6.6.2.

μ的0.95置信区间为 $\left[\bar{x}-\frac{u_{1-\frac{\alpha}{2}}\sigma}{\sqrt{n}}, \bar{x}+\frac{u_{1-\frac{\alpha}{2}}\sigma}{\sqrt{n}}\right]$

~~区间前~~ 区间长度减于k,

则 $\frac{2u_{1-\frac{\alpha}{2}}\sigma}{\sqrt{n}} \le k$

即 $n \ge \frac{4u_{1-\frac{\alpha}{2}}^2\sigma^2}{k^2} = \frac{15.37\sigma^2}{k^2}$

6.6.3.

(1) 取 $y_i = \ln x_i$

则 μ的0.95置信区间为：$\left[\bar{y}-u_{1-\frac{\alpha}{2}}\sigma/\sqrt{n}, \bar{y}+u_{1-\frac{\alpha}{2}}\sigma/\sqrt{n}\right]$

即 $[-0.98, 0.98]$

(2) $EX = e^{\mu+\frac{1}{2}}$

$\therefore$ EX的0.95置信区间为 $\left[e^{-0.98+\frac{1}{2}}, e^{0.98+\frac{1}{2}}\right] = [0.62, 4.39]$

6.6.9

(2) 当 $\sigma_1^2=\sigma_2^2$ 时，$\mu_1-\mu_2$ 的0.95置信区间为

$\left[\bar{x}-\bar{y}-\sqrt{\frac{n_1+n_2}{n_1n_2}}s_w t_{1-\frac{\alpha}{2}}(n_1+n_2-2), \bar{x}-\bar{y}+\sqrt{\frac{n_1+n_2}{n_1n_2}}s_w t_{1-\frac{\alpha}{2}}(n_1+n_2-2)\right]$

即 $[-0.2063, 12.2063]$

(4) $\frac{\sigma_1^2}{\sigma_2^2}$ 的 0.95 置信区间为

$$\left[\frac{S_x^2}{S_y^2}\frac{1}{F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)},\ \frac{S_x^2}{S_y^2}\frac{1}{F_{\frac{\alpha}{2}}(n_1-1,n_2-1)}\right]=[0.3359,4.0973]$$

11. $\sum_{i=1}^{n} x_i \sim Ga(n,\lambda)$

$\therefore 2\lambda\sum_{i=1}^{n} x_i \sim \chi^2(2n)$

则 $P\left(\chi^2_{\frac{\alpha}{2}}(2n) \le 2\lambda\sum_{i=1}^{n} x_i \le \chi^2_{1-\frac{\alpha}{2}}(2n)\right)=1-\alpha$

$\therefore$ λ 的 $1-\alpha$ 置信区间为 $\left[\frac{\chi^2_{\frac{\alpha}{2}}(2n)}{2\sum_{i=1}^{n} x_i},\ \frac{\chi^2_{1-\frac{\alpha}{2}}(2n)}{2\sum_{i=1}^{n} x_i}\right]$

19. (1) 证明：取 $y_i=x_i-\theta$，则 $y_i\sim Exp(1)$

则 $f_{Y_{(1)}}(y_{(1)})=ne^{-ny_{(1)}},\ y_{(1)}>0$ 与 θ 无关。

(2) $P(c\le x_{(1)}-\theta\le d)=\int_c^d ne^{-ny}dy=1-\alpha$

若使 $d-c$ 最小，则 $c=0$，$d=-\frac{\ln\alpha}{n}$

$\therefore$ θ 的 $1-\alpha$ 置信区间为 $\left[x_{(1)}+\frac{\ln\alpha}{n},\ x_{(1)}\right]$

7.1.1

(1).

$\alpha = P(\bar{x} \geq 2.6 | H_0) = P\left(\frac{\bar{x}-2}{\sqrt{1/20}} \geq \frac{2.6-2}{\sqrt{1/20}}\right) = 0.0037$

$\beta = P(\bar{x} < 2.6 | H_1) = P\left(\frac{\bar{x}-3}{\sqrt{1/20}} < \frac{2.6-3}{\sqrt{1/20}}\right) = 0.0367$

(2) $\beta = P\left(\frac{\bar{x}-3}{\sqrt{\frac{1}{n}}} < \frac{2.6-3}{\sqrt{\frac{1}{n}}}\right) = 0.01$, 解得：$n \geq 34$

(3) $\alpha = P\left(\frac{\bar{x}-2}{\sqrt{\frac{1}{n}}} \geq \frac{2.6-2}{\sqrt{\frac{1}{n}}}\right) = 1-\phi(0.6\sqrt{n})$

$\beta = 1-\phi(0.4\sqrt{n})$

当 $n \to \infty$ 时，α 和 β 都 $\to 0$

7.1.3. $P(|\bar{x}-6| \geq c | H_0) = 0.05 \Rightarrow \phi(2c) = 0.975$

$c = 0.98$

$\mu = 6.5$ 时，$\beta = P(|\bar{x}-6| < 0.98 | \mu = 6.5)$

$= P\left(-2.96 < \frac{\bar{x}-6.5}{0.5} < 0.96\right) = 0.83$

1. 7.1.4 $f_{X_{(n)}}(x) = \frac{n x^{n-1}}{\theta^n} I_{\{0<x<\theta\}}$

2. $\alpha(\theta) = P(X_{(n)} \le 2.5 | H_0) = (\frac{2.5}{\theta})^n$

$\alpha = \sup \alpha(\theta) = \alpha(3) = (\frac{5}{6})^n$

若 $\alpha \le 0.05$ ∴得：$n \ge 16.43$，即 $n \ge 17$

7.2.3. $H_0: \mu = 15$ $H_1: \mu \ne 15$

拒绝域为 $\{|u| \ge u_{1-\frac{\alpha}{2}}\}$，

而 $u = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = -4.90$，$4.90 > u_{1-\frac{\alpha}{2}}$

∴ 不能认为产品的平均质量仍为 15g

7.2.5 拒绝域为 $\{u \le u_\alpha\}$ $u_\alpha = -1.65$

$\therefore \beta = P(u > -1.65 | \mu \le 13) = 1 - \phi(-1.65 + \frac{15-\mu}{\sqrt{2.5}/n}) \le 0.05$

$\therefore \sup \beta = 1 - \phi(-1.65 + \frac{15-13}{\sqrt{2.5/n}}) \le 0.05$. $n \ge 7$

7.2.13. $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \ne \mu_2$

拒绝域为 $\{|t| \ge t_{1-\frac{\alpha}{2}}(m+n-2)\}$

而 $t = \frac{\bar{x} - \bar{y}}{s_w \sqrt{\frac{1}{m} + \frac{1}{n}}} = -0.2056 < t_{0.975}(15)$ ∴ 可以看作一样

7.2.20.

$H_0: \sigma_1^2 = \sigma_2^2 \quad H_1: \sigma_1^2 < \sigma_2^2$

拒绝域为 $F \le F_\alpha(m-1, n-1)$

而 $F = \frac{S_1^2}{S_2^2} = 2.572 > F_{0.05}(9, 9)$

∴ 不能认为方差冬季比夏季小.