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32、(1) 记X为检修前生产的一等品数，Y为检修前共生产数

$$P(X=i \mid Y=n) = C_{n-1}^{i}\left(\frac{p_1}{p_1+p_2}\right)^{i}\left(\frac{p_2}{p_1+p_2}\right)^{n-1-i}$$

$i \le n$

即 $X \mid Y=n \sim B(n-1, \frac{p_1}{p_1+p_2})$ ∴ $E(X \mid Y=n) = \frac{p_1}{p_1+p_2}(n-1)$

(2) $EX = \sum_{i=1}^{\infty} E(X \mid Y=i) P(Y=i) = \frac{p_1}{p_3}$

35.

X \ L	2	3	4
0	$\frac{3}{7}$	0	0
1	$\frac{1}{7}$	0	0
2	0	$\frac{1}{7}$	$\frac{2}{7}$

最佳均方预期即为 $E(L \mid X) = \begin{cases} 2 & X=0,1 \\ \frac{11}{3} & X=2 \end{cases}$

36. ~~$E(Y-E(Y \mid X) \mid X)=0$~~

~~即 $Z=Y-E(Y \mid X)$~~

由 $E((X-\varphi(Y))^2) = E((X-E(X \mid Y))^2) + E((E(X \mid Y)-\varphi(Y))^2)$

取 $\varphi(Y)=EX$，

则 $E(X-EX)^2 = DX$

$E((E(X \mid Y)-EX)^2)$

$=E((E(X \mid Y)-E(E(X \mid Y)))^2)$

$=D(E(X \mid Y))$

∴ $DX = E(D(X \mid Y)) + D(E(X \mid Y))$

37. (1) $P(X_n \geq 0, n=1,2,3,4)$

$= p(p + \cancel{p}qp) = p^2(1+\cancel{p}q)$

(4) $P(|X_n| \leq 2, n=1,2,3,4)$

$= 1 - P(\exists n=1,2,3,4, |X_n| > 2)$

$= 1 - P(\exists n=3,4, |X_n| > 2)$

$= 3pq$

39. (a) $P(X_{n+1}=i_{n+1} | X_n=i_n, \cdots X_0=i_0) = \dfrac{P(X_{n+1}=i_{n+1}, \cdots X_0=i_0)}{P(X_n=i_n, \cdots X_0=i_0)}$

$= \dfrac{P(X_{n+1}-X_n=i_{n+1}-i_n) \cdots P(X_1-X_0=i_1-i_0) P(X_0=i_0)}{P(X_n-X_{n-1}=i_n-i_{n-1}) \cdots P(X_0=i_0)}$

$= P(X_{n+1}-X_n=i_{n+1}-i_n)$

$= P(X_{n+1}=i_{n+1} | X_n=i_n)$

(b) 左式 $= \dfrac{P(X_{n+1}=i_{n+1}, X_{n-1}=i_{n-1}, X_n=i_n)}{P(X_n=i_n)}$

$= \dfrac{P(X_{n+1}-X_n=i_{n+1}-i_n) P(X_n=i_n) P(X_n-X_{n-1}=i_n-i_{n-1})}{P(X_n=i_n)}$

$= P(X_{n+1}-X_n=i_{n+1}-i_n) P(X_n-X_{n-1}=i_n-i_{n-1})$

$= P(X_{n+1}=i_{n+1} | X_n=i) P(X_{n-1}=i_{n-1} | X_n=i_n)$

补充题：

$\frac{2}{3}+\frac{5}{6}+\frac{7}{6}$

$2+\frac{10}{3}$

4

1、$P(X=-2)=\frac{1}{3}$

$P(X=5)=\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$

$P(X=7)=\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$

$P(X=10)=\frac{1}{3}$

$\therefore EX=6$

2、设 Y 为6点前~~出现的次~~执行的次数，则 $X|Y=i \sim B(i,\frac{1}{5})$

则 $E(X|Y=i)=\frac{i}{5}$

$i=0,1,\cdots$

$EX=\sum_{i=0}^{\infty}E(X|Y=i)P(Y=i)=1$