簡諧響應分析

- **ANSYS**之簡諧響應分析**(harmonic response analysis)**即強迫簡諧振動**(forced harmonic vibration)**分析，也就是指外力為簡諧激發型態，例如$F=F_0 \cos\omega t$這類型式之力，F_0為外力振幅。由於振動之位移和外力均假設為簡諧振動且頻率相同，而且為穩態**(steady-state)**，因此可以使用頻域來處理，不必以暫態**(transient)**分析。

Equation of Motion

- **General equation of motion:**

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\}$$

- **[F] and {u} are harmonic, with frequency ω:**

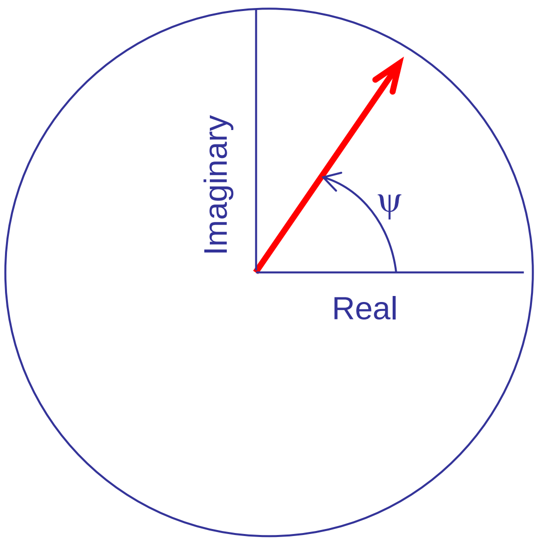
$$\{F\} = \{F_{max} e^{i\psi}\} e^{i\omega t} = (\{F_1\} + i\{F_2\}) e^{i\omega t}$$
$$\{u\} = \{u_{max} e^{i\phi}\} e^{i\omega t} = (\{u_1\} + i\{u_2\}) e^{i\omega t}$$

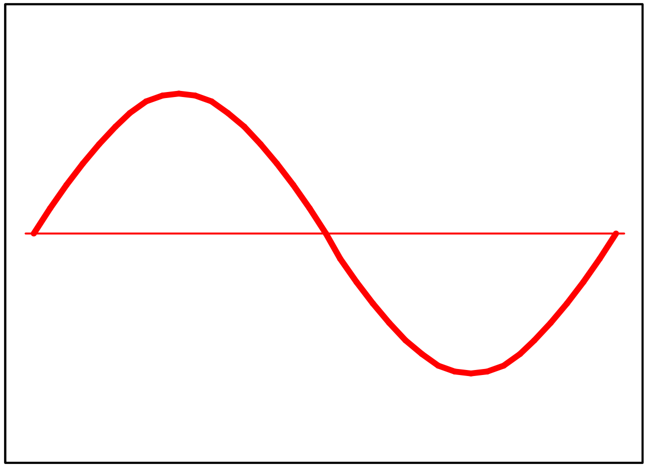
- **Equation of motion for harmonic analysis:**

$$(-\omega^2 [M] + i\omega [C] + [K])(\{u_1\} + i\{u_2\}) = (\{F_1\} + i\{F_2\})$$

Nature of Harmonic Loads

- **Sinusoidally varying, at known frequencies.**
- **Phase angle ψ allows multiple, out-of-phase loads to be applied. Defaults to zero.**
- **<u>All</u> applied loads are assumed to be harmonic, including temperatures and gravity.**

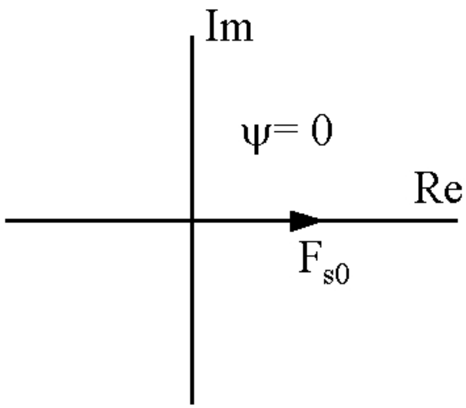




Complex Displacements

- **Calculated displacements will be complex if:**
  - **Damping is specified.**
  - **Applied load is complex (i.e, imaginary part is non-zero).**

F_{s0}= 0.001N
F, 2,FY, 0.001,0



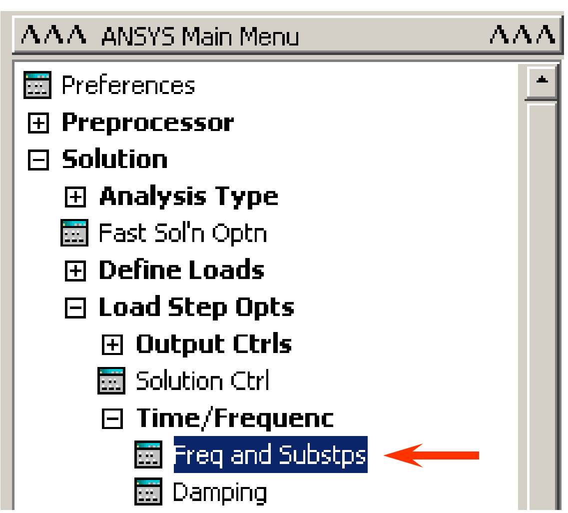
- **Complex displacements lag by phase angle ϕ (with respect to the applied force).**

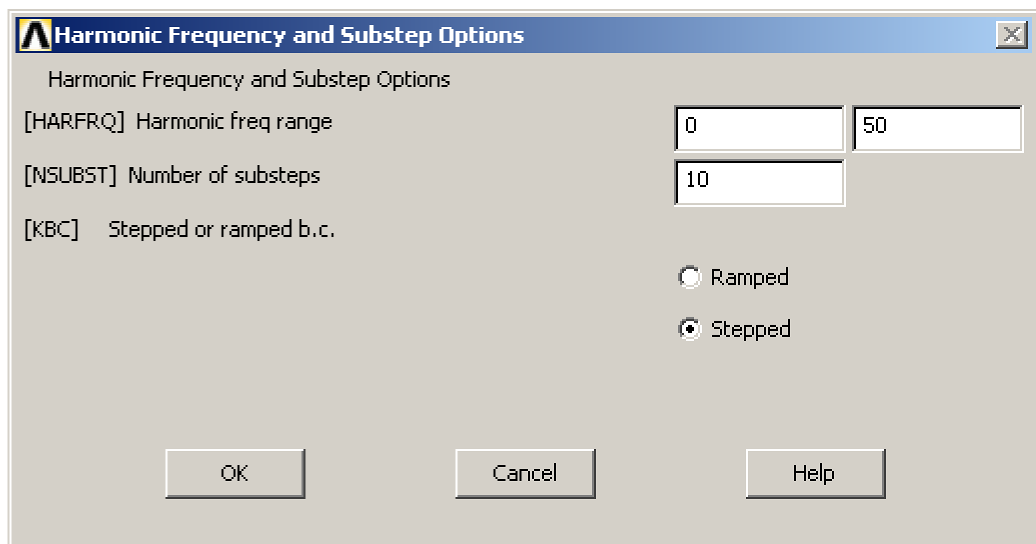
- **Results can be viewed in the form of real and imaginary parts or amplitude and phase angle.**

Apply Harmonic Loads and Solve

Frequency of harmonic load:

- **Specified in cycles per second (Hertz) by a frequency range and number of substeps within that range.**
- **For example, a range of 0-50 Hz with 10 substeps gives solutions at frequencies of 5, 10, 15, …, 45, and 50 Hz. Same range with 1 substep gives one solution at 50 Hz.**

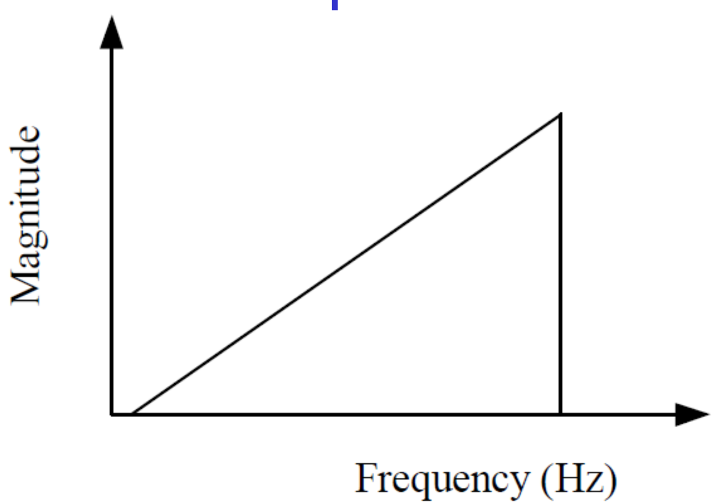




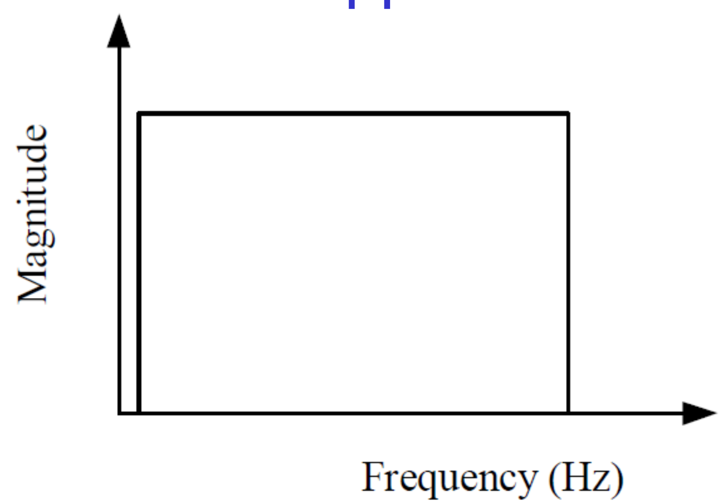
Stepped versus ramped loads:

- **With multiple substeps, loads can be applied gradually (*ramped*) or all at once in the first substep (*stepped*).**
- **Harmonic loads are usually stepped since the load value represents maximum amplitude.**

ramped



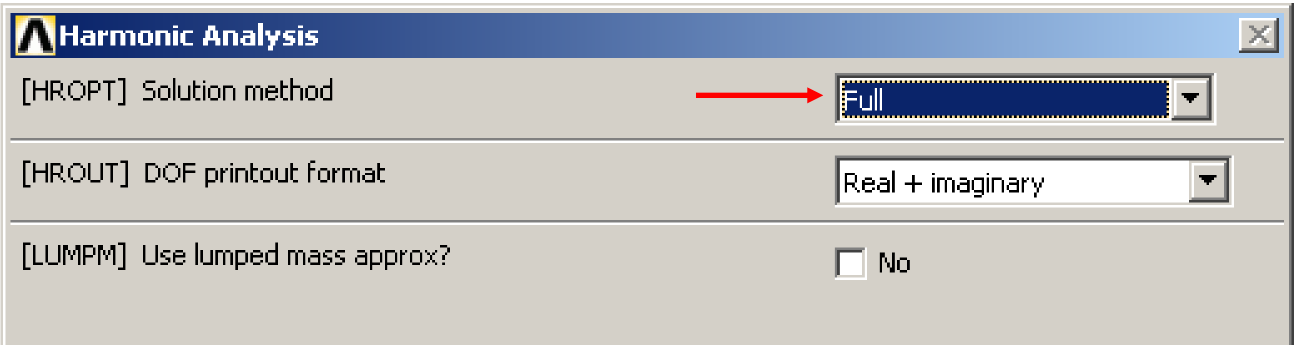
stepped



Solution Methods

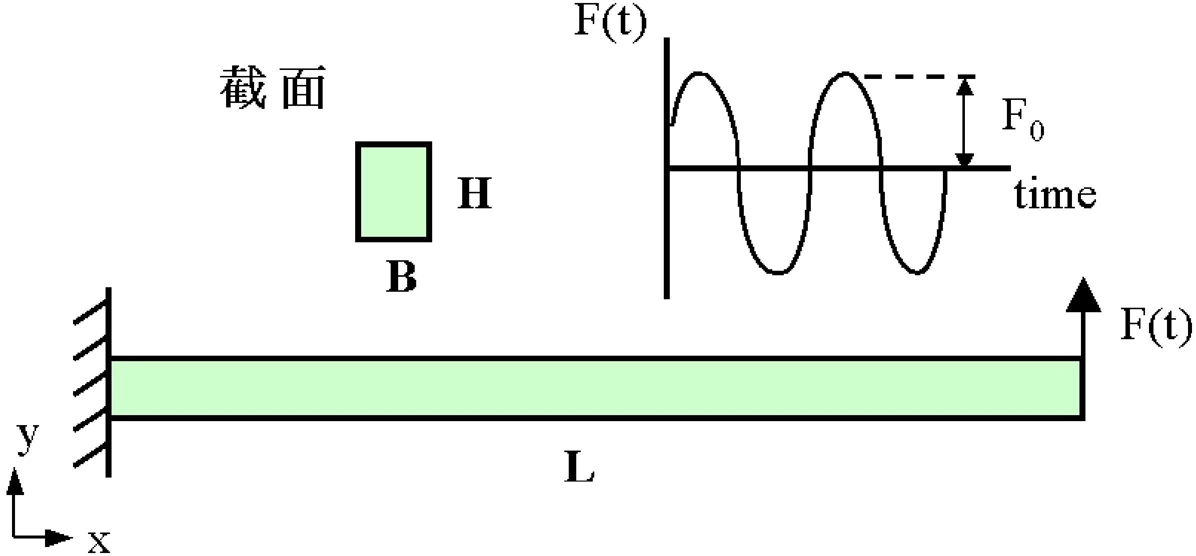
Three methods of solving the harmonic equation of motion:

- **Full method**
  - **Default method, easiest of all.**
  - **Uses full structure matrices. Unsymmetric matrices (e.g, acoustics) are allowed.**
- **Reduced method**
  - **Uses reduced matrices, faster than full method.**
  - **Requires master DOF selection, which results in approximate [M] and [C].**
- **Mode superposition**
  - **Sums factored mode shapes from a preceding modal analysis.**
  - **Fastest of all methods.**

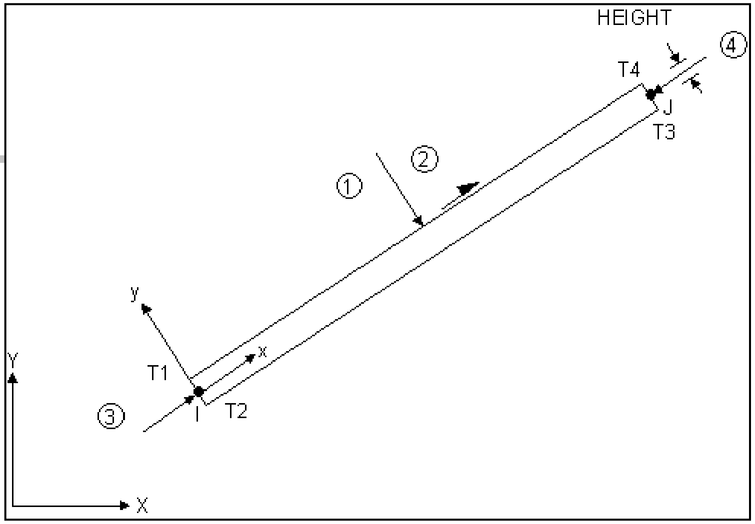


懸臂樑橫向振動之簡諧分析

❖ 如下圖所示之懸臂樑，長度**L=0.3m**，截面高**H=0.006m**，厚度**B=0.003m** ，楊氏模數**E=2200MPa**，密度**ρ=1100kg/m^3**，阻尼比**ζ=0.01**。若自由端給定一**y**方向簡諧力**F=F$_0$sinωt**，其振幅為**F$_0$=0.001N**，求其振動位移與外力激發頻率之關係。分析單位系統採用**SI**制：**m**、**N**、**Pa**、**kg**。



BEAM3 輸入資料

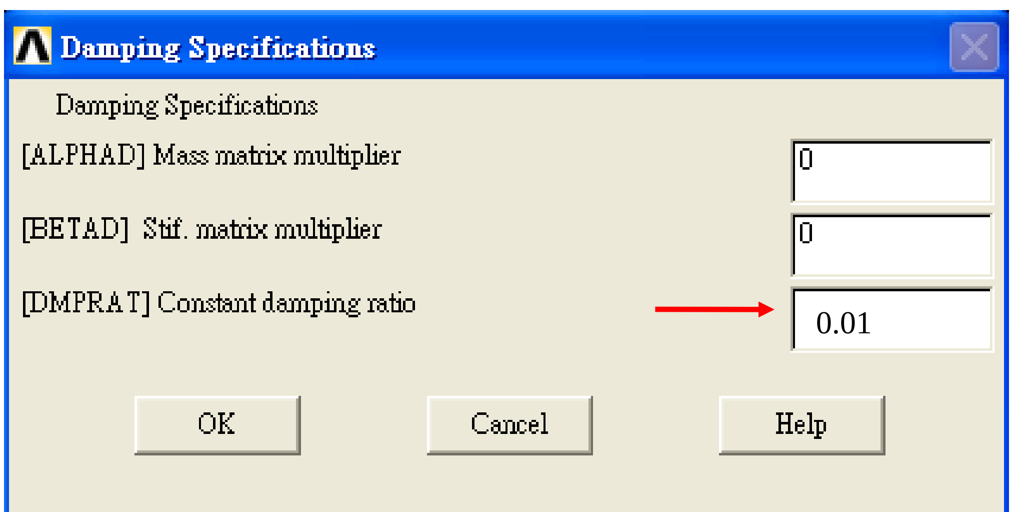


Element Name	BEAM3
Nodes	I, J
Degrees of Freedom	UX, UY, ROTZ
Real Constants	AREA, IZZ, HEIGHT, SHEARZ, ISTRN, ADDMAS
Material Properties	EX, NUXY, GXY, ALPX, DENS, DAMP
Surface Loads	Pressure face 1, face 2, face 3, face 4
Body Loads	Temperature -- T1, T2, T3, T4
Special Features	Stress stiffening, Large deflection, etc.

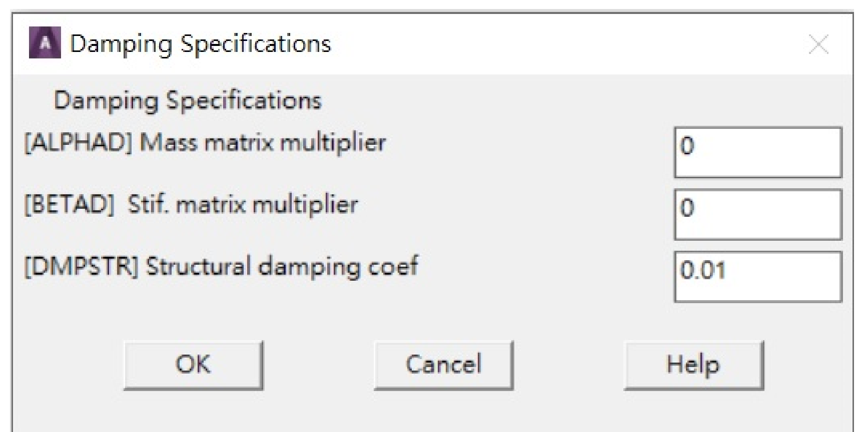
- **HROPT**指令在於設定求解法，如「 **HROPT,FULL**」中之**FULL**即代表**Full method**。
- 指令「**HROUT,OFF**」代表於計算結果檔**(*.out**檔**)**之輸出格式，位移**UX**和**UY**等自由度是以振幅和相位角表示，若設為**ON**則以複數型態表示。
- 「**HARFRQ,0,500**」表示計算範圍為**0~500Hz**。
- 「**NSUBST,100**」則是將**0~500Hz**頻率範圍分割為**100**等分來計算。
- 「**KBC,1**」是將負荷方式設為階梯式**(stepped)**，這是**ANSYS**簡諧分析之規定。
- 「**OUTRES,ALL,ALL**」 是表示於結果檔案**(*.rst)**中，每個頻率結果均儲存下來。
- 「**DMPRAT,0.01**」是設定整個結構之阻尼比為**0.01**。

- 材料內部之遲滯阻尼(**hysteretic damping**)為非黏性阻尼，非黏性阻尼在數學上較難處理，因此以等效黏性阻尼(**equivalent viscous damping**)的模式來模擬結構的非黏性阻尼，其等效條件必須為穩態簡諧振動，且符合非黏性阻尼較小的情況。
- 若要在**ANSYS**中模擬以上結構的非黏性阻尼現象，常用的做法是輸入阻尼比(**damping ratio**) ζ，可令$\zeta=\zeta_{eq}$，ζ_{eq}即為等效黏性阻尼比(**equivalent viscous damping ratio**)。

DMPRAT, 0.01



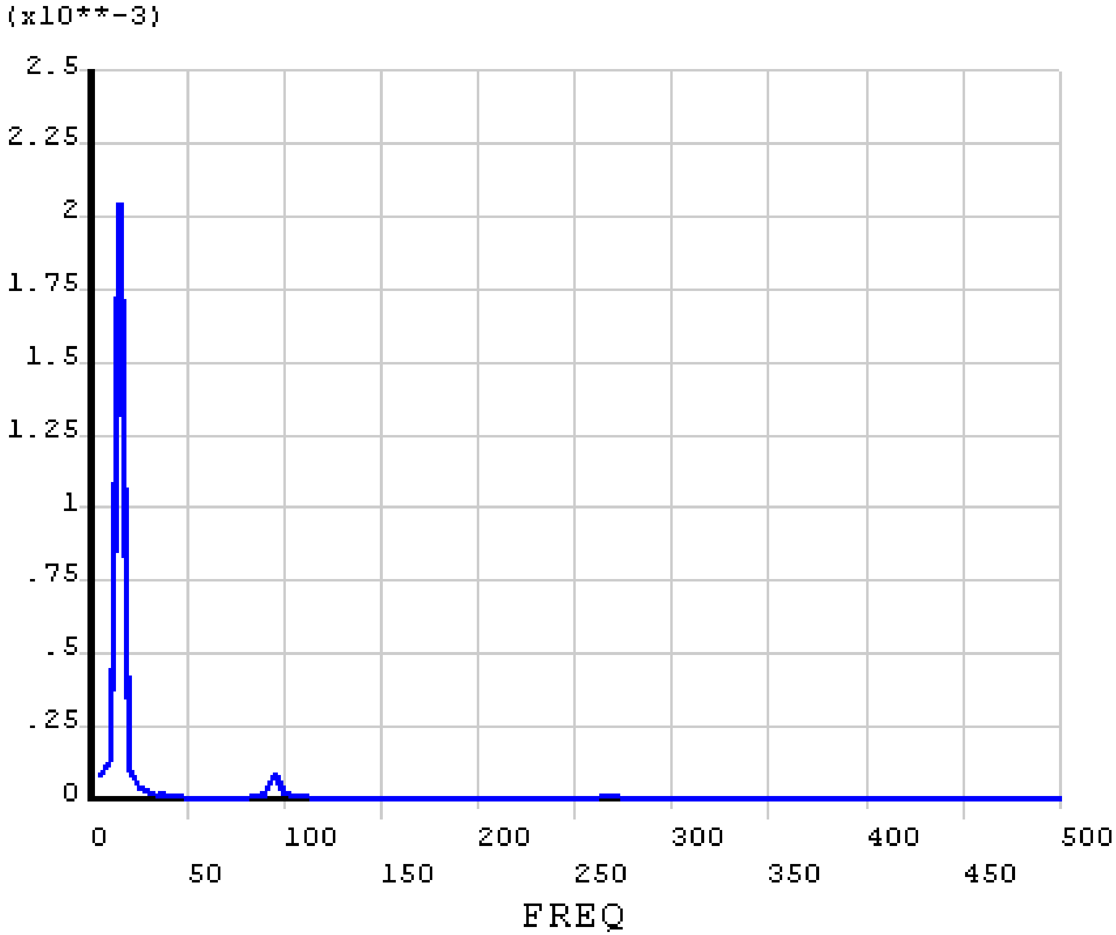
DMPSTR, 0.01



結果與討論

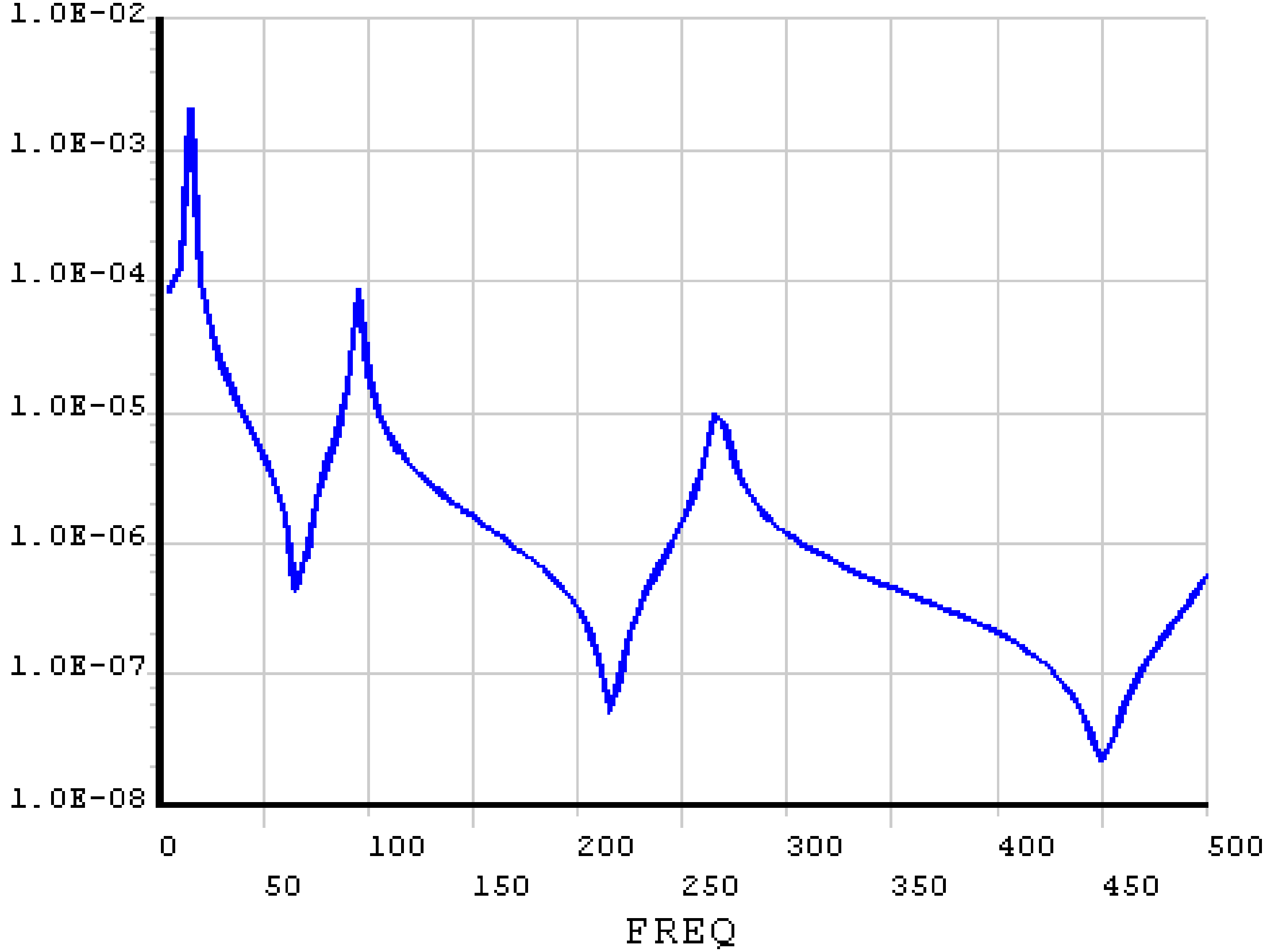
- 指令「**NSOL,2,2,U,Y,END-UY**」是將節點**2(**懸臂樑之右端自由端**)**的**UY**位移抓出，定義為變數**2**，取名為**END-UY**。在簡諧分析的**/POST26**中，其變數**1**為頻率。
- 指令「**PRCPLX,1**」是設定分析結果輸出為振幅和相位角**(Amplitude+Phase)**格式。
- 指令「**PLCPLX,0**」是設定分析結果的繪圖為振幅值。
- 指令「**PLVAR,2,**」是以變數**2(**自由端的**UY**位移，單位為**m)**為縱軸，以頻率值**(**單位為**Hz)**為橫軸，畫出位移振幅-頻率圖。
- 另一種繪圖方式是將縱軸變數，取為變數**2**振幅值之對數**(log)**值，即**log(|UY|)**值，以上方法是使用指令「**/GROPT,LOGY,ON**」來完成，再以指令「**PLVAR,2,**」畫出位移振幅**log**值-頻率圖。

位移振幅-頻率關係



最大振幅處之頻率為15Hz，其振幅UY=0.204887×10^{-2}m ，此15Hz頻率接近模態分析得到的第1自然頻率15.229Hz ，因此振幅最大

位移振幅log值-頻率關係



❖ 懸臂樑自由端位移**UY**最大振幅發生於頻率**15Hz**，其振幅 **UY=0.204887×10^{-2}m**，相位角為 **-33.7943**度，且**15Hz**為第**3**副增量**(substep)**，因此使用以下指令：

```
/POST1
SET,LIST
SET,1,3,
HRCPLX, 1, 3, -33.7943,
PLDISP,1
```

←下一頁

***** ANSYS POST26 VARIABLE LISTING *****

FREQ	2 UY END-UY AMPLITUDE	PHASE
5.0000	0.846265E-004	-1.28058
10.000	0.131442E-003	-1.99912
15.000	0.204887E-002	$\boxed{-33.7943}$
20.000	0.991282E-004	-178.355
25.000	0.410274E-004	-179.215
30.000	0.231013E-004	-179.429

❖ 以指令「**HRCPLX, 1, 3, -33.7943,**」將相位角為 **-33.7943**度的位移振幅算出，接著再畫出**15Hz**時的變形圖。

SET,LIST Command

File

***** INDEX OF DATA SETS ON RESULTS FILE *****

SET	TIME/FREQ	LOAD STEP	SUBSTEP	CUMULATIVE
1	5.0000	1	1	1
2	5.0000	1	1	1
3	10.000	1	2	2
4	10.000	1	2	2
5	15.000	1	3	3
6	15.000	1	3	3
7	20.000	1	4	4
8	20.000	1	4	4
9	25.000	1	5	5
10	25.000	1	5	5
11	30.000	1	6	6
12	30.000	1	6	6
13	35.000	1	7	7
14	35.000	1	7	7
15	40.000	1	8	8
16	40.000	1	8	8
17	45.000	1	9	9

15 Hz之位移振幅變形圖