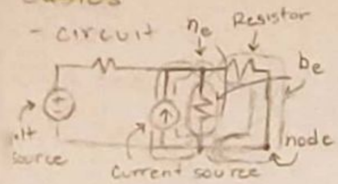
Principles of Electrical Engineering
Study Guide

Basics
- circuit



b_e = branches btwn n_e

n_e = node where 3+ elements join

- $\ell = b - n + 1$ (indep loops)

mesh - loop does not have other loops inside

Sources
		indep	dep
- voltage	$\frac{+}{-}$	(±)	◇ $v_s = \alpha v_x$
- current		(↑)	◇ $i_s = r i_x$

Basics (cont)
- Current (A, Amp)
  - ↳ $i(t) = \frac{d}{dt} q(t)$
  - $q(t) = \int_{t_0}^{t} i(x)\,dx + q(t_0)$
- Voltage (V, Volts)
  - ↳ $v = \frac{d}{dq} w$
- Power (W, Watt, J/s)
  - ↳ $p = \frac{dw}{dt} = iv$
- Energy / work (J)
  - ↳ $w(T) = \int_0^T p(x)\,dx$
- $p > 0$ → consume power
- $p < 0$ → produce power

* Σ nrg = constant

$P_{generated} = P_{dissipated}$

Passive Elements
- switch ⟶ "short" $R = 0$; ⟋ "open" $R = \infty$
- Resistance (Ω, ohm)
  - ↳ $R = \frac{\rho \cdot \ell}{A}$
  - ↳ $v = iR$
  - ↳ $p = iv = i^2 R = \frac{v^2}{R}$
- kirchoff's
  1. walk around a loop
  2. Add elements + → − depending on crossing
     - ↳ v + → − is +
     - ↳ i → is +

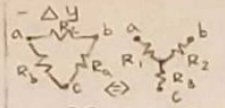
KCL n-1 eqns

\# loops = b − n + 1 = # eqns ← KVL

Equivalance
→ series
- ↳ $R_{eq} = R_1 + R_2 + \dots$
  - ↳ consistant current

→ parallel
- ↳ $R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right)^{-1}$
  - ↳ consistant voltage

- Δ Y



$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$

$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$

$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$

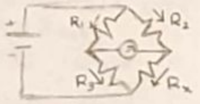
$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$

$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$

$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$

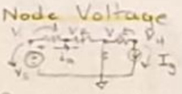
- volt divider (series)
  - $V_n = i R_n = V_{source} \frac{R_n}{\Sigma R}$
  - w/ ∞ resistors
- Current divider (parallel)
  - 2 resistor $I_n = I_s \frac{R_2}{R_1 + R_2}$
  - multiple $I_n = I_s \frac{R_1 \| R_2 \| \dots \| R_r}{R_n}$

Wheatstone
↳ used to find an unknown resistance



↳ $R_x = \frac{R_2 R_3}{R_1}$

Node Voltage

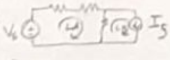


1. mark nodes
2. Indicate ground
3. Write eqns from node→out
   ie @ $V_1 \rightarrow -i_s + \frac{V_1 - V_2}{R_1} = 0$
   @ $V_4 \rightarrow I_g + \frac{V_4 - V_3}{R_3} = 0$
4. Constraints
   $V_1 - 0 = V_s$ so $V_1 = V_s$
5. Solve for node voltages

* Solving for current / volt across elements (+)−(−)
ie $i_a = \frac{V_1 - V_2}{R_1}$

* Supernode (combine node volts) to simplify dep sources

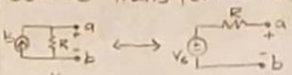
Mesh Current



1. Set up loops
2. Eqns based on loops (same dir / opp dir)
   ie $-V_s + R_1 I_A + R_2 I_A + R_3(I_A - I_B) = 0$
   $R_3(I_B - I_A) + I_g = 0$
3. Constraints
   $I_B = I_g$
4. Solve for current loops

* Supermesh, walk 2 loops ignoring dep source

Source Transformation

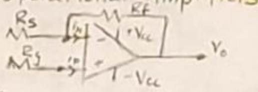


$I_s = \frac{V}{R}$ or $V_s = I_s R$

then current/voltage divider

Superposition
1. Set all sources but 1 to be 0 (±) → short & (↑) → open
2. Calculate values for modified circuits
3. Repeat 1 & 2 for all sources
4. Σ values for each element = actual value

Operational Amplifiers



$i_N = i_P = 0$ Amps

$-V_{cc} \le V_o \le V_{cc}$ ★ [min] [max]

$V_P = V_N$

→ noninverting
source → + terminal
$k = 1 + R_f/R_s$ gain

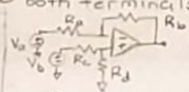
→ inverting
source → − terminal
$k = -R_f/R_s$ gain

$V_o = k \cdot V_g$

→ rec use node volt to solve

→ summing op amps
multiple sources @ one terminal
$V_o = -\left(\frac{R_f}{R_a} V_a + \frac{R_f}{R_b} V_b + \frac{R_f}{R_c} V_c\right)$

→ Difference op amps
source @ both terminals



$V_o = \underbrace{\frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}}_{k_1} V_b - \underbrace{\frac{R_b}{R_a}}_{k_2} V_a$

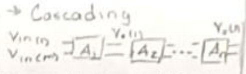
if $\frac{R_a}{R_b} = \frac{R_c}{R_d}$, $V_o = \frac{R_b}{R_a}(V_b - V_a)$

→ buffer op amps

$R_f = 0\,\Omega$

avoid impedence

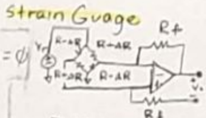
→ Cascading



$V_{o(n)} = (A_1 \cdot A_2 \cdot \ldots \cdot A_n) V_{in}$

★ Resistors can never be negative

Strain Guage

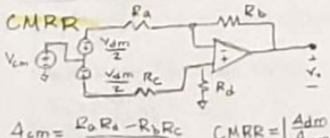


$V_o = \frac{R_f(2\Delta R)}{R^2 - (\Delta R)^2} \cdot V_r$

$\delta = \frac{\Delta R}{R} = \frac{R}{2R_f} \cdot \frac{V_o}{V_r}$

$V_o \approx \frac{R_f}{R} 2\delta \cdot V_r$

CMRR

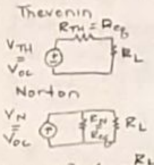


$A_{cm} = \frac{R_a R_d - R_b R_c}{R_a(R_c + R_d)}$ $\quad CMRR = \left|\frac{A_{dm}}{A_{cm}}\right|$ ideally ∞

$A_{dm} = \frac{R_d(R_a + R_b) + R_b(R_c + R_d)}{2R_a(R_c + R_d)}$

Thevenin + Norton
1. $V_{open\ circuit}$
   → replace R_L w/ V_{oc}
   → calculate w/ node volt
2. $i_{short\ circuit}$
   → replace R_L w/ i_{sc}
   → calculate w/ node volt or mesh current
3. R_{eq}
   ↳ indep only
   → replace R_L w/ a volt source
   → (±) = short & (↑) = open
   → Find R_{eq}
   ↳ w/ dep sources
   → add volt source $V_T = 1V$
   → $R_{eq} = \frac{V_T}{i_T}$

Now ↓



→ $P_L = V_{TH}^2 \left(\frac{R_L}{(R_L + R_{TH})^2}\right)$

or

$P_{L(max)} = \frac{V_{TH}^2}{4R_{TH}} = \frac{I_N^2 \cdot R_N}{4}$

b/c max when $R_L = R_{TH}$

Inductance & Capacitor
→ Capacitor

$q(t) = C \cdot v(t) = \frac{\epsilon A}{d} v(t)$

$i(t) = C \frac{d}{dt} v(t)$

$q(t) = \int_{t_0}^{t} i(\tau)\,d\tau + q(t_0)$

$w(t) = \frac{1}{2} C v^2(t) = \frac{q^2(t)}{2}$

$v(t) = \frac{1}{C} \int_0^t i\,d\tau + v(0)$

↳ ∥ $C_{eq} = \Sigma C$
$i_n(t) = C_n \frac{dv}{dt}$

↳ series $C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right)^{-1}$
$v(t_0) = v_1(t_0) + v_2(t_0) + \dots$

→ Inductor

$i(t) = \frac{1}{L} \int_{t_0}^{t} v(\tau)\,d\tau + v(t_0)$

$v(t) = L \frac{d}{dt} i$

$L = \frac{\mu N^2 A}{\ell}$

$w(t) = \frac{1}{2} L i^2(t)$

↳ ∥ $L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \dots\right)^{-1}$
$i(t_0) = \Sigma i_n(t_0)$

↳ series $L_{eq} = \Sigma L$

- Steady state
  - inductor → short $i = I$ $v = 0$
  - capacitor → open $i = 0$ $v = V$

Complex Numbers
$j = \sqrt{-1}$ so $-j = \frac{1}{j}$ & $j = \frac{-1}{j}$

$j = \cos\theta + j\sin\theta$ $\quad 1 + j = \sqrt{2}\angle 45°$

$-j = e^{-90j}$ $\quad j = e^{90j}$ $\quad$ *Note quadrant

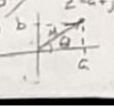
$Z = a + jb$ & $M\angle\theta$

$M = |Z| = \sqrt{a^2 + b^2}$

$\theta = \text{atan}(b/a)$

$a = M\cos\theta$

$b = M\sin\theta$



- complex conjugate

$Z = a + jb \rightarrow Z^* = a - jb$
$= M\angle\theta \rightarrow Z^* = M\angle -\theta$

$Z \cdot Z^* = |Z|^2 = M^2$

- phasor

$A\cos(\omega t + \theta) = A\angle\theta$
$= A\sin(\omega t + \theta - 90)$
↳ if +A → −90 ↗ −A → +90

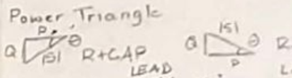
Impedence
- R → $Z_R = R\ \Omega$
- L → $Z_L = j\omega L\ \Omega$
- C → $Z_C = -j\frac{1}{\omega C}\ \Omega$

* Then use regular circuit rules

Phasor Power
$S = IV^*$ & $R = \frac{|V|^2}{S^*}$ [RMS]

↳ $S = \frac{VI^*}{2}$ not rms $\quad S = P + jQ$

Power Triangle



→ $\theta_{pf} = -\cos^{-1}(pf)°$ or $\cos^{-1}(pf)°$

→ $P = |S|\cos\theta$ Watts

→ $Q = |S|\sin\theta$ VAr $\quad$ Sin VA

$P_{max} = \frac{V_{TH\,rms}^2}{4R} = \frac{V_{TH\,m}^2}{8R}$

$S_{source} + S_{line} + S_{load} = 0$