1. 设强度为$\lambda$.

则 $P(X=1)=\lambda e^{-\lambda}$ $\quad P(X=2)=\frac{\lambda^2}{2!}e^{-\lambda}$

得 $\lambda=2$

$\therefore P(X=4)=\frac{2}{3}e^{-2}$

5. ~~由看~~ 记~~试~~成功次数为$Y$

则$Y$符合强度为$\lambda p$的Poisson分布

$\therefore P(Y=k)=\frac{(\lambda p)^k}{k!}e^{-\lambda p}$ $\quad EY=\lambda p$

6. 记$N$为虫数，$X$为虫卵数

则 $P(X=x)=P(\sum_{i=1}^{N}X_i=x)$

$=\sum_{k=0}^{\infty}P(\sum_{i=1}^{N}X_i=x \mid N=k)P(N=k)$

$=\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}e^{-\lambda}P(\sum_{i=1}^{k}X_i=x)$

$=\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}e^{-\lambda}\frac{(k\mu)^x}{x!}e^{-k\mu}$ $\quad$ ($\sum_{i=1}^{k}X_i$符合$k\mu$的泊松分布)

$=\frac{\mu^x}{x!}e^{-\lambda}\sum_{k=0}^{\infty}\frac{(\lambda e^{-\mu})^k k^x}{k!}$

7. $P(X=k)=P(X=k\mid\Lambda=\lambda_1)P(\Lambda=\lambda_1)+P(X=k\mid\Lambda=\lambda_2)P(\Lambda=\lambda_2)$

$$=p\frac{\lambda_1^k e^{-\lambda_1}}{k!}+(1-p)\frac{\lambda_2^k e^{-\lambda_2}}{k!}$$

10. (1) 易得: $X\sim \text{Poisson}_{\lambda p}$ $Y\sim \text{Poisson}_{\lambda(1-p)}$

(2) $P(X=x,Y=y)=\sum_{k=0}^{\infty}P(X=x,Y=y\mid N=k)P(N=k)$

$$=P(X=x,Y=y\mid N=x+y)P(N=x+y)$$

$$=C_{x+y}^{x}p^x(1-p)^y\cdot\frac{\lambda^{x+y}e^{-\lambda}}{(x+y)!}=\frac{1}{x!y!}p^x(1-p)^y\lambda^{x+y}e^{-\lambda}$$

$$P(X=x)P(Y=y)=\frac{(\lambda p)^x e^{-\lambda p}}{x!}\cdot\frac{(\lambda(1-p))^y e^{-\lambda(1-p)}}{y!}=\frac{1}{x!y!}p^x(1-p)^y\lambda^{x+y}e^{-\lambda}$$

$\therefore P(X=x,Y=y)=P(X=x)P(Y=y)$ $\therefore$ 独立

12. $P(X_1=x,N=n)=\begin{cases}0 & n>x\\ \frac{\lambda^x}{x!}e^{-\lambda}\sum_{i=0}^{x}\frac{\lambda^i e^{-\lambda}}{i!} & n=x\\ \frac{\lambda^x}{x!}e^{-\lambda}\cdot\frac{\lambda^n e^{-\lambda}}{n!} & n<x\end{cases}$

$$\therefore P(N=n)=\sum_{i=n}^{\infty}P(X_1=i,N=n)=2e^{-2\lambda}\frac{\lambda^n}{n!}\sum_{j=n+1}^{\infty}\frac{\lambda^j}{j!}+\frac{\lambda^{2n}e^{-2\lambda}}{(n!)^2}$$

$P(X_1=x,M=m)=\begin{cases}0 & m<x\\ \frac{\lambda^x}{x!}e^{-\lambda}\sum_{i=0}^{m}\frac{\lambda^i e^{-\lambda}}{i!} & m=x\\ \frac{\lambda^x}{x!}e^{-\lambda}\cdot\frac{\lambda^m e^{-\lambda}}{m!} & m>x\end{cases}$

$$\therefore P(M=m)=2e^{-2\lambda}\frac{\lambda^m}{m!}\sum_{j=0}^{m-1}\frac{\lambda^j}{j!}+\frac{\lambda^{2m}e^{-2\lambda}}{(m!)^2}$$

13. 不妨设 $s\le t$

$$E(N_sN_t)=E[N_s((N_t-N_s)+N_s)]$$

$$=E(N_s)E(N_t-N_s)+E(N_s^2)$$

$$=\lambda s(\lambda t-\lambda s)+\lambda s+\lambda^2s^2$$

$$=\lambda^2st+\lambda s$$

$$\therefore Cov(N_s,N_t)=E(N_sN_t)-EN_sEN_t=\lambda s$$

$$=\lambda\min\{s,t\}$$

15. ~~$EX=$~~

记营业额为 $Y$, $X\sim Poisson_{10800}$

则 $$E(Y)=\sum_{k=1}^{\infty}E\left(\sum_{i=1}^{X}\eta_i\,\middle|\,X=k\right)P(X=k)$$

$$=\sum_{k=1}^{\infty}k\times10\times0.15\times\frac{(10800)^k}{k!}e^{-10800}$$

$$=\cancel{10800}\ 16200$$

$$E(Y^2)=(\lambda^2t^2+\lambda t)E\eta_1^2$$

$$\therefore DY=EY^2-(EY)^2=\lambda tE\eta_1^2=38070$$

$$\therefore \sigma_Y=\sqrt{DY}=195.12$$

1. (a) $\int_{-1}^{1} \frac{c}{\sqrt{1-x^2}} dx = 1$

得: $c = \frac{1}{\pi}$

(b) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\pi\sqrt{1-x^2}} dx = \frac{1}{3}$

4. $P(X>c) = \int_{c}^{+\infty} \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = e^{-\frac{c}{\theta}} = \frac{1}{2}$

$\therefore c = \theta \ln 2$