Introductory Workshop

... Galloping Gertie

Objective

- To get an idea of the steps involved in a typical dynamic analysis.
- The Tacoma Narrows bridge, also known as the Galloping Gertie is famous for its spectacular collapse in 1940. In this workshop, we will examine a model of the bridge and calculate its natural frequencies and mode shapes. We will then simulate the wind storm and vortex shedding that caused the bridge's collapse by doing a harmonic analysis.

自由振動之特徵值問題

- 自然頻率(**natural frequencies**)與模態振型(**mode shapes**)是結構重要的動態特徵，若外界振動源的頻率與結構本身的自然頻率相等或非常接近，便會造成共振，形成大振幅的振動。若一系統之阻尼(**damping**)夠小時，其對於自然頻率之影響很小，因此便可將阻尼忽略，以特徵值問題(**eigenvalue problem**)來處理無阻尼之自由振動，進而求出系統的特徵值(**eigenvalues**)與特徵向量(**eigenvectors**)，它們分別代表自然頻率與模態振型。

Theory and Assumptions

- General equation of motion:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F(t)\}$$

- Assume free vibrations and ignore damping:

$$[M]\{\ddot{u}\}+[K]\{u\}=\{0\}$$

- Assume harmonic motion:

$$\{u\}=\{\phi\}_i\sin(\omega_i t+\theta_i)$$
$$\{\ddot{u}\}=-\omega_i^2\{\phi\}_i\sin(\omega_i t+\theta_i)$$

- Substituting $\{u\}$ and $\{\ddot{u}\}$ in the governing equation gives an eigenvalue equation:

$$\left([K]-\omega_i^2[M]\right)\{\phi_i\}=\{0\}$$

- The roots of this equation are ω_i^2, the eigenvalues, where i ranges from 1 to number of DOF. Corresponding vectors are $\{\phi_i\}$, the eigenvectors.

- The square roots of the eigenvalues are ω_i , the structure's natural circular frequencies (radians/sec). Natural frequencies f_i are then calculated as $f_i = \omega_i /2\pi$ (cycles/sec). It is the natural frequencies f_i that are input by the user and output by ANSYS.

- The eigenvectors $\{\phi_i\}$ represent the mode shapes - the shape assumed by the structure when vibrating at frequency f_i.

- Mode shapes $\{\phi_i\}$ are relative values, not absolute.

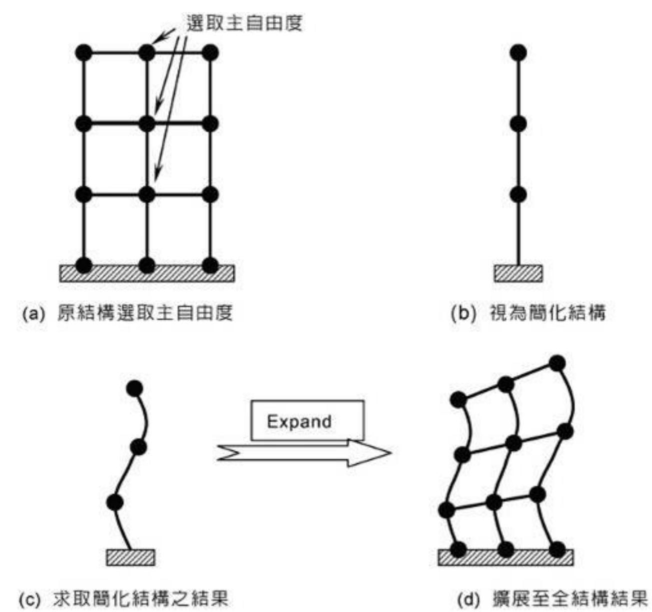
- **ANSYS**之模態分析**(modal analysis)**，主要就是利用有限元素法，求解無阻尼自由振動的結構特徵值問題。當然，若實際問題之阻尼無法忽略，**ANSYS**也提供含阻尼之模態分析方法**(**即**damped method)**。此外必須注意，不論是以上的數學分析或是**ANSYS**的模態分析，均被假設為線性問題，也就是結構變形很小，且材料性質在線性範圍。

Terminology & Concepts

- **Mode Extraction** is the term used to describe the calculation of eigenvalues and eigenvectors.

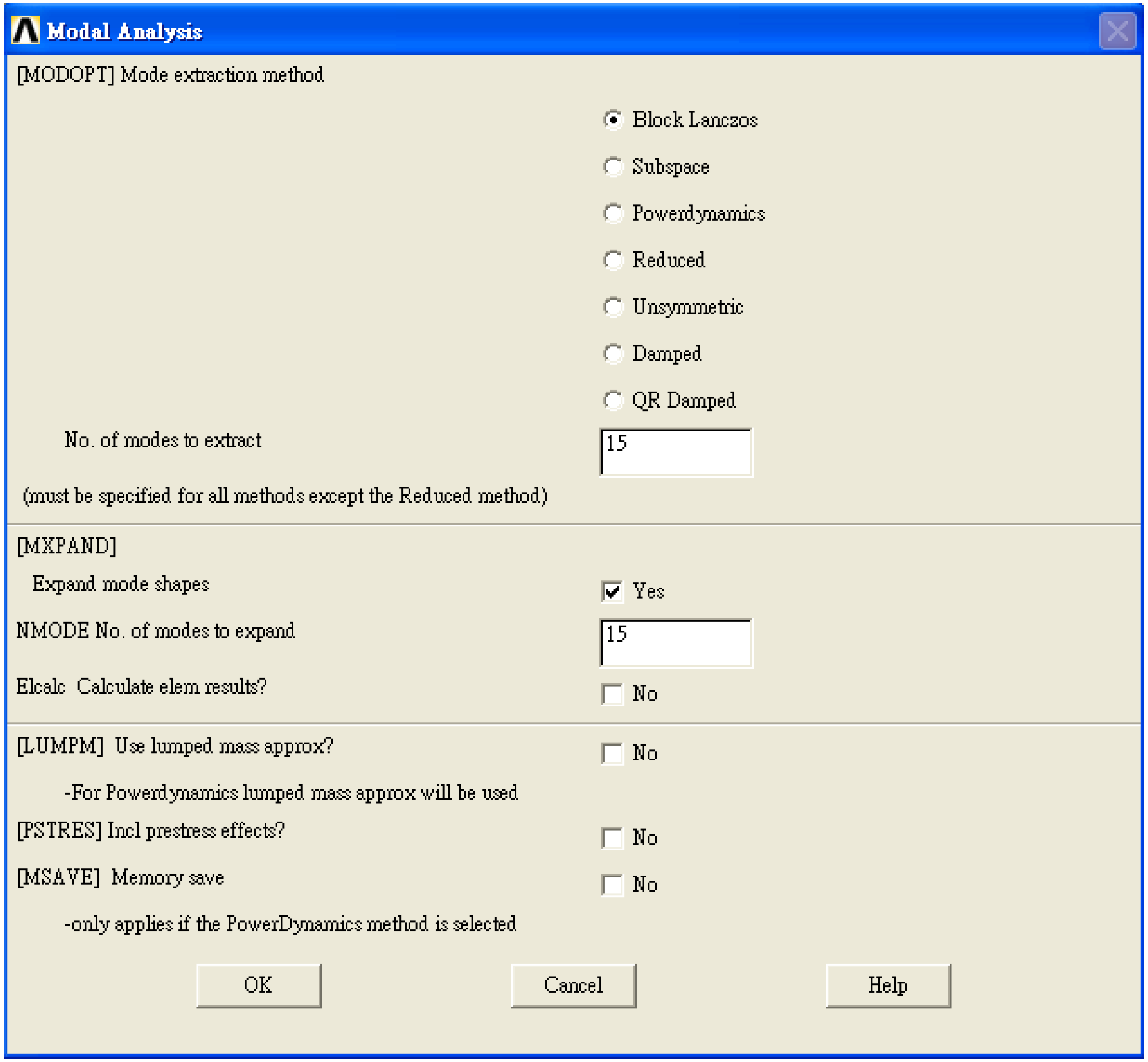
- **Mode Expansion** has a dual meaning. For the reduced method, mode expansion means calculating the full mode shapes from the reduced mode shapes. For all other methods, mode expansion simply means writing mode shapes to the results file.

降階法(reduced method)模態示意圖:



Mode Extraction Methods

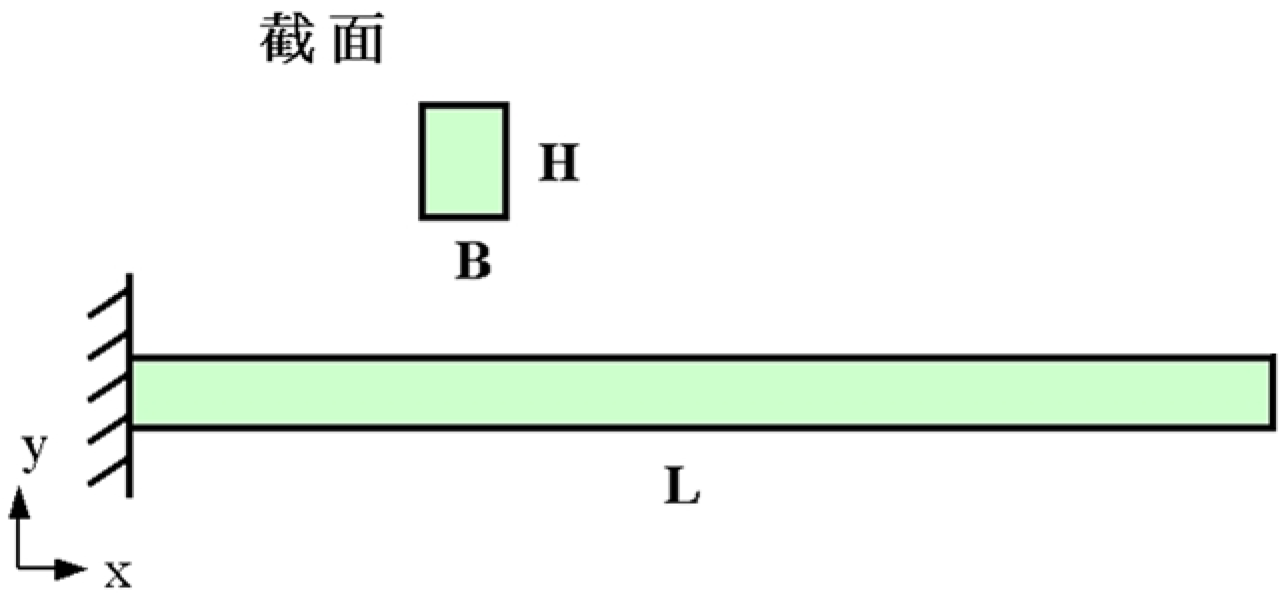
- Several mode extraction methods are available in ANSYS:



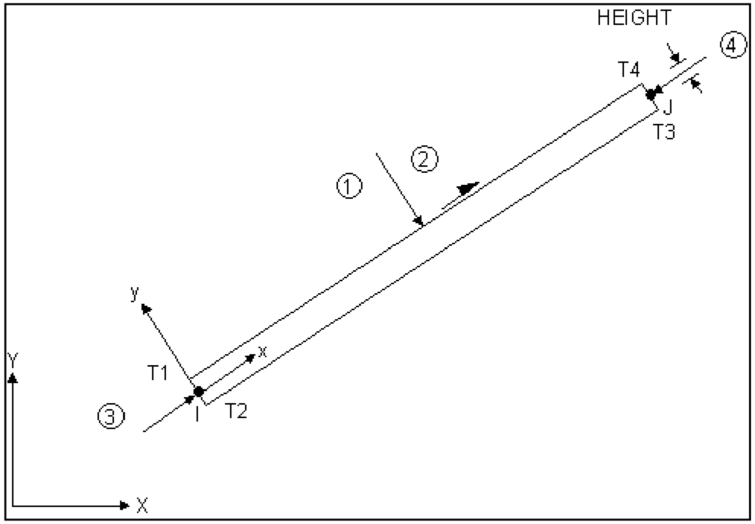
- Block Lanczos method:該方法用於大型結構對稱之質量及剛性矩陣，和次空間方法相似，但收斂性更快，此方法為軟體內定之模態抓取法。
- Subspace method:次空間法通常用於大型結構中，僅探討前幾個之振動頻率，所得到之結果較準確，不需定義主自由度，但需要較多的硬碟空間。
- Power dynamics method: 該方法用於非常大之結構(自由度大於10^5)且僅需得知最小之數個模態。
- Reduced method: 降階法要搭配主自由度來分析，藉由該主自由度以定義結構之質量矩陣及剛性矩陣，並求取其頻率及振動模態，進而將其結果擴展至全部結構。
- Unsymmetric method: 該方法使用於質量矩陣或剛性矩陣為非對稱，例如噪音與結構耦合。
- Damped method: 該方法使用於結構系統具有阻尼現象，例如轉子動力系統。

懸臂樑橫向振動之模態分析

- 如下圖所示之懸臂樑，長度**L=0.3m**，截面高**H=0.006m**，厚度**B=0.003m** ，楊氏模數**E=2200MPa**，密度**ρ=1100kg/m³**。求其無外力之橫向(**y**方向)振動自然頻率與模態振型。分析單位系統採用**SI**制：**m**、**N**、**Pa**、**kg**。



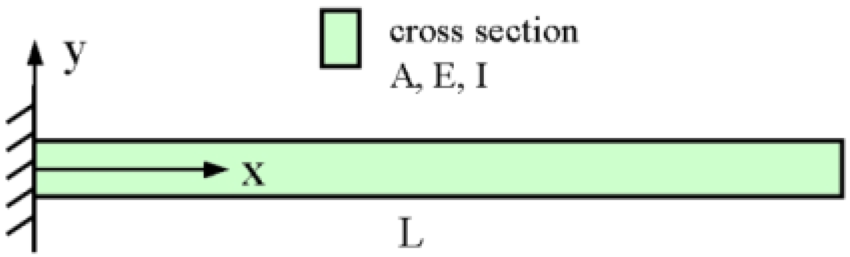
BEAM3 輸入資料

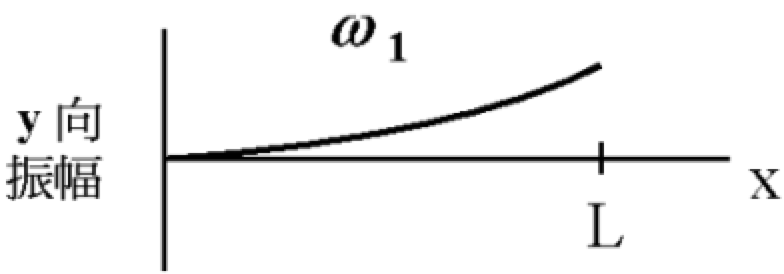


Element Name	BEAM3
Nodes	I, J
Degrees of Freedom	UX, UY, ROTZ
Real Constants	AREA, IZZ, HEIGHT, SHEARZ, ISTRN, ADDMAS
Material Properties	EX, NUXY, GXY, ALPX, DENS, DAMP
Surface Loads	Pressure face 1, face 2, face 3, face 4
Body Loads	Temperature -- T1, T2, T3, T4
Special Features	Stress stiffening, Large deflection, etc.

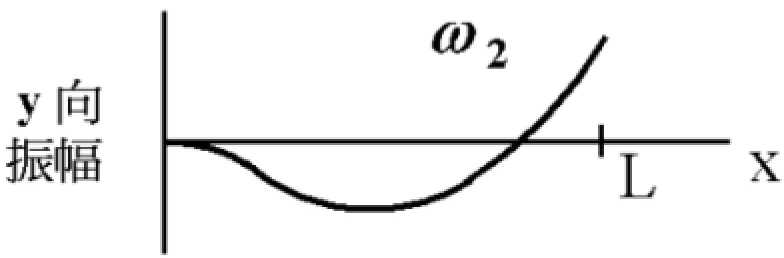
- 指令「**MODOPT,LANB,10,0,500**」是設定模態抓取法**(mode extraction method)**，**LANB**即代表**Block Lanczos method**，參數**10**代表模態抓取數目，接著兩個參數**0**和**500**代表模態抓取頻率範圍是**0~500Hz**。
- 指令「**MXPAND,10**」用來設定模態擴展**(expand)**數目為**10**個。
- 特別注意在**ANSYS**的振動分析中，頻率的輸入和輸出值，單位均為**Hz(**次**/**秒**)**。
- 「**LUMPM**」指令可設定是否使用簡化之集中質量**(lumped mass)** 。
- 「**PSTRES**」指令用來設定含預應力之模態分析 。

懸臂樑之橫向自由振動

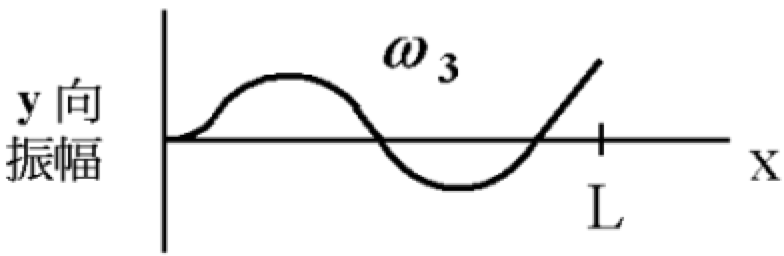




$$\omega_1 = 3.5160\sqrt{\frac{EI}{mL^4}}$$



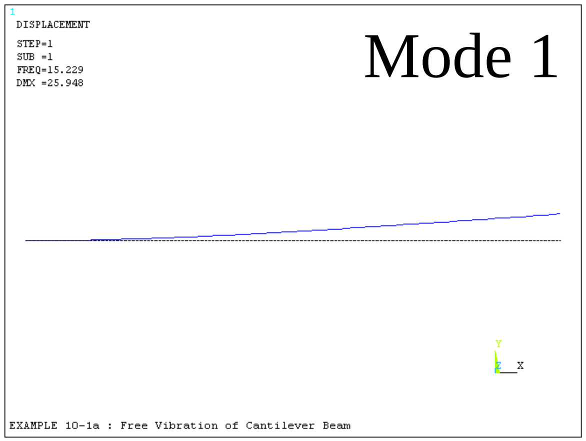
$$\omega_2 = 22.0345\sqrt{\frac{EI}{mL^4}}$$

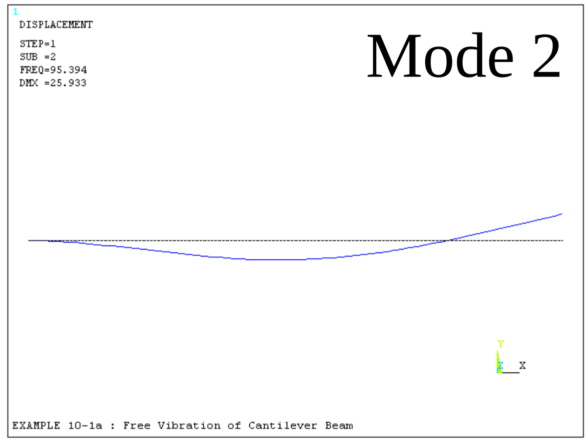


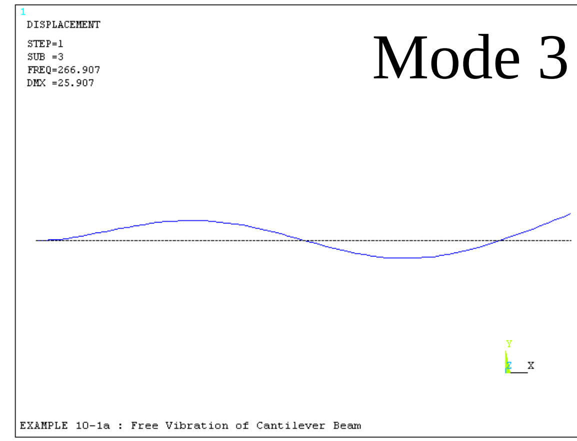
$$\omega_3 = 61.6972\sqrt{\frac{EI}{mL^4}}$$

結果與討論

ANSYS 振動模態分析:







自然頻率比較 ：

	ANSYS解	理論解	誤 差
第1 模態 ω_1	15.229 Hz	15.23 Hz	0.0066%
第2 模態 ω_2	95.394 Hz	95.447 Hz	0.056 %
第3 模態 ω_3	266.91 Hz	267.25 Hz	0.1272%

- 對於本題自然頻率之理論解，可將本範例之條件代入，得到以下結果：

$$E = 2200 \times 10^{6}\,Pa \qquad L = 0.3m \qquad I = \frac{1}{12} BH^{3} = 5.4 \times 10^{-11} m^{4}$$

$$m = \rho BH = 1.98 \times 10^{-2}\, kg/m$$

$$\omega_1 = 3.5160\sqrt{\frac{EI}{mL^4}} = 95.695\,rad/\sec = 15.23Hz$$

$$\omega_2 = 22.0345\sqrt{\frac{EI}{mL^4}} = 599.71\,rad/\sec = 95.447Hz$$

$$\omega_3 = 61.6972\sqrt{\frac{EI}{mL^4}} = 1679.2\,rad/\sec = 267.25Hz$$

Utility Menu: PlotCtrls > Animate > Deformed Shape

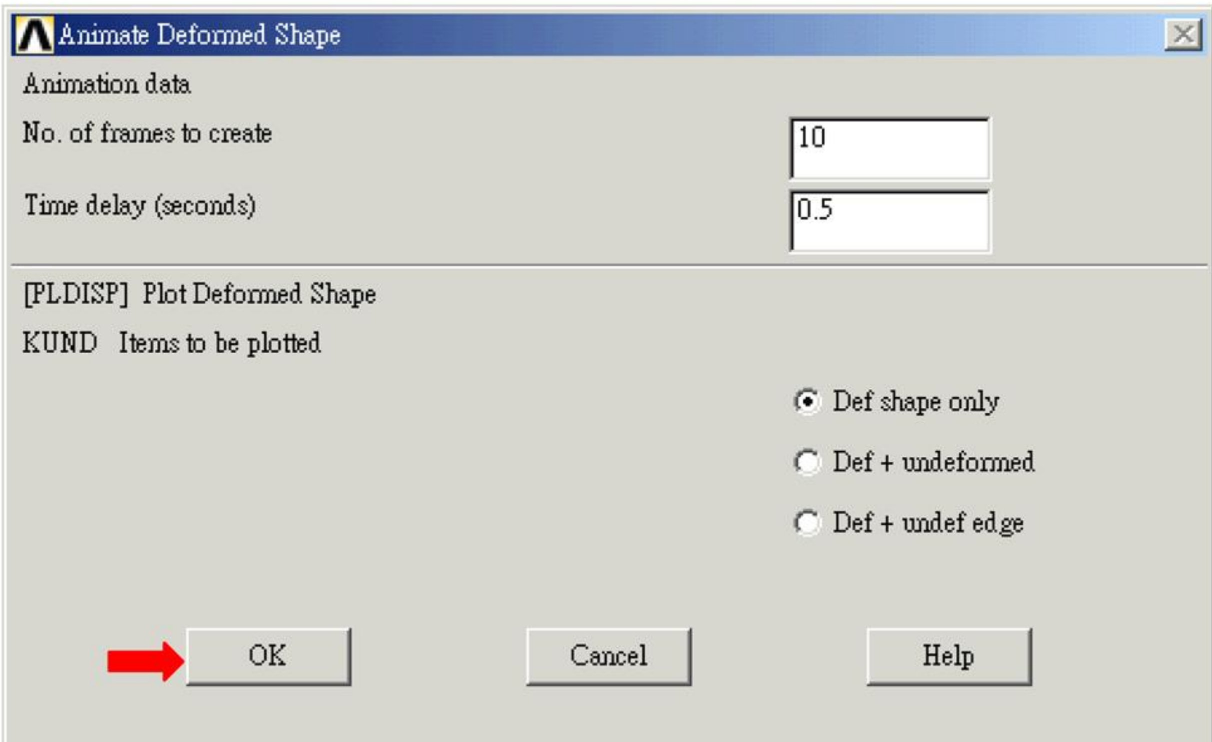


圖 3.4-2　動畫控制視窗