題目(材料力學)

本例如圖 5.5 的二維懸臂樑(cantilever beam)問題 ，樑之楊氏模數 E=12 GPa，矩形截面爲 400 mm×200 mm，試求 B 點的撓曲位移和斜率(slope)角度，和 A 點的彎曲應力(bending stress)。分析單位系統採用：m、N、Pa。

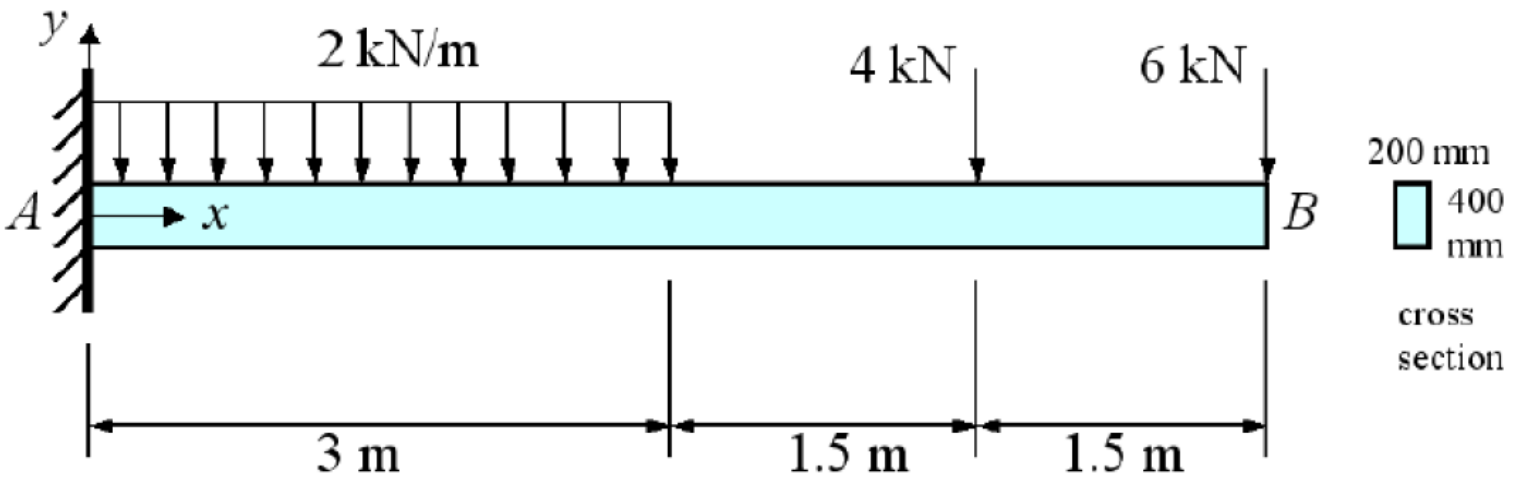
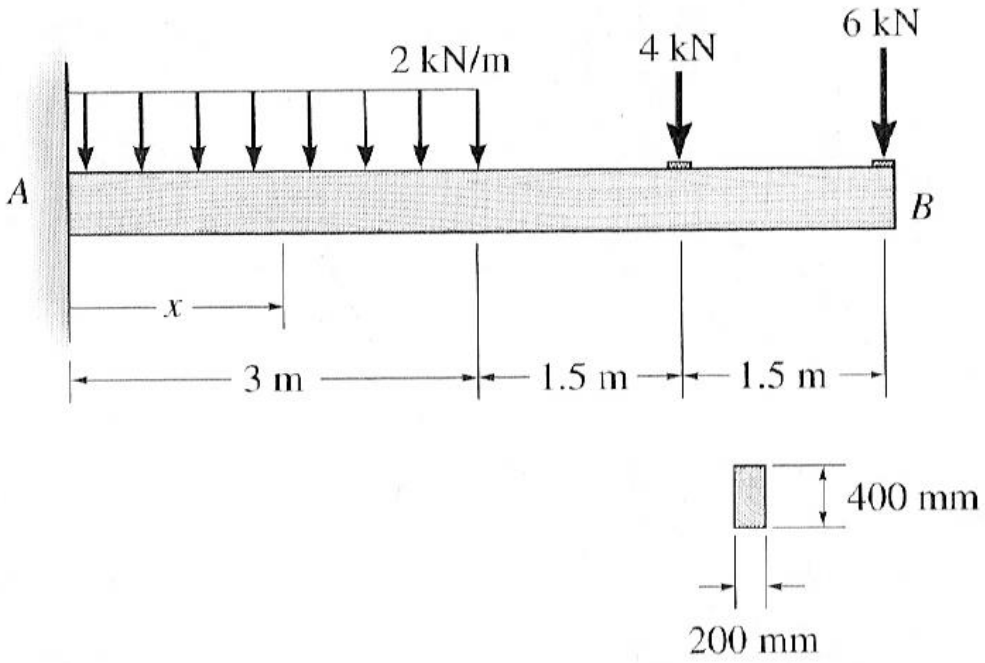


圖 5.5 懸臂樑問題

原題目

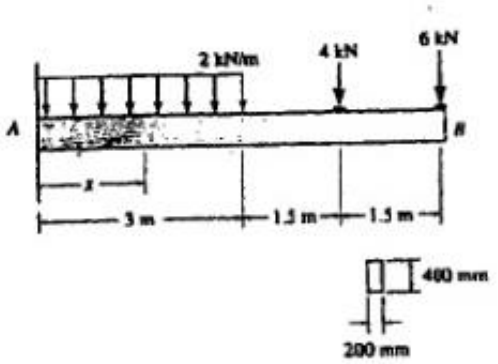
12–39. The wooden beam is subjected to the load shown. Determine the equation of the elastic curve. If $E_w = 12$ GPa, determine the deflection and the slope at end B.

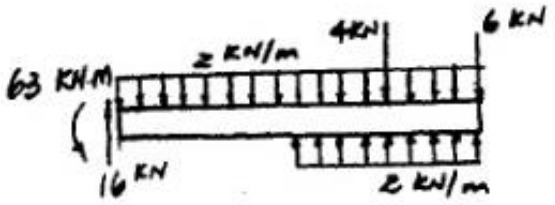


ANS: $\theta_B = -0.705°, v_B = -51.7$ mm

R.C. Hibbeler, *Mechanics of Materials*. 5th edition, New Jersey: Prentice Hall, 2003.

Solution





$$M = -63\langle x-0\rangle^0 - (-16)\langle x-0\rangle - \frac{2}{2}\langle x-0\rangle^2 - (-\frac{2}{2})\langle x-3\rangle^2 - 4\langle x-4.5\rangle$$

$$M = -63 + 16x - x^2 + \langle x-3\rangle^2 - 4\langle x-4.5\rangle$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M = 63 + 16x - x^2 + \langle x-3\rangle^2 - 4\langle x-4.5\rangle$$

$$EI\frac{dv}{dx} = -63x + 8x^2 - \frac{x^3}{3} + \frac{1}{3}\langle x-3\rangle^3 - 2\langle x-4.5\rangle^2 + C_1 \qquad (1)$$

$$EIv = -31.5x^2 + \frac{8}{3}x^3 - \frac{x^4}{12} + \frac{1}{12}\langle x-3\rangle^4 - \frac{2}{3}\langle x-4.5\rangle^3 + C_1x + C_2 \qquad (2)$$

Boundary conditions :

$\frac{dv}{dx} = 0$ at $x = 0$

From Eq. (1), $C_1 = 0$

$v = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$$\frac{dv}{dx} = \frac{1}{EI}\left[-63x + 8x^2 - \frac{x^3}{3} + \frac{1}{3}\langle x-3\rangle^3 - 2\langle x-4.5\rangle^2\right] \qquad (3)$$

$$v = \frac{1}{EI}\left[-31.5x^2 + \frac{8}{3}x^3 - \frac{x^4}{12} + \frac{1}{12}\langle x-3\rangle^4 - \frac{2}{3}\langle x-4.5\rangle^3\right] \text{kN}\cdot\text{m}^3 \qquad (4) \quad \text{Ans}$$

$$I = \frac{1}{12}(0.20)(0.40)^3 = 1.067(10^{-3})\text{m}^4$$

At point B, $x = 6$m

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=6\text{m}} = \frac{-157.5}{EI} = \frac{-157.5(10^3)}{12(10^9)(1.067)(10^{-3})} = -0.0123 \text{ rad} = -0.705^\circ \quad \text{Ans}$$

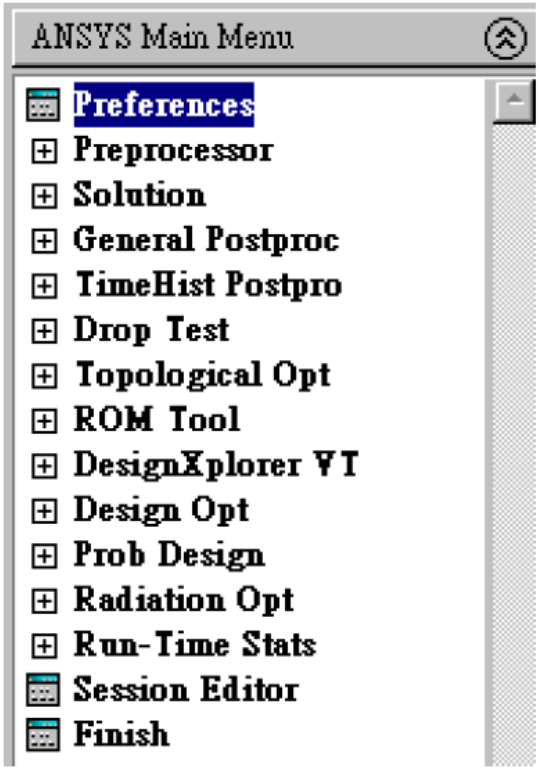
The negative sign indicates clockwise rotation.

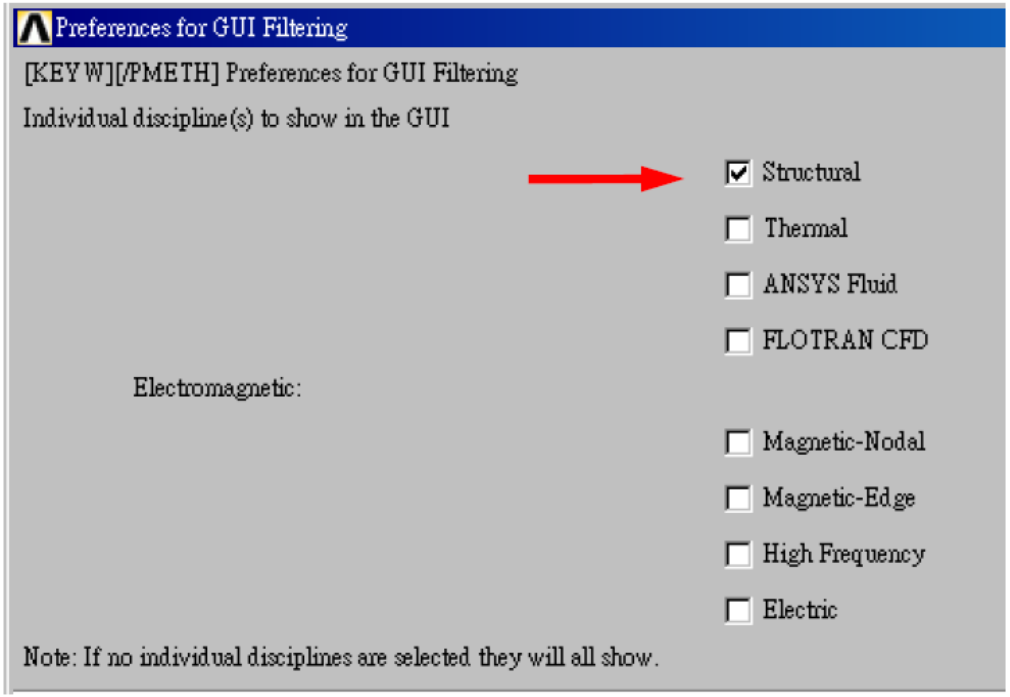
$$v_B = \frac{-661.5}{EI} = \frac{-661.5(10^3)}{12(10^9)(1.067)(10^{-3})} = -0.0517\text{m} = -51.7 \text{ mm} \quad \text{Ans}$$

The negative sign indicates downward displacement.

預先設定

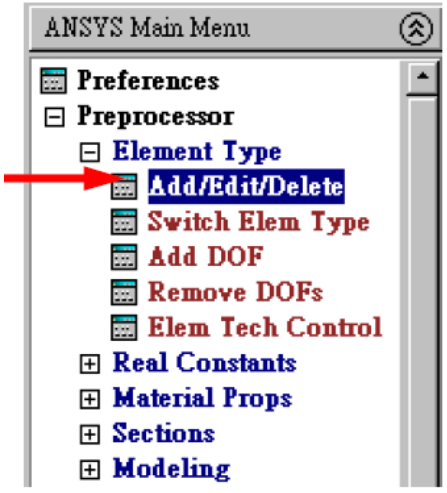
- GUI: Utility Menu > File > Change Jobname > 輸入 ex1
- GUI: Utility Menu > File > Change Title >輸入 ex1 - cantilever beam
- GUI: Main Menu > Preferences > Structural 打勾

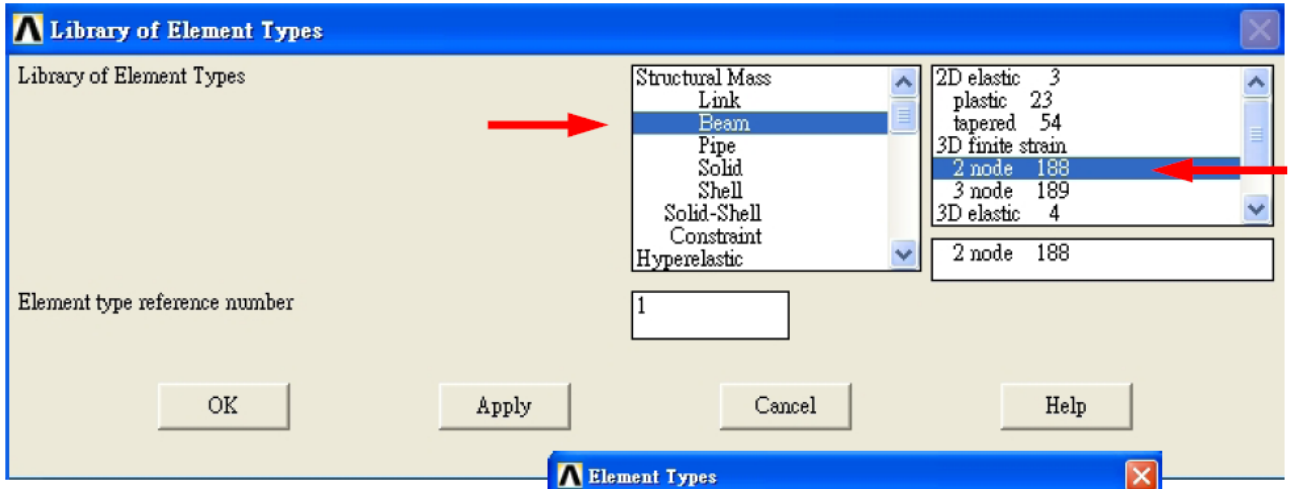


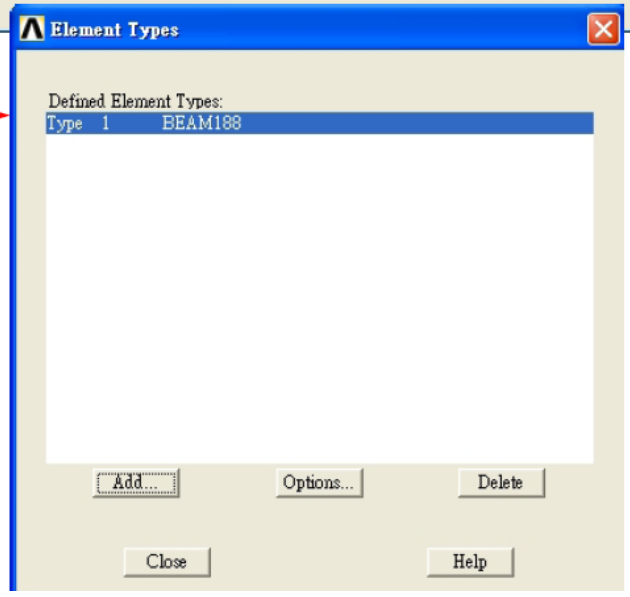


設定元素

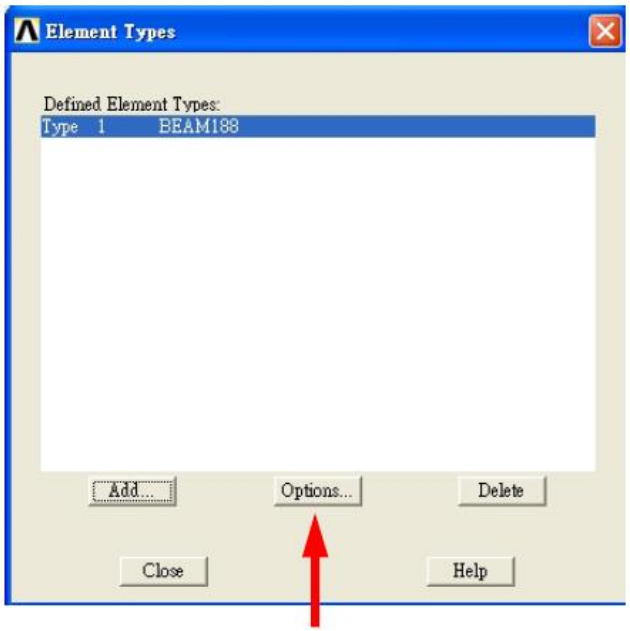
- GUI: Main Menu > Preprocessor > Element Type > Add/Edit/Delete >在Element Types 視窗按Add > 在另一視窗選 Beam 2 node 188 元素, 按OK >在Element Types 視窗按 Close



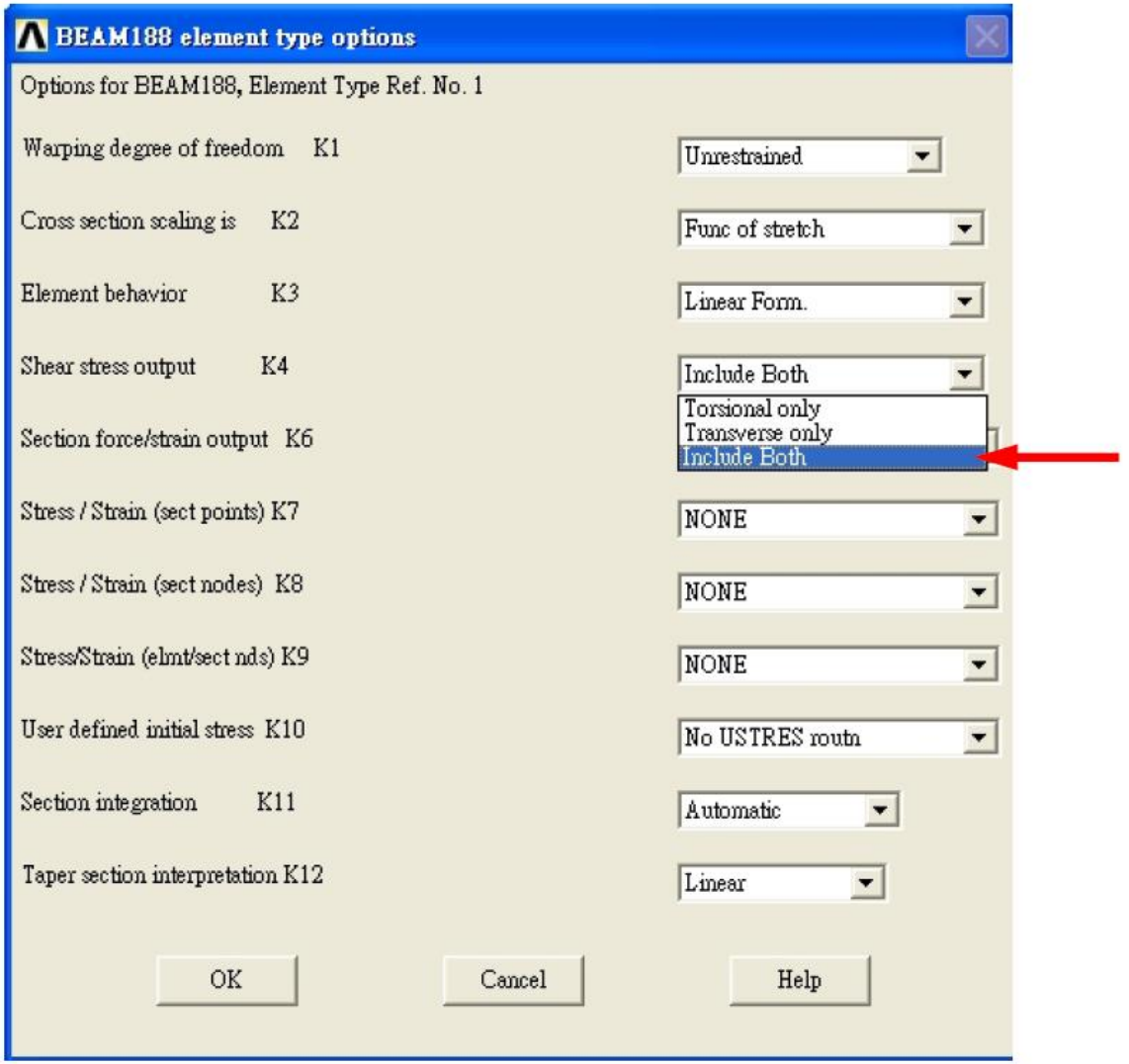




設定元素(續)



Options



設定元素(續)

ANSYS Release 10.0 Documentation for ANSYS

[◄] [►]

Element Reference | Part I. Element Library |

BEAM188

3-D Linear Finite Strain Beam

MP ME ST ◇ ◇ PR ◇ ◇ ◇ PP ED

BEAM188 Element Description

BEAM188 is suitable for analyzing slender to moderately stubby/thick beam structures. This element is based on Timoshenko beam theory. Shear deformation effects are included.

BEAM188 is a linear (2-node) or a quadratic beam element in 3-D. BEAM188 has six or seven degrees of freedom at each node, with the number of degrees of freedom depending on the value of KEYOPT(1). When KEYOPT(1) = 0 (the default), six degrees of freedom occur at each node. These include translations in the x, y, and z directions and rotations about the x, y, and z directions. When KEYOPT(1) = 1, a seventh degree of freedom (warping magnitude) is also considered. This element is well-suited for linear, large rotation, and/or large strain nonlinear applications.

BEAM188 includes stress stiffness terms, by default, in any analysis with NLGEOM,ON. The provided stress stiffness terms enable the elements to analyze flexural, lateral, and torsional stability problems (using eigenvalue buckling or collapse studies with arc length methods).

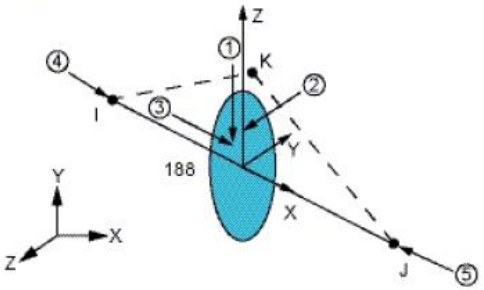
BEAM188 can be used with any beam cross-section defined via SECTYPE, SECDATA, SECOFFSET, SECWRITE, and SECREAD. The cross-section associated with the beam may be linearly tapered.

Elasticity, creep, and plasticity models are supported (irrespective of cross-section subtype). A cross-section associated with this element type can be a built-up section referencing more than one material.

BEAM188 ignores any real constant data beginning with Release 6.0. See the SECCONTROLS command for defining the transverse shear stiffness, and added mass.

For BEAM188, the element coordinate system (/PSYMB,ESYS) is not relevant.

Figure 188.1 BEAM188 Geometry

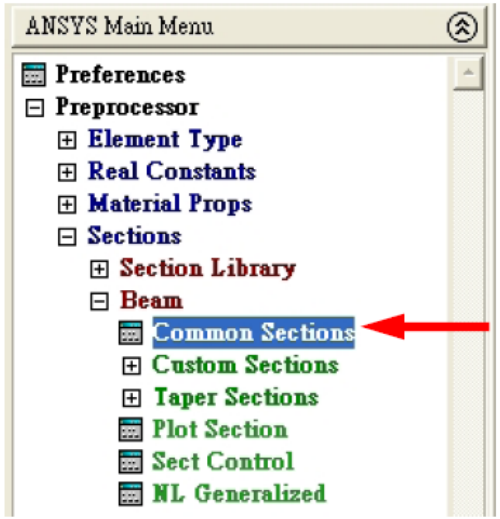


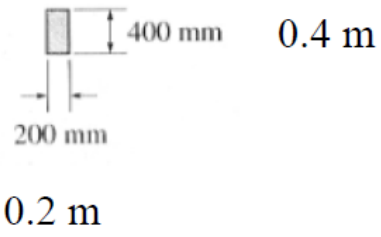
BEAM188 Input Data

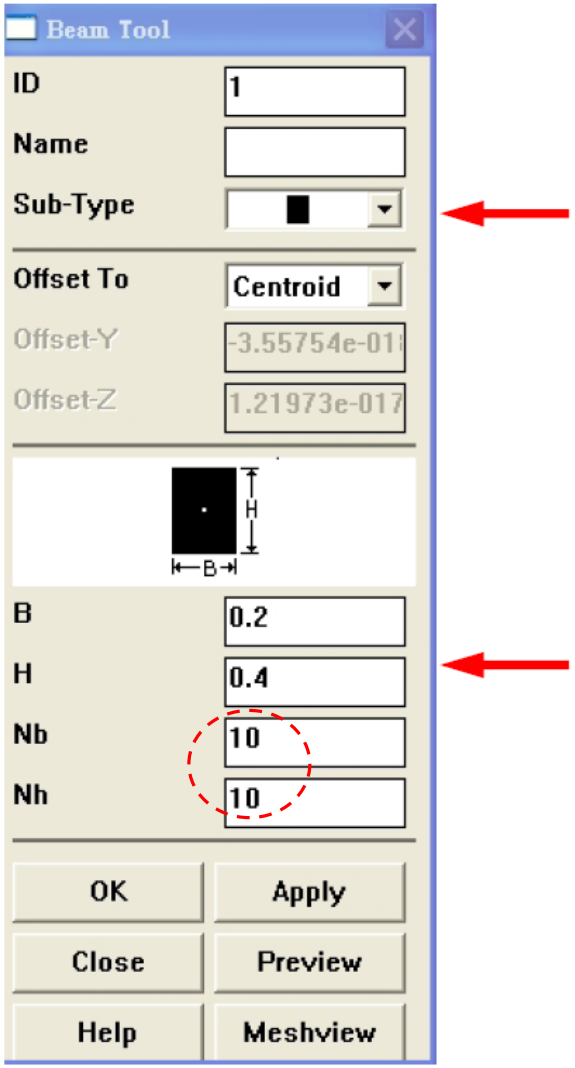
ANSYS Help

設定截面(section)

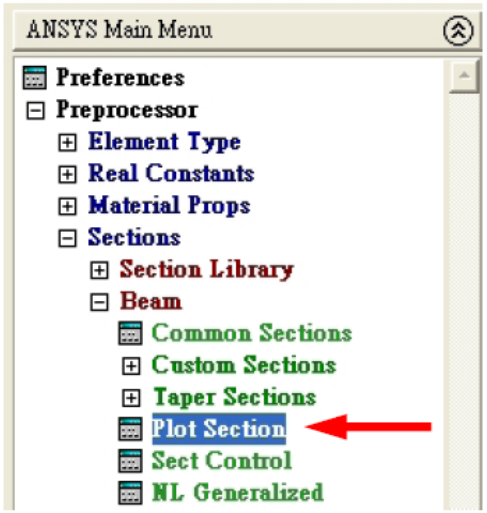
- GUI: Main Menu > Preprocessor > Sections > Beam > Common Sections

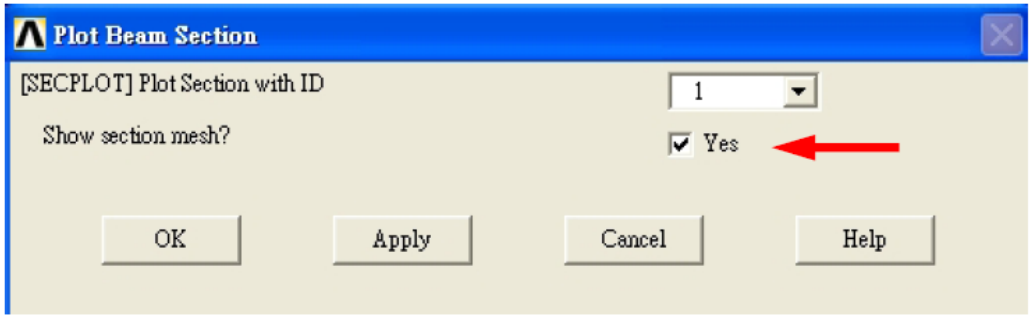


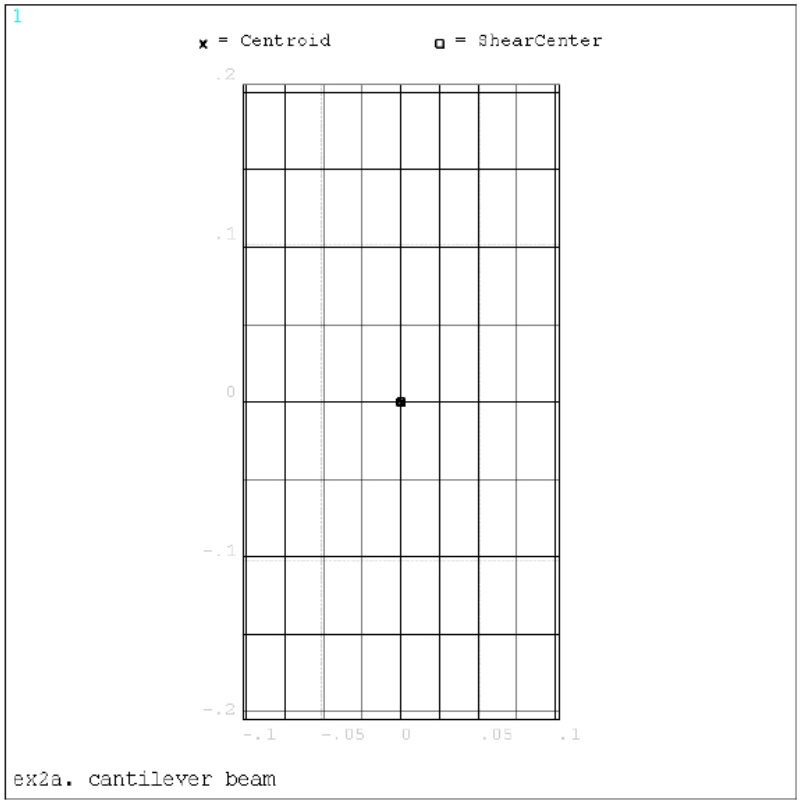




設定截面(section)（續）








```
SECTION ID 1
DATA SUMMARY

Section Name
  =
Area
  = .08
Iyy
  = .001067
Iyz
  = 0
Izz
  = .267E-03
Warping Constant
  = .130E-05
Torsion Constant
  = .732E-03
Centroid Y
  = -.731E-17
Centroid Z
  = .256E-16
Shear Center Y
  = -.826E-14
Shear Center Z
  = .173E-14
Shear Corr. YY
  = .833363
Shear Corr. YZ
  = .705E-13
Shear Corr. ZZ
  = .833363
```

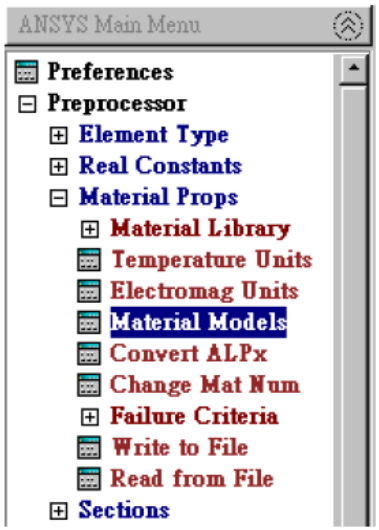
材料係數

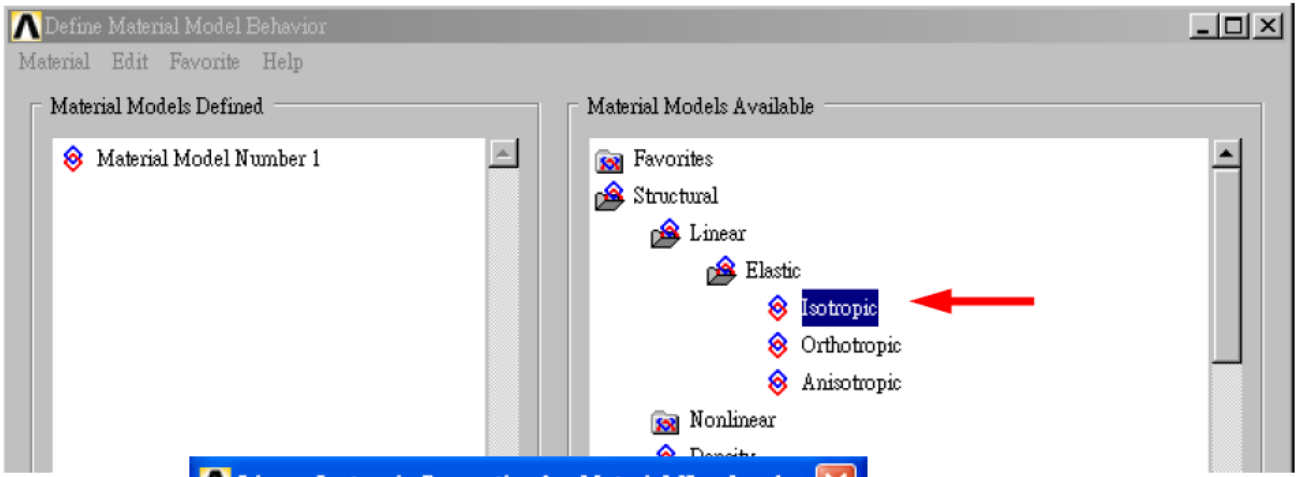
- ANSYS設定:
  - 材料: Young's modulus= $12*10^9$ Pa
  - Poisson's ratio: 0.3 (原題目沒有)

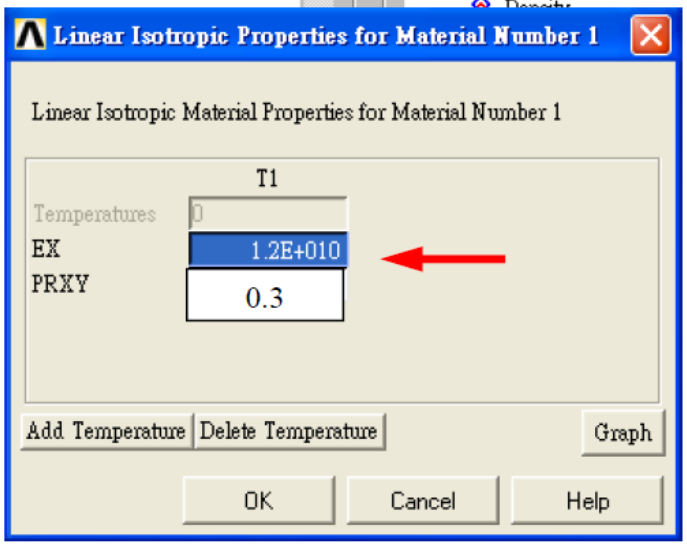
Pa= N/m^2
GPa= 10^9 Pa
MPa= 10^6 Pa
1 kgf = 9.81 N

材料係數(續)

- GUI: Main Menu > Preprocessor > Material Props > Material Models > 出現Define Material Model 視窗, 在右邊的 Structural, Linear, Elastic, Isotropic 各以滑鼠按2下 > 在Linear Isotropic 視窗輸入 **EX= 12E9, PRXY=0.3**



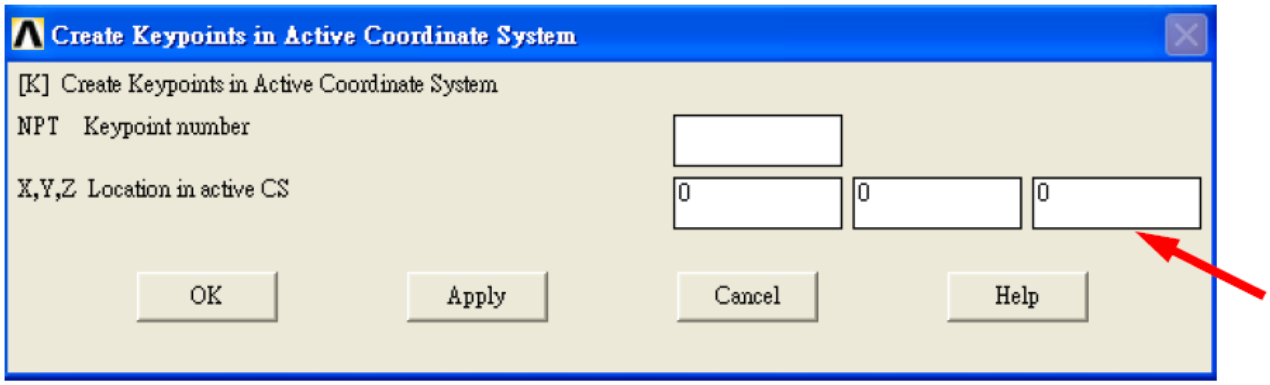




建立關鍵點(keypoints)

- GUI : Main Menu > Preprocessor > Modeling > Create > Keypoints > In Active CS > 在視窗上輸入keypoints座標, 輸入後按OK.

需輸入的座標點如下:
(0,0,0)　(3,0,0)
(4.5,0,0)　(6,0,0)
(0,1,0)



K,,0,0

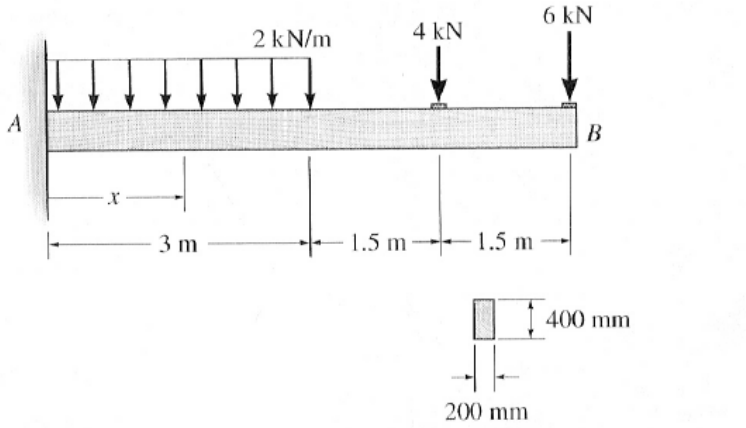
K,,3,0

K,,4.5,0

K,,6,0

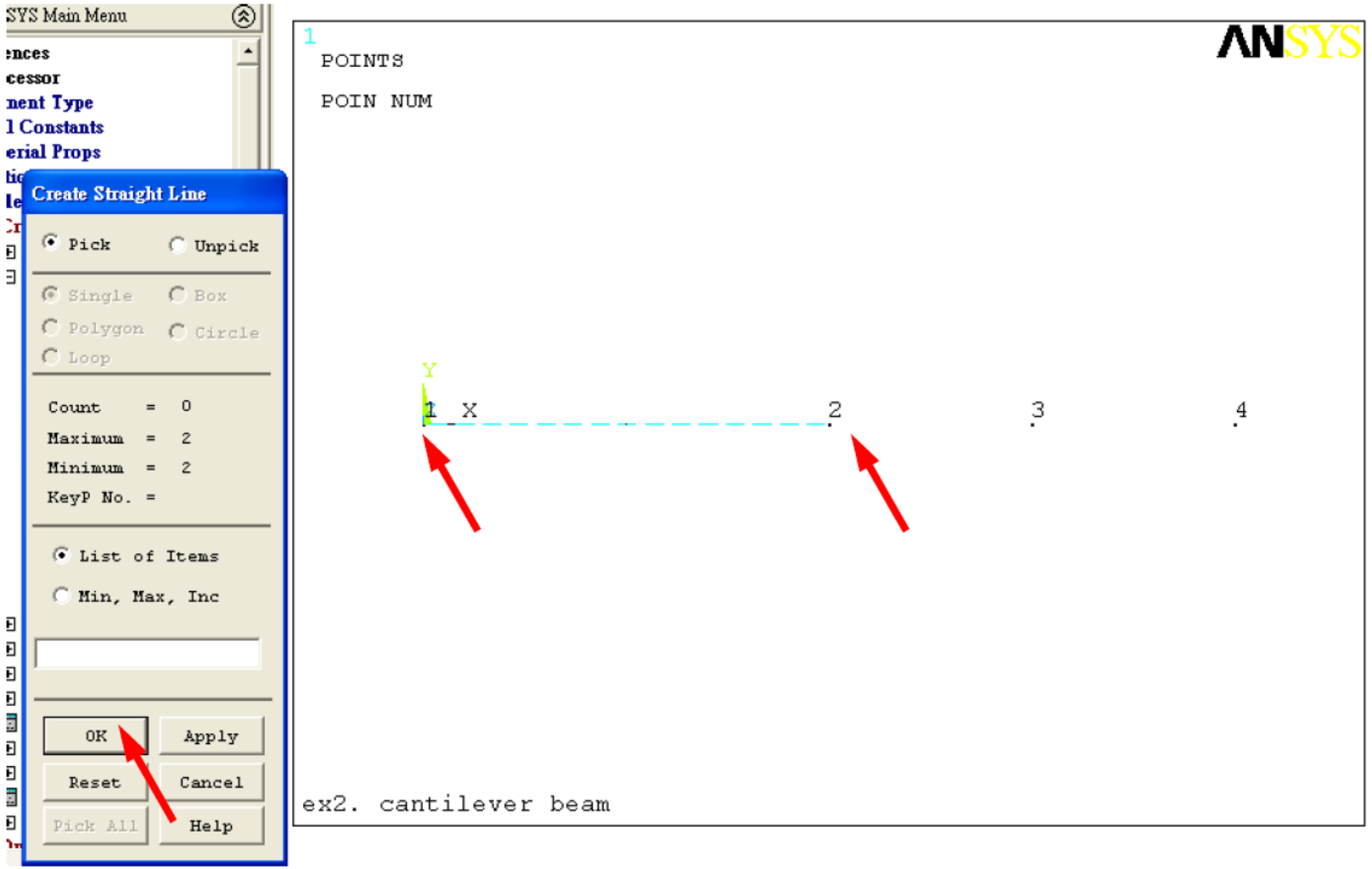
K,,0,1

12–39. The wooden beam is subjected to the load shown. Determine the equation of the elastic curve. If $E_w = 12$ GPa, determine the deflection and the slope at end B.

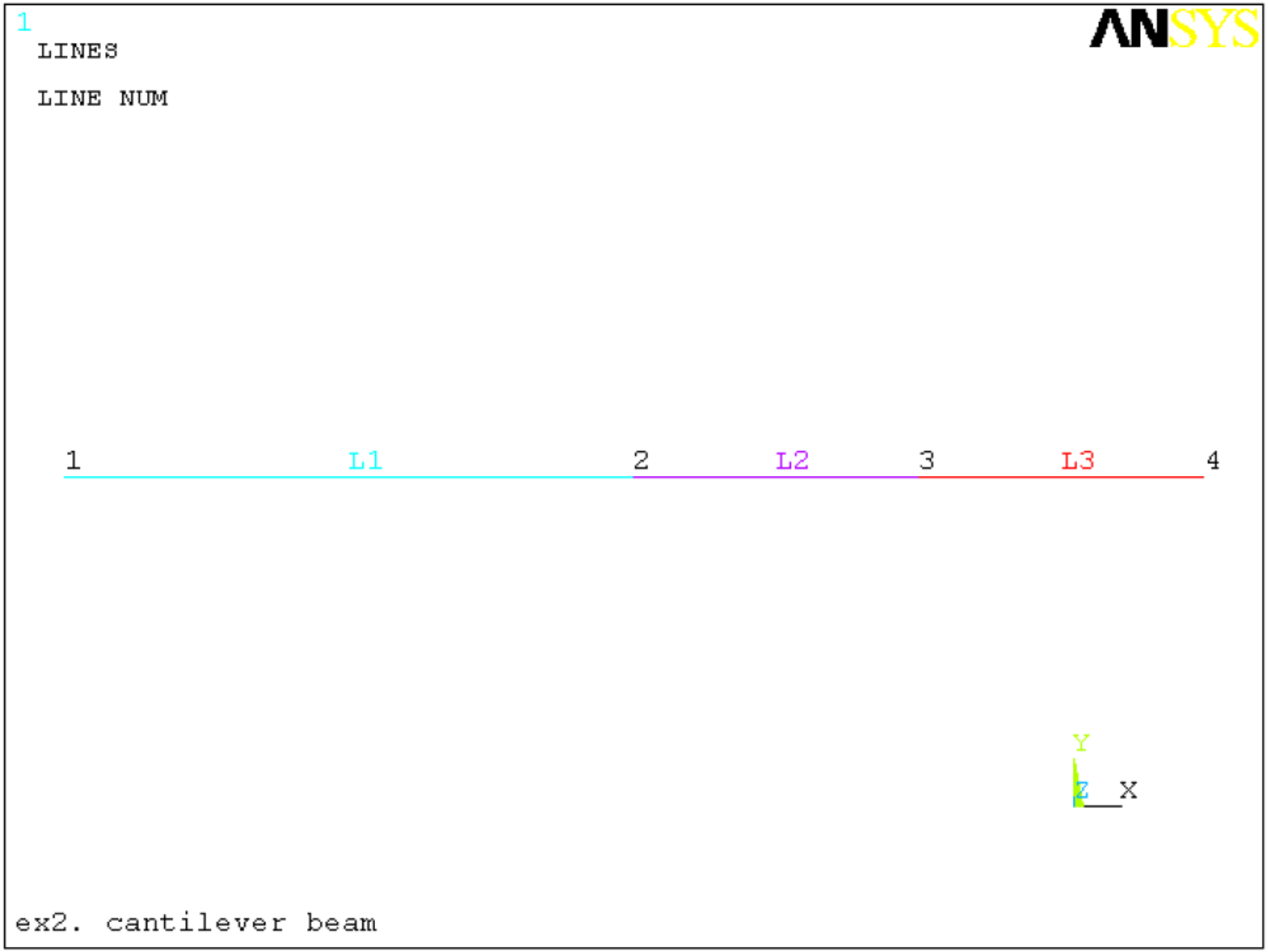


建立直線

- GUI : Main Menu > Preprocessor > Modeling > Create > Lines > Lines > Straight Line > 抓2點, 按左下角OK

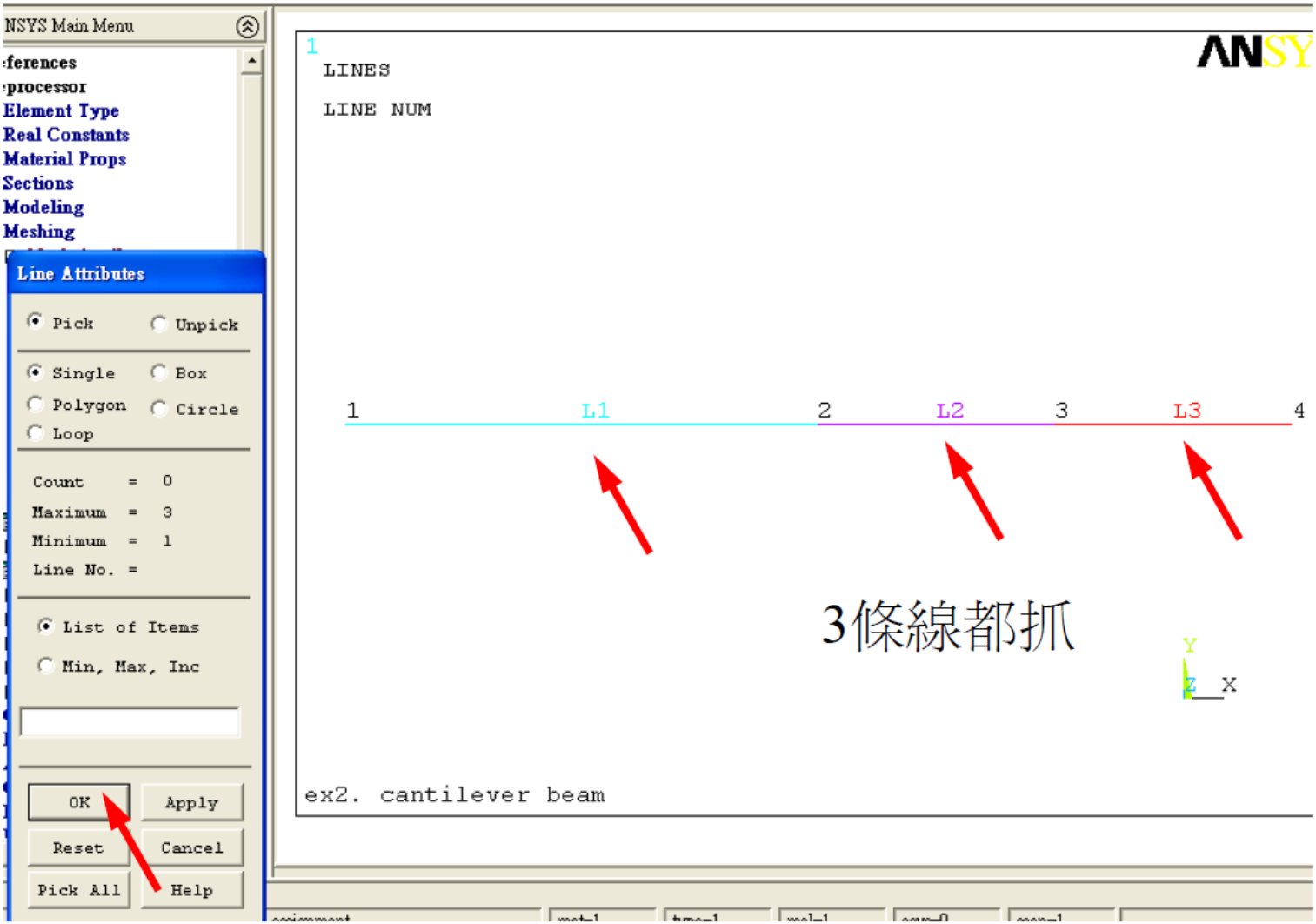


建立直線(續)

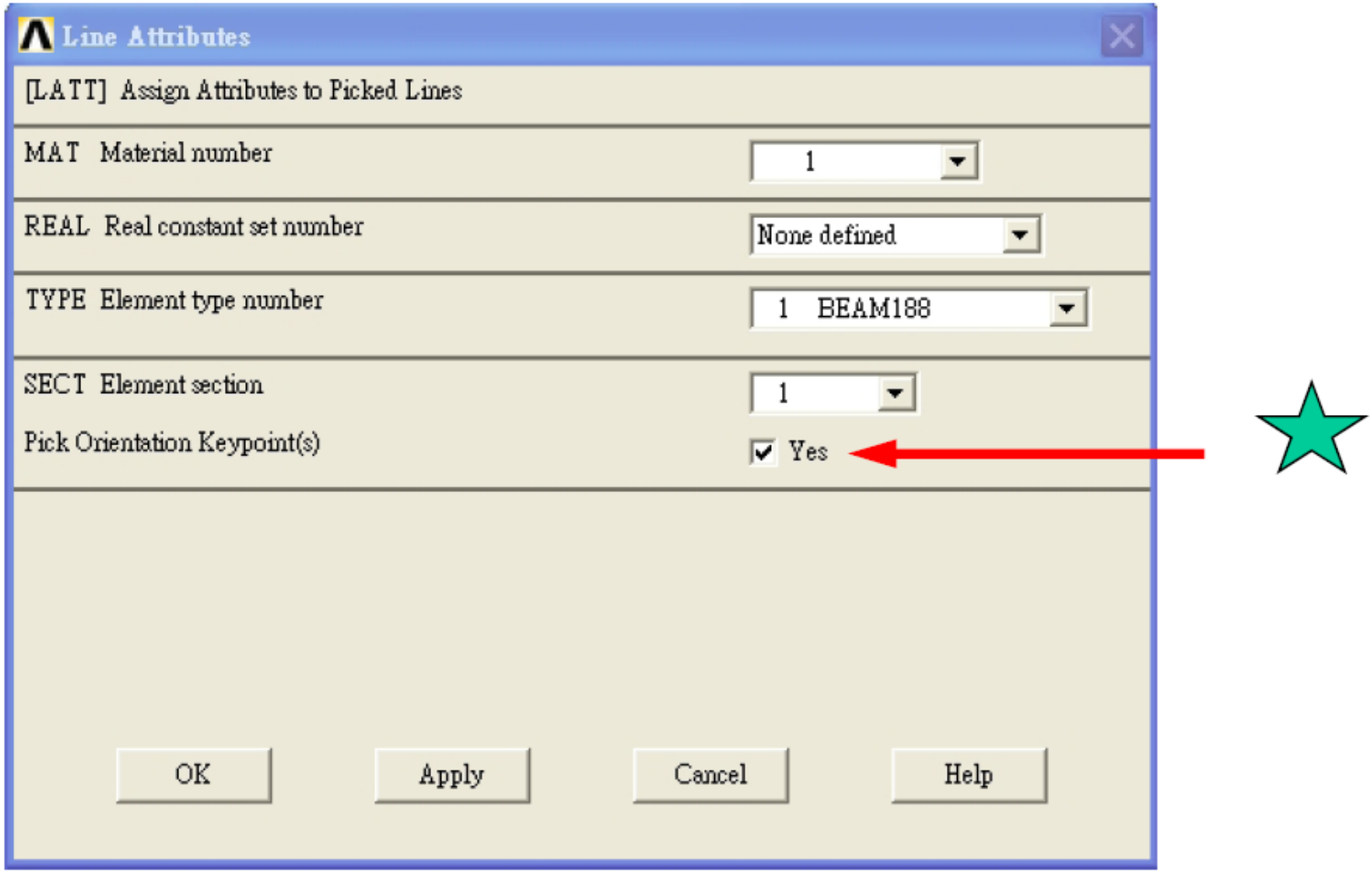


在各條線, 給定元素性質

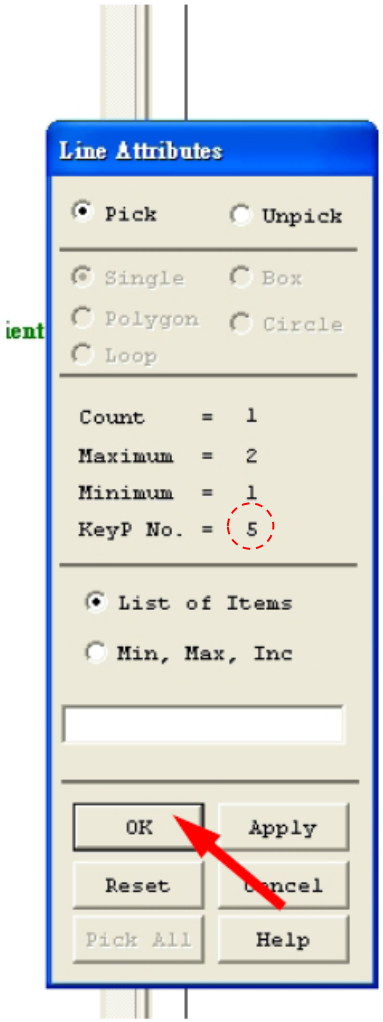
- GUI : Main Menu >Preprocessor > Meshing > Mesh Attributes > Picked Lines >

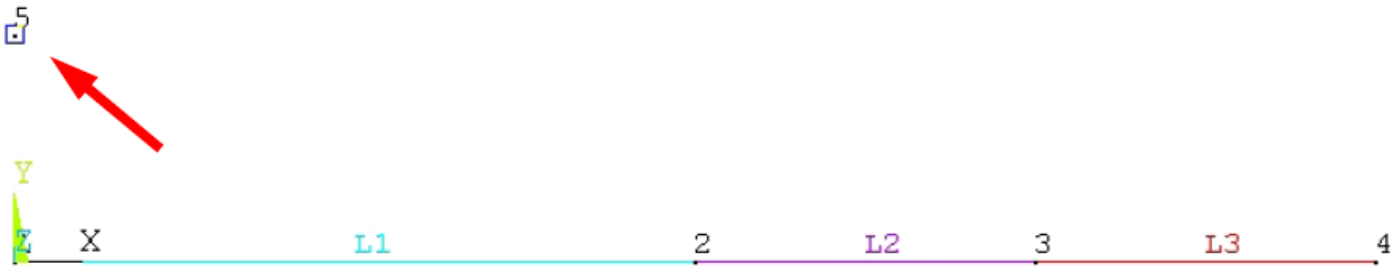


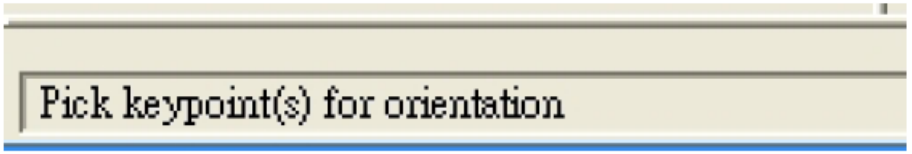
在各條線, 給定元素性質(續)



在各條線, 給定元素性質(續)

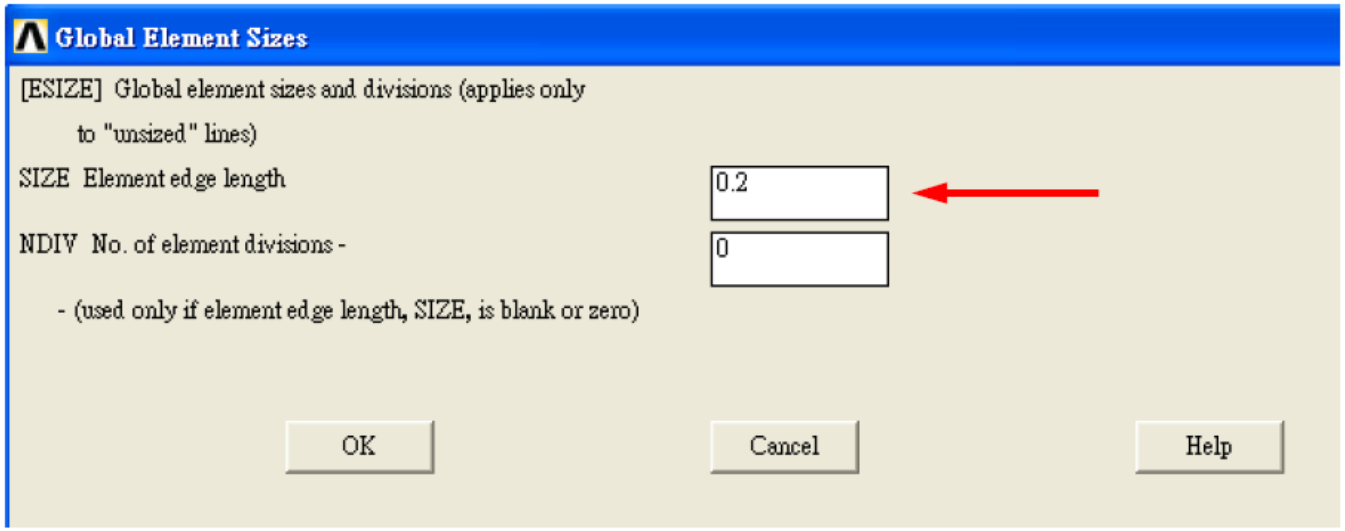






設定網格大小

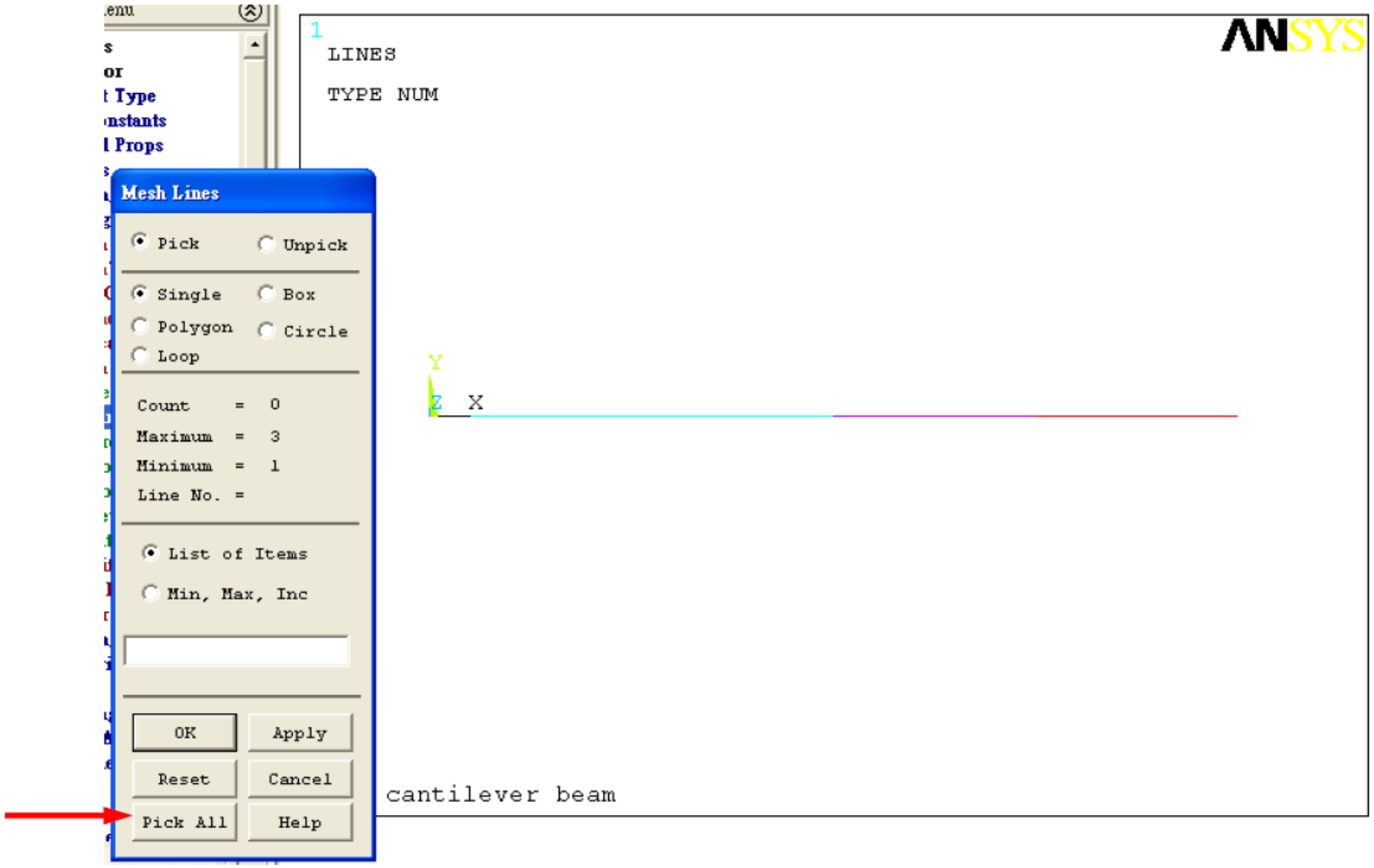
- 網格(mesh): elements + nodes
- GUI : Main Menu > Preprocessor > Meshing > Size Cntrls > ManualSize > Global > Size



建立網格

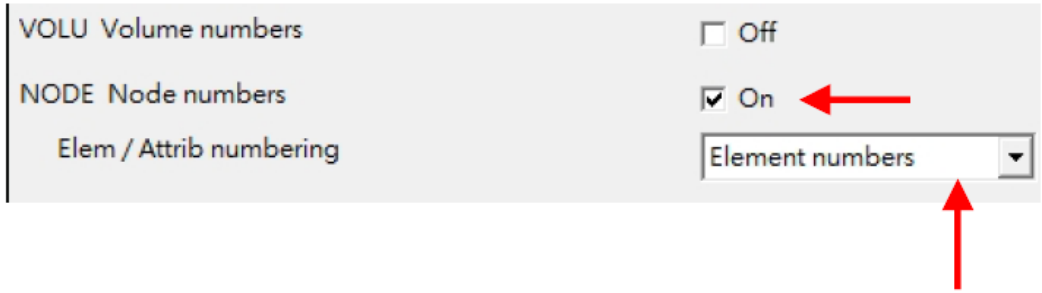
- GUI : Main Menu > Preprocessor > Meshing > Mesh > Lines >

  Pick all : 將所有線抓出

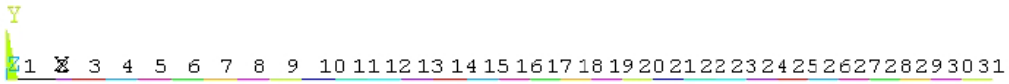


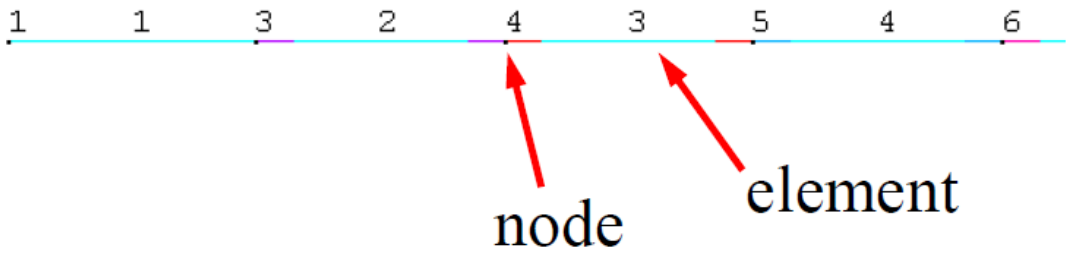
建立網格(續)

GUI : Utility Menu > PlotCtrls > Numbering.



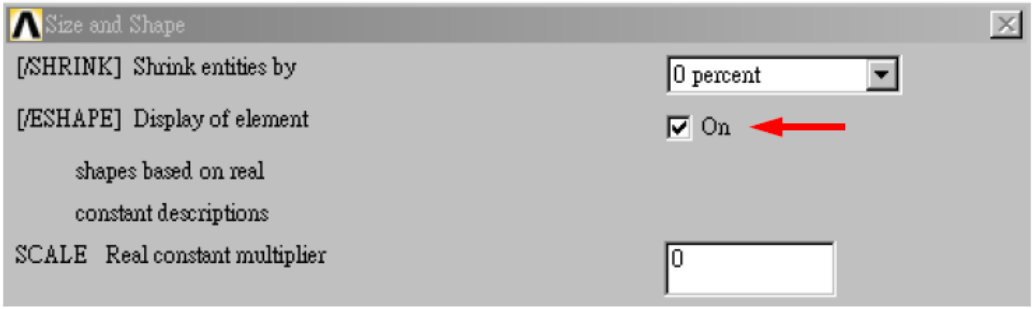
GUI : Utility Menu > Plot > Multi-Plots



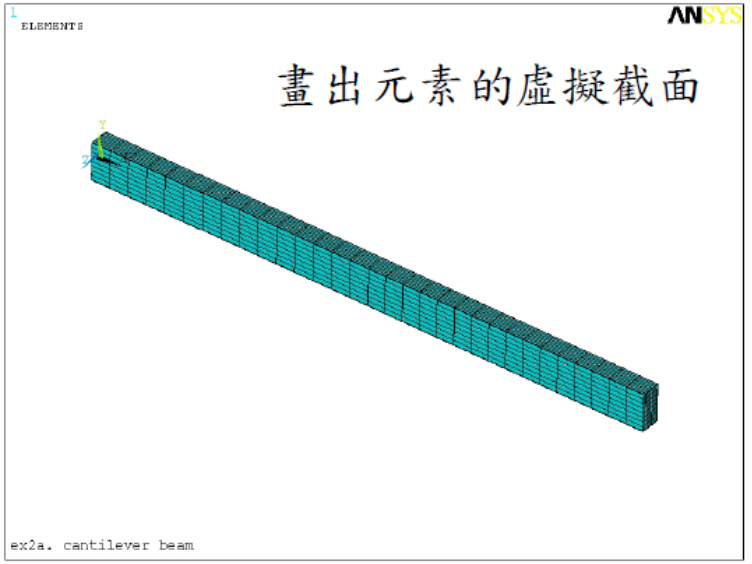


檢查元素截面

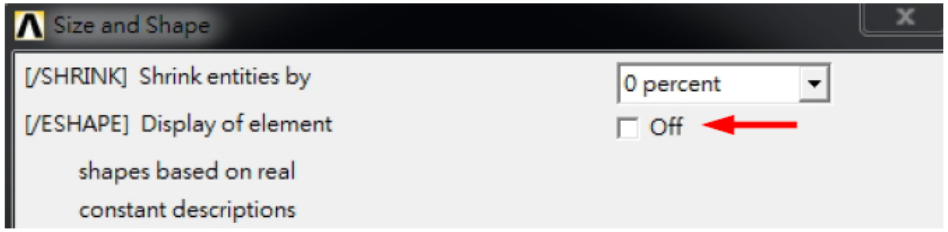
- GUI : Utility Menu > Plot Ctrls > Style > Size and Shape >



GUI : Utility Menu > Plot > Elements



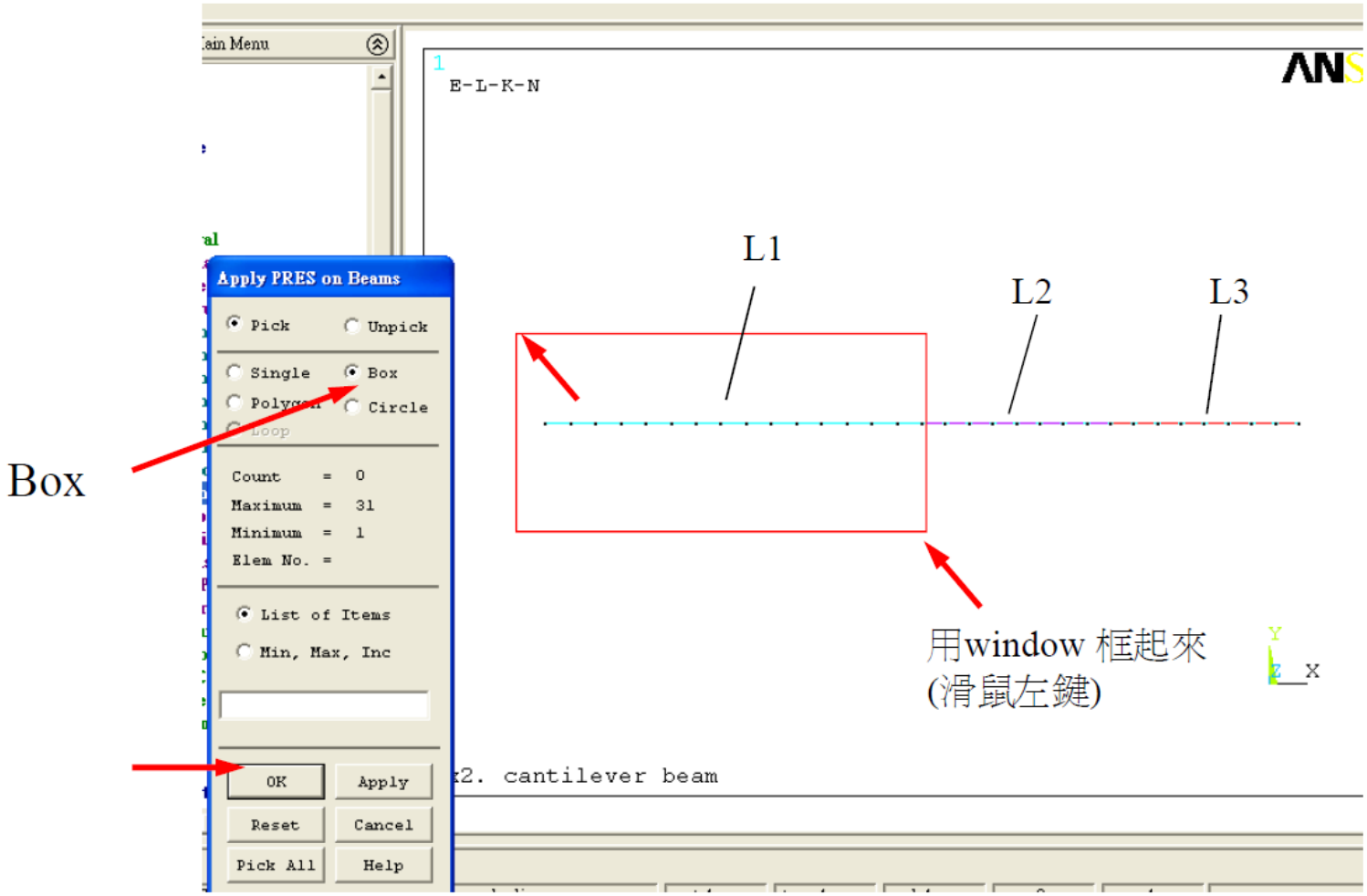
- GUI : Utility Menu > Plot Ctrls > Style > Size and Shape >



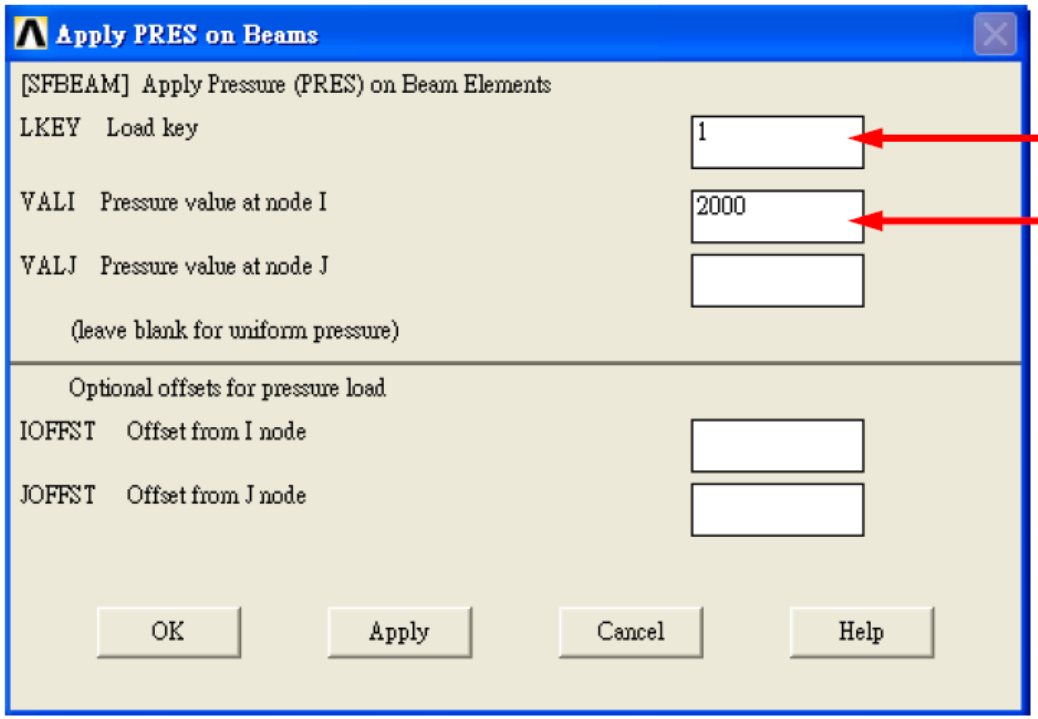
GUI : Utility Menu > Plot > Elements

施加分布力

- GUI : Main Menu > Solution > Analysis Type > New Analysis > 選 Static
- GUI : Main Menu > Solution > Define Loads > Apply > Structural > Pressure > On Beams >



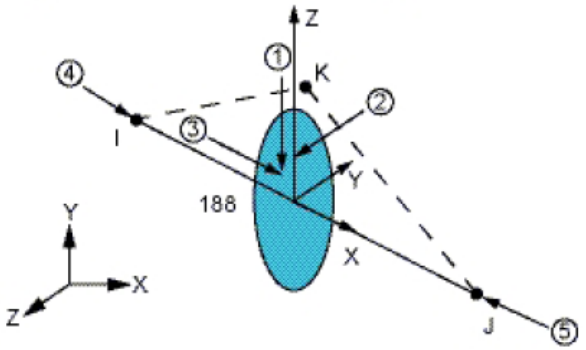
施加分布力(續)



Load key 1

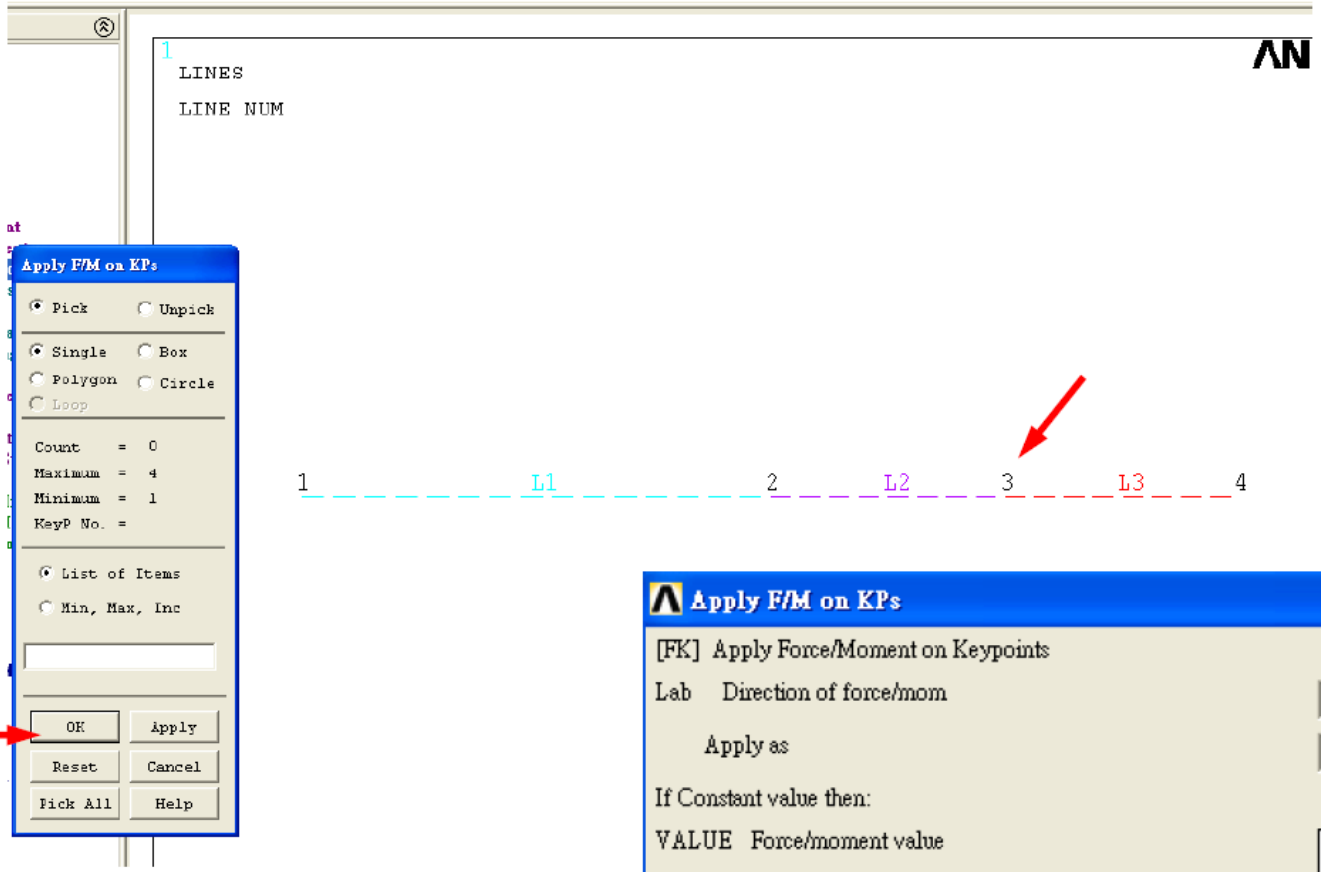
Pressure
(distributed load)
2000 N/m

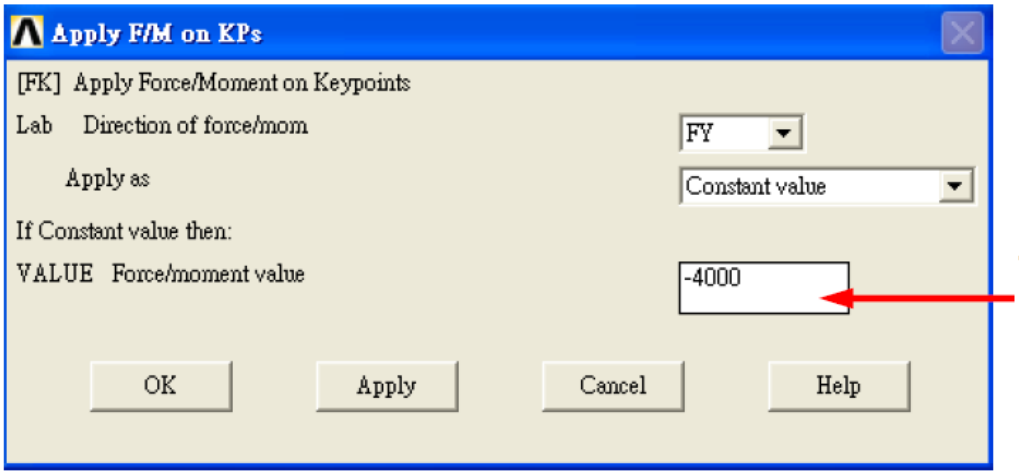
Figure 188.1 BEAM188 Geometry



施加集中力

- GUI : Main Menu > Solution > Define Loads > Apply > Structural > Force/Moment > On Keypoints >

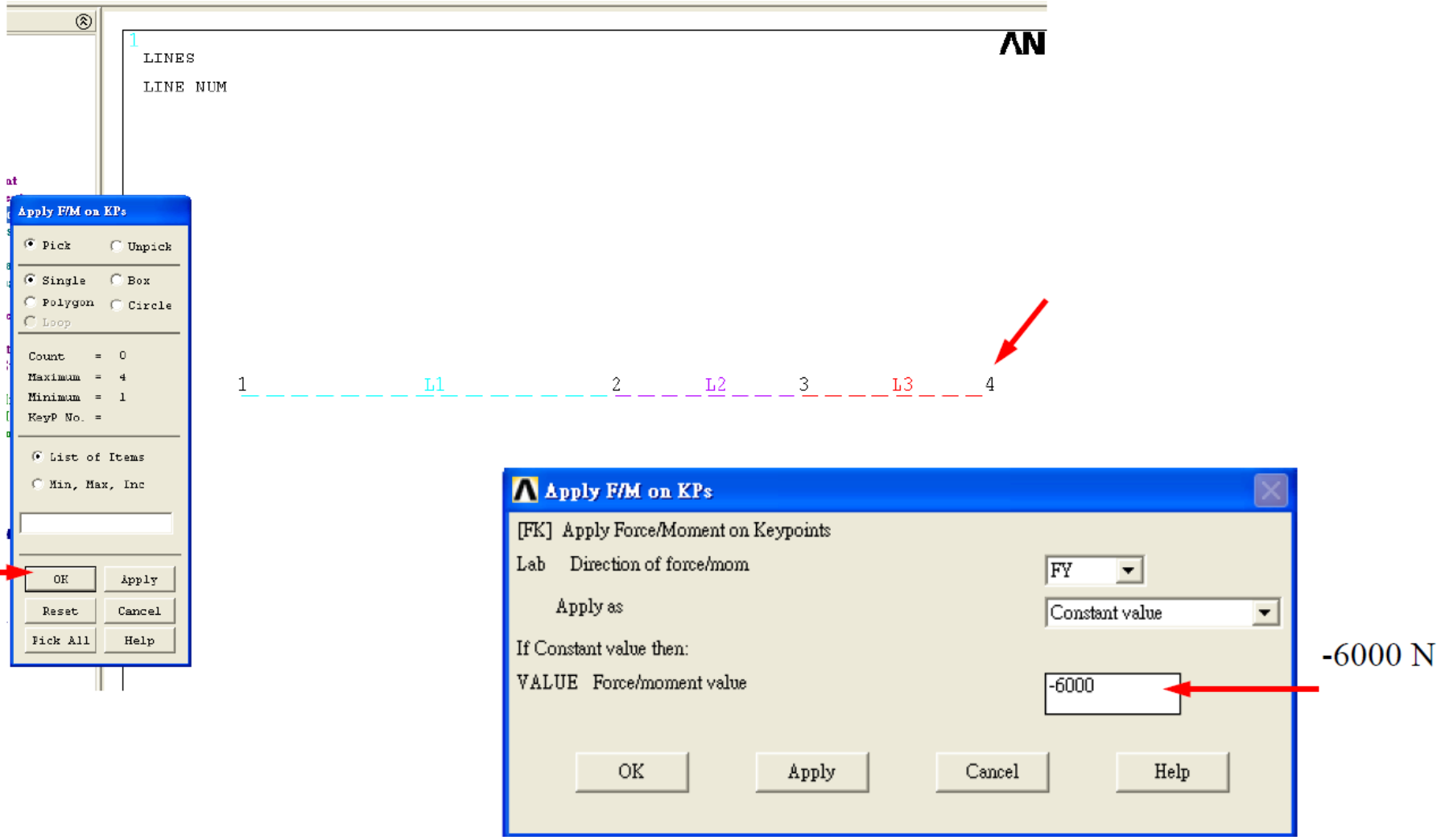




-4000 N

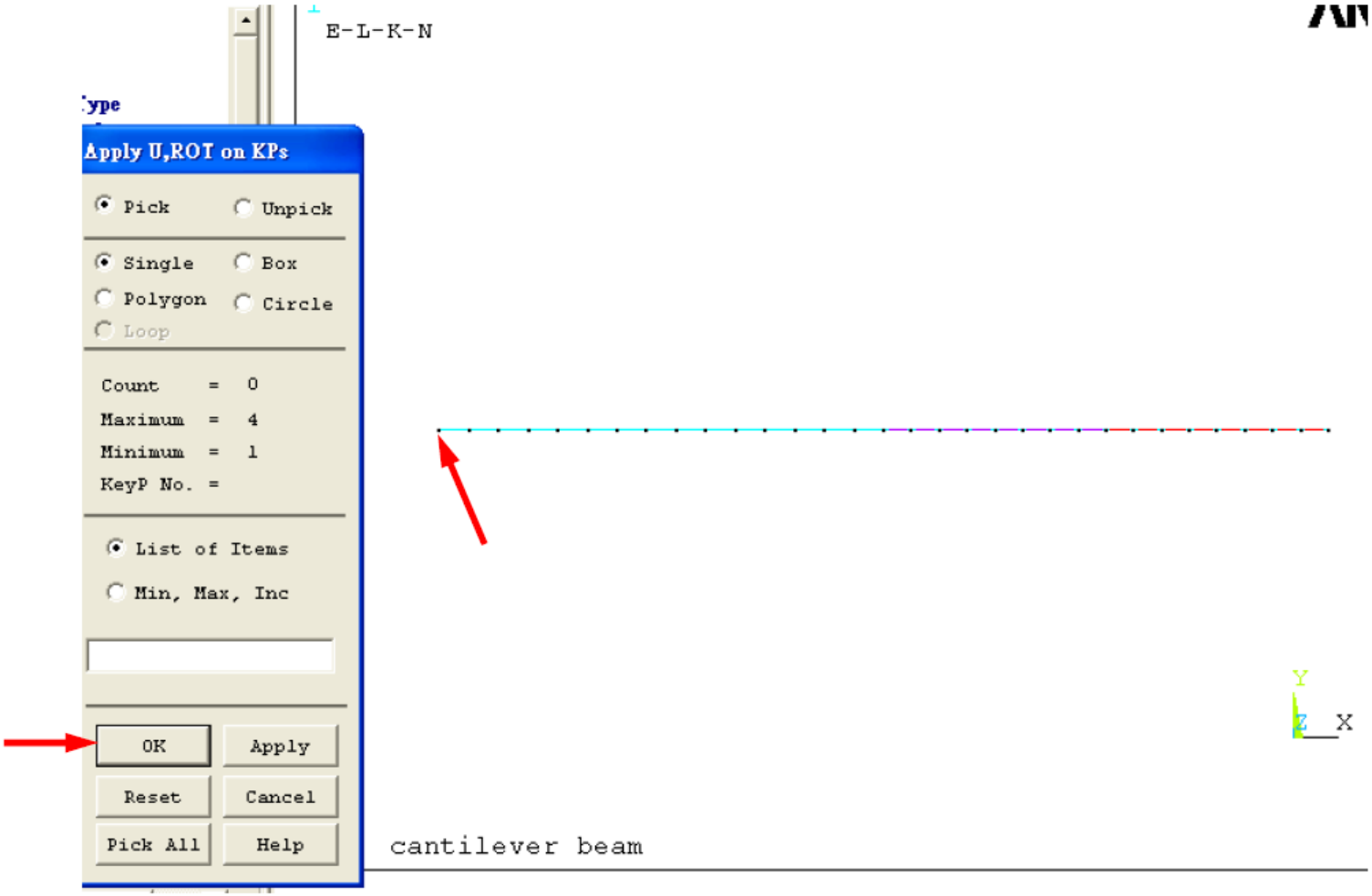
施加集中力(續)

- GUI : Main Menu > Solution > Define Loads > Apply > Structural > Force/Moment > On Keypoints >

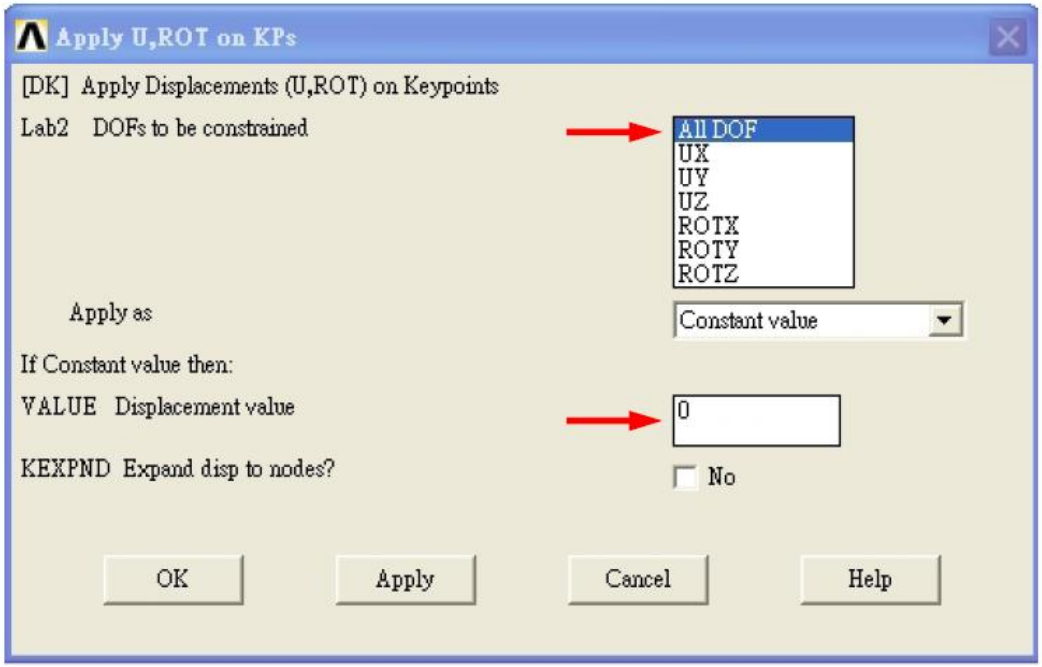


拘束條件

- GUI : Main Menu > Solution > Define Loads > Apply > Structural > Displacement > On Keypoints >



拘束條件(續)



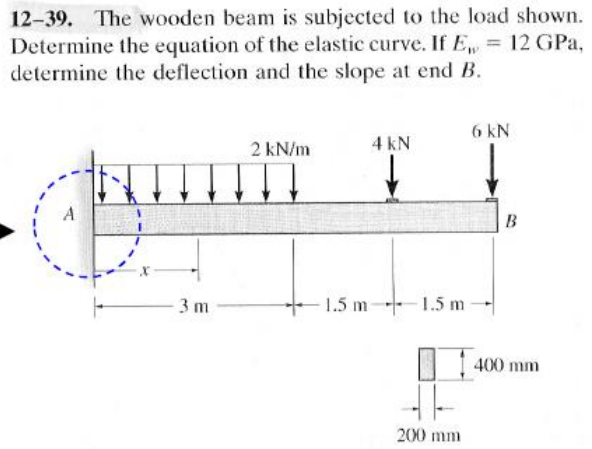
Fixed end

ALL DOFs = 0

that is,

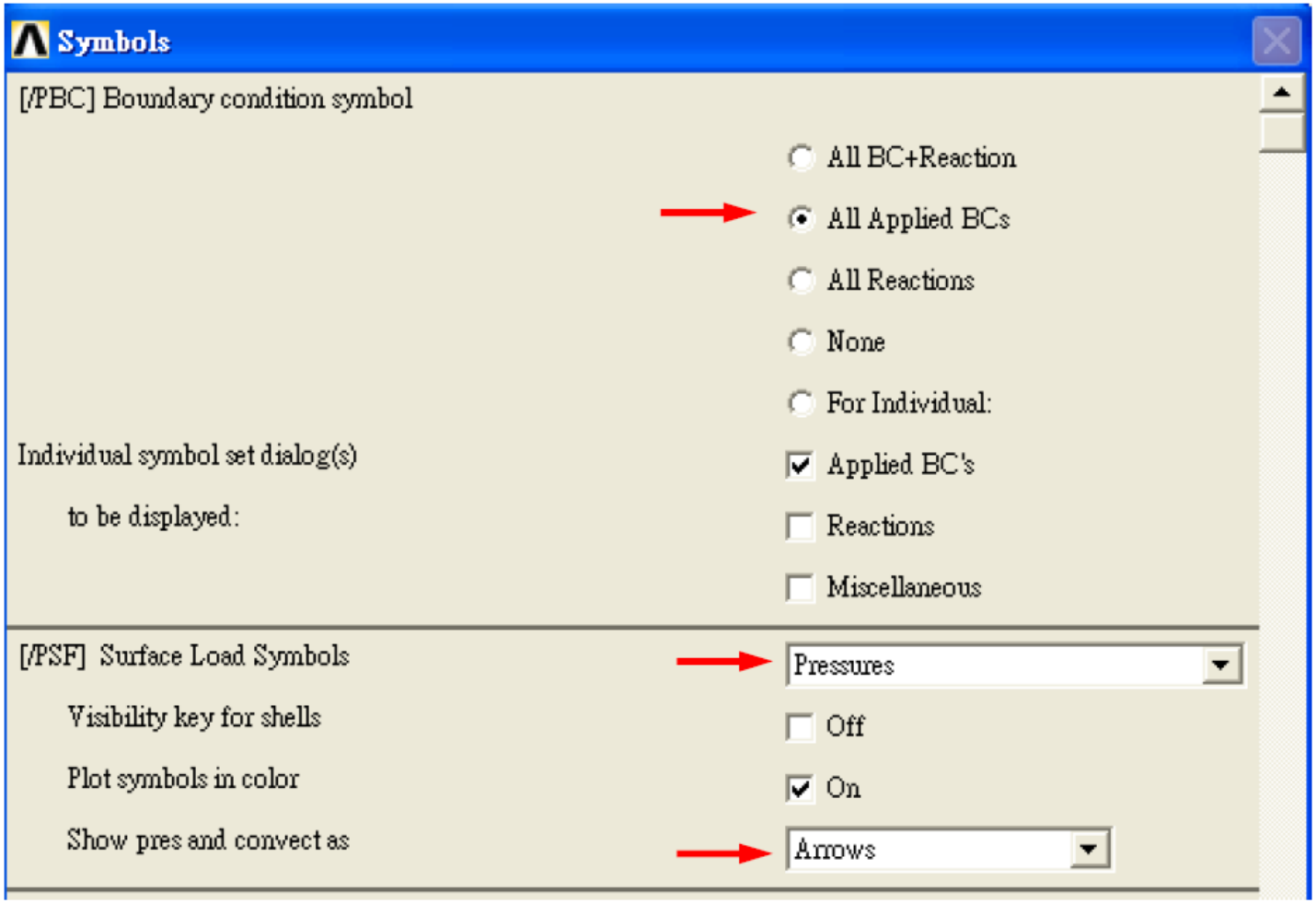
UX=UY=UZ=ROTX=

ROTY=ROTZ=0



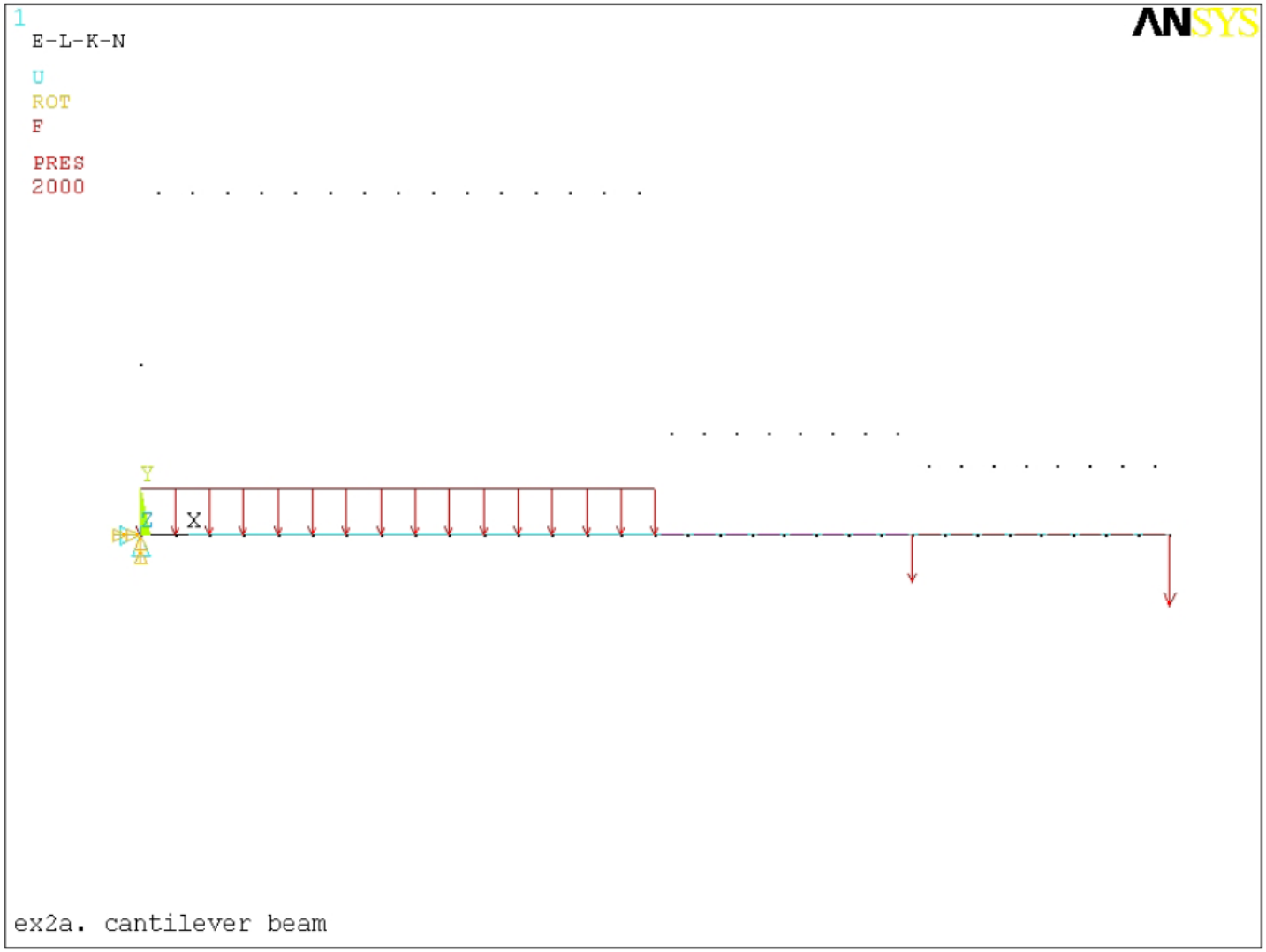
畫出邊界條件

- GUI : Utility Menu > Plot Ctrls > Symbols >

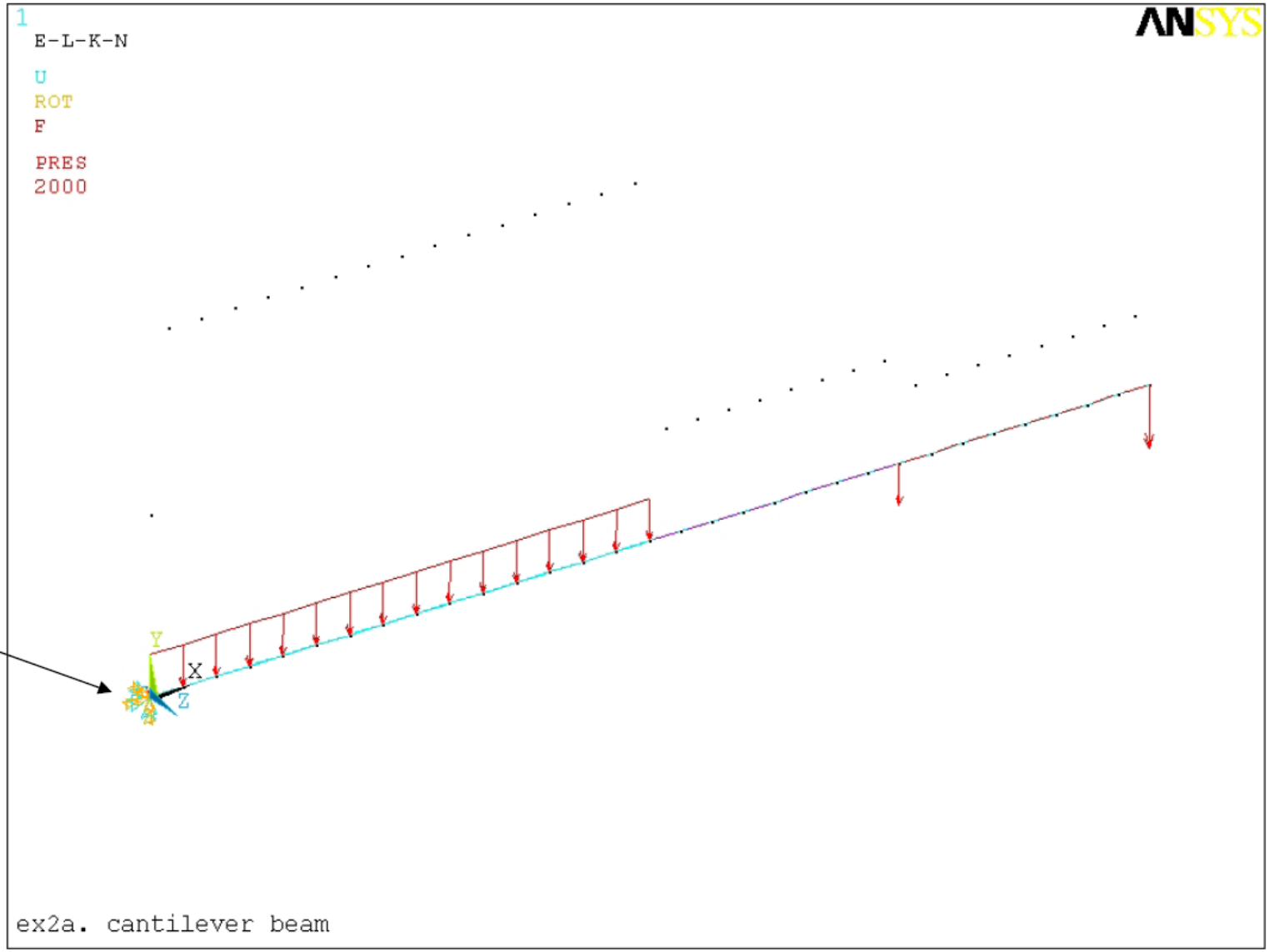


畫出邊界條件(續)

- GUI : Utility Menu > Plot > Multi-Plots



畫出邊界條件(續)



Symbols

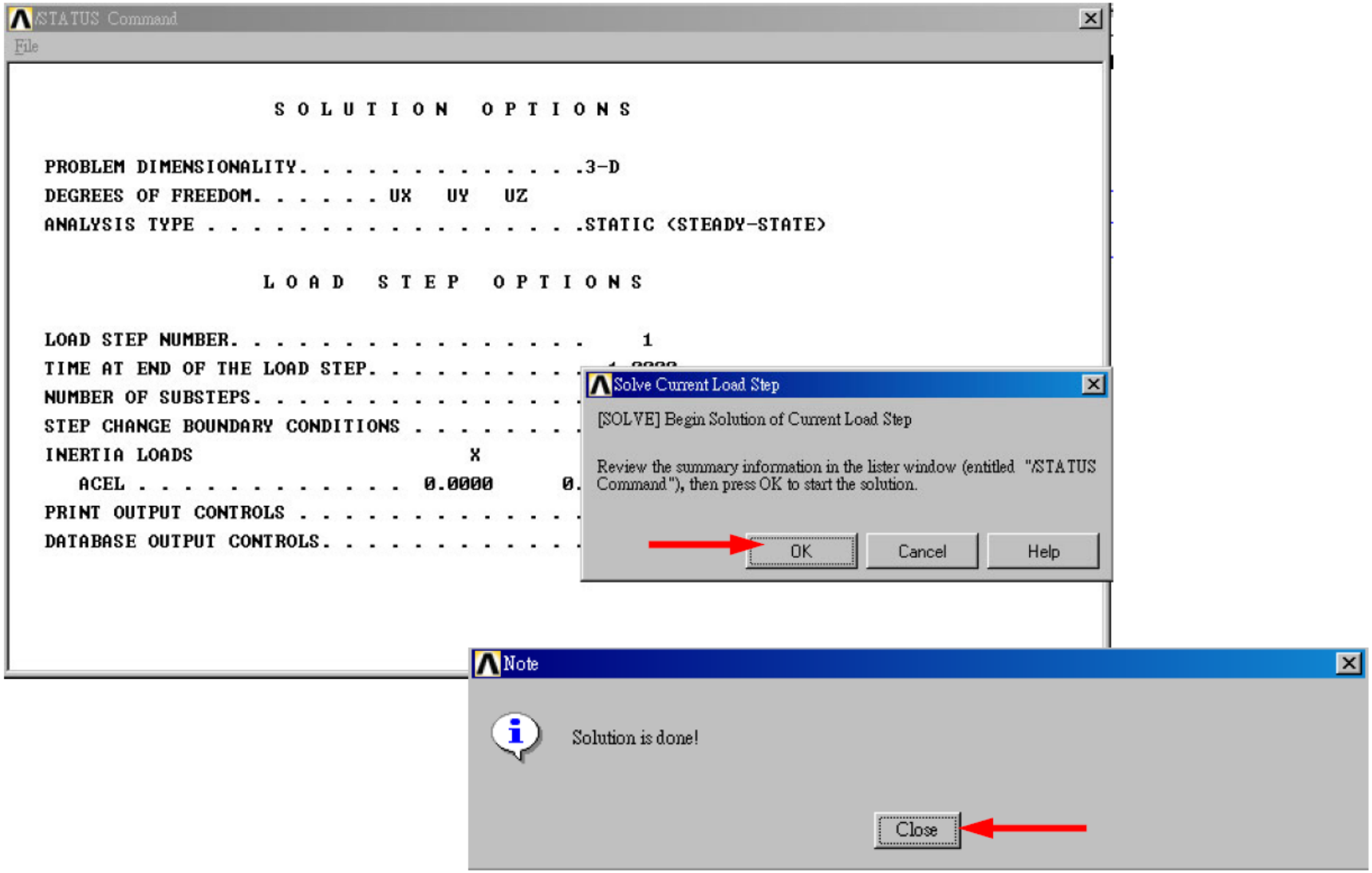
Fixed end

存檔

- GUI: ANSYS Toolbar > SAVE_DB

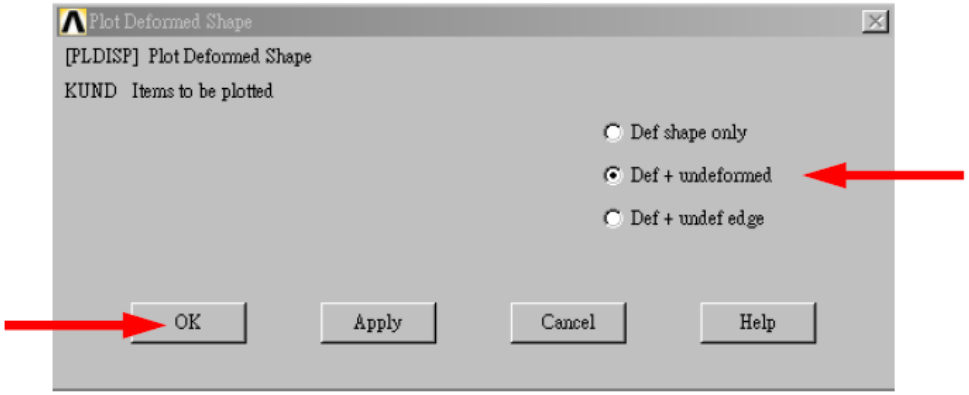
求解

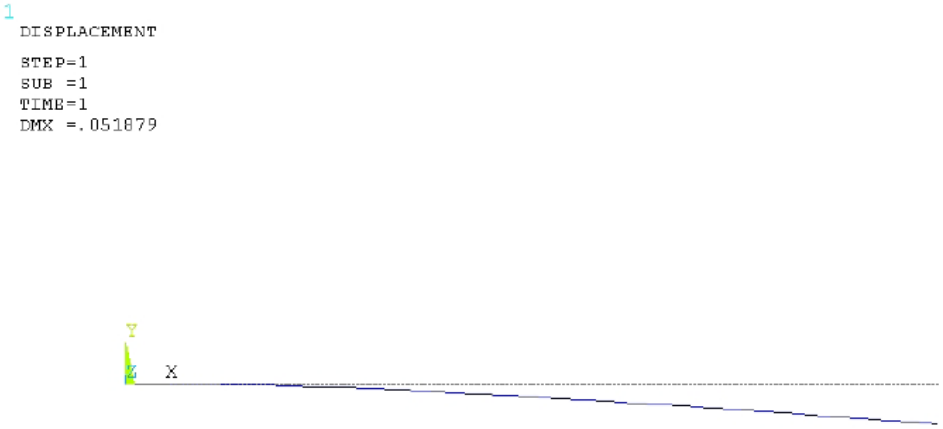
- GUI: Main Menu > Solution > Solve > Current LS >



觀察結果: 變形

- GUI: Main Menu > General Postproc > Plot Results > Deformed Shape >



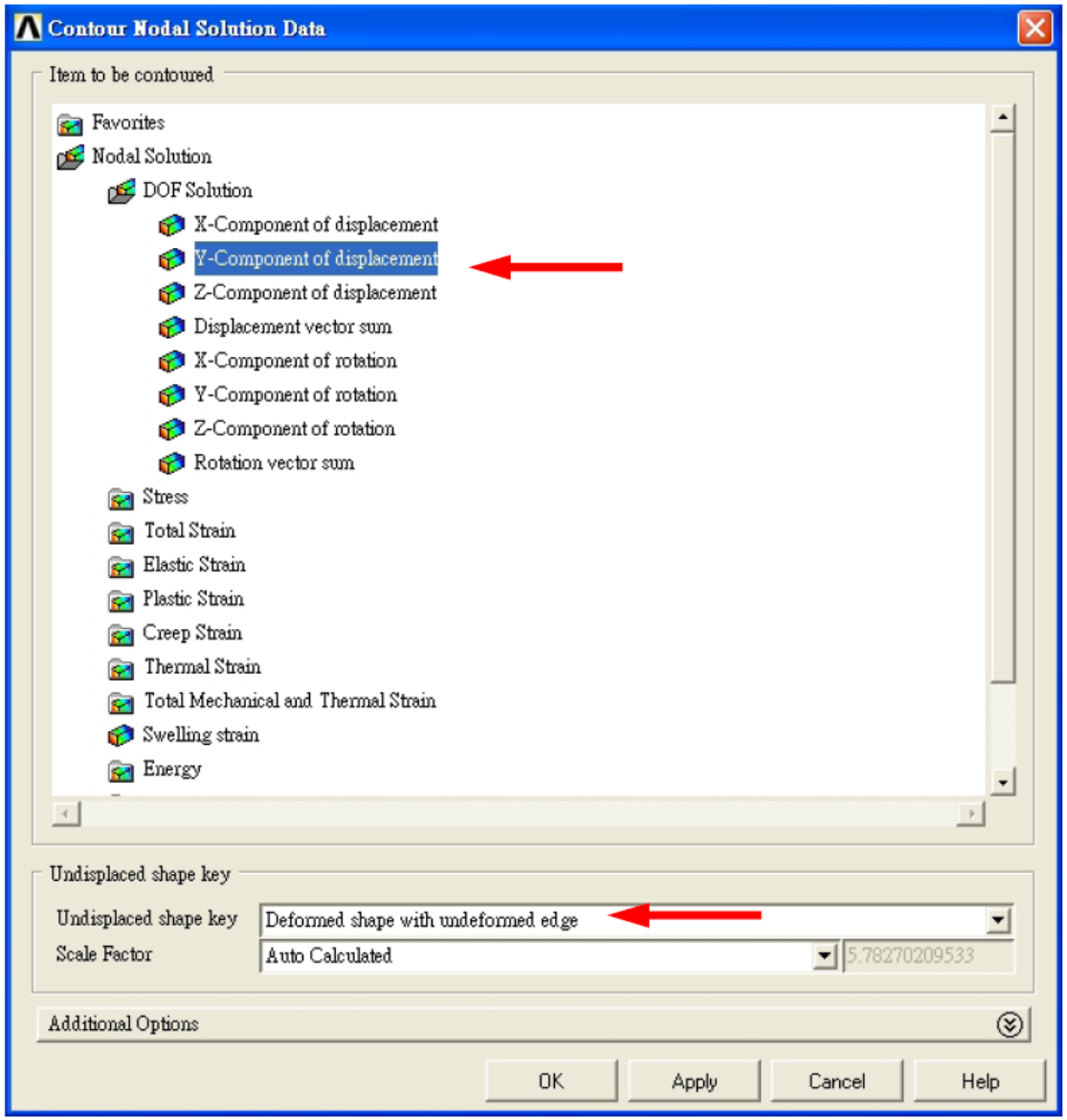


Max. displacement= 0.051879 m

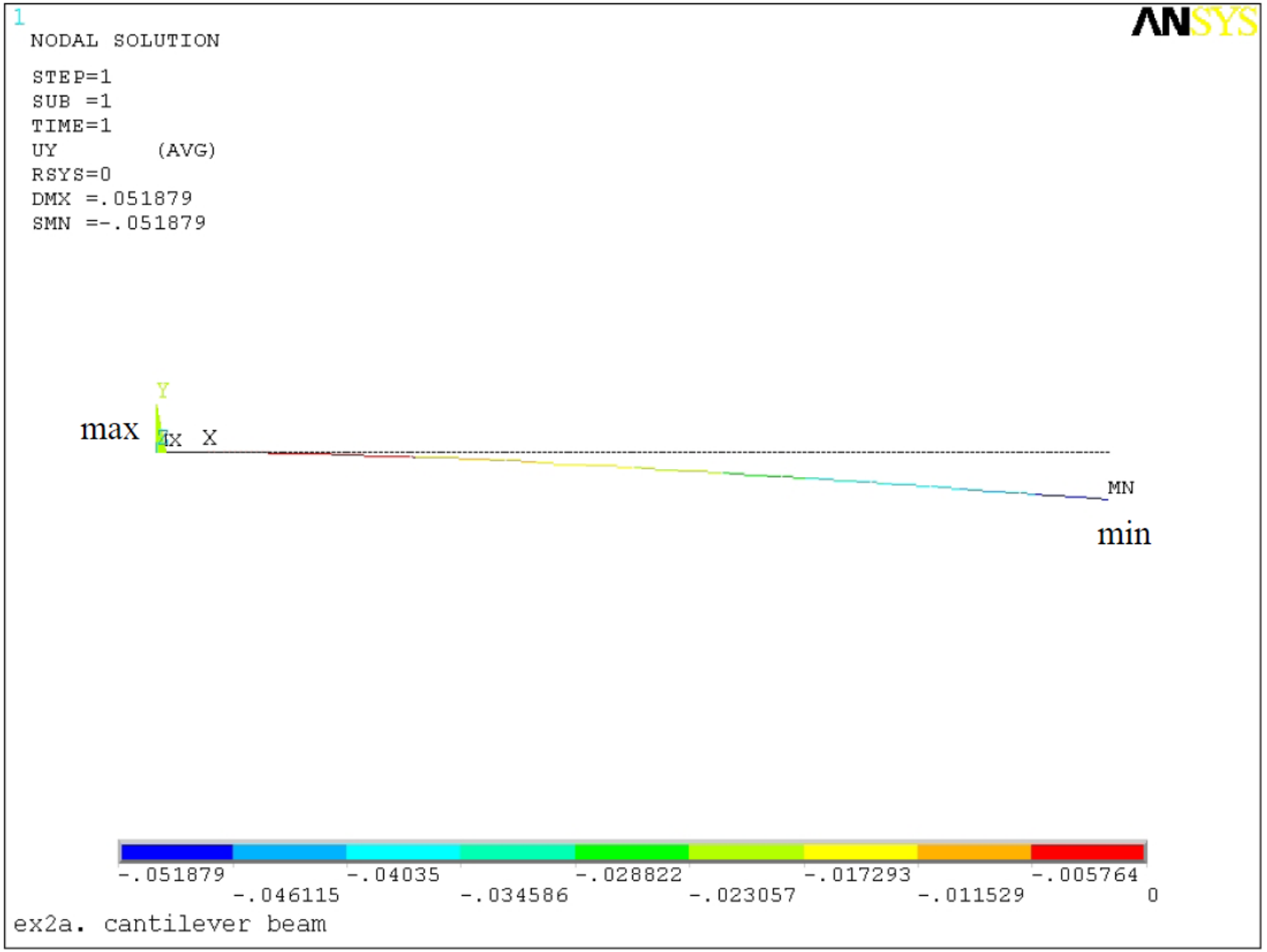
單位: m

觀察結果: UY

- GUI: Main Menu > General Postproc > Plot Results > Contour Plot > Nodal Solu >



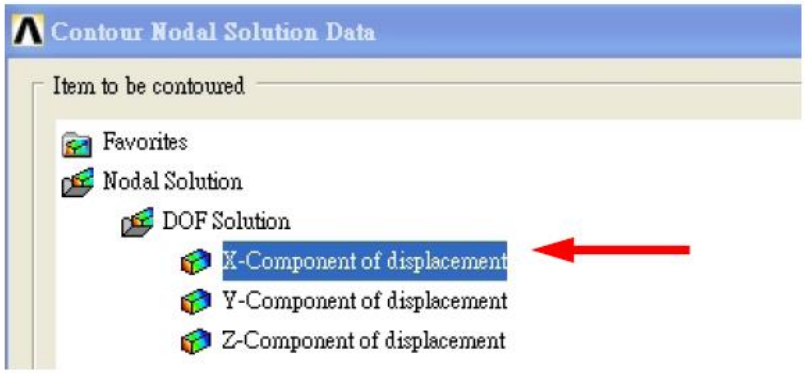
觀察結果: UY（續）


1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
DMX =.051879
SMN =-.051879
ANSYS
Y
max
1X X
MN
min
-.051879
-.046115
-.04035
-.034586
-.028822
-.023057
-.017293
-.011529
-.005764
0
ex2a. cantilever beam

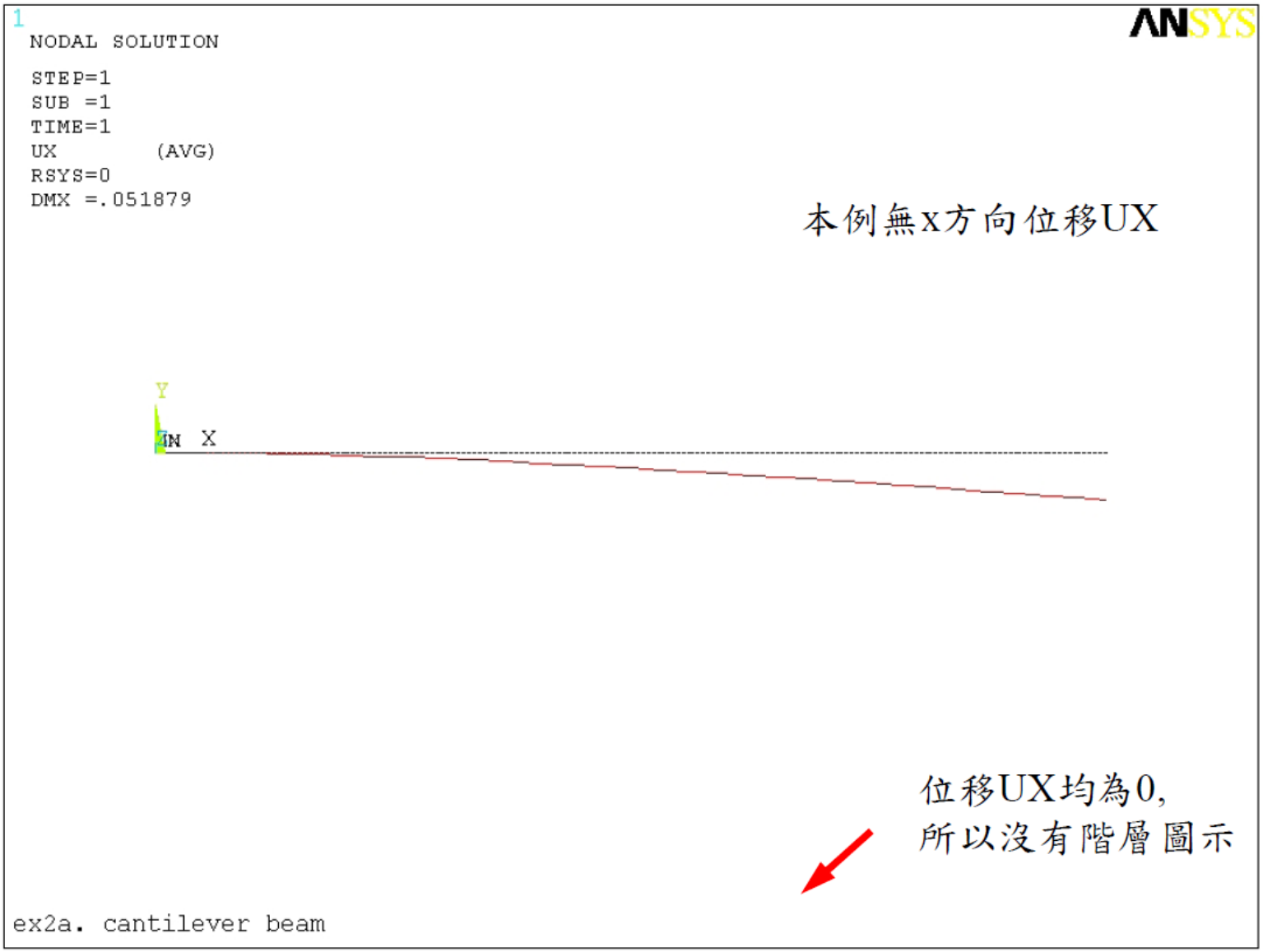
單位: m

觀察結果：UX

- GUI: Main Menu > General Postproc > Plot Results > Contour Plot > Nodal Solu >

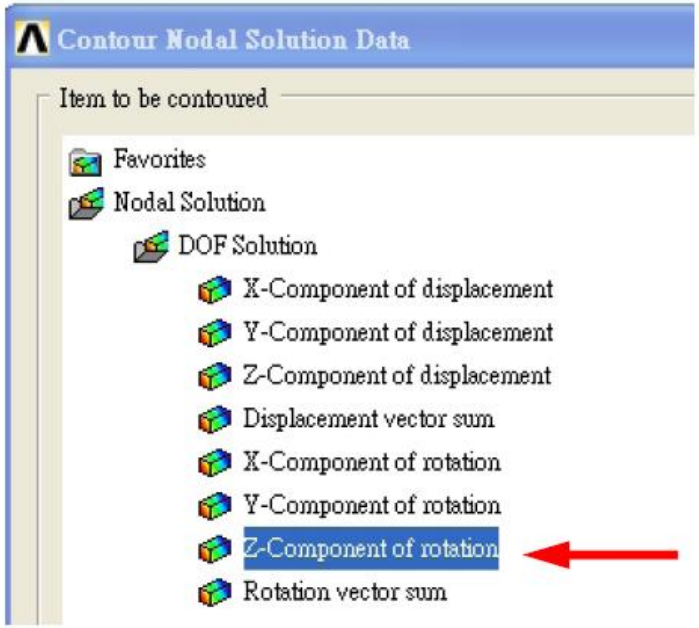


觀察結果: UX（續）

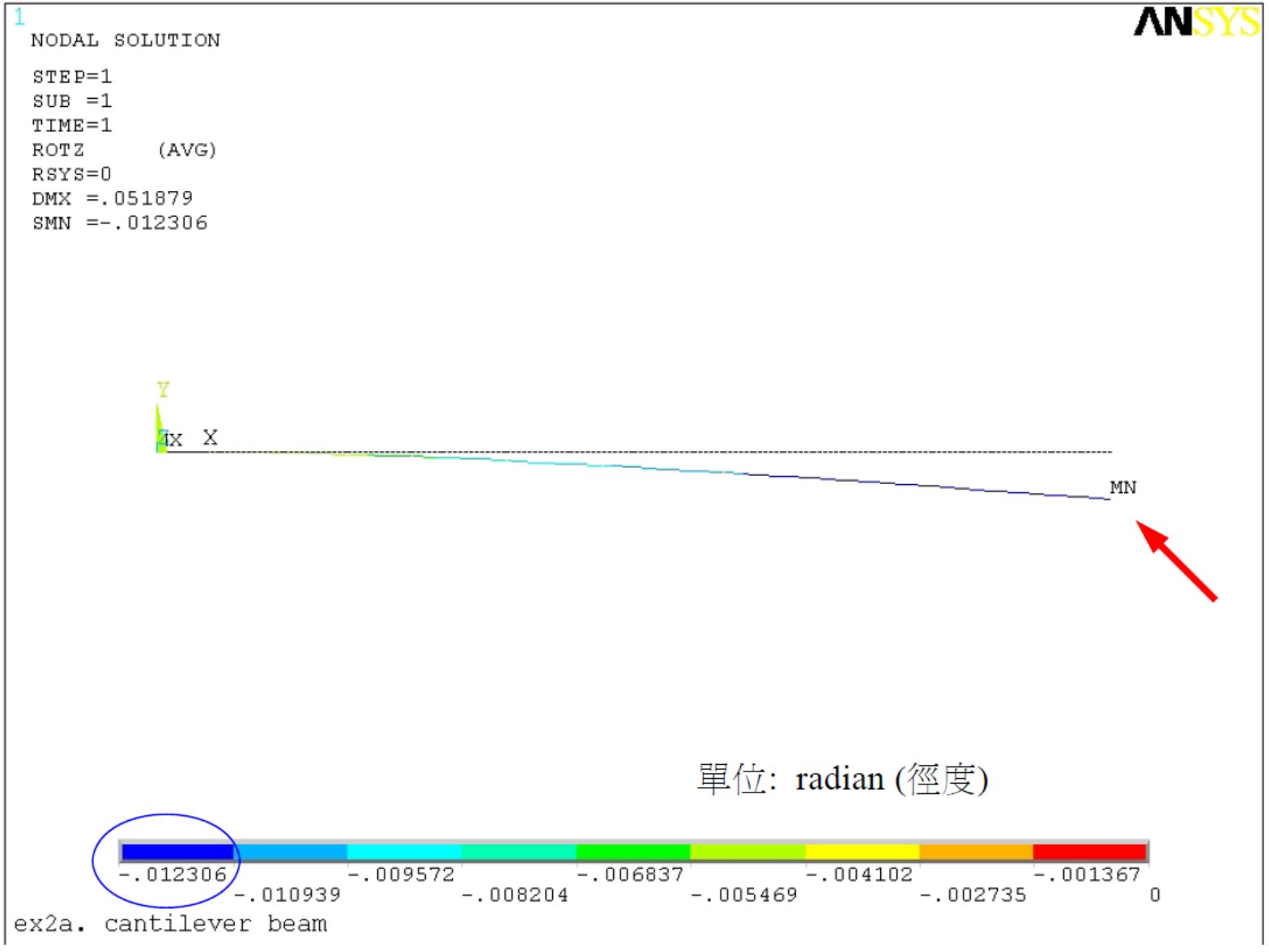

1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
DMX =.051879
ANSYS
本例無x方向位移UX
Y
MN X
位移UX均為0,
所以沒有階層圖示
ex2a. cantilever beam

觀察結果：ROTZ

- GUI: Main Menu > General Postproc > Plot Results > Contour Plot > Nodal Solu >



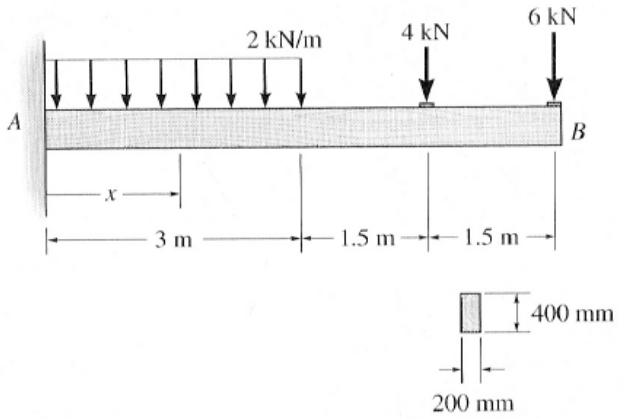
觀察結果：ROTZ（續）


1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
ROTZ (AVG)
RSYS=0
DMX =.051879
SMN =-.012306
ANSYS
Y
Z X
MN
單位: radian (徑度)
-.012306
-.010939
-.009572
-.008204
-.006837
-.005469
-.004102
-.002735
-.001367
0
ex2a. cantilever beam

UY和ROTZ 答案比較

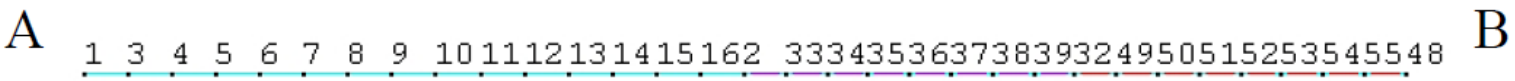
Original model:

12–39. The wooden beam is subjected to the load shown. Determine the equation of the elastic curve. If $E_w = 12$ GPa, determine the deflection and the slope at end B.



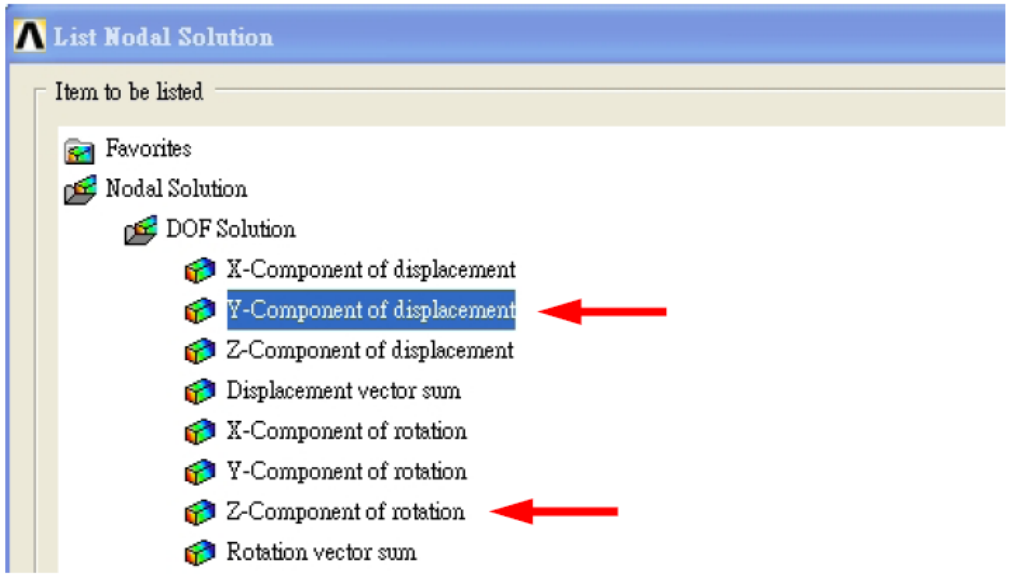
ANS: $\theta_B = -0.705°, v_B = -51.7$ mm

FE model:

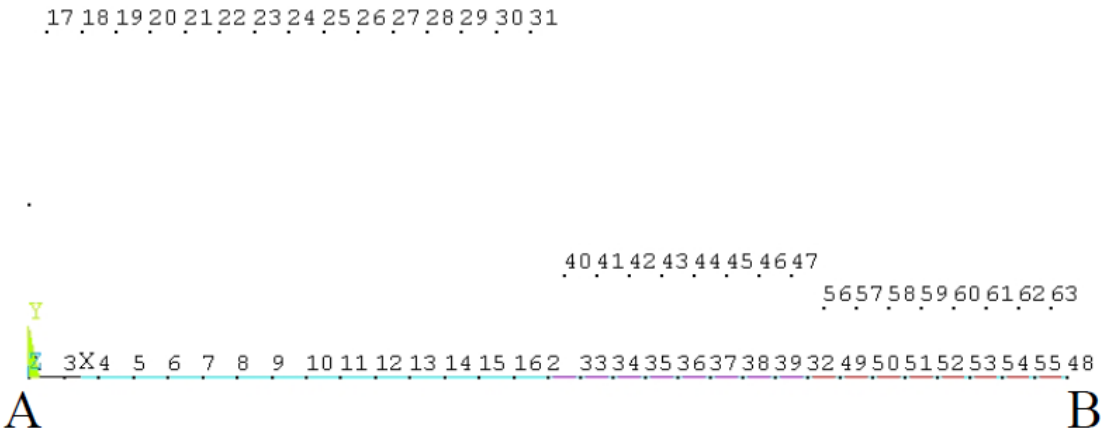


UY和ROTZ 答案比較(續)

- GUI: Main Menu > General Postproc > List Results > Nodal Solution

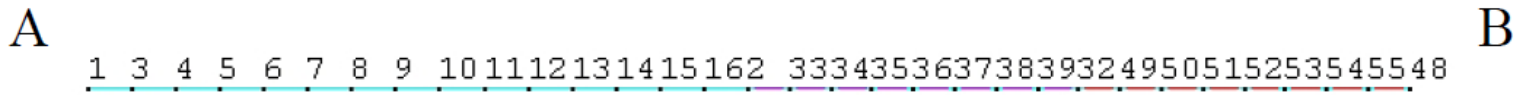


NODE	UY	NODE	ROTZ
1	0.0000	1	0.0000
2	-0.17172E-01	2	-0.98453E-02
3	-0.10640E-03	3	-0.95969E-03
4	-0.39960E-03	4	-0.18706E-02
5	-0.86997E-03	5	-0.27341E-02
37	-0.27147E-01	37	-0.11260E-01
38	-0.29283E-01	38	-0.11460E-01
39	-0.31454E-01	39	-0.11633E-01
48	-0.51879E-01	48	-0.12306E-01



Node 48
UY= -0.51879E-01 (m)
ROTZ= -0.12306E-01 (radian)

UY和ROTZ 答案比較(續)

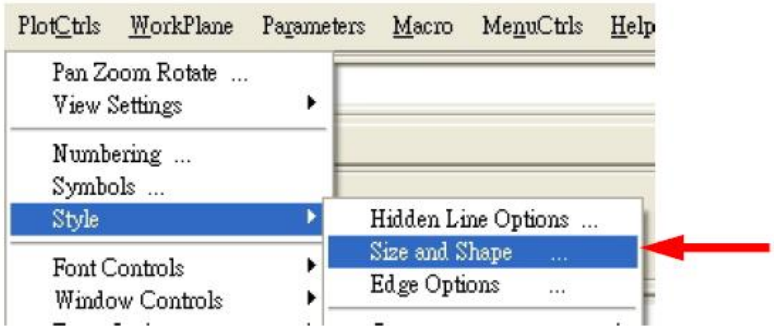


ROTZ= -0.12306E-01 radians = -0.012306*180/3.14 = -0.7054 degrees

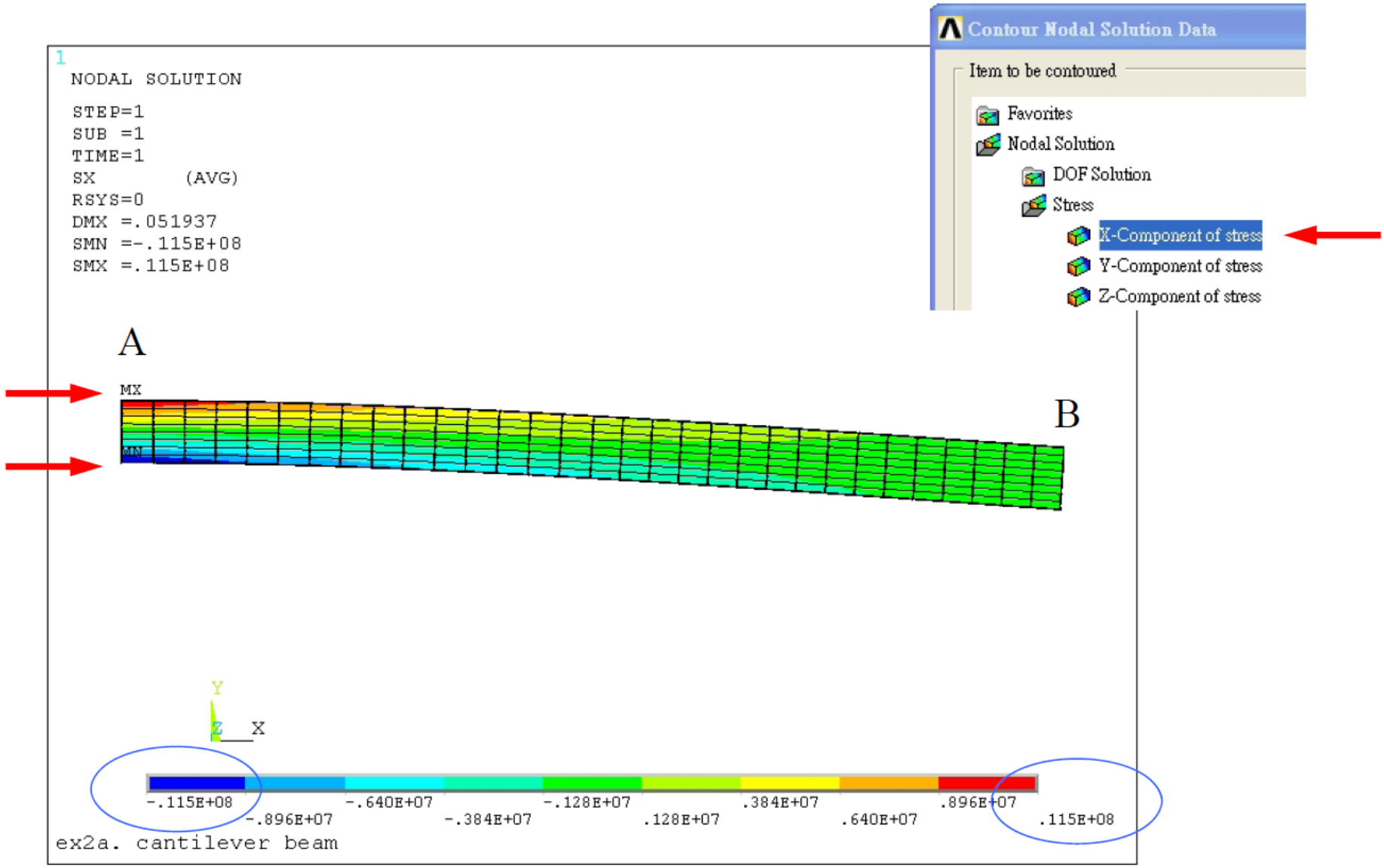
	UY (m)	ROTZ (degree)
ANSYS	-0.051879	-0.7054
Theory	-0.0517	-0.705
Error	0.35%	0.06%

誤差很小,ANSYS結果正確

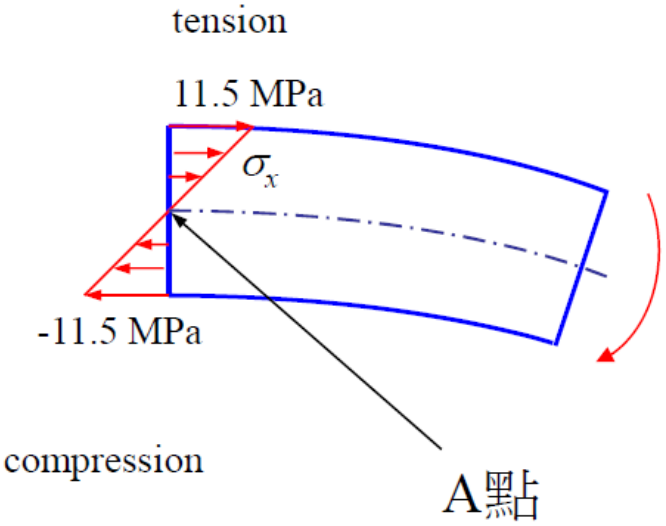
結果：彎曲應力σ_x

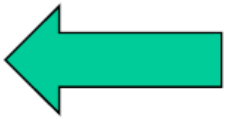


結果：彎曲應力σ_x

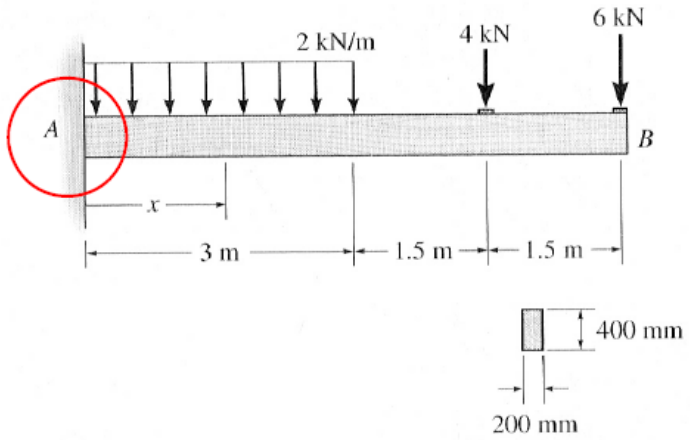


結果：彎曲應力σ_x





12–39. The wooden beam is subjected to the load shown. Determine the equation of the elastic curve. If $E_w = 12$ GPa, determine the deflection and the slope at end B.



結果：彎曲應力σ_x

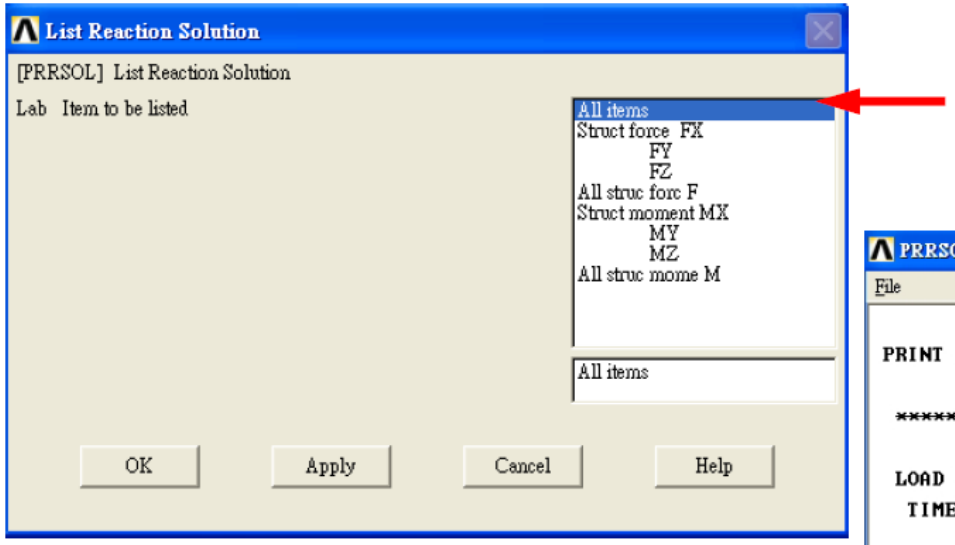
$$\sigma_b = \frac{Mc}{I} \qquad (5\text{-}1)$$

樑A點之彎矩M爲 63000 N-m，慣性矩I爲1.0667×10^{-3} m^4，c即爲0.2m和-0.2m，以上數值代入(5-1)式可得 $\sigma_b = 11.812\times10^6$ Pa和-11.812×10^6 Pa，可見ANSYS之結果與理論解吻合。

11.5 MPa vs. 11.812 MPa
2.64 % error

列出結果: 反作用力

- GUI: Main Menu > General Postproc > List Results > Reaction Solu

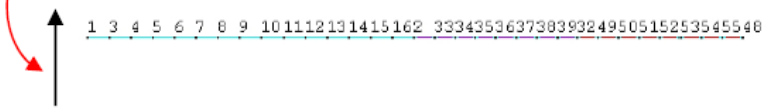


PRRSOL Command

File

```
PRINT REACTION SOLUTIONS PER NODE

 ***** POST1 TOTAL REACTION SOLUTION LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000      LOAD CASE=   0

 THE FOLLOWING X,Y,Z SOLUTIONS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE      FX          FY          FZ          MX          MY          MZ
      1   0.0000      16000.    -0.52832E-15-0.84148E-05 0.35881E-12  63000.

TOTAL VALUES
VALUE     0.0000      16000.    -0.52832E-15-0.84148E-05 0.35881E-12  63000.
```

63000 N-m



16000 N

Refining mesh