LINEAR PROGRAMMING QUESTION

Web organization sells many household products through an online catalog. The company needs substantial warehouse space for storing its goods. Plans now are being made for leasing warehouse storage space over the next 5 months. Just how much space will be required in each of these months is known. However, since these space requirements are quite different, it maybe most economical to lease only the amount needed each months is much less than for the first month, so it may be less expensive to lease the maximum amount needed for the entire 5 months. Another option is the intermediate approach of changing the total amount of space leased (by adding a new lease and/ or having an old lease expire) at least once but not every month. The space requirement and the leasing costs for the various leasing periods are given in the table. The objective is to minimize the total leasing cost for meeting the space requirements.

Formulate a linear programming (LP) model for this problem.

Month	Required Space (Sq. Ft.)
1	30,000
2	20,000
3	40,000
4	10,000
5	50,000

Leasing Period (Months)	**Cost Per Sq. Ft. Leased**
1	**₦65**
2	**₦100**
3	**₦135**
4	**₦160**
5	**₦190**

SOLUTION

A COMPREHENSIVE INTERPRETATION OF THE QUESTION ABOVE

From the question above, we aim develop a Linear Programming Model that aims to minimize the total leasing cost of meeting the space requirements for the web organization company. We are provided with the amount of space required for over the next five months and the corresponding cost for each leasing periods (months).

In order to obtain the minimum total cost, we need to put into consideration the following:

- **Decision Variable:**
  These are the unknown quantities that are expected to be estimated as an output of the LPP solution. In relation to our problem above, the decision variable is the amount of space leased in a month for a certain period of months. For instance, Let's say the company decides to lease 30,000 acres for just 1 month for a period of 2 months

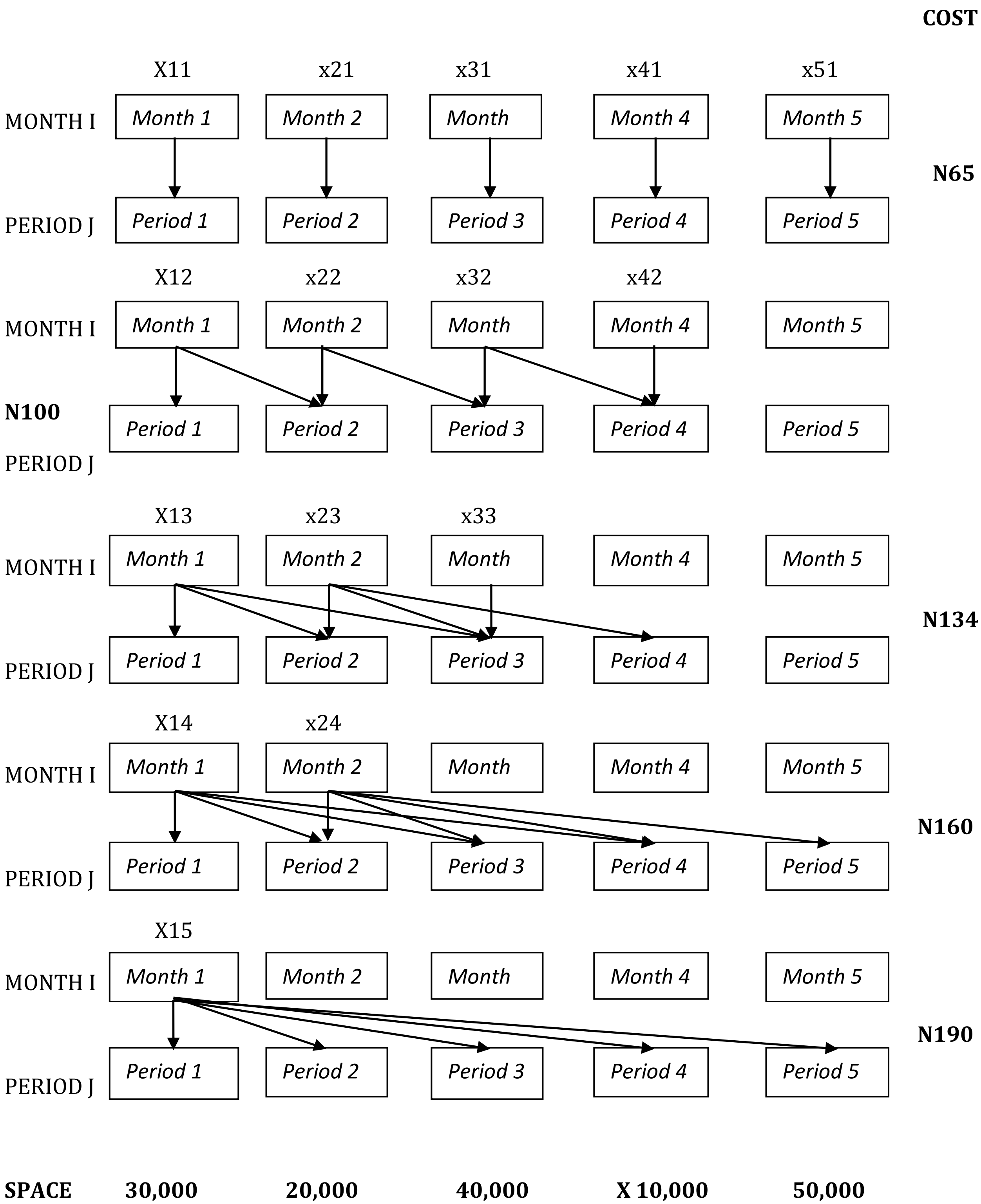
- **Objective function:**
  The main aim of this linear programming problem is to either minimize the cost of leasing the space required. The objective function evaluates the amount by which each decision variable would contribute to the net present value of the warehouse.

- **Objective Function coefficient:**
  This is denoted by the amount by which the objective function value would change when one unit of a decision variable (the amount of space leased) is altered.

- **Constraints:**
  The restrictions and the limitation on the total amount of the space required for the company's warehouse that would decide the level of optimization in the decision variables. In the standard form of a linear programming problem, all constraints are in the form of an inequality equation.

- **Non-negative constraints:**
  Here, each decision variable in the linear programming model must be positive irrespective of whether the object function is to minimize or maximize the net present value of the warehouse.

DIAGRAMMATIC REPRESENTATION OF THE ANAYSIS OF THE LINEAR PROGRAMMING PROBLEM (GRAPHICAL METHOD)



MAKING POSSIBLE ASSUMPTION AND CONSTRUCTING THE ENTITIES FOR POSSIBLE EQUATION FORMULATION.

POSSIBLE ASSUMPTIONS:

The decision variable:

a. Let x_{ij} = the amount of space based in month i for a period of j months for i = 1,….,5 and j = 1,….,6 – i.

ENTITIES FOR POSSIBLE EQUATION FORMULATION, HENCE THE LPP MODEL:

The Objective Function:

Minimize

$$z = 65\,(x_{11} + x_{21} + x_{31} + x_{41} + x_{51}) +$$
$$100\,(x_{12} + x_{22} + x_{32} + x_{42}) + 135\,(x_{13} + x_{23} + x_{33}) +$$
$$160(x_{14} + x_{24}) + 190x_{15}$$

Subject to

The Constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} \geq 30{,}000$$
$$x_{12} + x_{13} + x_{14} + x_{15} + x_{21} + x_{22} + x_{23} + x_{24} \geq 20{,}000$$
$$x_{13} + x_{14} + x_{15} + x_{22} + x_{23} + x_{24} + x_{31} + x_{32} + x_{33} \geq 40{,}000$$
$$x_{14} + x_{15} + x_{23} + x_{24} + x_{32} + x_{33} + x_{41} + x_{42} \geq 10{,}000$$
$$x_{15} + x_{24} + x_{33} + x_{42} + x_{51} \geq 50{,}000$$

and

Non-negative constraints:

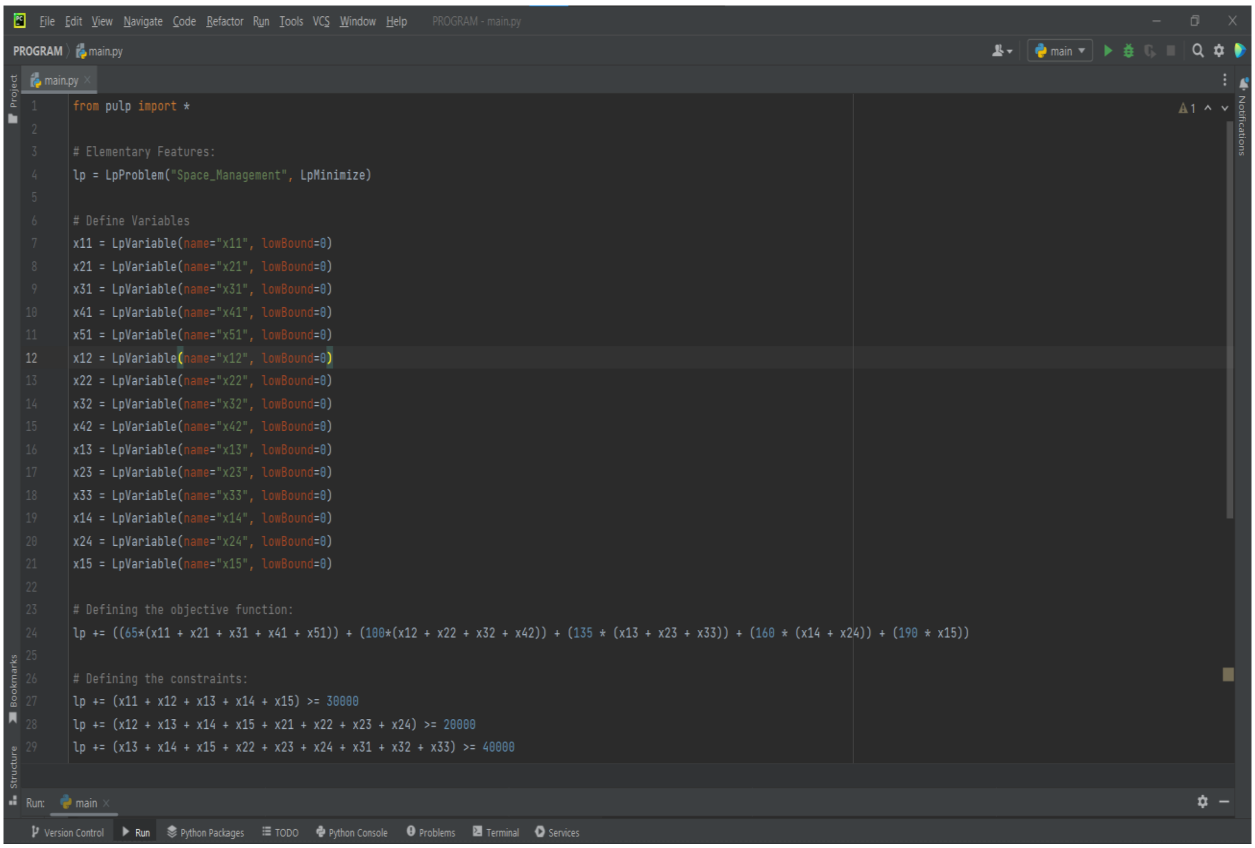
$$x_{ij} \geq 0, \qquad for\ i = 1, \dots, 5\ and\ j = 1, \dots, 6 - i.$$

TRANSLATING THE LPP MODEL INTO A PROGRAM

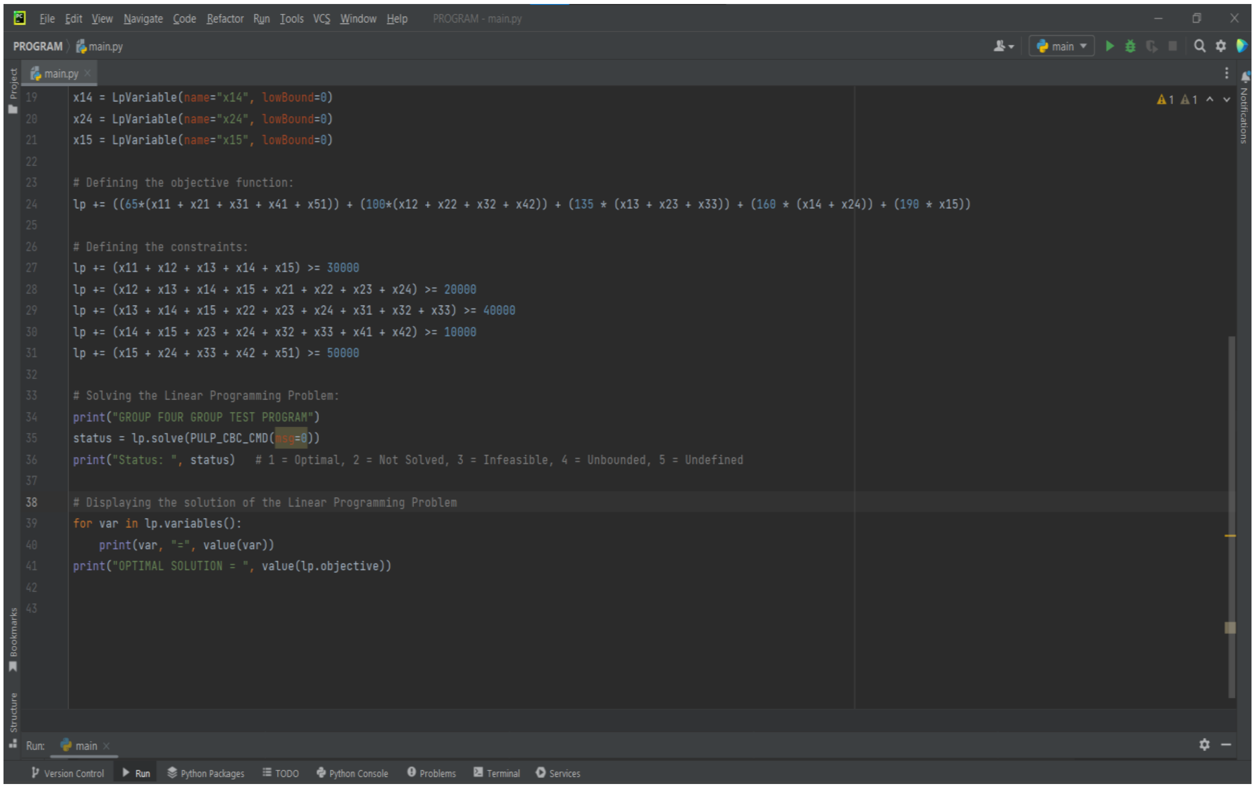
The linear programming model was developed using python programming language. PuLP library was used for the modeling. PuLP is a python library that can be used to solve linear programming problems. It enables users to describe mathematical program.

PuLP library works entirely within the syntax of the python programming language by providing python objects that represents optimization problems, and decision variables, and allowing constraints to be expressed in a way that is similar to the original mathematical expression.

THE PROGRAM SYNTAX

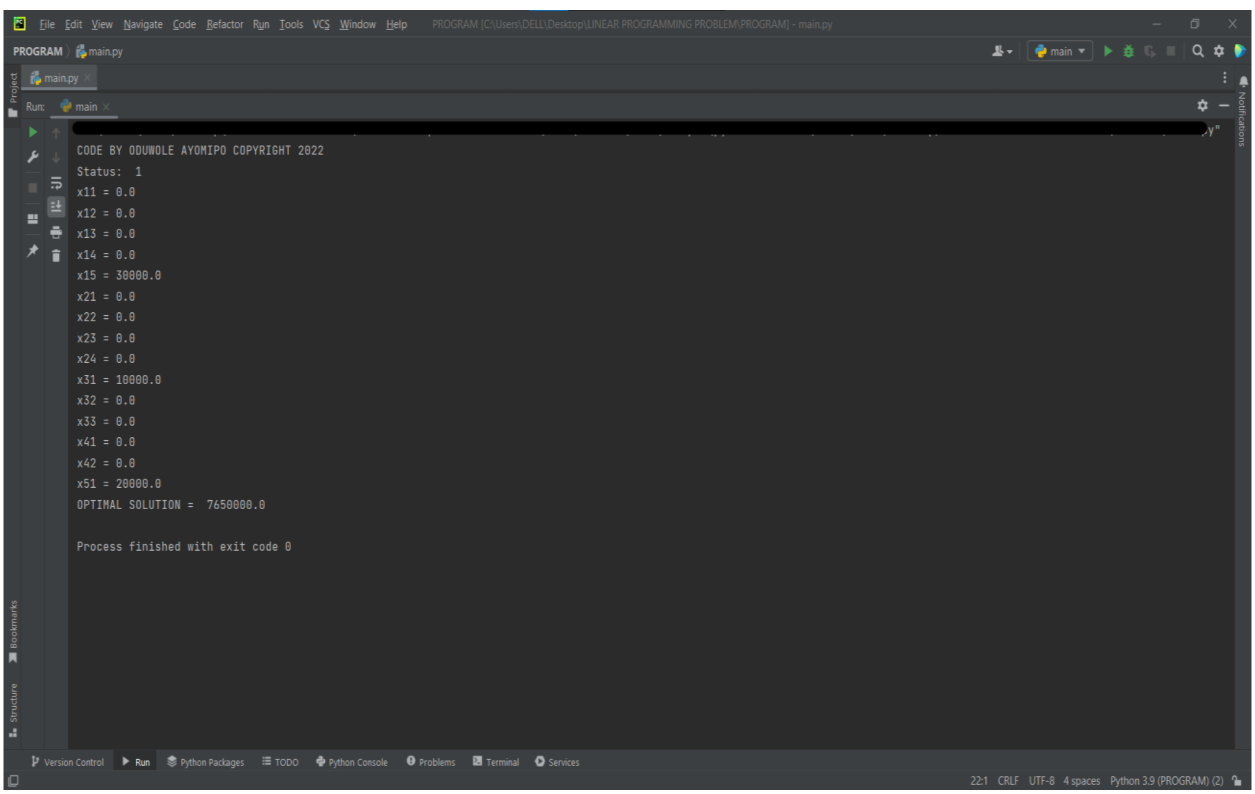


```python
from pulp import *

# Elementary Features:
lp = LpProblem("Space_Management", LpMinimize)

# Define Variables
x11 = LpVariable(name="x11", lowBound=0)
x21 = LpVariable(name="x21", lowBound=0)
x31 = LpVariable(name="x31", lowBound=0)
x41 = LpVariable(name="x41", lowBound=0)
x51 = LpVariable(name="x51", lowBound=0)
x12 = LpVariable(name="x12", lowBound=0)
x22 = LpVariable(name="x22", lowBound=0)
x32 = LpVariable(name="x32", lowBound=0)
x42 = LpVariable(name="x42", lowBound=0)
x13 = LpVariable(name="x13", lowBound=0)
x23 = LpVariable(name="x23", lowBound=0)
x33 = LpVariable(name="x33", lowBound=0)
x14 = LpVariable(name="x14", lowBound=0)
x24 = LpVariable(name="x24", lowBound=0)
x15 = LpVariable(name="x15", lowBound=0)

# Defining the objective function:
lp += ((65*(x11 + x21 + x31 + x41 + x51)) + (100*(x12 + x22 + x32 + x42)) + (135 * (x13 + x23 + x33)) + (160 * (x14 + x24)) + (190 * x15))

# Defining the constraints:
lp += (x11 + x12 + x13 + x14 + x15) >= 30000
lp += (x12 + x13 + x14 + x15 + x21 + x22 + x23 + x24) >= 20000
lp += (x13 + x14 + x15 + x22 + x23 + x24 + x31 + x32 + x33) >= 40000
```



```python
x14 = LpVariable(name="x14", lowBound=0)
x24 = LpVariable(name="x24", lowBound=0)
x15 = LpVariable(name="x15", lowBound=0)

# Defining the objective function:
lp += ((65*(x11 + x21 + x31 + x41 + x51)) + (100*(x12 + x22 + x32 + x42)) + (135 * (x13 + x23 + x33)) + (160 * (x14 + x24)) + (190 * x15))

# Defining the constraints:
lp += (x11 + x12 + x13 + x14 + x15) >= 30000
lp += (x12 + x13 + x14 + x15 + x21 + x22 + x23 + x24) >= 20000
lp += (x13 + x14 + x15 + x22 + x23 + x24 + x31 + x32 + x33) >= 40000
lp += (x14 + x15 + x23 + x24 + x32 + x33 + x41 + x42) >= 10000
lp += (x15 + x24 + x33 + x42 + x51) >= 50000

# Solving the Linear Programming Problem:
print("GROUP FOUR GROUP TEST PROGRAM")
status = lp.solve(PULP_CBC_CMD(msg=0))
print("Status: ", status)   # 1 = Optimal, 2 = Not Solved, 3 = Infeasible, 4 = Unbounded, 5 = Undefined

# Displaying the solution of the Linear Programming Problem
for var in lp.variables():
    print(var, "=", value(var))
print("OPTIMAL SOLUTION = ", value(lp.objective))
```

RESULT OF THE PROGRAM



[redacted]

```
CODE BY ODUWOLE AYOMIPO COPYRIGHT 2022
Status:  1
x11 = 0.0
x12 = 0.0
x13 = 0.0
x14 = 0.0
x15 = 30000.0
x21 = 0.0
x22 = 0.0
x23 = 0.0
x24 = 0.0
x31 = 10000.0
x32 = 0.0
x33 = 0.0
x41 = 0.0
x42 = 0.0
x51 = 20000.0
OPTIMAL SOLUTION =  7650000.0

Process finished with exit code 0
```

CONCLUSION

From the result above,

Objective Function Value (i.e. The Minimal Cost), C = ₦ 7,650,000.00

The Optimal solution:

X15 = N 30,000

X31 = N 10,000

X51 = N 20,000

Hence, in the second month, there will be an extra space of 10,000 sq.ft, and in the Fourth Month, there will be an extra space of 20,000 sq.ft.

REPORT

The source code and a complete file of the program is available on GitHub, via the link below
https://github.com/JustAyoo/Linear-Programming-Problem-Using-Python/