MAT 234, Spring 2021 Exam #1
M. Murray February 25, 2021

Name: _____

Each problem is worth 12 points total, except for #3 which is worth 16 points.

1. Which of the following sentences are statements? In the case of a statement, determine whether the statement is true or false and briefly explain your reasoning.

   (a) $6^2 + 8^2 = 10^2$.

   (b) $a^2 + b^2 = c^2$.

   (c) For each real number x, if $x^2 = 4$, then $x = 2$.

   (d) There exist real numbers x and y such that $3x + 5y = 8$.

2. Below is a statement of a theorem about certain cubic equations. For this theorem, b represents a real number.

Theorem 1 If f is a cubic function of the form $f(x) = x^3 - x + b$ and $b > 1$, the the function has f has exactly one x-intercept.

Theorem 2 If f and g are functions with $g(x) = k \cdot f(x)$, where k is a nonzero real number, then f and g have exactly the same x-intercepts.

Using **only** these theorems and some simple algebraic manipulations, what can be concluded about the functions given by the following formulas? (15 points)

(a) $f(x) = x^3 - x + 11$

(b) $g(x) = x^3 + x + 11$

(c) $h(x) = -x^3 + x - 5$

(d) $r(x) = x^4 - x + 11$

(e) $k(x) = 2x^3 - 2x + 11$

3. Write a formal proof of the following statement using the definitions of even and odd integers.

If x and y are odd integers, then $xy + 1$ is an even integer.

4. Let P and Q be statements.

   (a) Show that $P \to Q$ and $\neg P \vee Q$ are logically equivalent using a truth table.

   (b) Next, negate both $P \to Q$ and $\neg P \vee Q$ to fill in the blank here:

   $$\neg(P \to Q) \equiv \_\_\_\_\_$$

   with a different, logically equivalent form of $\neg(\neg P \vee Q)$. Briefly explain your work.

5. Suppose that P and Q are statements for which $P \to Q$ is false. What conclusions (if any) can be made about the truth value of each of the following statements? Briefly explain your work.

   (a) Q

   (b) $P \leftrightarrow Q$ (recall: $P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$)

   (c) $\neg P \to Q$

6. Let x be a real number. Consider the following conditional statement:

$$\text{If } x^2 - 5x + 4 = 0, \text{ then } x = 4 \text{ or } x = 1.$$

Which of the following statement have the same meaning as this conditional statement and which ones are negations of this conditional statement? Briefly explain your conclusion and feel free to use the logical equivalencies on pg. 48 in your text.

(a) $x^2 - 5x + 4 \neq 0$ or $x = 4$ or $x = 1$.

(b) If $x \neq 4$ and $x \neq 1$, then $x^2 - 5x + 4 \neq 0$.

(c) If $x = 4$ or $x = 1$, then $x^2 - 5x + 4 = 0$.

(d) $x^2 - 5x + 4 = 0$, $x \neq 4$, and $x \neq 1$.

7. Each of the following sets is defined using the roster method. Use set builder notation to describe each set.

   (a) $A = \{1, 4, 9, 16, 25, \dots\}$

   (b) $B = \{\dots, -\pi^4, -\pi^3, -\pi^2, -\pi\}$

   (c) $D = \{0, 4, 8, \dots, 96, 100\}$

   (d) Write the set of all even integers using the roster method and set builder notation.

8. Consider the following theorem concerning real numbers.

   Theorem For all real numbers a and b, if ab = 0, then a = 0 or b = 0.

   (a) Rewrite the theorem using the contrapositive of the given statement. Make sure your response is a clear English sentence.

   (b) Negate the theorem in a grammatically correct English sentence. (Note: You should negate the original version, rather than the contrapositive form.)