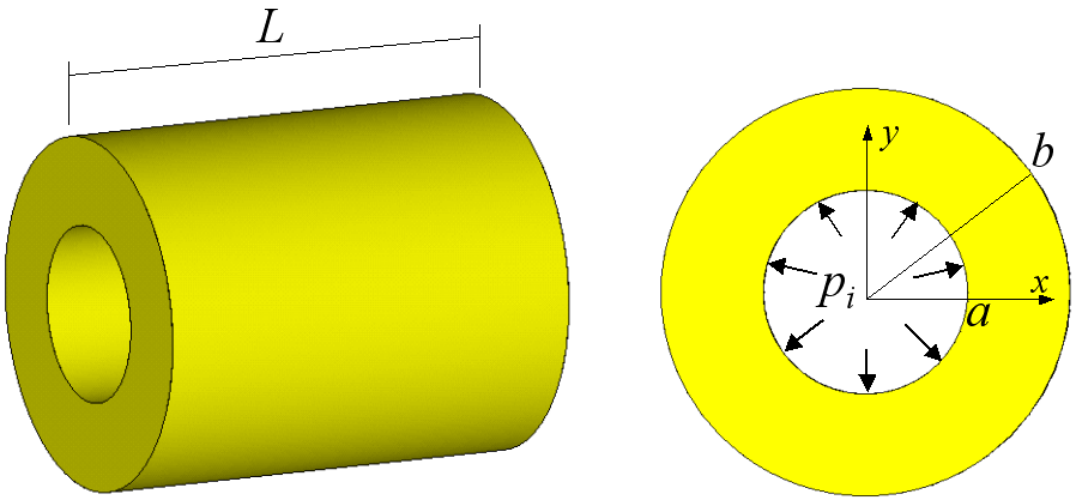
軸對稱問題

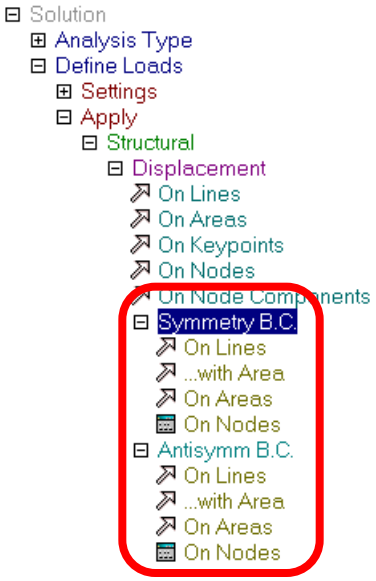
- 如下圖為一厚壁圓管，內外徑分別為**a=100mm**和**b=200mm**，其圓管長度**L**為**500mm**，內徑面受壓力p_i**=3MPa**。圓管材料之楊氏模數**E=70GPa**，普松比**v=0.33**，試求圓管應力分布。分析單位系統採用：**mm**、**N**、**MPa**。
- 本例使用**PLANE42**元素來模擬

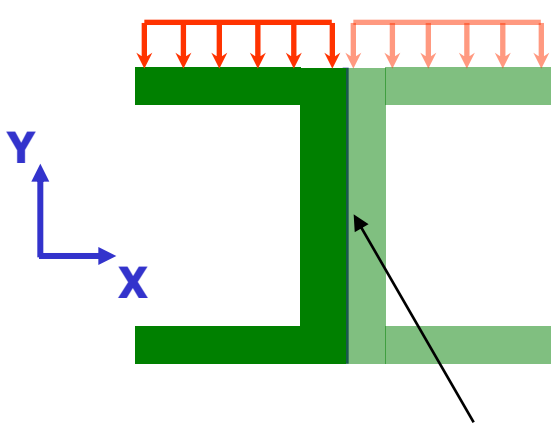


(a) (b)

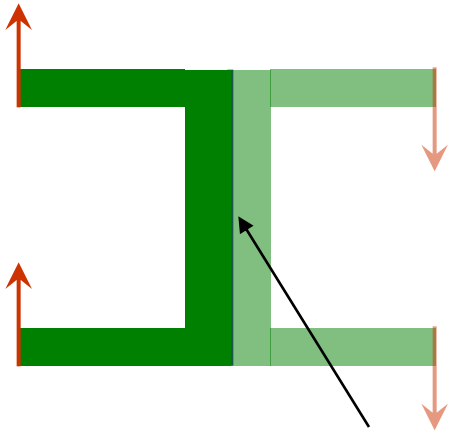
Displacement Constraints

- **Displacement constraints are also used to enforce *symmetry* or *antisymmetric* boundary conditions.**
  - **Symmetry BC: Out-of-plane displacements and in-plane rotations are fixed.**
  - **Antisymmetry BC: In-plane displacements and out-of-plane rotations are fixed.**

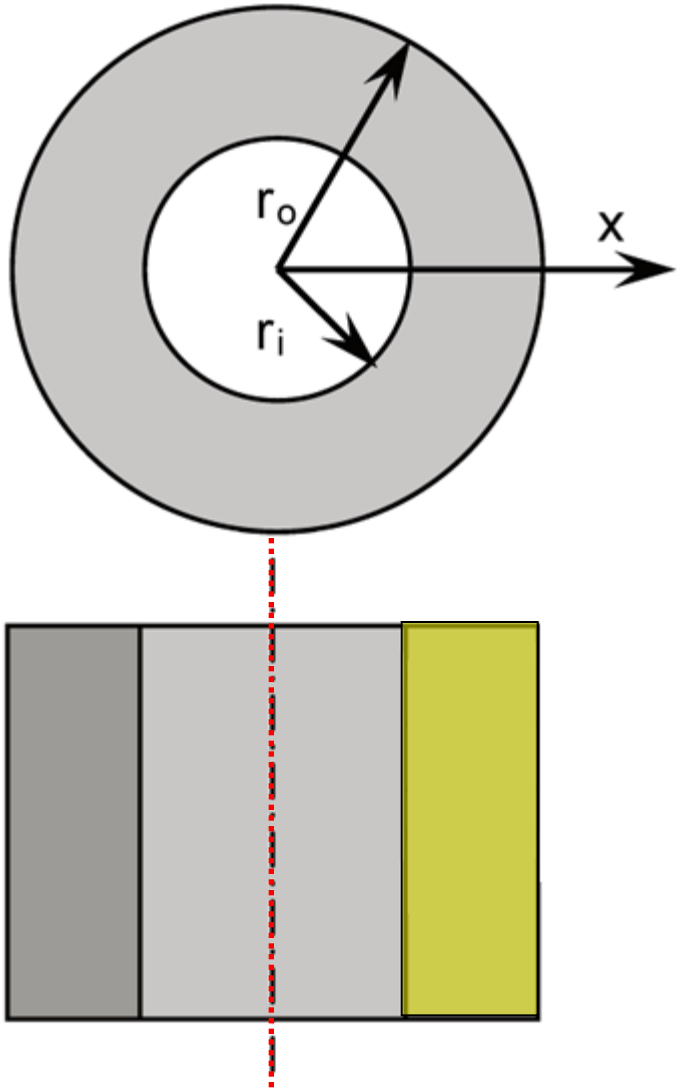




Symmetry Boundary
UX=0
ROTY=ROTZ=0



Antisymmetry Boundary
UY=UZ=0
ROTX=0



Axisymmetric axis

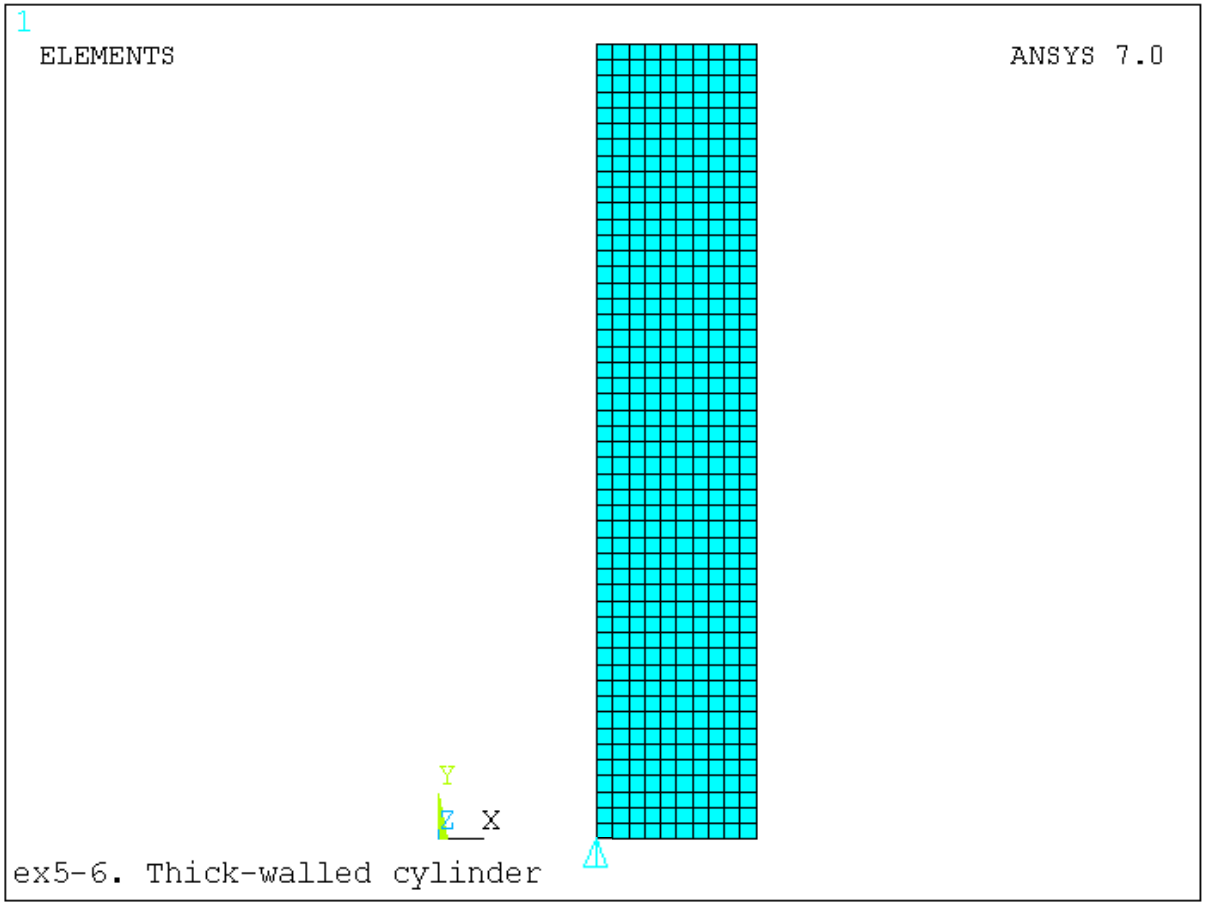
PLANE42 輸入資料

Element Name	PLANE42
Nodes	I, J, K, L
Degrees of Freedom	UX, UY
Real Constant	None, if KEYOPT (3) = 0, 1, 2 Thickness, if KEYOPT (3) = 3
Material Properties	EX, NUXY, GXY, ALPX, DENS, DAMP, etc.
Surface Loads	Pressure: face 1 (J-I), face 2 (K-J), face 3 (L-K), face 4 (I-L)
Body Loads	Temperature -- T(I), T(J), T(K), T(L)
Special Features	Plasticity, Creep, Stress stiffening, Large deflection, Large strain, etc.
KEYOPT(1)	Key for element coordinate system: 0 -- Element C.S. is parallel to the global C.S. 1 -- Element C.S. is based on the element I-J side
KEYOPT(2)	Key to include extra shapes: 0 -- Include extra displacement shapes 1 -- Suppress extra displacement shapes
KEYOPT(3)	0 -- Plane stress 1 -- Axisymmetric 2 -- Plane strain 3 -- Plane stress with thickness input

軸對稱有限元素模型

The axis of symmetry must coincide with the global Y-axis. The geometry has to lie on the positive X-axis of the X-Y plane.

The directions of X, Y, and Z are radial, axial, and circumferential, respectively.



指令說明

SFL, *LINE*, *lab*, *VALI*, *VALJ*

施加表面的負載於線段上。

LINE：表示線段的編號。可爲 ALL、P 或 Component 名稱。

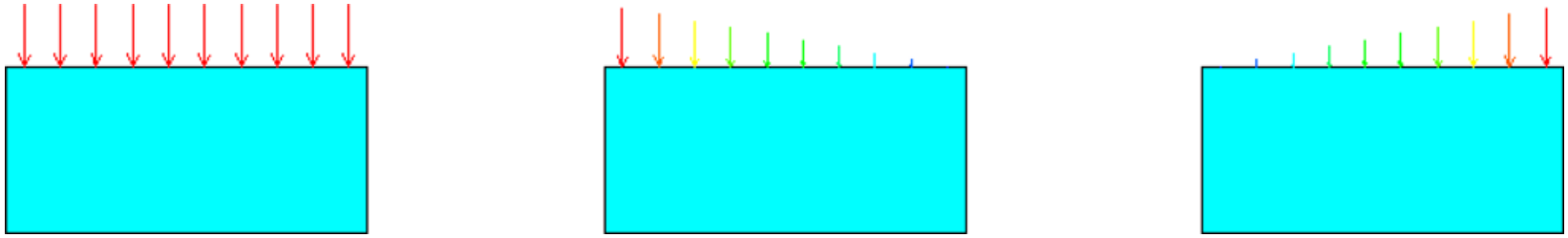
Lab：邊界條件的代號，於結構分析中是壓力，所以代號爲 PRES。

VALI, *VALJ*：分別指起始跟終止的壓力，只給 *VALI*，表示均布壓力。

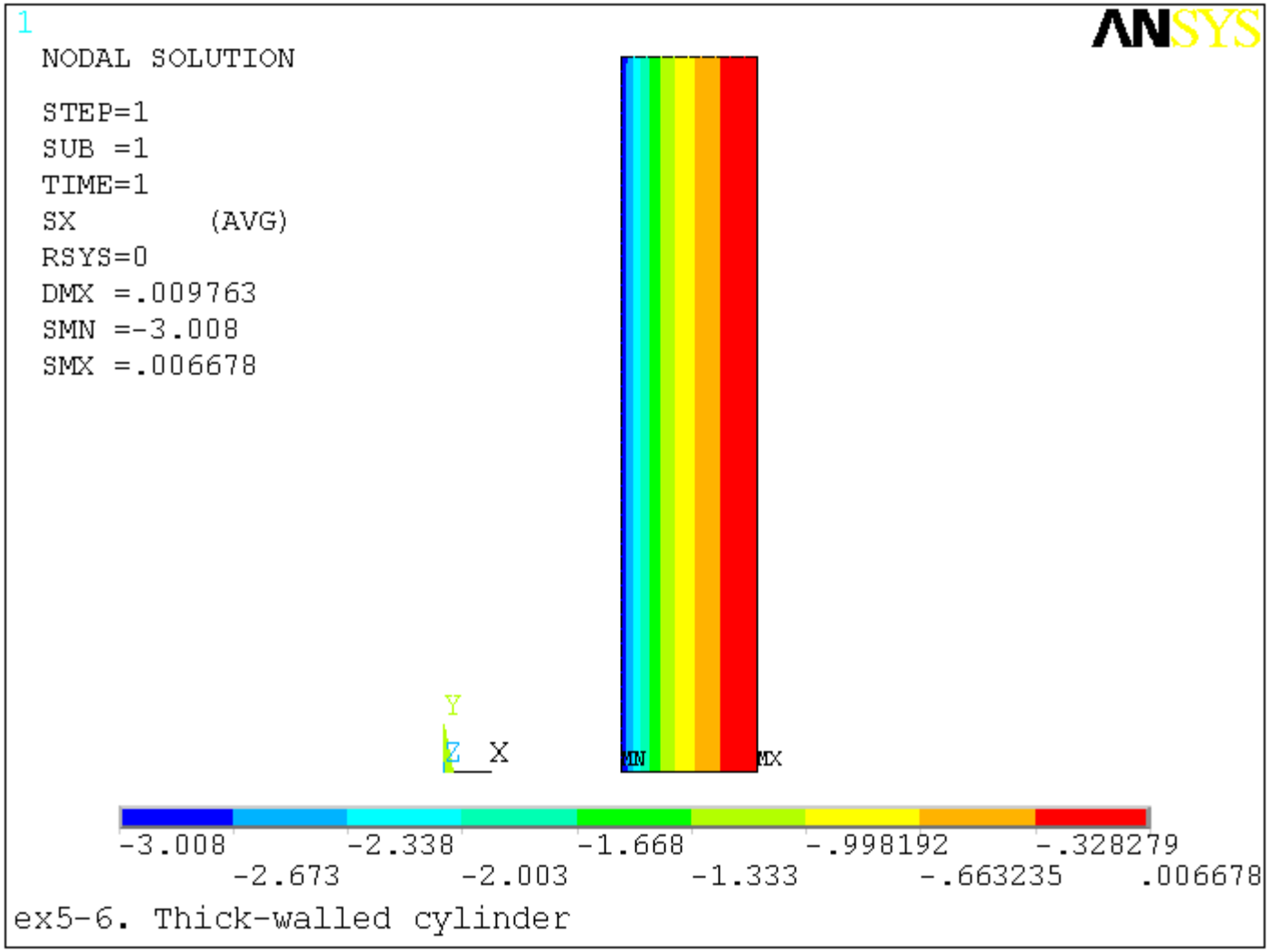
其他相關指令有 SFA 施加表面的負載於面積上，SF 施加表面的負載於節點上，SFE 施加表面的負載於元素上。

使用的指令設定壓力

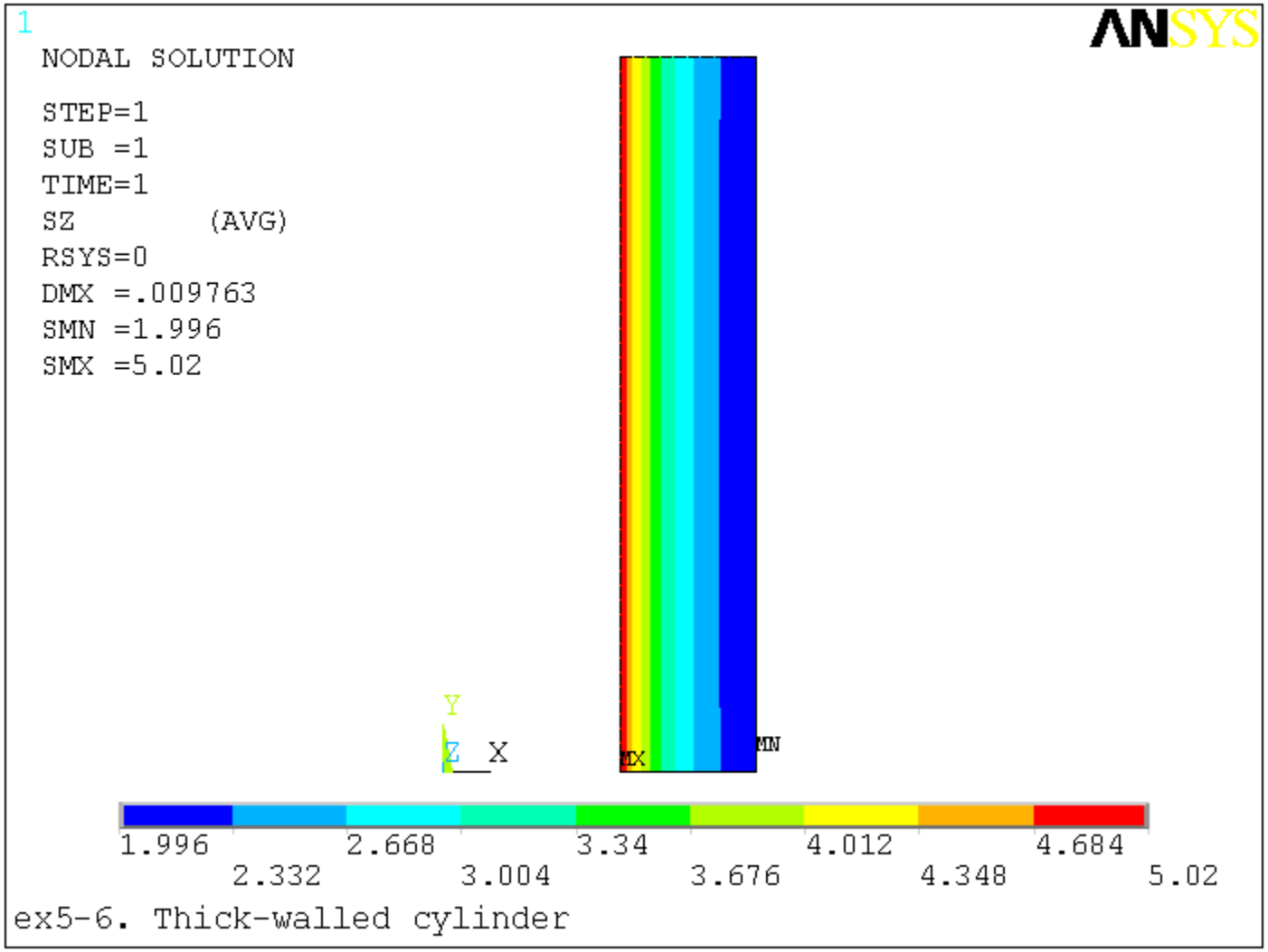
```
SFL, 3, PRES, 0.0533, 0
```



圓柱座標應力σ_r(SX) (MPa)



圓柱座標應力σ_θ(SZ) (MPa)



討論

- 本例於文獻中有解析解如下：

$$\sigma_r = -\frac{a^2 b^2 p_i}{(b^2 - a^2) r^2} + \frac{a^2 p_i}{b^2 - a^2}$$

$$\sigma_\theta = \frac{a^2 b^2 p_i}{(b^2 - a^2) r^2} + \frac{a^2 p_i}{b^2 - a^2}$$

- 將本例題目的數值代入上式可得

$$\sigma_r = \frac{-40000}{r^2} + 1 \quad \text{MPa}$$

$$\sigma_\theta = \frac{40000}{r^2} + 1 \quad \text{MPa}$$

A.P. Boresi and K.P. Chong, Elasticity in Engineering Mechanics. Second edition, New York: John Wiley & Sons, 2000

應力σ_r和σ_θ解析解與 ANSYS解的比較