angle 0^\circ V$$

$$\tilde{I}_R = \frac{\tilde{V}_R}{Z} = \frac{220 \angle 0^\circ}{20+j10} = 9.84 \angle -26.56^\circ A$$

$$\therefore \text{Average power (P)} = 3V_{ph} I_{ph} \cos\phi \\ = 3 \times 220 \times 9.84 \times \cos(0^\circ - (-26.56^\circ)) = 5809.02W$$

(b) Loads are wye-connected,

Phase voltage = Line voltage

Phase voltage = $\frac{\sqrt{3}}{\sqrt{3}}$

$$V_{ph} = \frac{V_1}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127.02V$$

$$\tilde{V}_R = 127.02 \angle 0^\circ V$$

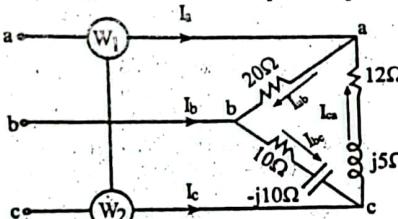
$$\tilde{I}_R = \frac{\tilde{V}_R}{Z} = \frac{127.02 \angle 0^\circ}{20+j10} = 5.68 \angle -26.56^\circ A$$

$$\therefore \text{Average power (P)} = 3V_{ph} I_{ph} \cos\phi \\ = 3 \times 127.02 \times 5.68 \times \cos(0^\circ - (-26.56^\circ)) = 1936W$$

8. In figure, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $V_{ab} = 208 \angle 0^\circ V$ with positive phase sequence.

a) Determine the reading of each wattmeter.

b) Calculate the total apparent power absorbed by the load.



Phase voltages:

$$\tilde{V}_{ab} = 208 \angle 0^\circ V$$

$$\tilde{V}_{bc} = 208 \angle -120^\circ V$$

$$\tilde{V}_{ca} = 208 \angle 120^\circ V$$

Phase currents:

$$\tilde{I}_{ab} = \frac{\tilde{V}_{ab}}{20} = \frac{208 \angle 0^\circ}{20} = 10.4 \angle 0^\circ A$$

$$\tilde{I}_{bc} = \frac{208 \angle -120^\circ}{10 - j10} = 14.71 \angle -75^\circ A$$

$$\tilde{I}_{ca} = \frac{208 \angle 120^\circ}{12 + j5} = 16 \angle 97.38^\circ A$$

Line Currents:

$$\tilde{I}_a + \tilde{I}_{ca} = \tilde{I}_{ab}$$

$$\therefore \tilde{I}_a = \tilde{I}_{ab} - \tilde{I}_{ca} = (10.4 \angle 0^\circ) - (16 \angle 97.38^\circ) = 20.17 \angle -51.86^\circ A$$

$$\tilde{I}_b + \tilde{I}_{ab} = \tilde{I}_{bc}$$

$$\therefore \tilde{I}_b = \tilde{I}_{bc} - \tilde{I}_{ab} = (14.71 \angle -75^\circ) - (10.4 \angle 0^\circ) = 15.66 \angle -114.89^\circ A$$

$$\tilde{I}_c + \tilde{I}_{bc} = \tilde{I}_{ca}$$

$$\therefore \tilde{I}_c = \tilde{I}_{ca} - \tilde{I}_{bc} = (16 \angle 97.38^\circ) - (14.71 \angle -75^\circ) = 30.64 \angle 101.03^\circ A$$

W₁ measures V_{ab} voltage and I_a line current.

$$\begin{aligned} W_1 &= V_{ab} I_a \cos\phi \\ &= 208 \times 20.17 \times \cos(0^\circ - (-51.86^\circ)) \\ &= 2590.99 W \end{aligned}$$

W₂ measures V_{cb} voltage and I_c line current.

$$\begin{aligned} \tilde{V}_{cb} &= -\tilde{V}_{bc} = -(208 \angle -120^\circ) = 208 \angle 60^\circ V \\ W_2 &= V_{cb} I_c \cos\phi \\ &= 208 \times 30.64 \times \cos(60^\circ - 101.03^\circ) \\ &= 4807.66 W \end{aligned}$$

Now,

Power absorbed by Phase A,

$$\begin{aligned} \text{Active power (P}_A\text{)} &= V_{ab} I_{ab} \cos\phi_a = 208 \times 10.4 \times \cos(0^\circ - 0^\circ) \\ &= 2163.2 W \end{aligned}$$

Reactive power (Q_A) = $V_{ab} I_{ab} \sin\phi_a = 208 \times 10.4 \times \sin 0^\circ = 0$

Power absorbed by phase B,

$$\begin{aligned} \text{Active power (P}_B\text{)} &= V_{bc} I_{bc} \cos\phi_b = 208 \times 14.71 \times \cos(-120^\circ - (-75^\circ)) \\ &= 2163.5 W \end{aligned}$$

$$\begin{aligned} \text{Reactive power (Q}_B\text{)} &= V_{bc} I_{bc} \sin\phi_b = 208 \times 14.71 \times \sin(-120^\circ - (-75^\circ)) \\ &= -2163.5 VAR \end{aligned}$$

Power absorbed by phase C,

$$\begin{aligned} \text{Active power (P}_C\text{)} &= V_{ca} I_{ca} \cos\phi_c = 208 \times 16 \times \cos(120^\circ - 97.38^\circ) \\ &= 3071.99 \approx 3072 W \end{aligned}$$

$$\begin{aligned} \text{Reactive power (Q}_C\text{)} &= V_{ca} I_{ca} \sin\phi_c = 208 \times 16 \times \sin(120^\circ - 97.38^\circ) \\ &= 1280 VAR \end{aligned}$$

$$\begin{aligned} \text{Total Active Power (P)} &= P_A + P_B + P_C \\ &= 2163.2 + 2163.5 + 3072 \\ &= 7398.7 W \end{aligned}$$

$$\begin{aligned} \text{Total Reactive Power (Q)} &= Q_A + Q_B + Q_C \\ &= 0 + (-2163.5) + 1280 \\ &= -883.5 VAR \end{aligned}$$

Total apparent Power absorbed by load

$$S = \sqrt{P^2 + Q^2} = \sqrt{(7398.7)^2 + (-883.5)^2} = 7451.26 VA$$

Some Theoretical Questions

1. Explain emf, potential difference and current with a circuit diagram.
2. Define electric circuit. Explain the constituent parts of an electric system.
3. State Ohm's law and write down its limitations.
4. On what factor does the resistance offered by a conductor depends upon?
5. Define the temperature coefficient of resistance and explain the effect of temperature on resistance of a substance.
6. What are ideal and practical voltage and current sources? Explain.
7. Explain how can we convert a voltage source into a current source and a current source into a voltage source?
8. Explain the following:
 - a. Series circuit
 - b. Parallel circuit
9. Compute equivalent resistance of three resistors connected in
 - i. Series
 - ii. Parallel
10. What is current divider rule? Explain with example.
11. What is voltage divider rule? Explain with example.
12. What do you understand by duality between series and parallel circuits?
13. State and explain Kirchhoff's laws.
14. Explain power and energy in dc circuit.
15. Explain the nodal method of solving a network.
16. Explain the loop current method of solving a network.
17. Explain and write the equations for delta-star conversion and for star-delta conversion.
18. State and explain Superposition theorem with a suitable example. Also mention its limitations.
19. State Norton's theorem with an example and list out the steps for nortonizing of a given circuit.
20. State Thevenin's theorem with an example and list out the steps for thevenizing of a given circuit. Also list out the major advantages offered by the use of this theorem.
21. Show that Thevenin's and Norton's theorems are dual to each other.
22. State and explain Maximum power transfer theorem.
23. State and explain reciprocity theorem with a suitable example.
24. Define the following terms
 - a. Resistance
 - b. Inductance
 - c. Capacitance
25. Describe Capacitance from circuit view point, and geometric view point.
26. State the definition of the capacitance and from it write an equation for the charge stored in a capacitor.
27. Derive the expression of energy stored in an inductive coil.
28. Derive the expression of energy stored in a capacitor.
29. Derive an expression for the equivalent inductance of two inductors when they are connected in parallel (i) Aiding combination (ii) Opposing combination.
30. Derive an expression for the equivalent inductance of two inductors when they are connected in series (i) Aiding combination (ii) Opposing combination.
31. Explain about series and parallel combination of capacitors.

32. Derive an equation for the capacitance of a parallel plate capacitor.
33. Explain the process of charging and discharging of capacitor with neat sketch.
34. List out the difference between ac and dc system.
35. Explain generation of sinusoidal emf with diagram.
36. Define the following terms for an alternating quantity.

a. Frequency	b. Cycle
c. Phase	d. Phase difference
e. Time period	f. Average value
g. RMS value	h. Form factor
i. Peak factor	j. Angular velocity
k. Instantaneous value	
37. Define the following terms with phasor and waveform (i) lagging (ii) leading (iii) in phase. How would you calculate the RMS value of a waveform?
38. Why do we express an ac voltage or current by its RMS value? Discuss.
39. Derive the equation for instantaneous current flowing through a pure resistor when excited by ac sinusoidal voltage $v = v_m \sin \omega t$. Draw the waveform of voltage and current and phasor diagram of the circuit. Show analytically and graphically that it consumes active power.
40. Derive the equation for instantaneous current flowing through a pure capacitor when excited by ac sinusoidal voltage $v = v_m \sin \omega t$. Draw the waveform of voltage and current and phasor diagram of the circuit. Show analytically and graphically that it does not consume any active power.
41. Derive the relationship between the voltage and current for a purely inductive circuit excited by ac voltage source. Also show that the average power consumed by a purely inductive circuit is zero.
42. Draw wave diagram and phasor diagram to illustrate clearly the relation between voltage and current in the case of
 - i. R - L series circuit.
 - ii. R - C series circuit.
 - iii. R - L - C series circuit.
43. What are active, reactive and apparent power? Draw the power triangle.
44. Derive the expression for instantaneous power in RL and RC series circuit.
45. Derive expressions for impedance and power factor for an R-L-C series circuit when applied with ac voltage. Draw also the phasor diagram.
46. What do you mean by complex power? Explain it with the help of R-L series circuit and power triangle.
47. What is power factor in ac circuit? Explain the disadvantages of low power factor.
48. Explain the requirement of power factor and the method of its correction.
49. What are the advantages of three phase ac system over single phase ac system.
50. What are the two ways of connection in a 3-phase system? Draw their phasor diagrams and write down the relationship between phase and line voltages and currents for these systems.
51. Compare the star and delta 3-phase connection.
52. Explain two wattmeter method for the measurement of power in a balanced three phase load. What is the variation of wattmeter readings with load power factor.
53. Describe the measurement of power in an unbalanced three phase load by two wattmeter method with neat sketch of circuit diagram.