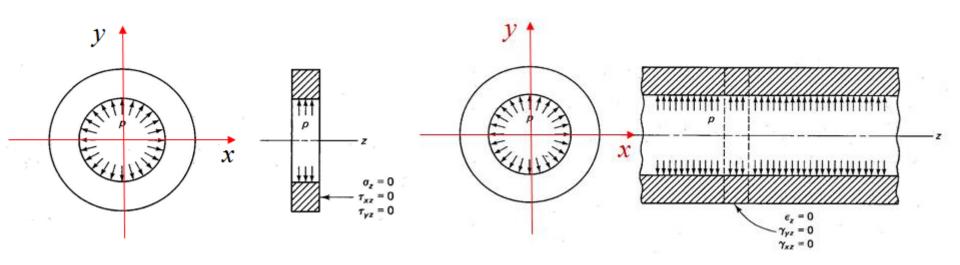


### Analysis of 2D Structural Solids

A thin planar body

A long body of uniform cross section



plane stress

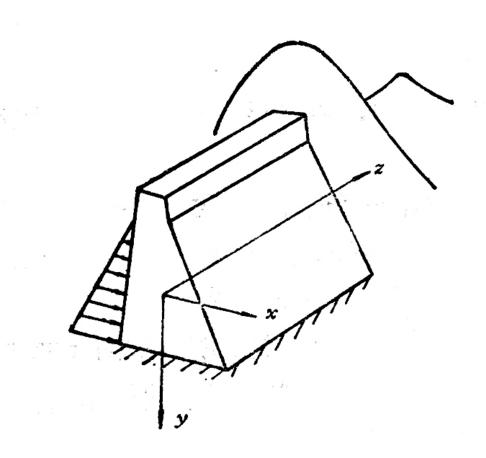
$$\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$$

plane strain

$$\varepsilon_{zz} = \gamma_{xz} = \gamma_{yz} = 0$$

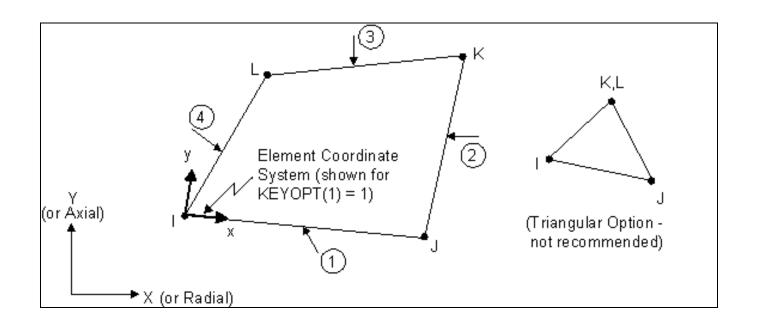


### 2D Plane strain problem



A concrete dam

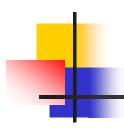
### PLANE42 元素描述





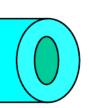
### PLANE42 輸入資料

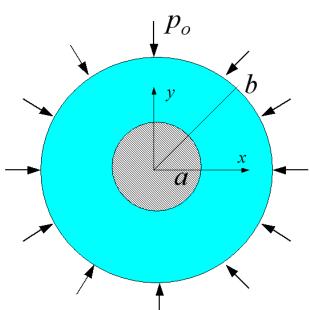
Element Name	PLANE42
Nodes	I, J, K, L
Degrees of Freedom	UX, UY
Real Constant	None, if KEYOPT (3) = 0, 1, 2 Thickness, if KEYOPT (3) = 3
Material Properties	EX, NUXY, GXY, ALPX, DENS, DAMP, etc.
Surface Loads	Pressure: face 1 (J-I), face 2 (K-J), face 3 (L-K), face 4 (I-L)
Body Loads	Temperature T(I), T(J), T(K), T(L)
Special Features	Plasticity, Creep, Stress stiffening, Large deflection, Large strain, etc.
KEYOPT(1)	Key for element coordinate system: 0 Element C.S. is parallel to the global C.S. 1 Element C.S. is based on the element I-J side
KEYOPT(2)	Key to include extra shapes: 0 Include extra displacement shapes 1 Suppress extra displacement shapes
KEYOPT(3)	0 Plane stress 1 Axisymmetric 2 Plane strain 3 Plane stress with thickness input



### Plane strain problem

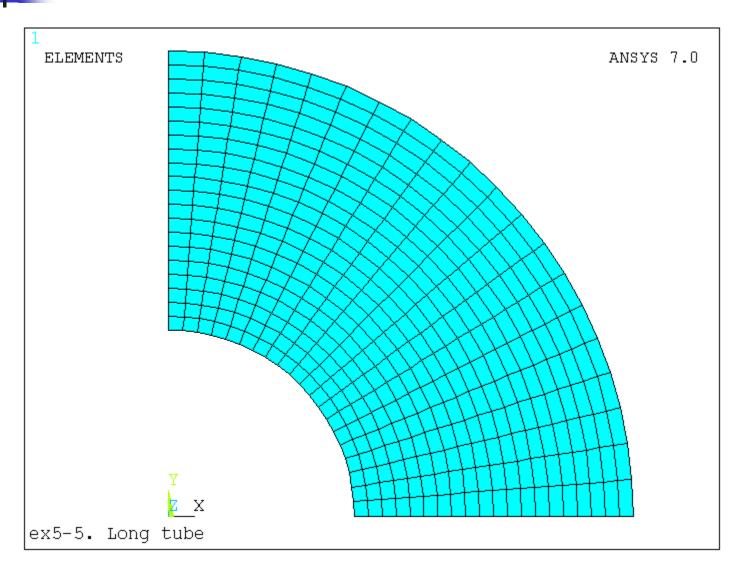
❖下圖為一長條狀之厚壁(thick-walled)圓管,內外徑分別為a=200mm和b=500mm,其圓管長度遠大於外徑,圓管內徑黏接於一剛性圓柱,外徑受壓力 p₀=1MPa。圓管材料之楊氏模數E=210GPa,普松比v=0.3,試求圓管應力分布。分析單位系統採用:mm、N、MPa。





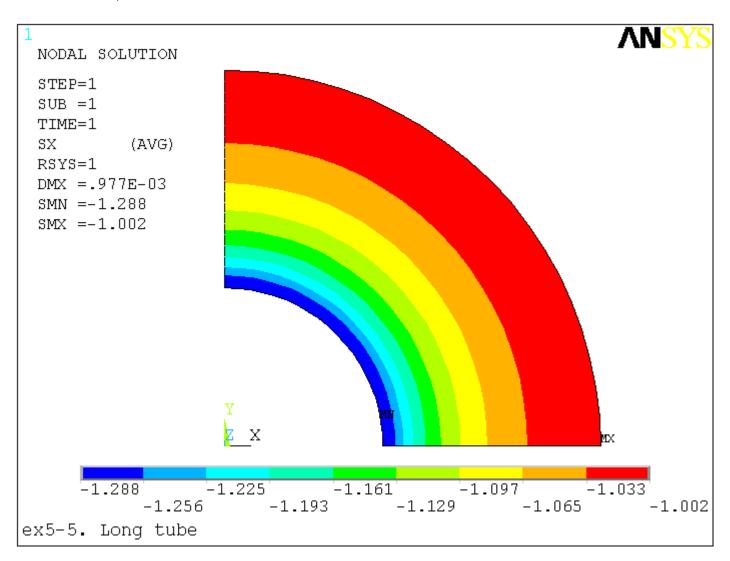
# 4

## 有限元素模型



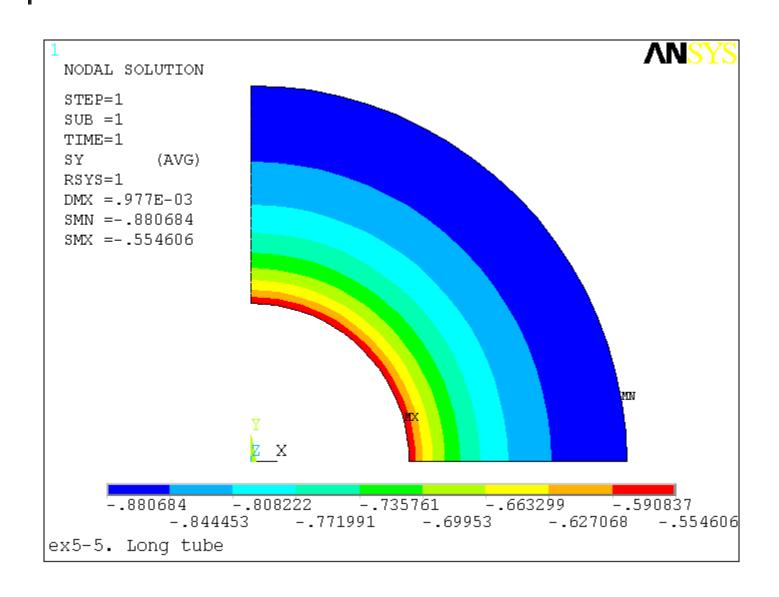
### 圓柱座標應力 $\sigma_r$ (SX) (MPa)

### RSYS, 1

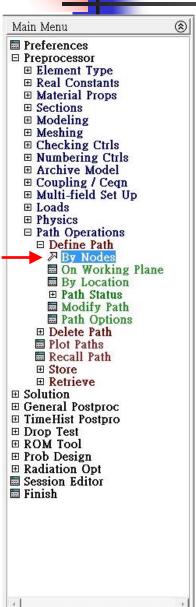


# -

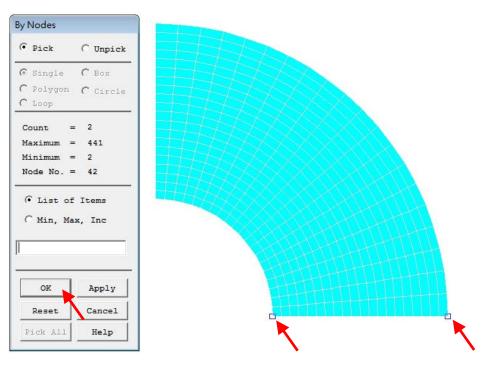
## 圓柱座標應力 $\sigma_{\theta}$ (SY) (MPa)



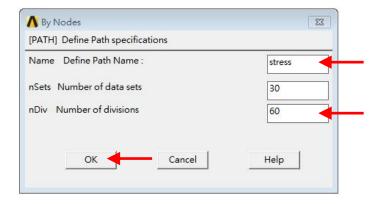
### Path operation for X-Y graph

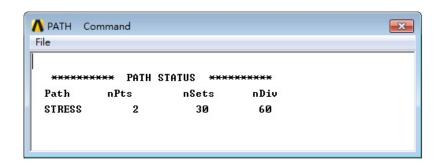












### ○指令說明◁◁

PATH, NAME, nPts, nSets, nDiv

定義一用於顯示資料的路徑。

NAME: 路徑的名稱,不得超過 8 個字元。如果所指定的名字重複則會將舊的覆蓋,如果只指定一已經存在的路徑名稱忽略其他項目,則表示使用此路徑 (Recall Path),如果 NAME=STATUS,則表示列出所有已定義的路徑。

nPts:用於定義路徑的點的數量,最多不可超過 1000 點,最少 2 點。

nSets:可儲存的資料數量,預設爲30組,最少要4組

nDiv:每兩點之間,欲細分的資料點數,預設爲20點。



[PDEF] Map Result Items onto Path		
Lab User label for item		
Item,Comp Item to be mapped	DOF solution	A-
	Stress Y-direction SY	
	Strain-total E Z-direction SZ	
	Energy XY-shear SXY	
	Strain-elastic YZ-shear SYZ	+
	Strain-thermal	- 3
	Strain-plastic + X-direction SX	
[AVPRIN] Eff NU for EQV strain		
Average results across element	<b>▽</b> Yes	
[/PBC] Show boundary condition symbol		
Show path on display	E 51-	
snow path on display	☐ No	
OK Appl	y Cancel Help	

PDEF, Lab, Item, Comp, Avglab

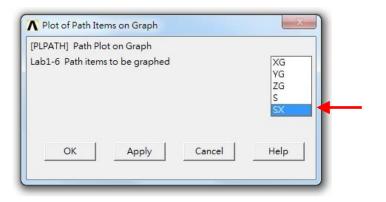
擷取路徑上的資料。

Lab: 資料的名稱,最多 8 個字元。如果為 STAT 表示列示出目前所使用的路 徑上所定義的所有資料,如果為 CLEAR,則表示清除所有使用者定義的 資料。

Item, Comp: 欲撷取的資料種類,其使用方式與指令 PLNSOL 相同。

Avglab: 用來設定擷取時是否平均元素間的資料, AVG 表示平均, NOAV 表示 不平均,預設爲 AVG。

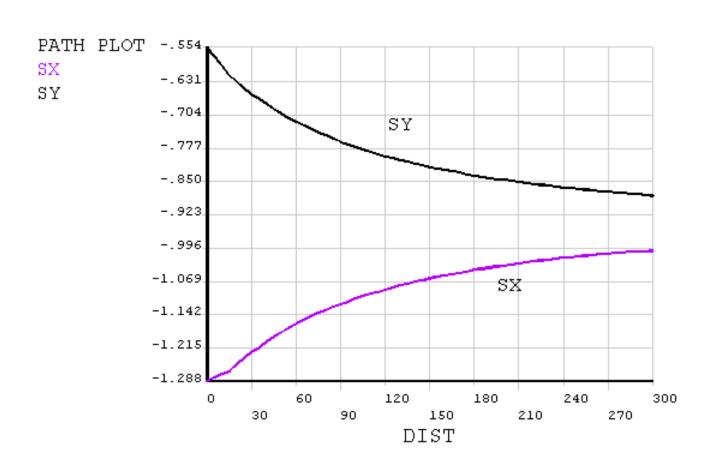




PLPATH, Lab1, Lab2, Lab3, Lab4, Lab5, Lab6 以線條圖的方式將資料畫出。

Lab1~ Lab6:資料的名稱。

### Path operation (plot)





### 討論

□ 本例在文獻中有解析解如下:

$$\sigma_r = -\frac{p_o b^2}{a^2 (1 - 2v) + b^2} \left[ 1 + (1 - 2v) \frac{a^2}{r^2} \right]$$

$$\sigma_{\theta} = -\frac{p_{o}b^{2}}{a^{2}(1-2v)+b^{2}} \left[1-(1-2v)\frac{a^{2}}{r^{2}}\right]$$

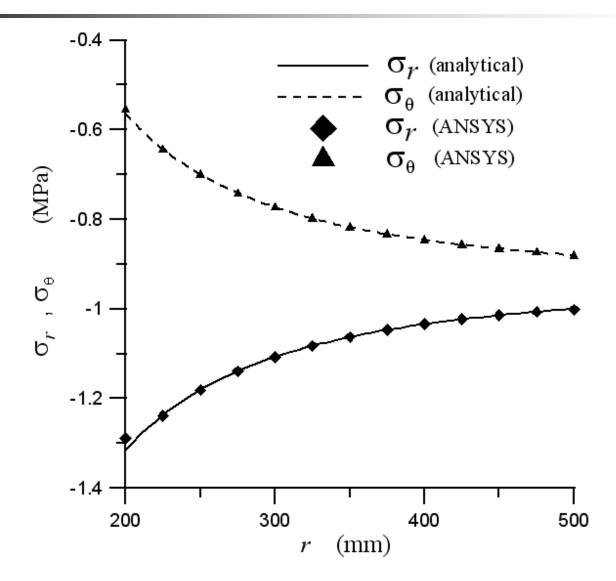
□ 將本例題目的數值代入上式可得

$$\sigma_r = -0.9399 \left[ 1 + \frac{16000}{r^2} \right]$$
 MPa

$$\sigma_{\theta} = -0.9399 \left[ 1 - \frac{16000}{r^2} \right]$$
 MPa

A.P. Boresi and K.P. Chong, Elasticity in Engineering Mechanics. Second edition, New York: John Wiley & Sons, 2000

應力 $\sigma_r$ 和 $\sigma_\theta$ 解析解與 ANSYS解的比較



ANSYS 計算之最大誤差發生於內徑位置 (r = a = 200 mm) 的  $\sigma_r$ ,爲 2.1%。