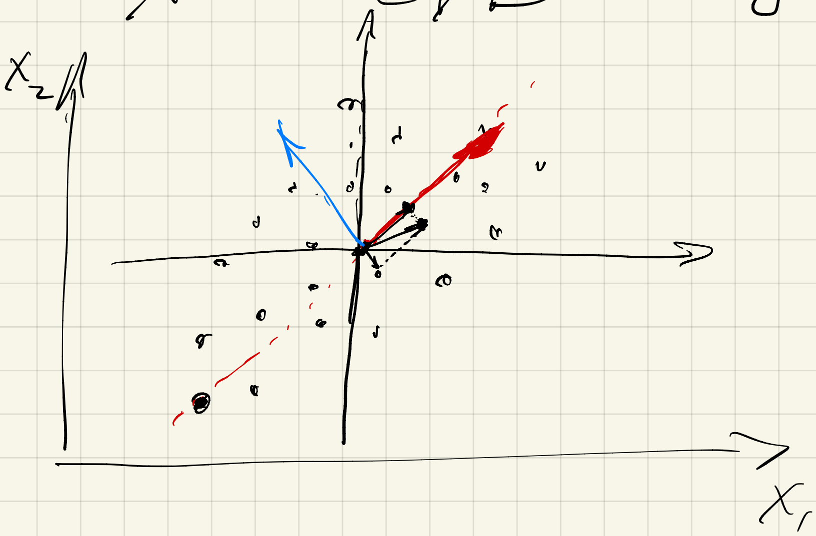
PCA = principal components analysis

$\underline{X} \in \mathbb{R}^D$

$X \to SVD$ Singular values decomposition

$$X_c = U \Sigma V^T$$

$$T = U \Sigma$$

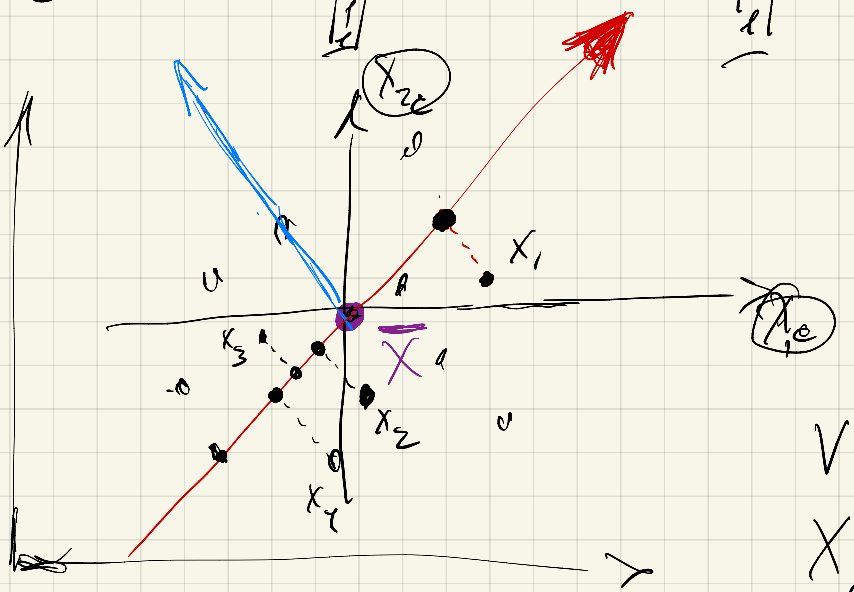


$$\underline{X} = \begin{bmatrix} | & | & | & & | \\ x_1 & x_2 & x_3 & \cdots & x_N \\ | & | & | & & | \end{bmatrix} = (SVD) = \overset{U}{\begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_N \\ | & | & & | \end{bmatrix}} \cdot \overset{\Sigma}{\begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_N \\ \hline & & 0 & \end{bmatrix}} \cdot \begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_N \\ | & | & & | \end{bmatrix}^T$$

$$UU^T = U^T U = \underline{I} = VV^T = V^T V$$

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \qquad \bar{X} \quad (1 \times f)$$

$$X_c = X - \begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\bar{X} \qquad \begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}\bar{X} \quad (N \times f)$$



$$X_c = T V^T$$

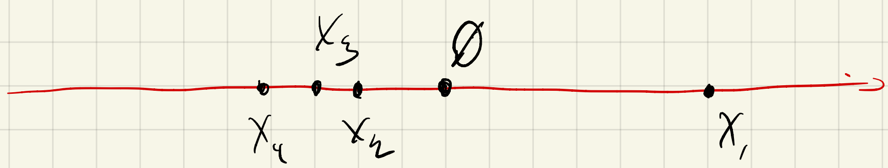
$$T = U\Sigma = X_c V$$

$V^T \quad (k \times f)$

$X_c \quad (N \times f)$

$$\tilde{X}_c = (\tilde{T}\tilde{V})\tilde{V}^T \overset{?}{\neq} X_c$$

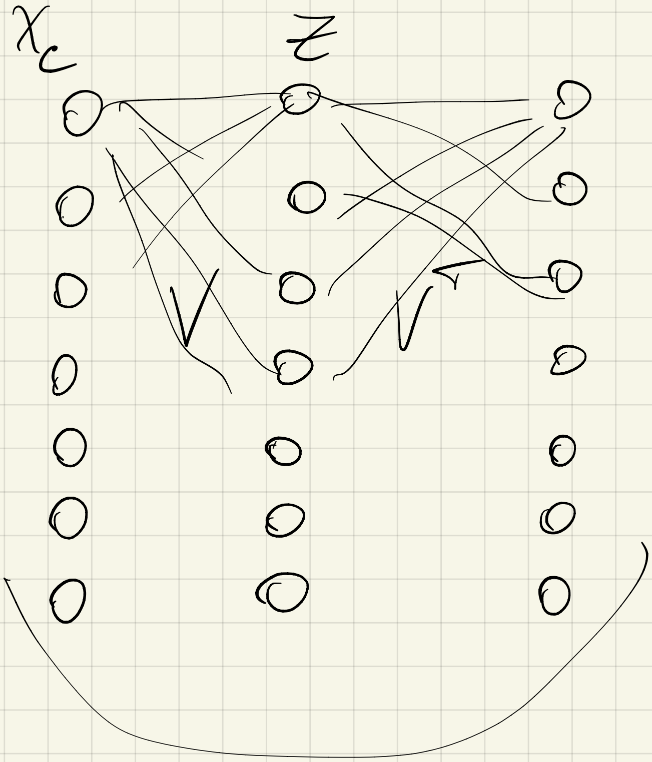
$T \quad (N \times k)$



$$Z = X_c V$$

$$\tilde{X}_c = \underbrace{X_c V^*}_{} V^{*T}$$

$$\tilde{X}_c = \tilde{Z} V^T$$



$h = X_c V \qquad \theta X_c$

$y = h V^T$

$V V^T = I$

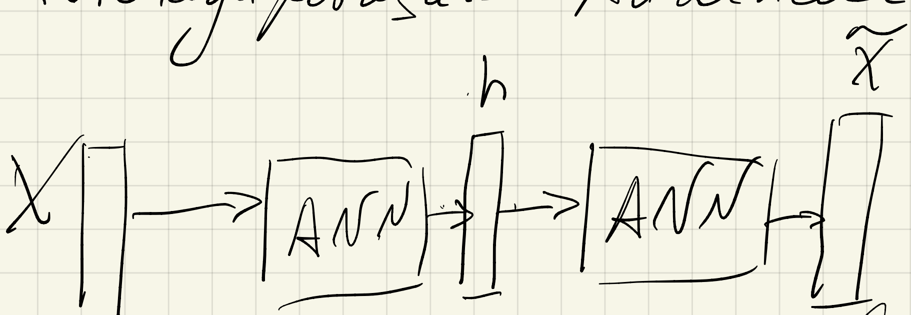
$X_c = X - \overline{X}$

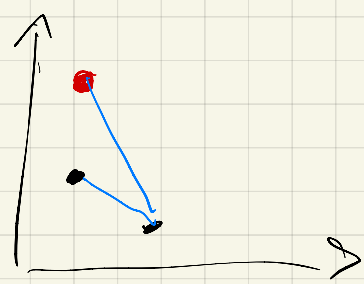
$z \in \mathbb{R}^k \qquad k < d$

$V \quad (d \times k)$

$V^T \quad (k \times d)$

Автокодировщик Autoencoder





$X \in \mathbb{R}^{784}$

$h \in \mathbb{R}^{M}$

X_1

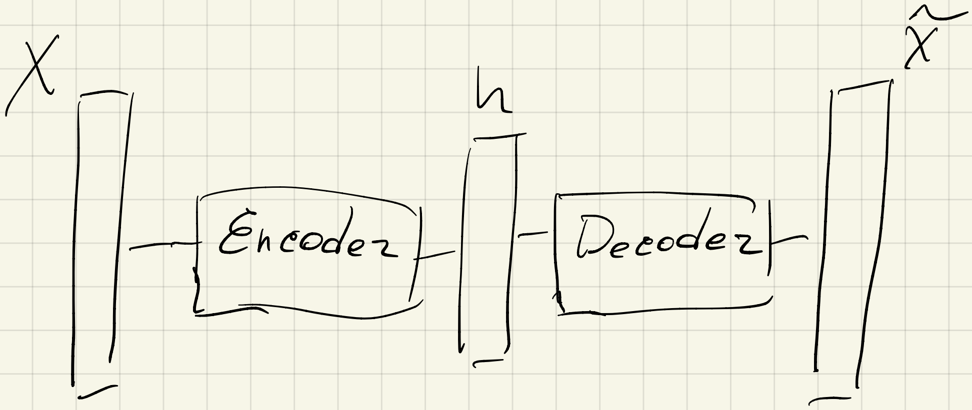
X_2

$$\sqrt{\sum_{i=1}^{D} \left(X_1^{(i)} - X_2^{(i)}\right)^2}$$

$D = 2$

h: hidden representation
embedding

$D = 10000$



$$\mathcal{E}(\cdot): \mathbb{X} \to \mathbb{H} \qquad \mathbb{H} = \mathbb{R}^H$$

$$\mathbb{X} = \mathbb{R}^D \qquad D = 784$$

260000

$$D(\cdot): \mathbb{H} \to \mathbb{X}$$

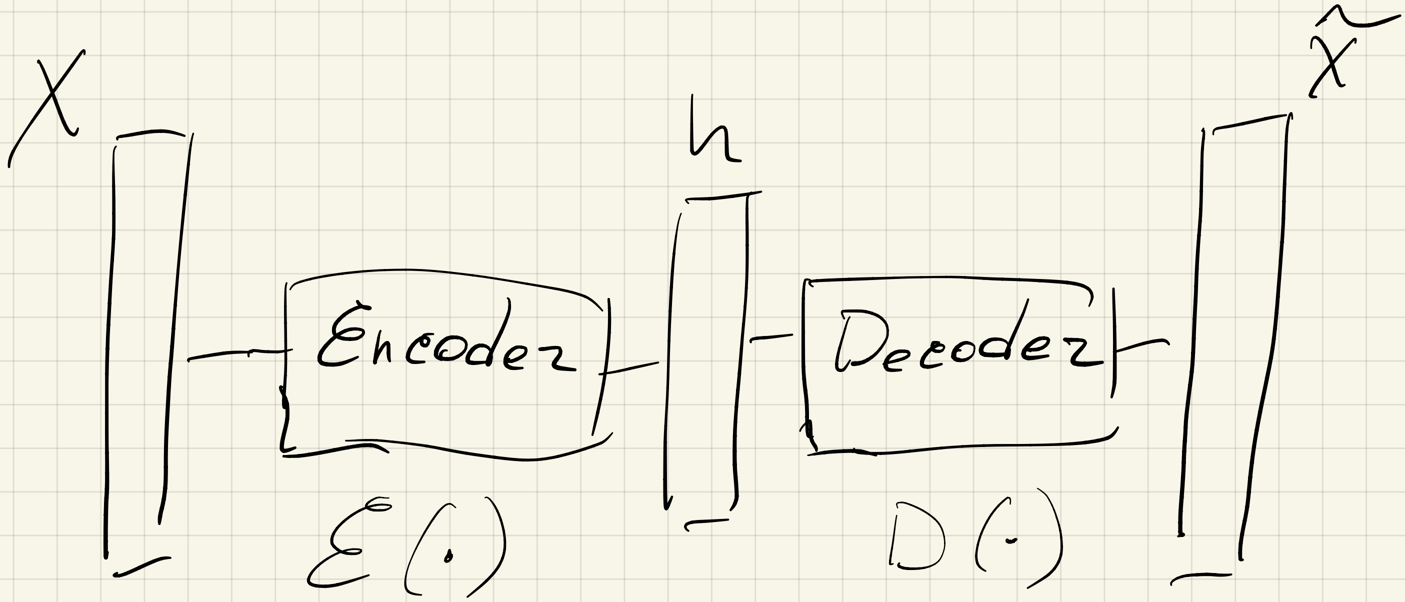
$$\tilde{X} = D(\mathcal{E}(X)) \qquad \tilde{X} = X$$

$$y = X \qquad p = D(\mathcal{E}(X))$$

$$\mathcal{L} = -\frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{784} \left(y \log(p) + (1-y)\log(1-p)\right)$$

$$\mathcal{L} = MAE(X, \tilde{X}) = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{D} |p_{ij} - x_{ij}|$$

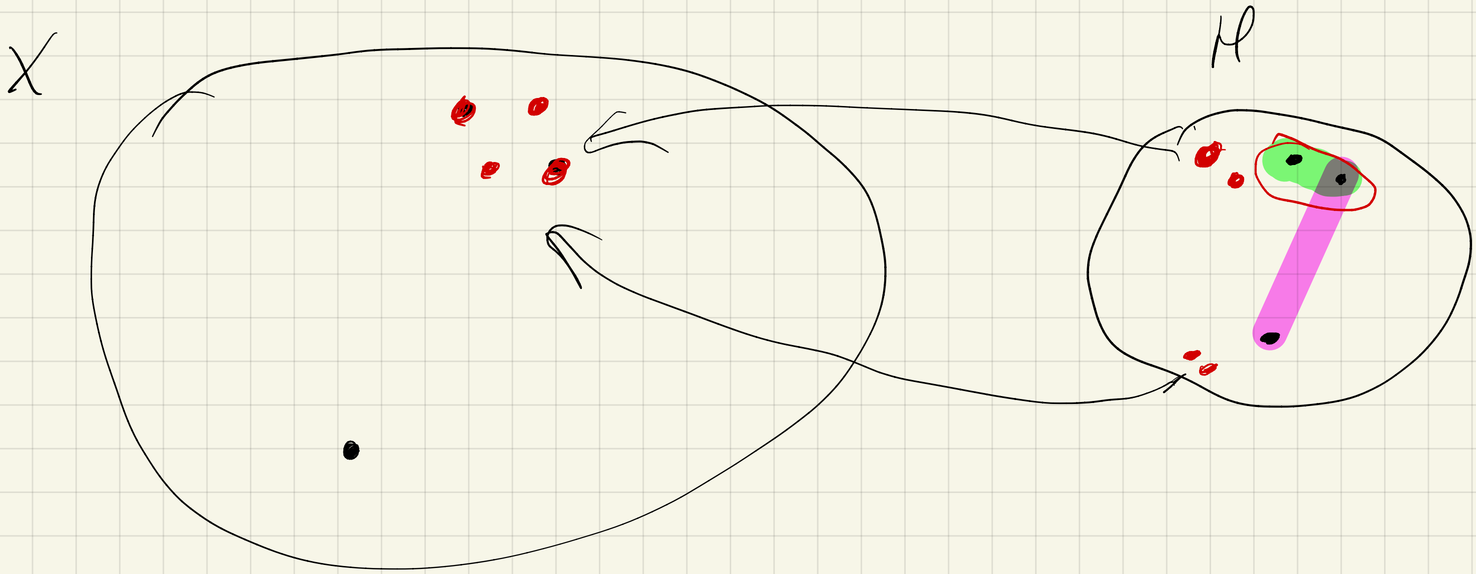
$$\mathcal{L} = MSE(X, \tilde{X}) = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{D} (p_{ij} - x_{ij})^2$$



$h = \mathcal{E}(X)$
$\tilde{X} = D(h)$

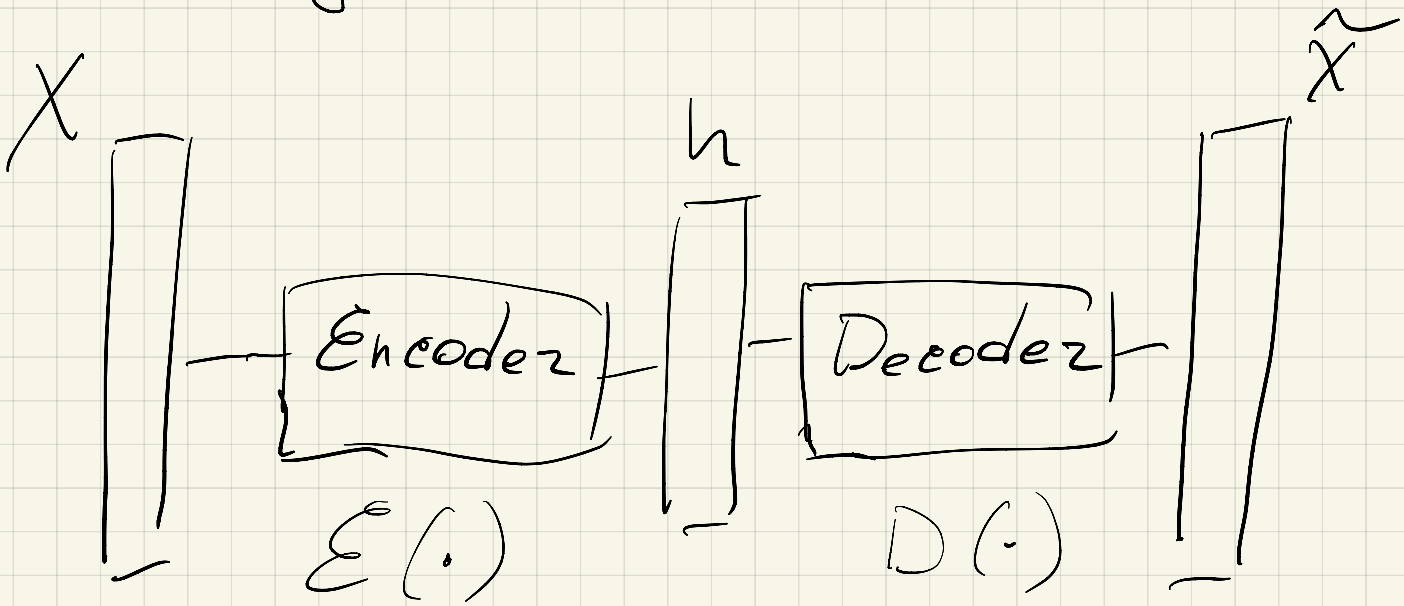
$D(\mathcal{E}(x)) = X$

$h = X\theta_1$
$\tilde{X} = h\theta_2$



$x \in \mathbb{R}^D$

Denoisinng Autoencoder



$X^* = X + \xi$

$\tilde{X} = D(\mathcal{E}(X^*))$

ξ – шум

$\mathcal{L} = MSE(X, D(\mathcal{E}(X^*)))$