Chapter 1: The Real Numbers

1.1 - Significance of the Pythagoras Theorem

(Refer: 1.1 Discussion: The Irrationality of $\sqrt{2}$ from Understanding Analysis by Stephen Abbott, Springer.)

Revisit the idea of Pythagoras Theorem. Ancients had the idea of rational numbers i.e. fractions, because they associated the numbers with lengths. However, Pythagoras brought up the length, $\sqrt{2}$ which couldn't be associated to any number class known by that point.

Theorem 1.1 - The Irrationality of $\sqrt{2}$: There is no rational number whose square is 2.

Motivate how difficult it is to think about irrationals, and real numbers in general. Ask the curious questions.

- What properties does the set of irrational numbers have?
- How do the sets of rational and irrational numbers fit together?
- Is there a kind of symmetry between the rationals and the irrationals, or is there some sense in which we can argue that one type of real number is more common than the other?
- Can all irrational numbers be expressed as algebraic combinations of nth roots and rational numbers, or are there still other irrational numbers beyond those of this form?