

**Problem Chosen**

**A**

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Summary Sheet**

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## **Input Your Article Title Here if it is too Long**

### **Summary**

This is abstract. This section should describe what problem the paper solves, what methods are applied, what results are obtained and summarize them.

This is the second line abstract. And if you look carefully you can see that the spacing within and between paragraphs is different, which facilitates our reading in paragraphs.

This is **the special** *special* **special** *special* fonts in abstract.

**Keywords:** Fighting Wildfires; Multi-Objective Optimization; Poisson Distribution; Tabu Search Algorithm; Sensitivity Analysis

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# 1 Ideal Battery

## 1.1 Assumptions

To consider an simplest ideal battery model, the following assumptions are made:

- The battery is kept at a constant environment, so the temperature effects are neglected.
- The battery has no energy loss during charging and discharging.
- The battery has infinite cycle life, so the degradation effects are neglected and all the parameters are constant.
- The Open Circuit Voltage (OCV) is only related to the SoC, so we can express the SoC as a function of  $U_{OC}$ :

$$SoC = f(U_{OC}) \quad (1)$$

in which  $f(\cdot)$  can be obtained through curve fitting based on experimental data.

- The battery behavior is same for charging and discharging. For smartphones, which mostly use LiCoO<sub>2</sub> batteries that has little or no hysteresis, this assumption is reasonable.

## 1.2 Thevenin Model

A Li-ion battery system can be extremely complex, involving electrochemical, thermal and mechanical processes. However, for system-level studies, an equivalent circuit model is often used to represent the battery behavior. The Thevenin model is a widely used equivalent circuit model that captures the dynamic response of the battery voltage during charge and discharge cycles. The model is described as follows:

The relationship between  $U_{OC}$  and other parameters in the Thevenin model can be expressed with Kirchhoff's laws:

$$U_{OC} = U_t + IR_0 + U_1 + U_2 \quad (2)$$

in which  $U_t$  is the terminal voltage.

For  $U_1, U_2$  we have:

$$I = C_1 \frac{dU_1}{dt} + \frac{U_1}{R_1} = C_2 \frac{dU_2}{dt} + \frac{U_2}{R_2} \quad (3)$$

Differentiateing the SoC's definition with respect to time, we get:

$$\frac{d(SoC)}{dt} = -\frac{I}{Q_{max}} \quad (4)$$

where  $Q_{max}$  is the maximum capacity of the battery.

Combining the above equations, we can derive the complete Thevenin model in matrix form:

$$\frac{d}{dt} \begin{bmatrix} SoC \\ U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{R_1 C_1} & 0 \\ 0 & 0 & -\frac{1}{R_2 C_2} \end{bmatrix} \begin{bmatrix} SoC \\ U_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{Q_{max}} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix} I \quad (5)$$

where  $R_0, R_1, R_2, C_1, C_2, Q_{max}$  are the model parameters that need to be estimated.

### 1.3 Parameter Estimation

The Hybrid Pulse Power Characterization (HPPC) test is commonly used for this purpose. The test is conducted with steps as follows:

1. The battery is first fully charged to 100% SoC in ways the manufacturer recommends.
2. After resting for a certain period(e.g. 1 hour), the battery is discharged with a constant current pulse (e.g.,  $0.5C$ ) for a short duration (e.g., 10s). The voltage response is recorded.
3. Then discharge the battery to another selected SoC point (e.g., 90%).
4. Repeat steps 2 and 3 until the battery reaches a low SoC point (e.g., 10%) or the maximum discharge limit the manufacturer specifies.
5. If needed, repeat similar steps for charging pulses.

With HPPC data, the model parameters can be estimated through curve fitting techniques. The process are as follows:

- **Estimate  $R_0$ :** The instantaneous voltage drop at the start of each pulse can be used to estimate the internal resistance  $R_0$ , at which point the capacitive effects are negligible.

$$R_0 = \frac{\Delta U_{instant}}{I_{pulse}} \quad (6)$$

- **Estimate  $R_1, C_1$  and  $R_2, C_2$ :** Solve the polarization equation (3), we can get the polarization voltage response:

$$U_{polar} = IR + (U_{init} - IR)e^{-\frac{t}{RC}} \quad (7)$$

where  $U_{init}$  is the voltage at the start of the pulse.

Without loss of generality, let  $R_1C_1 \leq R_2C_2$ , so that the faster electrochemical polarization are represented by  $R_1, C_1$  and the slower concentration polarization effects are represented by  $R_2, C_2$ . Then substitute them into (2), we have:

$$U_t = U_{OC} - I(R_0 + R_1 + R_2) - (U_{1,init} - IR_1)e^{-\frac{t}{R_1C_1}} - (U_{2,init} - IR_2)e^{-\frac{t}{R_2C_2}} \quad (8)$$

By least squares fitting of (8) to the voltage response data during each pulse, we can estimate the values of  $R_1, C_1$  and  $R_2, C_2$  at different SoC points.

- **Estimate  $Q_{max}$ :** The maximum capacity  $Q_{max}$  can be estimated by integrating the current over the full discharge cycle:

$$Q_{max} = \int_{t_0}^{t_f} I(t)dt \quad (9)$$

where  $t_0$  and  $t_f$  are the start and end times of the discharge cycle.  
Most of the time this value is provided by the manufacturer.

We use a Samsung INR21700 30T 3Ah Li-ion Battery Dataset to demonstrate the parameter estimation process. The fitting result are shown as follows:

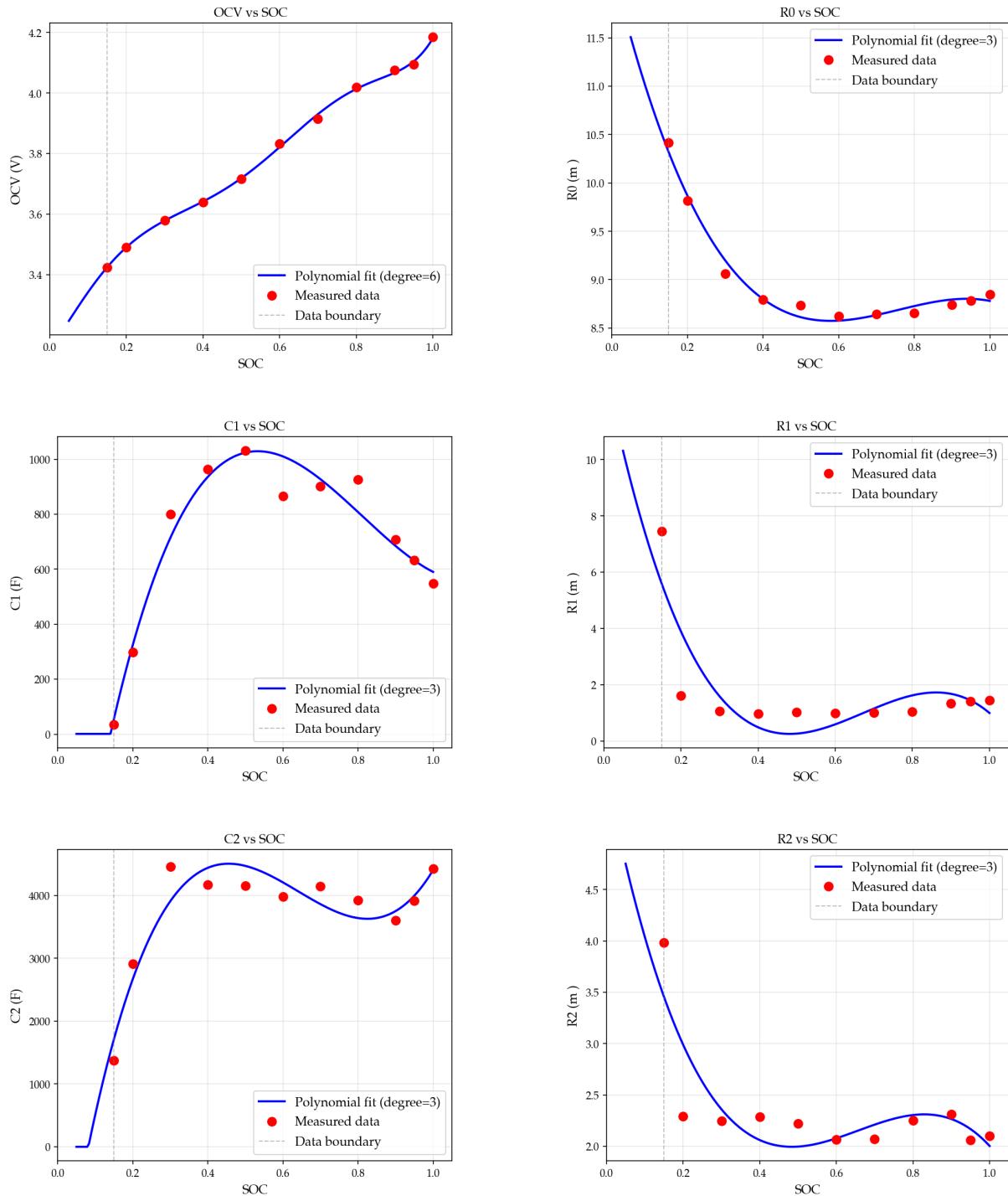


Figure 1: Fitting results of the parameter estimation process.

6-degree polynomial is used to fit the OCV-SOC curve, and cubic functions are used to fit the RC parameters.

As the fitting result shows, the Ohm resistance  $R_0$  increases significantly when SoC is low, which may be the main reason for voltage drop and device shutdown. Later we will verify this conclusion. Polarization resistances  $R_1, R_2$  also increase when SoC is low, indicating the battery's internal electrochemical processes are hindered. And

Polarization capacitances  $C_1, C_2$  decrease when SoC is low, meaning that the battery's ability to respond to load changes is weakened.

Noticed that  $C_1, C_2$  drop to zero when SoC is low, which means the Thevenin model is not valid in that region. However, the smartphone will actually shutdown before the battery reaches such low SoC, because the output voltage will be too low to power the device.

## 1.4 Simulations

With proper model parameters, we can do numerical simulations of the battery behavior under different load profiles. 2 simplified load profiles are considered here:

- Constant current;
- Constant power.

### 1.4.1 Constant current

Since the current is constant, equation (4) can be directly integrated to get the SoC at time  $t$ :

$$SoC(t) = SoC(0) - \frac{It}{Q_{max}} \quad (10)$$

Then perform Rk45 algorithm to the polarization voltages  $U_1, U_2$  with initial conditions  $U_1(0) = U_2(0) = 0$ , we can get the final result as follows. note that output power and loss are shown with different units.

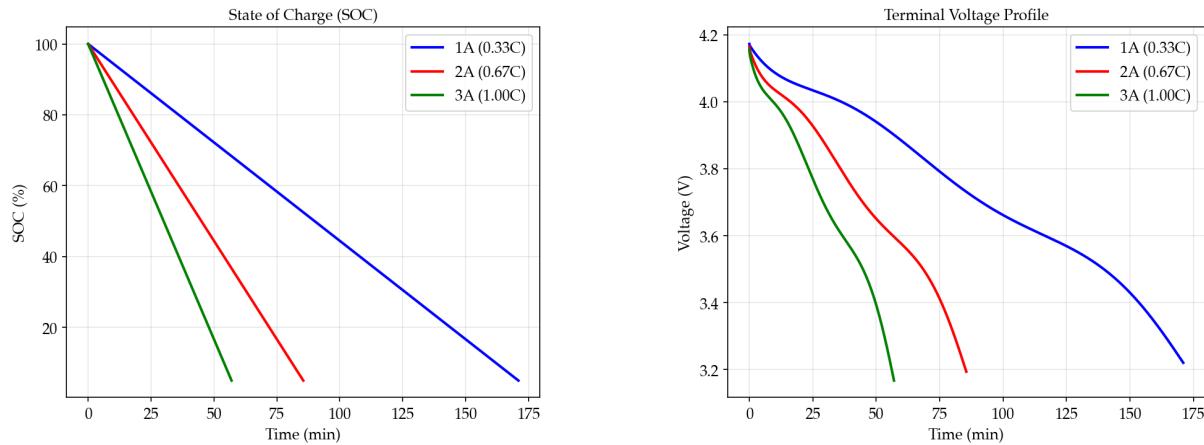


Figure 2: Simulation results under constant current load profile

### 1.4.2 Constant power

The output power

$$P = U_t I \quad (11)$$

is constant. In this case, analytic solutions are impossible, so we just use the Rk45 algorithm:

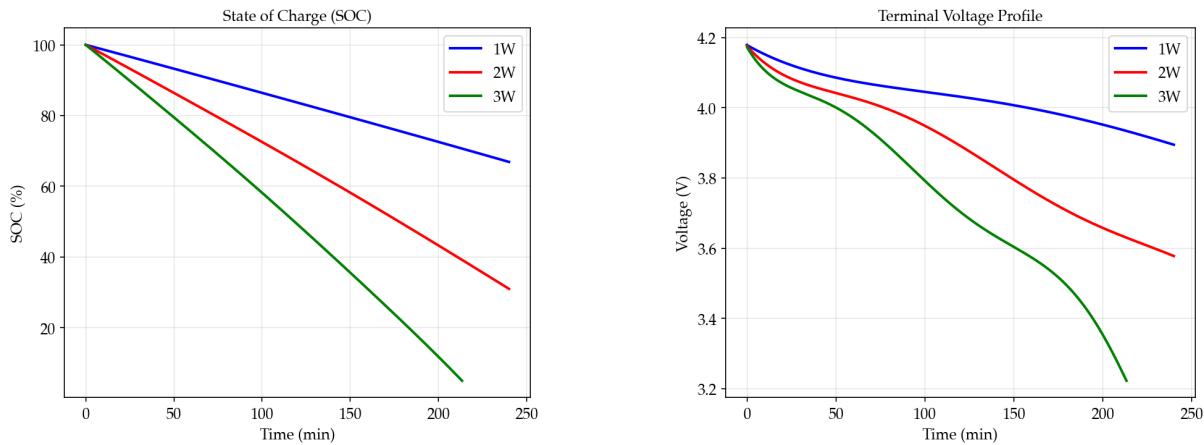


Figure 3: Simulation results under constant power load profile

It can be seen that under constant power load profile, the more the battery discharges, the larger the current it needs to provide to maintain constant power output, so the voltage drops faster. This is consistent with our daily experience that devices tend to shutdown quicker when the battery is low.

## 2 Actual Battery

### 2.1 Assumptions

An actual battery is much more complex than the idealized model presented in the previous section. In this section, we will consider more factors that affect the performance and behavior of real batteries:

- **Temperature:** The performance of a battery can vary significantly with temperature. At low temperatures, the internal resistance increases, leading to reduced capacity and power output. Conversely, high temperatures can enhance performance but may also accelerate degradation.
- **Complex Power Profile:** Real batteries often experience varying power demands, which can affect their efficiency and lifespan. High performance demands can lead to increased heat generation, which in turn affects battery temperature.

### 2.2 Temperature Behavior of Battery

Actual battery behavior is significantly influenced by temperature variations. The most common parameter affected by temperature is the internal resistance. Using the Arrhenius equation, we can model the temperature dependence of internal resistance as follows:

$$R(T) = R_0 \cdot e^{-\frac{E_a}{R_u} \left( \frac{1}{T} - \frac{1}{T_0} \right)} \quad (12)$$

where  $E_a$  is the activation energy,  $R_u$  is the universal gas constant and  $T_0$  is a reference temperature.

## **2.3 Heat Transfer Model in High-Performance Scenarios**

### **2.4 Answer to Question 1**

### **2.5 Answer to Question 2**