8. 此方程有实根 $\Leftrightarrow \Delta \geq 0 \Leftrightarrow k \geq 2$

$\therefore P(k \geq 2) = \frac{3}{5}$

10. $p = P(X > 10) = \int_{10}^{\infty} \frac{1}{5} e^{-\frac{x}{5}} dx = e^{-2}$

$Y \sim B(5, e^{-2})$ $\quad \therefore EY = 5e^{-2}$

$P(Y \geq 1) = 1 - P(Y=0) - P(Y=1)$

$= 0.5167$

14. $X^* = \frac{X-\mu}{6}$

则 $E|X^*| = 2\int_0^{+\infty} \frac{x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \sqrt{\frac{2}{\pi}}$

$\therefore E|X-\mu| = 6\sqrt{\frac{2}{\pi}}$

15. 记m为中位数.

则 $E|X-m| = \int_{-\infty}^{m} (m-x) f(x) dx + \int_{m}^{\infty} (x-m) f(x) dx$

$= -\int_{-\infty}^{m} x f(x) dx + \int_{m}^{+\infty} x f(x) dx$

若 $a < m$, 则 $E|X-a| = \int_{-\infty}^{m} (a-x) f(x) dx - \int_{a}^{m} (a-x) f(x) + \int_{m}^{\infty} (x-a) f(x) dx - \int_{a}^{m} (x-a) f(x) dx$

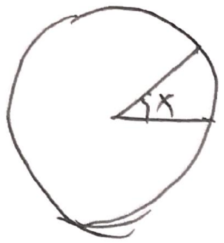
$= \int_{m}^{+\infty} x f(x) dx - \int_{-\infty}^{m} x f(x) dx + 2\int_{a}^{m} (x-a) f(x) dx$

$\geq E|X-m|$

若 $a<m$ 时，同理有 $E|X-a| \geq E|X-m|$

$\therefore E|X-m| = \min\limits_{a\in R} E|X-a|$

16.



则 $X \sim U(0,2\pi)$

$L = 2R\sin\frac{X}{2}$

$$EL = \int_0^{2\pi} 2R\sin\frac{x}{2}\cdot\frac{1}{2\pi}\cdot dx$$

$$= \frac{4R}{\pi}$$

22. 方程有实根 $\Longleftrightarrow Y \leq X^2$

~~而 $X\in(0,1)$~~

$f(x,y) = f_X(x)f_Y(y)$

$$\therefore P = \iint\limits_{\substack{Y\leq X^2,\ X\in(0,1),\\ Y>0}} f(x,y)\,dx\,dy = \frac{1}{e}$$

23. $\cancel{f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \{}$

$f_X(1) = \int_{-\infty}^{+\infty} f(1,y)\,dy = 0$ $\quad f_Y(1) = \int_{-\infty}^{+\infty} f(x,1)\,dx = \frac{1}{2}$

$f(1,1) = \frac{1}{2} \neq f_X(1)f_Y(1)$ $\quad\therefore$ 不独立

25.
(1) $EX = \int_{-\infty}^{+\infty} x \frac{e^{-|x|}}{2} dx = 0$

$EX^2 = \int_{-\infty}^{+\infty} \frac{x^2}{2} e^{-|x|} dx = 2\int_0^{+\infty} \frac{x^2 e^{-x}}{2} dx = 2$

$DX = EX^2 - (EX)^2 = 2$

(2) $E(X|X|) = \int_{-\infty}^{+\infty} x|x| \frac{e^{-|x|}}{2} dx = 0$

$\therefore Cov(X, |X|) = E(X|X|) - EX\,E|X| = 0$

$\therefore$ 不相关

(3) 对$\forall a > 0$,

$P(|X| < a, X < a) = P(|X| < a) \neq P(|X| < a) P(X < a)$

$\therefore$ 不独立

30. $EX = \int_{-\pi}^{\pi} \sin\theta \frac{1}{2\pi} d\theta = 0$

$EXY = \int_{-\pi}^{\pi} \sin\theta\cos\theta \frac{1}{2\pi} d\theta = 0$

$\therefore Cov(X,Y) = EXY - EXEY = 0$

$\therefore X, Y$ 不相关,

$P(X > \frac{\sqrt{2}}{2}, Y > \frac{\sqrt{2}}{2}) = 0$

$P(X > \frac{\sqrt{2}}{2}) = \frac{1}{4}$ $P(Y > \frac{\sqrt{2}}{2}) = \frac{1}{4}$

$P(X > \frac{\sqrt{2}}{2}, Y > \frac{\sqrt{2}}{2}) \neq P(X > \frac{\sqrt{2}}{2}) P(Y > \frac{\sqrt{2}}{2})$

$\therefore$ 不独立

34 $f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \begin{cases} \cancel{\lambda e^{-\lambda y}} \; \frac{1}{\lambda} e^{-\lambda x-\lambda y} & x > y \\ 0 & x \le y \end{cases}$

$E(X|Y=y) = \int_{y}^{+\infty} x \frac{\cancel{\lambda} e^{\lambda x-\lambda y}}{\lambda} dx = y + \frac{1}{\lambda}$

$E(X^2|Y=y) = \int_{y}^{+\infty} x^2 \frac{e^{\lambda x-\lambda y}}{\lambda} dx = y^2 + \frac{2y}{\lambda} + \frac{2}{\lambda^2}$

$\therefore D(X|Y=y) = E(X^2|Y=y) - (E(X|Y=y))^2 = \frac{1}{\lambda^2}$