23. (a)

X	1	2	3
p	$\frac{1}{9}$	$\frac{2}{3}$	$\frac{2}{9}$

Y	0	1	2	3
p	$\frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$

(b) 不独立

(c)

$Y\|X=1$	0	1	2	3
p	$\frac{2}{3}$	0	0	$\frac{1}{3}$

$X\|Y=0$	1	2	3
p	$\frac{1}{4}$	$\frac{3}{4}$	0

(d) $P(X=3|Y=2)=0$ $\quad$ $P(Y=2|X=3)=0$

25. $D(XY)=E(X^2Y^2)-E(XY)^2$

$$=E(X^2)E(Y^2)-E(X)^2E(Y)^2$$

$$=(D(X)+E(X)^2)(DY+(EY)^2)-(EX)^2(EY)^2$$

$$=DXDY+(EX)^2DY+(EY)^2DX$$

26. 设 ξ_i 分布的期望为 μ，方差为 σ^2

则 $EX=EY=n\mu$

$$EXY=E\left(\left(\sum_{k=1}^{n}\xi_k\right)\left(\sum_{k=1}^{n}\xi_{m+k}\right)\right)=E\left(\sum_{\substack{i\neq j\\ 1\le i\le n\\ m+1\le j\le n+m}}\xi_i\xi_j+\sum_{i=m+1}^{n}\xi_i^2\right)$$

$$=(n^2-n+m)E(\xi_1\xi_2)+(n-m)E(\xi_1^2)$$

$$=(n^2-n+m)\mu^2+(n-m)(\sigma^2+\mu^2)$$

$$=n^2\mu^2+(n-m)\sigma^2$$

$\sqrt{DX} = \sqrt{DY} = \sqrt{n}\,\sigma$

$$\therefore r_{X,Y} = \frac{EXY - EXEY}{\sqrt{DX}\sqrt{DY}} = \frac{n-m}{n}$$

27. 设记 $X_i = \begin{cases} 1 & \text{第}i\text{次为}1 \\ 0 & \text{第}i\text{次不为}1 \end{cases}$ $\quad Y_i = \begin{cases} 1 & \text{第}i\text{次为}6 \\ 0 & \text{第}i\text{次不为}6 \end{cases}$

$$\text{则 } EXY = E\left(\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i\right) = \sum_{i=1}^{n} EX_iY_i + 2\sum_{i<j} EX_iX_j$$

$$= \frac{n(n-1)}{36}$$

$$Cov(X,Y) = EXY - EXEY = -\frac{n}{36}$$

$$r_{X,Y} = \frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}} = -\frac{1}{5}$$

30.

$E(X\mid Y)$	$\frac{13}{7}$	$\frac{28}{15}$	$\frac{11}{5}$
P	$\frac{7}{27}$	$\frac{5}{9}$	$\frac{5}{27}$

$$EX = \sum P\, E(X\mid Y) = \frac{52}{27}$$

31.

$Y_3 \backslash Y_2$	-2	0	2
-3	q^3	0	0
-1	q^2p	$2pq^2$	0
1	0	$2p^2q$	p^2q
3	0	0	p^3

$\therefore$ ~~$E(Y_3\mid Y_2)$~~

$E(Y_3\mid Y_2)$	$-2+p-q$	$p-q$	$2+p-q$
P	q^2	$2pq$	p^2

$E(Y_{n+1} = Y_n+1 \mid Y_n) = p$

$E(Y_{n+1} = Y_n-1 \mid Y_n) = q$

$\therefore E(Y_{n+1} \mid Y_n) = Y_n + p - q$

补：1.

设袋里白球数目为X

则 $P(白) = \sum_{i=1}^{N} P(X=i) \cdot \frac{i}{N} = \frac{1}{N}\sum_{i=1}^{N} P(X=i) i = \frac{1}{N} EX = \frac{n}{N}$

2. $Cov(X-(\hat{a}Y+\hat{b}), Y) = E((X-(\hat{a}Y+\hat{b}))Y) - E(X-(\hat{a}Y+\hat{b}))EY$

$\because E(X-(\hat{a}Y+\hat{b})) = 0$

$E((X-(\hat{a}Y+\hat{b}))Y) = EXY - \hat{a}EY^2 - \hat{b}EY$

$= EXY - \hat{a}(DY+(EY)^2) - \hat{b}EY$

$= EXY - Cov(Y,Y) - \hat{a}(EY)^2 - \hat{b}EY$

$= EXEY - (\hat{a}EY+\hat{b})EY = 0$

$\therefore Cov(X-(\hat{a}Y+\hat{b}), Y) = 0$