5.4.

3. $\bar{x} \sim N(100, \frac{4}{15})$ $\bar{y} \sim N(100, \frac{1}{5})$

$\therefore \bar{x}-\bar{y} \sim N(0, \frac{7}{15})$

$\therefore P(|\bar{x}-\bar{y}|>0.2) = 2P(\bar{x}-\bar{y}>0.2) \simeq 0.7718$

8. $$p_Z(z) = \frac{\Gamma(\frac{m+n}{2})(\frac{n}{m})^{\frac{n}{2}}}{\Gamma(\frac{n}{2})\Gamma(\frac{m}{2})} \cdot \left(\frac{mz}{n(1-z)}\right)^{\frac{n}{2}-1} \cdot \left(1+\frac{z}{1-z}\right)^{-\frac{m+n}{2}} \cdot \frac{m}{n(1-z)^2}$$

$$= \frac{\Gamma(\frac{m+n}{2})}{\Gamma(\frac{n}{2})\Gamma(\frac{m}{2})} z^{\frac{n}{2}-1}(1-z)^{\frac{m}{2}-1} \sim Be(\frac{n}{2}, \frac{m}{2})$$

11. $c(\bar{x}-\mu_1) \sim N(0, \frac{c^2\sigma^2}{n})$ $d(\bar{y}-\mu_2) \sim N(0, \frac{d^2\sigma^2}{m})$

$\therefore c(\bar{x}-\mu_1)+d(\bar{y}-\mu_2) \sim N(0, \frac{c^2\sigma^2}{n}+\frac{d^2\sigma^2}{m})$

$\frac{(n-1)S_x^2}{\sigma^2} \sim \chi^2(n-1)$ $\frac{(m-1)S_y^2}{\sigma^2} \sim \chi^2(m-1)$

$\therefore \frac{(n+m-2)}{\sigma^2}S_w^2 \sim \chi^2(n+m-2)$

$$\therefore t = \frac{\dfrac{c(\bar{x}-\mu_1)+d(\bar{y}-\mu_2)}{\sqrt{\frac{c^2\sigma^2}{n}+\frac{d^2\sigma^2}{m}}}}{\sqrt{\dfrac{\frac{(n+m-2)S_w^2}{\sigma^2}}{n+m-2}}} \sim t(n+m-2)$$

19. 记 $Y_i = F(X_i)$ 则 $F_Y(y) = P(F(X) \le y) = P(X \le F^{-1}(y))$

$= y \quad y \in (0,1)$

$\therefore Y_i \sim U(0,1)$

记 $Z_i = -\ln Y_i$, 则 $F_Z(z) = P(-\ln Y \le z) = 1 - e^{-z} \quad (z > 0)$

$\therefore Z_i \sim \chi^2(2)$

$\therefore T \sim \chi^2(2n)$

5.5.

4. $T = \sum_{i=1}^{n} X_i \sim N(n\mu, n)$

$$\frac{P(X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n)}{P(T = t)} = \frac{\frac{1}{\sqrt{2\pi}^n} e^{-\frac{1}{2}\sum_{i=1}^{n}(x_i - \mu)^2}}{\frac{1}{\sqrt{2\pi n}} e^{-\frac{1}{2n}(t - \mu n)^2}}$$

$$= \frac{1}{\sqrt{2n}^{n-1} \cdot \sqrt{\pi}} \cdot e^{-\frac{1}{2}\left(\sum_{i=1}^{n} x_i^2 - \frac{t^2}{n}\right)}$$ 与 μ 无关

$\therefore T$ 为充分统计量

7. $f(x_1, x_2 \cdots x_n, \theta) = \theta^n a^n (x_1 x_2 \cdots x_n)^{-(\theta+1)}$

~~显然~~ 可得：$T = x_1 x_2 \cdots x_n$ 为θ的充分统计量

9.

(1) $f(x, \theta) = \left(\prod_{i=1}^{n} C_{x_i+r-1}^{r-1}\right) \theta^{nr} (1-\theta)^{\sum_{i=1}^{n} x_i}$

可得：$T = \sum_{i=1}^{n} x_i$ 为θ的充分统计量

(2) $p(x, m) = \frac{1}{m^n} I_{\{1 \le x_{(1)} \le x_{(n)} \le m\}}$

$\therefore x_{(n)}$ 为充分统计量

(3) $f(x, \mu, \sigma^2) = \frac{1}{(\sqrt{2\pi})^n x_1 \cdots x_n \sigma^n} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(\ln x_i)^2 + \frac{\mu}{\sigma^2}\sum_{i=1}^{n}\ln x_i - \frac{n\mu^2}{2\sigma^2}\right\}$

$\therefore T = \left(\sum_{i=1}^{n}(\ln x_i)^2, \sum_{i=1}^{n}(\ln x_i)\right)$ 为充分统计量

(4) $f(x, \lambda) = 2^n \left(\prod_{i=1}^{n} x_i\right) \cdot \lambda^n e^{-\lambda \sum_{i=1}^{n} x_i^2} I_{\{x_{(1)} \ge 0\}}$

$\therefore T = \sum_{i=1}^{n} x_i^2$ 为充分统计量

12. $f(x,\theta) = \left(\frac{1}{\theta}\right)^n I_{\theta \le x_{(1)} \le x_{(n)} \le 2\theta}$

$\therefore$ ~~$(x_{(1)}, x_{(n)})$~~ $(x_{(1)}, x_{(n)})$ 为θ的充分统计量

19. $p(x, \theta, \mu) = \left(\frac{1}{\theta}\right)^n e^{\frac{-n\bar{x}+n\mu}{\theta}} \cdot I_{x_{(1)} > \mu}$

$\therefore$ $(\bar{x}, x_{(1)})$ 为充分统计量

6.1.

6. $f_{x_{(1)}}(x) = \frac{3}{\theta^3}(\theta - x)^2 \quad 0 < x < \theta$

$f_{x_{(3)}}(x) = \frac{3}{\theta^3} x^2$

$\therefore E(4x_{(1)}) = 4Ex_{(1)} = \theta$

$E(\frac{4}{3}x_{(3)}) = \frac{4}{3}Ex_{(3)} = \theta$ 都为无偏估计,

$D(4x_{(1)}) = 16Dx_{(1)} = \frac{3}{5}\theta^2$

$D(\frac{4}{3}x_{(3)}) = \frac{16}{9}Dx_{(3)} = \frac{\theta^2}{15}$

$\therefore$ $\frac{4}{3}x_{(3)}$ 更有效

11、

$EY_1 = \frac{1}{n}\cdot n E|X_1-\bar{X}| = \sqrt{\frac{n-1}{n}}\frac{\sqrt{2}\sigma}{\sqrt{\pi}}$ $\quad (X_1-\bar{X} \sim N(0,\frac{n-1}{n}\sigma^2))$

$EY_2 = \frac{1}{n(n-1)}\cdot n(n-1) E|X_1-X_2| = \frac{2\sigma}{\sqrt{\pi}}$ $\quad (X_1-X_2 \sim N(0, 2\sigma^2))$

$\therefore C_1 = \sqrt{\frac{n\pi}{2(n-1)}} \quad C_2 = \frac{\sqrt{\pi}}{2}$

6、2

3、(1) $EX = \frac{N-1}{2}$, $\quad N = 2EX+1$

$\therefore \hat{N} = 2\bar{X}+1$

(2) $EX = \frac{2}{\theta}$ $\quad \therefore \theta = \frac{2}{EX}$

$\hat{\theta} = \frac{2}{\bar{X}}$

4、(1) $EX = \frac{1}{3}\theta$ $\quad \therefore \hat{\theta} = 3\bar{X}$

(2) $EX = \frac{\theta+1}{\theta+2}$ $\quad \therefore \hat{\theta} = \frac{1-2\bar{X}}{\bar{X}-1}$

6.3.6

1.

(1). $L(\theta) = \sqrt{\theta}^n (x_1 \cdots x_n)^{\sqrt{\theta}-1}$

$$\frac{\partial \ln L(\theta)}{\partial \theta} = \frac{n}{2\theta} + \frac{\ln x_1 + \ln x_2 + \cdots + \ln x_n}{2\sqrt{\theta}} = 0$$

得 $\hat{\theta} = \left(\frac{\sum_{i=1}^{n} \ln x_i}{n}\right)^{-2}$

此时，$\hat{\theta}$为最大值点 ∴ $\hat{\theta}$为θ的最大似然估计

(2) $L(\theta) = \theta^n c^{n\theta} (x_1 \cdots x_n)^{-(\theta+1)}$

$$\frac{\partial \ln L(\theta)}{\partial \theta} = \frac{n}{\theta} + n\ln c - (\ln x_1 + \cdots + \ln x_n) = 0$$

$$\hat{\theta} = \left(\frac{1}{n}\sum_{i=1}^{n} \ln x_i - \ln c\right)^{-1}$$

此时，$\hat{\theta}$为最大值点 ∴ $\hat{\theta}$为θ的最大似然估计.