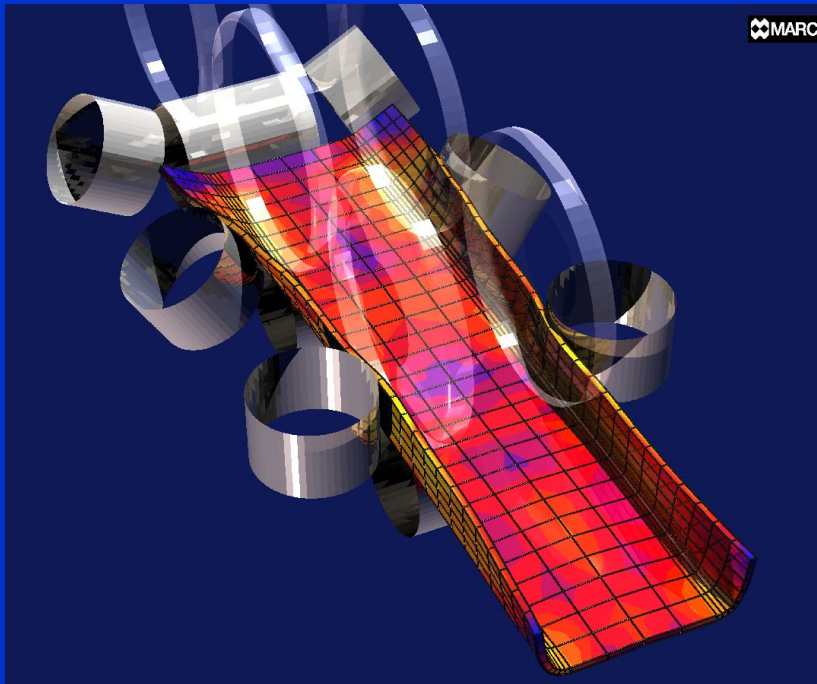




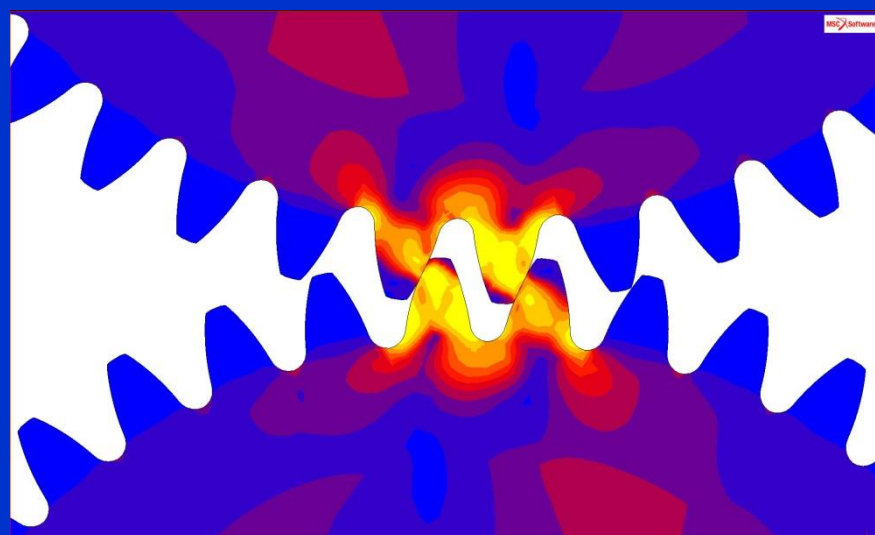
Module 9

Introduction to Nonlinear Analysis

Metal Forming



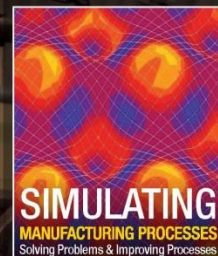
Gear Contact



simulating REALITY

MSC Software Magazine

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THE HADLEY GROUP

Simulation Helps Increase Sales by \$4M

Nonlinear FEA Validates New Cold Roll Forming Process

COMTES/PILSEN STEEL

Getting to the Root Cause of Cracks

Heat Transfer Analysis Helps Solve Tough Forging Problem

INNOVATING

MOMENTUM IN GROUND VEHICLES
New Tire Development & Optimized Engines

BIG TYRE

Designing the Right Tire

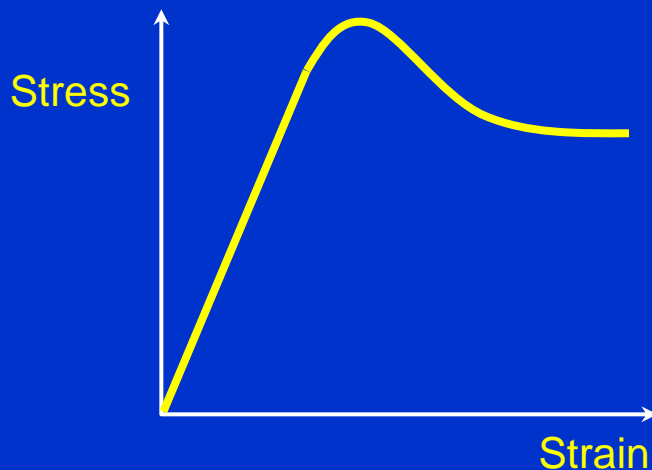
Nonlinear FEA Accelerates Development of Non-Pneumatic, Non-Solid Tire for Mining Industry

MANUFACTURING COMPANIES RELY ON MARC

The World is Nonlinear

9. Introduction to Nonlinear Analysis

- A **nonlinear** analysis is needed if the loading on a structure causes significant changes in stiffness. Typical reasons for stiffness to change significantly are:
 - Strains beyond the elastic limit (**plasticity**)
 - **Large deflections**, such as with a loaded fishing rod
 - **Contact** between two bodies

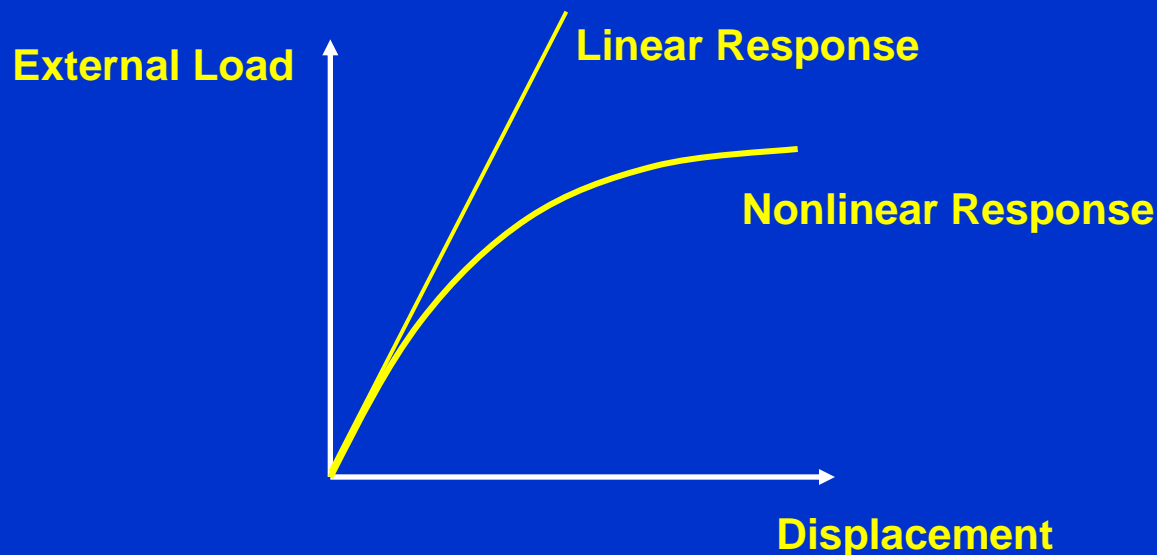


...Introduction to Nonlinear Analysis

- In this chapter, we will present a brief introduction to the basics of nonlinear solution via the following topics:
 - A. Basic Concepts
 - B. Typical Procedure
 - C. Workshop
- The purpose is to give you a "taste" of nonlinear analysis. There are many, many aspects of nonlinear analysis that are well beyond the scope of this training course and are covered in:
 - *ANSYS Structural Analysis Guide*
 - *Basic Structural Nonlinearities Training Manual*
 - *Advanced Contact and Bolt Pretension Training Manual*
 - *Advanced Structural Nonlinearities Training Manual*

A. Basic Concepts

- When a load causes significant changes in stiffness, the load-deflection curve becomes nonlinear.
- The challenge is to calculate the nonlinear displacement response using a linear set of equations.



Equations for a Nonlinear Structure

元素的力平衡方程式

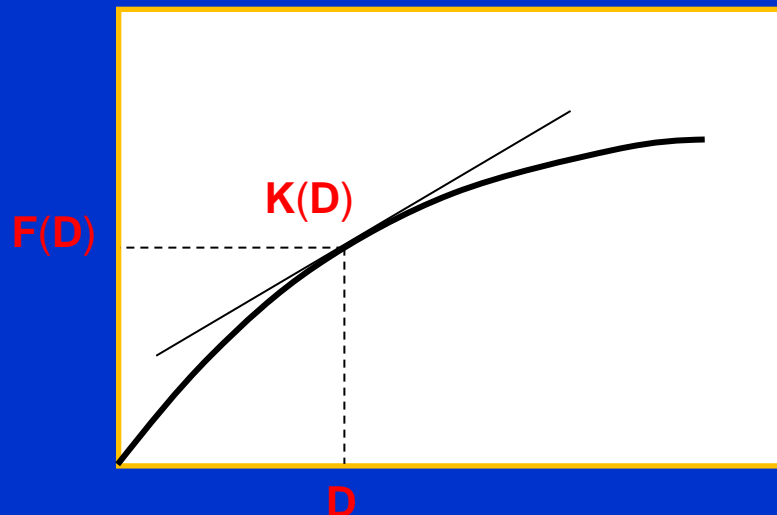
$$\mathbf{k} \times \mathbf{d} = \mathbf{f}$$

整體結構的力平衡方程式

$$\mathbf{K} \times \mathbf{D} = \mathbf{F}$$

非線性結構的力平衡方程式，其形式還是如上所描述，但是其中的 \mathbf{K} 及 \mathbf{F} 不再是常數，而是依 \mathbf{D} 的改變而改變：

$$\mathbf{K}(\mathbf{D}) \times \mathbf{D} = \mathbf{F}(\mathbf{D})$$

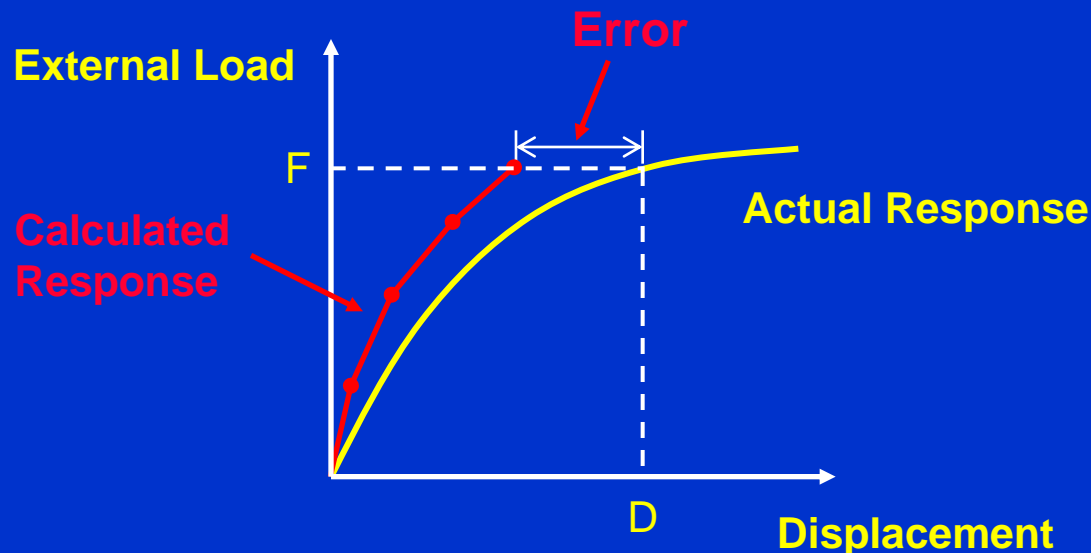


求解施力 \mathbf{F} 時之位移量 \mathbf{D}

Introduction to Nonlinear Analysis

...Basic Concepts

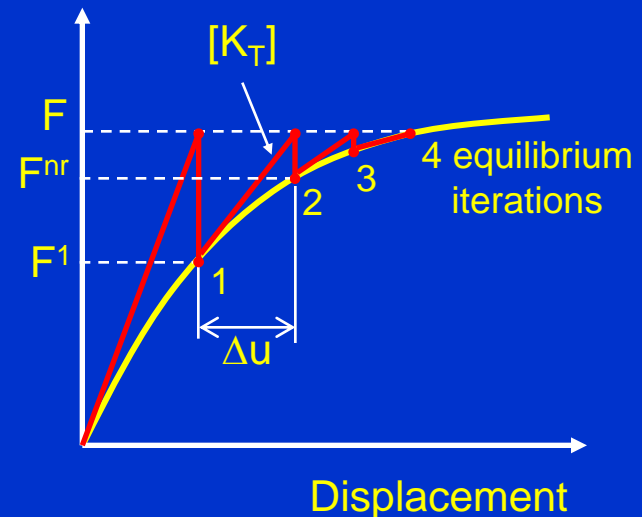
- Incremental method :
- This approach is to apply the load gradually by dividing it into a series of increments and **adjusting the stiffness matrix** at the end of each increment.
- The problem with this approach is that errors accumulate with each load increment, causing the final results to be out of equilibrium.



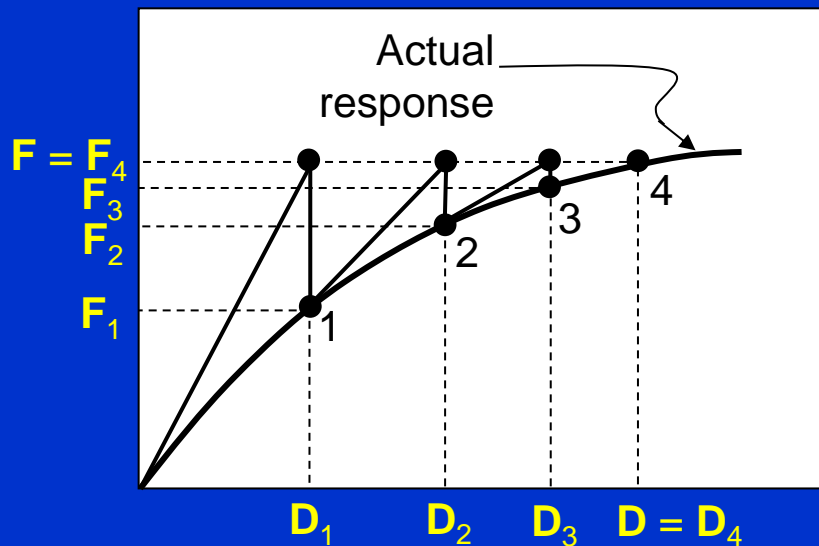
Introduction to Nonlinear Analysis

...Basic Concepts

- ANSYS uses the Newton-Raphson algorithm:
 - Applies the load gradually, in increments.
 - Also performs **equilibrium** iterations at each load increment to drive the incremental solution to equilibrium.
 - Solves the equation $[K_T]\{\Delta u\} = \{F\} - \{F^{nr}\}$
 - $[K_T]$ = tangent stiffness matrix
 - $\{\Delta u\}$ = displacement increment
 - $\{F\}$ = **external** load vector
 - $\{F^{nr}\}$ = **internal** force vector
 - Iterations continue until $\{F\} - \{F^{nr}\}$ (difference between external and internal loads) is within a tolerance.
 - Some nonlinear analyses have trouble **converging**. Advanced analysis techniques are available in such cases (covered in the *Advanced Structural Nonlinearities* training course).



Convergence Criteria in ANSYS



兩次疊代之間的變位差 ΔD

$$\|\Delta D\| \leq 0.05 \|\mathbf{D}_{\max}\|$$

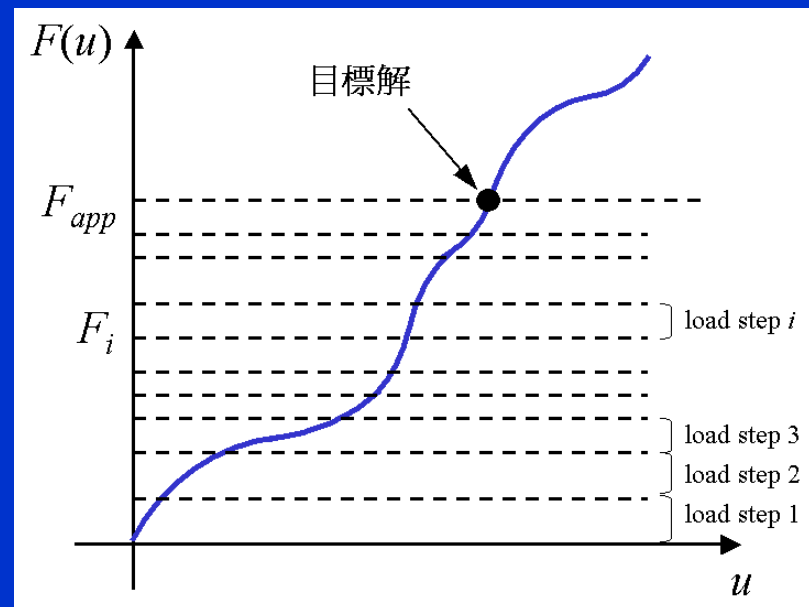
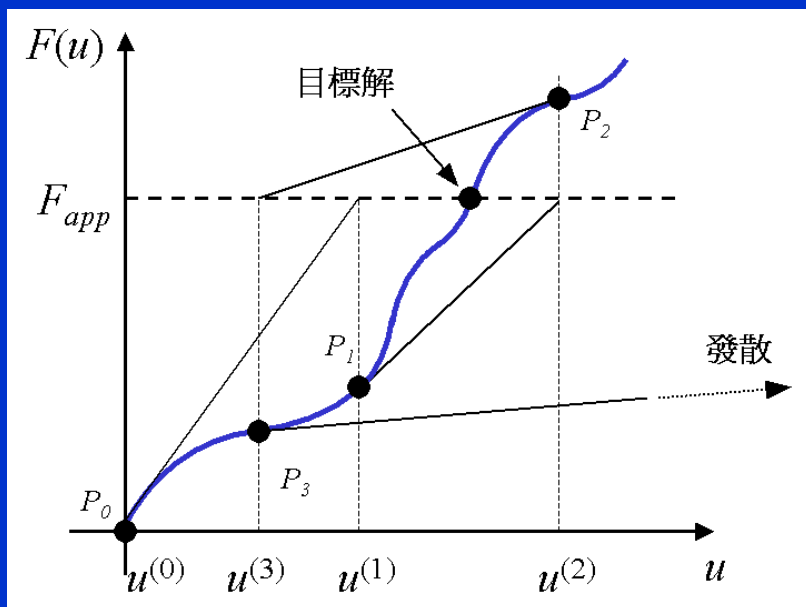
兩次疊代之間的外力差 ΔF

$$\|\Delta F\| \leq 0.005 \|\mathbf{F}_{\max}\|$$

Newton-Raphson method之疊代發散

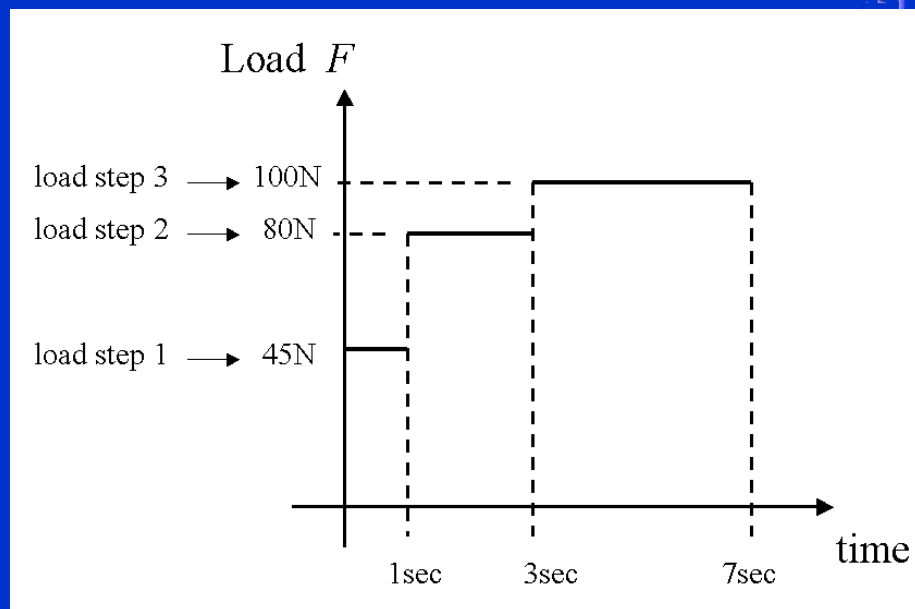
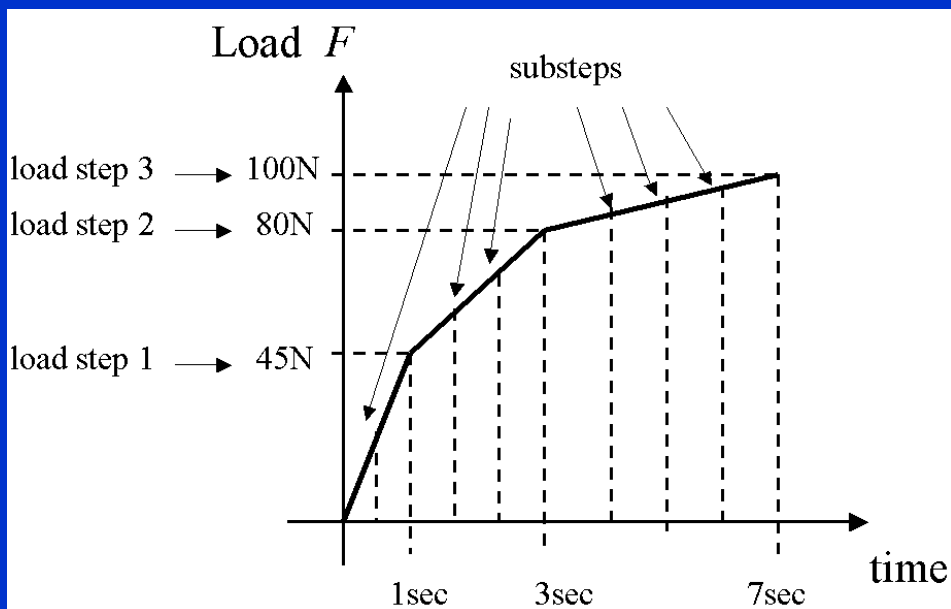
若 $F(u)$ 函數太複雜，或給定的起始點 P_0 之值不對，則疊代分析有可能會發散，即無法收斂至目標解。

將外力 F_{app} 分割成數個負荷增量 F_i ($i=1,2,3,\dots$)來接連計算， F_i 一步一步施加於結構，在每個增量範圍內均分別做牛頓-拉福森法疊代計算，由於每個增量區間之函數較單純，所以較容易收斂。



在靜態分析中無時間效應，使用者所定義之時間僅為負荷增量之參考點，並無物理意義，此「時間」並不參與計算。

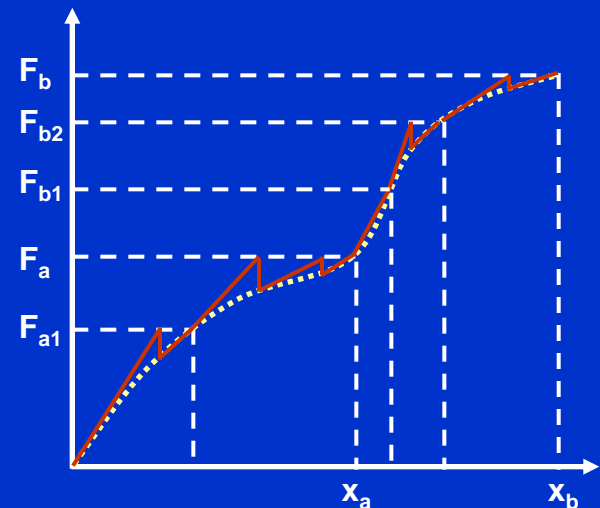
以**ANSYS**設定受力由**0N**增至**100N**為例，若將**100N**之力分為三個負荷增量(**load steps**)，再個別細分為**2**個、**3**個和**4**個副增量(**substeps**)，且在**ANSYS**設定中使用傾斜線(**ramped**)之加力方式，施力過程便如圖**8.12**所示，施力分別為「**0N→45N(0~1秒)**」，「**45N→80N(1~3秒)**」，「**80N→100N(3~7秒)**」。



Nonlinear Structural Analysis

... Nonlinear Solution

- It is useful to understand how loads are managed
 - **Load steps** are changes in general loading.
 - Simulation usually solves all nonlinear models with one load step, but, in the case of Pretension Bolt Loads, this is done in **two load steps**. The bolt preload is applied first, then all other loads are applied next. These *load steps* can be thought of as F_a and F_b .
 - **Substeps** apply the loads in an incremental fashion
 - Because of the complex response, it may be necessary to apply the load **incrementally**. For example, F_{a1} may be near 50% of the F_a load. After the load for F_{a1} is converged, then the full F_a load is applied. F_a has **2 substeps** while F_b has **3 substeps** in this example
 - **Equilibrium iterations** are the corrective solutions to obtain a converged substep
 - In the example on right, the iterations between the dotted white lines indicate equilibrium iterations.

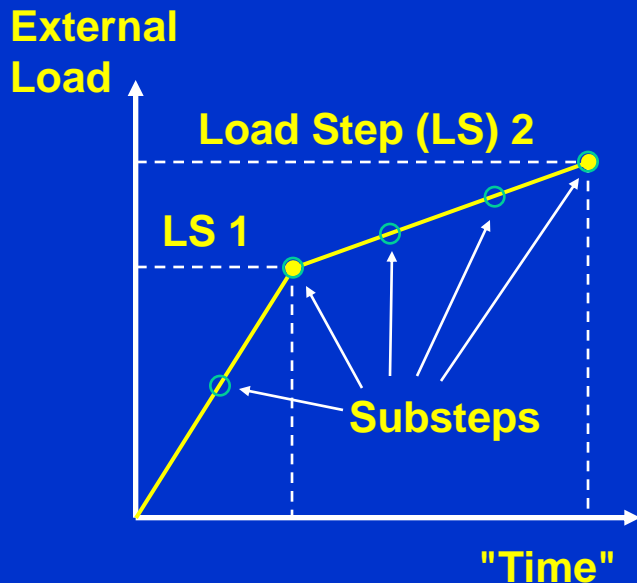


Introduction to Nonlinear Analysis

...Basic Concepts

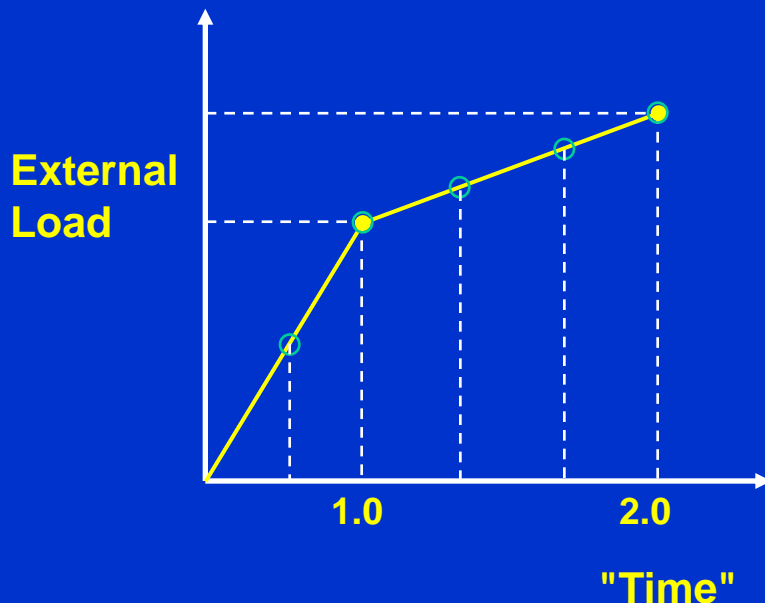
- This process is repeated for each load increment until the full external load has been applied.
- Thus a nonlinear solution typically involves the following:

- 一個 SOLVE 命令代表一個 load step
- **One** or **more load steps** to apply the external loads and boundary conditions. (This is true of linear analyses too.)
- Multiple **substeps** to apply the load gradually. Each substep represents one load increment. (A linear analysis needs just one substep per load step.)
- **Equilibrium iterations** to obtain equilibrium (or *convergence*) at each substep. (Does not apply to linear analyses.)



Time and Time Step

- Each load step and substep is associated with a value of **time**.
- **Time** in most nonlinear static analyses is simply used as a counter and does not mean actual, chronological time.



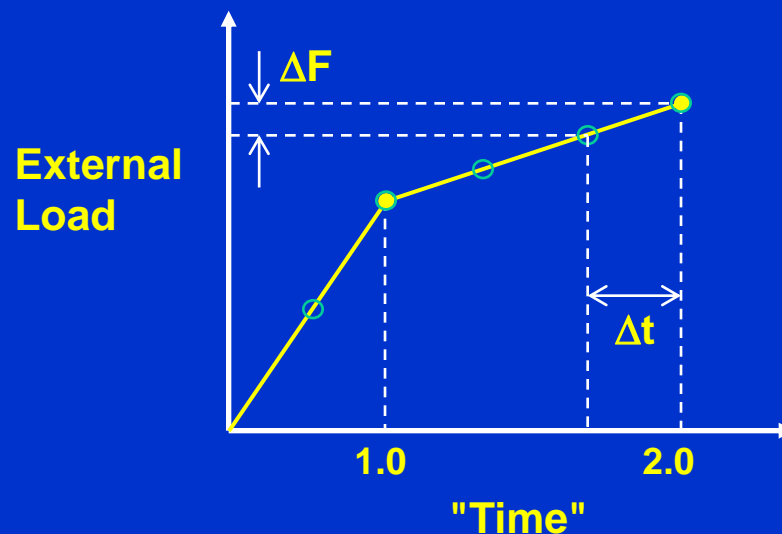
- By default, time = 1.0 at the end of **load step 1**, 2.0 at the end of **load step 2**, and so on.
- For rate-independent analyses, you can set it to any desired value for convenience. For example, by setting time equal to the load magnitude, you can easily plot the load-deflection curve.

- The **ends** of load steps or substeps can be identified by time.
- For dynamic problem, time is used as a real-world clock.
- For **static** problem, time is used as a **counter**.

Introduction to Nonlinear Analysis

...Basic Concepts

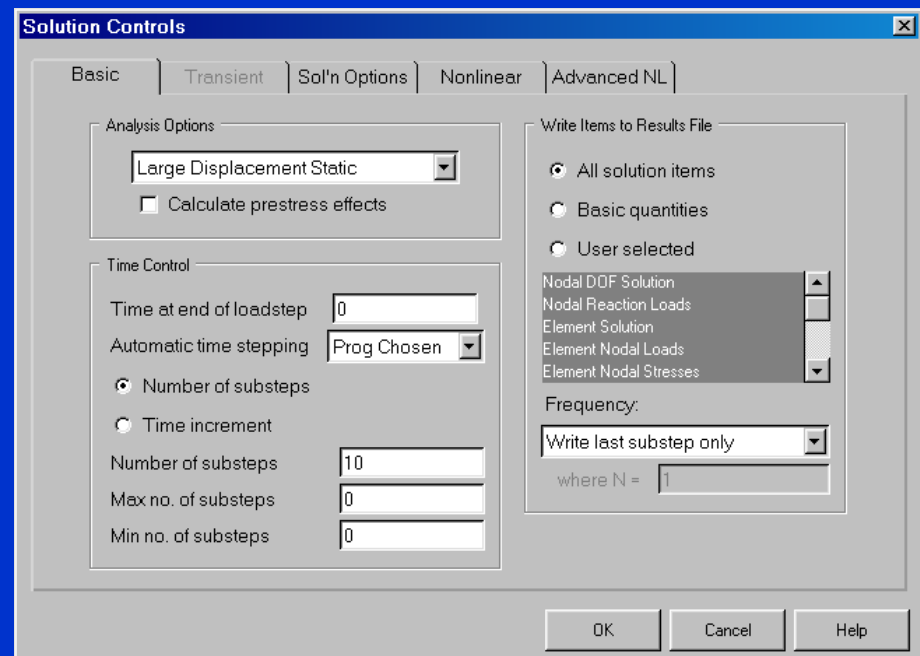
- The "time increment" between each substep is the **time step Δt** .
- Time step Δt determines the load increment ΔF over a substep. The higher the value of Δt , the larger the ΔF , so Δt has a direct effect on the accuracy of the solution.
- ANSYS has an **automatic time stepping** algorithm that predicts and controls the time step size for all substeps in a load step.



Introduction to Nonlinear Analysis

B. Typical Procedure

- Assuming that the geometry and meshing step have been completed, typical steps for a nonlinear solution are:
 1. Specify analysis type (usually static).
 2. Specify solution controls using **Solution > Analysis Type > Sol'n Controls**. Many controls are available, but the common ones are:
 - small or **large deflection**
 - time and ΔT or number of substeps
 - output controls
 3. Apply the loading.
 4. Save the database.
 5. Solve.



Nonlinear Geometry (NLGEOM)

- **NLGEOM turns ON/OFF the effects of geometric nonlinearity.**

NSUBST, NSBSTP, NSBMX, NSBMN, Carry

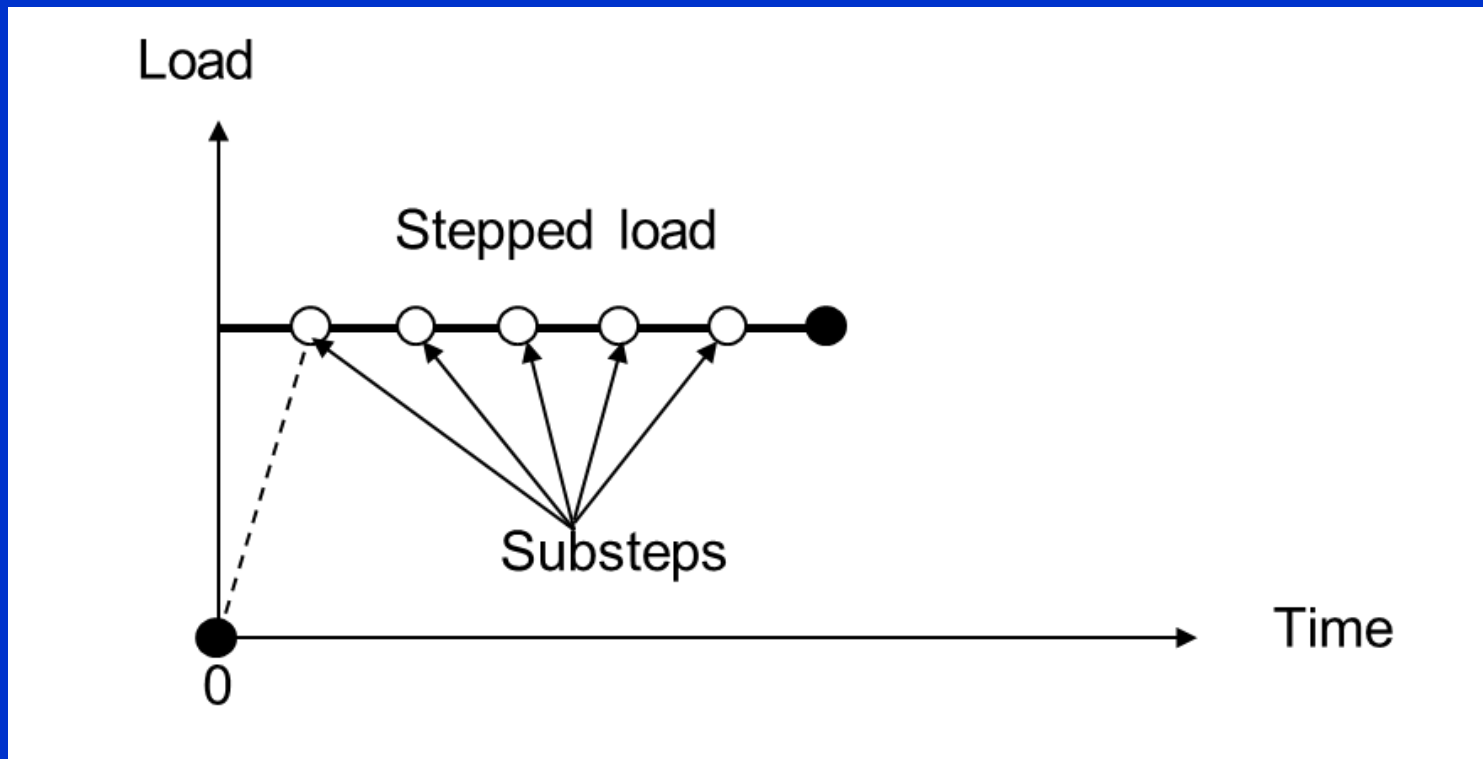
DELTIM, DTIME, DTMIN, DTMAX, Carry

- **NSUBST** specifies number of substeps for the following load step.
- **DELTIM** specifies time for each substep for the following load step.
- The two commands are equivalent, i.e., **$\text{TIME} = \text{DELTIM} \times \text{NSUBST}$**

Automatic Time Stepping (AUTOTS)

- For nonlinear problem, the user-input Δt is used as initial incremental time.
- ANSYS adjusts Δt automatically according to the convergence behavior of the solution.
- AUTOTS turns ON/OFF the functions of **auto time stepping**.

Ramped/Stepped Loading (KBC)



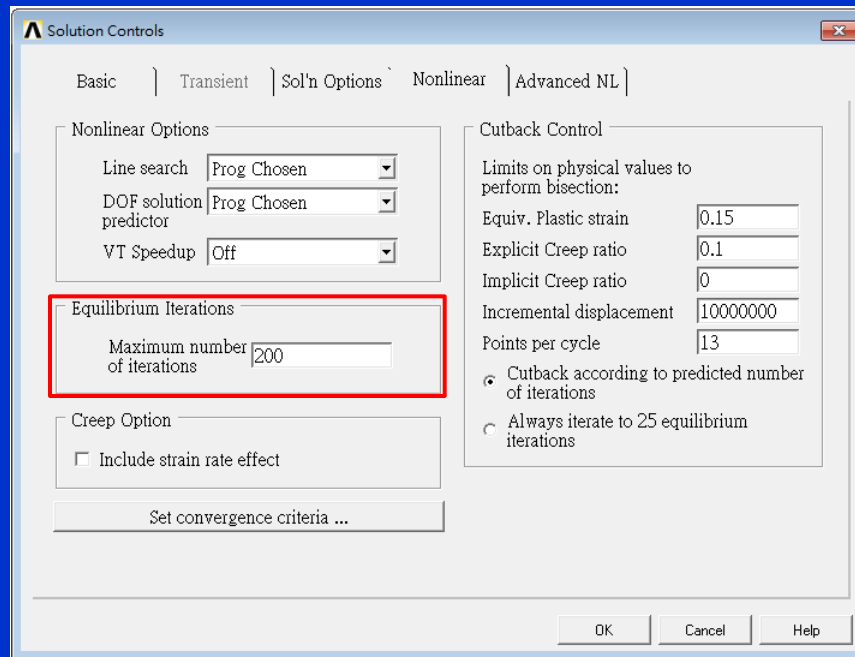
當load step是非常stepped時，必須取非常細的substeps

Time Step Prediction Based on Contact Status (SOLCONTROL)

`SOLCONTROL, Key1, Key2`

- SOLCONTROL command can be used to activate automatic nonlinear solution control algorithm.
- The default is ON.
- The second key of SOLCONTROL command is to ask ANSYS to adjust time steps according to contact status.

- NEQIT specifies the number of iterations beyond which ANSYS would start another “attempt”.
- The default is between 15 and 26.



- This workshop consists of the following problem:


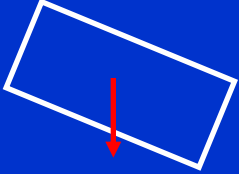


W9. Arched Beam

Please refer to your *Workshop Supplement* for instructions.

Nonlinear Structural Analysis

... Load Orientation

- It is important to note the orientation of loads and its effect on the structure in large-deflection analyses:

Load	Direction Before Deflection	Direction After Deflection
Acceleration (constant direction)		
Force, Moment, Bolt Load (constant direction)		
Pressure (always normal to surface)	