MA323 Lab09 Report

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1a)

Since E_{10} has 4,8,9 as predecessors,
$E_{10} = T_{10} + \max(E_4, E_8, E_9)$

$E_4 = T_4 + E_2 = T_4 + T_2 + T_1$

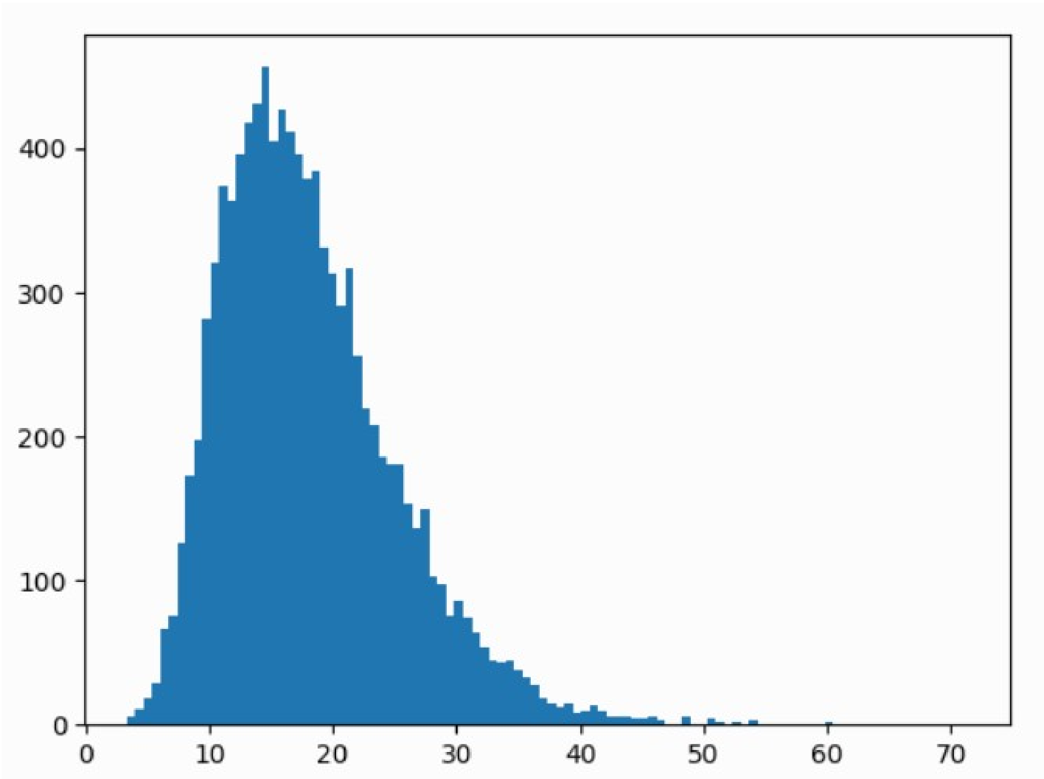
Similarly,
$E_8 = T_8 + T_3 + T_1$
$E_9 = \max(E_5, E_6, E_7) + T_9$

$$\therefore E_{10} = T_{10} + \max(T_4 + T_2 + T_1, \max(T_8 + T_3 + T_1, T_9 + \max(T_5 + T_2 + T_1, \max(T_6 + T_3 + T_1, T_7 + T_3 + T_1))))$$

1b,c,d)

Taking sample size, n = 10000

Using Simple Monte Carlo yields an estimated mean E_{10} as 18.1153



Histogram of E_{10} samples generated

The graph is positively skewed, and skewness measured = 1.0737, this means that the probabilities of values greater than the mean are very small.

The estimated probability = 0.0001 (1 E_{10} greater than 70)
The Standard deviation of the probability = 0.009

The standard deviation is very high, thus this is an unreliable method for sampling the probability.
Also if we used a different seed, the number of E_{10}>70 might've been 0, this is another reason this method is unreliable.

1e)

To Estimate probability using importance method:

$$\widehat{\mu}_{\text{imp}} = \frac{1}{n}\sum_{i=1}^{n}\frac{f(\boldsymbol{X}_i)p(\boldsymbol{X}_i)}{q(\boldsymbol{X}_i)} = \frac{1}{n}\sum_{i=1}^{n} h(\boldsymbol{X}_i)$$

which in this case is (Note that X is 10-dimensional):

p(X) = $\prod_{j=1}^{n} e^{-T_{ij}/\theta_j} / \theta_j$

q(X) = $\prod_{j=1}^{n} e^{-T_{ij}/\lambda_j} / \lambda_j$

f(X) = $I\{E_{i,10} > 70\}$ (Indicator for when $E_{10} > 70$)

$$\therefore\ \widetilde{\mu}_{imp} = {}^{1}/_{n} \sum_{i=1}^{n} I\{E_{i,10} > 70\} \prod_{j=1}^{10} \frac{e^{-T_{ij}/\theta_j}/\theta_j}{e^{-T_{ij}/\lambda_j}/\lambda_j} \quad \sigma_{imp}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(h(X_i) - \widetilde{\mu}_{imp}\right)^2$$

Where I is indicator function for even $E_{10} > 70$
θ_j is the mean time of the i'th process
$\lambda_j = \kappa * \theta_j$

The product calculated is the likeliness ratio(weight).

Estimated probability using Importance method=2.879×10^{-5}
Standard Deviation = 0.00086
Estimated Sample Size = 3.2701

The sample size is too small, this is because multiplying 4 to all the means is causing too much of a distortion. Thus this sampling is unreliable.

1f,g,h)

The formula for calculating the probability will remain same, except that when $j \in \{3, 5, 6, 7, 8, 9\}$, $\kappa=1$

κ	Estimated Probability	Number of $E_{10}>70$	Standard Deviation	99% Confidence Interval for probability	Estimated Sample Size
3	3.04889×10^{-5}	1447	0.00017206	$[2.6049 \times 10^{-5}, 3.4927 \times 10^{-5}]$	917.95
4	2.82657×10^{-5}	3142	0.00014025	$[2.4647 \times 10^{-5}, 3.1884 \times 10^{-5}]$	327.47
5	3.38229×10^{-5}	4793	0.00017824	$[2.9224 \times 10^{-5}, 3.8422 \times 10^{-5}]$	141.05

Since only the processes on critical path, I.e, {1,2,4,10}, contribute to E_{10}, while the the rest do not contribute as much, so we do not have to increase the mean for all the T_j, that is the reason the estimated sample sizes here are greater than the value obtained in (e).

$\kappa=5$ has the smallest estimated sample size and the 99% confidence interval of probability = $[2.9224 \times 10^{-5}, 3.8422 \times 10^{-5}]$