Intro to Logic Study Guide

Connectives

(¬) Not
(^) And
(v) Or
(⊕) XOR
(→) If... then
(↔) Biconditional
- "but" = ^
- "P if Q" = Q→P
- "P only if Q" = P→Q
- "unless" = v/⊕/¬↔

Validity

- modus ponens - proof by affirmation

① Truth table
↳ look for premises = T but conclusion = F
- valid if no counterex

② Truth trees
↳ CE = trace open branches

¬[¬P ↔ ¬S] even # ¬s

Truth Trees

(P^Q) (PvQ) (P→Q) (P↔Q)

¬(^) ¬(v) ¬(→) ¬(↔)

① negate conclusion
② Breakdown
↳ stack 1st
↳ then split
③ Look for contradictions
↳ on open branches
↳ no open branch? valid argument

Natural deduction

* Only w/ valid args

¬E
1. ¬¬P
P

^E
1. P^Q
2. P
3. Q

^I
4. P^Q

vE
1. PvQ
2. ¬P
3. Q

vI
4. Q v [anything]

↔E
1. P↔Q
2. P→Q
3. Q→P

↔I
4. P↔Q

→E
1. P→Q
2. P
3. Q

① Breakdow
② → in conclusion? Do →I
③ Argument by cases
↳ need or statement
④ If ^ or ↔ conclusion work backwards
⑤ D ⊥I

→I
1. P→Q
2. Q→R //P→R
ass conclusion
3. P
4. P→Q
5. Q
6. Q→R
derive consequent
7. R
8. P→R

¬I
1. P→Q
2. ¬Q //¬P
looking to negate
3. P
4. P→Q
5. Q
look for ⊥
6. ¬Q
7. ⊥
8. ¬P

AC
1. AvB
2. A→C
3. B→C //C
ass.
4. A
5. A→C
6. C
derive conclusion
7. B
2x
8. B→C
9. C
10. C

- ~~PREDICATE~~ LOGIC

Quantifiers

(∀) universal
(∃) existential

Truth Trees

∀x Px
P[anything]

∃x Px
Pa → var not already in problem

Natural Deduction

∀E
1. ∀x Px
Pâ

∃I
1. Pa
2. ∃Px

∀I
1. ∀(Px ^ Qx)
2. Pâ ^ Qâ
3. Qâ
4. ∀Qx

∃E
1. ∃x(Px ^ Qx) Ass
2. (A) Pa ^ Qa
3. Qa unrelated
4. ∃Qx
5. ∃xQx

① Proposition logic
② ∃E
③ ∀E
④ If ans has ∀, work backwards

Identity

x = y
↳ x same as y

=E
1. ∀x (Wx→Mx)
2. Wv ^ v=t //Wt ^ Mt
3. Wt
4. v=t
5. Wt =E 3,4
6. Wt̂ → Mt̂
7. Mt
8. Wt ^ Mt

=I
1. a=b //b=a
2. a=a =I
3. b=a =E 1,2
no #