CHAPTER

4

ENERGY TRANSFER BY HEAT, WORK, AND MASS

Energy can be transferred to or from a closed system (a fixed mass) in two distinct forms: *heat* and *work.* For control volumes, energy can also be transferred by mass. An energy transfer to or from a closed system is *heat* if it is caused by a temperature difference between the system and its surroundings. Otherwise it is *work,* and it is caused by a force acting through a distance. We start this chapter with a discussion of energy transfer by *heat.* We then introduce various forms of *work,* with particular emphasis on the *moving boundary work* or *P dV work* commonly encountered in reciprocating devices such as automotive engines and compressors. We continue with the *flow work,* which is the work associated with forcing a fluid into or out of a control volume, and show that the combination of the internal energy and the flow work gives the property *enthalpy.* Then we discuss the *conservation of mass principle* and apply it to various systems. Finally, we show that h + ke + pe represents the energy of a flowing fluid per unit of its mass.

CONTENTS

4–1 ▪ HEAT TRANSFER

Energy can cross the boundary of a closed system in two distinct forms: *heat* and *work* (Fig. 4–1). It is important to distinguish between these two forms of energy. Therefore, they will be discussed first, to form a sound basis for the development of the principles of thermodynamics.

We know from experience that a can of cold soda left on a table eventually warms up and that a hot baked potato on the same table cools down (Fig. 4–2). When a body is left in a medium that is at a different temperature, energy transfer takes place between the body and the surrounding medium until thermal equilibrium is established, that is, the body and the medium reach the same temperature. The direction of energy transfer is always from the higher temperature body to the lower temperature one. Once the temperature equality is established, energy transfer stops. In the processes described above, energy is said to be transferred in the form of heat.

Heat is defined as *the form of energy that is transferred between two systems (or a system and its surroundings) by virtue of a temperature difference* (Fig. 4–3). That is, an energy interaction is heat only if it takes place because of a temperature difference. Then it follows that there cannot be any heat transfer between two systems that are at the same temperature.

In daily life, we frequently refer to the sensible and latent forms of internal energy as *heat*, and we talk about the heat content of bodies. In thermodynamics, however, we usually refer to those forms of energy as *thermal energy* to prevent any confusion with *heat transfer*.

Several phrases in common use today—such as heat flow, heat addition, heat rejection, heat absorption, heat removal, heat gain, heat loss, heat storage, heat generation, electrical heating, resistance heating, frictional heating, gas heating, heat of reaction, liberation of heat, specific heat, sensible heat, latent heat, waste heat, body heat, process heat, heat sink, and heat source—are not consistent with the strict thermodynamic meaning of the term *heat*, which limits its use to the *transfer* of thermal energy during a process. However, these phrases are deeply rooted in our vocabulary, and they are used by both ordinary people and scientists without causing any misunderstanding since they are usually interpreted properly instead of being taken literally. (Besides, no acceptable alternatives exist for some of these phrases.) For example, the phrase *body heat* is understood to mean *the thermal energy content* of a body. Likewise, *heat flow* is understood to mean *the transfer of thermal energy,* not

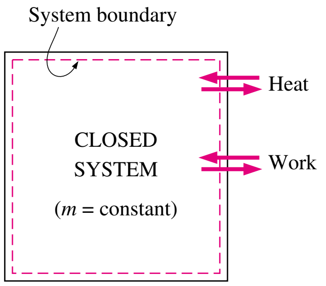

FIGURE 4–1
Energy can cross the boundaries of a closed system in the form of heat and work.

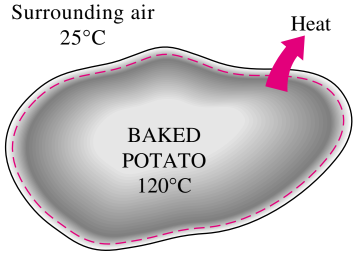

FIGURE 4–2
Heat is transferred from hot bodies to colder ones by virtue of a temperature difference.

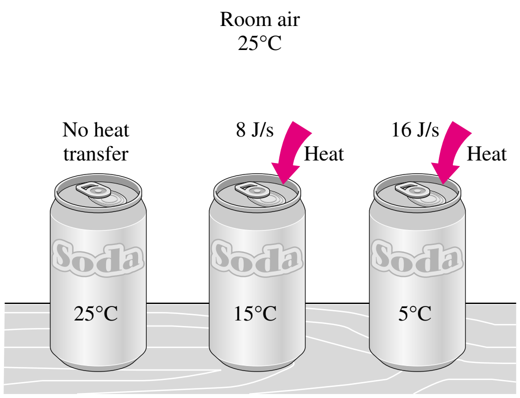

FIGURE 4–3
Temperature difference is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer.

the flow of a fluidlike substance called heat, although the latter incorrect interpretation, which is based on the caloric theory, is the origin of this phrase. Also, the transfer of heat into a system is frequently referred to as *heat addition* and the transfer of heat out of a system as *heat rejection*. Perhaps there are thermodynamic reasons for being so reluctant to replace *heat* by *thermal energy*: It takes less time and energy to say, write, and comprehend *heat* than it does *thermal energy*.

Heat is energy in transition. It is recognized only as it crosses the boundary of a system. Consider the hot baked potato one more time. The potato contains energy, but this energy is heat transfer only as it passes through the skin of the potato (the system boundary) to reach the air, as shown in Fig. 4–4. Once in the surroundings, the transferred heat becomes part of the internal energy of the surroundings. Thus, in thermodynamics, the term *heat* simply means *heat transfer.*

A process during which there is no heat transfer is called an **adiabatic process** (Fig. 4–5). The word *adiabatic* comes from the Greek word *adiabatos,* which means *not to be passed.* There are two ways a process can be adiabatic: Either the system is well insulated so that only a negligible amount of heat can pass through the boundary, or both the system and the surroundings are at the same temperature and therefore there is no driving force (temperature difference) for heat transfer. An adiabatic process should not be confused with an isothermal process. Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means such as work.

As a form of energy, heat has energy units, kJ (or Btu) being the most common one. The amount of heat transferred during the process between two states (states 1 and 2) is denoted by Q_{12}, or just Q. Heat transfer *per unit mass* of a system is denoted q and is determined from

$$q = \frac{Q}{m} \qquad \text{(kJ/kg)} \tag{4–1}$$

Sometimes it is desirable to know the *rate of heat transfer* (the amount of heat transferred per unit time) instead of the total heat transferred over some time interval (Fig. 4–6). The heat transfer rate is denoted $\dot{Q}$, where the overdot stands for the time derivative, or "per unit time." The heat transfer rate $\dot{Q}$ has the unit kJ/s, which is equivalent to kW. When $\dot{Q}$ varies with time, the amount of heat transfer during a process is determined by integrating $\dot{Q}$ over the time interval of the process:

$$Q = \int_{t_1}^{t_2} \dot{Q}\, dt \qquad \text{(kJ)} \tag{4–2}$$

When $\dot{Q}$ remains constant during a process, this relation reduces to

$$Q = \dot{Q}\, \Delta t \qquad \text{(kJ)} \tag{4–3}$$

where $\Delta t = t_2 - t_1$ is the time interval during which the process occurs.

Historical Background on Heat

Heat has always been perceived to be something that produces in us a sensation of warmth, and one would think that the nature of heat is one of the first

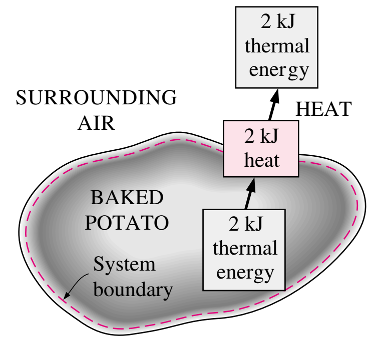

FIGURE 4–4
Energy is recognized as heat transfer only as it crosses the system boundary.

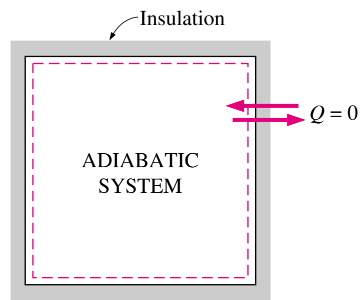

FIGURE 4–5
During an adiabatic process, a system exchanges no heat with its surroundings.

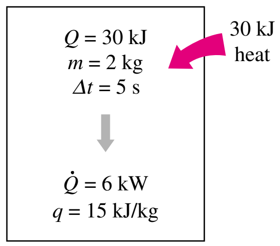

FIGURE 4–6
The relationships among q, Q, and $\dot{Q}$.

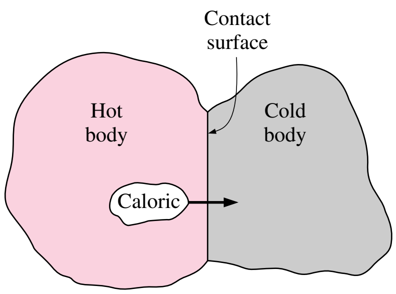

FIGURE 4–7
In the early nineteenth century, heat was thought to be an invisible fluid called the *caloric* that flowed from warmer bodies to the cooler ones.

things understood by mankind. However, it was only in the middle of the nineteenth century that we had a true physical understanding of the nature of heat, thanks to the development at that time of the **kinetic theory,** which treats molecules as tiny balls that are in motion and thus possess kinetic energy. Heat is then defined as the energy associated with the random motion of atoms and molecules. Although it was suggested in the eighteenth and early nineteenth centuries that heat is the manifestation of motion at the molecular level (called the *live force*), the prevailing view of heat until the middle of the nineteenth century was based on the caloric theory proposed by the French chemist Antoine Lavoisier (1744–1794) in 1789. The caloric theory asserts that heat is a fluidlike substance called the **caloric** that is a massless, colorless, odorless, and tasteless substance that can be poured from one body into another (Fig. 4–7). When caloric was added to a body, its temperature increased; and when caloric was removed from a body, its temperature decreased. When a body could not contain any more caloric, much the same way as when a glass of water could not dissolve any more salt or sugar, the body was said to be saturated with caloric. This interpretation gave rise to the terms *saturated liquid* and *saturated vapor* that are still in use today.

The caloric theory came under attack soon after its introduction. It maintained that heat is a substance that could not be created or destroyed. Yet it was known that heat can be generated indefinitely by rubbing one's hands together or rubbing two pieces of wood together. In 1798, the American Benjamin Thompson (Count Rumford) (1754–1814) showed in his papers that heat can be generated continuously through friction. The validity of the caloric theory was also challenged by several others. But it was the careful experiments of the Englishman James P. Joule (1818–1889) published in 1843 that finally convinced the skeptics that heat was not a substance after all, and thus put the caloric theory to rest. Although the caloric theory was totally abandoned in the middle of the nineteenth century, it contributed greatly to the development of thermodynamics and heat transfer.

Heat is transferred by three mechanisms: conduction, convection, and radiation. **Conduction** is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interaction between particles. **Convection** is the transfer of energy between a solid surface and the adjacent fluid that is in motion, and it involves the combined effects of conduction and fluid motion. **Radiation** is the transfer of energy due to the emission of electromagnetic waves (or photons).

4–2 ▪ ENERGY TRANSFER BY WORK

Work, like heat, is an energy interaction between a system and its surroundings. As mentioned earlier, energy can cross the boundary of a closed system in the form of heat or work. Therefore, *if the energy crossing the boundary of a closed system is not heat, it must be work*. Heat is easy to recognize: Its driving force is a temperature difference between the system and its surroundings. Then we can simply say that an energy interaction that is not caused by a temperature difference between a system and its surroundings is work. More specifically, *work is the energy transfer associated with a force acting through a distance*. A rising piston, a rotating shaft, and an electric wire crossing the system boundaries are all associated with work interactions.

Work is also a form of energy transferred like heat and, therefore, has energy units such as kJ. The work done during a process between states 1 and 2 is denoted by W_{12}, or simply W. The work done *per unit mass* of a system is denoted by w and is expressed as

$$w = \frac{W}{m} \qquad \text{(kJ/kg)} \tag{4–4}$$

The work done *per unit time* is called **power** and is denoted $\dot{W}$ (Fig. 4–8). The unit of power is kJ/s, or kW.

Heat and work are *directional quantities*, and thus the complete description of a heat or work interaction requires the specification of both the *magnitude* and *direction*. One way of doing that is to adopt a sign convention. The generally accepted **formal sign convention** for heat and work interactions is as follows: *heat transfer to a system and work done by a system are positive; heat transfer from a system and work done on a system are negative*. Another way is to use the subscripts *in* and *out* to indicate direction (Fig. 4–9). For example, a work input of 5 kJ can be expressed as $W_{in} = 5$ kJ, while a heat loss of 3 kJ can be expressed as $Q_{out} = 3$ kJ. When the direction of a heat or work interaction is not known, we can simply *assume* a direction for the interaction (using the subscript *in* or *out*) and solve for it. A positive result indicates the assumed direction is right. A negative result, on the other hand, indicates that the direction of the interaction is the opposite of the assumed direction. This is just like assuming a direction for an unknown force when solving a statics problem, and reversing the direction when a negative result is obtained for the force. We will use this *intuitive approach* in this book as it eliminates the need to adopt a formal sign convention and the need to carefully assign negative values to some interactions.

Note that a quantity that is transferred to or from a system during an interaction is not a property since the amount of such a quantity depends on more than just the state of the system. Heat and work are *energy transfer mechanisms* between a system and its surroundings, and there are many similarities between them:

1. Both are recognized at the boundaries of a system as they cross the boundaries. That is, both heat and work are *boundary* phenomena.
2. Systems possess energy, but not heat or work.
3. Both are associated with a *process,* not a state. Unlike properties, heat or work has no meaning at a state.
4. Both are *path functions* (i.e., their magnitudes depend on the path followed during a process as well as the end states).

Path functions have **inexact differentials** designated by the symbol δ. Therefore, a differential amount of heat or work is represented by δQ or δW, respectively, instead of dQ or dW. Properties, however, are **point functions** (i.e., they depend on the state only, and not on how a system reaches that state), and they have **exact differentials** designated by the symbol d. A small change in volume, for example, is represented by dV, and the total volume change during a process between states 1 and 2 is

$$\int_1^2 dV = V_2 - V_1 = \Delta V$$

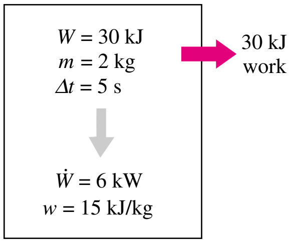

FIGURE 4–8
The relationships among w, W, and $\dot{W}$.

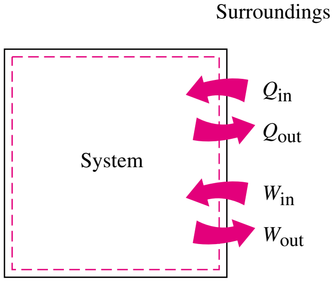

FIGURE 4–9
Specifying the directions of heat and work.

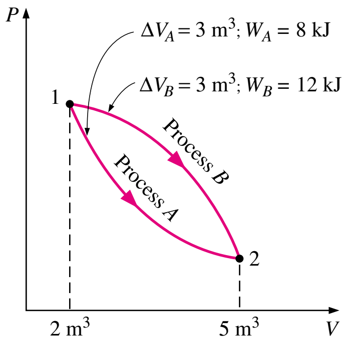

FIGURE 4–10
Properties are point functions; but heat and work are path functions (their magnitudes depend on the path followed).

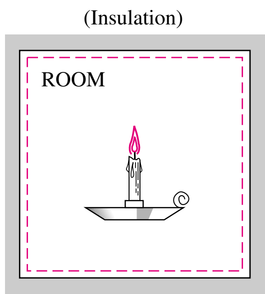

FIGURE 4–11
Schematic for Example 4–1.

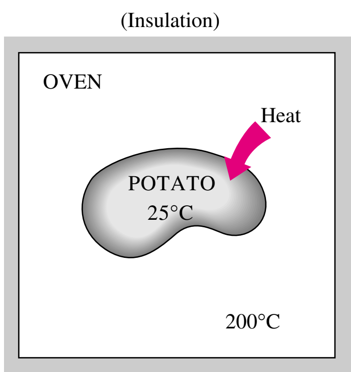

FIGURE 4–12
Schematic for Example 4–2.

That is, the volume change during process 1–2 is always the volume at state 2 minus the volume at state 1, regardless of the path followed (Fig. 4–10). The total work done during process 1–2, however, is

$$\int_1^2 \delta W = W_{12} \qquad (not\ \Delta W)$$

That is, the total work is obtained by following the process path and adding the differential amounts of work (δW) done along the way. The integral of δW *is not* $W_2 - W_1$ (i.e., the work at state 2 minus work at state 1), which is meaningless since work is not a property and systems do not possess work at a state.

EXAMPLE 4–1 Burning of a Candle in an Insulated Room

A candle is burning in a well-insulated room. Taking the room (the air plus the candle) as the system, determine (*a*) if there is any heat transfer during this burning process and (*b*) if there is any change in the internal energy of the system.

SOLUTION (*a*) The interior surfaces of the room form the system boundary, as indicated by the dashed lines in Fig. 4–11. As pointed out earlier, heat is recognized as it crosses the boundaries. Since the room is well insulated, we have an adiabatic system and no heat will pass through the boundaries. Therefore, $Q = 0$ for this process.

(*b*) The internal energy involves energies that exist in various forms (sensible, latent, chemical, nuclear). During the process just described, part of the chemical energy is converted to sensible energy. Since there is no increase or decrease in the total internal energy of the system, $\Delta U = 0$ for this process.

EXAMPLE 4–2 Heating of a Potato in an Oven

A potato initially at room temperature (25°C) is being baked in an oven that is maintained at 200°C, as shown in Fig. 4–12. Is there any heat transfer during this baking process?

SOLUTION This is not a well-defined problem since the system is not specified. Let us assume that we are observing the potato, which will be our system. Then the skin of the potato can be viewed as the system boundary. Part of the energy in the oven will pass through the skin to the potato. Since the driving force for this energy transfer is a temperature difference, this is a heat transfer process.

EXAMPLE 4–3 Heating of an Oven by Work Transfer

A well-insulated electric oven is being heated through its heating element. If the entire oven, including the heating element, is taken to be the system, determine whether this is a heat or work interaction.

SOLUTION For this problem, the interior surfaces of the oven form the system boundary, as shown in Fig. 4–13. The energy content of the oven obviously increases during this process, as evidenced by a rise in temperature. This energy transfer to the oven is not caused by a temperature difference between the oven and the surrounding air. Instead, it is caused by *electrons* crossing the system boundary and thus doing work. Therefore, this is a work interaction.

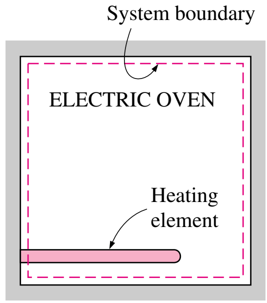

FIGURE 4–13
Schematic for Example 4–3.

EXAMPLE 4–4 Heating of an Oven by Heat Transfer

Answer the question in Example 4–3 if the system is taken as only the air in the oven without the heating element.

SOLUTION This time, the system boundary will include the outer surface of the heating element and will not cut through it, as shown in Fig. 4–14. Therefore, no electrons will be crossing the system boundary at any point. Instead, the energy generated in the interior of the heating element will be transferred to the air around it as a result of the temperature difference between the heating element and the air in the oven. Therefore, this is a heat transfer process.

Discussion For both cases, the amount of energy transfer to the air is the same. These two examples show that the same interaction can be heat or work depending on how the system is selected.

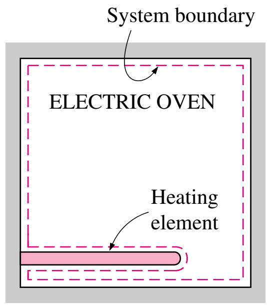

FIGURE 4–14
Schematic for Example 4–4.

Electrical Work

It was pointed out in Example 4–3 that electrons crossing the system boundary do electrical work on the system. In an electric field, electrons in a wire move under the effect of electromotive forces, doing work. When N coulombs of electrical charge move through a potential difference V, the electrical work done is

$$W_e = VN$$

which can also be expressed in the rate form as

$$\dot{W}_e = VI \qquad \text{(W)} \tag{4–5}$$

where $\dot{W}_e$ is the **electrical power** and I is the number of electrical charges flowing per unit time, that is, the *current* (Fig. 4–15). In general, both V and I vary with time, and the electrical work done during a time interval Δt is expressed as

$$W_e = \int_1^2 VI\,dt \qquad \text{(kJ)} \tag{4–6}$$

When both V and I remain constant during the time interval Δt, it reduces to

$$W_e = VI\,\Delta t \qquad \text{(kJ)} \tag{4–7}$$

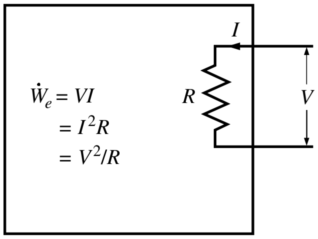

FIGURE 4–15
Electrical power in terms of resistance R, current I, and potential difference V.

4–3 ▪ MECHANICAL FORMS OF WORK

There are several different ways of doing work, each in some way related to a force acting through a distance (Fig. 4–16). In elementary mechanics, the

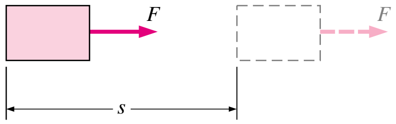

FIGURE 4–16
The work done is proportional to the force applied (F) and the distance traveled (s).

work done by a constant force F on a body displaced a distance s in the direction of the force is given by

$$W = Fs \qquad \text{(kJ)} \tag{4–8}$$

If the force F is not constant, the work done is obtained by adding (i.e., integrating) the differential amounts of work,

$$W = \int_1^2 F\,ds \qquad \text{(kJ)} \tag{4–9}$$

Obviously one needs to know how the force varies with displacement to perform this integration. Equations 4–8 and 4–9 give only the magnitude of the work. The sign is easily determined from physical considerations: The work done on a system by an external force acting in the direction of motion is negative, and work done by a system against an external force acting in the opposite direction to motion is positive.

There are two requirements for a work interaction between a system and its surroundings to exist: (1) there must be a *force* acting on the boundary, and (2) the boundary must *move.* Therefore, the presence of forces on the boundary without any displacement of the boundary does not constitute a work interaction. Likewise, the displacement of the boundary without any force to oppose or drive this motion (such as the expansion of a gas into an evacuated space) is not a work interaction since no energy is transferred.

FIGURE 4–17
If there is no movement, no work is done. (Reprinted with special permission of King Features Syndicate.)

In many thermodynamic problems, mechanical work is the only form of work involved. It is associated with the movement of the boundary of a system or with the movement of the entire system as a whole (Fig. 4–17). Some common forms of mechanical work are discussed next.

1 Moving Boundary Work

One form of mechanical work frequently encountered in practice is associated with the expansion or compression of a gas in a piston-cylinder device. During this process, part of the boundary (the inner face of the piston) moves back and forth. Therefore, the expansion and compression work is often called **moving boundary work,** or simply **boundary work** (Fig. 4–18). Some call it the $P\,dV$ work for reasons explained later. Moving boundary work is the primary form of work involved in *automobile engines*. During their expansion, the combustion gases force the piston to move, which in turn forces the crankshaft to rotate.

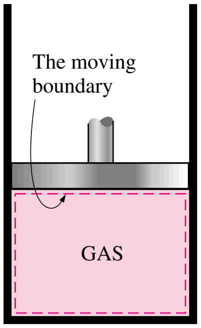

FIGURE 4–18
The work associated with a moving boundary is called *boundary work.*

The moving boundary work associated with real engines or compressors cannot be determined exactly from a thermodynamic analysis alone because the piston usually moves at very high speeds, making it difficult for the gas inside to maintain equilibrium. Then the states through which the system passes during the process cannot be specified, and no process path can be drawn. Work, being a path function, cannot be determined analytically without a knowledge of the path. Therefore, the boundary work in real engines or compressors is determined by direct measurements.

In this section, we analyze the moving boundary work for a *quasi-equilibrium process*, a process during which the system remains in equilibrium at all times. A quasi-equilibrium process, also called a *quasi-static process*, is closely approximated by real engines, especially when the piston moves at low velocities. Under identical conditions, the work output of the engines is found to be a maximum, and the work input to the compressors to be a minimum when quasi-equilibrium processes are used in place of nonquasi-equilibrium processes. Below, the work associated with a moving boundary is evaluated for a quasi-equilibrium process.

Consider the gas enclosed in the piston-cylinder device shown in Fig. 4–19. The initial pressure of the gas is P, the total volume is V, and the cross-sectional area of the piston is A. If the piston is allowed to move a distance ds in a quasi-equilibrium manner, the differential work done during this process is

$$\delta W_b = F\,ds = PA\,ds = P\,dV \tag{4–10}$$

That is, the boundary work in the differential form is equal to the product of the absolute pressure P and the differential change in the volume dV of the system. This expression also explains why the moving boundary work is sometimes called the $P\,dV$ work.

Note in Eq. 4–10 that P is the absolute pressure, which is always positive. However, the volume change dV is positive during an expansion process (volume increasing) and negative during a compression process (volume decreasing). Thus, the boundary work is positive during an expansion process and negative during a compression process. Therefore, Eq. 4–10 can be viewed as an expression for boundary work output, $W_{b,\text{ out}}$. A negative result indicates boundary work input (compression).

The total boundary work done during the entire process as the piston moves is obtained by adding all the differential works from the initial state to the final state:

$$W_b = \int_1^2 P\,dV \qquad \text{(kJ)} \tag{4–11}$$

This integral can be evaluated only if we know the functional relationship between P and V during the process. That is, $P = f(V)$ should be available. Note that $P = f(V)$ is simply the equation of the process path on a P-V diagram.

The quasi-equilibrium expansion process described above is shown on a P-V diagram in Fig. 4–20. On this diagram, the differential area dA is equal to $P\,dV$, which is the differential work. The total area A under the process curve 1–2 is obtained by adding these differential areas:

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P\,dV \tag{4–12}$$

A comparison of this equation with Eq. 4–11 reveals that *the area under the process curve on a P-V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.* (On the P-v diagram, it represents the boundary work done per unit mass.)

A gas can follow several different paths as it expands from state 1 to state 2. In general, each path will have a different area underneath it, and since this area represents the magnitude of the work, the work done will be different for each process (Fig. 4–21). This is expected, since work is a path function (i.e., it depends on the path followed as well as the end states). If work were not a path function, no cyclic devices (car engines, power plants) could operate as work-producing devices. The work produced by these devices during one part

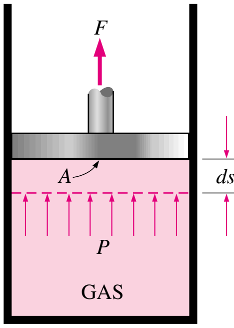

FIGURE 4–19
A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount ds.

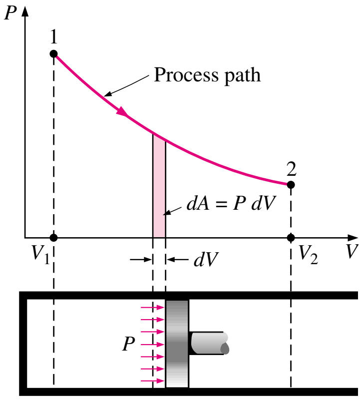

FIGURE 4–20
The area under the process curve on a P-V diagram represents the boundary work.

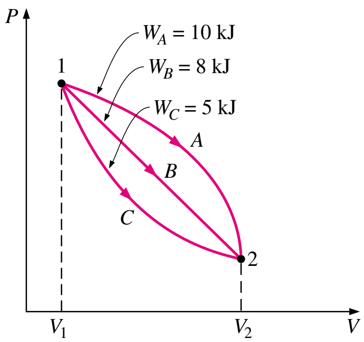

FIGURE 4–21
The boundary work done during a process depends on the path followed as well as the end states.

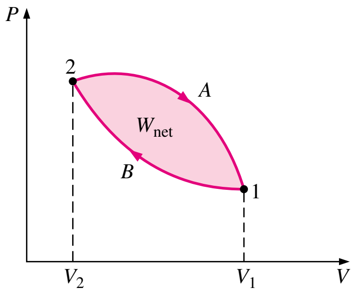

FIGURE 4–22
The net work done during a cycle is the difference between the work done by the system and the work done on the system.

of the cycle would have to be consumed during another part, and there would be no net work output. The cycle shown in Fig. 4–22 produces a net work output because the work done by the system during the expansion process (area under path A) is greater than the work done on the system during the compression part of the cycle (area under path B), and the difference between these two is the net work done during the cycle (the colored area).

If the relationship between P and V during an expansion or a compression process is given in terms of experimental data instead of in a functional form, obviously we cannot perform the integration analytically. But we can always plot the P-V diagram of the process, using these data points, and calculate the area underneath graphically to determine the work done.

Strictly speaking, the pressure P in Eq. 4–11 is the pressure at the inner surface of the piston. It becomes equal to the pressure of the gas in the cylinder only if the process is quasi-equilibrium and thus the entire gas in the cylinder is at the same pressure at any given time. Equation 4–11 can also be used for nonquasi-equilibrium processes provided that the pressure *at the inner face of the piston* is used for P. (Besides, we cannot speak of the pressure of a *system* during a nonquasi-equilibrium process since properties are defined for equilibrium states.) Therefore, we can generalize the boundary work relation by expressing it as

$$W_b = \int_1^2 P_i\, dV \qquad \textbf{(4–13)}$$

where P_i is the pressure at the inner face of the piston.

Note that work is a mechanism for energy interaction between a system and its surroundings, and W_b represents the amount of energy transferred from the system during an expansion process (or to the system during a compression process). Therefore, it has to appear somewhere else and we must be able to account for it since energy is conserved. In a car engine, for example, the boundary work done by the expanding hot gases is used to overcome friction between the piston and the cylinder, to push atmospheric air out of the way, and to rotate the crankshaft. Therefore,

$$W_b = W_{\text{friction}} + W_{\text{atm}} + W_{\text{crank}} = \int_1^2 (F_{\text{friction}} + P_{\text{atm}} A + F_{\text{crank}})\, dx \qquad \textbf{(4–14)}$$

Of course the work used to overcome friction will appear as frictional heat and the energy transmitted through the crankshaft will be transmitted to other components (such as the wheels) to perform certain functions. But note that the energy transferred by the system as work must equal the energy received by the crankshaft, the atmosphere, and the energy used to overcome friction.

The use of the boundary work relation is not limited to the quasi-equilibrium processes of gases only. It can also be used for solids and liquids.

EXAMPLE 4–5 **Boundary Work during a Constant-Volume Process**

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

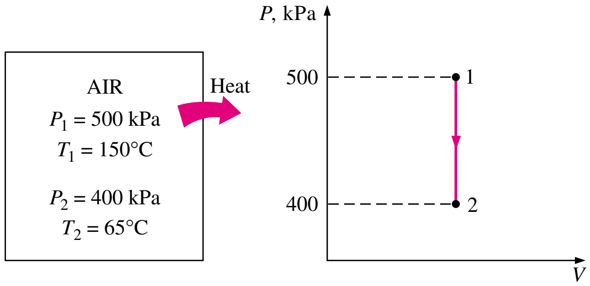

FIGURE 4–23
Schematic and P-V diagram for Example 4–5.

SOLUTION A sketch of the system and the P-V diagram of the process are shown in Fig. 4–23.

Analysis The boundary work can be determined from Eq. 4–11 to be

$$W_b = \int_1^2 P\,\cancel{dV}^{\,0} = 0$$

This is expected since a rigid tank has a constant volume and $dV = 0$ in this equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant-volume process is always zero. This is also evident from the P-V diagram of the process (the area under the process curve is zero).

EXAMPLE 4–6 Boundary Work for a Constant-Pressure Process

A frictionless piston-cylinder device contains 10 lbm of water vapor at 60 psia and 320°F. Heat is now transferred to the steam until the temperature reaches 400°F. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process.

SOLUTION A sketch of the system and the P-v diagram of the process are shown in Fig. 4–24.

Assumption The expansion process is quasi-equilibrium.

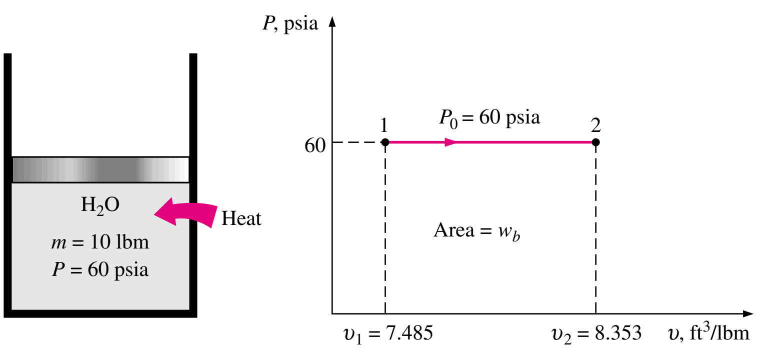

FIGURE 4–24
Schematic and P-v diagram for Example 4–6.

Analysis Even though it is not explicitly stated, the pressure of the steam within the cylinder remains constant during this process since both the atmospheric pressure and the weight of the piston remain constant. Therefore, this is a constant-pressure process, and, from Eq. 4–11

$$W_b = \int_1^2 P\,dV = P_0 \int_1^2 dV = P_0(V_2 - V_1) \qquad \textbf{(4–15)}$$

or

$$W_b = mP_0(v_2 - v_1)$$

since $V = mv$. From the superheated vapor table (Table A–6E), the specific volumes are determined to be $v_1 = 7.485$ ft^3/lbm at state 1 (60 psia, 320°F) and $v_2 = 8.353$ ft^3/lbm at state 2 (60 psia, 400°F). Substituting these values yields

$$W_b = (10\text{ lbm})(60\text{ psia})[(8.353 - 7.485)\text{ ft}^3\text{/lbm}]\left(\frac{1\text{ Btu}}{5.404\text{ psia}\cdot\text{ft}^3}\right)$$

$$= \mathbf{96.4\ Btu}$$

Discussion The positive sign indicates that the work is done by the system. That is, the steam used 96.4 Btu of its energy to do this work. The magnitude of this work could also be determined by calculating the area under the process curve on the *P-V* diagram, which is simply $P_0\,\Delta V$ for this case.

EXAMPLE 4–7 Isothermal Compression of an Ideal Gas

A piston-cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

SOLUTION A sketch of the system and the *P-V* diagram of the process are shown in Fig. 4–25.

Assumptions **1** The compression process is quasi-equilibrium. **2** At the specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.

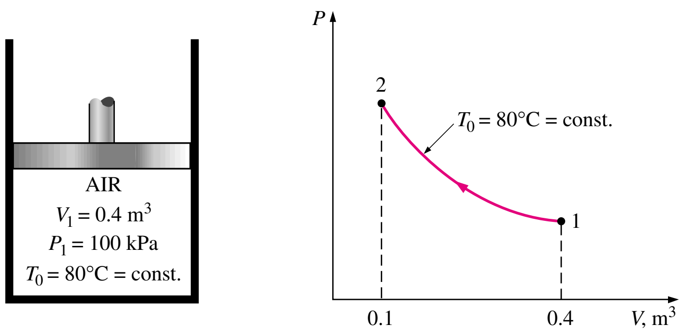

FIGURE 4–25
Schematic and *P-V* diagram for Example 4–7.

Analysis For an ideal gas at constant temperature T_0,

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

where C is a constant. Substituting this into Eq. 4–11, we have

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}\,dV = C\int_1^2 \frac{dV}{V} = C\ln\frac{V_2}{V_1} = P_1V_1\ln\frac{V_2}{V_1} \qquad \textbf{(4–16)}$$

In Eq. 4–16, P_1V_1 can be replaced by P_2V_2 or mRT_0. Also, V_2/V_1 can be replaced by P_1/P_2 for this case since $P_1V_1 = P_2V_2$.

Substituting the numerical values into Eq. 4–16 yields

$$W_b = (100\text{ kPa})(0.4\text{ m}^3)\left(\ln\frac{0.1}{0.4}\right)\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right)$$
$$= \mathbf{-55.45\ kJ}$$

Discussion The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.

Polytropic Process

During actual expansion and compression processes of gases, pressure and volume are often related by $PV^n = C$, where n and C are constants. A process of this kind is called a **polytropic process** (Fig. 4–26). Below we develop a general expression for the work done during a polytropic process. The pressure for a polytropic process can be expressed as

$$P = CV^{-n} \qquad \textbf{(4–17)}$$

Substituting this relation into Eq. 4–11, we obtain

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n} \qquad \textbf{(4–18)}$$

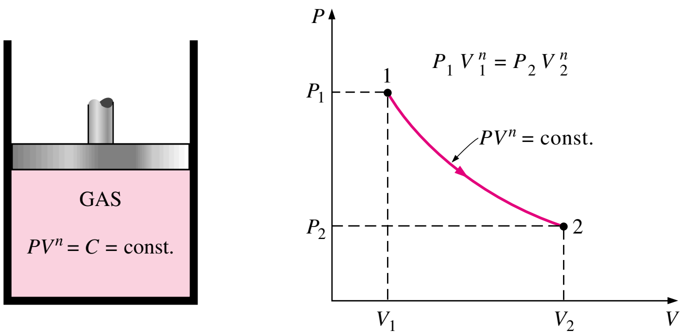

FIGURE 4–26
Schematic and P-V diagram for a polytropic process.

since $C = P_1V_1^n = P_2V_2^n$. For an ideal gas ($PV = mRT$), this equation can also be written as

$$W_b = \frac{mR(T_2 - T_1)}{1 - n} \qquad n \neq 1 \qquad \text{(kJ)} \tag{4–19}$$

For the special case of $n = 1$ the boundary work becomes

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-1}\,dV = PV\ln\left(\frac{V_2}{V_1}\right)$$

For an ideal gas this result is equivalent to the isothermal process discussed in the previous example.

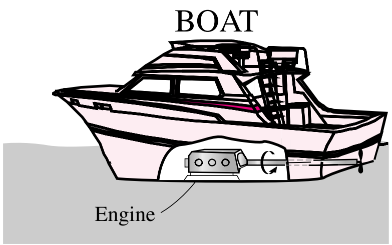

FIGURE 4–27
Energy transmission through rotating shafts is commonly encountered in practice.

2 Shaft Work

Energy transmission with a rotating shaft is very common in engineering practice (Fig. 4–27). Often the torque **T** applied to the shaft is constant, which means that the force F applied is also constant. For a specified constant torque, the work done during n revolutions is determined as follows: A force F acting through a moment arm r generates a torque **T** of (Fig. 4–28)

$$\mathbf{T} = Fr \longrightarrow F = \frac{\mathbf{T}}{r} \tag{4–20}$$

This force acts through a distance s, which is related to the radius r by

$$s = (2\pi r)n \tag{4–21}$$

Then the shaft work is determined from

$$W_{sh} = Fs = \left(\frac{T}{r}\right)(2\pi rn) = 2\pi n\mathbf{T} \qquad \text{(kJ)} \tag{4–22}$$

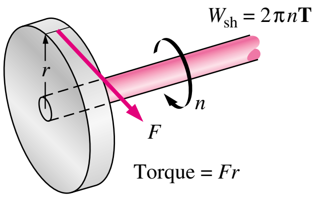

FIGURE 4–28
Shaft work is proportional to the torque applied and the number of revolutions of the shaft.

The power transmitted through the shaft is the shaft work done per unit time, which can be expressed as

$$\dot{W}_{sh} = 2\pi\dot{n}\mathbf{T} \qquad \text{(kW)} \tag{4–23}$$

where $\dot{n}$ is the number of revolutions per unit time.

EXAMPLE 4–8 Power Transmission by the Shaft of a Car

Determine the power transmitted through the shaft of a car when the torque applied is 200 N · m and the shaft rotates at a rate of 4000 revolutions per minute (rpm).

SOLUTION The torque and the rpm for a car engine are given. The power transmitted is to be determined.

Analysis A sketch of the car is given in Fig. 4–29. The shaft power is determined directly from

$$\dot{W}_{sh} = 2\pi\dot{n}\mathbf{T} = (2\pi)\left(4000\,\frac{1}{\text{min}}\right)(200\text{ N}\cdot\text{m})\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right)$$
$$= \mathbf{83.8\ kW}\text{ (or 112.3 hp)}$$

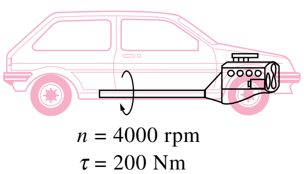

FIGURE 4–29
Schematic for Example 4–8.

3 Spring Work

It is common knowledge that when a force is applied on a spring, the length of the spring changes (Fig. 4–30). When the length of the spring changes by a differential amount dx under the influence of a force F, the work done is

$$\delta W_{\text{spring}} = F\,dx \qquad \textbf{(4–24)}$$

To determine the total spring work, we need to know a functional relationship between F and x. For linear elastic springs, the displacement x is proportional to the force applied (Fig. 4–31). That is,

$$F = kx \qquad \text{(kN)} \qquad \textbf{(4–25)}$$

where k is the spring constant and has the unit kN/m. The displacement x is measured from the undisturbed position of the spring (that is, $x = 0$ when $F = 0$). Substituting Eq. 4–25 into Eq. 4–24 and integrating yield

$$W_{\text{spring}} = \tfrac{1}{2}k(x_2^2 - x_1^2) \qquad \text{(kJ)} \qquad \textbf{(4–26)}$$

where x_1 and x_2 are the initial and the final displacements of the spring, respectively, measured from the undisturbed position of the spring.

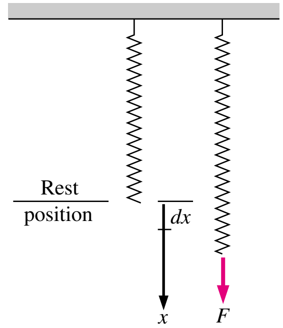

FIGURE 4–30
Elongation of a spring under the influence of a force.

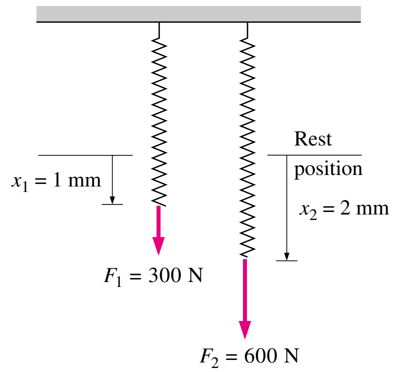

FIGURE 4–31
The displacement of a linear spring doubles when the force is doubled.

EXAMPLE 4–9 **Expansion of a Gas against a Spring**

A piston-cylinder device contains 0.05 m^3 of a gas initially at 200 kPa. At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2, determine (*a*) the final pressure inside the cylinder, (*b*) the total work done by the gas, and (*c*) the fraction of this work done against the spring to compress it.

SOLUTION A sketch of the system and the *P-V* diagram of the process are shown in Fig. 4–32.

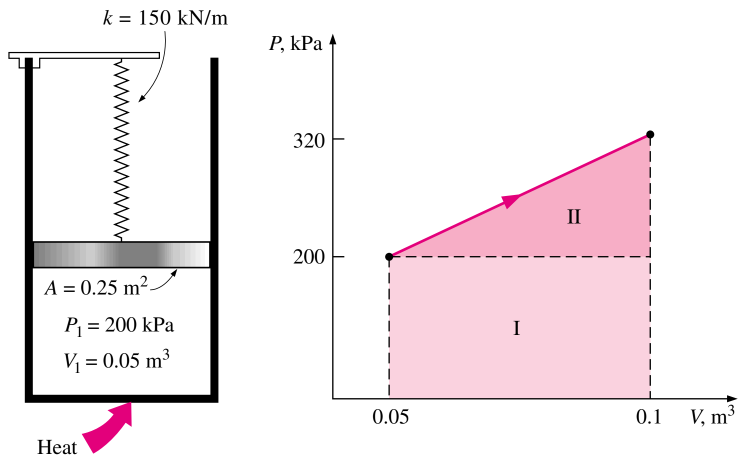

FIGURE 4–32
Schematic and *P-V* diagram for Example 4–9.

Assumptions **1** The expansion process is quasi-equilibrium. **2** The spring is linear in the range of interest.

Analysis (*a*) The enclosed volume at the final state is

$$V_2 = 2V_1 = (2)(0.05\ \text{m}^3) = 0.1\ \text{m}^3$$

Then the displacement of the piston (and of the spring) becomes

$$x = \frac{\Delta V}{A} = \frac{(0.1 - 0.05)\ \text{m}^3}{0.25\ \text{m}^2} = 0.2\ \text{m}$$

The force applied by the linear spring at the final state is

$$F = kx = (150\ \text{kN/m})(0.2\ \text{m}) = 30\ \text{kN}$$

The additional pressure applied by the spring on the gas at this state is

$$P = \frac{F}{A} = \frac{30\ \text{kN}}{0.25\ \text{m}^2} = 120\ \text{kPa}$$

Without the spring, the pressure of the gas would remain constant at 200 kPa while the piston is rising. But under the effect of the spring, the pressure rises linearly from 200 kPa to

$$200 + 120 = \mathbf{320\ kPa}$$

at the final state.

(*b*) An easy way of finding the work done is to plot the process on a *P-V* diagram and find the area under the process curve. From Fig. 4–32 the area under the process curve (a trapezoid) is determined to be

$$W = \text{area} = \frac{(200 + 320)\ \text{kPa}}{2}\,[(0.1 - 0.05)\ \text{m}^3]\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) = \mathbf{13\ kJ}$$

Note that the work is done by the system.

(*c*) The work represented by the rectangular area (region I) is done against the piston and the atmosphere, and the work represented by the triangular area (region II) is done against the spring. Thus,

$$W_{\text{spring}} = \tfrac{1}{2}[(320 - 200)\ \text{kPa}](0.05\ \text{m}^3)\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) = \mathbf{3\ kJ}$$

This result could also be obtained from Eq. 4–26:

$$W_{\text{spring}} = \tfrac{1}{2}k(x_2^2 - x_1^2) = \tfrac{1}{2}(150\ \text{kN/m})[(0.2\ \text{m})^2 - 0^2]\left(\frac{1\ \text{kJ}}{1\ \text{kN}\cdot\text{m}}\right) = 3\ \text{kJ}$$

4 Other Mechanical Forms of Work

There are many other forms of mechanical work. Next we introduce some of them briefly.

Work Done on Elastic Solid Bars

Solids are often modeled as linear springs because under the action of a force they contract or elongate, as shown in Fig. 4–33, and when the force is lifted, they return to their original lengths, like a spring. This is true as long as the force is in the elastic range, that is, not large enough to cause permanent (plastic) deformations. Therefore, the equations given for a linear spring can also be used for elastic solid bars. Alternately, we can determine the work associated with the expansion or contraction of an elastic solid bar by replacing pressure P by its counterpart in solids, *normal stress* $\sigma_n = F/A$, in the boundary work expression:

$$W_{\text{elastic}} = \int_1^2 \sigma_n \, dV = \int_1^2 \sigma_n A \, dx \qquad \text{(kJ)} \tag{4–27}$$

where A is the cross-sectional area of the bar. Note that the normal stress has pressure units.

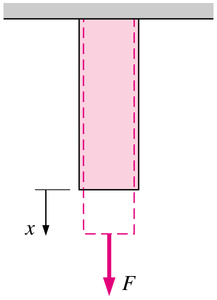

FIGURE 4–33
Solid bars behave as springs under the influence of a force.

Work Associated with the Stretching of a Liquid Film

Consider a liquid film such as soap film suspended on a wire frame (Fig. 4–34). We know from experience that it will take some force to stretch this film by the movable portion of the wire frame. This force is used to overcome the microscopic forces between molecules at the liquid–air interfaces. These microscopic forces are perpendicular to any line in the surface, and the force generated by these forces per unit length is called the **surface tension** σ_s, whose unit is N/m. Therefore, the work associated with the stretching of a film is also called *surface tension work.* It is determined from

$$W_{\text{surface}} = \int_1^2 \sigma_s \, dA \qquad \text{(kJ)} \tag{4–28}$$

where $dA = 2b\,dx$ is the change in the surface area of the film. The factor 2 is due to the fact that the film has two surfaces in contact with air. The force acting on the movable wire as a result of surface tension effects is $F = 2b\sigma_s$ where σ_s is the surface tension force per unit length.

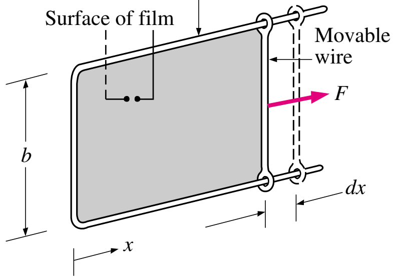

FIGURE 4–34
Stretching a liquid film with a movable wire.

Work Done to Raise or to Accelerate a Body

When a body is raised in a gravitational field, its potential energy increases. Likewise, when a body is accelerated, its kinetic energy increases. The conservation of energy principle requires that an equivalent amount of energy must be transferred to the body being raised or accelerated. Remember that energy can be transferred to a given mass by heat and work, and the energy transferred in this case obviously is not heat since it is not driven by a temperature difference. Therefore, it must be work. Then we conclude that (1) the work transfer needed to raise a body is equal to the change in the potential energy of the body, and (2) the work transfer needed to accelerate a body is equal to the change in the kinetic energy of the body (Fig. 4–35). Similarly, the potential or kinetic energy of a body represents the work that can be obtained from the body as it is lowered to the reference level or decelerated to zero velocity.

This discussion together with the consideration for friction and other losses form the basis for determining the required power rating of motors used to

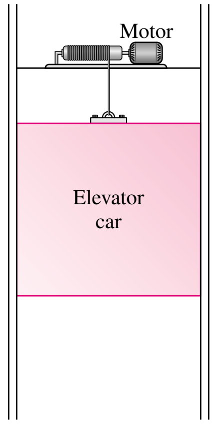

FIGURE 4–35
The energy transferred to a body while being raised is equal to the change in its potential energy.

drive devices such as elevators, escalators, conveyor belts, and ski lifts. It also plays a primary role in the design of automotive and aircraft engines, and in the determination of the amount of hydroelectric power that can be produced from a given water reservoir, which is simply the potential energy of the water relative to the location of the hydraulic turbine.

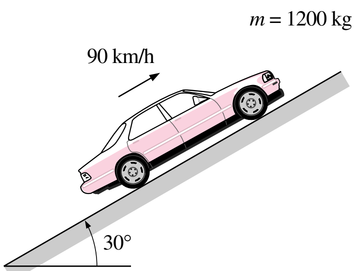

FIGURE 4–36
Schematic for Example 4–10.

EXAMPLE 4–10 Power Needs of a Car to Climb a Hill

Consider a 1200-kg car cruising steadily on a level road at 90 km/h. Now the car starts climbing a hill that is sloped 30° from the horizontal (Fig. 4–36). If the velocity of the car is to remain constant during climbing, determine the additional power that must be delivered by the engine.

SOLUTION A car is to climb a hill while maintaining a constant velocity. The additional power needed is to be determined.

Analysis The additional power required is simply the work that needs to be done per unit time to raise the elevation of the car, which is equal to the change in the potential energy of the car per unit time:

$$\dot{W}_g = mg\,\Delta z/\Delta t = mg\,\mathcal{V}_{\text{vertical}}$$
$$= (1200\text{ kg})(9.81\text{ m/s}^2)(90\text{ km/h})(\sin 30°)\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)$$
$$= 147\text{ kJ/s} = \mathbf{147\text{ kW}} \qquad (\text{or } 197\text{ hp})$$

Discussion Note that the car engine will have to produce almost 200 hp of additional power while climbing the hill if the car is to maintain its velocity.

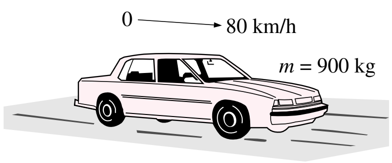

FIGURE 4–37
Schematic for Example 4–11.

EXAMPLE 4–11 Power Needs of a Car to Accelerate

Determine the power required to accelerate a 900-kg car shown in Fig. 4–37 from rest to a velocity of 80 km/h in 20 s on a level road.

SOLUTION The power required to accelerate a car to a specified velocity is to be determined.

Analysis The work needed to accelerate a body is simply the change in the kinetic energy of the body,

$$W_a = \tfrac{1}{2}m(\mathcal{V}_2^2 - \mathcal{V}_1^2) = \tfrac{1}{2}(900\text{ kg})\left[\left(\frac{80{,}000\text{ m}}{3600\text{ s}}\right)^2 - 0^2\right]\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)$$
$$= 222\text{ kJ}$$

The average power is determined from

$$\dot{W}_a = \frac{W_a}{\Delta t} = \frac{222\text{ kJ}}{20\text{ s}} = \mathbf{11.1\text{ kW}} \qquad (\text{or } 14.9\text{ hp})$$

Discussion This is in addition to the power required to overcome friction, rolling resistance, and other imperfections.

4–4 ▪ NONMECHANICAL FORMS OF WORK

The treatment in Section 4–3 represents a fairly comprehensive coverage of mechanical forms of work. But some work modes encountered in practice are not mechanical in nature. However, these nonmechanical work modes can be treated in a similar manner by identifying a *generalized force F* acting in the direction of a *generalized displacement x.* Then the work associated with the differential displacement under the influence of this force is determined from $\delta W = F\,dx$.

Some examples of nonmechanical work modes are **electrical work,** where the generalized force is the *voltage* (the electrical potential) and the generalized displacement is the *electrical charge,* as discussed earlier; **magnetic work,** where the generalized force is the *magnetic field strength* and the generalized displacement is the total *magnetic dipole moment;* and **electrical polarization work,** where the generalized force is the *electric field strength* and the generalized displacement is the *polarization of the medium* (the sum of the electric dipole rotation moments of the molecules). Detailed consideration of these and other nonmechanical work modes can be found in specialized books on these topics.

4–5 ▪ CONSERVATION OF MASS PRINCIPLE

The conservation of mass principle is one of the most fundamental principles in nature. We are all familiar with this principle, and it is not difficult to understand. As the saying goes, you cannot have your cake and eat it, too! A person does not have to be an engineer to figure out how much vinegar-and-oil dressing he is going to have if he mixes 100 g of oil with 25 g of vinegar. Even chemical equations are balanced on the basis of the conservation of mass principle. When 16 kg of oxygen reacts with 2 kg of hydrogen, 18 kg of water is formed (Fig. 4–38). In an electrolysis process, this water will separate back to 2 kg of hydrogen and 16 kg of oxygen.

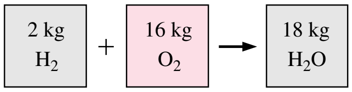

FIGURE 4–38
Mass is conserved even during chemical reactions.

Mass, like energy, is a conserved property, and it cannot be created or destroyed. However, mass m and energy E can be converted to each other according to the famous formula proposed by Einstein:

$$E = mc^2 \qquad \textbf{(4–29)}$$

where c is the speed of light. This equation suggests that the mass of a system will change when its energy changes. However, for all energy interactions encountered in practice, with the exception of nuclear reactions, the change in mass is extremely small and cannot be detected by even the most sensitive devices. For example, when 1 kg of water is formed from oxygen and hydrogen, the amount of energy released is 15,879 kJ, which corresponds to a mass of 1.76×10^{-10} kg. A mass of this magnitude is beyond the accuracy required by practically all engineering calculations and thus can be disregarded.

For *closed systems,* the conservation of mass principle is implicitly used by requiring that the mass of the system remain constant during a process. For *control volumes,* however, mass can cross the boundaries, and so we must keep track of the amount of the mass entering and leaving the control volume (Fig. 4–39).

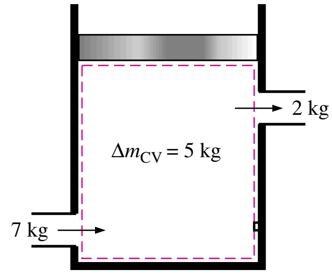

FIGURE 4–39
Conservation of mass principle for a control volume.

Mass and Volume Flow Rates

The amount of mass flowing through a cross section per unit time is called the **mass flow rate** and is denoted $\dot{m}$. Again the dot over a symbol is used to indicate a *quantity per unit time.*

A fluid flows in or out of a control volume through pipes (or ducts). The mass flow rate of a fluid flowing in a pipe is proportional to the cross-sectional area A of the pipe, the density ρ, and the velocity $\mathcal{V}$ of the fluid. The mass flow rate through a differential area dA can be expressed as

$$d\dot{m} = \rho \mathcal{V}_n \, dA \qquad \textbf{(4–30)}$$

where $\mathcal{V}_n$ is the velocity component normal to dA. The mass flow rate through the entire cross-sectional area of the pipe or duct is obtained by integration:

$$\dot{m} = \int_A \rho \mathcal{V}_n \, dA \qquad \text{(kg/s)} \qquad \textbf{(4–31)}$$

In most practical applications, the flow of a fluid through a pipe or duct can be approximated to be *one-dimensional flow,* and thus the properties can be assumed to vary in *one* direction only (the direction of flow). As a result, all properties are *uniform* at any cross section normal to the flow direction, and the properties are assumed to have *bulk average values* over the cross section. However, the values of the properties at a cross section *may* change with time unless the flow is steady.

The one-dimensional-flow approximation has little impact on most properties of a fluid flowing in a pipe or duct such as temperature, pressure, and density since these properties usually remain constant over the cross section. This is not the case for *velocity,* however, whose value varies from zero at the wall to a maximum at the center because of the viscous effects (friction between fluid layers). Under the one-dimensional-flow assumption, the velocity is assumed to be constant across the entire cross section at some equivalent average value (Fig. 4–40). Then the integration in Eq. 4–31 can be performed for one-dimensional flow to yield

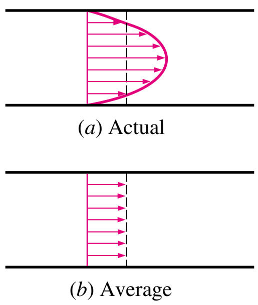

FIGURE 4–40
Actual and mean velocity profiles for flow in a pipe (the mass flow rate is the same for both cases).

$$\dot{m} = \rho \mathcal{V}_m A \qquad \text{(kg/s)} \qquad \textbf{(4–32)}$$

where

ρ = density of fluid, kg/m^3 ($= 1/v$)
$\mathcal{V}_m$ = mean fluid velocity normal to A, m/s
A = cross-sectional area normal to flow direction, m^2

The volume of the fluid flowing through a cross section per unit time is called the **volume flow rate** $\dot{V}$ (Fig. 4–41) and is given by

$$\dot{V} = \int_A \mathcal{V}_n \, dA = \mathcal{V}_m A \qquad \text{(m}^3\text{/s)} \qquad \textbf{(4–33)}$$

The mass and volume flow rates are related by

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v} \qquad \textbf{(4–34)}$$

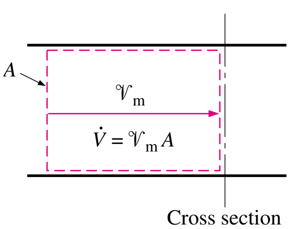

FIGURE 4–41
The volume flow rate is the volume of fluid flowing through a cross section per unit time.

This relation is analogous to $m = V/v$, which is the relation between the mass and the volume of a fluid in a container.

For simplicity, we drop the subscript on the mean velocity. Unless otherwise stated, $\mathcal{V}$ denotes the mean velocity in the flow direction. Also, A denotes the cross-sectional area normal to the flow direction.

Conservation of Mass Principle

The **conservation of mass principle** can be expressed as: *net mass transfer to or from a system during a process is equal to the net change (increase or decrease) in the total mass of the system during that process.* That is,

$$\left(\begin{array}{c}\text{Total mass}\\ \text{entering the system}\end{array}\right) - \left(\begin{array}{c}\text{Total mass}\\ \text{leaving the system}\end{array}\right) = \left(\begin{array}{c}\text{Net change in mass}\\ \text{within the system}\end{array}\right)$$

or

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \qquad \text{(kg)} \tag{4–35}$$

where $\Delta m_{\text{system}} = m_{\text{final}} - m_{\text{initial}}$ is the change in the mass of the system during the process (Fig. 4–42). It can also be expressed in the *rate form* as

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = dm_{\text{system}}/dt \qquad \text{(kg/s)} \tag{4–36}$$

where $\dot{m}_{\text{in}}$ and $\dot{m}_{\text{out}}$ are the total rates of mass flow into and out of the system and dm_{system}/dt is the rate of change of mass within the system boundaries. The relations above are often referred to as the **mass balance** and are applicable to any system undergoing any kind of process.

The mass balance for a control volume can also be expressed more explicitly as

$$\sum m_i - \sum m_e = (m_2 - m_1)_{\text{system}} \tag{4–37}$$

and

$$\sum \dot{m}_i - \sum \dot{m}_e = dm_{\text{system}}/dt \tag{4–38}$$

where i = inlet; e = exit; 1 = initial state and 2 = final state of the control volume; and the summation signs are used to emphasize that all the inlets and exits are to be considered.

When the properties at the inlets and the exits as well as within the control volume are not uniform, the mass flow rate can be expressed in the differential form as $d\dot{m} = \rho \mathcal{V}_n\, dA$. Then the general rate form of the mass balance (Eq. 4–38) can be expressed as

$$\sum \int_{A_i} (\rho \mathcal{V}_n\, dA)_i - \sum \int_{A_e} (\rho \mathcal{V}_n\, dA)_e = \frac{d}{dt}\int_V (\rho\, dV)_{CV} \tag{4–39}$$

to account for the variation of properties. The integration of $dm_{\text{CV}} = \rho\, dV$ on the right-hand side over the volume of the control volume gives the total mass contained within the control volume at time t.

The conservation of mass principle is based on experimental observations and requires every bit of mass to be accounted for during a process. A person who can balance a checkbook (by keeping track of deposits and withdrawals, or simply by observing the "conservation of money" principle) should have no difficulty in applying the conservation of mass principle to engineering systems. The conservation of mass equation is often referred to as the **continuity equation** in fluid mechanics.

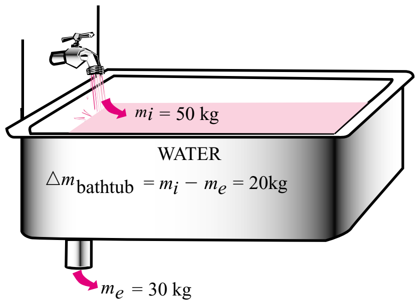

FIGURE 4–42
Conservation of mass principle for an ordinary bathtub.

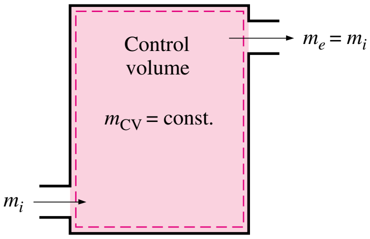

FIGURE 4–43
During a steady-flow process, the amount of mass entering a control volume equals the amount of mass leaving.

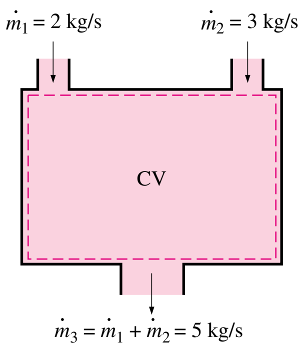

FIGURE 4–44
Conservation of mass principle for a two-inlet–one-exit steady-flow system.

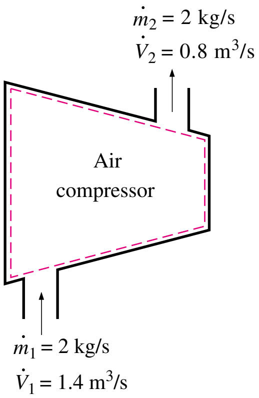

FIGURE 4–45
During a steady-flow process, volume flow rates are not necessarily conserved.

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant). Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it (Fig. 4–43). For a garden hose nozzle, for example, the amount of water entering the nozzle is equal to the amount of water leaving it in steady operation.

When dealing with steady-flow processes, we are not interested in the amount of mass that flows in or out of a device over time; instead, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate* $\dot{m}$. The **conservation of mass principle** for a general steady-flow system with multiple inlets and exits can be expressed in the rate form as (Fig. 4–44)

$$\begin{pmatrix}\text{Total mass entering CV}\\ \text{per unit time}\end{pmatrix} = \begin{pmatrix}\text{Total mass leaving CV}\\ \text{per unit time}\end{pmatrix}$$

or

Steady Flow: $$\sum \dot{m}_i = \sum \dot{m}_e \qquad \text{(kg/s)} \tag{4–40}$$

where the subscript i stands for inlet and e for exit. Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one exit). For these cases, we denote the inlet state by the subscript 1 and the exit state by the subscript 2. We also drop the summation signs. Then Eq. 4–40 reduces, for *single-stream steady-flow systems*, to

Steady Flow (single stream): $$\dot{m}_1 = \dot{m}_2 \longrightarrow \rho_1 \mathcal{V}_1 A_1 = \rho_2 \mathcal{V}_2 A_2 \tag{4–41}$$

Special Case: Incompressible Flow (ρ = constant)

The conservation of mass relations above can be simplified even further when the fluid is incompressible, which is usually the case for liquids, and sometimes for gases. Canceling the density from both sides of the steady-flow relations gives

Steady Incompressible Flow: $$\sum \dot{V}_i = \sum \dot{V}_e \qquad (\text{m}^3/\text{s}) \tag{4–42}$$

For single-stream steady-flow systems it becomes

Steady Incompressible Flow (single stream): $$\dot{V}_1 = \dot{V}_2 \longrightarrow \mathcal{V}_1 A_1 = \mathcal{V}_2 A_2 \tag{4–43}$$

It should always be kept in mind that there is no such thing as a "conservation of volume" principle. Therefore, the volume flow rates into and out of a steady-flow device may be different. The volume flow rate at the exit of an air compressor will be much less than that at the inlet even though the mass flow rate of air through the compressor is constant (Fig. 4–45). This is due to the higher density of air at the compressor exit. For liquid flow, however, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible (constant-density) substances. Water flow through the nozzle of a garden hose is an example for the latter case.

EXAMPLE 4–12 Water Flow through a Garden Hose Nozzle

A garden hose attached with a nozzle is used to fill a 10-gallon bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit (Fig. 4–46). If it takes 50 s to fill the bucket with water, determine (*a*) the volume and mass flow rates of water through the hose, and (*b*) the mean velocity of water at the nozzle exit.

SOLUTION A garden hose is used to fill water buckets. The volume and mass flow rates of water and the exit velocity are to be determined.

Assumptions **1** Water is an incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis (*a*) Noting that 10 gallons of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}}\left(\frac{3.7854 \text{ L}}{1 \text{ gal}}\right) = \mathbf{0.757\ L/s}$$
$$\dot{m} = \rho\dot{V} = (1 \text{ kg/L})\,(0.757 \text{ L/s}) = \mathbf{0.757\ kg/s}$$

(*b*) The cross-sectional area of the nozzle exit is

$$A_e = \pi r_e^2 = \pi\,(0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the velocity of water at the nozzle exit becomes

$$\mathcal{V}_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = \mathbf{15.1\ m/s}$$

Discussion It can be shown that the mean velocity in the hose is 2.4 m/s. Therefore, the nozzle increases the water velocity by over 6 times.

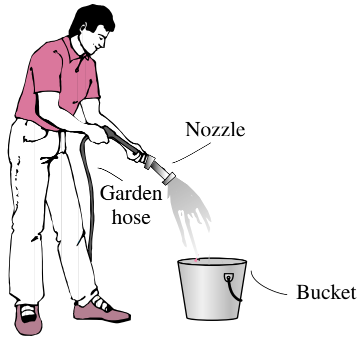

FIGURE 4–46
Schematic for Example 4–12.

EXAMPLE 4–13 Discharge of Water from a Tank

A 4-ft-high 4-ft-diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams out (Fig. 4–47). The mean velocity of the jet is given by $\mathcal{V} = \sqrt{2gh}$ where *h* is the height of water in the tank measured from the center of the hole (a variable) and *g* is the gravitational acceleration. Determine how long it will take for the water level in the tank to drop to 2 ft level from the bottom.

SOLUTION The plug near the bottom of a water tank is pulled out. The time it will take for half of the water in the tank to empty is to be determined.

Assumptions **1** Water is an incompressible substance. **2** The distance between the bottom of the tank and the center of the hole is negligible compared to the total water height. **3** The gravitational acceleration is 32.2 ft/s^2.

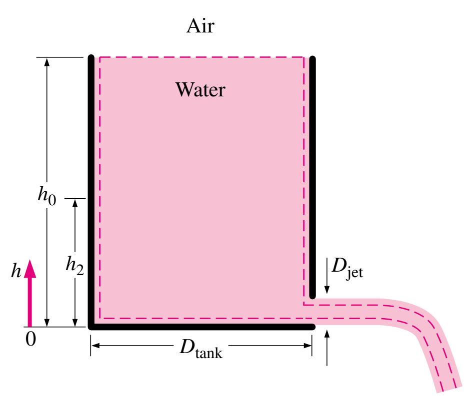

FIGURE 4–47
Schematic for Example 4–13.

Analysis We take the volume occupied by water as the control volume. The size of the control volume will decrease in this case as the water level drops, and thus this is a variable control volume. (We could also treat this as a fixed control volume which consists of the interior volume of the tank by disregarding the air that replaces the space vacated by the water.) This is obviously an unsteady-flow problem since the properties (such as the amount of mass) within the control volume change with time.

The conservation of mass relation for any system undergoing any process is given in the rate form as

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{system}}{dt} \qquad \textbf{(1)}$$

During this process no mass enters the control volume ($\dot{m}_{\text{in}} = 0$), and the mass flow rate of discharged water can be expressed as

$$\dot{m}_{\text{out}} = (\rho \mathcal{V} A)_{\text{out}} = \rho\sqrt{2gh}\,A_{\text{jet}} \qquad \textbf{(2)}$$

where $A_{\text{jet}} = \pi D_{\text{jet}}^2/4$ is the cross-sectional area of the jet, which is constant. Noting that the density of water is constant, the mass of water in the tank at any time is

$$m_{\text{system}} = \rho V = \rho A_{\text{tank}} h \qquad \textbf{(3)}$$

where $A_{\text{tank}} = \pi D_{\text{tank}}^2/4$ is the base area of the cylindrical tank. Substituting Eqs. (2) and (3) into the mass balance relation (1) gives

$$-\rho\sqrt{2gh}\,A_{\text{jet}} = \frac{d(\rho A_{\text{tank}}\,h)}{dt} \quad \rightarrow \quad -\rho\sqrt{2gh}\,(\pi D_{\text{jet}}^2/4) = \frac{\rho(\pi D_{\text{tank}}^2/4)\,dh}{dt}$$

Canceling the densities and other common terms and separating the variables give

$$dt = -\frac{D_{\text{tank}}^2}{D_{\text{jet}}^2}\frac{dh}{\sqrt{2gh}}$$

Integrating from $t = 0$ at which $h = h_0$ to $t = t$ at which $h = h_2$ gives

$$\int_0^t dt = -\frac{D_{\text{tank}}^2}{D_{\text{jet}}^2\sqrt{2g}}\int_{h_0}^{h_2}\frac{dh}{\sqrt{h}} \quad \rightarrow \quad t = \frac{\sqrt{h_0} - \sqrt{h_2}}{\sqrt{g/2}}\left(\frac{D_{\text{tank}}}{D_{\text{jet}}}\right)^2$$

Substituting, the time of discharge is determined to be

$$t = \frac{\sqrt{4\text{ ft}} - \sqrt{2\text{ ft}}}{\sqrt{32.2/2\text{ ft/s}^2}}\left(\frac{3 \times 12\text{ in}}{0.5\text{ in}}\right)^2 = 757\text{ s} = \mathbf{12.6\ min}$$

Therefore, half of the tank will be emptied in 12.6 min after the discharge hole is unplugged.

Discussion Using the same relation with $h_2 = 0$ gives $t = 43.1$ min for the discharge of the entire water in the tank. Therefore, emptying the bottom half of the tank will take much longer than emptying the top half. This is due to the decrease in the average discharge velocity of water with decreasing h.

4–6 ▪ FLOW WORK AND THE ENERGY OF A FLOWING FLUID

Unlike closed systems, control volumes involve mass flow across their boundaries, and some work is required to push the mass into or out of the control volume. This work is known as the **flow work,** or **flow energy,** and is necessary for maintaining a continuous flow through a control volume.

To obtain a relation for flow work, consider a fluid element of volume V as shown in Fig. 4–48. The fluid immediately upstream will force this fluid element to enter the control volume; thus, it can be regarded as an imaginary piston. The fluid element can be chosen to be sufficiently small so that it has uniform properties throughout.

If the fluid pressure is P and the cross-sectional area of the fluid element is A (Fig. 4–49), the force applied on the fluid element by the imaginary piston is

$$F = PA \qquad \textbf{(4–44)}$$

To push the entire fluid element into the control volume, this force must act through a distance L. Thus, the work done in pushing the fluid element across the boundary (i.e., the flow work) is

$$W_{\text{flow}} = FL = PAL = PV \qquad \text{(kJ)} \qquad \textbf{(4–45)}$$

The flow work per unit mass is obtained by dividing both sides of this equation by the mass of the fluid element:

$$w_{\text{flow}} = Pv \qquad \text{(kJ/kg)} \qquad \textbf{(4–46)}$$

The flow work relation is the same whether the fluid is pushed into or out of the control volume (Fig. 4–50).

It is interesting that unlike other work quantities, flow work is expressed in terms of properties. In fact, it is the product of two properties of the fluid. For that reason, some people view it as a *combination property* (like enthalpy) and refer to it as *flow energy*, *convected energy*, or *transport energy* instead of flow work. Others, however, argue rightfully that the product Pv represents energy for flowing fluids only and does not represent any form of energy for nonflow (closed) systems. Therefore, it should be treated as work. This controversy is not likely to end, but it is comforting to know that both arguments yield the same result for the energy equation. In the discussions that follow, we consider the flow energy to be part of the energy of a flowing fluid, since this greatly simplifies the energy analysis of control volumes.

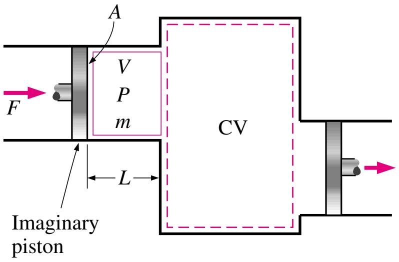

FIGURE 4–48
Schematic for flow work.

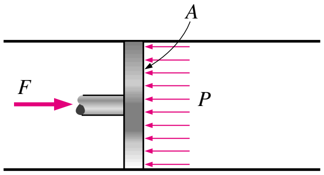

FIGURE 4–49
In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.

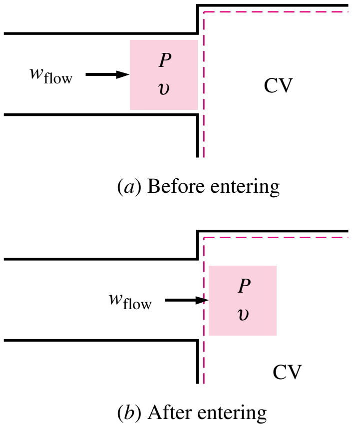

FIGURE 4–50
Flow work is the energy needed to push a fluid into or out of a control volume, and it is equal to Pv.

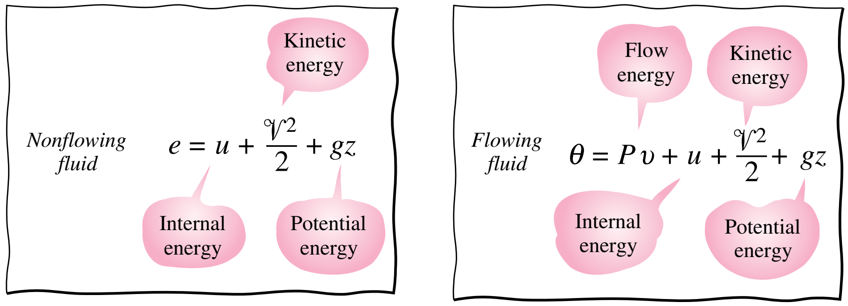

FIGURE 4–51
The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

Total Energy of a Flowing Fluid

As we discussed in Chap. 1, the total energy of a simple compressible system consists of three parts: internal, kinetic, and potential energies (Fig. 4–51). On a unit-mass basis, it is expressed as

$$e = u + \text{ke} + \text{pe} = u + \frac{\mathcal{V}^2}{2} + gz \qquad \text{(kJ/kg)} \tag{4–47}$$

where $\mathcal{V}$ is the velocity and z is the elevation of the system relative to some external reference point.

The fluid entering or leaving a control volume possesses an additional form of energy—the *flow energy* Pv, as already discussed. Then the total energy of a **flowing fluid** on a unit-mass basis (denoted by θ) becomes

$$\theta = Pv + e = Pv + (u + \text{ke} + \text{pe}) \tag{4–48}$$

But the combination $Pv + u$ has been previously defined as the enthalpy h. So the relation in Eq. 4–48 reduces to

$$\theta = h + \text{ke} + \text{pe} = h + \frac{\mathcal{V}^2}{2} + gz \qquad \text{(kJ/kg)} \tag{4–49}$$

By using the enthalpy instead of the internal energy to represent the energy of a flowing fluid, one does not need to be concerned about the flow work. The energy associated with pushing the fluid into or out of the control volume is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy. From now on, the energy of a fluid stream flowing into or out of a control volume is represented by Eq. 4–49, and no reference will be made to flow work or flow energy.

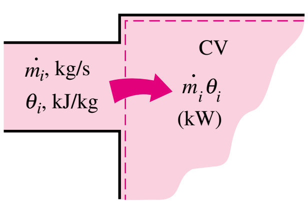

FIGURE 4–52
The product $\dot{m}_i\theta_i$ is the energy transported into the control volume by mass per unit time.

Energy Transport by Mass

Noting that θ is total energy per unit mass, the total energy of a flowing fluid of mass m is simply $m\theta$, provided that the properties of the mass m are uniform. Also, when a fluid stream with uniform properties is flowing at a mass flow rate of $\dot{m}$, the rate of energy flow with that stream is $\dot{m}\theta$ (Fig. 4–52). That is, *Amount of Energy Transport:*

$$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \quad \text{(kJ)} \tag{4–50}$$

Rate of Energy Transport:

$$\dot{E}_{mass} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \quad \text{(kW)} \tag{4–51}$$

When the kinetic and potential energies of a fluid stream are negligible, as is usually the case, these relations simplify to $E_{\text{mass}} = mh$ and $\dot{E}_{\text{mass}} = \dot{m}h$.

In general, the total energy transported by mass into or out of the control volume is not easy to determine since the properties of the mass at each inlet or exit may be changing with time as well as over the cross section. Thus, the only way to determine the energy transport through an opening as a result of mass flow is to consider sufficiently small differential masses δm that have uniform properties and to add their total energies during flow.

Again noting that θ is total energy per unit mass, the total energy of a flowing fluid of mass δm is $\theta\ \delta m$. Then the total energy transported by mass through an inlet or exit ($m_i\theta_i$ and $m_e\theta_e$) is obtained by integration. At an inlet, for example, it becomes

$$E_{\text{in, mass}} = \int_{m_i} \theta_i\, \delta m_i = \int_{m_i}\left(h_i + \frac{V_i^2}{2} + gz_i\right)\delta m_i \tag{4–52}$$

Most flows encountered in practice can be approximated as being steady and one-dimensional, and thus the simple relations in Eqs. 4–50 and 4–51 can be used to represent the energy transported by a fluid stream.

EXAMPLE 4–14 Energy Transport by Mass

Steam is leaving a 4-L pressure cooker whose operating pressure is 150 kPa (Fig. 4–53). It is observed that the amount of liquid in the cooker has decreased by 0.6 L in 40 minutes after the steady operating conditions are established, and the cross-sectional area of the exit opening is 8 mm^2. Determine (*a*) the mass flow rate of the steam and the exit velocity, (*b*) the total and flow energies of the steam per unit mass, and (*c*) the rate at which energy is leaving the cooker by steam.

SOLUTION Steam is leaving a pressure cooker at a specified pressure. The velocity, flow rate, the total and flow energies, and the rate of energy transfer by mass are to be determined.

Assumptions **1** The flow is steady, and the initial start-up period is disregarded. **2** The kinetic and potential energies are negligible, and thus they are not considered. **3** Saturation conditions exist within the cooker at all times so that steam leaves the cooker as a saturated vapor at the cooker pressure.

Properties The properties of saturated liquid water and water vapor at 150 kPa are v_f = 0.001053 m^3/kg, v_g = 1.1593 m^3/kg, u_g = 2519.7 kJ/kg, and h_g = 2693.6 kJ/kg (Table A–5).

Analysis (*a*) Saturation conditions exist in a pressure cooker at all times after the steady operating conditions are established. Therefore, the liquid has the

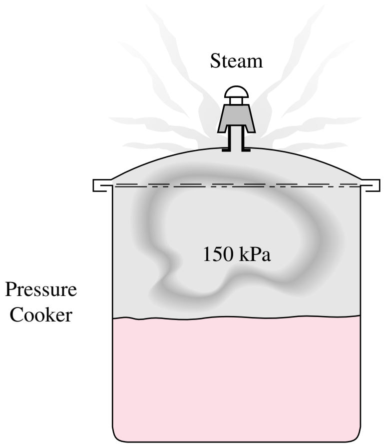

FIGURE 4–53
Schematic for Example 4–14.

properties of saturated liquid and the exiting steam has the properties of saturated vapor at the operating pressure. The amount of liquid that has evaporated, the mass flow rate of the exiting steam, and the exit velocity are

$$m = \frac{\Delta V_{\text{liquid}}}{v_f} = \frac{0.6 \text{ L}}{0.001053 \text{ m}^3/\text{kg}}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = 0.570 \text{ kg}$$

$$\dot{m} = \frac{m}{\Delta t} = \frac{0.570 \text{ kg}}{40 \text{ min}} = 0.0142 \text{ kg/min} = \mathbf{2.37 \times 10^{-4} \text{ kg/s}}$$

$$\mathcal{V} = \frac{\dot{m}}{\rho_g A_c} = \frac{\dot{m} v_g}{A_c} = \frac{(2.37 \times 10^{-4} \text{ kg/s})(1.1593 \text{ m}^3/\text{kg})}{8 \times 10^{-6} \text{ m}^2} = \mathbf{34.3 \text{ m/s}}$$

(*b*) Noting that $h = u + Pv$ and that the kinetic and potential energies are disregarded, the flow and total energies of the exiting steam are

$$e_{\text{flow}} = Pv = h - u = 2693.6 - 2519.7 = \mathbf{173.9 \text{ kJ/kg}}$$

$$\theta = h + \text{ke} + \text{pe} \cong h = \mathbf{2693.6 \text{ kJ/kg}}$$

Note that the kinetic energy in this case is $\text{ke} = \mathcal{V}^2/2 = (34.3 \text{ m/s})^2/2 = 588 \text{ m}^2/\text{s}^2 = 0.588 \text{ kJ/kg}$, which is small compared to enthalpy.

(*c*) The rate at which energy is leaving the cooker by mass is simply the product of the mass flow rate and the total energy of the exiting steam per unit mass,

$$\dot{E}_{\text{mass}} = \dot{m}\theta = (2.37 \times 10^{-4} \text{ kg/s})(2693.6 \text{ kJ/kg}) = 0.638 \text{ kJ/s} = \mathbf{0.638 \text{ kW}}$$

Discussion The numerical value of the energy leaving the cooker with steam alone does not mean much since this value depends on the reference point selected for enthalpy (it could even be negative). The significant quantity is the difference between the enthalpies of the exiting vapor and the liquid inside (which is h_{fg}) since it relates directly to the amount of energy supplied to the cooker, as we will discuss in Chap. 5.

SUMMARY

Energy can cross the boundaries of a closed system in the form of heat or work. For control volumes, energy can also be transported by mass. If the energy transfer is due to a temperature difference between a closed system and its surroundings, it is *heat;* otherwise, it is *work.*

Work is the energy transferred as a force acts on a system through a distance. The most common form of mechanical work is the *boundary work,* which is the work associated with the expansion and compression of substances. On a *P-V* diagram, the area under the process curve represents the boundary work for a quasi-equilibrium process. Various forms of work are expressed as follows:

Electrical work: $W_e = VI\,\Delta t$

Boundary work:

(1) General $W_b = \int_1^2 P\,dV$

(2) Isobaric process $W_b = P_0(V_2 - V_1)$
($P_1 = P_2 = P_0 =$ constant)

(3) Polytropic process $W_b = \dfrac{P_2V_2 - P_1V_1}{1 - n}$ $(n \neq 1)$
($Pv^n =$ constant)

(4) Isothermal process of an ideal gas ($PV = mRT_0 = \text{constant}$) $\quad W_b = P_1V_1 \ln \dfrac{V_2}{V_1} = mRT_0 \ln \dfrac{V_2}{V_1}$

Shaft work: $\quad W_{sh} = 2\pi n\mathbf{T}$

Spring work: $\quad W_{spring} = \dfrac{1}{2} k_s (x_2^2 - x_1^2)$

The *conservation of mass principle* states that the net mass transfer to or from a system during a process is equal to the net change (increase or decrease) in the total mass of the system during that process, and is expressed as

$$m_{in} - m_{out} = \Delta m_{system} \quad \text{and} \quad \dot{m}_{in} - \dot{m}_{out} = dm_{system}/dt$$

where $\Delta m_{system} = m_{final} - m_{initial}$ is the change in the mass of the system during the process, $\dot{m}_{in}$ and $\dot{m}_{out}$ are the total rates of mass flow into and out of the system, and dm_{system}/dt is the rate of change of mass within the system boundaries. The relations above are also referred to as the *mass balance* or *continuity equation*, and are applicable to any system undergoing any kind of process.

The amount of mass flowing through a cross section per unit time is called the *mass flow rate,* and is expressed as

$$\dot{m} = \rho \mathcal{V} A$$

where ρ = density of fluid, $\mathcal{V}$ = mean fluid velocity normal to A, and A = cross-sectional area normal to flow direction. The volume of the fluid flowing through a cross section per unit time is called the *volume flow rate* and is expressed as

$$\dot{V} = \mathcal{V} A = \dot{m}/\rho$$

For steady-flow systems, the conservation of mass principle is expressed as

Steady Flow: $\quad \sum \dot{m}_i = \sum \dot{m}_e$

Steady Flow (single stream):
$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 \mathcal{V}_1 A_1 = \rho_2 \mathcal{V}_2 A_2$

Steady Incompressible Flow: $\quad \sum \dot{V}_i = \sum \dot{V}_e$

Steady Incompressible Flow (single stream):
$\dot{V}_1 = \dot{V}_2 \quad \rightarrow \quad \mathcal{V}_1 A_1 = \mathcal{V}_2 A_2$

The work required to push a unit mass of fluid into or out of a control volume is called *flow work* or *flow energy,* and is expressed as $w_{flow} = Pv$. In the analysis of control volumes, it is convenient to combine the flow energy and internal energy into *enthalpy*. Then the total energy of a flowing fluid is expressed as

$$\theta = h + \text{ke} + \text{pe} = h + \frac{\mathcal{V}^2}{2} + gz$$

The total energy transported by a flowing fluid of mass m with uniform properties is $m\theta$. The rate of energy transport by a fluid with a mass flow rate of $\dot{m}$ is $\dot{m}\theta$. When the kinetic and potential energies of a fluid stream are negligible, the amount and rate of energy transport become $E_{mass} = mh$ and $\dot{E}_{mass} = \dot{m}h$, respectively.

REFERENCES AND SUGGESTED READINGS

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2. A. Bejan. *Advanced Engineering Thermodynamics.* New York: Wiley, 1988.
3. M. J. Moran and H. N. Shapiro. *Fundamentals of Engineering Thermodynamics.* 4th ed. New York: Wiley, 2000.
4. K. Wark and D. E. Richards. *Thermodynamics.* 6th ed. New York: McGraw-Hill, 1999.

PROBLEMS*

Heat Transfer and Work

4–1C In what forms can energy cross the boundaries of a closed system?

4–2C When is the energy crossing the boundaries of a closed system heat and when is it work?

4–3C What is an adiabatic process? What is an adiabatic system?

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with a CD-EES icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

4–4C A gas in a piston-cylinder device is compressed, and as a result its temperature rises. Is this a heat or work interaction?

4–5C A room is heated by an iron that is left plugged in. Is this a heat or work interaction? Take the entire room, including the iron, as the system.

4–6C A room is heated as a result of solar radiation coming in through the windows. Is this a heat or work interaction for the room?

4–7C An insulated room is heated by burning candles. Is this a heat or work interaction? Take the entire room, including the candles, as the system.

4–8C What are point and path functions? Give some examples.

4–9C What is the caloric theory? When and why was it abandoned?

Boundary Work

4–10C On a P-υ diagram, what does the area under the process curve represent?

4–11C Is the boundary work associated with constant-volume systems always zero?

4–12C An ideal gas at a given state expands to a fixed final volume first at constant pressure and then at constant temperature. For which case is the work done greater?

4–13C Show that 1 kPa · m^3 = 1 kJ.

4–14 A mass of 5 kg of saturated water vapor at 200 kPa is heated at constant pressure until the temperature reaches 300°C. Calculate the work done by the steam during this process. *Answer:* 430.5 kJ

4–15 A frictionless piston-cylinder device initially contains 200 L of saturated liquid refrigerant-134a. The piston is free to move, and its mass is such that it maintains a pressure of 800 kPa on the refrigerant. The refrigerant is now heated until its temperature rises to 50°C. Calculate the work done during this process. *Answer:* 5227 kJ

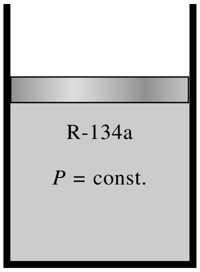

FIGURE P4–15

4–16 Reconsider Prob. 4–15. Using EES (or other) software, investigate the effect of pressure on the work done. Let the pressure vary from 400 kPa to 1200 kPa. Plot the work done versus the pressure, and discuss the results. Explain why the plot is not linear. Also plot the process described in Prob. 4–15 on the P-υ diagram.

4–17E A frictionless piston-cylinder device contains 12 lbm of superheated water vapor at 60 psia and 500°F. Steam is now cooled at constant pressure until 70 percent of it, by mass, condenses. Determine the work done during this process.

4–18 A mass of 2.4 kg of air at 150 kPa and 12°C is contained in a gas-tight, frictionless piston-cylinder device. The air is now compressed to a final pressure of 600 kPa. During the process, heat is transferred from the air such that the temperature inside the cylinder remains constant. Calculate the work input during this process. *Answer:* 272 kJ

4–19 Nitrogen at an initial state of 300 K, 150 kPa, and 0.2 m^3 is compressed slowly in an isothermal process to a final pressure of 800 kPa. Determine the work done during this process.

4–20 A gas is compressed from an initial volume of 0.42 m^3 to a final volume of 0.12 m^3. During the quasi-equilibrium process, the pressure changes with volume according to the relation $P = aV + b$, where $a = -1200$ kPa/m^3 and $b = 600$ kPa. Calculate the work done during this process (*a*) by plotting the process on a P-V diagram and finding the area under the process curve and (*b*) by performing the necessary integrations.

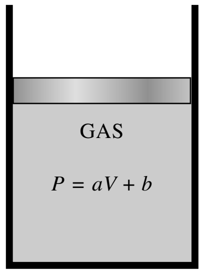

FIGURE P4–20

4–21E During an expansion process, the pressure of a gas changes from 15 to 100 psia according to the relation $P = aV + b$, where $a = 5$ psia/ft^3 and b is a constant. If the initial volume of the gas is 7 ft^3, calculate the work done during the process. *Answer:* 181 Btu

4–22 During some actual expansion and compression processes in piston-cylinder devices, the gases have been observed to satisfy the relationship $PV^n = C$, where n and C are constants. Calculate the work done when a gas expands from 150 kPa and 0.03 m^3 to a final volume of 0.2 m^3 for the case of $n = 1.3$.

4–23 Reconsider Prob. 4–22. Using the EES software, plot the process described in the problem on a P-V diagram, and investigate the effect of the polytropic exponent n on the boundary work. Let the polytropic exponent vary from 1.1 to 1.6. Plot the boundary work versus the polytropic exponent, and discuss the results.

4–24 A frictionless piston-cylinder device contains 2 kg of nitrogen at 100 kPa and 300 K. Nitrogen is now compressed slowly according to the relation $PV^{1.4}$ = constant until it reaches a final temperature of 360 K. Calculate the work input during this process. *Answer:* 89 kJ

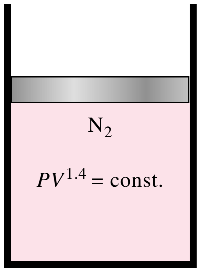

FIGURE P4–24

4–25 The equation of state of a gas is given as $\bar{v}(P + 10/\bar{v}^2) = R_uT$, where the units of $\bar{v}$ and P are m^3/kmol and kPa, respectively. Now 0.5 kmol of this gas is expanded in a quasi-equilibrium manner from 2 to 4 m^3 at a constant temperature of 300 K. Determine (*a*) the unit of the quantity 10 in the equation and (*b*) the work done during this isothermal expansion process.

4–26 Reconsider Prob. 4–25. Using the integration feature of the EES software, calculate the work done, and compare your result with the "hand calculated" result obtained in Prob. 4–25. Plot the process described in the problem on a *P-V* diagram.

4–27 Carbon dioxide contained in a piston-cylinder device is compressed from 0.3 to 0.1 m^3. During the process, the pressure and volume are related by $P = aV^{-2}$, where $a = 8\ \text{kPa} \cdot m^6$. Calculate the work done on the carbon dioxide during this process. *Answer:* 53.3 kJ

4–28E Hydrogen is contained in a piston-cylinder device at 14.7 psia and 15 ft^3. At this state, a linear spring ($F \propto x$) with a spring constant of 15,000 lbf/ft is touching the piston but exerts no force on it. The cross-sectional area of the piston is 3 ft^2. Heat is transferred to the hydrogen, causing it to expand until its volume doubles. Determine (*a*) the final pressure, (*b*) the total work done by the hydrogen, and (*c*) the fraction of this work done against the spring. Also, show the process on a *P-V* diagram.

4–29 A piston-cylinder device contains 50 kg of water at 150 kPa and 25°C. The cross-sectional area of the piston is 0.1 m^2. Heat is now transferred to the water, causing part of it to evaporate and expand. When the volume reaches 0.2 m^3, the piston reaches a linear spring whose spring constant is 100 kN/m. More heat is transferred to the water until the piston rises 20 cm more. Determine (*a*) the final pressure and temperature and (*b*) the work done during this process. Also, show the process on a *P-V* diagram.
Answers: (*a*) 350 kPa, 138.88°C; (*b*) 27.5 kJ

4–30 Reconsider Prob. 4–29. Using the EES software, investigate the effect of the spring constant on the final pressure in the cylinder and the boundary work done. Let the spring constant vary from 50 kN/m to 500 kN/m. Plot the final pressure and the boundary work against the spring constant, and discuss the results.

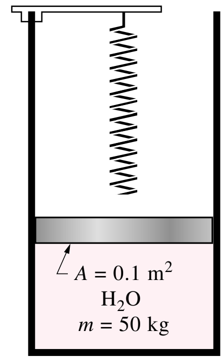

FIGURE P4–29

4–31 A piston-cylinder device with a set of stops contains 10 kg of refrigerant-134a. Initially, 8 kg of the refrigerant is in the liquid form, and the temperature is −8°C. Now heat is transferred slowly to the refrigerant until the piston hits the stops, at which point the volume is 400 L. Determine (*a*) the temperature when the piston first hits the stops and (*b*) the work done during this expansion process. Also, show the process on a *P-V* diagram. *Answers:* (*a*) −8°C, (*b*) 45.6 kJ

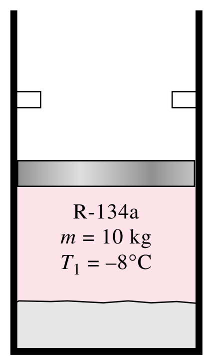

FIGURE P4–31

4–32 A frictionless piston-cylinder device contains 10 kg of saturated refrigerant-134a vapor at 50°C. The refrigerant is then allowed to expand isothermally by gradually decreasing the pressure in a quasi-equilibrium manner to a final value of 500 kPa. Determine the work done during this expansion process (*a*) by using the experimental specific volume data from the tables and (*b*) by treating the refrigerant vapor as an ideal gas. Also, determine the error involved in the latter case.

4–33 Reconsider Prob. 4–32. Using the integration feature of the EES software and the built-in property functions, calculate the work done and compare it to the result obtained by the ideal-gas assumption. Plot the process described in the problem on a *P-v* diagram.

4–34 Determine the boundary work done by a gas during an expansion process if the pressure and volume values at various states are measured to be 300 kPa, 1 L; 290 kPa, 1.1 L; 270 kPa, 1.2 L; 250 kPa, 1.4 L; 220 kPa, 1.7 L; and 200 kPa, 2 L.

Other Forms of Mechanical Work

4–35C A car is accelerated from rest to 85 km/h in 10 s. Would the work energy transferred to the car be different if it were accelerated to the same speed in 5 s?

4–36C Lifting a weight to a height of 20 m takes 20 s for one crane and 10 s for another. Is there any difference in the amount of work done on the weight by each crane?

4–37 Determine the energy required to accelerate an 800-kg car from rest to 100 km/h on a level road. *Answer:* 308.6 kJ

4–38 Determine the energy required to accelerate a 2000-kg car from 20 to 70 km/h on an uphill road with a vertical rise of 40 m.

4–39E Determine the torque applied to the shaft of a car that transmits 450 hp and rotates at a rate of 3000 rpm.

4–40 Determine the work required to deflect a linear spring with a spring constant of 70 kN/m by 20 cm from its rest position.

4–41 The engine of a 1500-kg automobile has a power rating of 75 kW. Determine the time required to accelerate this car from rest to a speed of 85 km/h at full power on a level road. Is your answer realistic?

4–42 A ski lift has a one-way length of 1 km and a vertical rise of 200 m. The chairs are spaced 20 m apart, and each chair can seat three people. The lift is operating at a steady speed of 10 km/h. Neglecting friction and air drag and assuming that the average mass of each loaded chair is 250 kg, determine the power required to operate this ski lift. Also estimate the power required to accelerate this ski lift in 5 s to its operating speed when it is first turned on.

4–43 Determine the power required for a 2000-kg car to climb a 100-m-long uphill road with a slope of 30° (from horizontal) in 10 s (*a*) at a constant velocity, (*b*) from rest to a final velocity of 30 m/s, and (*c*) from 35 m/s to a final velocity of 5 m/s. Disregard friction, air drag, and rolling resistance. *Answers:* (*a*) 98.1 kW, (*b*) 188.1 kW, (*c*) −21.9 kW

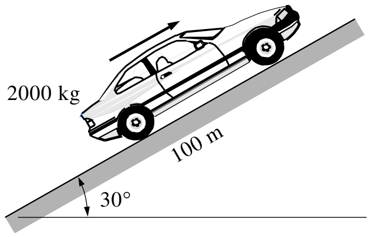

FIGURE P4–43

4–44 A damaged 1200-kg car is being towed by a truck. Neglecting the friction, air drag, and rolling resistance, determine the extra power required (*a*) for constant velocity on a level road, (*b*) for constant velocity of 50 km/h on a 30° (from horizontal) uphill road, and (*c*) to accelerate on a level road from stop to 90 km/h in 12 s.
Answers: (*a*) 0, (*b*) 81.7 kW, (*c*) 31.25 kW

Conservation of Mass

4–45C Define mass and volume flow rates. How are they related to each other?

4–46C Does the amount of mass entering a control volume have to be equal to the amount of mass leaving during an unsteady-flow process?

4–47C When is the flow through a control volume steady?

4–48C Consider a device with one inlet and one exit. If the volume flow rates at the inlet and at the exit are the same, is the flow through this device necessarily steady? Why?

4–49E A garden hose attached with a nozzle is used to fill a 20-gallon bucket. The inner diameter of the hose is 1 in and it reduces to 0.5 in at the nozzle exit. If the mean velocity in the hose is 8 ft/s, determine (*a*) the volume and mass flow rates of water through the hose, (*b*) how long it will take to fill the bucket with water, and (*c*) the mean velocity of water at the nozzle exit.

4–50 Air enters a nozzle steadily at 2.21 kg/m^3 and 30 m/s and leaves at 0.762 kg/m^3 and 180 m/s. If the inlet area of the nozzle is 80 cm^2, determine (*a*) the mass flow rate through the nozzle, and (*b*) the exit area of the nozzle.
Answers: (*a*) 0.5304 kg/s, (*b*) 38.7 cm^2

4–51 A hair dryer is basically a duct of constant diameter in which a few layers of electric resistors are placed. A small fan pulls the air in and forces it through the resistors where it is heated. If the density of air is 1.20 kg/m^3 at the inlet and 1.05 kg/m^3 at the exit, determine the percent increase in the velocity of air as it flows through the dryer.

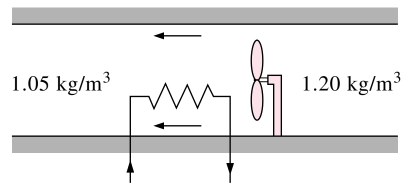

FIGURE P4–51

4–52E Air whose density is 0.078 lbm/ft^3 enters the duct of an air-conditioning system at a volume flow rate of 450 ft^3/min. If the diameter of the duct is 10 in, determine the velocity of the air at the duct inlet and the mass flow rate of air.

4–53 A 1-m^3 rigid tank initially contains air whose density is 1.18 kg/m^3. The tank is connected to a high-pressure supply line through a valve. The valve is opened, and air is allowed to enter the tank until the density in the tank rises to 7.20 kg/m^3. Determine the mass of air that has entered the tank.
Answer: 6.02 kg

4–54 The ventilating fan of the bathroom of a building has a volume flow rate of 30 L/s and runs continuously. If the density of air inside is 1.20 kg/m^3, determine the mass of air vented out in one day.

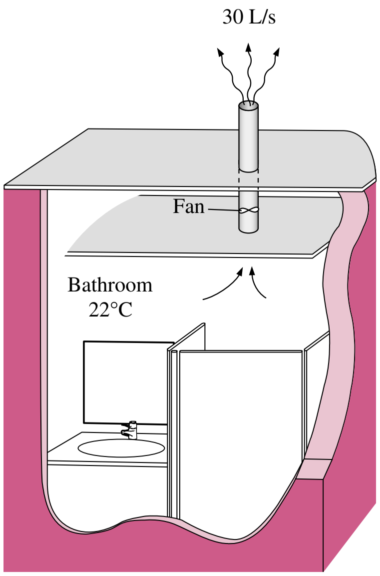

FIGURE P4–54

4–55E Chickens with an average mass of 4.5 lbm are to be cooled by chilled water in a continuous-flow-type immersion chiller. Chickens are dropped into the chiller at a rate of 500 chickens per hour. Determine the mass flow rate of chickens through the chiller.

4–56 A desktop computer is to be cooled by a fan whose flow rate is 0.34 m^3/min. Determine the mass flow rate of air through the fan at an elevation of 3400 m where the air density is 0.7 kg/m^3. Also, if the mean velocity of air is not to exceed 110 m/min, determine the diameter of the casing of the fan. *Answers:* 0.238 kg/min, 0.063 m

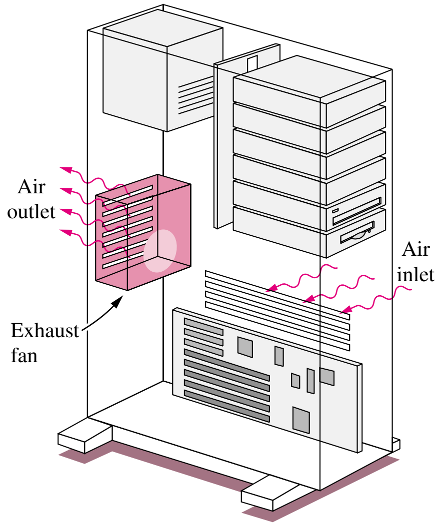

FIGURE P4–56

Flow Work and Energy Transfer by Mass

4–57C What are the different mechanisms for transferring energy to or from a control volume?

4–58C What is flow energy? Do fluids at rest possess any flow energy?

4–59C How do the energies of a flowing fluid and a fluid at rest compare? Name the specific forms of energy associated with each case.

4–60E Steam is leaving a pressure cooker whose operating pressure is 30 psia. It is observed that the amount of liquid in the cooker has decreased by 0.4 gal in 45 minutes after the steady operating conditions are established, and the cross-sectional area of the exit opening is 0.15 in^2. Determine (*a*) the mass flow rate of the steam and the exit velocity, (*b*) the total and flow energies of the steam per unit mass, and (*c*) the rate at which energy is leaving the cooker by steam.

4–61 Refrigerant-134a enters the compressor of a refrigeration system as saturated vapor at 0.14 MPa, and leaves as superheated vapor at 0.8 MPa and 50°C at a rate of 0.04 kg/s. Determine the rates of energy transfers by mass into and out of the compressor. Assume the kinetic and potential energies to be negligible.

4–62 A house is maintained at 1 atm and 24°C, and warm air inside a house is forced to leave the house at a rate of 150 m^3/h as a result of outdoor air at 5°C infiltrating into the house through the cracks. Determine the rate of net energy loss of the house due to mass transfer. *Answer:* 0.945 kW

Review Problems

4–63 Consider a vertical elevator whose cabin has a total mass of 800 kg when fully loaded and 150 kg when empty. The weight of the elevator cabin is partially balanced by a 400-kg counterweight that is connected to the top of the cabin by cables that pass through a pulley located on top of the elevator well. Neglecting the weight of the cables and assuming the guide rails and the pulleys to be frictionless, determine (*a*) the power required while the fully loaded cabin is rising at a constant speed of 2 m/s and (*b*) the power required while the empty cabin is descending at a constant speed of 2 m/s.

What would your answer be to (*a*) if no counterweight were used? What would your answer be to (*b*) if a friction force of 1200 N has developed between the cabin and the guide rails?

4–64 A frictionless piston-cylinder device initially contains air at 200 kPa and 0.2 m^3. At this state, a linear spring ($F \propto x$) is touching the piston but exerts no force on it. The air is now heated to a final state of 0.5 m^3 and 800 kPa. Determine (*a*) the total work done by the air and (*b*) the work done against the spring. Also, show the process on a P-υ diagram.
Answers: (*a*) 150 kJ, (*b*) 90 kJ

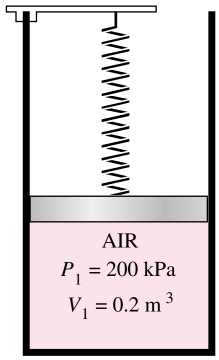

FIGURE P4–64

4–65 A mass of 5 kg of saturated liquid–vapor mixture of water is contained in a piston-cylinder device at 100 kPa. Initially, 2 kg of the water is in the liquid phase and the rest is in the vapor phase. Heat is now transferred to the water, and the piston, which is resting on a set of stops, starts moving when the pressure inside reaches 200 kPa. Heat transfer continues until the total volume increases by 20 percent. Determine (*a*) the initial and final temperatures, (*b*) the mass of liquid water when the piston first starts moving, and (*c*) the work done during this process. Also, show the process on a *P*-υ diagram.

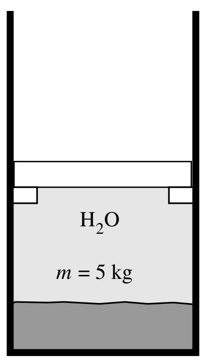

FIGURE P4–65

4–66E A spherical balloon contains 10 lbm of air at 30 psia and 800 R. The balloon material is such that the pressure inside is always proportional to the square of the diameter. Determine the work done when the volume of the balloon doubles as a result of heat transfer. *Answer:* 715 Btu

4–67E Reconsider Prob. 4–66E. Using the integration feature of the EES software, determine the work done. Compare the result with your "hand-calculated" result.

4–68 A $D_0 = 10$ m diameter tank is initially filled with water 2 m above the center of a $D = 10$ cm diameter valve near the bottom. The tank surface is open to the atmosphere, and the tank drains through a $L = 100$ m long pipe connected to the valve. The friction coefficient of the pipe is given to be $f = 0.015$, and the discharge velocity is expressed as $\mathcal{V} = \sqrt{\dfrac{2gz}{1.5 + fL/D}}$ where z is the water height above the center of the valve. Determine (*a*) the initial discharge velocity from the tank and (*b*) the time required to empty the tank. The tank can be considered to be empty when the water level drops to the center of the valve.

4–69 Milk is to be transported from Texas to California for a distance of 2100 km in a 7-m-long, 2-m-external-diameter cylindrical tank. The walls of the tank are constructed of 5-cm-thick urethane insulation sandwiched between two metal sheets of negligible thickness. Determine the amount of milk in the tank in kg and in gallons.

4–70 Underground water is being pumped into a pool whose cross section is 3 m × 4 m while water is discharged through a 5-cm-diameter orifice at a constant mean velocity of 5 m/s. If the water level in the pool rises at a rate of 1.5 cm/min, determine the rate at which water is supplied to the pool, in m^3/s.

4–71 The velocity of a liquid flowing in a circular pipe of radius R varies from zero at the wall to a maximum at the pipe center. The velocity distribution in the pipe can be represented as $V(r)$, where r is the radial distance from the pipe center. Based on the definition of mass flow rate $\dot{m}$, obtain a relation for the mean velocity in terms of $V(r)$, R, and r.

4–72 Air at 4.18 kg/m^3 enters a nozzle that has an inlet-to-exit area ratio of 2:1 with a velocity of 120 m/s and leaves with a velocity of 380 m/s. Determine the density of air at the exit. *Answer:* 2.64 kg/m^3

4–73 A long roll of 1-m-wide and 0.5-cm-thick 1-Mn manganese steel plate ($\rho = 7854$ kg/m^3) coming off a furnace is to be quenched in an oil bath to a specified temperature. If the metal sheet is moving at a steady velocity of 10 m/min, determine the mass flow rate of the steel plate through the oil bath.

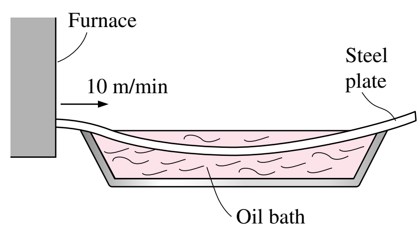

FIGURE P4–73

4–74 The air in a 6 m × 5 m × 4 m hospital room is to be completely replaced by conditioned air every 20 min. If the average air velocity in the circular air duct leading to the room is not to exceed 5 m/s, determine the minimum diameter of the duct.

4–75E It is well-established that indoor air quality (IAQ) has a significant effect on general health and productivity of employees at a workplace. A recent study showed that enhancing IAQ by increasing the building ventilation from 5 cfm (cubic feet per minute) to 20 cfm increased the productivity by 0.25 percent, valued at $90 per person per year, and decreased the respiratory illnesses by 10 percent for an average annual savings of $39 per person while increasing the annual energy consumption by $6 and the equipment cost by about $4 per person per year (*ASHRAE Journal,* December 1998). For a workplace with 120 employees, determine the net monetary benefit of installing an enhanced IAQ system to the employer per year. *Answer:* $14,280/yr

Design and Essay Problems

4–76 Design a reciprocating compressor capable of supplying compressed air at 800 kPa at a rate of 15 kg/min. Also specify the size of the electric motor capable of driving this compressor. The compressor is to operate at no more than 2000 rpm (revolutions per minute).

4–77 A considerable fraction of energy loss in residential buildings is due to the cold outdoor air infiltrating through the cracks mostly around the doors and windows of the building. Write an essay on infiltration losses, their cost to homeowners, and the measures to prevent them.

4–78 Using a large bucket whose volume is known and measuring the time it takes to fill the bucket with water from a garden hose, determine the mass flow rate and the average velocity of water through the hose.