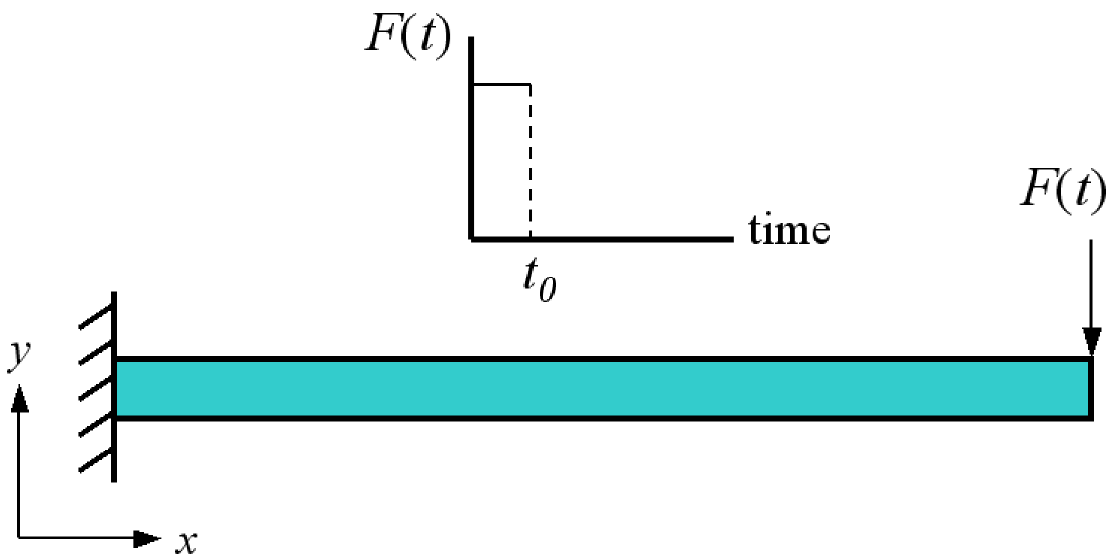
暫態動力學分析

❖ 暫態動力學**(transient dynamic)**分析是關於時間和加速度相關的應力與變形分析。典型的暫態動力學分析如下圖所示，當一結構受到衝擊力**(impact force) F(t)**作用時，它的反應會與時間有關，這與靜態分析大不相同，而且衝擊力作用時間$\mathbf{t_0}$的長短也會造成不同的結構反應與應力場。



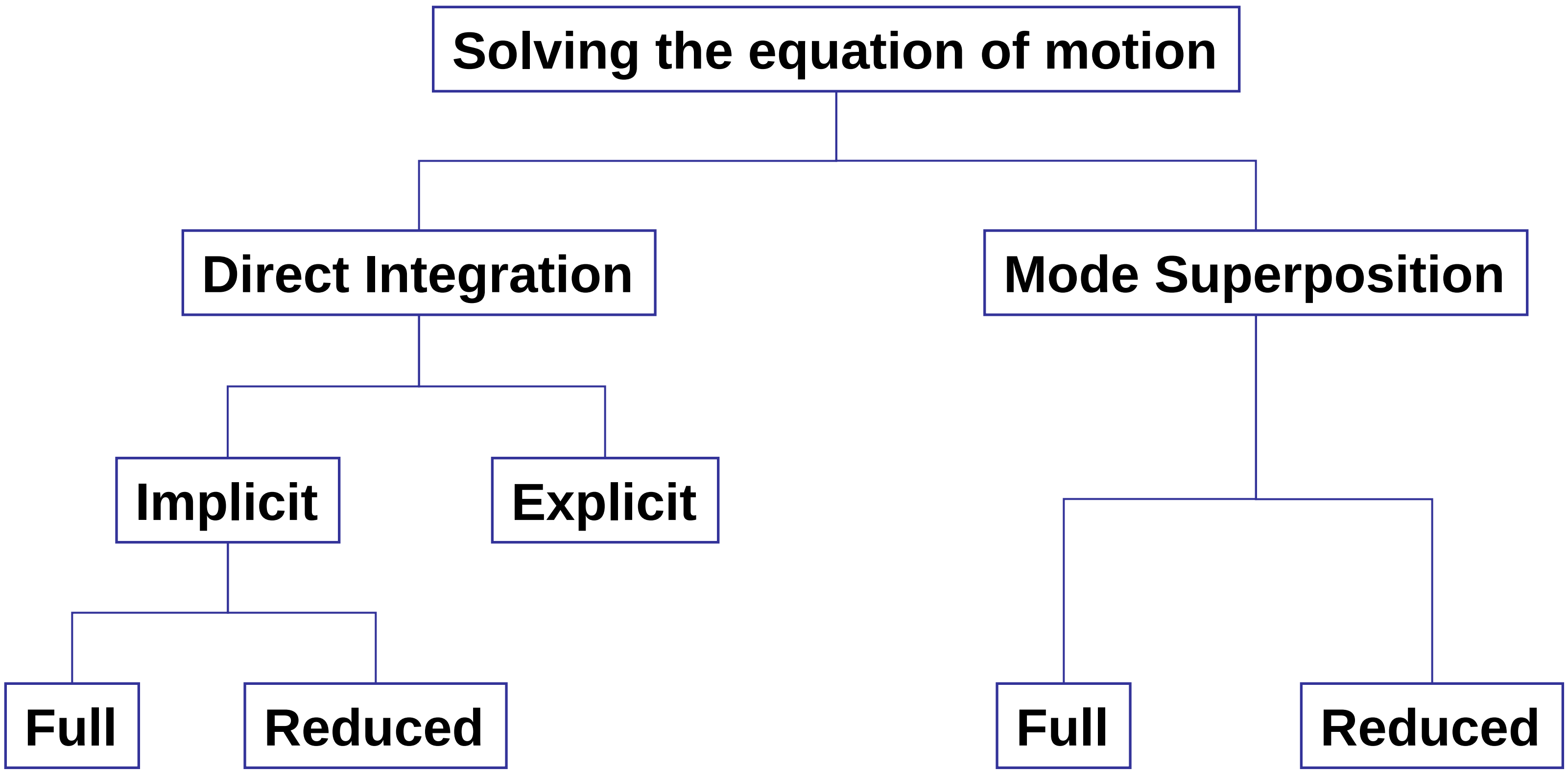
Equation of Motion

- **Equation of motion for a transient dynamic analysis is the same as the general equation of motion.**

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F(t)\}$$

- **This is the most general form of dynamic analysis. Loading may be any arbitrary function of time.**
- **Depending on the method of solution, ANSYS allows all types of nonlinearities to be included in a transient dynamic analysis - large deformation, contact, plasticity, etc.**

Solution Methods



discussion of implicit and explicit

Implicit Method

- Matrix inversion is required.
- Nonlinearities require equilibrium iterations (convergence problems).
- Integration time step Δt can be large but may be restricted by convergence issues.
- Efficient for most problems except where Δt needs to be very small.

Explicit Method

- No matrix inversion.
- Can handle nonlinearities easily (no convergence issues).
- Integration time step Δt must be small (1e-6 second is typical).
- Useful for short duration transients such as wave propagation, shock loading, and highly nonlinear problems such as metal forming.
- ANSYS-LS/DYNA uses this method.

Implicit method

- **ANSYS uses Newmark integration scheme.**

$$[M]\{\ddot{u}_{t+\Delta t}\}+[C]\{\dot{u}_{t+\Delta t}\}+[K]\{u_{t+\Delta t}\}=\{F(t)\}$$

$$u_{t+\Delta t}=u_t+\dot{u}_t\Delta t+[(1/2-\alpha)\ddot{u}_t+\alpha\ddot{u}_{t+\Delta t}]\Delta t^2$$

$$\dot{u}_{t+\Delta t}=\dot{u}_t+[(1-\beta)\ddot{u}_t+\beta\ddot{u}_{t+\Delta t}]\Delta t$$

- **Varying values of α and β causes integration scheme to change (implicit / explicit / average acceleration).**
- **Newmark is an implicit scheme.**
- **LS-DYNA uses explicit scheme.**

Integration Time Step

- **The integration time step (also ITS or Δt) size should be small enough to capture the following:**
  - **the response frequency**
  - **the contact frequency (if applicable)**
  - **wave propagation effects (if applicable)**
  - **Nonlinear response (plasticity, creep, contact status)**

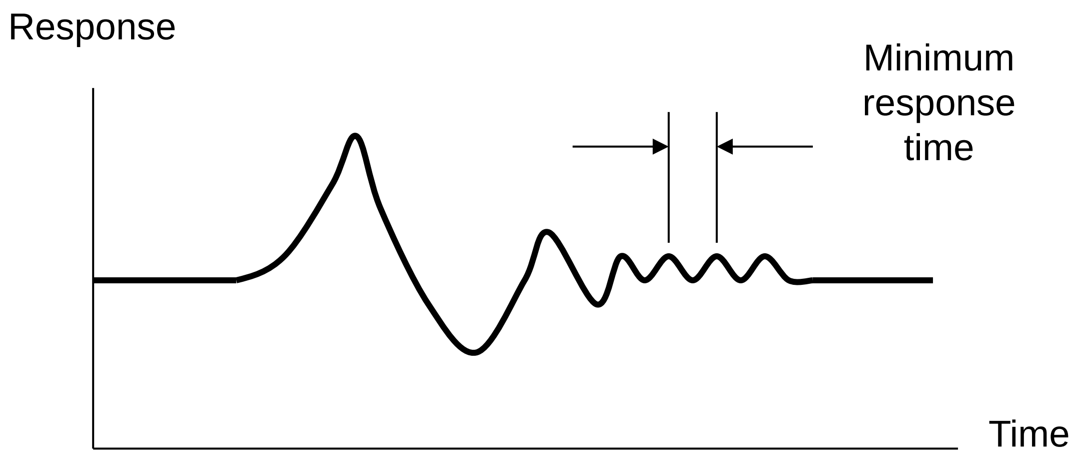
反應頻率(response frequency)

- 針對一般的暫態動力學問題，**Δt**概略的選取判斷如下式：

$$\Delta t = \frac{1}{20f}$$

  其中**Δt**單位為秒，**f**為結構的反應頻率**(response frequency)**，單位為**Hz**，這是概指分析案例的最高頻率。

- 至於反應頻率**f**值，則必須先經由初步的**ANSYS**暫態動力學分析後才能得到，**ANSYS**在求解時會於**Output Window**列印出**f**值，以供參考。接著利用**ANSYS**提供的**f**值，再以上式求出適當的**Δt**，以此**Δt**再重做一次分析。



Δt要足夠小到能捕捉結構反應的細節，上圖表示結構的反應**(structural response)**，假設最小的反應周期是**T**，則**Δt**大致上不要超過**T/20**。

$$\Delta t \le \frac{T}{20}$$

接觸頻率(contact frequency)

- 若暫態動力學分析包含了接觸問題，則需要考慮**Δt**是否能精確模擬出兩物體接觸時的動量轉移，若**Δt**不夠小則會造成誤差。對於這類問題的**Δt**概略取法如下式：

$$\Delta t = \frac{1}{30 f_c}$$

$$f_c = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

其中**f_c**稱為接觸頻率**(contact frequency)**，**k**為間隙剛度**(gap stiffness)**，**m**為接觸面的等效質量**(effective mass)**。

- 不過，上式的**k**和**m**較不易明確定義，而**ANSYS**在求解時會於**Output Window**列印出反應頻率**(response frequency)**，即**f_c**值，以供使用者參考。

應力波傳遞(wave propagation)

- 若要求解應力波傳遞問題，其**Δt**概略取法如下式：

$$\Delta t \leq \frac{\Delta x}{3c}$$

$$c = \sqrt{E/\rho}$$

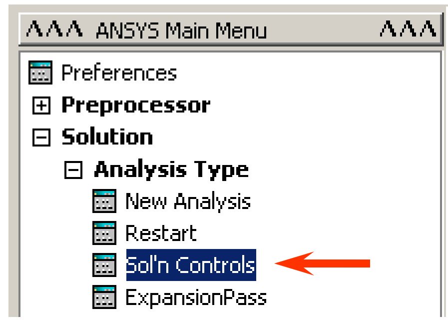
其中**Δx**為元素尺寸，若波傳遞方向的結構長度為**L**，則**Δx**須小於**L/20**，**c**則為彈性波速**(elastic wave speed)**，而**E**和**ρ**分別為楊氏模數和密度。以上計算須注意單位的相容性。

- **Δt**的設定需由**ANSYS**使用者給定，而**ANSYS**提供了指令**AUTOTS**和**DELTIM**，可自動調整時間增量。

Choose Analysis Type & Options

Solution options

- **Choose large displacement transient or small displacement transient .**
  - **When in doubt, choose large displacement transient**



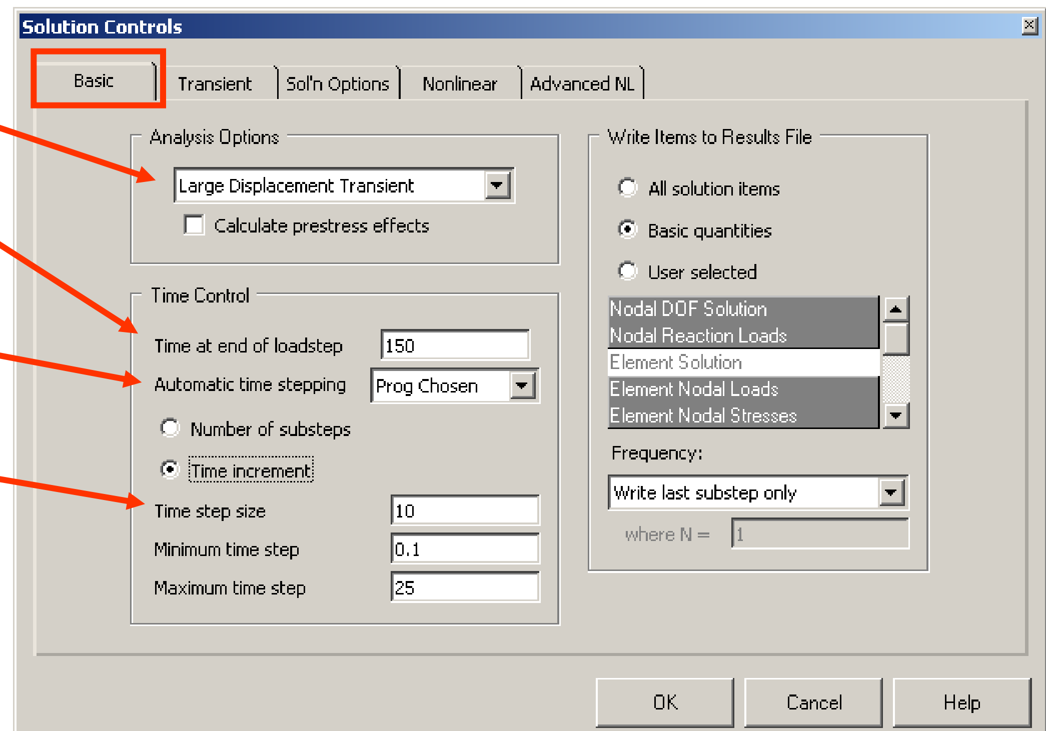
Specify time at end of load step.

Automatic time stepping

— **If nonlinearities are present, use the "Program Chosen" option.**

Specify initial, min and max values of Δt for this load step.

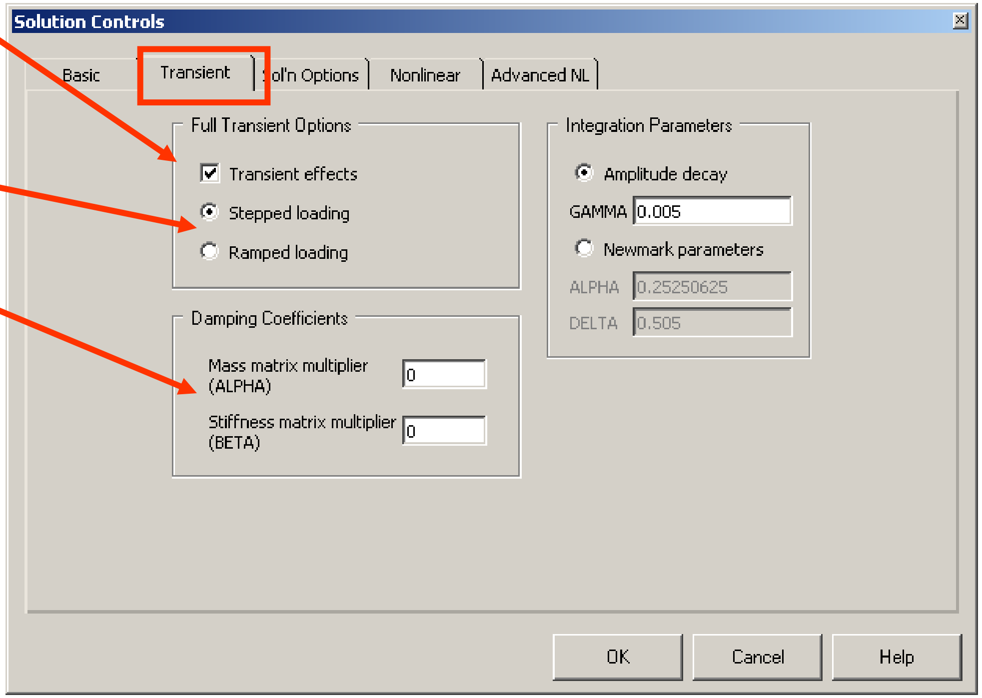
Specify output controls



Turn transient effects on/off

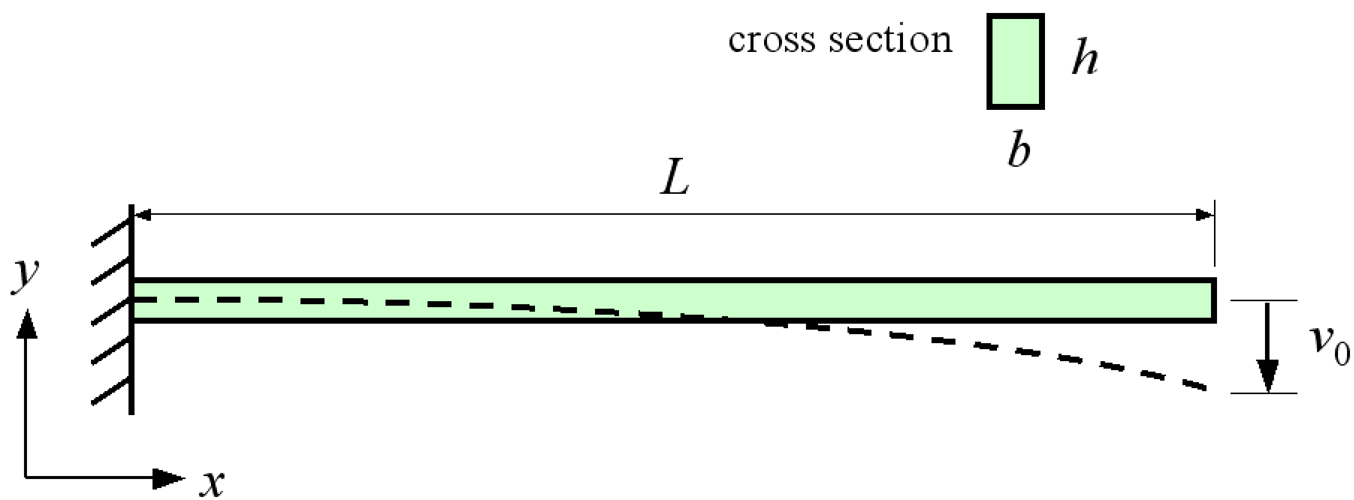
- useful for setting up initial conditions (discussed later)

- Ramp or Step apply load
- Specify damping (discussed later)
- Use default values for time integration parameters



懸臂樑暫態反應分析

如 下圖 的二維懸臂樑問題，樑之長度 $L = 6$ m，矩形截面 $b = 200$ mm 和 $h = 400$ mm，楊氏模數 $E = 12$ GPa，密度 $\rho = 1500$ kg/m^3，假設其 Rayleigh 阻尼係數 $\alpha = 0$ 和 $\beta = 0.006$。懸臂樑初始條件爲施加一負 y 方向位移 $v_0 = 40$ mm 於右端，且其初始速度 $\dot{v}_0 = 0$。試求右端的初始拘束釋放後，右端在 2 秒內的暫態位移響應，分析單位系統採用：m、N、Pa、kg。



Rayleigh damping

- 又稱為比例阻尼**(proportional damping)**，它是假設整個結構之阻尼模式均相同，如下為結構之運動方程式：

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F\}$$

其中阻尼矩陣**[C]**可以下列方式表示：

$$[C]=\alpha\,[M]+\beta[K]$$

α[M]和**β[K]**的意義是分別用來表現**viscous damping**(結構和周遭流體的摩擦)和**hysteresis damping**(材料本身內部分子間的摩擦)。

Specify BC's & Initial Conditions

Example - "Plucking" the free end of a cantilever beam

- **In this case $u_0 \neq 0$ at one end of the beam, and $v_0=0$.**
- **Use the static load step method.**
- **Load step 1:**
  - **Transient effects OFF. Use TIMINT,OFF command or**

    Solution > Sol'n Control

    Select the "Transient" Tab and unselect Transient effects
  - **Small time interval, e.g, 0.001.**
  - **2 substeps, stepped loads. (If ramped or with one substep, v_0 will be non-zero.)**
  - **Apply the desired non-zero displacement at the free end of the beam.**
  - **SOLVE.**
- **Load step 2:**
  - **Transient effects ON.**
  - **Delete the imposed displacement.**
  - **Specify ending time and continue with the transient.**

Example: Plucking a Cantilever Beam

```
/SOLU
  ANTYPE, TRANS
  ...
  TIMINT, OFF   ! Transient effects off
  TIME, 0.001   ! Small time interval
  D, ...        ! Apply displacement at desired nodes
  KBC, 1        ! Stepped loads
  NSUBST, 2     ! To avoid non-zero velocity
  SOLVE
  如果跳出/SOLU，再進入/SOLU，靜力分析的結果會因重新進入/SOLU後被清除掉
  TIMINT, ON    ! Transient effects on
  TIME, ...     ! Actual time at end of load
  DDELE, ...    ! Delete the applied displacement
  SOLVE
```

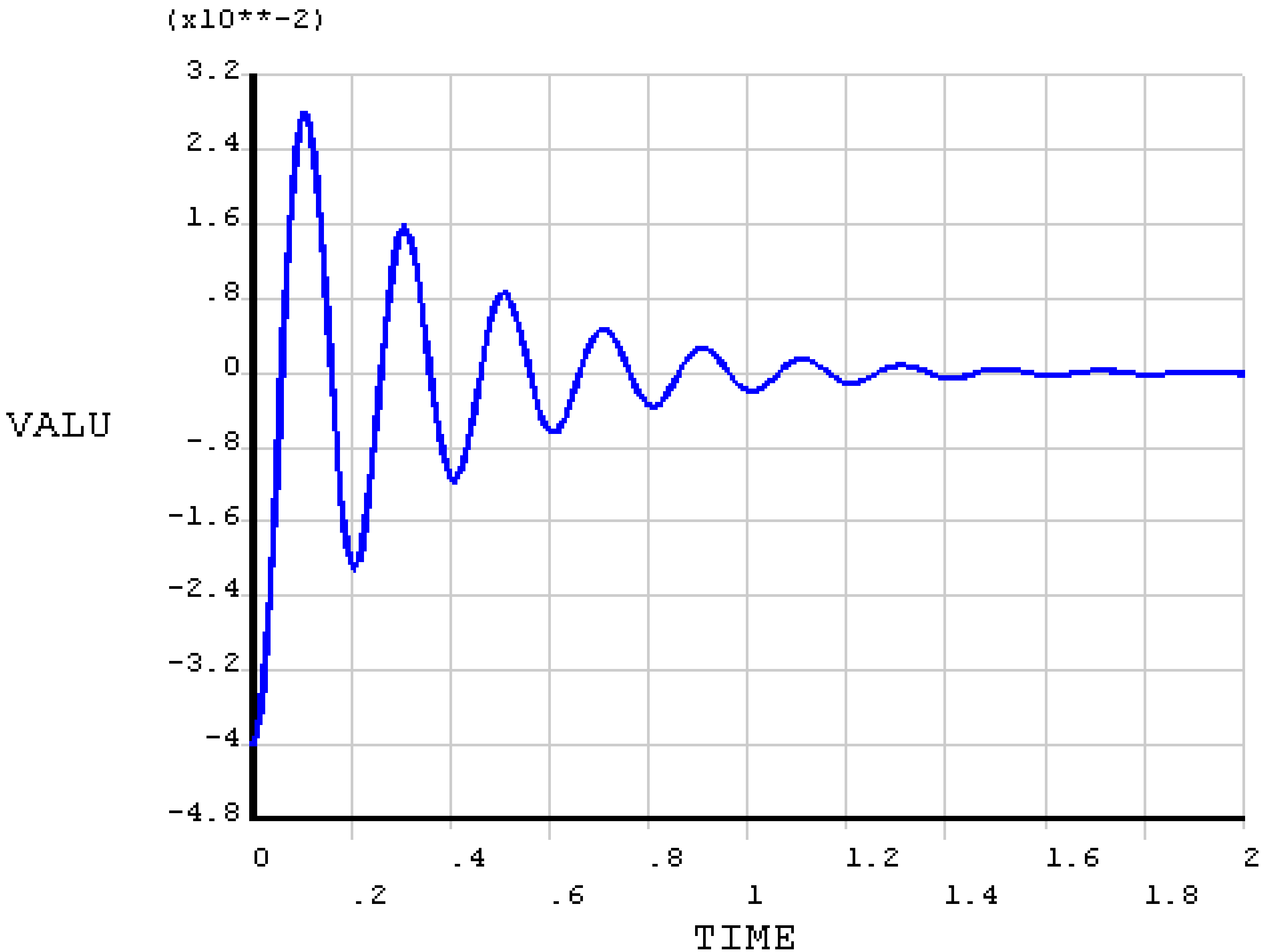
Output Window 印出的反應頻率

ANSYS 7.0 Output Window

```
*** LOAD STEP     2   SUBSTEP   131  COMPLETED.    CUM ITER =    133
*** TIME =   1.67169          TIME INC =  0.128846E-01  OLD TRIANG MATRIX
*** RESPONSE FREQ =  5.072         PERIOD=  0.1971       PTS/CYC =  15.
*** AUTO STEP TIME:  NEXT TIME INC = 0.12885E-01  UNCHANGED

*** LOAD STEP     2   SUBSTEP   132  COMPLETED.    CUM ITER =    134
*** TIME =   1.68457          TIME INC =  0.128846E-01  OLD TRIANG MATRIX
*** RESPONSE FREQ =  5.072         PERIOD=  0.1971       PTS/CYC =  15.
*** AUTO STEP TIME:  NEXT TIME INC = 0.12885E-01  UNCHANGED

*** LOAD STEP     2   SUBSTEP   133  COMPLETED.    CUM ITER =    135
*** TIME =   1.69746          TIME INC =  0.128846E-01  OLD TRIANG MATRIX
*** RESPONSE FREQ =  5.072         PERIOD=  0.1971       PTS/CYC =  15.
*** AUTO STEP TIME:  NEXT TIME INC = 0.12885E-01  UNCHANGED

*** LOAD STEP     2   SUBSTEP   134  COMPLETED.    CUM ITER =    136
*** TIME =   1.71034          TIME INC =  0.128846E-01  OLD TRIANG MATRIX
*** RESPONSE FREQ =  5.072         PERIOD=  0.1971       PTS/CYC =  15.
*** AUTO STEP TIME:  NEXT TIME INC = 0.12885E-01  UNCHANGED

*** LOAD STEP     2   SUBSTEP   135  COMPLETED.    CUM ITER =    137
*** TIME =   1.72323          TIME INC =  0.128846E-01  OLD TRIANG MATRIX
*** RESPONSE FREQ =  5.072         PERIOD=  0.1971       PTS/CYC =  15.
*** AUTO STEP TIME:  NEXT TIME INC = 0.12885E-01  UNCHANGED

*** LOAD STEP     2   SUBSTEP   136  COMPLETED.    CUM ITER =    138
*** TIME =   1.73611          TIME INC =  0.128846E-01  OLD TRIANG MATRIX
```

反應頻率為5.072Hz，由此算出Δt為1/(20*5.072)≈0.0099秒，約為0.01秒，因此DELTIM指令中的起始副增量可設定為0.01秒。

樑右端在2秒內的位移變化



位移單位：m
時間單位：sec