

MA323 Lab-02 Report

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Q1)

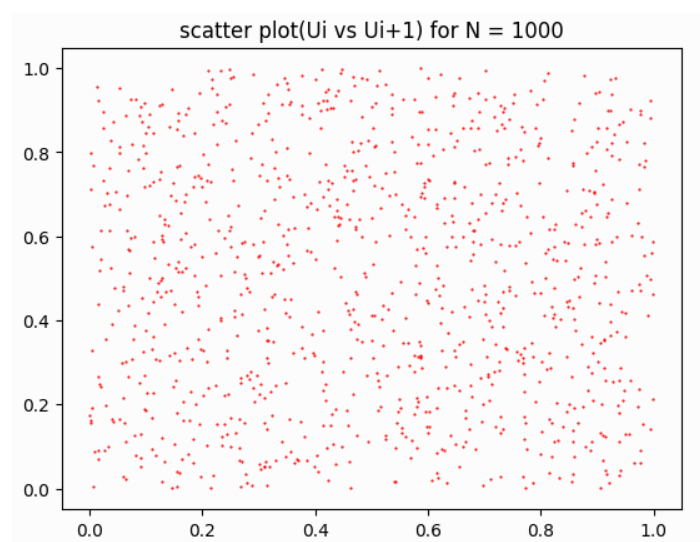
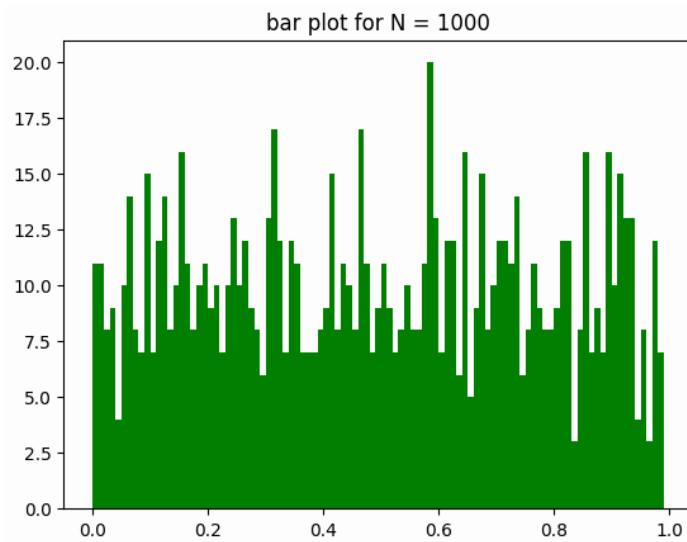
Linear congruence generator:

$$x_{i+1} = (ax_i + b) \bmod m$$

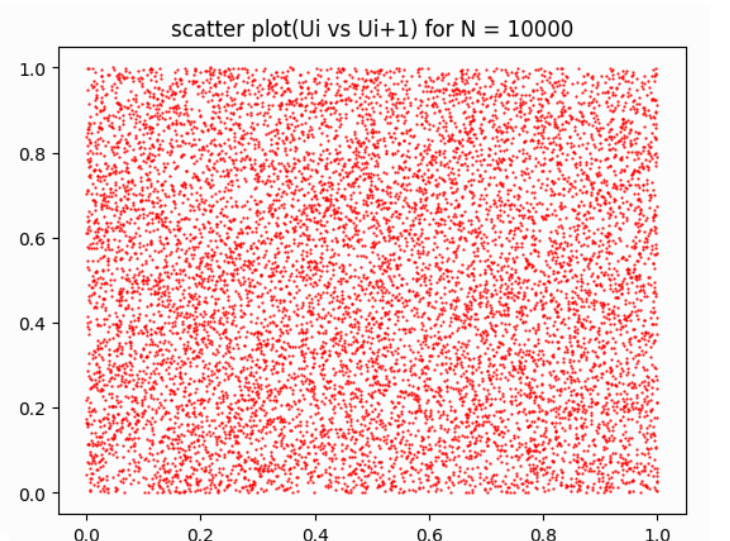
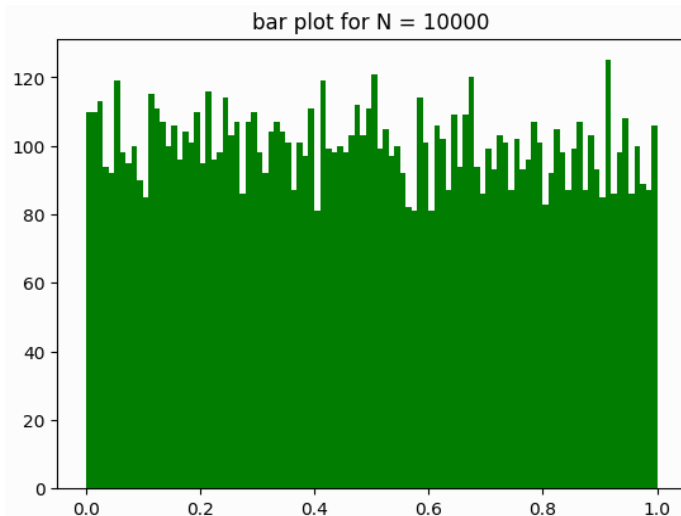
$$u_i = x_i/m$$

$$a = 1597, b = 1, m = 244944$$

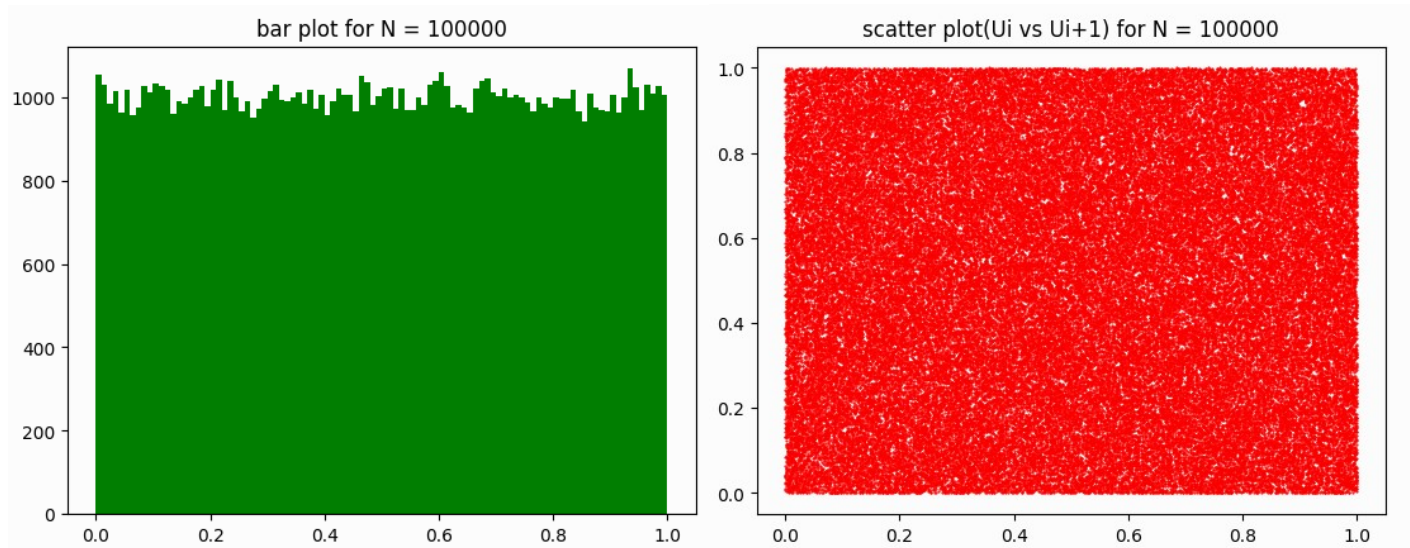
(I) Plot for N = 1000 values:



(II) Plot for N = 10000 values:



(III) Plot for N = 100000 values:



Observations:

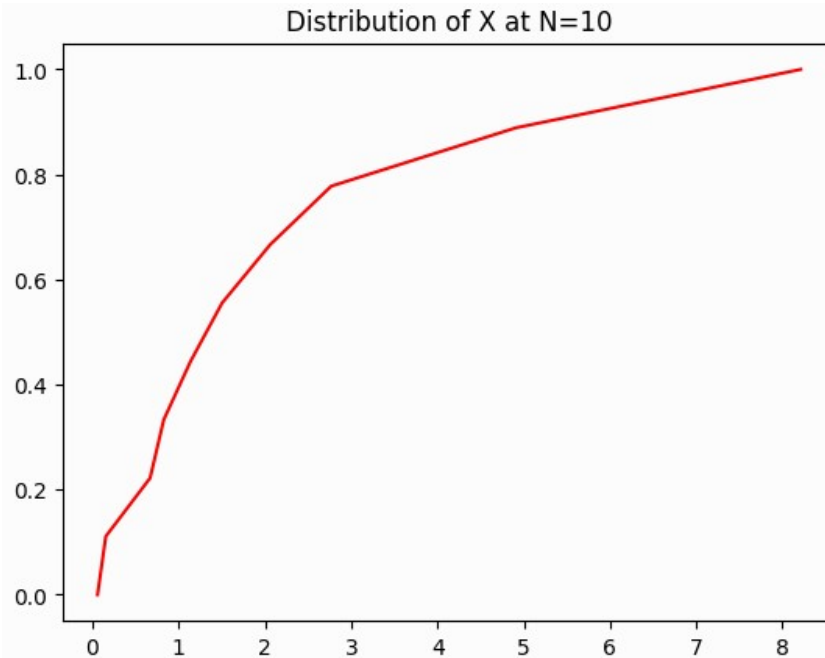
- As seen in the scatter plot, since the U_i 's do not follow any pattern, they are almost completely random, hence the random generator behaves as a good random generator.

Q2)

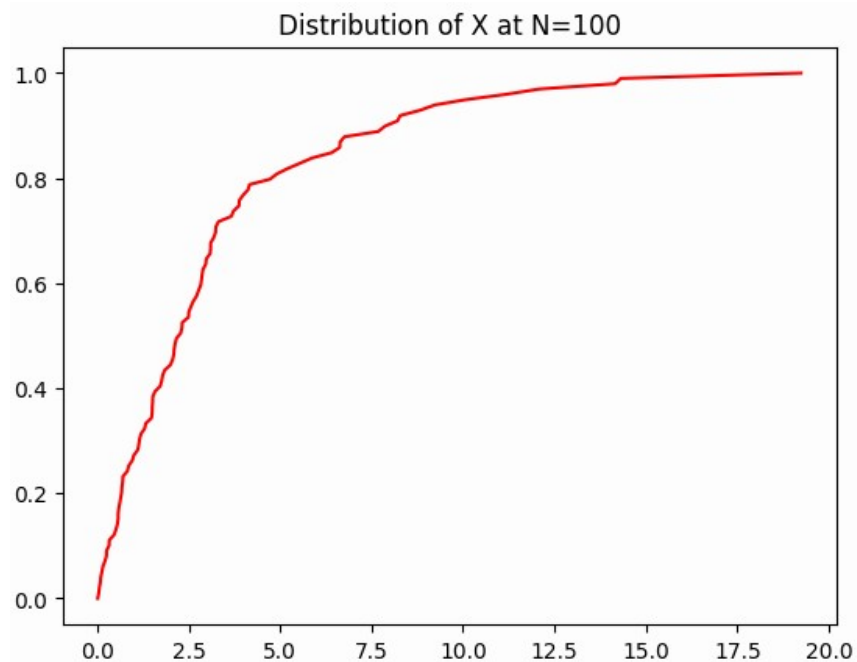
CDF: $F(x) = 1 - e^{-x/\theta}$

Assume Mean (θ) = π

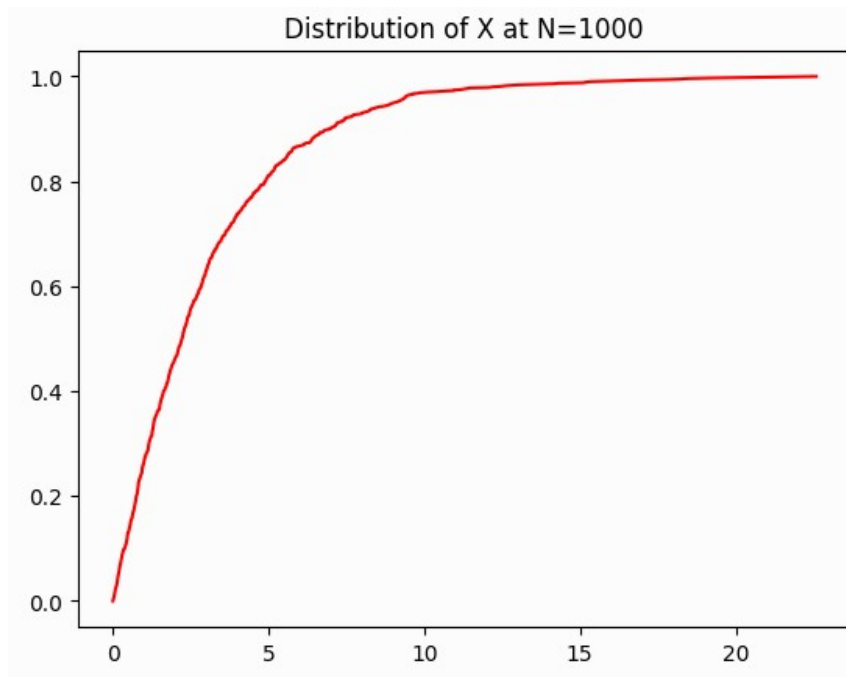
(I) Distribution of X for N = 10 values:



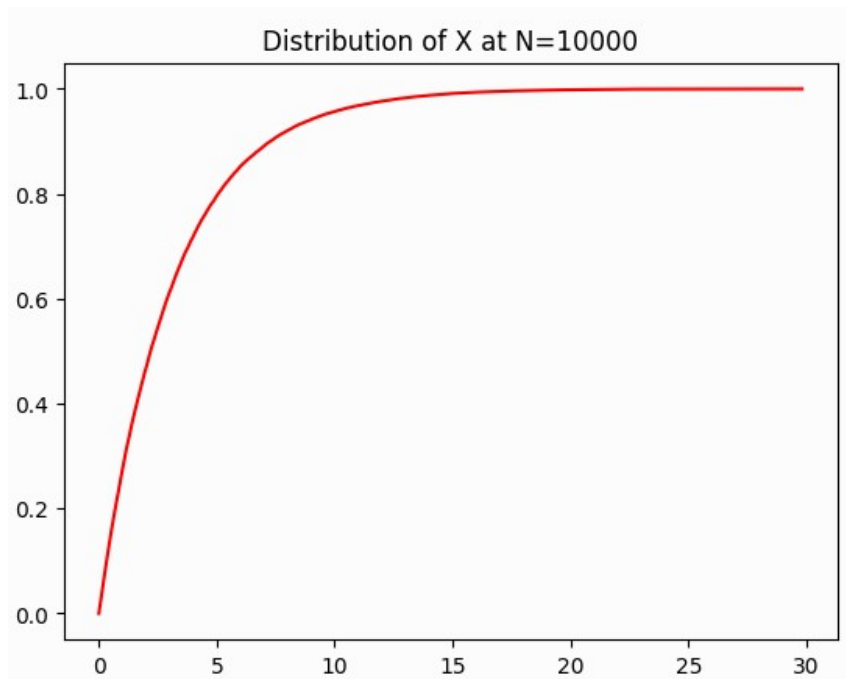
(II) Distribution of X for N = 100 values:



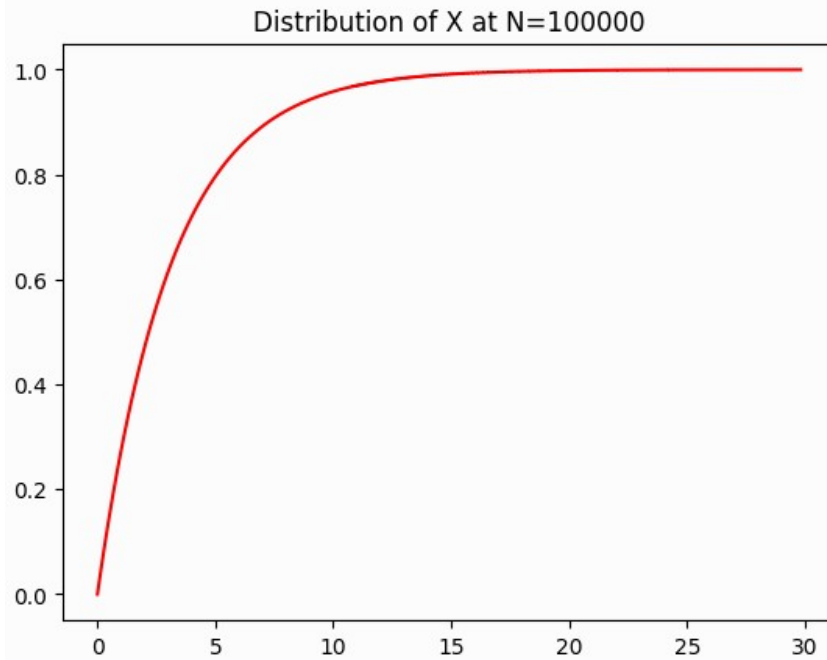
(III) Distribution of X for N = 1000 values:



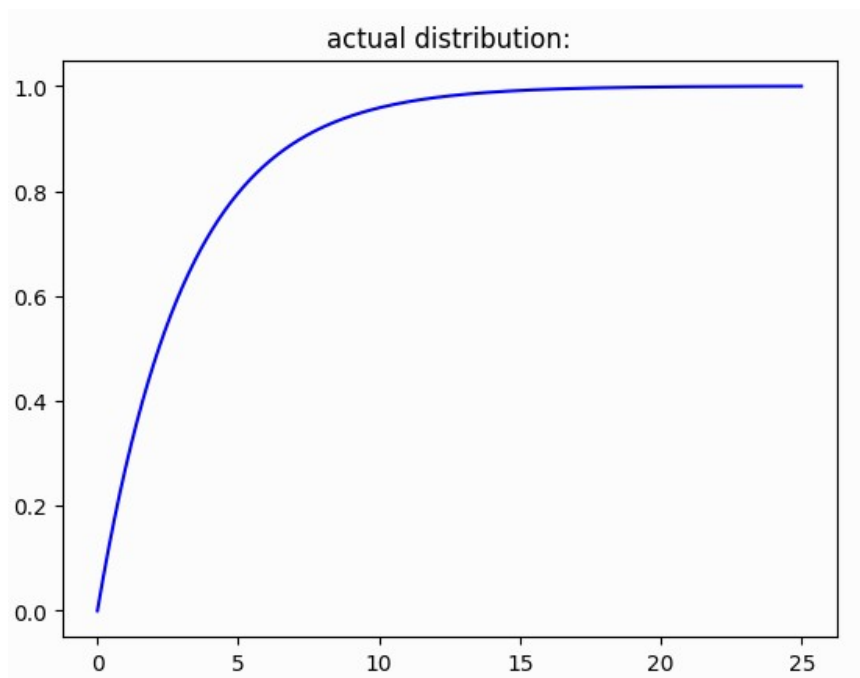
(IV) Distribution of X for N = 10000 values:



(V) Distribution of X for N = 100000 values:



Actual Distribution:



N	Sample Mean	Sample Variance
10	2.2305096009221885	5.842095983802902
100	3.2247396125776895	11.98085780828704
1000	3.081161111374821	9.59241179375116
10000	3.1563771607678928	10.017771389934223
100000	3.1416862866656436	9.85951207447013

Actual Mean = 3.141592653589793

Actual Variance = 9.869604401089358

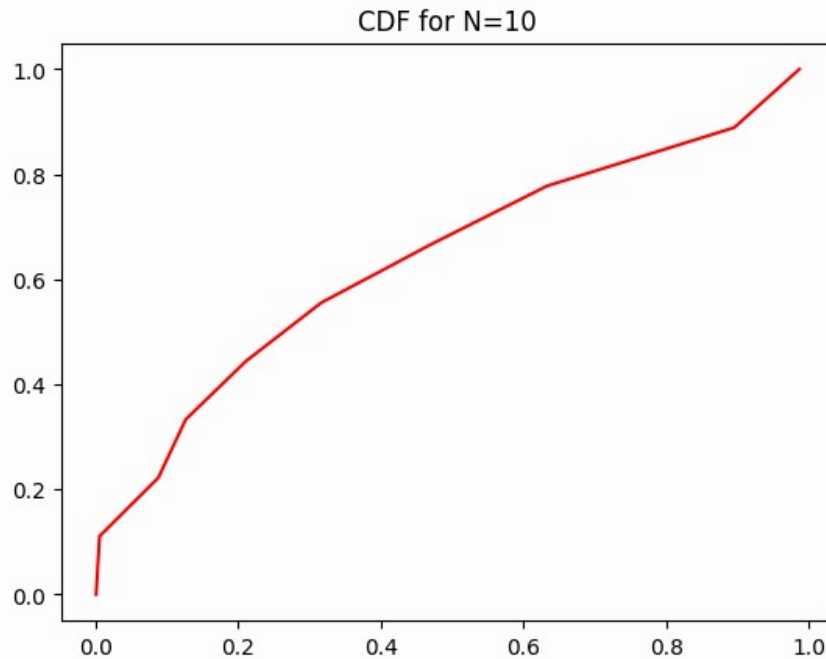
Observations:

- As we increase the number of values generated the mean and variance of the generated numbers converge to the actual mean and variance.
- The distribution of X is identical to the CDF ($F(x)$) which shows the inverse transform method.

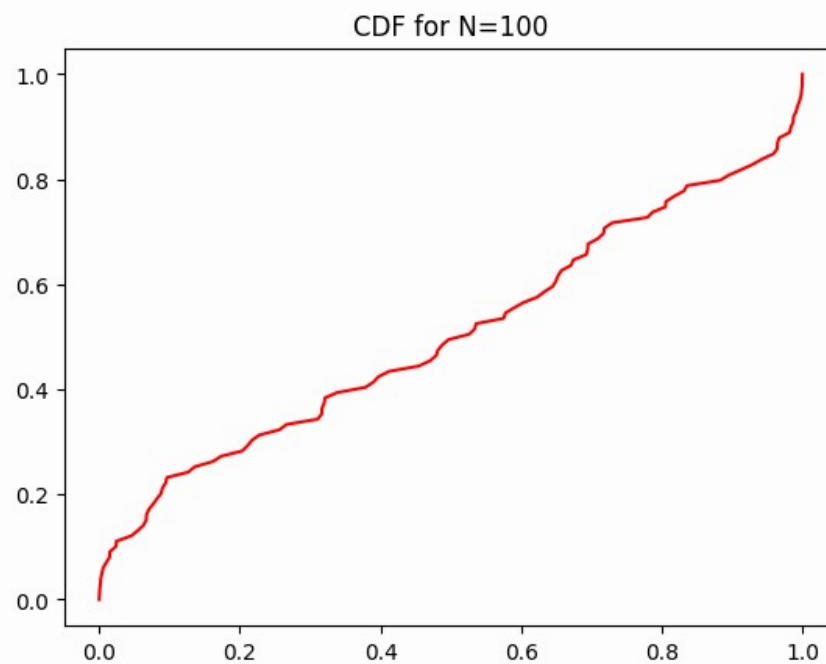
Q3)

CDF: $F(x) = 2/\pi \sin^{-1}(x^{1/2})$

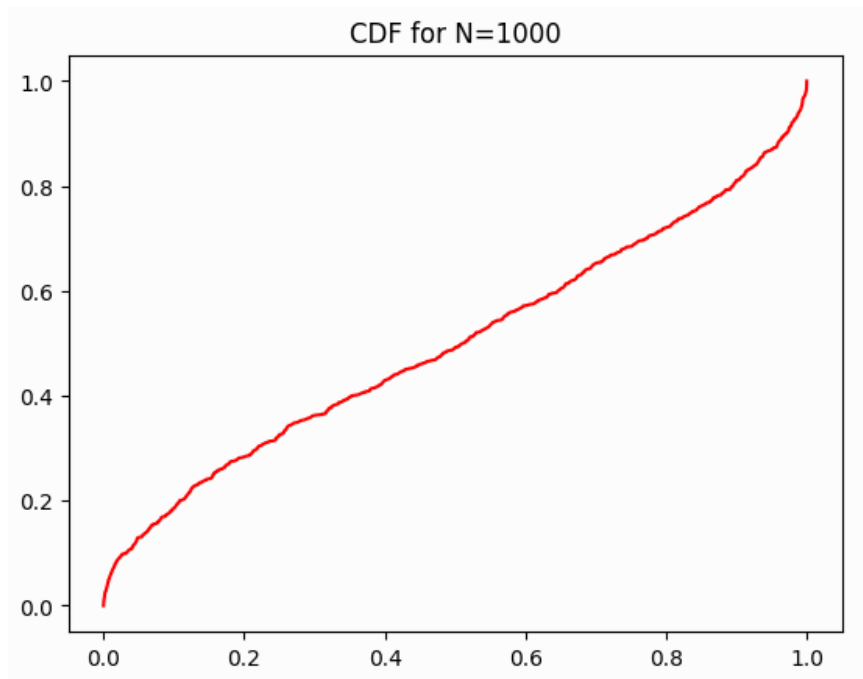
(I) Distribution of X for N = 10 values:



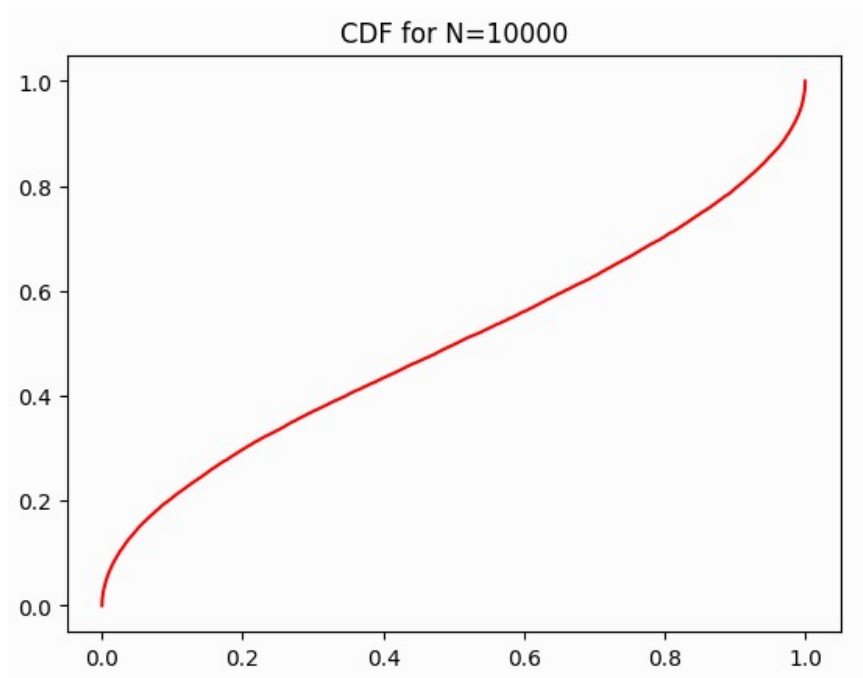
(II) Distribution of X for N = 100 values:



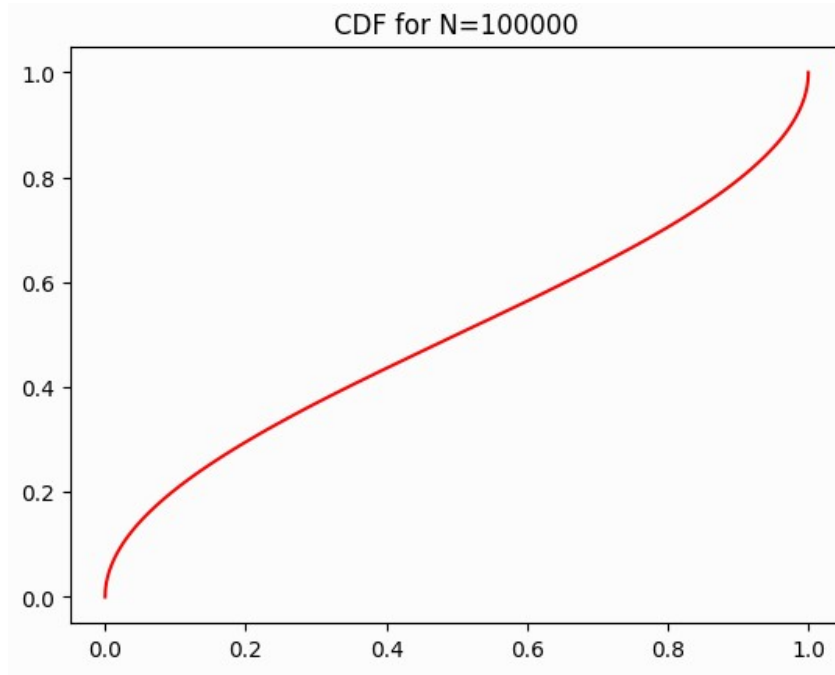
(III) Distribution of X for N = 1000 values:



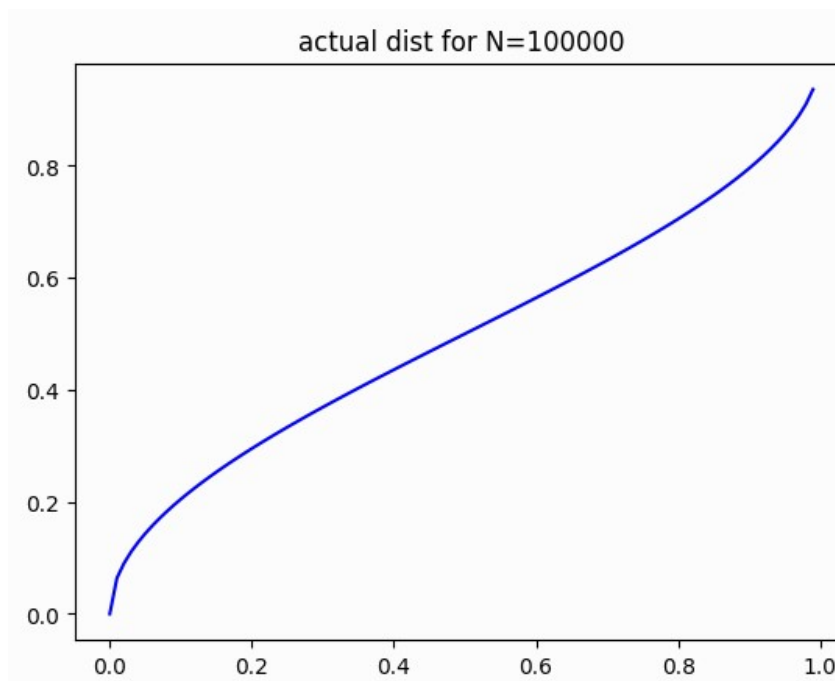
(IV) Distribution of X for N = 10000 values:



(V) Distribution of X for N = 100000 values:



Actual Distribution:



N	Sample Mean	Sample Variance
10	0.37347798353615164	0.11717232248515157
100	0.4945329062603193	0.12066235544391767
1000	0.4989849034206795	0.11906164185832245
10000	0.500682895969179	0.12555147882558407
100000	0.4998735233307937	0.12507022069752055

Q4)

Frequencies of values generated by discrete random number generation:

Ranges	Frequencies
1-1000	10026
1001-2000	10015
2001-3000	9958
3001-4000	9960
4001-5000	10159
5001-6000	9898
6001-7000	9946
7001-8000	10062
8001-9000	10083
9001-10000	9893