2. 设 $P(X=k)=\frac{c}{k(k+1)}$，则 $\sum_{n=1}^{\infty}\frac{c}{n(n+1)}=1$ 得：$c=1$

$\therefore P(X=k)=\frac{1}{k(k+1)}$

3. $P(X=1)=\frac{C_3^1}{C_4^2}=\frac{1}{2}$ $P(X=2)=\frac{C_2^1}{C_4^2}=\frac{1}{3}$

$P(X=3)=\frac{C_1^1}{C_4^2}=\frac{1}{6}$

X	1	2	3
P	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

5. $P(X=i,Y=i)=\frac{1}{6^n}$

$P(X=i,Y=j)$ ~~(i<j)~~ $(i<j)$

$=\frac{(j-i+1)^n-2(j-i)^n+(j-i-1)^n}{6^n}$

$\therefore$ $P(X=i)=\frac{(7-i)^n-(6-i)^n}{6^n}$

$P(Y=i)=\frac{i^n-(i-1)^n}{6^n}$

$P(X=2,Y=5)=\frac{4^n-2\times3^n+2^n}{6^n}$

6. $P(X=0)=\frac{3}{4}$

$P(X=1)=\frac{1}{4}\times\frac{9}{11}=\frac{9}{44}$

$P(X=2)=\frac{1}{4}\times\frac{2}{11}\times\frac{9}{10}=\frac{9}{220}$

$P(X=3)=1-(P(X=1)+P(X=2)+P(X=3))=\frac{1}{220}$

7. $P(X=2,Y=1)=P(X=2,Y=3)=0$ $\quad P(X=2,Y=2)=\frac{2\times C_3^2}{3^3}=\frac{6}{27}$

$P(X=3,Y=2)=P(X=3,Y=3)=0$, $\quad P(X=3,Y=1)=\frac{1}{3^3}=\frac{1}{27}$

$P(X=0,Y=1)=\frac{2}{27}$ $\quad P(X=0,Y=2)=\frac{6}{27}$ $\quad P(X=0,Y=3)=0$

$P(X=1,Y=1)=0$ $\quad P(X=1,Y=2)=\frac{6}{27}$ $\quad P(X=1,Y=3)=\frac{6}{27}$

∴

$X \backslash Y$	1	2	3	p_i^X
0	$\frac{2}{27}$	$\frac{6}{27}$	0	$\frac{8}{27}$
1	0	$\frac{6}{27}$	$\frac{6}{27}$	$\frac{12}{27}$
2	0	$\frac{6}{27}$	0	$\frac{6}{27}$
3	$\frac{1}{27}$	0	0	$\frac{1}{27}$
p_j^Y	$\frac{1}{9}$	$\frac{2}{3}$	$\frac{2}{9}$	1

9. 设 X_i 为第 i 次取出卡片的号码.

显然，放回不放回，$P(X_i=k)=\frac{1}{n}$ 不变，$EX_i=\frac{n+1}{2}$

$\therefore EX=E(X_1+X_2+\cdots+X_k)=EX_1+EX_2+\cdots EX_k=\frac{k(n+1)}{2}$

12. $p=\frac{3^3+3\times 3^3}{6^3}=\frac{1}{2}$

设试验所需次数为 X，显然 $X\sim Geo(p)$

$\therefore EX=2$，即平均次数为2

16. $EY = 1\cdot C_n^{0} p^{0} q^{n-0} + (-1)\cdot C_n^{1} p q^{n-1} + \cdots$

$= \sum_{k=0}^{n} C_n^{k} (-p)^k q^{n-k}$

$= (-p+q)^n = (1-2p)^n$

19 (a) $EX = \sum_{k=1}^{\infty} kP(X=k) = \sum_{k=1}^{\infty} \sum_{n=1}^{k} P(X=k)$

$= \sum_{n=1}^{\infty} \sum_{k=n}^{\infty} P(X=k) = \sum_{n=1}^{\infty} P(X \geq n)$

20. $X \geq k$，意味着前$k-1$局一直是两人轮流获胜

∴ $P(X\geq 1)=1$

$P(X\geq 2k+1) = 2p^k q^k$

$P(X \geq 2k) = p^k q^{k-1} + p^{k-1} q^k = (pq)^{k-1}$ $\quad k=1,2,\cdots$

$EX = \sum_{n=1}^{\infty} P(X\geq n) = \frac{2+pq}{1-pq}$

23. (a)

X	1	2	3
P	$\frac{1}{9}$	$\frac{2}{3}$	$\frac{2}{9}$

Y	0	1	2	3
P	$\frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$

(b) 不独立

(c)

$Y\mid X=1$	0	1	2	3
P	$\frac{2}{3}$	0	0	$\frac{1}{3}$

$X\mid Y=0$	1	2	3
P	$\frac{1}{4}$	$\frac{3}{4}$	0

(d) $P(X=3 \mid Y=2) = 0$

$P(Y=2 \mid X=3) = 0$

补充题:

1. 即前 $m+n-1$ 次实验, 失败了 $m-1$ 次, 成功了 n 次, 且第 $m+n$ 次失败

$$p = C_{m+n-1}^{m-1} q^{m-1} p^{n} \cdot q = C_{m+n-1}^{m-1} q^{m} p^{n}$$

2. $$P(X=k) = \frac{k^n - (k-1)^n}{N^n}$$

$$EX = \sum_{k=1}^{N} k P(X=k) = \sum_{k=1}^{N} k \frac{k^n - (k-1)^n}{N^n} = \frac{1}{N^n}\left(N^{n+1} - \sum_{k=1}^{N-1} k^n\right)$$

$$= N - \cancel{\frac{(1-k)}{1-k}} \frac{1}{N^n} \sum_{k=1}^{N-1} k^n$$

其中: $\sum_{k=1}^{N-1} k^n$ 可由递推得到.