RC RL RLC series RLC parallel

$\tau = RC = \frac{L}{R} = \frac{2L}{R} = 2RC$

Principles of Electrical Engineering 2

$w_{diss} = \frac{1}{2}LI^2(1-e^{-2t/\tau})$

Components

- resistor (Ω) $V = IR$, $P = IV = I^2R$
- inductor (Henry) $V_L(t) = L\frac{di_L(t)}{dt}$, $w_L = \frac{1}{2}LI_L^2$
- capacitor (Farad) $i_C(t) = C\frac{dv_C(t)}{dt}$, $w_C = \frac{1}{2}CV_C^2$

→ Series

$\Sigma R = R_1 + R_2 + \dots$

$\Sigma L = L_1 + L_2 + \dots$

$\Sigma C = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right)^{-1}$

→ Parallel

$\Sigma R = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right)^{-1}$

$\Sigma L = \left(\frac{1}{L_1} + \frac{1}{L_2} + \dots\right)^{-1}$

$\Sigma C = C_1 + C_2 + \dots$

RC Circuit $\tau = RC$ sec

→ Natural

$V_C(t) = V_0 e^{-t/\tau}$

$i_C(t) = -C\frac{dv(t)}{dt} = I_0 e^{-t/\tau}$

→ Step

$V_C(t) = E(1-e^{-t/\tau})$

$i_C(t) = \frac{E}{R}e^{-t/\tau}$ (use thevenin) ↳ I_0, R_{eq}

RL Circuit $\tau = \frac{L}{R}$

→ Natural

$V_L(t) = L\frac{di(t)}{dt}$

$i_L(t) = I_0 e^{-t/\tau}$

→ Step

$V_L(t) = Ee^{-t/\tau}$

$i_L(t) = \frac{E}{R}(1-e^{-t/\tau})$ ↳ I_0

General RC-RL

$x(t) = (x_0 - x_f)e^{\frac{-(t-t_0)}{\tau}} + x_f$

any vari t_0 = time switch start

Op Amps

→ differentiator $i_n = i_p = 0A$ $-V_{cc} \le V_{out} \le V_{cc}$

$V_o = \frac{-1}{RC}\frac{dv(t)}{dt}$

→ integrator $i_1 = -i_2$ $i_1 = \frac{V_{in}}{R}$ $V_o = -RC\frac{dv(t)}{dt}$

→ Cascading

→ noninverting

$K = 1 + \frac{R_f}{R_s}$ $V_o = kV_{in}$

→ inverting

$K = -R_f/R_s$

$-V_C = \frac{-1}{RC}\int_0^t V_{in}(t)\,dt + V_o$

Cascading

$-V_{out2} = \frac{-1}{R_1R_2C_1C_2}\int_0^t\int_0^t V_{in}(t)\,dt + V_o$

$-V_{out} = -RC\frac{dv(t)}{dt}$

RCL Circuits

$i_R(t) = \frac{v(t)}{R}$

$i_C(t) = C\frac{dv(t)}{dt}$

$i_L(t) = \frac{1}{L}\int_0^t v(t)dt + I_L(0)$

– rec to use KCL / KVL

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$\alpha_{series} = R/2L$ in $\frac{Rad}{sec}$ $\frac{1}{\sqrt{LC}}$

$\alpha_{para} = \frac{1}{2RC}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

Overdamped ($\omega_0^2 < \alpha^2$) step

$x(t) = A_1e^{s_1t} + A_2e^{s_2t} + x_f$

Crit damped ($\omega_0^2 = \alpha^2$)

$x(t) = B_1e^{\alpha t} + B_2 t e^{\alpha t} + x_f$

↳ use derivative to find 2nd term @ t=0

Underdamped ($\omega_0^2 > \alpha^2$)

$x(t) = D_1e^{-\alpha t}\cos(\omega_n t) + D_2e^{-\alpha t}\sin(\omega_n t) + x_f$

V_{in} parallel, I_{in} series ↳ $\omega_n = \sqrt{\omega_0^2 - \alpha^2}$

① Parallel/series?
Natural/step?
② $\alpha^2 > \omega_0^2$?
③ use eqn id'd above
④ find unknowns using initial cond. & eqn ③

Partial Frac Decomp

① proper rational?
highest deg num < high denom
② Factor denom & num
③ Breakdown by factor

$\frac{(x)(y)}{(w)(z)(u)} = \frac{A}{w} + \frac{B}{z} + \frac{C}{u}$

④ Solve for consts

→ $\frac{xy}{zu} = A + \frac{wB}{z} + \frac{wC}{u}$

↳ make $w=0$, solve for A

→ repeat for others

⑤ translate $F(s) \to f(t)$

→ Improper
↳ Long division
↳ pf decomp remainder
↳ add const from div to pf decomp to get F(s) then → f(t)

→ Repeated roots

$\frac{xy}{w^2 z} = \frac{A}{z} + \frac{B}{w^2} + \frac{C}{w}$

↳ use derivative or system of eqns to solve

* if denom is complex conj → num is complex conj

La Place

→ in parallel

$V_C(s) = \frac{I_{dc}}{s}\left(\frac{1}{R} + \frac{1}{Ls} + Cs\right)^{-1}$

→ in series

$I_C(s) = \frac{V_0}{s}\left(R + Ls + \frac{1}{Cs}\right)^{-1}$

→ in general (series)
time → s domain

$V_s \to V_s/s$ $L \to Ls$

$R \to R$ $C \to 1/Cs + \frac{V_C(0^-)}{s}$

* $V(s) = I(s)R$ (volt-sec, Amp-sec, Ω)

t	s
$\delta(t)$	1
$u(t)$	$1/s$
t	$1/s^2$
e^{at}	$1/(s-a)$
$\sin\omega t$	$\omega/(s^2+\omega^2)$
$\cos\omega t$	$s/(s^2+\omega^2)$
te^{-at}	$1/(s+a)^2$
$e^{-\alpha t}\sin(\omega t)$	$\omega/[(s+a)^2+\omega^2]$
$e^{-\alpha t}\cos(\omega t)$	$(s+a)/[(s+a)^2+\omega^2]$
$Ke^{-\alpha t}$	$K/(s+a)$
$Kte^{-\alpha t}$	$K/(s+a)^2$
$\frac{K}{2}t^2e^{-\alpha t}$	$K/(s+a)^3$
$2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)$	$K/(s+\alpha-j\beta) + K^*/(s+\alpha+j\beta)$
$2t\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)$	$K/(s+\alpha-j\beta)^2 + K^*/(s+\alpha+j\beta)^2$

$K = \sqrt{a^2+b^2}$ $\theta = \tan^{-1}(\frac{b}{a})$ $a \pm jb$

$df(t)/dt$	$sF(s) - f(0^-)$
$d^nf(t)/dt^n$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} \dots$
$\int_0^t f(x)dx$	$F(s)/s$
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$e^{-at}f(t)$	$F(s+a)$
$f(at), a>0$	$\frac{1}{a}F(s/a)$
$tf(t)$	$-dF(s)/ds$
$t^nf(t)$	$(-1)^n d^nF(s)/ds^n$
$f(t)/t$	$\int_s^\infty F(u)du$

Volt divider (series)

$V_n = R_n V_{source}\frac{R_n}{\Sigma R}$

Current divider (para)

$I_n = I_{source}\frac{R_1 \| R_2 \| \dots}{R_n} = I_s\frac{1/R_n}{1/R_1 + 1/R_2 + \dots}$

Complete the Square

$as^2 + bs + c \to$

$as^2 + bs + c + (\frac{b}{2})^2 = (\frac{b}{2})^2$

$e^{-j\theta} + e^{j\theta} = 2\cos\theta$

$\cos A\cos B + \sin A\sin B = \cos(A-B)$

$\frac{dv(0^+)}{dt} = s_1A_1 + s_2A_2 = \frac{1}{C}(\frac{-V_0}{R} - I_0)$

$\frac{di(0^+)}{dt} = s_1A_1 + s_2A_2 = \frac{1}{L}(-RI_0 - V_0)$

$\frac{dv(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{C}(\frac{-V_0}{R} - I_0)$

$\frac{di(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{L}(-RI_0 - V_0)$

$\frac{dv(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{C}(\frac{-V_0}{R} - I_0)$

$\frac{di(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{L}(-RI_0 - V_0)$

Initial & Final Values

→ Initial Theorem

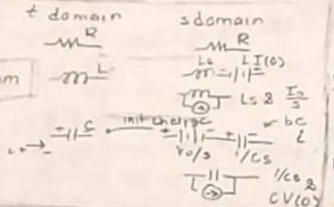
$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$

→ Final Theorem

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Conversions

t domain   s domain

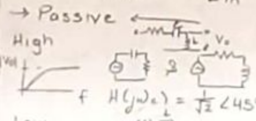


Filters

$H(s) = \frac{V_{out}}{V_{in}} = \frac{I_{out}}{I_{in}}$

→ Passive

High



$H(j\omega_c) = \frac{1}{\sqrt{2}}\angle 45°$

Low

$H(j\omega_c) = \frac{1}{\sqrt{2}}\angle -45°$ -3dB cutoff

$\omega_c = \frac{1}{RC} = \frac{R}{L}$ rad/s $f_c = \frac{\omega_c}{2\pi}$ Hz

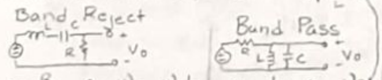
$a + bj = M\angle\theta$ $M = \sqrt{a^2+b^2}$ $\theta = \tan^{-1}(\frac{b}{a})$

$\cdot H(j\omega)_{max}$ → minimize deonominator

$V_{steady\ state} = (\text{Amplitude}) \cdot H(j\omega) \cdot \cos(\omega t + \phi)$

· resonance when $\omega_c^2 = \frac{1}{LC}$ $= \sqrt{\frac{RC}{L}}$

Band Reject   Band Pass



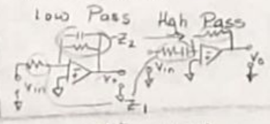
$\beta = \frac{R}{L}$ $Q = \frac{\omega_0}{\beta} = \frac{\omega_0 L}{R}$ $\beta = \frac{1}{RC}$ $Q = \frac{\omega_0}{\beta}$

$\omega_{c1,c2} = \omega_0\left[\frac{\pm 1}{2Q} + \sqrt{1 + \frac{1}{2Q}^2}\right]$ all in rad/s

→ Active

- require power source
- internal gain
- cascading ok & signal processing
- R & C (no L)

Low Pass   High Pass



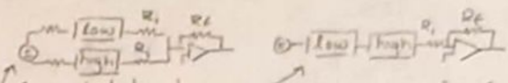
$H(s) = -\frac{Z_2}{Z_1}$

gain = $\pm\frac{R_2}{R_1}$

$\omega_{Lc} = \frac{1}{R_2C}$

$\omega_{Hc} = \frac{1}{R_1C}$

$K_{dB} = 20\log(\frac{R_2}{R_1})$



Cascaded bandpass $f_{c1} < f_{c2}$

f_{c2} pairs w/ lowpass & f_{c1} w/ highpass

Cascaded band reject

f_{c1} pairs w/ lowpass & f_{c2} w/ high pass

$R_f = (\text{amp})R_i$

For active low → $\frac{-k\beta s}{s^2 + \beta s + \omega_o^2} = H(s)$

$V_{ss} = (\text{Amplitude } V_i)(\text{gain})$

For active high → $\frac{-k\omega_c}{s + \omega_c} = H(s)$

Fourier Series

$T = 2L$

$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)]$

$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)dt$, DC term

→ cos wave $b_n = \phi$ → sin wave $a_n = 0$

$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(k\omega_0 t)dt$ = a_n but cos

2 Port Networks

Impedance Parameters (Ω)

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$ $Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$ $Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$

Admittance (S)

$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ $y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$ $y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ $y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

Transmission ($a \to \Omega$ $b \to S$)

$a_{11} = \frac{V_1}{V_2}\Big|_{I_2=0}$ $a_{12} = \frac{-V_1}{I_2}\Big|_{V_2=0}$ $a_{21} = \frac{I_1}{V_2}\Big|_{I_2=0}$ $a_{22} = \frac{-I_1}{I_2}\Big|_{V_2=0}$

$b_{11} = \frac{V_2}{V_1}\Big|_{I_1=0}$ $b_{12} = \frac{-V_2}{I_1}\Big|_{V_1=0}$ $b_{21} = \frac{I_2}{V_1}\Big|_{I_1=0}$ $b_{22} = -\frac{I_2}{I_1}\Big|_{V_1=0}$

Hybrid ($h \to \Omega$ $g \to S$)

$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ $h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$ $h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ $h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$

$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}$ $g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0}$ $g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$ $g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0}$

For $I_{1/2} = 0$, make open

For $V_{1/2} = 0$, make short

3 Phase

- balanced → 120° apart
- Y system Y

resistor Ω, induct $j\Omega$, cond $-j\Omega$

$|V_{L-L}| = \sqrt{3}\,V_\phi$

$|I_L| = |I_\phi|$

$I_\phi = \frac{V_{L-N}}{Z_\phi}$

120° apart & 180° shift

- Delta system

$|V_{L-L}| = |V_\phi|$ & $|I_L| = \sqrt{3}\,I_\phi$

Integration by parts

$\int u\,dv = uv - \int v\,du$

$u \to$ Log, Inverse, Alg, Trig, Exp

Fourier Cont

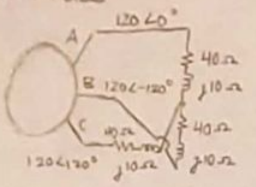
$T = 2L$ T = period

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n\cos(n\pi\frac{x}{L}) + b_n\sin(n\pi\frac{x}{L})]$

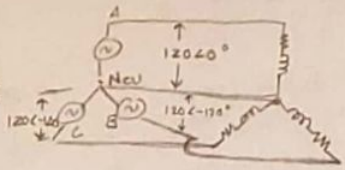
$a_n = \frac{2}{L}\int_0^{T/2} f(x)\cos(n\pi\frac{x}{L})$

take from graph ($y = mx + b$)

3 Phase



In Y $I_L = I_\phi = \frac{V_\phi}{Z_\phi}$ OR



$I_A = \frac{V_{A\text{-}N}}{Z_\phi} = \frac{120\angle 0°}{40+j10} = \frac{120\angle 0°}{41.23\angle 14.04} = 2.91\angle -14.01° A$

$I_B = \frac{V_{B\text{-}N}}{Z_\phi} = \frac{120\angle -120}{40+j10} = 2.91\angle -134.4° A$

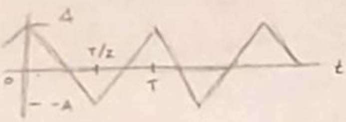
$I_C = \frac{V_{C\text{-}N}}{Z_\phi} = \frac{120\angle 120}{40+j10} = 2.91\angle 105.96° A$

$|V_{L\text{-}L}| = \sqrt{3}\,|V_\phi| \rightarrow V_{L\text{-}L} = \sqrt{3}(120) = 207.85V$

Fourier Series

$f(t) = A - \frac{4A}{T}t \quad 0 \le t \le T/2$

$f(t) = A + \frac{4A}{T}t \quad -T/2 \le t \le 0$



$y = mx + b$

even symmetry, cos $\rightarrow b_n = 0$

$a_0 = 0$ b/c + & - areas equal $(A \leftrightarrow -A)$

halfway symmetry $\rightarrow f(t) = -f(t - \frac{T}{2})$ so $a_n = 0$ for even n

$a_n = \frac{2A}{T}\int_0^{T/2}(1 - \frac{4A}{T}t)\cos(2\pi n f_0 t)dt + \frac{2A}{T}\int_{-T/2}^{0}(1 + \frac{4A}{T}t)\cos(2\pi n f_0 t)dt$

distribute & integrate

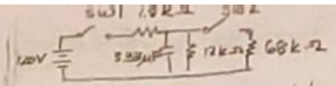
$a_n = \frac{-8A}{T^2}\int_0^{T/2} t\cos(2\pi n f_0 t)\,dt + \int_{-T/2}^{0} t\cos(2\pi n f_0 t)dt$

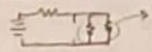
$\int x\cos(Ax)dx = \frac{1}{A^2}\cos(Ax) + \frac{x}{A}\sin(Ax)$

$a_n = \frac{-8A}{T^2}\left[\frac{1}{(2\pi n f_0)^2}\cos(2\pi n t f_0) + \frac{t}{2\pi n f_0}\sin(2\pi n f_0 t)\right] + \frac{8A}{T^2}[\ldots]$

$a_n = \frac{8A}{(n\pi)^2} \quad f(t) = \cancel{a_0}^0 + a_1\cos\omega t + \cancel{a_2\cos 2\omega t}^0 + a_3\cos 3\omega t + \ldots$

$f(t) = \frac{8A}{(1\pi)^2}\cos\omega t + \frac{8A}{(3\pi)^2}\cos 3\omega t + \ldots \quad n = \text{ODD}$



in ss $\rightarrow$  $\rightarrow (\frac{1}{12k} + \frac{1}{68k})^{-1} = 10.2k$

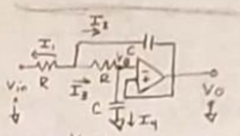
volt divider $\rightarrow 120\left(\frac{10.2k}{10.2k + 1.8k}\right) = 102V$ $V_C(0^+)$

switches open $\rightarrow$

$\tau = RC = (12k)(3.33\mu) = 40ms$

gen form $V_0 e^{-t/\tau} = 102e^{-(18ms)/(40ms)} = 65V$

@ 18ms



$I_3 = I_4$

$V_B = V_{out}$

$I_1 = \frac{V_A - V_{in}}{R} \quad I_2 = \frac{V_A - V_o}{1/Cs} \quad I_3 = \frac{V_A - V_o}{R} \quad I_4 = \frac{V_o}{1/Cs}$

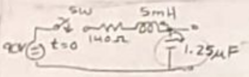
$I_3 = I_4 \rightarrow \frac{V_A - V_o}{R} = CsV_o \rightarrow V_A = V_o(RCs + 1)$

$I_1 + I_2 + I_3 = 0 \rightarrow \frac{V_A - V_{in}}{R} + \frac{V_A - V_o}{1/Cs} + \frac{V_A - V_o}{R} = 0$

Simplify $\rightarrow V_{in} = V_o[RCs + 1 + R^2C^2s^2 + RCs]$

$H(s) = \frac{V_o}{V_{in}} = \frac{1}{2RCs + 1 + R^2C^2s^2}$

Simplify $\rightarrow \frac{1/R^2C^2}{s^2 + \frac{2}{RC}s + \frac{1}{R^2C^2}} \longleftrightarrow H(j\omega) = \frac{1/R^2C^2}{(j\omega)^2 + \frac{2}{RC}j\omega + \frac{1}{R^2C^2}}$



initial charge on capacitor 150V

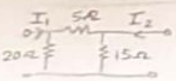
$\frac{V_o - \frac{90}{s}}{140 + 0.005s} + \frac{s(V_o - \frac{150}{s})}{8 \cdot 10^5} = 0$

simplify

$V_o = \frac{150(s^2 + 2800s + 96 \cdot 10^6)}{s(s + 8000)(s + 20000)} = \frac{A}{s} + \frac{B}{s + 8000} + \frac{C}{s + 20000}$

$V(s) = \frac{90}{s} + \frac{100}{s + 8000} + \frac{-40}{s + 20000}$

$v(t) = [90 + 100e^{-8000t} - 40e^{-20000t}]\,u(t)$



$Z_{11} \rightarrow$

$R_{tot} = 20 \| (5 + 15) = 10\Omega$

$Z_{11} = \frac{V_1}{I_1} = \frac{(1A)(10\Omega)}{1A} = 10\Omega$

$Z_{12} \rightarrow$

$I_a = \frac{15}{15 + 20 + 5} = \frac{15}{40}$

$Z_{12} = \frac{(I_a)(20\Omega)}{1A} = 7.5\Omega$

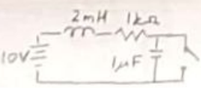
$Z_{21} \rightarrow$

$I_a = \frac{15 + 5}{20 + 15 + 5} = \frac{1}{2}A$

$Z_{21} = \frac{V_2}{I_1} = \frac{\frac{1}{2}(15)}{1} = 7.5\Omega$

$Z_{22} \rightarrow$

$Z_{22} = \frac{V_2}{I_2} = \frac{(\frac{25 \cdot 15}{40})(1A)}{1A} = 9.375\Omega$



sw closed, opens $t = 0$

Time domain

$i_L(0) = \frac{10V}{1000\Omega} = 10mA$

$V_L(t) = L\frac{di(t)}{dt}$

$\hookrightarrow V_R(0) = 1000\Omega(10mA) = 10V$

10V source vs 10V over R $\rightarrow V_c(0) = 0$

also inductor shorts

$\omega_0^2 = \frac{1}{LC} = 500(10^6)\ R^2/s^2$

$\alpha = \frac{R}{2L} = 250(10^3)\ R/s \rightarrow \alpha^2 = 62.5(10)^9\ R^2/s^2$

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega^2} = -(10)^3, -499(10)^3$

$i(t) = \cancel{i_{(\infty)}}^0 + A_1e^{s_1t} + A_2e^{s_2t}$

$\frac{di(t)}{dt} = s_1A_1e^{s_1t} + s_2A_2e^{s_2t}$

solve for $t = 0$

$A_1 = 10(10)^{-3} \quad A_2 = -20.1(10)^{-6}$

plug into gen equation