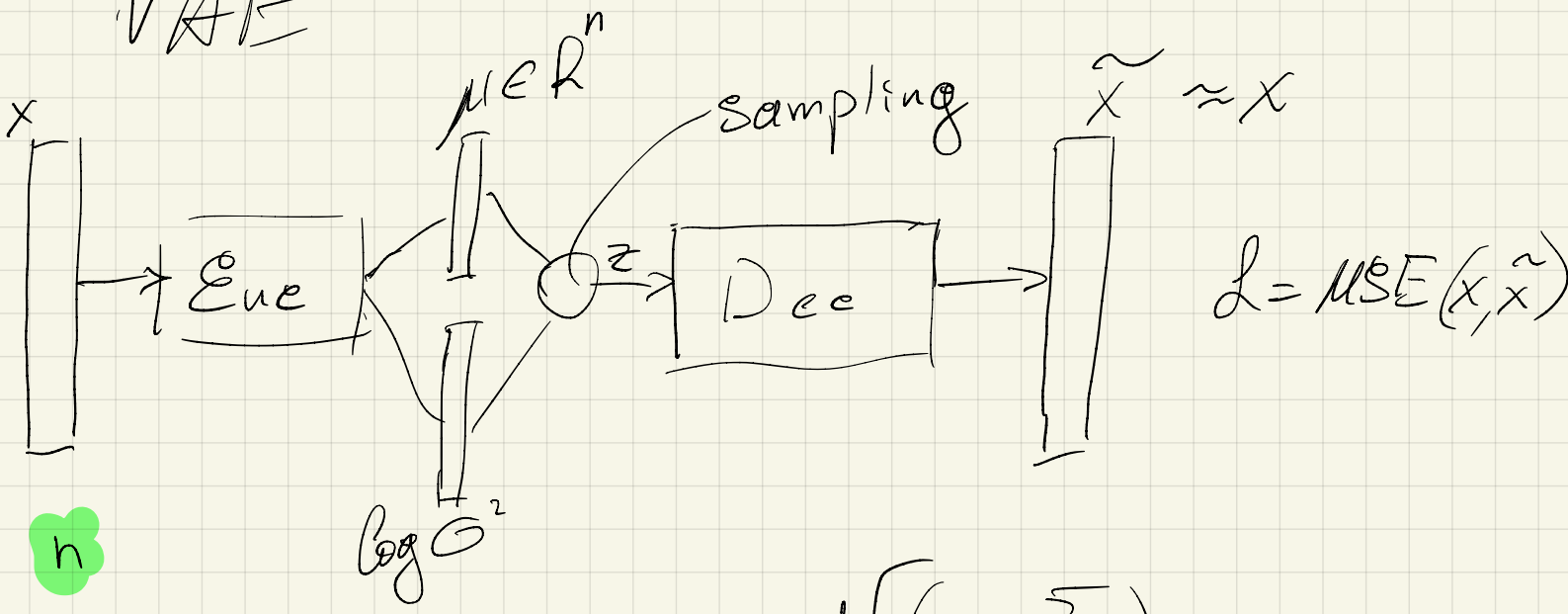
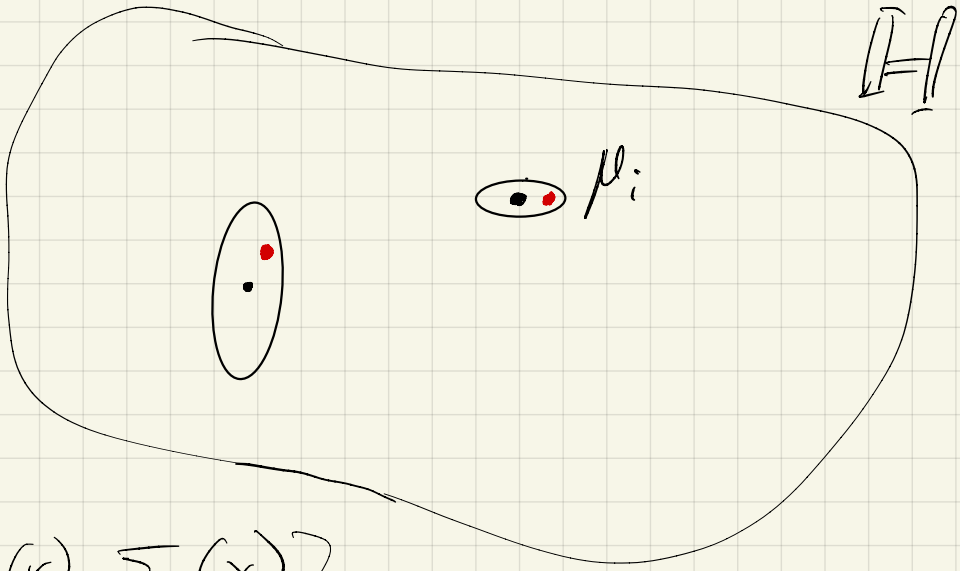
VAE



h

$$z \sim \mathcal{N}(\mu, \Sigma)$$

$$\Sigma = \text{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2)$$



$$p(z) = \mathcal{N}(0, I)$$

$$q(z|x) = \mathcal{N}(\mu(x), \Sigma(x))$$

$D_{KL}(p_1 \| p_2) - ?$ $\quad p_1 = \mathcal{N}(\mu_1, \Sigma_1)$ $\quad n$

$p_2 = \mathcal{N}(\mu_2, \Sigma_2)$

$$D_{KL}(p_1 \| p_2) = \frac{1}{2}\left[\log\frac{|\Sigma_2|}{|\Sigma_1|} - n + tr\left(\Sigma_2^{-1}\Sigma_1\right) + (\mu_2-\mu_1)^T\Sigma_2^{-1}(\mu_2-\mu_1)\right]$$

$p(z) = \mathcal{N}(\vec{0}, I)$

$q(z|x) = \mathcal{N}(\mu, \Sigma)$

$$D_{KL}(q(z|x), p(z)) = \frac{1}{2}\left[\log\frac{|I|}{|\Sigma|} - n + tr(I^{-1}\Sigma) + (\vec{0}-\mu)^T I^{-1}(\vec{0}-\mu)\right] =$$

$$= \frac{1}{2}\left[-\log|\Sigma| - n + tr\,\Sigma + \mu^T\mu\right] =$$

$$= \frac{1}{2}\left[-\log\prod_{i=1}^{n}\sigma_i^2 - n + \sum_{i=1}^{n}\sigma_i^2 + \sum_{i=1}^{n}\mu_i^2\right] =$$

$$= \frac{1}{2}\left[-\sum_{i=1}^{n}(\log\sigma_i^2 + 1) + \sum_{i=1}^{n}\sigma_i^2 + \sum_{i=1}^{n}\mu_i^2\right]$$