Chapter 22
RADIATION HEAT TRANSFER

View Factors

22-1C The view factor $F_{i \to j}$ represents the fraction of the radiation leaving surface i that strikes surface j directly. The view factor from a surface to itself is non-zero for concave surfaces.

22-2C The pair of view factors $F_{i \to j}$ and $F_{j \to i}$ are related to each other by the reciprocity rule $A_i F_{ij} = A_j F_{ji}$ where A_i is the area of the surface i and A_j is the area of the surface j. Therefore,

$$A_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \frac{A_2}{A_1} F_{21}$$

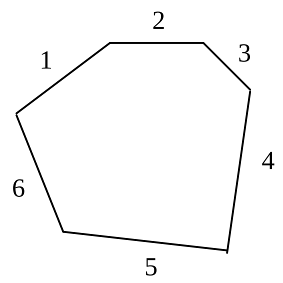
22-3C The summation rule for an enclosure and is expressed as $\sum_{j=1}^{N} F_{i \to j} = 1$ where N is the number of surfaces of the enclosure. It states that the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself must be equal to unity.

The superposition rule is stated as the view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j, $F_{1 \to (2,3)} = F_{1 \to 2} + F_{1 \to 3}$.

22-4C The cross-string method is applicable to geometries which are very long in one direction relative to the other directions. By attaching strings between corners the Crossed-Strings Method is expressed as

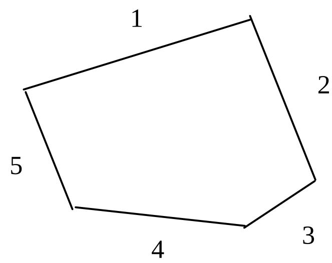
$$F_{i \to j} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{string on surface } i}$$

22-5 An enclosure consisting of six surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



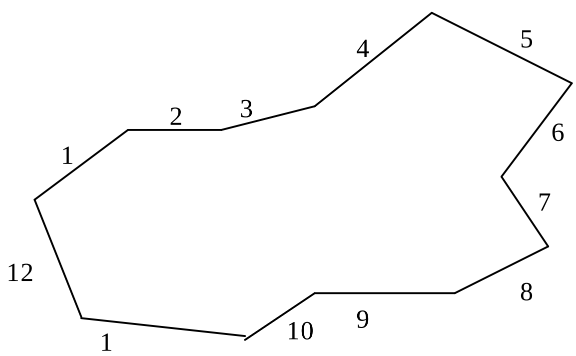
Analysis A seven surface enclosure (N=6) involves $N^2 = 6^2 = \mathbf{36}$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{6(6-1)}{2} = 15$ view factors directly. The remaining 36-15 = **21** of the view factors can be determined by the application of the reciprocity and summation rules.

22-6 An enclosure consisting of five surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



Analysis A five surface enclosure (N=5) involves $N^2 = 5^2 = \mathbf{25}$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{5(5-1)}{2} = 10$ view factors directly. The remaining 25-10 = **15** of the view factors can be determined by the application of the reciprocity and summation rules.

22-7 An enclosure consisting of twelve surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



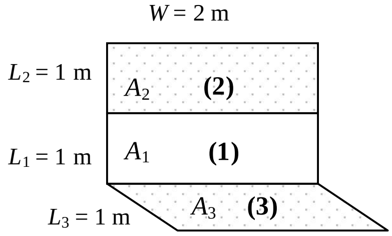
Analysis A twelve surface enclosure (N=12) involves $N^2 = 12^2 = \mathbf{144}$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{12(12-1)}{2} = 66$ view factors directly. The remaining 144-66 = **78** of the view factors can be determined by the application of the reciprocity and summation rules.

22-8 The view factors between the rectangular surfaces shown in the figure are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis From Fig. 22-6,

$$\left.\begin{array}{l}\dfrac{L_3}{W}=\dfrac{1}{2}=0.5\\[2mm]\dfrac{L1}{W}=\dfrac{1}{2}=0.5\end{array}\right\}F_{31}=0.24$$



and

$$\left.\begin{array}{l}\dfrac{L_3}{W}=\dfrac{1}{2}=0.5\\[2mm]\dfrac{L_1+L_2}{W}=\dfrac{2}{2}=1\end{array}\right\}F_{3\to(1+2)}=0.29$$

We note that $A_1 = A_3$. Then the reciprocity and superposition rules gives

$$\mathrm{A}_1F_{13}=A_3F_{31}\longrightarrow F_{13}=F_{31}=\mathbf{0.24}$$

$$F_{3\to(1+2)}=F_{31}+F_{32}\longrightarrow\quad 0.29=0.24+F_{32}\longrightarrow F_{32}=0.05$$

Finally, $A_2=A_3\longrightarrow F_{23}=F_{32}=\mathbf{0.05}$

22-9 A cylindrical enclosure is considered. The view factor from the side surface of this cylindrical enclosure to its base surface is to be determined.

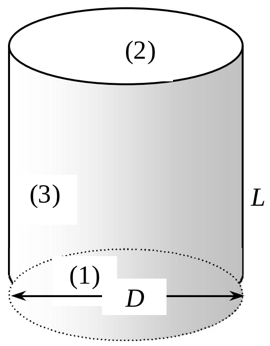
Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We designate the surfaces as follows:

Base surface by (1),

top surface by (2), and

side surface by (3).



Then from Fig. 22-7 (or Table 22-1 for better accuracy)

$$\left.\begin{array}{l}\dfrac{L}{r_1}=\dfrac{r_1}{r_1}=1\\[2ex] \dfrac{r_2}{L}=\dfrac{r_2}{r_2}=1\end{array}\right\} F_{12}=F_{21}=0.38$$

$$\text{summation rule}: F_{11}+F_{12}+F_{13}=1$$

$$0+0.38+F_{13}=1 \longrightarrow F_{13}=0.62$$

$$\text{reciprocity rule}: A_1F_{13}=A_3F_{31} \longrightarrow F_{31}=\frac{A_1}{A_3}F_{13}=\frac{\pi r_1^2}{2\pi r_1 L}F_{13}=\frac{\pi r_1^2}{2\pi r_1(r_1)}F_{13}=\frac{1}{2}(0.62)=\mathbf{0.31}$$

Discussion This problem can be solved more accurately by using the view factor relation from Table 22-1 to be

$$R_1=\frac{r_1}{L}=\frac{r_1}{r_1}=1$$

$$R_2=\frac{r_2}{L}=\frac{r_2}{r_2}=1$$

$$S=1+\frac{1+R_2^2}{R_1^2}=1+\frac{1+1^2}{1^2}=3$$

$$F_{12}=\frac{1}{2}\left\{S-\left[S^2-4\left(\frac{R_2}{R_1}\right)^2\right]^{0.5}\right\}=\frac{1}{2}\left\{3-\left[3^2-4\left(\frac{1}{1}\right)^2\right]^{0.5}\right\}=0.382$$

$$F_{13}=1-F_{12}=1-0.382=0.618$$

$$\text{reciprocity rule}: A_1F_{13}=A_3F_{31} \longrightarrow F_{31}=\frac{A_1}{A_3}F_{13}=\frac{\pi r_1^2}{2\pi r_1 L}F_{13}=\frac{\pi r_1^2}{2\pi r_1(r_1)}F_{13}=\frac{1}{2}(0.618)=\mathbf{0.309}$$

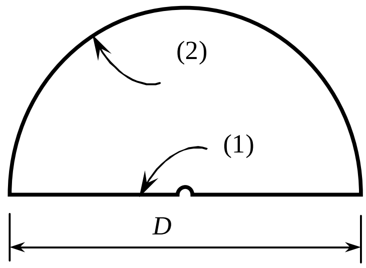
22-10 A semispherical furnace is considered. The view factor from the dome of this furnace to its flat base is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We number the surfaces as follows:

(1): circular base surface

(2): dome surface



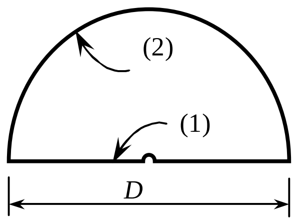
Surface (1) is flat, and thus $F_{11} = 0$.

$$\text{Summation rule: } F_{11} + F_{12} = 1 \rightarrow F_{12} = 1$$

$$\text{reciprocity rule: } A_1F_{12} = A_2F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2}F_{12} = \frac{A_1}{A_2}(1) = \frac{\frac{\pi D^2}{4}}{\frac{\pi D^2}{2}} = \frac{1}{2} = \mathbf{0.5}$$

22-11 Two view factors associated with three very long ducts with different geometries are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

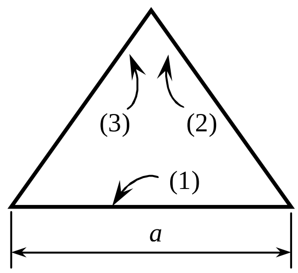


Analysis (*a*) Surface (1) is flat, and thus $F_{11} = 0$.

$$\text{summation rule: } F_{11} + F_{12} = 1 \rightarrow F_{12} = \mathbf{1}$$

$$\text{reciprocity rule: } A_1F_{12} = A_2F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2}F_{12} = \frac{Ds}{\left(\frac{\pi D}{2}\right)s}(1) = \frac{2}{\pi} = \mathbf{0.64}$$

(*b*) Noting that surfaces 2 and 3 are symmetrical and thus $F_{12} = F_{13}$, the summation rule gives



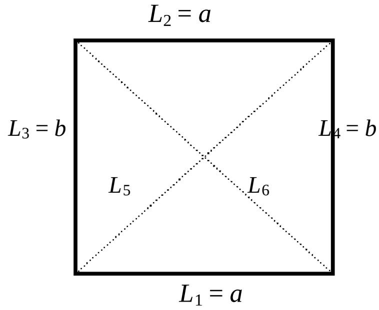
$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow 0 + F_{12} + F_{13} = 1 \longrightarrow F_{12} = \mathbf{0.5}$$

Also by using the equation obtained in Example 22-4,

$$F_{12} = \frac{L_1 + L_2 - L_3}{2L_1} = \frac{a + b - b}{2a} = \frac{a}{2a} = \frac{1}{2} = \mathbf{0.5}$$

$$\text{reciprocity rule: } A_1F_{12} = A_2F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2}F_{12} = \frac{a}{b}\left(\frac{1}{2}\right) = \mathbf{\frac{a}{2b}}$$

(*c*) Applying the crossed-string method gives



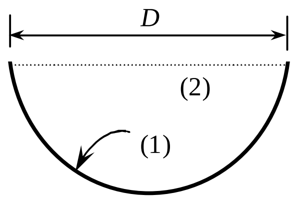
$$F_{12} = F_{21} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

$$= \frac{2\sqrt{a^2 + b^2} - 2b}{2a} = \mathbf{\frac{\sqrt{a^2 + b^2} - b}{a}}$$

22-12 View factors from the very long grooves shown in the figure to the surroundings are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis (*a*) We designate the circular dome surface by (1) and the imaginary flat top surface by (2). Noting that (2) is flat,

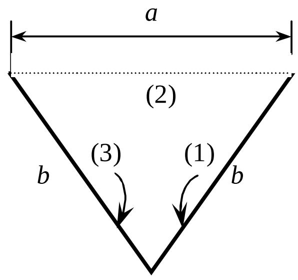


$$F_{22} = 0$$

$$\text{summation rule}: \; F_{21} + F_{22} = 1 \longrightarrow F_{21} = 1$$

$$\text{reciprocity rule}: \mathrm{A}_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \frac{A_2}{A_1} F_{21} = \frac{D}{\frac{\pi D}{2}}(1) = \frac{2}{\pi} = \mathbf{0.64}$$

(*b*) We designate the two identical surfaces of length *b* by (1) and (3), and the imaginary flat top surface by (2). Noting that (2) is flat,



$$F_{22} = 0$$

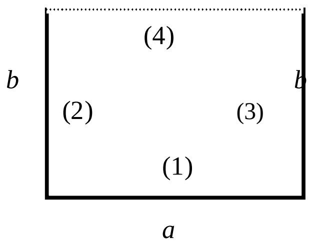
$$\text{summation rule}: \; F_{21} + F_{22} + F_{23} = 1 \longrightarrow F_{21} = F_{23} = 0.5 \quad \text{(symmetry)}$$

$$\text{summation rule}: F_{22} + F_{2\to(1+3)} = 1 \longrightarrow F_{2\to(1+3)} = 1$$

$$\text{reciprocity rule}: \mathrm{A}_2 F_{2\to(1+3)} = A_{(1+3)} F_{(1+3)\to 2}$$

$$\longrightarrow F_{(1+3)\to 2} = F_{(1+3)\to surr} = \frac{A_2}{A_{(1+3)}}(1) = \frac{\mathbf{a}}{\mathbf{2b}}$$

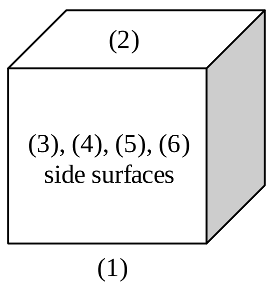
(*c*) We designate the bottom surface by (1), the side surfaces by (2) and (3), and the imaginary top surface by (4). Surface 4 is flat and is completely surrounded by other surfaces. Therefore, $F_{44} = 0$ and $F_{4\to(1+2+3)} = 1$.



$$\text{reciprocity rule}: \mathrm{A}_4 F_{4\to(1+2+3)} = A_{(1+2+3)} F_{(1+2+3)\to 4}$$

$$\longrightarrow F_{(1+2+3)\to 4} = F_{(1+2+3)\to surr} = \frac{A_4}{A_{(1+2+3)}}(1) = \frac{\mathbf{a}}{\mathbf{a+2b}}$$

22-13 The view factors from the base of a cube to each of the other five surfaces are to be determined.



Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Noting that $L_1 / w = L_2 / w = 1$, from Fig. 22-6 we read

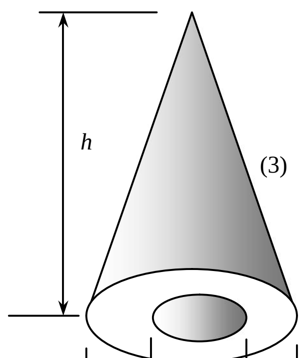
$$F_{12} = 0.2$$

Because of symmetry, we have

$$F_{12} = F_{13} = F_{14} = F_{15} = F_{16} = \mathbf{0.2}$$

22-14 The view factor from the conical side surface to a hole located at the center of the base of a conical enclosure is to be determined.

Assumptions The conical side surface is diffuse emitter and reflector.



Analysis We number different surfaces as

the hole located at the center of the base (1)

the base of conical enclosure (2)

conical side surface (3)

Surfaces 1 and 2 are flat , and they have no direct view of each other. Therefore,

$$F_{11} = F_{22} = F_{12} = F_{21} = 0$$

summation rule: $F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1$

reciprocity rule: $A_1 F_{13} = A_3 F_{31} \longrightarrow \frac{\pi d^2}{4}(1) = \frac{\pi D h}{2} F_{31} \longrightarrow F_{31} = \mathbf{\frac{d^2}{2Dh}}$

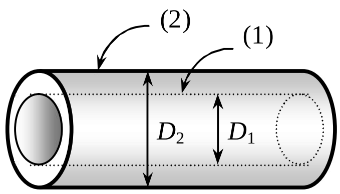
22-15 The four view factors associated with an enclosure formed by two very long concentric cylinders are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis We number different surfaces as

the outer surface of the inner cylinder (1)

the inner surface of the outer cylinder (2)



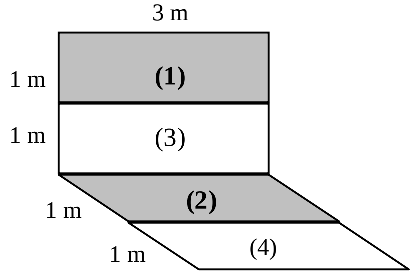
No radiation leaving surface 1 strikes itself and thus $F_{11} = \mathbf{0}$

All radiation leaving surface 1 strikes surface 2 and thus $F_{12} = \mathbf{1}$

reciprocity rule: $A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1 h}{\pi D_2 h}(1) = \mathbf{\frac{D_1}{D_2}}$

summation rule: $F_{21} + F_{22} = 1 \longrightarrow F_{22} = 1 - F_{21} = \mathbf{1 - \frac{D_1}{D_2}}$

22-16 The view factors between the rectangular surfaces shown in the figure are to be determined.
Assumptions The surfaces are diffuse emitters and reflectors.
Analysis We designate the different surfaces as follows:
shaded part of perpendicular surface by (1),
bottom part of perpendicular surface by (3),
shaded part of horizontal surface by (2), and
front part of horizontal surface by (4).



(*a*) From Fig.22-6

$$\left.\begin{array}{l}\frac{L_2}{W}=\frac{1}{3}\\[4pt] \frac{L_1}{W}=\frac{1}{3}\end{array}\right\}F_{23}=0.25 \quad \text{and} \quad \left.\begin{array}{l}\frac{L_2}{W}=\frac{2}{3}\\[4pt] \frac{L_1}{W}=\frac{1}{3}\end{array}\right\}F_{2\to(1+3)}=0.32$$

$$\text{superposition rule}: F_{2\to(1+3)}=F_{21}+F_{23}\longrightarrow F_{21}=F_{2\to(1+3)}-F_{23}=0.32-0.25=0.07$$

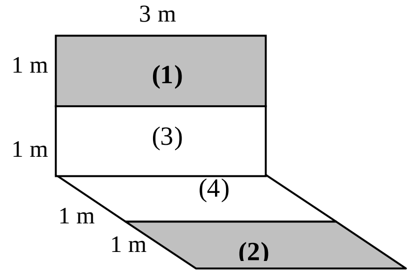
$$\text{reciprocity rule}: A_1=A_2\longrightarrow A_1F_{12}=A_2F_{21}\longrightarrow F_{12}=F_{21}=\mathbf{0.07}$$

(*b*) From Fig.22-6,

$$\frac{L_2}{W}=\frac{1}{3} \text{ and } \left.\frac{L_1}{W}=\frac{2}{3}\right\}F_{(4+2)\to 3}=0.15 \quad \text{and} \quad \frac{L_2}{W}=\frac{2}{3} \text{ and } \left.\frac{L_1}{W}=\frac{2}{3}\right\}F_{(4+2)\to(1+3)}=0.22$$

$$\text{superposition rule}: F_{(4+2)\to(1+3)}=F_{(4+2)\to 1}+F_{(4+2)\to 3}\longrightarrow F_{(4+2)\to 1}=0.22-0.15=0.07$$

$$\text{reciprocity rule}: A_{(4+2)}F_{(4+2)\to 1}=A_1F_{1\to(4+2)}$$



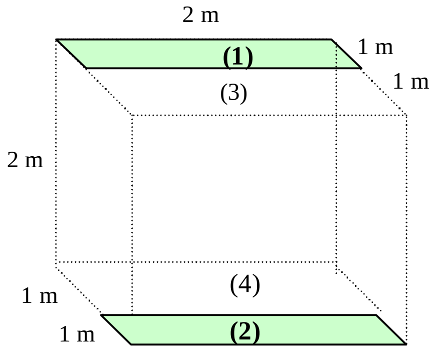
$$\longrightarrow F_{1\to(4+2)}=\frac{A_{(4+2)}}{A_1}F_{(4+2)\to 1}=\frac{6}{3}(0.07)=0.14$$

$$\text{superposition rule}: F_{1\to(4+2)}=F_{14}+F_{12}$$

$$\longrightarrow F_{14}=0.14-0.07=\mathbf{0.07}$$

since F_{12} = 0.07 (from part *a*). Note that F_{14} in part (*b*) is equivalent to F_{12} in part (*a*).

(*c*) We designate
shaded part of top surface by (1),
remaining part of top surface by (3),
remaining part of bottom surface by (4), and
shaded part of bottom surface by (2).



From Fig.22-5,

$$\left.\begin{array}{l}\frac{L_2}{D}=\frac{2}{2}\\[4pt] \frac{L_1}{D}=\frac{2}{2}\end{array}\right\}F_{(2+4)\to(1+3)}=0.20 \quad \text{and} \quad \left.\begin{array}{l}\frac{L_2}{D}=\frac{2}{2}\\[4pt] \frac{L_1}{D}=\frac{1}{2}\end{array}\right\}F_{14}=0.12$$

$$\text{superposition rule}: F_{(2+4)\to(1+3)}=F_{(2+4)\to 1}+F_{(2+4)\to 3}$$

$$\text{symmetry rule}: F_{(2+4)\to 1}=F_{(2+4)\to 3}$$

Substituting symmetry rule gives

$$F_{(2+4)\to 1}=F_{(2+4)\to 3}=\frac{F_{(2+4)\to(1+3)}}{2}=\frac{0.20}{2}=0.10$$

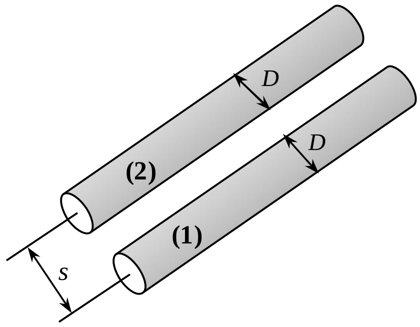
$$\text{reciprocity rule}: A_1F_{1\to(2+4)}=A_{(2+4)}F_{(2+4)\to 1}\longrightarrow(2)F_{1\to(2+4)}=(4)(0.10)\longrightarrow F_{1\to(2+4)}=0.20$$

$$\text{superposition rule}: F_{1\to(2+4)}=F_{12}+F_{14}\longrightarrow 0.20=F_{12}+0.12\longrightarrow F_{12}=0.20-0.12=\mathbf{0.08}$$

22-17 The view factor between the two infinitely long parallel cylinders located a distance *s* apart from each other is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Using the crossed-strings method, the view factor between two cylinders facing each other for *s/D* > 3 is determined to be

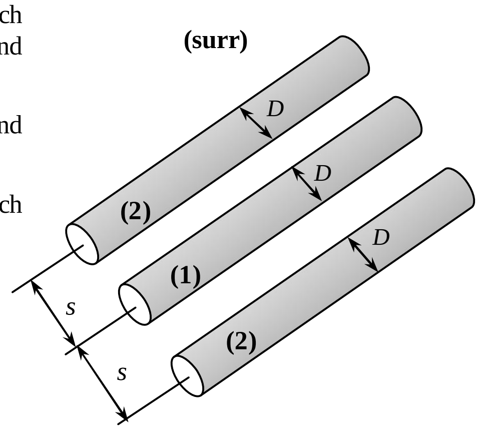


$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{String on surface 1}}$$

$$= \frac{2\sqrt{s^2 + D^2} - 2s}{2(\pi D/2)}$$

or $$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$

22-18 Three infinitely long cylinders are located parallel to each other. The view factor between the cylinder in the middle and the surroundings is to be determined.



Assumptions The cylinder surfaces are diffuse emitters and reflectors.

Analysis The view factor between two cylinder facing each other is, from Prob. 22-17,

$$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$

Noting that the radiation leaving cylinder 1 that does not strike the cylinder will strike the surroundings, and this is also the case for the other half of the cylinder, the view factor between the cylinder in the middle and the surroundings becomes

$$F_{1-surr} = 1 - 2F_{1-2} = 1 - \frac{4\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$

Radiation Heat Transfer Between Surfaces

22-19C The analysis of radiation exchange between black surfaces is relatively easy because of the absence of reflection. The rate of radiation heat transfer between two surfaces in this case is expressed as $\dot{Q} = A_1 F_{12} \sigma (T_1^{\ 4} - T_2^{\ 4})$ where A_1 is the surface area, F_{12} is the view factor, and T_1 and T_2 are the temperatures of two surfaces.

22-20C Radiosity is the total radiation energy leaving a surface per unit time and per unit area. Radiosity includes the emitted radiation energy as well as reflected energy. Radiosity and emitted energy are equal for blackbodies since a blackbody does not reflect any radiation.

22-21C Radiation surface resistance is given as $R_i = \dfrac{1-\varepsilon_i}{A_i \varepsilon_i}$ and it represents the resistance of a surface to the emission of radiation. It is zero for black surfaces. The space resistance is the radiation resistance between two surfaces and is expressed as $R_i = \dfrac{1-\varepsilon_i}{A_i \varepsilon_i}$

22-22C The two methods used in radiation analysis are the matrix and network methods. In matrix method, equations 22-34 and 22-35 give N linear algebraic equations for the determination of the N unknown radiosities for an N -surface enclosure. Once the radiosities are available, the unknown surface temperatures and heat transfer rates can be determined from these equations respectively. This method involves the use of matrices especially when there are a large number of surfaces. Therefore this method requires some knowledge of linear algebra.

The network method involves drawing a surface resistance associated with each surface of an enclosure and connecting them with space resistances. Then the radiation problem is solved by treating it as an electrical network problem where the radiation heat transfer replaces the current and the radiosity replaces the potential. The network method is not practical for enclosures with more than three or four surfaces due to the increased complexity of the network.

22-23C Some surfaces encountered in numerous practical heat transfer applications are modeled as being adiabatic as the back sides of these surfaces are well insulated and net heat transfer through these surfaces is zero. When the convection effects on the front (heat transfer) side of such a surface is negligible and steady-state conditions are reached, the surface must lose as much radiation energy as it receives. Such a surface is called reradiating surface. In radiation analysis, the surface resistance of a reradiating surface is taken to be zero since there is no heat transfer through it.

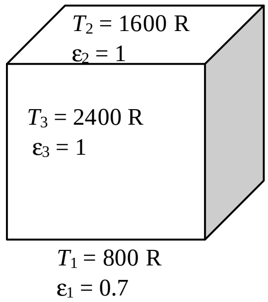
22-24E Top and side surfaces of a cubical furnace are black, and are maintained at uniform temperatures. Net radiation heat transfer rate to the base from the top and side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities are given to be ε = 0.7 for the bottom surface and 1 for other surfaces.

Analysis We consider the base surface to be surface 1, the top surface to be surface 2 and the side surfaces to be surface 3. The cubical furnace can be considered to be three-surface enclosure with a radiation network shown in the figure. The areas and blackbody emissive powers of surfaces are

$$A_1 = A_2 = (10\text{ ft})^2 = 100\text{ ft}^2 \qquad A_3 = 4(10\text{ ft})^2 = 400\text{ ft}^2$$



$$E_{b1} = \sigma T_1^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(800\text{ R})^4 = 702\text{ Btu/h.ft}^2$$

$$E_{b2} = \sigma T_2^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(1600\text{ R})^4 = 11{,}233\text{ Btu/h.ft}^2$$

$$E_{b3} = \sigma T_3^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(2400\text{ R})^4 = 56{,}866\text{ Btu/h.ft}^2$$

The view factor from the base to the top surface of the cube is $F_{12} = 0.2$. From the summation rule, the view factor from the base or top to the side surfaces is

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Then the radiation resistances become

$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.7}{(100\text{ ft}^2)(0.7)} = 0.0043\text{ ft}^{-2} \qquad R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(100\text{ ft}^2)(0.2)} = 0.0500\text{ ft}^{-2}$$

$$R_{13} = \frac{1}{A_1F_{13}} = \frac{1}{(100\text{ ft}^2)(0.8)} = 0.0125\text{ ft}^{-2}$$

Note that the side and the top surfaces are black, and thus their radiosities are equal to their emissive powers. The radiosity of the base surface is determined

$$\frac{E_{b1}-J_1}{R_1} + \frac{E_{b2}-J_1}{R_{12}} + \frac{E_{b3}-J_1}{R_{13}} = 0$$

Substituting,
$$\frac{702-J_1}{0.0043} + \frac{11{,}233-J_1}{0.500} + \frac{56{,}866-J_1}{0.0125} = 0 \longrightarrow J_1 = 15{,}054\text{ W/m}^2$$

(*a*) The net rate of radiation heat transfer between the base and the side surfaces is

$$\dot{Q}_{31} = \frac{E_{b3}-J_1}{R_{13}} = \frac{(56{,}866-15{,}054)\text{ Btu/h.ft}^2}{0.0125\text{ ft}^{-2}} = \mathbf{3.345\times10^6\ Btu/h}$$

(*b*) The net rate of radiation heat transfer between the base and the top surfaces is

$$\dot{Q}_{12} = \frac{J_1-E_{b2}}{R_{12}} = \frac{(15{,}054-11{,}233)\text{ Btu/h.ft}^2}{0.05\text{ ft}^{-2}} = \mathbf{7.642\times10^4\ Btu/h}$$

The net rate of radiation heat transfer to the base surface is finally determined from

$$\dot{Q}_1 = \dot{Q}_{21} + \dot{Q}_{31} = -76{,}420 + 3{,}344{,}960 = \mathbf{3.269\times10^6\ Btu/h}$$

Discussion The same result can be found form

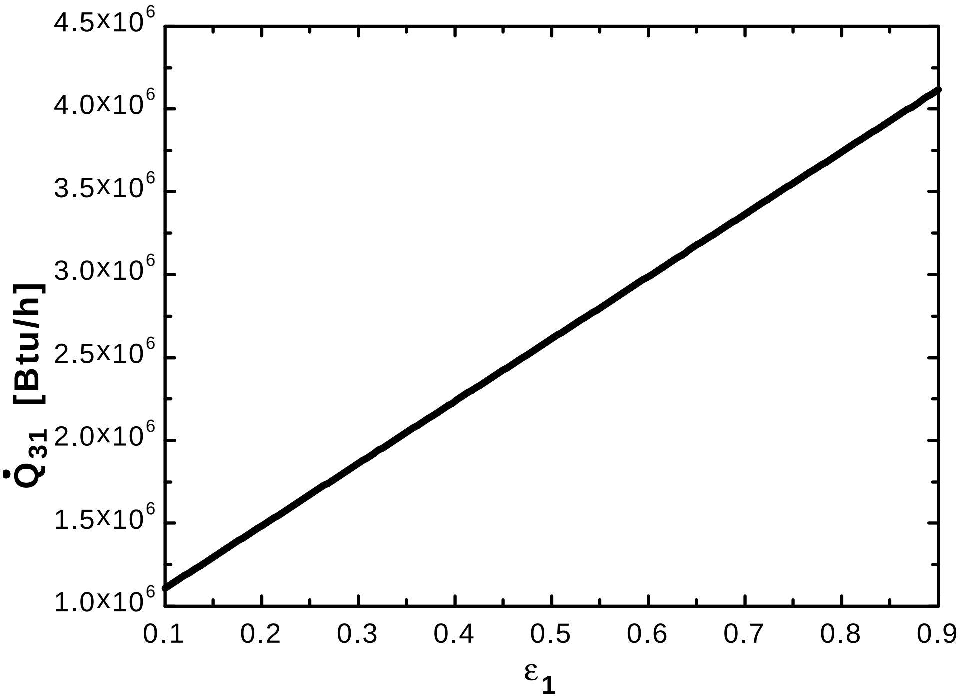
$$\dot{Q}_1 = \frac{J_1-E_{b1}}{R_1} = \frac{(15{,}054-702)\text{ Btu/h.ft}^2}{0.0043\text{ ft}^{-2}} = 3.338\times10^6\text{ Btu/h}$$

The small difference is due to round-off error.

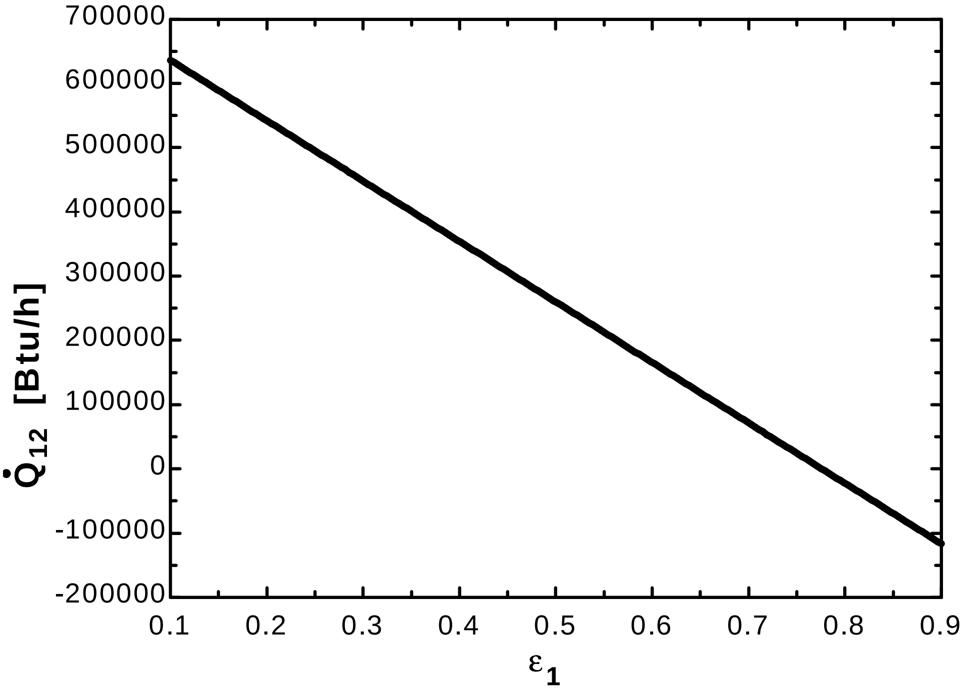
22-25E

```
"GIVEN"
a=10 "[ft]"
"epsilon_1=0.7 parameter to be varied"
T_1=800 "[R]"
T_2=1600 "[R]"
T_3=2400 "[R]"
sigma=0.1714E-8 "[Btu/h-ft^2-R^4], Stefan-Boltzmann constant"

"ANALYSIS"
"Consider the base surface 1, the top surface 2, and the side surface 3"
E_b1=sigma*T_1^4
E_b2=sigma*T_2^4
E_b3=sigma*T_3^4
A_1=a^2
A_2=A_1
A_3=4*a^2
F_12=0.2 "view factor from the base to the top of a cube"
F_11+F_12+F_13=1 "summation rule"
F_11=0 "since the base surface is flat"
R_1=(1-epsilon_1)/(A_1*epsilon_1) "surface resistance"
R_12=1/(A_1*F_12) "space resistance"
R_13=1/(A_1*F_13) "space resistance"
(E_b1-J_1)/R_1+(E_b2-J_1)/R_12+(E_b3-J_1)/R_13=0 "J_1 : radiosity of base surface"
"(a)"
Q_dot_31=(E_b3-J_1)/R_13
"(b)"
Q_dot_12=(J_1-E_b2)/R_12
Q_dot_21=-Q_dot_12
Q_dot_1=Q_dot_21+Q_dot_31
```

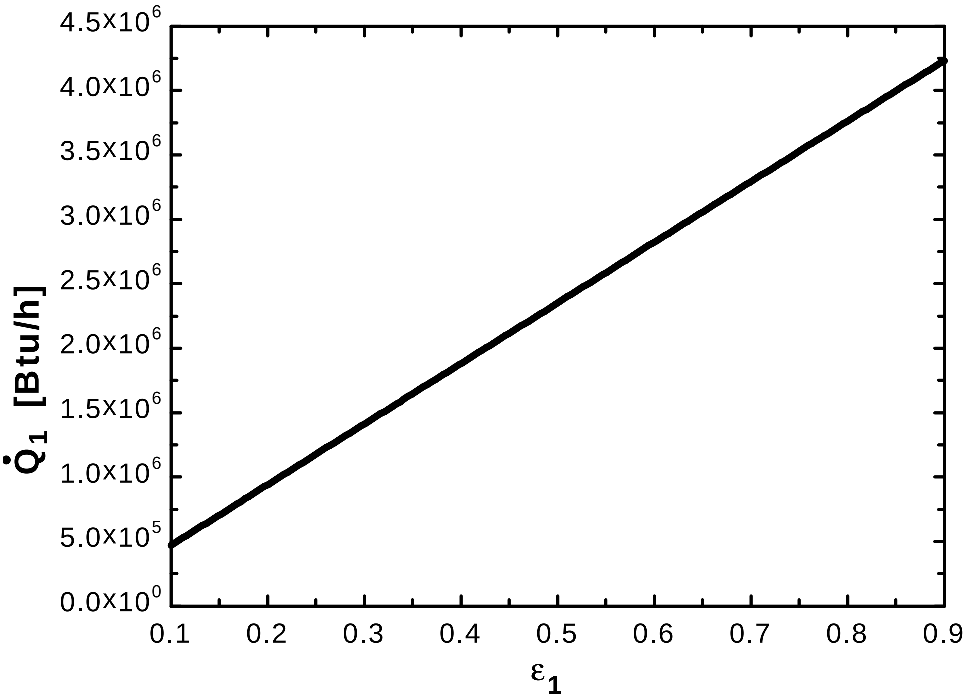
ε_1	Q_{31} [Btu/h]	Q_{12} [Btu/h]	Q_1 [Btu/h]
0.1	1.106E+06	636061	470376
0.15	1.295E+06	589024	705565
0.2	1.483E+06	541986	940753
0.25	1.671E+06	494948	1.176E+06
0.3	1.859E+06	447911	1.411E+06
0.35	2.047E+06	400873	1.646E+06
0.4	2.235E+06	353835	1.882E+06
0.45	2.423E+06	306798	2.117E+06
0.5	2.612E+06	259760	2.352E+06
0.55	2.800E+06	212722	2.587E+06
0.6	2.988E+06	165685	2.822E+06
0.65	3.176E+06	118647	3.057E+06
0.7	3.364E+06	71610	3.293E+06
0.75	3.552E+06	24572	3.528E+06
0.8	3.741E+06	-22466	3.763E+06
0.85	3.929E+06	-69503	3.998E+06
0.9	4.117E+06	-116541	4.233E+06

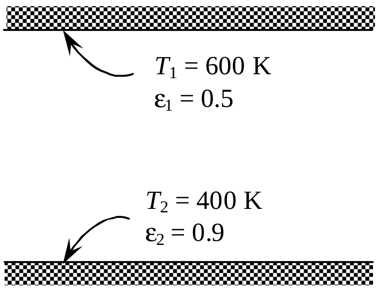


4.5x10^6
4.0x10^6
3.5x10^6
3.0x10^6
2.5x10^6
2.0x10^6
1.5x10^6
1.0x10^6
$\dot{Q}_{31}$ [Btu/h]
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ε_1



700000
600000
500000
400000
300000
200000
100000
0
-100000
-200000
$\dot{Q}_{12}$ [Btu/h]
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ε_1


4.5x10^6
4.0x10^6
3.5x10^6
3.0x10^6
2.5x10^6
2.0x10^6
1.5x10^6
1.0x10^6
5.0x10^5
0.0x10^0
Q̇1 [Btu/h]
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ε1



22-26 Two very large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities ε of the plates are given to be 0.5 and 0.9.

Analysis The net rate of radiation heat transfer between the two surfaces per unit area of the plates is determined directly from

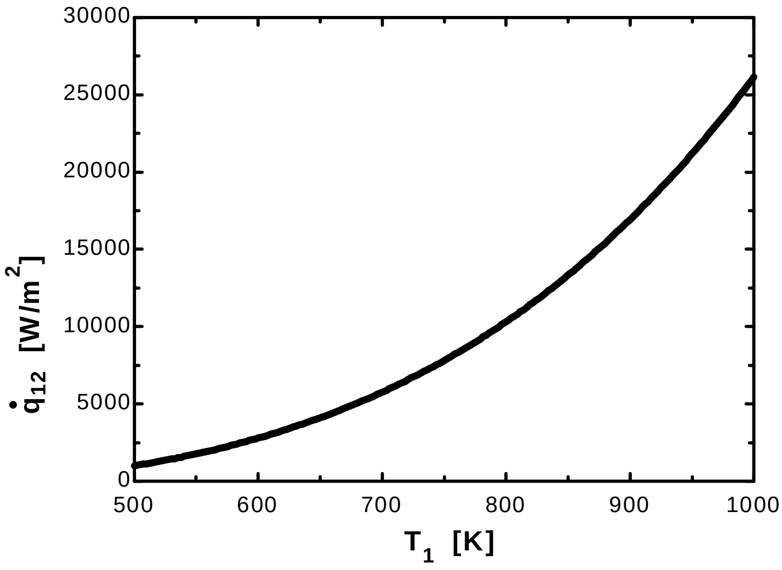
$$\frac{\dot{Q}_{12}}{A_s} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (400\ \text{K})^4]}{\frac{1}{0.5} + \frac{1}{0.9} - 1} = \mathbf{2795\ W/m^2}$$

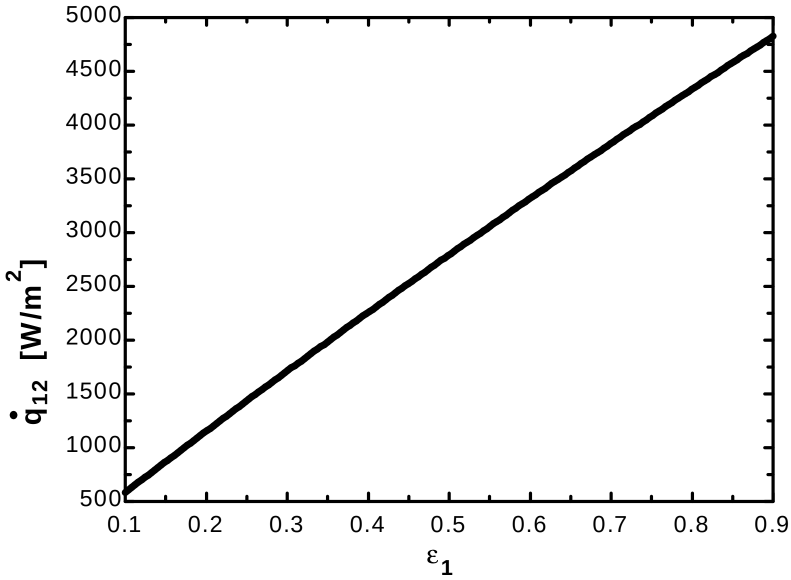
22-27

```
"GIVEN"
T_1=600 "[K], parameter to be varied"
T_2=400 "[K]"
epsilon_1=0.5 "parameter to be varied"
epsilon_2=0.9
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
q_dot_12=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
```

T_1 [K]	q_{12} [W/m^2]
500	991.1
525	1353
550	1770
575	2248
600	2793
625	3411
650	4107
675	4888
700	5761
725	6733
750	7810
775	9001
800	10313
825	11754
850	13332
875	15056
900	16934
925	18975
950	21188
975	23584
1000	26170

ε_1	q_{12} [W/m^2]
0.1	583.2
0.15	870
0.2	1154
0.25	1434
0.3	1712
0.35	1987
0.4	2258
0.45	2527
0.5	2793
0.55	3056
0.6	3317
0.65	3575
0.7	3830
0.75	4082
0.8	4332
0.85	4580
0.9	4825

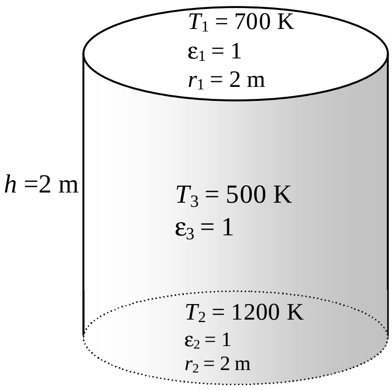

30000
25000
20000
15000
10000
5000
0
500
600
700
800
900
1000
$\dot{q}_{12}$ [W/m^2]
T_1 [K]


5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
$\dot{q}_{12}$ [W/m^2]
ε_1

22-28 The base, top, and side surfaces of a furnace of cylindrical shape are black, and are maintained at uniform temperatures. The net rate of radiation heat transfer to or from the top surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Properties The emissivity of all surfaces are ε = 1 since they are black.



Analysis We consider the top surface to be surface 1, the base surface to be surface 2 and the side surfaces to be surface 3. The cylindrical furnace can be considered to be three-surface enclosure. We assume that steady-state conditions exist. Since all surfaces are black, the radiosities are equal to the emissive power of surfaces, and the net rate of radiation heat transfer from the top surface can be determined from

$$\dot{Q} = A_1 F_{12}\sigma(T_1^{\,4} - T_2^{\,4}) + A_1 F_{13}\sigma(T_1^{\,4} - T_3^{\,4})$$

and $\quad A_1 = \pi r^2 = \pi(2\text{ m})^2 = 12.57\text{ m}^2$

The view factor from the base to the top surface of the cylinder is $F_{12} = 0.38$ (From Figure 22-44). The view factor from the base to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.38 = 0.62$$

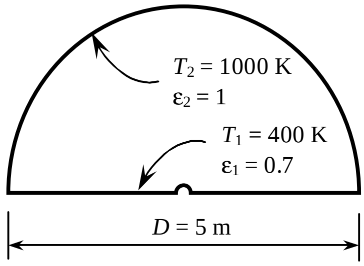
Substituting,

$$\begin{aligned}\dot{Q} &= A_1 F_{12}\sigma(T_1^{\,4} - T_2^{\,4}) + A_1 F_{13}\sigma(T_1^{\,4} - T_3^{\,4})\\ &= (12.57\text{ m}^2)(0.38)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(700\text{ K}^4 - 500\text{ K}^4)\\ &\quad + (12.57\text{ m}^2)(0.62)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(700\text{ K}^4 - 1200\text{ K}^4)\\ &= -7.62\times10^5\text{ W} = \mathbf{-762\ kW}\end{aligned}$$

Discussion The negative sign indicates that net heat transfer is to the top surface.

22-29 The base and the dome of a hemispherical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Analysis The view factor is first determined from

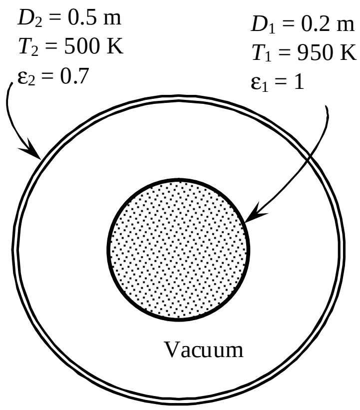
$$F_{11} = 0 \quad \text{(flat surface)}$$
$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \quad \text{(summation rule)}$$

Noting that the dome is black, net rate of radiation heat transfer from dome to the base surface can be determined from

$$\begin{aligned}\dot{Q}_{21} &= -\dot{Q}_{12} = -\varepsilon A_1 F_{12} \sigma (T_1^{\,4} - T_2^{\,4}) \\ &= -(0.7)[\pi(5\text{ m})^2/4](1)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(400\text{ K})^4 - (1000\text{ K})^4] \\ &= 7.594\times10^5\text{ W} \\ &= \mathbf{759.4\ kW}\end{aligned}$$

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

22-30 Two very long concentric cylinders are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined.



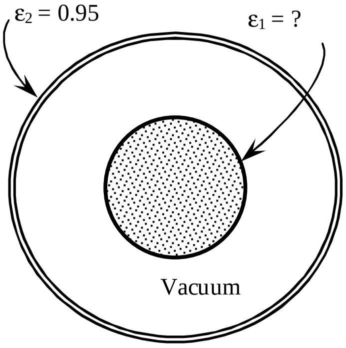
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.7$.

Analysis The net rate of radiation heat transfer between the two cylinders per unit length of the cylinders is determined from

$$\begin{aligned}\dot{Q}_{12} &= \frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)} = \frac{[\pi(0.2\text{ m})(1\text{ m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(950\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1}{1} + \dfrac{1-0.7}{0.7}\left(\dfrac{2}{5}\right)} \\ &= 22{,}870\text{ W} = \mathbf{22.87\ kW}\end{aligned}$$

22-31 A long cylindrical rod coated with a new material is placed in an evacuated long cylindrical enclosure which is maintained at a uniform temperature. The emissivity of the coating on the rod is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivity of the enclosure is given to be ε_2 = 0.95.

Analysis The emissivity of the coating on the rod is determined from

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

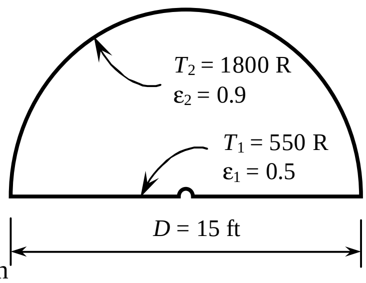
$$8\text{ W} = \frac{[\pi(0.01\text{m})(1\text{m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(500\text{ K})^4 - (200\text{ K})^4]}{\dfrac{1}{\varepsilon_1} + \dfrac{1-0.95}{0.95}\left(\dfrac{1}{10}\right)}$$

which gives

ε_1 = **0.074**

22-32E The base and the dome of a long semicylindrical duct are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivities of surfaces are given to be ε_1 = 0.5 and ε_2 = 0.9.

Analysis The view factor from the base to the dome is first determined from

$$F_{11} = 0 \quad \text{(flat surface)}$$
$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \quad \text{(summation rule)}$$

The net rate of radiation heat transfer from dome to the base surface can be determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = -\frac{(0.1714\times10^{-8}\text{ Btu/h.ft}^2\cdot\text{R}^4)[(550\text{ R})^4 - (1800\text{ R})^4]}{\dfrac{1-0.5}{(15\text{ ft}^2)(0.5)} + \dfrac{1}{(15\text{ ft}^2)(1)} + \dfrac{1-0.9}{\left[\dfrac{\pi(15\text{ ft})(1\text{ ft})}{2}\right](0.9)}}$$

$$= \mathbf{1.311\times10^6\ Btu/h}$$

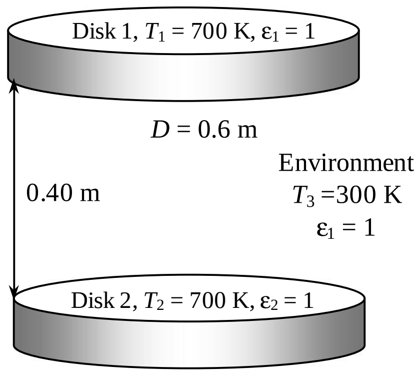
The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

22-33 Two parallel disks whose back sides are insulated are black, and are maintained at a uniform temperature. The net rate of radiation heat transfer from the disks to the environment is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are ε = 1 since they are black.

Analysis Both disks possess same properties and they are black. Noting that environment can also be considered to be blackbody, we can treat this geometry as a three surface enclosure. We consider the two disks to be surfaces 1 and 2 and the environment to be surface 3. Then from Figure 22-7, we read



$$F_{12} = F_{21} = 0.26$$
$$F_{13} = 1 - 0.26 = 0.74 \quad \text{(summation rule)}$$

The net rate of radiation heat transfer from the disks into the environment then becomes

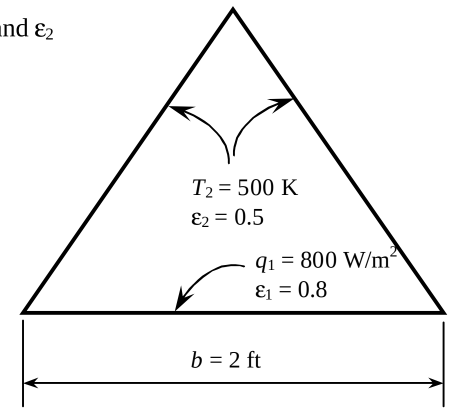
$$\dot{Q}_3 = \dot{Q}_{13} + \dot{Q}_{23} = 2\dot{Q}_{13}$$
$$\begin{aligned}\dot{Q}_3 &= 2F_{13}A_1\sigma(T_1^{\,4} - T_3^{\,4}) \\ &= 2(0.74)[\pi(0.3\,\text{m})^2](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(700\,\text{K})^4 - (300\,\text{K})^4] \\ &= \mathbf{5505\ W}\end{aligned}$$

22-34 A furnace shaped like a long equilateral-triangular duct is considered. The temperature of the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** End effects are neglected.

Properties The emissivities of surfaces are given to be ε_1 = 0.8 and ε_2 = 0.5.

Analysis This geometry can be treated as a two surface enclosure since two surfaces have identical properties. We consider base surface to be surface 1 and other two surface to be surface 2. Then the view factor between the two becomes $F_{12} = 1$. The temperature of the base surface is determined from



$$\dot{Q}_{12} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$800\ \text{W} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(T_1)^4 - (500\,\text{K})^4]}{\dfrac{1-0.8}{(1\,\text{m}^2)(0.8)} + \dfrac{1}{(1\,\text{m}^2)(1)} + \dfrac{1-0.5}{(2\,\text{m}^2)(0.5)}} \longrightarrow \mathbf{T_1 = 543\,K}$$

Note that $A_1 = 1\,\text{m}^2$ and $A_2 = 2\,\text{m}^2$.

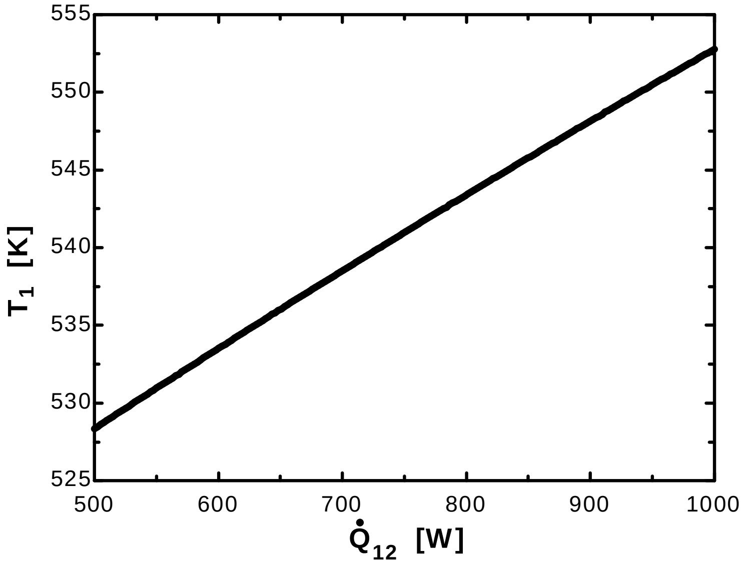
22-35

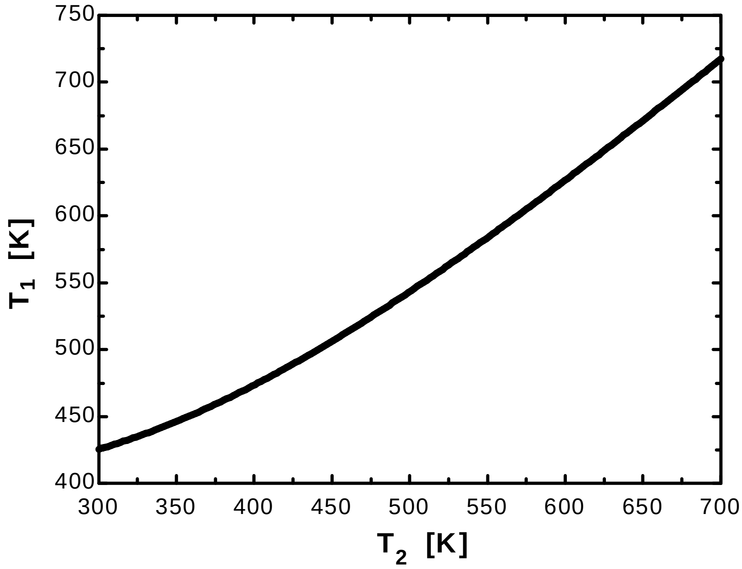
```
"GIVEN"
a=2 "[m]"
epsilon_1=0.8
epsilon_2=0.5
Q_dot_12=800 "[W], parameter to be varied"
T_2=500 "[K], parameter to be varied"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
"Consider the base surface to be surface 1, the side surfaces to be surface 2"
Q_dot_12=(sigma*(T_1^4-T_2^4))/((1-epsilon_1)/(A_1*epsilon_1)+1/(A_1*F_12)+(1-
epsilon_2)/(A_2*epsilon_2))
F_12=1
A_1=1 "[m^2], since rate of heat supply is given per meter square area"
A_2=2*A_1
```

Q_{12} [W]	T_1 [K]
500	528.4
525	529.7
550	531
575	532.2
600	533.5
625	534.8
650	536
675	537.3
700	538.5
725	539.8
750	541
775	542.2
800	543.4
825	544.6
850	545.8
875	547
900	548.1
925	549.3
950	550.5
975	551.6
1000	552.8

T_2 [K]	T_1 [K]
300	425.5
325	435.1
350	446.4
375	459.2
400	473.6
425	489.3
450	506.3
475	524.4
500	543.4
525	563.3
550	583.8
575	605
600	626.7

625	648.9
650	671.4
675	694.2
700	717.3





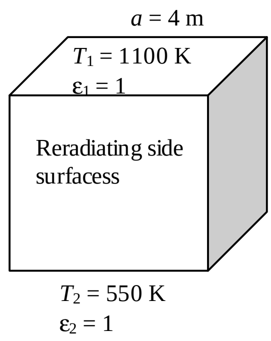
22-36 The floor and the ceiling of a cubical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer between the floor and the ceiling is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are $\varepsilon = 1$ since they are black or reradiating.

Analysis We consider the ceiling to be surface 1, the floor to be surface 2 and the side surfaces to be surface 3. The furnace can be considered to be three-surface enclosure with a radiation network shown in the figure. We assume that steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. The view factor from the ceiling to the floor of the furnace is $F_{12} = 0.2$. Then the rate of heat loss from the ceiling can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1}}$$



where

$$E_{b1} = \sigma T_1^4 = (5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(1100\text{ K})^4 = 83{,}015\text{ W/m}^2$$

$$E_{b2} = \sigma T_2^4 = (5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(550\text{ K})^4 = 5188\text{ W/m}^2$$

and

$$A_1 = A_2 = (4\text{ m})^2 = 16\text{ m}^2$$

$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(16\text{ m}^2)(0.2)} = 0.3125\text{ m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1F_{13}} = \frac{1}{(16\text{ m}^2)(0.8)} = 0.078125\text{ m}^{-2}$$

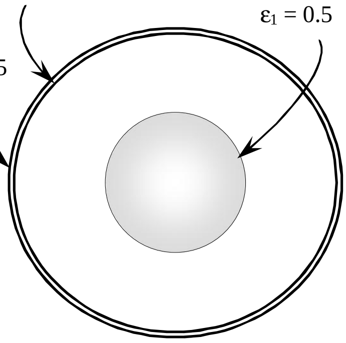
Substituting,

$$\dot{Q}_{12} = \frac{(83{,}015 - 5188)\text{ W/m}^2}{\left(\frac{1}{0.3125\text{ m}^{-2}} + \frac{1}{2(0.078125\text{ m}^{-2})}\right)^{-1}} = 7.47\times10^5\text{ W} = \mathbf{747\ kW}$$

22-37 Two concentric spheres are maintained at uniform temperatures. The net rate of radiation heat transfer between the two spheres and the convection heat transfer coefficient at the outer surface are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.



Analysis The net rate of radiation heat transfer between the two spheres is

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^2}{r_2^2}\right)}$$

$$= \frac{[\pi(0.3\,\text{m})^2](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(700\,\text{K})^4 - (400\,\text{K})^4]}{\dfrac{1}{0.5} + \dfrac{1-0.7}{0.7}\left(\dfrac{0.15\,\text{m}}{0.4\,\text{m}}\right)^2}$$

$$= \mathbf{1669\ W}$$

Radiation heat transfer rate from the outer sphere to the surrounding surfaces are

$$\dot{Q}_{rad} = \varepsilon F A_2 \sigma(T_2^4 - T_{surr}^4)$$
$$= (0.35)(1)[\pi(0.8\,\text{m})^2](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(400\,\text{K})^4 - (30+273\,\text{K})^4] = 685\,\text{W}$$

The convection heat transfer rate at the outer surface of the cylinder is determined from requirement that heat transferred from the inner sphere to the outer sphere must be equal to the heat transfer from the outer surface of the outer sphere to the environment by convection and radiation. That is,

$$\dot{Q}_{conv} = \dot{Q}_{12} - \dot{Q}_{rad} = 1669 - 685 = 9845\,\text{W}$$

Then the convection heat transfer coefficient becomes

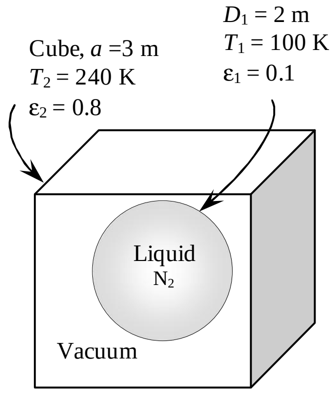
$$\dot{Q}_{conv.} = hA_2(T_2 - T_\infty)$$
$$984\,\text{W} = h[\pi(0.8\,\text{m})^2](400\,\text{K} - 303\,\text{K}) \longrightarrow h = \mathbf{5.04\ W/m^2\cdot{}^\circ C}$$

22-38 A spherical tank filled with liquid nitrogen is kept in an evacuated cubic enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis We take the sphere to be surface 1 and the surrounding cubic enclosure to be surface 2. Noting that $F_{12} = 1$, for this two-surface enclosure, the net rate of radiation heat transfer to liquid nitrogen can be determined from



$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{A_1}{A_2}\right)}$$

$$= -\frac{\left[\pi(2\text{ m})^2\right]\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left[(100\text{ K})^4 - (240\text{ K})^4\right]}{\dfrac{1}{0.1} + \dfrac{1-0.8}{0.8}\left[\dfrac{\pi(2\text{ m})^2}{6(3\text{ m})^2}\right]}$$

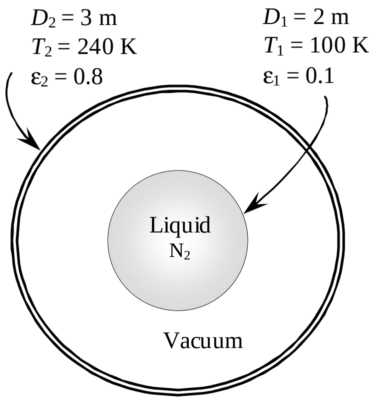
$$= \mathbf{228\ W}$$

22-39 A spherical tank filled with liquid nitrogen is kept in an evacuated spherical enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis The net rate of radiation heat transfer to liquid nitrogen can be determined from



$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^2}{r_2^2}\right)}$$

$$= \frac{[\pi(2\text{ m})^2](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(240\text{ K})^4 - (100\text{ K})^4]}{\dfrac{1}{0.1} + \dfrac{1-0.8}{0.8}\left(\dfrac{(1\text{ m})^2}{(1.5\text{ m})^2}\right)}$$

$$= \mathbf{227\ W}$$

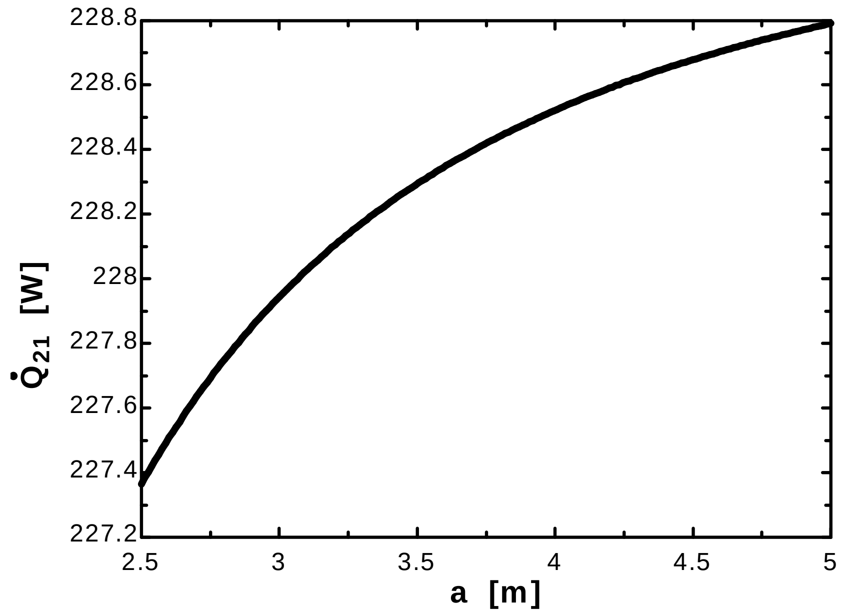
22-40

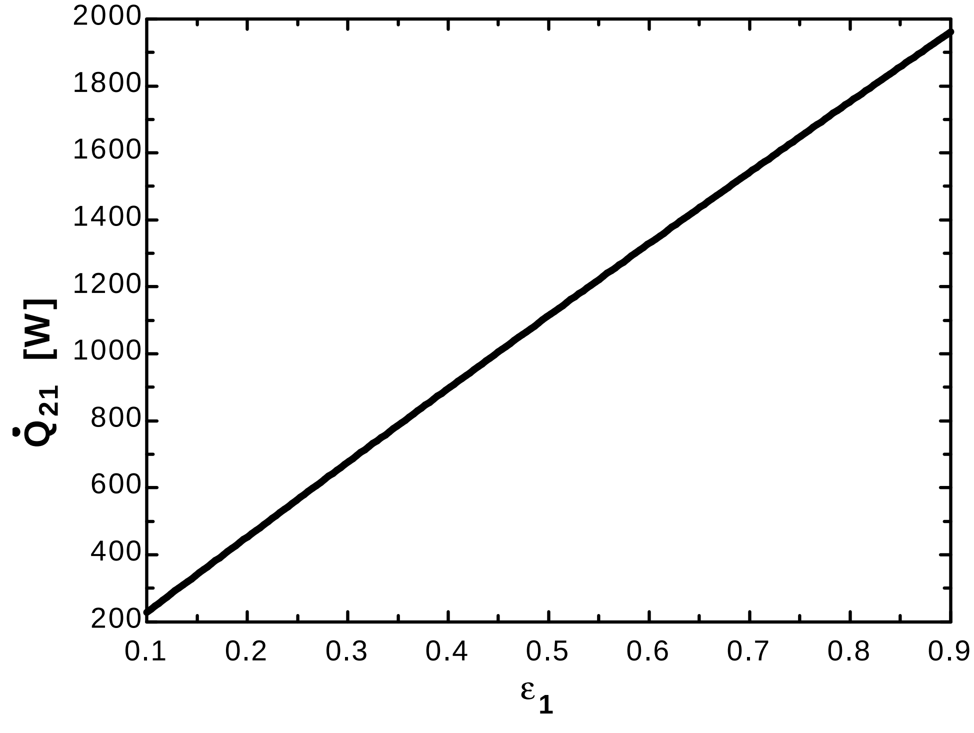
```
"GIVEN"
D=2 "[m]"
a=3 "[m], parameter to be varied"
T_1=100 "[K]"
T_2=240 "[K]"
epsilon_1=0.1 "parameter to be varied"
epsilon_2=0.8 "parameter to be varied"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
"Consider the sphere to be surface 1, the surrounding cubic enclosure to be surface 2"
Q_dot_12=(A_1*sigma*(T_1^4-T_2^4))/(1/epsilon_1+(1-epsilon_2)/epsilon_2*(A_1/A_2))
Q_dot_21=-Q_dot_12
A_1=pi*D^2
A_2=6*a^2
```

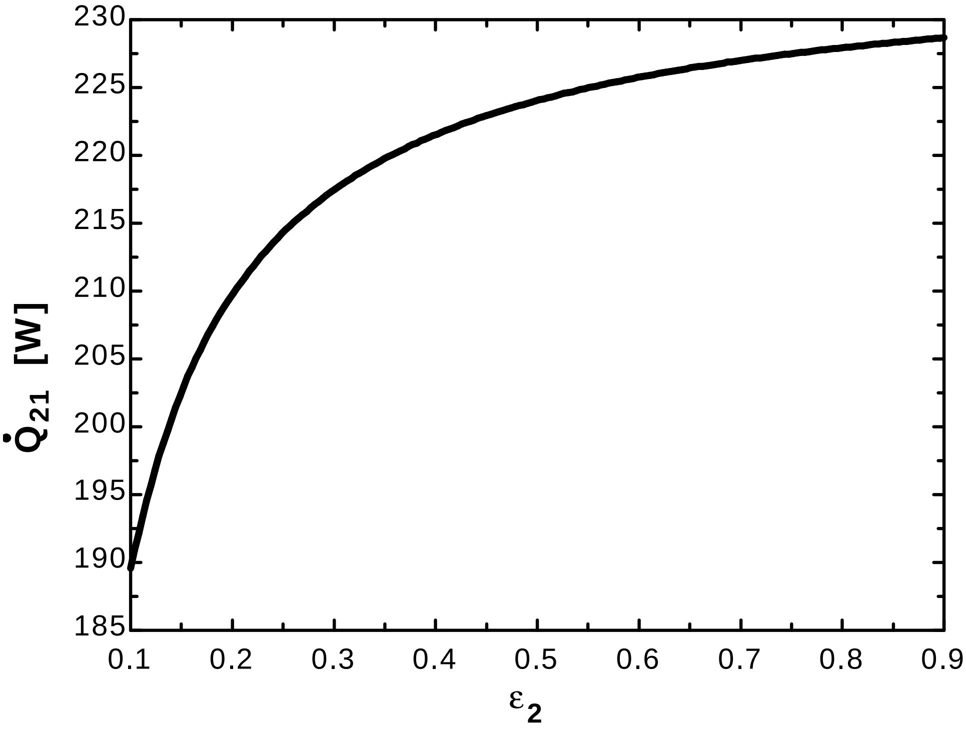
a [m]	Q_{21} [W]
2.5	227.4
2.625	227.5
2.75	227.7
2.875	227.8
3	227.9
3.125	228
3.25	228.1
3.375	228.2
3.5	228.3
3.625	228.4
3.75	228.4
3.875	228.5
4	228.5
4.125	228.6
4.25	228.6
4.375	228.6
4.5	228.7
4.625	228.7
4.75	228.7
4.875	228.8
5	228.8

ε_1	Q_{21} [W]
0.1	227.9
0.15	340.9
0.2	453.3
0.25	565
0.3	676
0.35	786.4
0.4	896.2
0.45	1005
0.5	1114
0.55	1222
0.6	1329
0.65	1436
0.7	1542
0.75	1648
0.8	1753
0.85	1857
0.9	1961

ε_2	Q_{21} [W]
0.1	189.6
0.15	202.6
0.2	209.7
0.25	214.3
0.3	217.5
0.35	219.8
0.4	221.5
0.45	222.9
0.5	224.1
0.55	225
0.6	225.8
0.65	226.4
0.7	227
0.75	227.5
0.8	227.9
0.85	228.3
0.9	228.7


228.8
228.6
228.4
228.2
228
227.8
227.6
227.4
227.2
2.5
3
3.5
4
4.5
5
a [m]
$\dot{Q}_{21}$ [W]

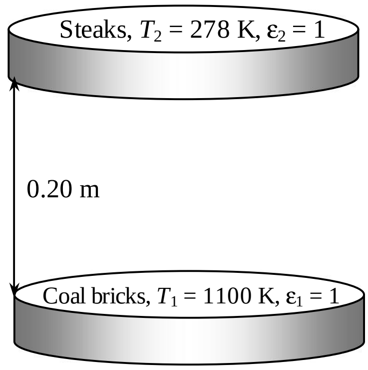

2000
1800
1600
1400
1200
1000
800
600
400
200
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ε_1
$\dot{Q}_{21}$ [W]


230
225
220
215
210
205
200
195
190
185
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
$\dot{Q}_{21}$ [W]
ε_2

22-41 A circular grill is considered. The bottom of the grill is covered with hot coal bricks, while the wire mesh on top of the grill is covered with steaks. The initial rate of radiation heat transfer from coal bricks to the steaks is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities are $\varepsilon = 1$ for all surfaces since they are black or reradiating.



Analysis We consider the coal bricks to be surface 1, the steaks to be surface 2 and the side surfaces to be surface 3. First we determine the view factor between the bricks and the steaks (Table 22-1),

$$R_i = R_j = \frac{r_i}{L} = \frac{0.15\,\text{m}}{0.20\,\text{m}} = 0.75$$

$$S = 1 + \frac{1 + R_j^{\,2}}{R_i^{\,2}} = \frac{1+0.75^2}{0.75^2} = 3.7778$$

$$F_{12} = F_{ij} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{R_j}{R_i}\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{3.7778 - \left[3.7778^2 - 4\left(\frac{0.75}{0.75}\right)^2\right]^{1/2}\right\} = 0.2864$$

(It can also be determined from Fig. 22-7).

Then the initial rate of radiation heat transfer from the coal bricks to the stakes becomes

$$\begin{aligned}\dot{Q}_{12} &= F_{12}A_1\sigma(T_1^{\,4} - T_2^{\,4}) \\ &= (0.2864)[\pi(0.3\,\text{m})^2/4](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1100\,\text{K})^4 - (278\,\text{K})^4] \\ &= \mathbf{1674\ W}\end{aligned}$$

When the side opening is closed with aluminum foil, the entire heat lost by the coal bricks must be gained by the stakes since there will be no heat transfer through a reradiating surface. The grill can be considered to be three-surface enclosure. Then the rate of heat loss from the room can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}}$$

where

$$E_{b1} = \sigma T_1^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1100\ \text{K})^4 = 83{,}015\ \text{W/m}^2$$

$$E_{b2} = \sigma T_2^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(18 + 273\ \text{K})^4 = 407\ \text{W/m}^2$$

and

$$A_1 = A_2 = \frac{\pi(0.3\ \text{m})^2}{4} = 0.07069\ \text{m}^2$$

$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(0.07069\,\text{m}^2)(0.2864)} = 49.39\,\text{m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1F_{13}} = \frac{1}{(0.07069\,\text{m}^2)(1 - 0.2864)} = 19.82\,\text{m}^{-2}$$

Substituting,

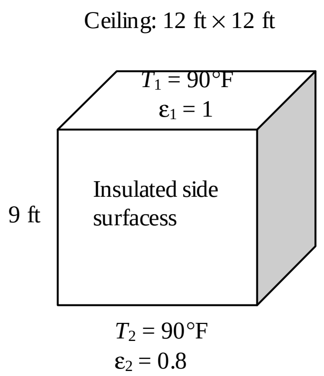
$$\dot{Q}_{12} = \frac{(83{,}015 - 407)\ \text{W/m}^2}{\left(\dfrac{1}{49.39\,\text{m}^{-2}} + \dfrac{1}{2(19.82\,\text{m}^{-2})}\right)^{-1}} = \mathbf{3757\ \ W}$$

22-42E A room is heated by lectric resistance heaters placed on the ceiling which is maintained at a uniform temperature. The rate of heat loss from the room through the floor is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** There is no heat loss through the side surfaces.

Properties The emissivities are ε = 1 for the ceiling and ε = 0.8 for the floor. The emissivity of insulated (or reradiating) surfaces is also 1.

Analysis The room can be considered to be three-surface enclosure with the ceiling surface 1, the floor surface 2 and the side surfaces surface 3. We assume steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. Then the rate of heat loss from the room through its floor can be determined from



$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1} + R_2}$$

where

$$E_{b1} = \sigma T_1^{\,4} = (0.1714 \times 10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(90 + 460\ \text{R})^4 = 157\ \text{Btu/h.ft}^2$$
$$E_{b2} = \sigma T_2^{\,4} = (0.1714 \times 10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(65 + 460\ \text{R})^4 = 130\ \text{Btu/h.ft}^2$$

and

$$A_1 = A_2 = (12\ \text{ft})^2 = 144\ \text{ft}^2$$

The view factor from the floor to the ceiling of the room is $F_{12} = 0.27$ (From Figure 22-5). The view factor from the ceiling or the floor to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.27 = 0.73$$

since the ceiling is flat and thus $F_{11} = 0$. Then the radiation resistances which appear in the equation above become

$$R_2 = \frac{1 - \varepsilon_2}{A_2 \varepsilon_2} = \frac{1 - 0.8}{(144\ \text{ft}^2)(0.8)} = 0.00174\ \text{ft}^{-2}$$
$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(144\ \text{ft}^2)(0.27)} = 0.02572\ \text{ft}^{-2}$$
$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(144\ \text{ft}^2)(0.73)} = 0.009513\ \text{ft}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(157 - 130)\ \text{Btu/h.ft}^2}{\left(\dfrac{1}{0.02572\ \text{ft}^{-2}} + \dfrac{1}{2(0.009513\ \text{ft}^{-2})}\right)^{-1} + 0.00174\ \text{ft}^{-2}} = \mathbf{2130\ Btu/h}$$

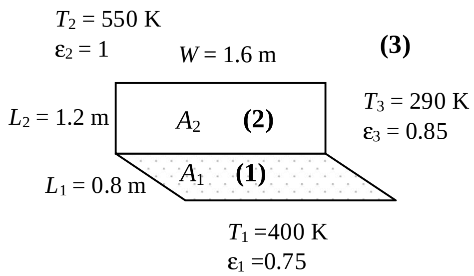
22-43 Two perpendicular rectangular surfaces with a common edge are maintained at specified temperatures. The net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of the horizontal rectangle and the surroundings are ε = 0.75 and ε = 0.85, respectively.

Analysis We consider the horizontal rectangle to be surface 1, the vertical rectangle to be surface 2 and the surroundings to be surface 3. This system can be considered to be a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from

$$\left.\begin{aligned}\frac{L_1}{W} &= \frac{0.8}{1.6} = 0.5\\ \frac{L_2}{W} &= \frac{1.2}{1.6} = 0.75\end{aligned}\right\} F_{12} = 0.27 \qquad \text{(Fig. 22-6)}$$



The surface areas are

$$A_1 = (0.8\text{ m})(1.6\text{ m}) = 1.28\text{ m}^2$$

$$A_2 = (1.2\text{ m})(1.6\text{ m}) = 1.92\text{ m}^2$$

$$A_3 = 2\times\frac{1.2\times 0.8}{2} + \sqrt{0.8^2+1.2^2}\times 1.6 = 3.268\text{ m}^2$$

Note that the surface area of the surroundings is determined assuming that surroundings forms flat surfaces at all openings to form an enclosure. Then other view factors are determined to be

$$A_1F_{12} = A_2F_{21} \longrightarrow (1.28)(0.27) = (1.92)F_{21} \longrightarrow F_{21} = 0.18 \qquad \text{(reciprocity rule)}$$

$$F_{11}+F_{12}+F_{13} = 1 \longrightarrow 0+0.27+F_{13} = 1 \longrightarrow F_{13} = 0.73 \qquad \text{(summation rule)}$$

$$F_{21}+F_{22}+F_{23} = 1 \longrightarrow 0.18+0+F_{23} = 1 \longrightarrow F_{23} = 0.82 \qquad \text{(summation rule)}$$

$$A_1F_{13} = A_3F_{31} \longrightarrow (1.28)(0.73) = (3.268)F_{31} \longrightarrow F_{31} = 0.29 \qquad \text{(reciprocity rule)}$$

$$A_2F_{23} = A_3F_{32} \longrightarrow (1.92)(0.82) = (3.268)F_{32} \longrightarrow F_{32} = 0.48 \qquad \text{(reciprocity rule)}$$

We now apply Eq. 9-52b to each surface to determine the radiosities.

Surface 1:

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1-J_2)+F_{13}(J_1-J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(400\text{ K})^4 = J_1 + \frac{1-0.75}{0.75}\left[0.27(J_1-J_2)+0.73(J_1-J_3)\right]$$

Surface 2:

$$\sigma T_2^4 = J_2 \longrightarrow (5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(550\text{ K})^4 = J_2$$

Surface 3:

$$\sigma T_3^4 = J_3 + \frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3-J_1)+F_{32}(J_3-J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(290\text{ K})^4 = J_3 + \frac{1-0.85}{0.85}\left[0.29(J_1-J_2)+0.48(J_1-J_3)\right]$$

Solving the above equations, we find

$$J_1 = 1587\text{ W/m}^2,\quad J_2 = 5188\text{ W/m}^2,\quad J_3 = 811.5\text{ W/m}^2$$

Then the net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are determined to be

$$\dot{Q}_{21} = -\dot{Q}_{12} = -A_1F_{12}(J_1-J_2) = -(1.28\text{ m}^2)(0.27)(1587-5188)\text{W/m}^2 = \mathbf{1245\text{ W}}$$

$$\dot{Q}_{13} = A_1F_{13}(J_1-J_3) = (1.28\text{ m}^2)(0.73)(1587-811.5)\text{W/m}^2 = \mathbf{725\text{ W}}$$

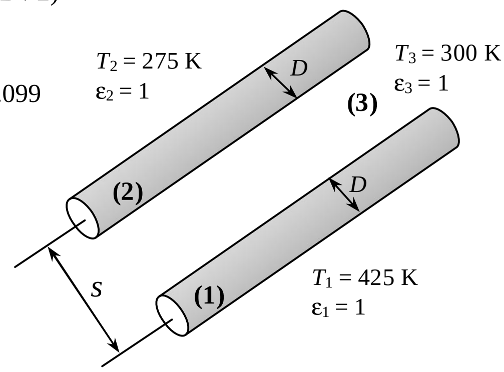
22-44 Two long parallel cylinders are maintained at specified temperatures. The rates of radiation heat transfer between the cylinders and between the hot cylinder and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis We consider the hot cylinder to be surface 1, cold cylinder to be surface 2, and the surroundings to be surface 3. Using the crossed-strings method, the view factor between two cylinders facing each other is determined to be

$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{String on surface 1}} = \frac{2\sqrt{s^2 + D^2} - 2s}{2(\pi D/2)}$$

$$\text{or} \quad F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D} = \frac{2\left(\sqrt{0.5^2 + 0.16^2} - 0.5\right)}{\pi(0.16)} = 0.099$$



The view factor between the hot cylinder and the surroundings is

$$F_{13} = 1 - F_{12} = 1 - 0.099 = 0.901 \text{ (summation rule)}$$

The rate of radiation heat transfer between the cylinders per meter length is

$$A = \pi DL/2 = \pi(0.16\text{ m})(1\text{ m})/2 = 0.2513\text{ m}^2$$

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^{\,4} - T_2^{\,4}) = (0.2513\text{ m}^2)(0.099)(5.67\times10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 275^4)\text{K}^4 = \textbf{38.0 W}$$

Note that half of the surface area of the cylinder is used, which is the only area that faces the other cylinder. The rate of radiation heat transfer between the hot cylinder and the surroundings per meter length of the cylinder is

$$A_1 = \pi DL = \pi(0.16\text{ m})(1\text{ m}) = 0.5027\text{ m}^2$$

$$\dot{Q}_{13} = A_1 F_{13}\sigma(T_1^{\,4} - T_3^{\,4}) = (0.5027\text{ m}^2)(0.901)(5.67\times10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 300^4)\text{K}^4 = \textbf{629.8 W}$$

22-45 A long semi-cylindrical duct with specified temperature on the side surface is considered. The temperature of the base surface for a specified heat transfer rate is to be determined.

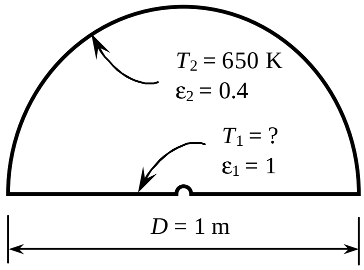
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the side surface is $\varepsilon = 0.4$.

Analysis We consider the base surface to be surface 1, the side surface to be surface 2. This system is a two-surface enclosure, and we consider a unit length of the duct. The surface areas and the view factor are determined as

$$A_1 = (1.0\text{ m})(1.0\text{ m}) = 1.0\text{ m}^2$$

$$A_2 = \pi DL/2 = \pi(1.0\text{ m})(1\text{ m})/2 = 1.571\text{ m}^2$$



$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$

The temperature of the base surface is determined from

$$\dot{Q}_{12} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$1200\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot K^4)[T_1^{\,4} - (650\text{ K})^4]}{\dfrac{1}{(1.0\text{ m}^2)(1)} + \dfrac{1-0.4}{(1.571\text{ m}^2)(0.4)}} \longrightarrow T_1 = \mathbf{684.8\ K}$$

22-46 A hemisphere with specified base and dome temperatures and heat transfer rate is considered. The emissivity of the dome is to be determined.

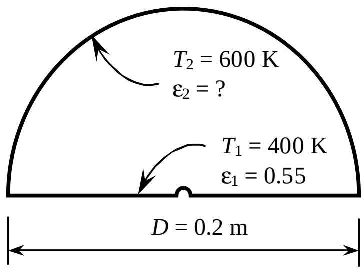
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the base surface is $\varepsilon = 0.55$.

Analysis We consider the base surface to be surface 1, the dome surface to be surface 2. This system is a two-surface enclosure. The surface areas and the view factor are determined as

$$A_1 = \pi D^2/4 = \pi(0.2\text{ m})^2/4 = 0.0314\text{ m}^2$$

$$A_2 = \pi D^2/2 = \pi(0.2\text{ m})^2/2 = 0.0628\text{ m}^2$$



$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$

The emissivity of the dome is determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$50\text{ W} = -\frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(400\text{ K})^4 - (600\text{ K})^4]}{\dfrac{1-0.55}{(0.0314\text{ m}^2)(0.55)} + \dfrac{1}{(0.0314\text{ m}^2)(1)} + \dfrac{1-\varepsilon_2}{(0.0628\text{ m}^2)\varepsilon_2}} \longrightarrow \varepsilon_2 = \mathbf{0.21}$$

Radiation Shields and The Radiation Effect

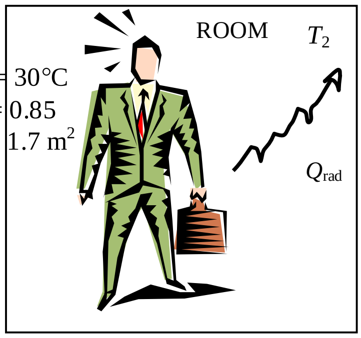
22-47C Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high reflectivity(low emissivity) sheet of material between the two surfaces. Such highly reflective thin plates or shells are known as radiation shields. Multilayer radiation shields constructed of about 20 shields per cm. thickness separated by evacuated space are commonly used in cryogenic and space applications to minimize heat transfer. Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect.

22-48C The influence of radiation on heat transfer or temperature of a surface is called the radiation effect. The radiation exchange between the sensor and the surroundings may cause the thermometer to indicate a different reading for the medium temperature. To minimize the radiation effect, the sensor should be coated with a material of high reflectivity (low emissivity).

22-49C A person who feels fine in a room at a specified temperature may feel chilly in another room at the same temperature as a result of radiation effect if the walls of second room are at a considerably lower temperature. For example most people feel comfortable in a room at 22°C if the walls of the room are also roughly at that temperature. When the wall temperature drops to 5°C for some reason, the interior temperature of the room must be raised to at least 27°C to maintain the same level of comfort. Also, people sitting near the windows of a room in winter will feel colder because of the radiation exchange between the person and the cold windows.

22-50 The rate of heat loss from a person by radiation in a large room whose walls are maintained at a uniform temperature is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivity of the person is given to be $\varepsilon_1 =$ 0.7.

Analysis (*a*) Noting that the view factor from the person to the walls $F_{12} = 1$, the rate of heat loss from that person to the walls at a large room which are at a temperature of 300 K is

$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1^{\,4} - T_2^{\,4}) \\ &= (0.85)(1)(1.7\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(303\,\text{K})^4 - (300\,\text{K})^4] \\ &= \mathbf{26.9\ W}\end{aligned}$$

(*b*) When the walls are at a temperature of 280 K,

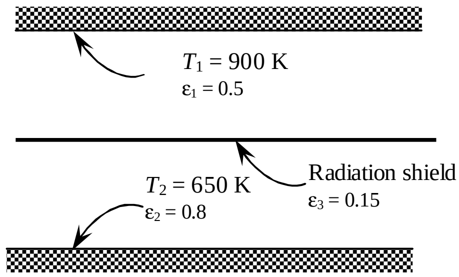
$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1^{\,4} - T_2^{\,4}) \\ &= (0.85)(1)(1.7\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(303\,\text{K})^4 - (280\,\text{K})^4] \\ &= \mathbf{187\ W}\end{aligned}$$

22-51 A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined for the cases of with and without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.8$, and $\varepsilon_3 = 0.15$.

Analysis The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is



$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_{3,1}}+\frac{1}{\varepsilon_{3,2}}-1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4-(650\ \text{K})^4]}{\left(\frac{1}{0.5}+\frac{1}{0.8}-1\right)+\left(\frac{1}{0.15}+\frac{1}{0.15}-1\right)}$$

$$= \mathbf{1857\ W/m^2}$$

The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\ \text{shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4-(650\ \text{K})^4]}{\left(\frac{1}{0.5}+\frac{1}{0.8}-1\right)} = 12{,}035\ \text{W/m}^2$$

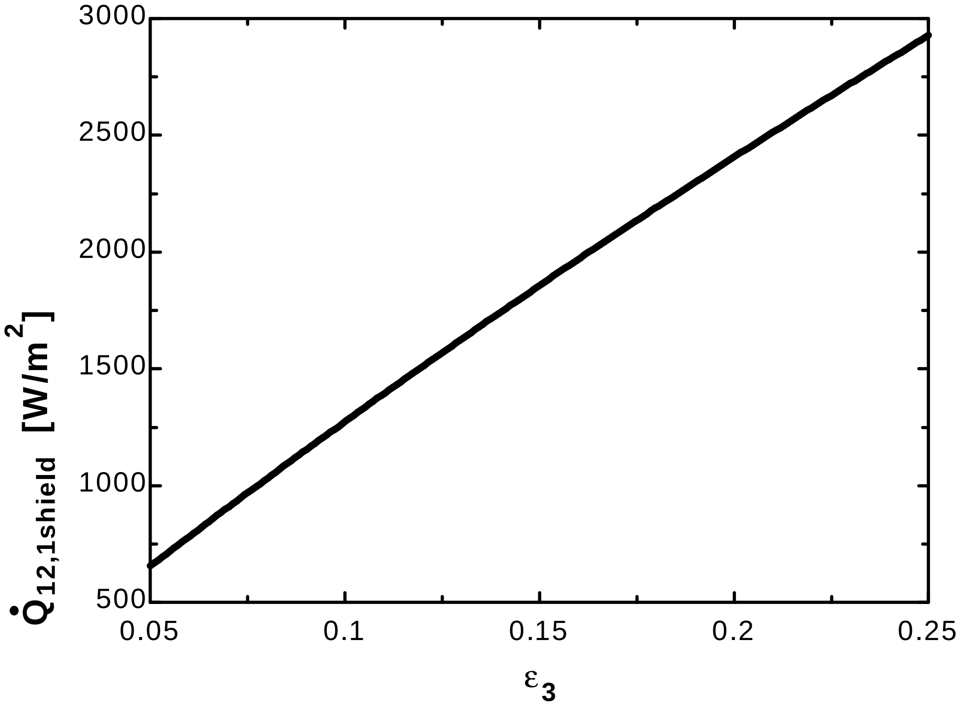
Then the ratio of radiation heat transfer for the two cases becomes

$$\frac{\dot{Q}_{12,\text{one shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{1857\ \text{W}}{12{,}035\ \text{W}} \cong \mathbf{\frac{1}{6}}$$

22-52

```
"GIVEN"
"epsilon_3=0.15 parameter to be varied"
T_1=900 "[K]"
T_2=650 "[K]"
epsilon_1=0.5
epsilon_2=0.8
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1/epsilon_1+1/epsilon_2-
1)+(1/epsilon_3+1/epsilon_3-1))
```

ε_3	$Q_{12,1\ shield}$ [W/m^2]
0.05	656.5
0.06	783
0.07	908.1
0.08	1032
0.09	1154
0.1	1274
0.11	1394
0.12	1511
0.13	1628
0.14	1743
0.15	1857
0.16	1969
0.17	2081
0.18	2191
0.19	2299
0.2	2407
0.21	2513
0.22	2619
0.23	2723
0.24	2826
0.25	2928


3000
2500
2000
1500
1000
500
0.05
0.1
0.15
0.2
0.25
Q12,1shield [W/m2]
ε3

22-53 Two very large plates are maintained at uniform temperatures. The number of thin aluminum sheets that will reduce the net rate of radiation heat transfer between the two plates to one-fifth is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

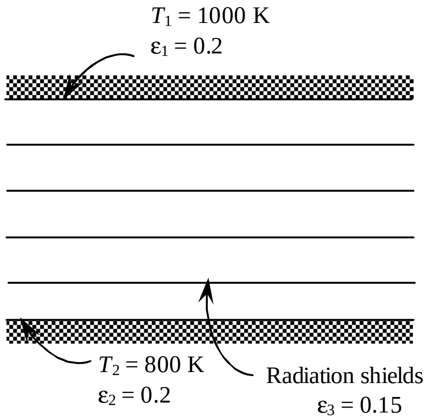
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$, and $\varepsilon_3 = 0.15$.

Analysis The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\,\text{shield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\,\text{K})^4 - (800\,\text{K})^4]}{\left(\dfrac{1}{0.2} + \dfrac{1}{0.2} - 1\right)}$$

$$= 3720\,\text{W/m}^2$$



The number of sheets that need to be inserted in order to reduce the net rate of heat transfer between the two plates to onefifth can be determined from

$$\dot{Q}_{12,\text{shields}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

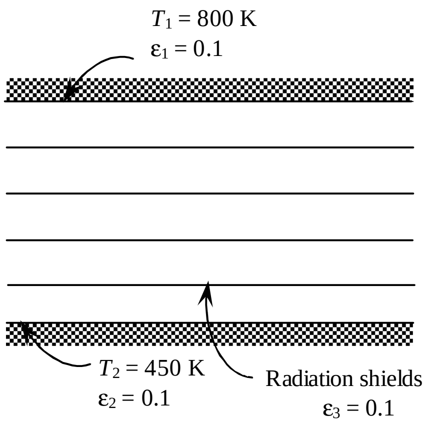
$$\frac{1}{5}(3720\,\text{W/m}^2) = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\,\text{K})^4 - (800\,\text{K})^4]}{\left(\dfrac{1}{0.2} + \dfrac{1}{0.2} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{0.15} + \dfrac{1}{0.15} - 1\right)} \longrightarrow N_{\text{shield}} = 2.92 \cong \mathbf{3}$$

22-54 Five identical thin aluminum sheets are placed between two very large parallel plates which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.1$ and $\varepsilon_3 = 0.1$.

Analysis Since the plates and the sheets have the same emissivity value, the net rate of radiation heat transfer with 5 thin aluminum shield can be determined from



$$\dot{Q}_{12,5\,\text{shield}} = \frac{1}{N+1}\dot{Q}_{12,\,\text{no shield}} = \frac{1}{N+1}\frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{1}{5+1}\frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\,\text{K})^4 - (450\,\text{K})^4]}{\left(\dfrac{1}{0.1} + \dfrac{1}{0.1} - 1\right)}$$

$$= \mathbf{183\ W/m^2}$$

The net rate of radiation heat transfer without the shield is

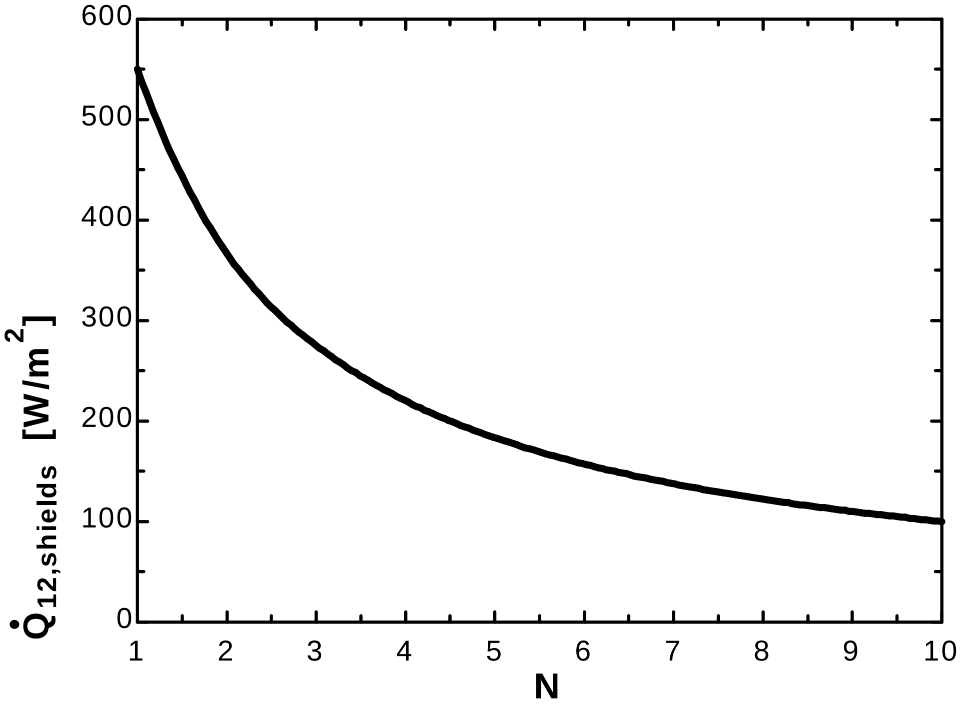
$$\dot{Q}_{12,5\,\text{shield}} = \frac{1}{N+1}\dot{Q}_{12,\,\text{no shield}} \longrightarrow \dot{Q}_{12,\,\text{no shield}} = (N+1)\dot{Q}_{12,5\,\text{shield}} = 6\times183\,\text{W} = \mathbf{1098\ W}$$

22-55

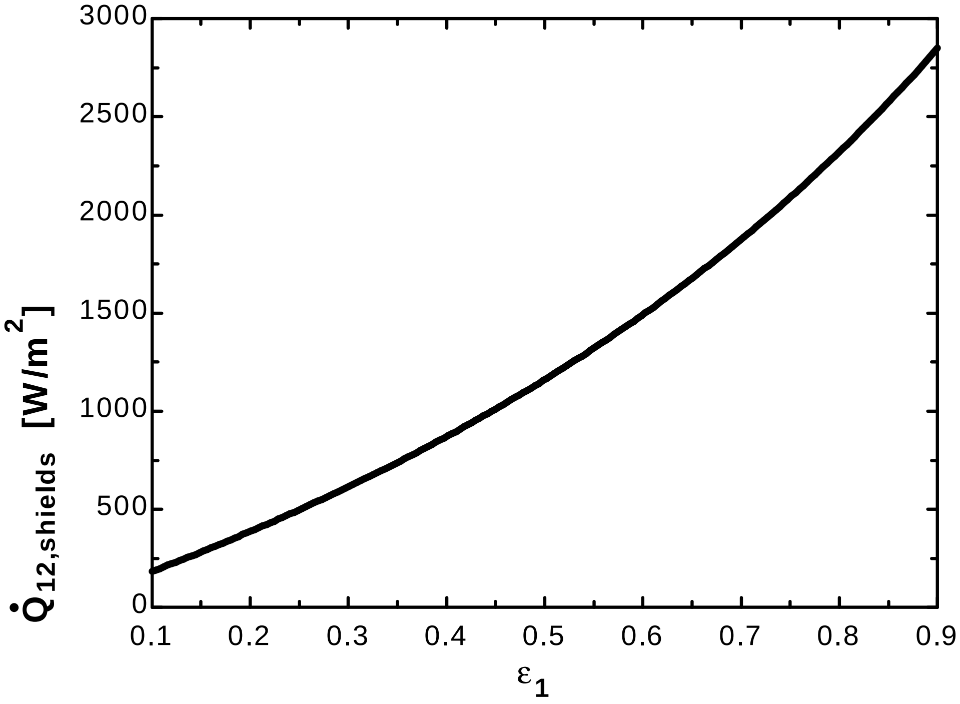
```
"GIVEN"
N=5 "parameter to be varied"
epsilon_3=0.1
"epsilon_1=0.1 parameter to be varied"
epsilon_2=epsilon_1
T_1=800 "[K]"
T_2=450 "[K]"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
Q_dot_12_shields=1/(N+1)*Q_dot_12_NoShield
Q_dot_12_NoShield=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
```

N	$Q_{12,shields}$ [W/m^2]
1	550
2	366.7
3	275
4	220
5	183.3
6	157.1
7	137.5
8	122.2
9	110
10	100

ε_1	$Q_{12,shields}$ [W/m^2]
0.1	183.3
0.15	282.4
0.2	387
0.25	497.6
0.3	614.7
0.35	738.9
0.4	870.8
0.45	1011
0.5	1161
0.55	1321
0.6	1493
0.65	1677
0.7	1876
0.75	2090
0.8	2322
0.85	2575
0.9	2850

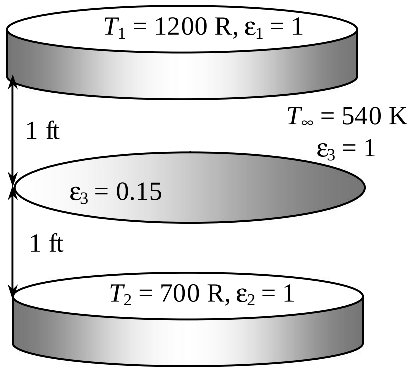


0
100
200
300
400
500
600
1
2
3
4
5
6
7
8
9
10
N
Q̇12,shields [W/m2]



0
500
1000
1500
2000
2500
3000
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ε1
Q̇12,shields [W/m2]

22-56E A radiation shield is placed between two parallel disks which are maintained at uniform temperatures. The net rate of radiation heat transfer through the shields is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 1$ and $\varepsilon_3 = 0.15$.

Analysis From Fig. 22-7 we have $F_{32} = F_{13} = 0.52$. Then $F_{34} = 1 - 0.52 = 0.48$. The disk in the middle is surrounded by black surfaces on both sides. Therefore, heat transfer between the top surface of the middle disk and its black surroundings can expressed as

$$\begin{aligned}\dot{Q}_3 &= \varepsilon A_3 \sigma [F_{31}(T_3^4 - T_1^4)] + \varepsilon A_3 \sigma [F_{32}(T_3^4 - T_2^4)] \\ &= 0.15(7.069\ \text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\{0.52[(T_3^4 - (1200\ \text{R})^4] + 0.48[T_3^4 - (540\ \text{K})^4]\}\end{aligned}$$

Similarly, for the bottom surface of the middle disk, we have

$$\begin{aligned}-\dot{Q}_3 &= \varepsilon A_3 \sigma [F_{34}(T_3^4 - T_4^4)] + \varepsilon A_3 \sigma [F_{35}(T_3^4 - T_5^4)] \\ &= 0.15(7.069\ \text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\{0.48[(T_3^4 - (700\ \text{R})^4] + 0.52[T_3^4 - (540\ \text{K})^4]\}\end{aligned}$$

Combining the equations above, the rate of heat transfer between the disks through the radiation shield (the middle disk) is determined to be

$$\dot{Q} = \mathbf{866\ Btu/h} \quad \text{and} \quad T_3 = 895\ \text{K}$$

22-57 A radiation shield is placed between two large parallel plates which are maintained at uniform temperatures. The emissivity of the radiation shield is to be determined if the radiation heat transfer between the plates is reduced to 15% of that without the radiation shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

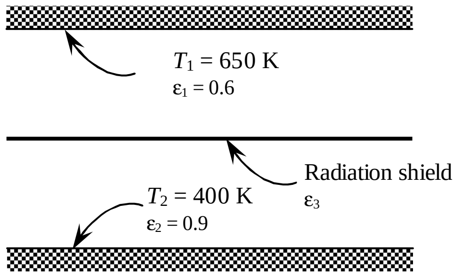
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.6$ and $\varepsilon_2 = 0.9$.

Analysis First, the net rate of radiation heat transfer between the two large parallel plates per unit area without a shield is

$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\ \text{K})^4 - (400\ \text{K})^4]}{\dfrac{1}{0.6} + \dfrac{1}{0.9} - 1} = 4877\ \text{W/m}^2$$

The radiation heat transfer in the case of one shield is

$$\dot{Q}_{12,\text{one shield}} = 0.15\times\dot{Q}_{12,\text{no shield}}$$
$$= 0.15\times4877\ \text{W/m}^2 = 731.6\ \text{W/m}^2$$



Then the emissivity of the radiation shield becomes

$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

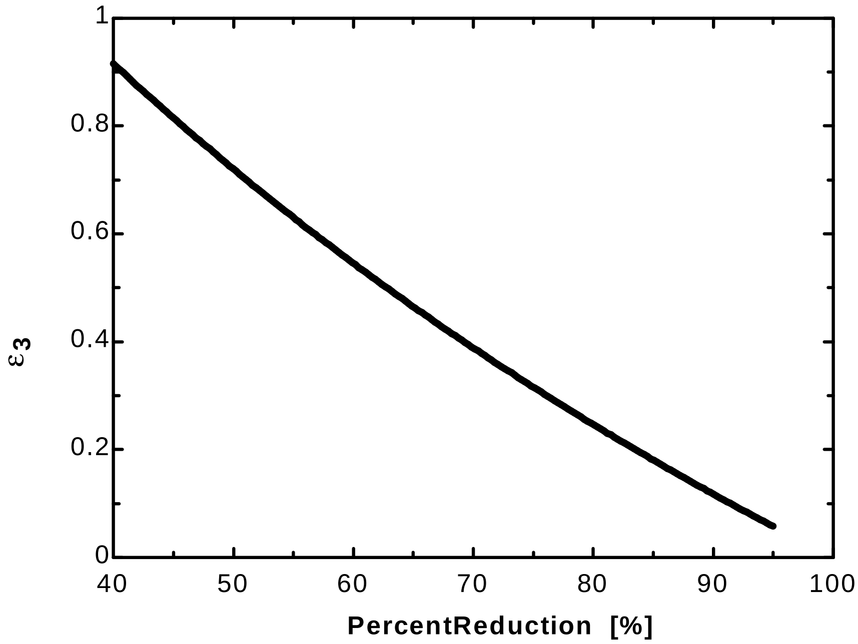
$$731.6\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\ \text{K})^4 - (400\ \text{K})^4]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.9} - 1\right) + \left(\dfrac{2}{\varepsilon_3} - 1\right)}$$

which gives $\varepsilon_3 = \mathbf{0.18}$

22-58

```
"GIVEN"
T_1=650 "[K]"
T_2=400 "[K]"
epsilon_1=0.6
epsilon_2=0.9
"PercentReduction=85 [%], parameter to be varied"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
Q_dot_12_NoShield=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1/epsilon_1+1/epsilon_2-
1)+(1/epsilon_3+1/epsilon_3-1))
Q_dot_12_1shield=(1-PercentReduction/100)*Q_dot_12_NoShield
```

Percent Reduction [%]	ε_3
40	0.9153
45	0.8148
50	0.72
55	0.6304
60	0.5455
65	0.4649
70	0.3885
75	0.3158
80	0.2466
85	0.1806
90	0.1176
95	0.05751



22-59 A coaxial radiation shield is placed between two coaxial cylinders which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.4$. and $\varepsilon_3 = 0.2$.

Analysis The surface areas of the cylinders and the shield per unit length are

$$A_{\text{pipe,inner}} = A_1 = \pi D_1 L = \pi(0.2\text{ m})(1\text{ m}) = 0.628\text{ m}^2$$
$$A_{\text{pipe,outer}} = A_2 = \pi D_2 L = \pi(0.1\text{ m})(1\text{ m}) = 0.314\text{ m}^2$$
$$A_{\text{shield}} = A_3 = \pi D_3 L = \pi(0.3\text{ m})(1\text{ m}) = 0.942\text{ m}^2$$

The net rate of radiation heat transfer between the two cylinders with a shield per unit length is

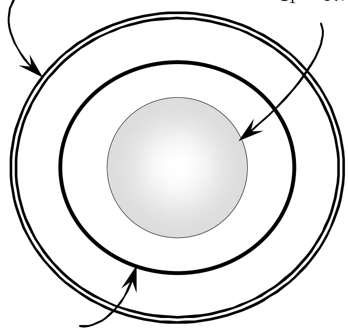
$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{13}} + \dfrac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \dfrac{1}{A_3 F_{3,2}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$
$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(750\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1-0.7}{(0.314)(0.7)} + \dfrac{1}{(0.314)(1)} + 2\dfrac{1-0.2}{(0.628)(0.2)} + \dfrac{1}{(0.628)(1)} + \dfrac{1-0.4}{(0.942)(0.4)}}$$
$$= \mathbf{703\ W}$$

If there was no shield,

$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$
$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(750\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1}{0.7} + \dfrac{1-0.4}{0.4}\left(\dfrac{0.1}{0.3}\right)}$$
$$= \mathbf{7465\ W}$$

Then their ratio becomes

$$\frac{\dot{Q}_{12,\text{one shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{703\text{ W}}{7465\text{ W}} = \mathbf{0.094}$$

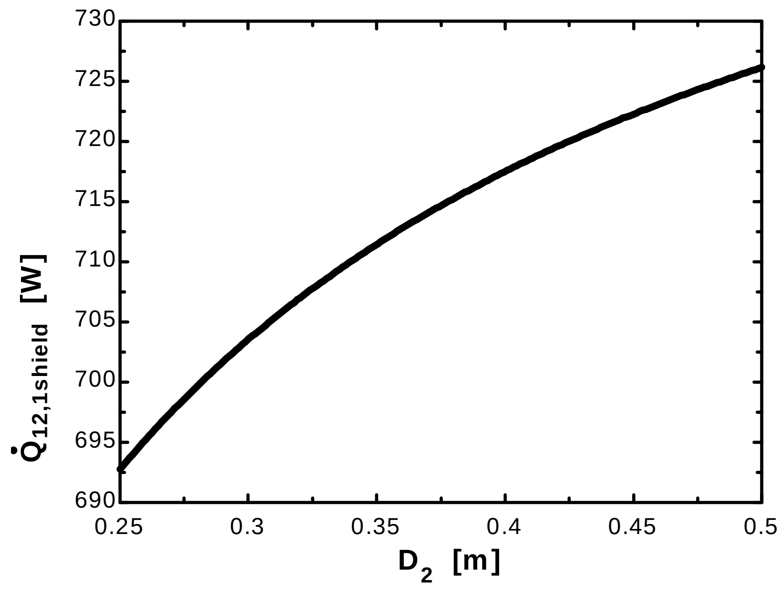


22-60

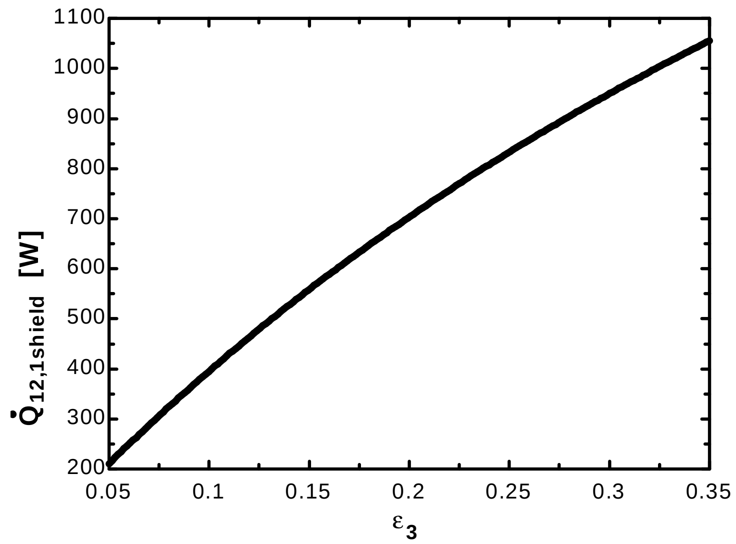
```
"GIVEN"
D_1=0.10 "[m]"
D_2=0.30 "[m], parameter to be varied"
D_3=0.20 "[m]"
epsilon_1=0.7
epsilon_2=0.4
epsilon_3=0.2 "parameter to be varied"
T_1=750 "[K]"
T_2=500 "[K]"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
L=1 "[m], a unit length of the cylinders is considered"
A_1=pi*D_1*L
A_2=pi*D_2*L
A_3=pi*D_3*L
F_13=1
F_32=1
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1-
epsilon_1)/(A_1*epsilon_1)+1/(A_1*F_13)+(1-epsilon_3)/(A_3*epsilon_3)+(1-
epsilon_3)/(A_3*epsilon_3)+1/(A_3*F_32)+(1-epsilon_2)/(A_2*epsilon_2))
```

D_2 [m]	$Q_{12,1\ shield}$ [W]
0.25	692.8
0.275	698.6
0.3	703.5
0.325	707.8
0.35	711.4
0.375	714.7
0.4	717.5
0.425	720
0.45	722.3
0.475	724.3
0.5	726.1

ε_3	$Q_{12,1\ shield}$ [W]
0.05	211.1
0.07	287.8
0.09	360.7
0.11	429.9
0.13	495.9
0.15	558.7
0.17	618.6
0.19	675.9
0.21	730.6
0.23	783
0.25	833.1
0.27	881.2
0.29	927.4
0.31	971.7
0.33	1014
0.35	1055



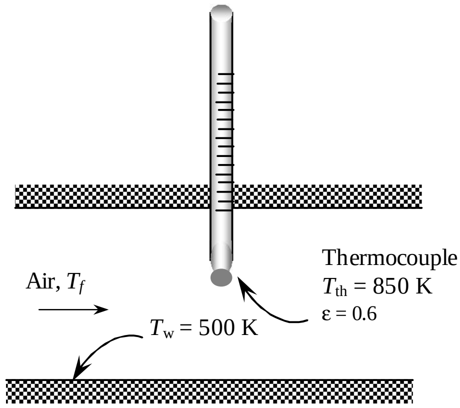
730
725
720
715
710
705
700
695
690
0.25
0.3
0.35
0.4
0.45
0.5
$\dot{Q}_{12,1shield}$ [W]
D_2 [m]



1100
1000
900
800
700
600
500
400
300
200
0.05
0.1
0.15
0.2
0.25
0.3
0.35
$\dot{Q}_{12,1shield}$ [W]
ε_3

Review Problems

22-61 The temperature of air in a duct is measured by a thermocouple. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.



Assumptions The surfaces are opaque, diffuse, and gray.

Properties The emissivity of thermocouple is given to be ε=0.6.

Analysis The actual temperature of the air can be determined from

$$T_f = T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}$$

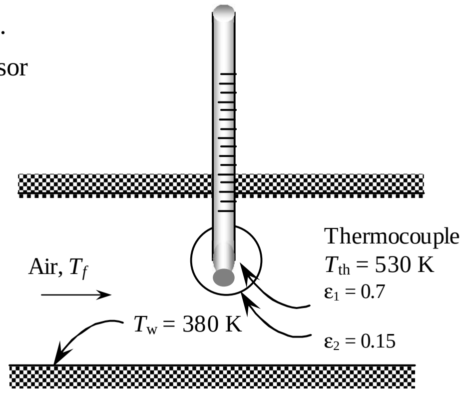
$$= 850\text{ K} + \frac{(0.6)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(850\text{ K})^4 - (500\text{ K})^4]}{60\text{ W/m}^2\cdot{}^\circ\text{C}} = \mathbf{1111 K}$$

22-62 The temperature of hot gases in a duct is measured by a thermocouple. The actual temperature of the gas is to be determined, and compared with that without a radiation shield.

Assumptions The surfaces are opaque, diffuse, and gray.

Properties The emissivity of the thermocouple is given to be ε =0.7.

Analysis Assuming the area of the shield to be very close to the sensor of the thermometer, the radiation heat transfer from the sensor is determined from



$$\dot{Q}_{\text{rad, from sensor}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} - 1\right) + \left(2\frac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(530\text{ K})^4 - (380\text{ K})^4]}{\left(\frac{1}{0.7} - 1\right) + \left(2\frac{1}{0.15} - 1\right)}$$

$$= 257.9\text{ W/m}^2$$

Then the actual temperature of the gas can be determined from a heat transfer balance to be

$$\dot{q}_{\text{conv, to sensor}} = \dot{q}_{\text{conv, from sensor}}$$

$$h(T_f - T_{th}) = 257.9\text{ W/m}^2$$

$$120\text{ W/m}^2\cdot{}^\circ\text{C}(T_f - 530) = 257.9\text{ W/m}^2 \longrightarrow T_f = \mathbf{532 K}$$

Without the shield the temperature of the gas would be

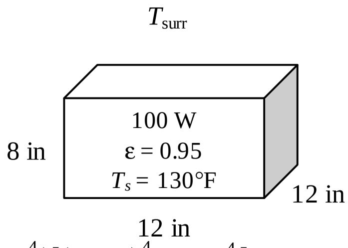
$$T_f = T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}$$

$$= 530\text{ K} + \frac{(0.7)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(530\text{ K})^4 - (380\text{ K})^4]}{120\text{ W/m}^2\cdot{}^\circ\text{C}} = \mathbf{549.2 K}$$

22-63E A sealed electronic box is placed in a vacuum chamber. The highest temperature at which the surrounding surfaces must be kept if this box is cooled by radiation alone is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** Heat transfer from the bottom surface of the box is negligible.

Properties The emissivity of the outer surface of the box is ε = 0.95.



Analysis The total surface area is

$$A_s = 4\times(8\times1/12)+(1\times1) = 3.67\ \text{ft}^2$$

Then the temperature of the surrounding surfaces is determined to be

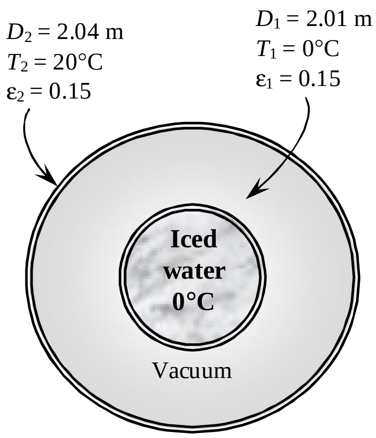
$$\dot{Q}_{rad} = \varepsilon A_s \sigma (T_s^{\,4} - T_{surr}^{\,4})$$

$$(100\times3.41214)\ \text{Btu/h} = (0.95)(3.67\ \text{m}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)[(590\ \text{R})^4 - T_{surr}^{\,4}]$$

$$\longrightarrow T_{surr} = 503\ \text{R} = \mathbf{43°F}$$

22-64 A double-walled spherical tank is used to store iced water. The air space between the two walls is evacuated. The rate of heat transfer to the iced water and the amount of ice that melts a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.



Properties The emissivities of both surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.15$.

Analysis (*a*) Assuming the conduction resistance s of the walls to be negligible, the rate of heat transfer to the iced water in the tank is determined to be

$$A_1 = \pi D_1^{\,2} = \pi(2.01\ \text{m})^2 = 12.69\ \text{m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^{\,4} - T_1^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

$$= \frac{(12.69\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(20+273\ \text{K})^4 - (0+273\ \text{K})^4]}{\dfrac{1}{0.15} + \dfrac{1-0.15}{0.15}\left(\dfrac{2.01}{2.04}\right)^2}$$

$$= \mathbf{107.4\ W}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.1074\ \text{kJ/s})(24\times3600\ \text{s}) = 9275\ \text{kJ}$$

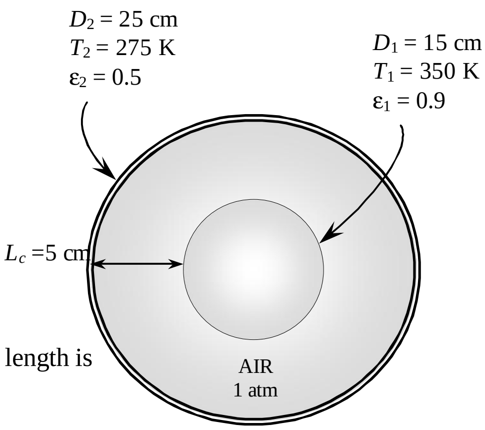
The amount of ice that melts during this period then becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{9275\ \text{kJ}}{333.7\ \text{kJ/kg}} = \mathbf{27.8\ kg}$$

22-65 Two concentric spheres which are maintained at uniform temperatures are separated by air at 1 atm pressure. The rate of heat transfer between the two spheres by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

Properties The emissivities of the surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.5$. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (350+275)/2 = 312.5$ K = 39.5°C are (Table A-15)



$$k = 0.02658 \text{ W/m.°C}$$
$$\upsilon = 1.697\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7256$$
$$\beta = \frac{1}{312.5 \text{ K}} = 0.0032 \text{ K}^{-1}$$

Analysis (*a*) Noting that $D_i = D_1$ and $D_o = D_2$, the characteristic length is

$$L_c = \frac{1}{2}(D_o - D_i) = \frac{1}{2}(0.25 \text{ m} - 0.15 \text{ m}) = 0.05 \text{ m}$$

Then

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003200 \text{ K}^{-1})(350-275 \text{ K})(0.05 \text{ m})^3}{(1.697\times10^{-5} \text{ m}^2/\text{s})^2}(0.7256) = 7.415\times10^5$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5}+D_o^{-7/5})^5} = \frac{0.05 \text{ m}}{[(0.15 \text{ m})(0.25 \text{ m})]^4\left[(0.15 \text{ m})^{-7/5}+(0.25 \text{ m})^{-7/5}\right]^5} = 0.005900$$

$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{sph}Ra)^{1/4}$$
$$= 0.74(0.02658 \text{ W/m.°C})\left(\frac{0.7256}{0.861+0.7256}\right)^{1/4}\left[(0.00590)(7.415\times10^5)\right]^{1/4} = 1315 \text{ W/m.°C}$$

Then the rate of heat transfer between the spheres becomes

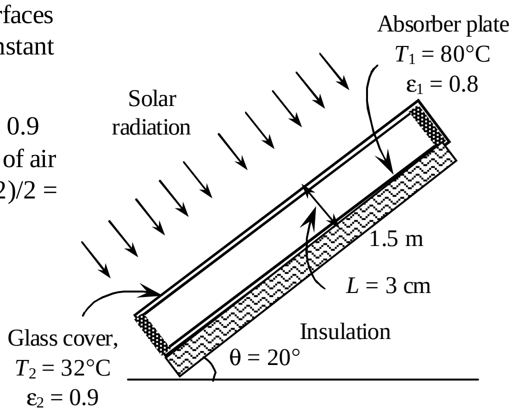
$$\dot{Q} = k_{\text{eff}}\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o) = (0.1315 \text{ W/m.°C})\pi\left[\frac{(0.15 \text{ m})(0.25 \text{ m})}{(0.05 \text{ m})}\right](350-275)\text{K} = \mathbf{23.3\ W}$$

(*b*) The rate of heat transfer by radiation is determined from

$$A_1 = \pi D_1^2 = \pi(0.15 \text{ m})^2 = 0.0707 \text{ m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon_1}+\dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2} = \frac{(0.0707 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(350 \text{ K})^4-(275 \text{ K})^4]}{\dfrac{1}{0.9}+\dfrac{1-0.9}{0.9}\left(\dfrac{0.15}{0.25}\right)^2} = \mathbf{32.3\ W}$$

22-66 A solar collector is considered. The absorber plate and the glass cover are maintained at uniform temperatures, and are separated by air. The rate of heat loss from the absorber plate by natural convection and radiation is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.9$ for glass and $\varepsilon_2 = 0.8$ for the absorber plate. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (80+32)/2 =$ 56°C are (Table A-15)

$$k = 0.02779\ \text{W/m.°C}$$

$$\upsilon = 1.857\times10^{-5}\ \text{m}^2/\text{s}$$

$$\text{Pr} = 0.7212$$

$$\beta = \frac{1}{T_f} = \frac{1}{(56+273)\text{K}} = 0.003040\ \text{K}^{-1}$$

Analysis For $\theta = 0°$, we have horizontal rectangular enclosure. The characteristic length in this case is the distance between the two glasses $L_c = L = 0.03$ m Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.00304\ \text{K}^{-1})(80-32\ \text{K})(0.03\ \text{m})^3}{(1.857\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7212) = 8.083\times10^4$$

$$A_s = H\times W = (1.5\ \text{m})(3\ \text{m}) = 4.5\ \text{m}^2$$

$$\text{Nu} = 1+1.44\left[1-\frac{1708}{\text{Ra}\cos\theta}\right]^+\left[1-\frac{1708(\sin 1.8\theta)^{1.6}}{\text{Ra}\cos\theta}\right]+\left[\frac{(\text{Ra}\cos\theta)^{1/3}}{18}-1\right]^+$$

$$= 1+1.44\left[1-\frac{1708}{(8.083\times10^4)\cos(20)}\right]^+\left[1-\frac{1708[\sin(1.8\times20)]^{1.6}}{(8.083\times10^4)\cos(20)}\right]+\left[\frac{[(8.083\times10^4)\cos(20)]^{1/3}}{18}-1\right]^+$$

$$= 3.747$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779\ \text{W/m.°C})(3.747)(4.5\ \text{m}^2)\frac{(80-32)\text{°C}}{0.03\ \text{m}} = \mathbf{750\ W}$$

Neglecting the end effects, the rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^{\ 4}-T_2^{\ 4})}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} = \frac{(4.5\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(80+273\ \text{K})^4-(32+273\ \text{K})^4]}{\frac{1}{0.8}+\frac{1}{0.9}-1} = \mathbf{1289\ W}$$

Discussion The rates of heat loss by natural convection for the horizontal and vertical cases would be as follows (Note that the Ra number remains the same):

Horizontal:

$$\text{Nu} = 1+1.44\left[1-\frac{1708}{\text{Ra}}\right]^+ + \left[\frac{\text{Ra}^{1/3}}{18}-1\right]^+ = 1+1.44\left[1-\frac{1708}{8.083\times10^4}\right]^+ + \left[\frac{(8.083\times10^4)^{1/3}}{18}-1\right]^+ = 3.812$$

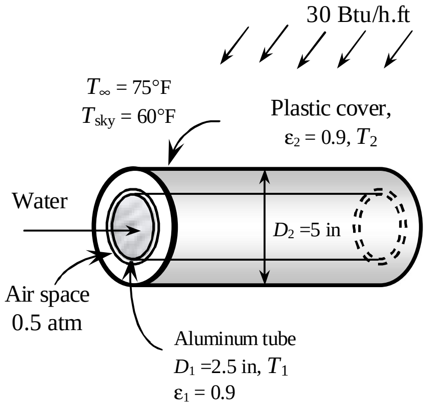
$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779\ \text{W/m.°C})(3.812)(6\ \text{m}^2)\frac{(80-32)\text{°C}}{0.03\ \text{m}} = \mathbf{1017\ W}$$

Vertical:

$$Nu = 0.42Ra^{1/4}\ \mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(8.083\times10^4)^{1/4}(0.7212)^{0.012}\left(\frac{2\ \mathrm{m}}{0.03\ \mathrm{m}}\right)^{-0.3} = 2.001$$

$$\dot{Q} = kNuA_s\frac{T_1 - T_2}{L} = (0.02779\ \mathrm{W/m.°C})(2.001)(6\ \mathrm{m}^2)\frac{(80-32)\mathrm{°C}}{0.03\ \mathrm{m}} = \mathbf{534\ W}$$

22-67E The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.



Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** The surfaces are opaque, diffuse, and gray for infrared radiation. **5** The glass cover is transparent to solar radiation.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 85°F, and use properties at an anticipated average temperature of (75+85)/2 =80°F (Table A-15E),

$$k = 0.01481 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$

$$\upsilon = 0.6110 \text{ ft}^2/\text{h} = 1.697\times10^{-4} \text{ ft}^2/s$$

$$\text{Pr} = 0.7290$$

$$\beta = \frac{1}{T_{\text{ave}}} = \frac{1}{540 \text{ R}}$$

Analysis We have a horizontal cylindrical enclosure filled with air at 0.5 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30 \text{ Btu/h} \quad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{glass} = (\pi D_o W) = \pi(5/12 \text{ ft})(1 \text{ ft}) = 1.309 \text{ ft}^2 \quad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, solution will require a trial-and-error approach. Assuming the glass cover temperature to be 85°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\text{Ra}_{Do} = \frac{g\beta(T_o - T_\infty)D_o^3}{\nu^2}\text{Pr}$$
$$= \frac{(32.2 \text{ ft/s}^2)[1/(540 \text{ R})](85-75 \text{ R})(5/12 \text{ ft})^3}{(1.675\times10^{-4} \text{ ft}^2/\text{s})^2}(0.7290) = 1.092\times10^6$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.092\times10^6)^{1/6}}{\left[1+(0.559/0.7290)^{9/16}\right]^{8/27}}\right\}^2$$
$$= 14.95$$

$$h_o = \frac{k}{D_0}\text{Nu} = \frac{0.01481 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{5/12 \text{ ft}}(14.95) = 0.5315 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

$$\dot{Q}_{o,\text{conv}} = h_o A_o(T_o - T_\infty) = (0.5315 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(1.309 \text{ ft}^2)(85-75)°\text{F} = 6.96 \text{ Btu/h}$$

Also,

$$\dot{Q}_{o,\text{rad}} = \varepsilon_o \sigma A_o(T_o^4 - T_{\text{sky}}^4)$$
$$= (0.9)(0.1714\times10^{-8} \text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(1.309 \text{ ft}^2)\left[(545 \text{ R})^4 - (535 \text{ R})^4\right]$$
$$= 30.5 \text{ Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{o,\text{total}} = \dot{Q}_{o,\text{conv}} + \dot{Q}_{o,\text{rad}} = 7.0 + 30.5 = 37.5 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 85°F for the glass cover is high. Repeating the calculations with lower temperatures (including the evaluation of properties), the glass cover temperature corresponding to 30 Btu/h is determined to be **81.5°F**.

The temperature of the aluminum tube is determined in a similar manner using the natural convection and radiation relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (5 - 2.5)/2 = 1.25 \text{ in} = 1.25/12 \text{ ft}$$

Also,

$$A_i = A_{tube} = (\pi D_i W) = \pi(2.5/12 \text{ ft})(1 \text{ ft}) = 0.6545 \text{ ft}^2 \quad \text{(per foot of tube)}$$

We start the calculations by assuming the tube temperature to be 118.5°F, and thus an average temperature of (81.5+118.5)/2 = 100°F=640 R. Using properties at 100°F,

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)[1/(640 \text{ R})](118.5 - 81.5 \text{ R})(1.25/12 \text{ ft})^3}{(1.809\times10^{-4} \text{ ft}^2/\text{s})^2}(0.726) = 1.334\times10^4$$

The effective thermal conductivity is

$$F_{\text{cyc}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{[\ln(5/2.5)]^4}{(1.25/12 \text{ ft})^3[(2.5/12 \text{ ft})^{-3/5} + (5/12 \text{ ft})^{-3/5}]^5} = 0.1466$$

$$\begin{aligned} k_{\text{eff}} &= 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyc}}\text{Ra}_L)^{1/4} \\ &= 0.386(0.01529 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})\left(\frac{0.726}{0.861+0.726}\right)(0.1466\times1.334\times10^4)^{1/4} \\ &= 0.03227 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F} \end{aligned}$$

Then the rate of heat transfer between the cylinders by convection becomes

$$\dot{Q}_{i,\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.03227 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})}{\ln(5/2.5)}(118.5 - 81.5)°\text{F} = 10.8 \text{ Btu/h}$$

Also,

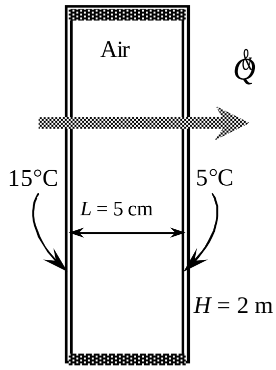
$$\dot{Q}_{i,\text{rad}} = \frac{\sigma A_i(T_i^4 - T_o^4)}{\frac{1}{\varepsilon_i} + \frac{1-\varepsilon_o}{\varepsilon_o}\left(\frac{D_i}{D_o}\right)} = \frac{(0.1714\times10^{-8} \text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(0.6545 \text{ ft}^2)\left[(578.5 \text{ R})^4 - (541.5 \text{ R})^4\right]}{\frac{1}{0.9} + \frac{1-0.9}{0.9}\left(\frac{2.5 \text{ in}}{5 \text{ in}}\right)} = 25.0 \text{ Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{i,\text{total}} = \dot{Q}_{i,\text{conv}} + \dot{Q}_{i,\text{rad}} = 10.8 + 25.0 = 35.8 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 118.5°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **113.2°F**. Therefore, the tube will reach an equilibrium temperature of 113.2°F when the pump fails.

22-68 A double-pane window consists of two sheets of glass separated by an air space. The rates of heat transfer through the window by natural convection and radiation are to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats. **4** Heat transfer through the window is one-dimensional and the edge effects are negligible.

Properties The emissivities of glass surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air at 0.3 atm and the average temperature of $(T_1+T_2)/2 = (15+5)/2 = 10$°C are (Table A-15)

$$k = 0.02439\ \text{W/m.°C}$$

$$\upsilon = \upsilon_{1atm}/0.3 = 1.426\times10^{-5}/0.3 = 4.753\times10^{-5}\ \text{m}^2/\text{s}$$

$$\text{Pr} = 0.7336$$

$$\beta = \frac{1}{(10+273)\ \text{K}} = 0.003534\ \text{K}^{-1}$$

Analysis The characteristic length in this case is the distance between the glasses, $L_c = L = 0.05$ m

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.003534\ \text{K}^{-1})(15-5)\text{K}(0.05\ \text{m})^3}{(4.753\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7336) = 1.918\times10^4$$

$$Nu = 0.197Ra^{1/4}\left(\frac{H}{L}\right)^{-1/9} = 0.197(1.918\times10^4)^{1/4}\left(\frac{2}{0.05}\right)^{-1/9} = 1.539$$

$$A_s = (2\ \text{m})(3\ \text{m}) = 6\ \text{m}^2$$

Then the rate of heat transfer by natural convection becomes

$$\dot{Q}_{conv} = kNuA_s\frac{T_1-T_2}{L} = (0.02439\ \text{W/m.°C})(1.539)(6\ \text{m}^2)\frac{(15-5)\text{°C}}{0.05\ \text{m}} = \mathbf{45.0\ W}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^4-T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}$$

$$= \frac{(6\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(15+273\ \text{K})^4-(5+273\ \text{K})^4]}{\frac{1}{0.9}+\frac{1}{0.9}-1}$$

$$= \mathbf{252\ W}$$

Then the rate of total heat transfer becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 45 + 252 = \mathbf{297\ W}$$

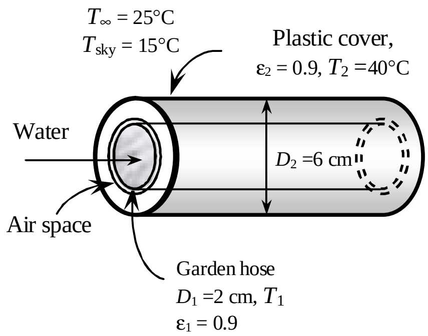
Discussion Note that heat transfer through the window is mostly by radiation.

22-69 A simple solar collector is built by placing a clear plastic tube around a garden hose. The rate of heat loss from the water in the hose by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air are at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+25)/2 = 32.5°C$ are (Table A-15)

$$k = 0.02607\ \text{W/m.°C}$$
$$\upsilon = 1.632\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.7275$$
$$\beta = \frac{1}{(32.5+273)\ \text{K}} = 0.003273\text{K}^{-1}$$



Analysis Under steady conditions, the heat transfer rate from the water in the hose equals to the rate of heat loss from the clear plastic tube to the surroundings by natural convection and radiation. The characteristic length in this case is the diameter of the plastic tube, $L_c = D_{plastic} = D_2 = 0.06\ \text{m}$.

$$Ra = \frac{g\beta(T_s - T_\infty)D_2^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.003273\ \text{K}^{-1})(40-25)\text{K}(0.06\ \text{m})^3}{(1.632\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7275) = 2.842\times10^5$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.842\times10^5)^{1/6}}{\left[1+(0.559/0.7241)^{9/16}\right]^{8/27}}\right\}^2 = 10.30$$

$$h = \frac{k}{D_2}Nu = \frac{0.02607\ \text{W/m.°C}}{0.06\ \text{m}}(10.30) = 4.475\ \text{W/m}^2\text{.°C}$$

$$A_{plastic} = A_2 = \pi D_2 L = \pi(0.06\ \text{m})(1\ \text{m}) = 0.1885\ \text{m}^2$$

Then the rate of heat transfer from the outer surface by natural convection becomes

$$\dot{Q}_{conv} = hA_2(T_s - T_\infty) = (4.475\ \text{W/m}^2\text{.°C})(0.1885\ \text{m}^2)(40-25)\text{°C} = \mathbf{12.7\ W}$$

The rate of heat transfer by radiation from the outer surface is determined from

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_2\sigma(T_s^{\ 4} - T_{sky}^{\ 4}) \\ &= (0.90)(0.1885\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(40+273\ \text{K})^4 - (15+273\ \text{K})^4] \\ &= \mathbf{26.2\ W}\end{aligned}$$

Finally,

$$\dot{Q}_{total,loss} = 12.7 + 26.2 = 38.9\ \text{W}$$

Discussion Note that heat transfer is mostly by radiation.

22-70 A solar collector consists of a horizontal copper tube enclosed in a concentric thin glass tube. The annular space between the copper and the glass tubes is filled with air at 1 atm. The rate of heat loss from the collector by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

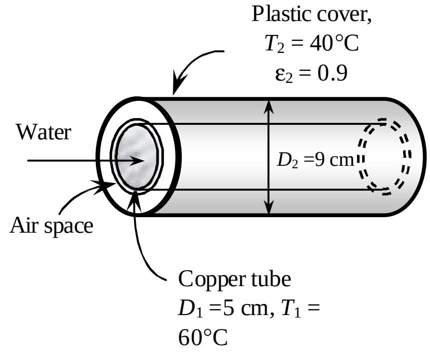
Properties The emissivities of surfaces are given to be ε_1 = 0.85 for the tube surface and ε_2 = 0.9 for glass cover. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2$ = (60+40)/2 = 50°C are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$

$$\upsilon = 1.798\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7228$$

$$\beta = \frac{1}{(50+273)\text{ K}} = 0.003096 \text{ K}^{-1}$$



Analysis The characteristic length in this case is

$$L_c = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.09\text{ m} - 0.05\text{ m}) = 0.02\text{ m}$$

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003096\text{ K}^{-1})(60-40)\text{K}(0.02\text{ m})^3}{(1.798\times10^{-5}\text{ m}^2\text{/s})^2}(0.7228) = 10{,}850$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{[\ln(0.09/0.05)]^4}{(0.02\text{ m})^3\left[(0.09\text{ m})^{-3/5} + (0.05\text{ m})^{-3/5}\right]^5} = 0.1303$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.02735\text{ W/m.°C})\left(\frac{0.7228}{0.861+0.7228}\right)^{1/4}[(0.1303)(10{,}850)]^{1/4} = 0.05321\text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q}_{\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.05321\text{ W/m.°C})}{\ln(0.09/0.05)}(60-40)\text{°C} = \mathbf{11.4\ W} \qquad \text{(Eq. 1)}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)}$$

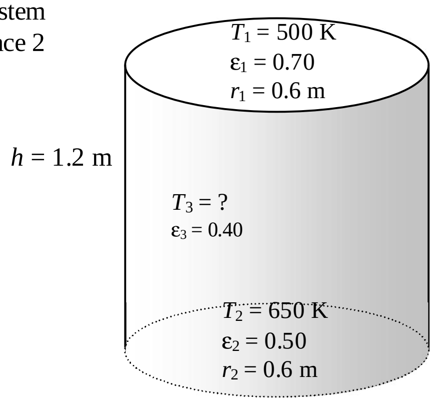
$$= \frac{(0.1571\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(60+273\text{ K})^4 - (40+273\text{ K})^4]}{\frac{1}{0.85} + \frac{1-0.9}{0.9}\left(\frac{5}{9}\right)}$$

$$= \mathbf{13.4\ W}$$

Finally,

$$\dot{Q}_{total,loss} = 11.4 + 13.4 = 24.8\text{ W} \quad \text{(per m length)}$$

22-71 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The temperature of the side surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.
Properties The emissivities of the top, bottom, and side surfaces are 0.70, 0.50, and 0.40, respectively.
Analysis We consider the top surface to be surface 1, the bottom surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from



$$\left.\begin{aligned} \frac{L}{r} &= \frac{1.2}{0.6} = 2 \\ \frac{r}{L} &= \frac{0.6}{1.2} = 0.5 \end{aligned}\right\} F_{12} = 0.17 \qquad \text{(Fig. 22-7)}$$

The surface areas are

$$A_1 = A_2 = \pi D^2/4 = \pi(1.2\text{ m})^2/4 = 1.131\text{ m}^2$$

$$A_3 = \pi D L = \pi(1.2\text{ m})(1.2\text{ m}) = 4.524\text{ m}^2$$

Then other view factors are determined to be

$$F_{12} = F_{21} = 0.17$$

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow 0 + 0.17 + F_{13} = 1 \longrightarrow F_{13} = 0.83 \quad \text{(summation rule)}, \qquad F_{23} = F_{13} = 0.83$$

$$A_1 F_{13} = A_3 F_{31} \longrightarrow (1.131)(0.83) = (4.524) F_{31} \longrightarrow F_{31} = 0.21 \quad \text{(reciprocity rule)}, \qquad F_{32} = F_{31} = 0.21$$

We now apply Eq. 22-35 to each surface

Surface 1:

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(500\text{ K})^4 = J_1 + \frac{1-0.70}{0.70}\left[0.17(J_1 - J_2) + 0.83(J_1 - J_3)\right]$$

Surface 2:

$$\sigma T_2^4 = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(500\text{ K})^4 = J_2 + \frac{1-0.50}{0.50}\left[0.17(J_2 - J_1) + 0.83(J_2 - J_3)\right]$$

Surface 3:

$$\sigma T_3^4 = J_3 + \frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3 - J_1) + F_{32}(J_3 - J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)T_3^4 = J_3 + \frac{1-0.40}{0.40}\left[0.21(J_1 - J_2) + 0.21(J_1 - J_3)\right]$$

We now apply Eq. 22-34 to surface 2

$$\dot{Q}_2 = A_2\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right] = (1.131\text{ m}^2)\left[0.17(J_2 - J_1) + 0.83(J_2 - J_3)\right]$$

Solving the above four equations, we find

$$T_3 = \mathbf{631\ K}, \quad J_1 = 4974\text{ W/m}^2, \quad J_2 = 8883\text{ W/m}^2, \quad J_3 = 8193\text{ W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$\dot{Q}_{21} = A_2 F_{21}(J_2 - J_1) = (1.131\text{ m}^2)(0.17)(8883 - 4974)\text{W/m}^2 = \mathbf{751.6\ W}$$

The rate of heat transfer between the bottom and the side surface is

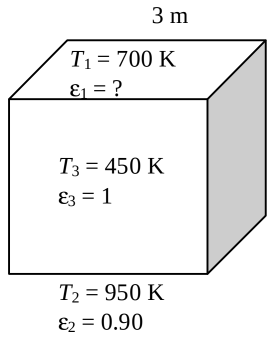
$$\dot{Q}_{23} = A_2 F_{23}(J_2 - J_3) = (1.131\text{ m}^2)(0.83)(8883 - 8197)\text{W/m}^2 = \mathbf{644.0\ W}$$

Discussion The sum of these two heat transfer rates are 751.6 + 644 = 1395.6 W, which is practically equal to 1400 W heat supply rate from surface 2. This must be satisfied to maintain the surfaces at the specified temperatures under steady operation. Note that the difference is due to round-off error.

22-72 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The emissivity of the top surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the bottom surface is 0.90.



Analysis We consider the top surface to be surface 1, the base surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from the base to the top surface of the cube is $F_{12} = 0.2$. The view factor from the base or the top to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Other view factors are

$$F_{21} = F_{12} = 0.20, \qquad F_{23} = F_{13} = 0.80, \qquad F_{31} = F_{32} = 0.20$$

We now apply Eq. 9-35 to each surface

Surface 1:

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)\right]$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(700\ \text{K})^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[0.20(J_1 - J_2) + 0.80(J_1 - J_3)\right]$$

Surface 2:

$$\sigma T_2^4 = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(950\ \text{K})^4 = J_2 + \frac{1-0.90}{0.90}\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Surface 3:

$$\sigma T_3^4 = J_3$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(450\ \text{K})^4 = J_3$$

We now apply Eq. 9-34 to surface 2

$$\dot{Q}_2 = A_2\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right] = (9\ \text{m}^2)\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Solving the above four equations, we find

$$\varepsilon_1 = \mathbf{0.44}, \quad J_1 = 11{,}736\ \text{W/m}^2, \quad J_2 = 41{,}985\ \text{W/m}^2, \quad J_3 = 2325\ \text{W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$A_1 = A_2 = (3\ \text{m})^2 = 9\ \text{m}^2$$

$$\dot{Q}_{21} = A_2 F_{21}(J_2 - J_1) = (9\ \text{m}^2)(0.20)(41{,}985 - 11{,}736)\ \text{W/m}^2 = \mathbf{54.4\ kW}$$

The rate of heat transfer between the bottom and the side surface is

$$A_3 = 4A_1 = 4(9\ \text{m}^2) = 36\ \text{m}^2$$

$$\dot{Q}_{23} = A_2 F_{23}(J_2 - J_3) = (9\ \text{m}^2)(0.8)(41{,}985 - 2325)\ \text{W/m}^2 = \mathbf{285.6\ kW}$$

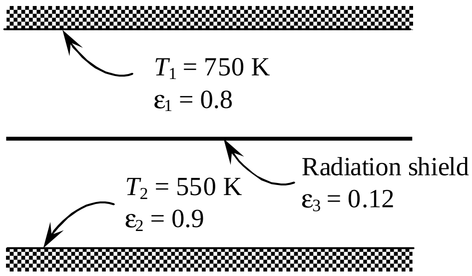
Discussion The sum of these two heat transfer rates are 54.4 + 285.6 = 340 kW, which is equal to 340 kW heat supply rate from surface 2.

22-73 A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates and the temperature of the radiation shield are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$, $\varepsilon_2 = 0.9$, and $\varepsilon_3 = 0.12$.

Analysis The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is



$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\ \text{K})^4 - (550\ \text{K})^4]}{\left(\frac{1}{0.8} + \frac{1}{0.9} - 1\right) + \left(\frac{1}{0.12} + \frac{1}{0.12} - 1\right)}$$

$$= \mathbf{748.9\ W/m^2}$$

The equilibrium temperature of the radiation shield is determined from

$$\dot{Q}_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right)}$$

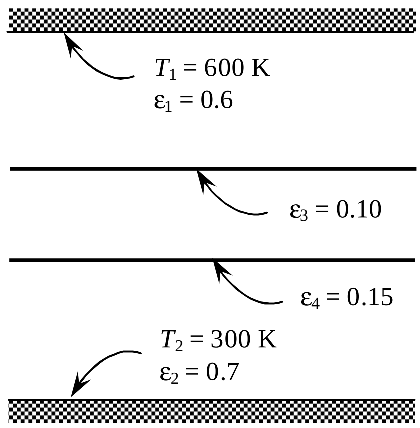
$$748.9\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\ \text{K})^4 - T_3^4]}{\left(\frac{1}{0.8} + \frac{1}{0.12} - 1\right)} \longrightarrow T_3 = \mathbf{671.3\ K}$$

22-74 Two thin radiation shields are placed between two large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates with and without the shields, and the temperatures of radiation shields are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.7$, $\varepsilon_3 = 0.10$, and $\varepsilon_4 = 0.15$.

Analysis The net rate of radiation heat transfer without the shields per unit area of the plates is



$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (300\ \text{K})^4]}{\dfrac{1}{0.6} + \dfrac{1}{0.7} - 1}$$

$$= \mathbf{3288\ W/m^2}$$

The net rate of radiation heat transfer with two thin radiation shields per unit area of the plates is

$$\dot{Q}_{12,\text{two-shields}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_3} - 1\right) + \left(\dfrac{1}{\varepsilon_4} + \dfrac{1}{\varepsilon_4} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (300\ \text{K})^4]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.7} - 1\right) + \left(\dfrac{1}{0.10} + \dfrac{1}{0.10} - 1\right) + \left(\dfrac{1}{0.15} + \dfrac{1}{0.15} - 1\right)}$$

$$= \mathbf{206\ W/m^2}$$

The equilibrium temperatures of the radiation shields are determined from

$$\dot{Q}_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right)}$$

$$206\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - T_3^4]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.10} - 1\right)} \longrightarrow T_3 = \mathbf{549\ K}$$

$$\dot{Q}_{42} = \frac{\sigma(T_4^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_4} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

$$206\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[T_4^4 - (300\ \text{K})^4]}{\left(\dfrac{1}{0.15} + \dfrac{1}{0.7} - 1\right)} \longrightarrow T_4 = \mathbf{429\ K}$$

22-75 ….. 22-77 Design and Essay Problems

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