CHAPTER 14

FLOW IN PIPES

Fluid flow in circular and noncircular pipes is commonly encountered in practice. The hot and cold water that we use in our homes is pumped through pipes. Water in a city is distributed by extensive piping networks. Oil and natural gas are transported hundreds of miles by large pipelines. Blood is carried throughout our bodies by veins. The cooling water in an engine is transported by hoses to the pipes in the radiator where it is cooled as it flows. Thermal energy in a hydronic space heating system is transferred to the circulating water in the boiler, and then it is transported to the desired locations in pipes.

Fluid flow is classified as *external* and *internal,* depending on whether the fluid is forced to flow over a surface or in a conduit. Internal and external flows exhibit very different characteristics. In this chapter we consider *internal flow* where the conduit is completely filled with the fluid, and flow is driven primarily by a pressure difference. This should not be confused with *open-channel flow* where the conduit is partially filled by the fluid and thus the flow is partially bounded by solid surfaces, as in an irrigation ditch, and flow is driven by gravity alone.

We start this chapter with a general physical description of internal flow and the *velocity boundary layer.* We continue with the discussion of the dimensionless *Reynolds number* and its physical significance. We then discuss the characteristics of flow inside pipes and introduce the *pressure drop* correlations associated with it for both laminar and turbulent flows. Finally, we present the minor losses and determine the pressure drop and pumping power requirements for piping systems.

CONTENTS

14–1 ▪ INTRODUCTION

Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications, and fluid distribution networks. The fluid in such applications is usually forced to flow by a fan or pump through a flow section. We pay particular attention to *friction,* which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts. The pressure drop is then used to determine the pumping power requirement. A typical piping system involves pipes of different diameters connected to each other by various fittings or elbows to direct the fluid, valves to control the flow rate, and pumps to pressurize the fluid.

The terms *pipe, duct,* and *conduit* are usually used interchangeably for flow sections. In general, flow sections of circular cross section are referred to as *pipes* (especially when the fluid is a liquid), and flow sections of noncircular cross section as *ducts* (especially when the fluid is a gas). Small-diameter pipes are usually referred to as *tubes.* Given this uncertainty, we will use more descriptive phrases (such as *a circular pipe* or *a rectangular duct*) whenever necessary to avoid any misunderstandings.

You have probably noticed that most fluids, especially liquids, are transported in *circular pipes.* This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing significant distortion. *Noncircular pipes* are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small, the manufacturing and installation costs are lower, and the available space is limited for duct work (Fig. 14–1).

Although the theory of fluid flow is reasonably well understood, theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe. Therefore, we must rely on experimental results and empirical relations for most fluid-flow problems rather than closed-form analytical solutions. Noting that the experimental results are obtained under carefully controlled laboratory conditions, and that no two systems are exactly alike, we must not be so naive as to view the results obtained as "exact." An error of 10 percent (or more) in friction factors calculated using the relations in this chapter is the "norm" rather than the "exception."

The fluid velocity in a pipe changes from *zero* at the surface because of the no-slip condition to a maximum at the pipe center. In fluid flow, it is convenient to work with an *average* or *mean* velocity $\mathcal{V}_m$, which remains constant in incompressible flow when the cross-sectional area of the pipe is constant (Fig. 14–2). The mean velocity in heating and cooling applications may change somewhat because of changes in density with temperature. But, in practice, we evaluate the fluid properties at some average temperature and treat them as constants. The convenience of working with constant properties usually more than justifies the slight loss in accuracy.

Also, the friction between the fluid layers in a pipe does cause a slight rise in fluid temperature as a result of the mechanical energy being converted to sensible thermal energy. But this temperature rise due to *fictional heating* is usually too small to warrant any consideration in calculations and thus is disregarded. For example, in the absence of any heat transfer, no noticeable difference can be detected between the inlet and exit temperatures of water flowing in a pipe. The primary consequence of friction in fluid flow is pressure drop, and thus any significant temperature change in the fluid is due to heat transfer.

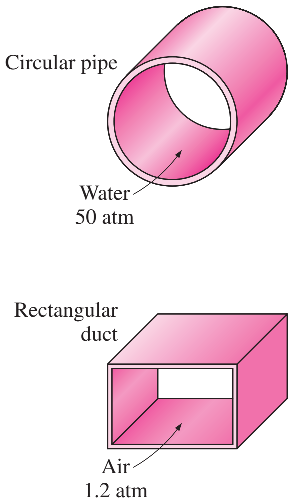


FIGURE 14–1
Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but noncircular pipes cannot.

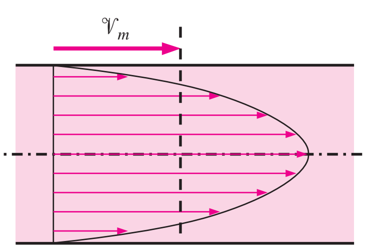


FIGURE 14–2
Mean velocity $\mathcal{V}_m$ is defined as the average speed through a cross section. For fully developed laminar pipe flow, $\mathcal{V}_m$ is half of maximum velocity.

The value of the mean velocity $\mathcal{V}_m$ is determined from the requirement that the *conservation of mass* principle be satisfied (Fig. 14–2). That is,

$$\dot{m} = \rho \mathcal{V}_m A_c = \int_{A_c} \rho u(r, x)\, dA_c \qquad \textbf{(14–1)}$$

where $\dot{m}$ is the mass flow rate, ρ is the density, A_c is the cross-sectional area, and $u(r, x)$ is the velocity profile. Then the mean velocity for incompressible flow in a circular pipe of radius R can be expressed as

$$\mathcal{V}_m = \frac{\int_{A_c} \rho u(r, x)\, dA_c}{\rho A_c} = \frac{\int_{A_c} \rho u(r, x) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r, x) r\, dr \qquad \textbf{(14–2)}$$

Therefore, when we know the mass flow rate or the velocity profile, the mean velocity can be determined easily.

14–2 ▪ LAMINAR AND TURBULENT FLOWS

If you have been around smokers, you probably noticed that the cigarette smoke rises in a smooth plume for the first few centimeters and then starts fluctuating randomly in all directions as it continues its journey toward the lungs of others (Fig. 14–3). Likewise, a careful inspection of flow in a pipe reveals that the fluid flow is streamlined at low velocities but turns chaotic as the velocity is increased above a critical value, as shown in Fig. 14–4. The flow regime in the first case is said to be **laminar,** characterized by *smooth streamlines* and *highly-ordered motion,* and **turbulent** in the second case, where it is characterized by *velocity fluctuations* and *highly-disordered motion.* The **transition** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent. Most flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.

We can verify the existence of these laminar, transitional, and turbulent flow regimes by injecting some dye streaks into the flow in a glass pipe, as the British scientist Osborne Reynolds (1842–1912) did over a century ago. We observe that the dye streak forms a *straight and smooth line* at low velocities when the flow is laminar (we may see some blurring because of molecular diffusion), has *bursts of fluctuations* in the transitional regime, and *zigzags rapidly and randomly* when the flow becomes fully turbulent. These zigzags and the dispersion of the dye are indicative of the fluctuations in the main flow and the rapid mixing of fluid particles from adjacent layers.

The *intense mixing* of the fluid in turbulent flow as a result of rapid fluctuations enhances momentum transfer between fluid particles, which increases the friction force on the surface and thus the required pumping power. The friction factor reaches a maximum when the flow becomes fully turbulent.

Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry, surface roughness, flow velocity, surface temperature,* and *type of fluid,* among other things. After exhaustive experiments in the 1880s, Osborne Reynolds

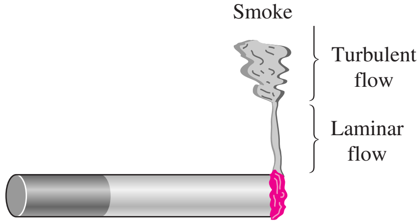


FIGURE 14–3
Laminar and turbulent flow regimes of cigarette smoke.

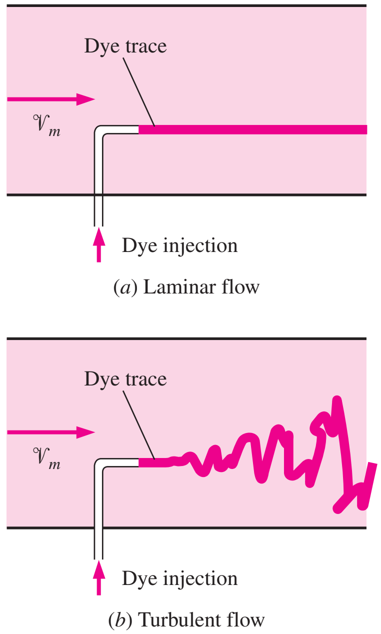


FIGURE 14–4
The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.

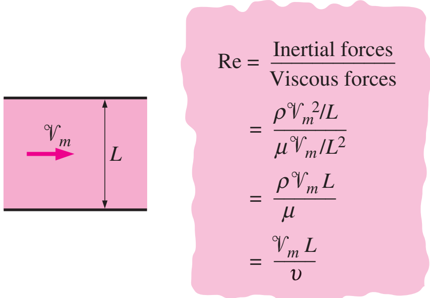


FIGURE 14–5
The Reynolds number can be viewed as the ratio of the inertial forces to viscous forces acting on a fluid volume element.

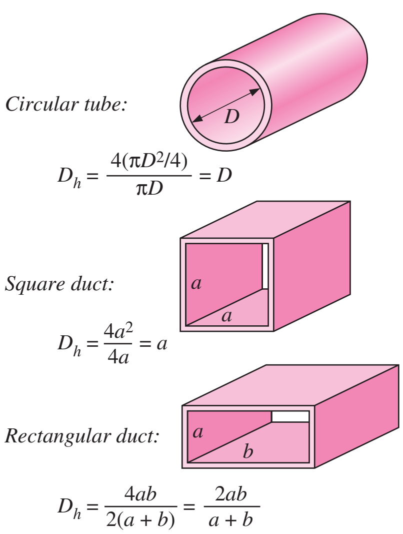


FIGURE 14–6
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

discovered that the flow regime depends mainly on the ratio of the *inertial forces* to *viscous forces* in the fluid. This ratio is called the **Reynolds number** and is expressed for internal flow in a circular pipe as (Fig. 14–5)

$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{\mathcal{V}_m D}{\upsilon} = \frac{\rho\mathcal{V}_m D}{\mu} \qquad \textbf{(14–3)}$$

where

$\mathcal{V}_m$ = mean flow velocity, m/s
D = characteristic length of the geometry (diameter in this case), m
$\upsilon = \mu/\rho$ = kinematic viscosity of the fluid, m^2/s.

Note that Reynolds number is a *dimensionless* quantity. Also, kinematic viscosity has the unit m^2/s, and can be viewed as *viscous diffusivity* or *diffusivity for momentum.*

At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. At *small* Reynolds numbers, however, the viscous forces are large enough to overcome the inertial forces and to keep the fluid "in line." Thus the flow is *turbulent* in the first case and *laminar* in the second.

The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds number,** Re_{cr}. The value of the critical Reynolds number is different for different geometries and flow conditions. For internal flow in a circular pipe, the generally accepted value of the critical Reynolds number is $\text{Re}_{cr} = 2300$.

For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter** D_h defined as (Fig. 14–6)

Hydraulic diameter:
$$D_h = \frac{4A_c}{p} \qquad \textbf{(14–4)}$$

where A_c is the cross-sectional area of the pipe and p is its perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular pipes,

Circular pipes:
$$D_h = \frac{4A_c}{p} = \frac{4(\pi D^2/4)}{\pi D} = D$$

It certainly is desirable to have precise values of Reynolds number for laminar, transitional, and turbulent flows, but this is not the case in practice. This is because the transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations,* and *fluctuations in the flow.* Under most practical conditions, the flow in a circular pipe is laminar for Re < 2300, turbulent for Re > 4000, and transitional in between. That is,

$$\begin{aligned} \text{Re} &< 2300 && \text{laminar flow} \\ 2300 \le \text{Re} &\le 4000 && \text{transitional flow} \\ \text{Re} &> 4000 && \text{turbulent flow} \end{aligned}$$

In transitional flow, the flow switches between laminar and turbulent randomly (Fig. 14–7). It should be kept in mind that laminar flow can be maintained at much higher Reynolds numbers in very smooth pipes by avoiding flow disturbances and pipe vibrations. In such carefully controlled experiments, laminar flow has been maintained at Reynolds numbers of up to 100,000. For flows approximated as inviscid flow, the Reynolds number is "infinity" since the viscosity is assumed to be zero.

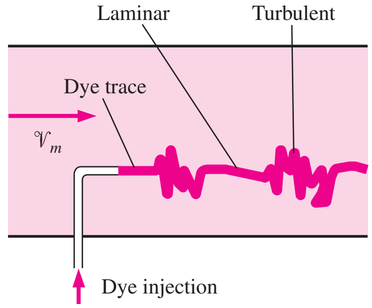


FIGURE 14–7
In the transitional flow region of $2300 \leq \text{Re} \leq 4000$, the flow switches between laminar and turbulent randomly.

14–3 ▪ THE ENTRANCE REGION

Consider a fluid entering a circular pipe at a uniform velocity. Because of the no-slip condition, the fluid particles in the layer in contact with the surface of the pipe come to a complete stop. This layer also causes the fluid particles in the adjacent layers to slow down gradually as a result of friction. To make up for this velocity reduction, the velocity of the fluid at the midsection of the pipe has to increase to keep the mass flow rate through the pipe constant. As a result, a velocity gradient develops along the pipe.

The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the **velocity boundary layer** or just the **boundary layer.** The hypothetical boundary surface divides the flow in a pipe into two regions: the **boundary layer region,** in which the viscous effects and the velocity changes are significant, and the **inviscid flow region,** in which the frictional effects are negligible and the velocity remains essentially constant in the radial direction.

The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the pipe center and thus fills the entire pipe, as shown in Fig. 14–8. The region from the pipe inlet to the point at which the boundary layer merges at the centerline is called the **hydrodynamic entrance region,** and the length of this region is called the **hydrodynamic entry length** L_h. Flow in the entrance region is called *hydrodynamically developing flow* since this is the region where the velocity profile develops. The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the **hydrodynamically fully developed region.** The flow is said to be **fully developed** when the normalized temperature profile also remains unchanged. Hydrodynamically developed flow is equivalent to fully developed flow when the fluid in the pipe is not heated or cooled since the fluid temperature in this case remains essentially constant throughout. The velocity profile in the fully developed region is *parabolic* in laminar flow and

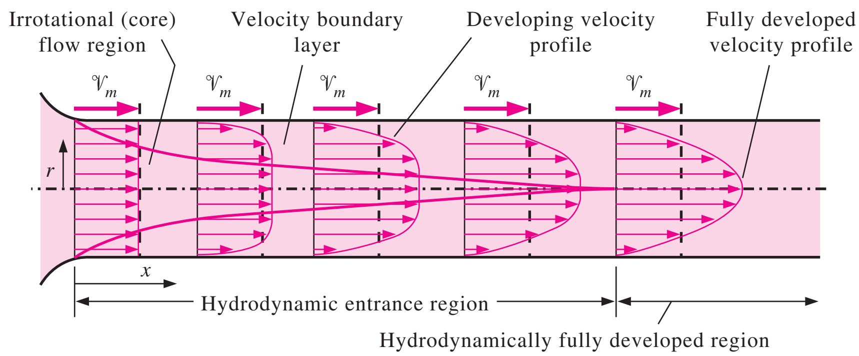


FIGURE 14–8
The development of the velocity boundary layer in a pipe. (The developed mean velocity profile is parabolic in laminar flow, as shown, but somewhat blunt in turbulent flow.)

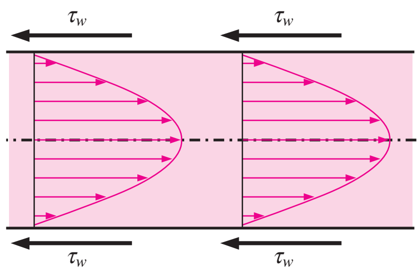


FIGURE 14–9
In the fully developed region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.

somewhat *flatter* (or *fuller*) in turbulent flow due to eddy motion and more vigorous mixing in the radial direction. The time-averaged velocity profile remains unchanged when the flow is fully developed, and thus

Hydrodynamically fully developed:
$$\frac{\partial u(r, x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r) \qquad \textbf{(14–5)}$$

The shear stress at the pipe wall τ_w is related to the shear stress at the surface, which is related to the slope of the velocity profile at the surface. Noting that the velocity profile remains unchanged in the hydrodynamically fully developed region, the wall shear stress also remains constant in that region (Fig. 14–9).

Consider fluid flow in the hydrodynamic entrance region of a pipe. The wall shear stress is the *highest* at the pipe inlet where the thickness of the boundary layer is zero, and decreases gradually to the fully developed value, as shown in Fig. 14–10. Therefore, the pressure drop is *higher* in the entrance regions of a pipe, and the effect of the entrance region is always to *increase* the average friction factor for the entire pipe. This increase can be significant for short pipes but negligible for long ones.

Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance where the friction factor reaches within about 2 percent of the fully developed value. In *laminar flow,* the hydrodynamic entry length is given approximately as [see Kays and Crawford (1993), and Shah and Bhatti (1987)]

$$L_{h,\text{ laminar}} \approx 0.05\ \text{Re}_D\ D \qquad \textbf{(14–6)}$$

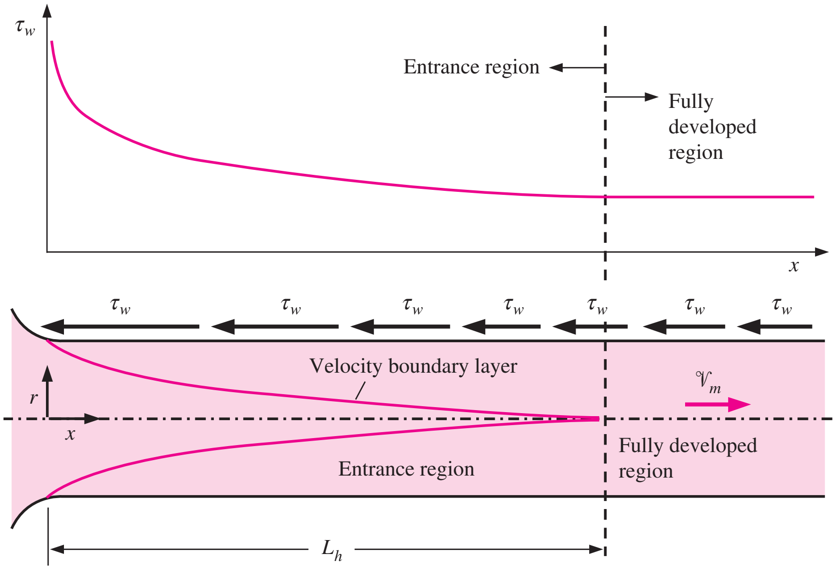


FIGURE 14–10
The variation of wall shear stress in the flow direction for flow in a pipe from the entrance region into the fully developed region.

For Re $= 20$, the hydrodynamic entry length is about the size of the diameter, but increases linearly with velocity. In the limiting laminar case of Re $= 2300$, the hydrodynamic entry length is $115D$.

In *turbulent flow,* the intense mixing during random fluctuations usually overshadows the effects of molecular diffusion. The hydrodynamic entry length for turbulent flow can be approximated as [see Bhatti and Shah (1987), and Zhi-qing (1982)]

$$L_{h,\,\text{turbulent}} = 1.359\,\text{Re}_D^{1/4} \tag{14–7}$$

The entry length is much shorter in turbulent flow, as expected, and its dependence on the Reynolds number is weaker. It is $11D$ at Re $= 10{,}000$, and increases to $43D$ at Re $= 10^5$. In many pipe flows of practical engineering interest, the entrance effects become insignificant beyond a pipe length of 10 diameters, and the hydrodynamic entry length is approximately taken to be

$$L_{h,\,\text{turbulent}} \approx 10D \tag{14–8}$$

Precise correlations for calculating the frictional head losses in entrance region are available in the literature. However, the pipes used in practice are usually several times the length of the entrance region, and thus the flow through the pipes is often assumed to be fully developed for the entire length of the pipe. This simplistic approach gives *reasonable* results for long pipes and *conservative* results for short ones since it underpredicts the friction factor.

14–4 ▪ LAMINAR FLOW IN PIPES

We mentioned earlier that flow in pipes is laminar for Re < 2300, and that the flow is fully developed if the pipe is sufficiently long (relative to the entry length) so that the entrance effects are negligible. In this section we consider the steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular pipe. We obtain the momentum equation by applying a momentum balance to a differential volume element, and obtain the velocity profile by solving it. Then we use it to obtain a relation for the friction factor. An important aspect of the analysis here is that it is one of the few available for viscous flow.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $u(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to flow is everywhere zero. There is no acceleration since the flow is steady and fully developed.

Now consider a ring-shaped differential volume element of radius r, thickness dr, and length dx oriented coaxially with the pipe, as shown in Fig. 14–11. The volume element involves only pressure and viscous effects, and thus the pressure and shear forces must balance each other. The pressure force acting on a submerged plane surface is the product of the pressure at the centroid of the surface and the surface area. A force balance on the volume element in the flow direction gives

$$(2\pi r\,drP)_x - (2\pi r\,drP)_{x+dx} + (2\pi r\,dx\tau)_r - (2\pi r\,dx\tau)_{r+dr} = 0 \tag{14–9}$$

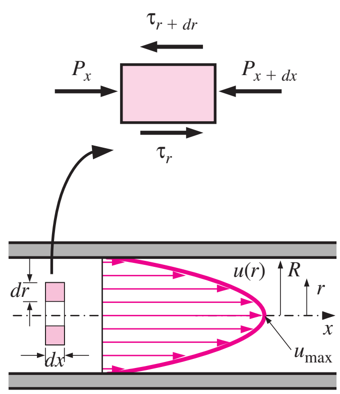


FIGURE 14–11
Free-body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal pipe in fully developed laminar flow.

which indicates that in fully developed flow in a horizontal pipe, the viscous and pressure forces balance each other. Dividing by $2\pi\, dr\, dx$ and rearranging,

$$r\frac{P_{x+dx}-P_x}{dx}+\frac{(r\tau)_{r+dr}-(r\tau)_r}{dr}=0 \tag{14–10}$$

Taking the limit as $dr, dx \to 0$ gives

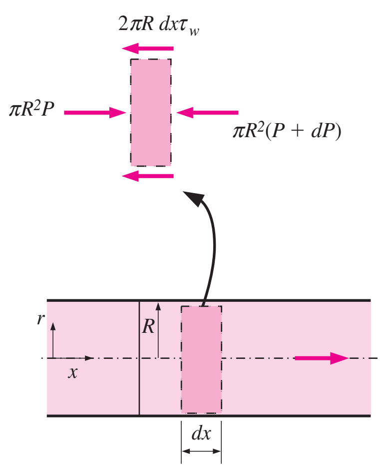
$$r\frac{dP}{dx}+\frac{d(r\tau)}{dr}=0 \tag{14–11}$$

Substituting $\tau = -\mu(du/dr)$ and rearranging gives the desired equation,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)=\frac{dP}{dx} \tag{14–12}$$

The quantity du/dr is negative in pipe flow, and the negative sign is included to obtain positive values for τ. (Or, $du/dr = -du/dy$ since $y = R - r$.) The left side of Eq. 14–12 is a function of r and the right side is a function of x. The equality must hold for any value of r and x, and an equality of the form $f(r) = g(x)$ can happen only if both $f(r)$ and $g(x)$ are equal to the same constant. Thus we conclude that dP/dx = constant. This can be verified by writing a force balance on a volume element of radius R and thickness dx (a slice of the pipe), which gives (Fig. 14–12)

$$\frac{dP}{dx}=-\frac{2\tau_w}{R} \tag{14–13}$$



Force balance:

$$\pi R^2 P - \pi R^2 (P + dR) - 2\pi R\, dx\tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

FIGURE 14–12
Free-body diagram of a fluid disk element of radius R and length dx in fully developed laminar flow in a horizontal pipe.

Here τ_w is constant since the viscosity and the velocity profile are constants in the fully developed region. Therefore, dP/dx = constant.

Equation 14–12 can be solved by rearranging and integrating it twice to give

$$u(r)=\frac{1}{4\mu}\left(\frac{dP}{dx}\right)+C_1\ln r+C_2 \tag{14–14}$$

The velocity profile $u(r)$ is obtained by applying the boundary conditions $\partial u/\partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $u = 0$ at $r = R$ (the no-slip condition at the pipe surface). We get

$$u(r)=-\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right) \tag{14–15}$$

Therefore, the velocity profile in fully developed laminar flow in a pipe is *parabolic* with a maximum at the centerline and minimum (zero) at the pipe wall. Also, the axial velocity u is positive for any r, and thus the axial pressure gradient dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).

The mean velocity is determined from its definition by substituting Eq. 14–15 into Eq. 14–2, and performing the integration. It gives

$$\mathcal{V}_m=\frac{2}{R^2}\int_0^R u(r)r\,dr=\frac{-2}{R^2}\int_0^R\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right)r\,dr=-\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right) \tag{14–16}$$

Combining the last two equations, the velocity profile is re-written as

$$u(r) = 2\mathcal{V}_m\left(1 - \frac{r^2}{R^2}\right) \tag{14–17}$$

This is a convenient form for the velocity profile since $\mathcal{V}_m$ can be determined easily from the flow rate information.

The maximum velocity occurs at the centerline, and is determined from Eq. 14–17 by substituting $r = 0$,

$$u_{\max} = 2\mathcal{V}_m \tag{14–18}$$

Therefore, *the mean velocity in laminar pipe flow is one-half of the maximum velocity*.

Pressure Drop and Head Loss

A quantity of interest in the analysis of pipe flow is the *pressure drop* ΔP since it is directly related to the power requirements of the fan or pump to maintain flow. We note that dP/dx = constant, and integrating from $x = x_1$ where the pressure is P_1 to $x = x_1 + L$ where the pressure is P_2 gives

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} \tag{14–19}$$

Substituting Eq. 14–19 into the $\mathcal{V}_m$ expression in Eq. 14–16, the pressure drop can be expressed as

Laminar flow: $$\Delta P = P_1 - P_2 = \frac{8\mu L\mathcal{V}_m}{R^2} = \frac{32\mu L\mathcal{V}_m}{D^2} \tag{14–20}$$

The symbol Δ is typically used to indicate the difference between the final and initial values, like $\Delta y = y_2 - y_1$. But in fluid flow, ΔP is used to designate pressure drop, and thus it is $P_1 - P_2$. A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss** ΔP_L to emphasize that it is a *loss* (just like the head loss h_L, which is proportional to it).

Note from Eq. 14–20 that the pressure drop is proportional to the viscosity μ of the fluid, and ΔP would be zero if there were no friction. Therefore, the drop of pressure from P_1 to P_2 in this case is due entirely to viscous effects, and Eq. 14–20 represents the pressure loss ΔP_L when a fluid of viscosity μ flows through a pipe of constant diameter D and length L at mean velocity $\mathcal{V}_m$.

In practice, it is found convenient to express the pressure loss for all types of fully developed internal flows (laminar or turbulent flows, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes) as (Fig. 14–13)

Pressure loss: $$\Delta P_L = f\frac{L}{D}\frac{\rho\mathcal{V}_m^2}{2} \tag{14–21}$$

where $\rho\mathcal{V}_m^2/2$ is the *dynamic pressure* and the dimensionless quantity f is the **friction factor** (also called the *Darcy friction factor* after French engineer Henry Darcy, 1803–1858, who first experimentally studied the effects of roughness on pipe resistance),

Darcy friction factor: $$f = \frac{8\tau_w}{\rho\mathcal{V}_m^2} \tag{14–22}$$

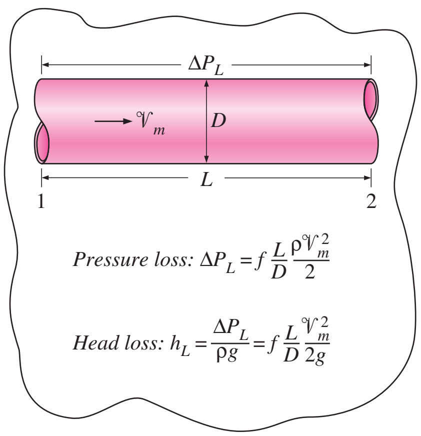


FIGURE 14–13
The relation for pressure loss (and head loss) is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and smooth or rough surfaces.

It should not be confused with the *friction coefficient* C_f (also called the *Fanning friction factor*) which is defined as $C_f = 2\tau_w/(\rho \mathcal{V}_m^2) = f/4$.

Setting Eqs. 14–20 and 14–21 equal to each other and solving for f gives the friction factor for fully developed laminar flow in a circular pipe,

Circular pipe, laminar:
$$f = \frac{64\mu}{\rho D \mathcal{V}_m} = \frac{64}{\text{Re}} \tag{14–22}$$

This equation shows that *in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface*.

In the analysis of piping systems, pressure losses are commonly expressed in terms of the *equivalent fluid column height*, called the **head loss** h_L. Noting from fluid statics that $\Delta P = \rho g h$ and thus a pressure difference of ΔP corresponds to a fluid height of $h = \Delta P/\rho g$, the *pipe head loss* is obtained by dividing ΔP_L by ρg to give

Head loss:
$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{\mathcal{V}_m^2}{2g} \tag{14–23}$$

The head loss h_L represents *the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe*. The head loss is caused by viscosity, and it is directly related to the wall shear stress. Equations 14–21 and 14–23 are valid for both laminar and turbulent flows in both circular and noncircular pipes.

Once the pressure loss (or head loss) is available, the required pumping power *to overcome the pressure loss* is determined from

$$\dot{W}_{\text{pump}, L} = \dot{V}\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L \tag{14–24}$$

where $\dot{V}$ is the volume flow rate and $\dot{m}$ is the mass flow rate.

The mean velocity for laminar flow in a horizontal pipe is, from Eq. 14–20,

Horizontal pipe:
$$\mathcal{V}_m = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P D^2}{32\mu L} \tag{14–25}$$

Then the volume flow rate for laminar flow through a horizontal pipe of diameter D and length L becomes

Horizontal pipe:
$$\dot{V} = \mathcal{V}_m A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P \pi D^4}{128\mu L} \tag{14–26}$$

This equation is known as **Poiseuille's Law,** and this flow is called *Hagen-Poiseuille flow* in honor of the works of G. Hagen (1797–1839) and J. Poiseuille (1799–1869) on the subject. Note from Eq. 14–26 that *for a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius* (*or diameter*) *of the pipe*. Therefore, the pumping power requirement for a piping system can be reduced by a factor of 16 by doubling the pipe diameter (Fig. 14–14). Of course the benefits of the reduction in the energy costs must be weighed against the increased cost of construction due to using a larger diameter pipe.

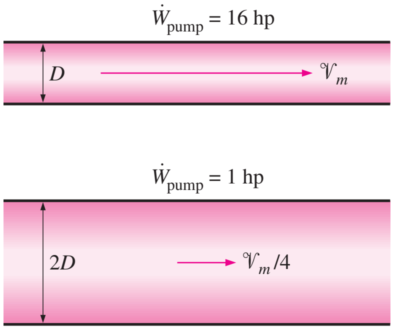


FIGURE 14–14
The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

The pressure drop ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe, but this is not the case for inclined pipes or pipes with variable cross-sectional area. This can be demonstrated by writing the energy equation for steady incompressible one-dimensional flow in terms of heads as (see Chap. 12)

$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \qquad \textbf{(14–27)}$$

where $h_{\text{pump, u}}$ is the useful pump head delivered to the fluid, $h_{\text{turbine, e}}$ is the turbine head extracted from the fluid, and h_L is the irreversible head loss between sections 1 and 2, and $\mathcal{V}_1$ and $\mathcal{V}_2$ are the mean velocities at sections 1 and 2, respectively (the subscript m has been dropped for convenience). Equation 14–27 can be rearranged as

$$P_1 - P_2 = \rho(\mathcal{V}_2^2 - \mathcal{V}_1^2)/2 + \rho g[(z_2 - z_1) + h_{\text{turbine, e}} - h_{\text{pump, u}} + h_L] \qquad \textbf{(14–28)}$$

Therefore, the pressure drop $\Delta P = P_1 - P_2$ and pressure loss $\Delta P_L = \rho g h_L$ for a given flow section are equivalent if (1) the flow section is horizontal so that there are no hydrostatic or gravity effects ($z_1 = z_2$), (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure ($h_{\text{pump, u}} = h_{\text{turbine, e}} = 0$), and (3) the cross-sectional area of the flow section is constant and thus the mean flow velocity is constant ($\mathcal{V}_1 = \mathcal{V}_2$).

Inclined Pipes

Relations for inclined pipes can be obtained in a similar manner from a force balance in the direction of flow. The only additional force in this case is the component of the fluid weight in the flow direction, whose magnitude is

$$W_x = W \sin\theta = \rho g V_{\text{element}} \sin\theta = \rho g(2\pi r\, dr\, dx) \sin\theta \qquad \textbf{(14–29)}$$

where θ is the angle between the horizontal and the flow direction (Fig. 14–15). The force balance in Eq. 14–9 now becomes

$$(2\pi r\, dr\, P)_x - (2\pi r\, drP)_{x+dx} + (2\pi r\, dx\,\tau)_r - (2\pi r\, dx\,\tau)_{r+dr} - \rho g(2\pi r\, dr\, dx)\sin\theta = 0 \qquad \textbf{(14–30)}$$

which results in the differential equation

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta \qquad \textbf{(14–31)}$$

Following the same solution procedure, the velocity profile can be shown to be

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right) \qquad \textbf{(14–32)}$$

It can also be shown that the *mean velocity* and the *volume flow rate* relations for laminar flow through inclined pipes are

$$\mathcal{V}_m = \frac{(\Delta P - \rho g L \sin\theta)D^2}{32\mu L} \quad \text{and} \quad \dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L} \qquad \textbf{(14–33)}$$

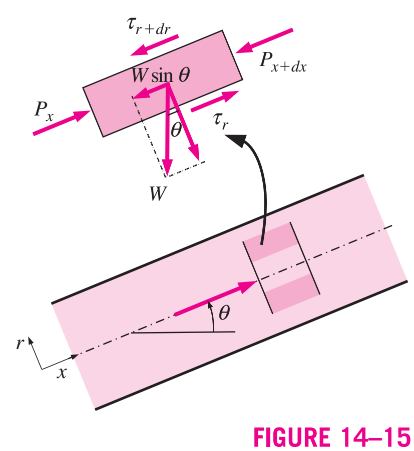


FIGURE 14–15
Free body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with an inclined pipe in fully developed laminar flow.

which are identical to the corresponding relations for horizontal pipes, except that ΔP is replaced by $\Delta P - \rho g L \sin\theta$. Therefore, the results already obtained for horizontal pipes can also be used for inclined pipes provided that ΔP is replaced by $\Delta P - \rho g L \sin\theta$ (Fig. 14–16). Note that $\theta > 0$ and thus $\sin\theta > 0$ for uphill flow, and $\theta < 0$ and thus $\sin\theta < 0$ for downhill flow.

In inclined pipes, the combined effect of pressure difference and gravity drives the flow. Gravity helps downhill flow but opposes uphill flow. Therefore, much greater pressure differences need to be applied to maintain a specified flow rate in uphill flow although this becomes important only for liquids, because the density of gases is generally low. In the special case of *no flow* ($\dot{V} = 0$), we have $\Delta P = \rho g L \sin\theta$, which is what we would obtain from fluid statics.

Horizontal pipe: $\dot{V} = \dfrac{\Delta P \pi D^4}{128\mu L}$

Inclined pipe: $\dot{V} = \dfrac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$

Uphill flow: $\theta > 0$ and $\sin\theta > 0$
Downhill flow: $\theta < 0$ and $\sin\theta < 0$

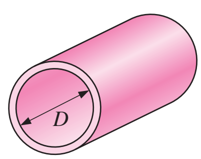
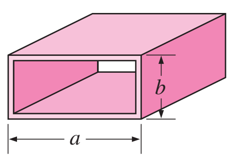
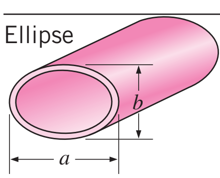
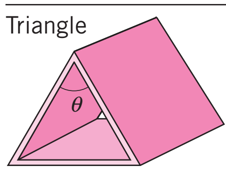
FIGURE 14–16
The relations developed for fully developed laminar flow through horizontal pipes can also be used for inclined pipes by replacing ΔP with $\Delta P - \rho g L \sin\theta$.

Laminar Flow in Noncircular Pipes

The friction factor f relations are given in Table 14–1 for *fully developed laminar flow* in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h = 4A_c/p$ where A_c is the cross-sectional area of the pipe and p is its perimeter.

TABLE 14–1

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\mathrm{Re} = \mathcal{V}_m D_h/\nu$)

Tube Geometry	a/b or $\theta°$	Friction Factor f
Circle 	—	64.00/Re
Rectangle 	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse 	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Triangle 	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re

EXAMPLE 14–1 Flow Rates in Horizontal and Inclined Pipes

Oil at 20°C (ρ = 888 kg/m^3 and μ = 0.8 kg/m · s) is flowing through a 5-cm-diameter 40-m-long pipe steadily (Fig. 14–17). During the flow, the pressure at the pipe inlet and exit are measured to be 745 kPa and 97 kPa, respectively. Determine the flow rate of oil through the pipe assuming the pipe is (*a*) horizontal, (*b*) inclined 15° upward, (*c*) inclined 15° downward. Also verify that the flow through the pipe is laminar.

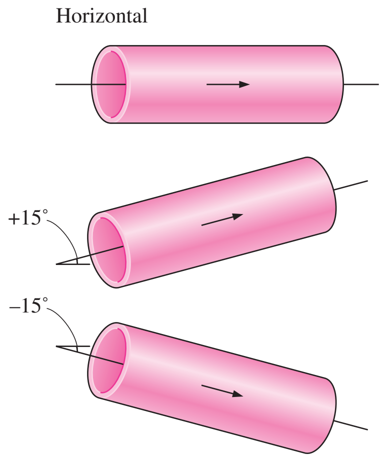


FIGURE 14–17
Schematic for Example 14–1.

SOLUTION The pressure readings at the inlet and exit of a pipe are given. The flow rates are to be determined for three different orientations, and the flow is to be shown to be laminar.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as a pump or a turbine.

Properties The density and dynamic viscosity of oil are given to be ρ = 888 kg/m^3 and μ = 0.8 kg/m · s, respectively.

Analysis The pressure drop across the pipe and the pipe cross-sectional area are

$$\Delta P = P_1 - P_2 = 745 - 97 = 648 \text{ kPa}$$
$$A_c = \pi D^2/4 = \pi(0.05 \text{ m})^2/4 = 0.001963 \text{ m}^2$$

(*a*) The flow rate for all three cases can be determined from

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ = 0 and thus sin θ = 0. Therefore,

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} = \frac{(648 \text{ kPa})\pi(0.05 \text{ m})^4}{128(0.8 \text{ kg/m}\cdot\text{s})(40 \text{ m})}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right) = \mathbf{0.00311\ m^3/s}$$

(*b*) For uphill flow with an inclination of 15°, we have θ = +15°, and

$$\begin{aligned}\dot{V}_{\text{uphill}} &= \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}\\ &= \frac{[(648{,}000 \text{ Pa} - (888 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(40 \text{ m})\sin 15°]\pi(0.05 \text{ m})^4}{128(0.8 \text{ kg/m}\cdot\text{s})(40 \text{ m})}\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{Pa}\cdot\text{m}^2}\right)\\ &= \mathbf{0.00267\ m^3/s}\end{aligned}$$

(*c*) For downhill flow with an inclination of 15°, we have θ = −15°, and

$$\begin{aligned}\dot{V}_{\text{downhill}} &= \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}\\ &= \frac{[(648{,}000 \text{ Pa} - (888 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(40 \text{ m})\sin(-15°)]\pi(0.05\text{m})^4}{128(0.8 \text{ kg/m}\cdot\text{s})(40 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ Pa}\cdot\text{m}^2}\right)\\ &= \mathbf{0.00354\ m^3/s}\end{aligned}$$

The flow rate is the highest for downhill flow case, as expected. The mean fluid velocity and the Reynolds number in this case are

$$\mathcal{V}_m = \frac{\dot{V}}{A_c} = \frac{0.00354 \text{ m}^3/\text{s}}{0.001963 \text{ m}^2} = 1.80 \text{ m/s}$$

$$\text{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{(888 \text{ kg/m}^3)(1.80 \text{ m/s})(0.05 \text{ m})}{0.8 \text{ kg/m} \cdot \text{s}} = 100$$

which is less than 2300. Therefore, the flow is *laminar* for all three cases, and the above analysis is valid.

Discussion Note that the flow is driven by the combined effect of pressure difference and gravity. As can be seen from the rates we calculated, gravity opposes uphill flow, but enhances downhill flow. Gravity has no effect on the flow rate in the horizontal case. Downhill flow can occur even in the absence of an applied pressure difference. For the case of $P_1 = P_2 = 97$ kPa (i.e., no applied pressure difference), the pressure throughout the entire pipe would remain constant at 97 Pa, and the fluid would flow through the pipe at a rate of 0.00043 m^3/s under the influence of gravity. The flow rate increases as the tilt angle of the pipe from the horizontal is increased in the negative direction, and would reach its maximum value when the pipe is vertical.

EXAMPLE 14–2 Pressure Drop and Head Loss in a Pipe

Water at 40°F ($\rho = 62.42$ lbm/ft^3 and $\mu = 3.74$ lbm/ft · h) is flowing through a 0.12-in (= 0.01 ft) diameter 30-ft long horizontal pipe steadily at an average velocity of 3 ft/s (Fig. 14–18). Determine (*a*) the head loss, (*b*) pressure drop, and (*c*) the pumping power requirement to overcome this pressure drop.

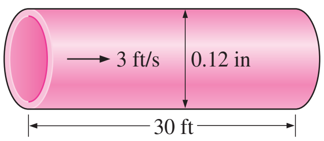


FIGURE 14–18
Schematic for Example 14–2.

SOLUTION The average flow velocity in a pipe is given. The head loss, the pressure drop, and the pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors.

Properties The density and dynamic viscosity of water are given to be $\rho = 62.42$ lbm/ft^3 and $\mu = 3.74$ lbm/ft · h, respectively.

Analysis (*a*) First we need to determine the flow regime. The Reynolds number is

$$\text{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})(0.01 \text{ ft})}{3.74 \text{ lbm/ft} \cdot \text{h}} \left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 1803$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the head loss become

$$f = \frac{64}{\text{Re}} = \frac{64}{1803} = 0.0355$$

$$h_L = f\frac{L}{D}\frac{\mathcal{V}_m^2}{2g} = 0.0355 \frac{30 \text{ ft}}{0.01 \text{ ft}} \frac{(3 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = \mathbf{29.8 \text{ ft}}$$

(*b*) Noting that the pipe is horizontal and its diameter is constant, the pressure drop in the pipe is due entirely to the frictional losses, and is equivalent to the pressure loss,

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho \mathcal{V}_m^2}{2} = 0.0355\,\frac{30\text{ ft}}{0.01\text{ ft}}\,\frac{(62.42\text{ lbm/ft}^3)(3\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= \mathbf{930\ lbf/ft^2} = \mathbf{6.46\ psi}$$

(*c*) The volume flow rate and the pumping power requirements are

$$\dot{V} = \mathcal{V}_m A_c = \mathcal{V}_m(\pi D^2/4) = (3\text{ ft/s})[\pi(0.01\text{ ft})^2/4] = 0.000236\text{ ft}^3\text{/s}$$
$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.000236\text{ ft}^3\text{/s})(930\text{ lbf/ft}^2)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.30\ W}$$

Therefore, power input in the amount of 0.30 W is needed to overcome the frictional losses in the flow due to viscosity.

14–5 TURBULENT FLOW IN PIPES

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress. However, turbulent flow is a complex mechanism dominated by fluctuations, and despite tremendous amounts of work done in this area by researchers, the theory of turbulent flow remains largely undeveloped. Therefore, we must rely on experiments and the empirical or semi-empirical correlations developed for various situations.

Turbulent flow is characterized by random and rapid fluctuations of swirling fluid particles, called *eddies*, throughout the flow. These fluctuations provide an additional mechanism for momentum and energy transfer. In laminar flow, fluid particles flow in an orderly manner along streamlines, and momentum and energy are transferred across streamlines by molecular diffusion. In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer. As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients (Fig. 14–19).

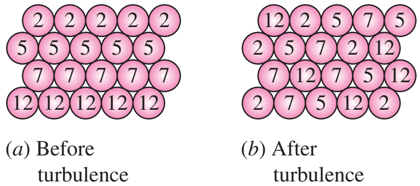


FIGURE 14–19
The intense mixing in turbulent flow brings fluid particles at different momentums into close contact, and thus enhances momentum transfer.

Even when the mean flow is steady, the eddy motion in turbulent flow causes significant fluctuations in the values of velocity, temperature, pressure, and even density (in compressible flow). Figure 14–20 shows the variation of the instantaneous velocity component u with time at a specified location, as can be measured with a hot-wire anemometer probe or other sensitive device. We observe that the instantaneous values of the velocity fluctuate about a mean value, which suggests that the velocity can be expressed as the sum of a *mean value* $\bar{u}$ and a *fluctuating component* u',

$$u = \bar{u} + u' \tag{14–34}$$

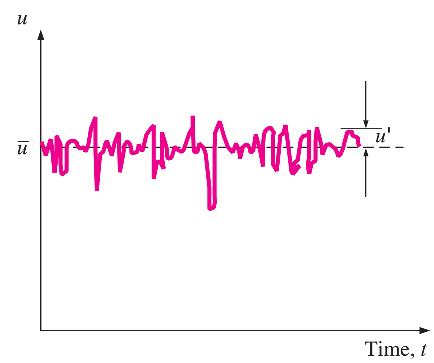
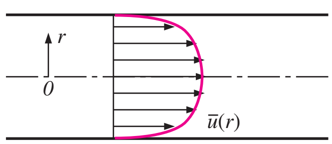


FIGURE 14–20
Fluctuations of the velocity component u with time at a specified location in turbulent flow.

This is also the case for other properties such as the velocity component υ in the y direction, and thus $\upsilon = \bar{\upsilon} + \upsilon'$, $P = \bar{P} + P'$, and $T = \bar{T} + T'$. The mean value of a property at some location is determined by averaging it over a time interval that is sufficiently large so that the time average levels off to a constant. Therefore, the time average of fluctuating components is zero, e.g.,

$\bar{u}' = 0$. The magnitude of u' is usually just a few percent of $\bar{u}$, but the high frequencies of eddies (in the order of a thousand per second) makes them very effective for the transport of momentum, thermal energy, and mass. In *steady* turbulent flow, the mean values of properties (indicated by an overbar) are independent of time. The chaotic fluctuations of fluid particles play a dominant role in pressure drop, and these random motions must be considered in analysis together with the mean velocity.

Perhaps the first thought that comes to mind is to determine the shear stress in an analogous manner to laminar flow from $\tau = -\mu\, d\bar{u}/dr$, where $\bar{u}(r)$ is the mean velocity profile for turbulent flow. But the experimental studies show that this is not the case, and the shear stress is much larger due to the turbulent fluctuations. Therefore, it is convenient to think of the turbulent shear stress as consisting of two parts: the *laminar component,* which accounts for the friction between layers in the flow direction (expressed as $\tau_{\text{lam}} = -\mu\, d\bar{u}/dr$), and the *turbulent component,* which accounts for the friction between the fluctuating fluid particles and the fluid body (denoted as τ_{turb} and is related to the fluctuation components of velocity). Then the *total shear stress* in turbulent flow can be expressed as



$$\tau_{\text{total}} = \tau_{\text{lam}} + \tau_{\text{turb}} \tag{14–35}$$

The typical mean velocity profile and relative magnitudes of laminar and turbulent components of shear stress for turbulent flow in a pipe are given in Fig. 14–21. Note that although the velocity profile is approximately parabolic in laminar flow, it becomes flatter or "fuller" in turbulent flow, with a sharp drop near the pipe wall. The fullness increases with the Reynolds number, and the velocity profile becomes more nearly uniform, lending support to the commonly utilized uniform velocity profile approximation.

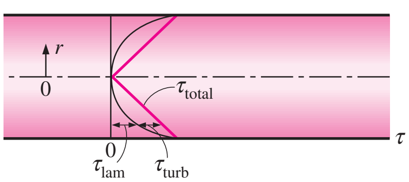


FIGURE 14–21
The velocity profile and the variation of shear stress with radial distance for turbulent flow in a pipe.

Turbulent Shear Stress

Consider turbulent flow in a horizontal pipe, and the upward eddy motion of fluid particles in a layer of lower velocity to an adjacent layer of higher velocity through a differential area dA as a result of the velocity fluctuation v', as shown in Fig. 14–22. The mass flow rate of the eddying fluid particles through dA is $\rho v' dA$, and its net effect on the layer above dA is a reduction in its mean flow velocity because of momentum transfer to the fluid particles with lower mean flow velocity. This momentum transfer will cause the horizontal velocity of the fluid particles to increase by u', and thus its momentum in the horizontal direction to increase at a rate of $(\rho v' dA)u'$, which must be equal to the decrease in the momentum of the upper fluid layer. Noting that force in a given direction is equal to the rate of change of momentum in that direction, the horizontal force acting on a fluid element above dA due to the passing of fluid particles through dA is $\delta F = (\rho v' dA)(-u') = -\rho u' v' dA$. Therefore, the shear force per unit area due to the eddy motion of fluid particles $dF/dA = -\rho u' v'$ can be viewed as the instantaneous turbulent shear stress. Then the **turbulent shear stress** can be expressed as

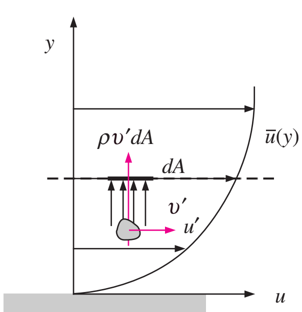


FIGURE 14–22
Fluid particles moving upward through a differential area dA as a result of the velocity fluctuation v'.

$$\tau_{\text{turb}} = -\rho \overline{u' v'} \tag{14–36}$$

where $\overline{u' v'}$ is the time average of the product of the fluctuating velocity components u' and v'. Note that $\overline{u' v'} \neq 0$ even though $\overline{u'} = 0$ and $\overline{v'} = 0$ (and thus $\overline{u'} \times \overline{v'} = 0$), and experimental results show that $\overline{u'v'}$ is usually a negative quantity. Terms such as $-\rho\overline{u'v'}$ or $-\rho\overline{u'^2}$ are called **Reynolds stresses** or **turbulent stresses.**

Many semi-empirical formulations have been developed that model the Reynolds stress in terms of mean velocity gradients in order to provide mathematical *closure* to the equations of motion. Such models are called **turbulence models.**

The random eddy motion of groups of particles resembles the random motion of molecules in a gas—colliding with each other after traveling a certain distance and exchanging momentum in the process. Therefore, momentum transport by eddies in turbulent boundary layers is analogous to the molecular momentum diffusion. In many of the simpler turbulence models, turbulent shear stress is expressed in an analogous manner as suggested by the French scientist J. Boussinesq in 1877 as

$$\tau_{\text{turb}} = -\rho\overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y} \qquad \textbf{(14–37)}$$

where μ_t is the **eddy viscosity** or **turbulent viscosity,** which accounts for momentum transport by turbulent eddies. Then the total shear stress can be expressed conveniently as

$$\tau_{\text{total}} = (\mu + \mu_t)\frac{\partial \bar{u}}{\partial y} = \rho(v + v_t)\frac{\partial \bar{u}}{\partial y} \qquad \textbf{(14–38)}$$

where $v_t = \mu_t/\rho$ is the **kinematic eddy viscosity** or **kinematic turbulent viscosity** (also called the *eddy diffusivity of momentum*). The concept of eddy viscosity is very appealing, but it is of no practical use unless its value can be determined. In other words, eddy viscosity must be modeled as a function of the mean flow variables; we call this *eddy viscosity closure*. For example, in the early 1900s, the German scientist L. Prandtl introduced the concept of **mixing length** l_m which is the distance a particle travels before colliding with other particles, and expressed the turbulent shear stress as

$$\tau_{\text{turb}} = \mu_t \frac{\partial \bar{u}}{\partial y} = \rho l_m^2 \left(\frac{\partial \bar{u}}{\partial y}\right)^2 \qquad \textbf{(14–39)}$$

But this concept is also of limited use since l_m is not a constant for a given flow (in the vicinity of the wall, for example, l_m is nearly proportional to distance from the wall), and its determination is not easy. Final mathematical closure is obtained only when l_m is written as a function of mean flow variables, distance from the wall, etc.

Eddy motion and thus eddy diffusivities are much larger than their molecular counterparts in the core region of a turbulent boundary layer. The eddy motion loses its intensity close to the wall, and diminishes at the wall because of the no-slip condition (u' and v' are identically zero at a stationary wall). Therefore, the velocity and temperature profiles are very slowly changing in the core region of a turbulent boundary layer, but very steep in the thin layer adjacent to the wall, resulting in large velocity and temperature gradients at the wall surface. So it is no surprise that the wall shear stress is much larger in turbulent flow than it is in laminar flow (Fig. 14–23).

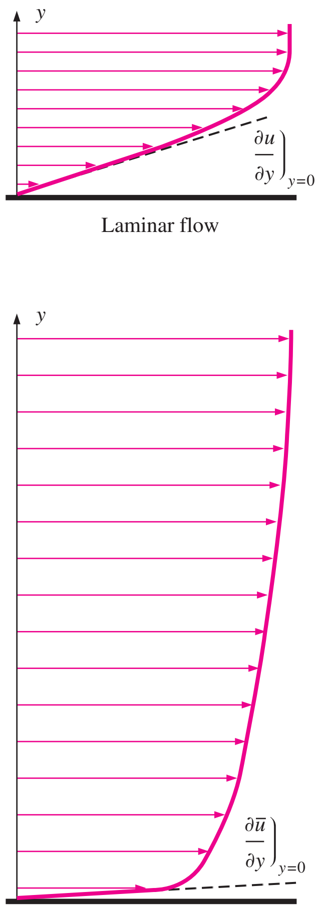


FIGURE 14–23
The velocity gradients at the wall, and thus the wall shear stress, are much larger for turbulent flow than they are for laminar flow, even though the turbulent boundary layer is thicker than the laminar one for the same value of free-stream velocity.

Note that molecular diffusivity of momentum υ (as well as μ) is a fluid property, and its value can be found listed in fluid handbooks. Eddy diffusivity υ_t (as well as μ_t), however, is *not* a fluid property, and its value depends on flow conditions. Eddy diffusivity υ_t decreases towards the wall, becoming zero at the wall. Its value ranges from zero at the wall to several thousand times the value of the molecular diffusivity in the core region.

Turbulent Velocity Profile

Unlike laminar flow, the expressions for the velocity profile in a turbulent flow are based on both analysis and measurements, and thus they are semi-empirical in nature with constants determined from experimental data. Consider fully developed turbulent flow in a pipe, and let u denote the time-averaged velocity in the axial direction (and thus drop the overbar from $\bar{u}$ for simplicity).

Typical velocity profiles for fully developed laminar and turbulent flows are given in Fig. 14–24. Note that the velocity profile is parabolic in laminar flow but is much fuller in turbulent flow, with a sharp drop near the pipe wall. Turbulent flow along a wall can be considered to consist of four regions, characterized by the distance from the wall. The very thin layer next to the wall where viscous effects are dominant is the **viscous** (or **laminar** or **linear** or **wall**) sublayer. The velocity profile in this layer is very nearly *linear*, and the flow is streamlined. Next to the viscous sublayer is the **buffer layer,** in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects. Above the buffer layer is the **overlap** (or **transition**) **layer,** also called the **inertial sublayer,** in which the turbulent effects are much more significant, but still not dominant. Above that is the **outer** (or **turbulent**) **layer** in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

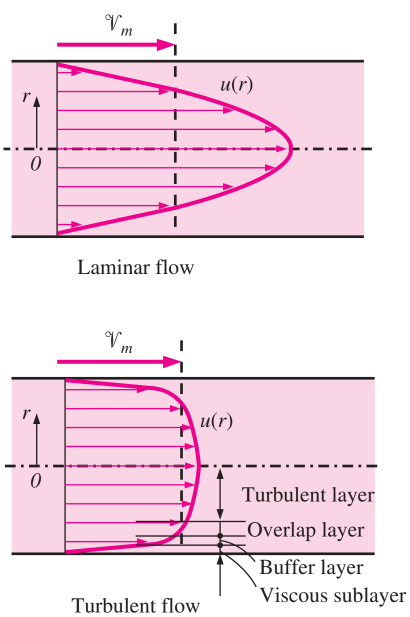


FIGURE 14–24
The velocity profile is parabolic in laminar flow, but nearly flat in turbulent flow.

Flow characteristics are quite different in different regions, and thus it is difficult to come up with an analytic relation for the velocity profile for the entire flow as we did for laminar flow. The best approach in the turbulent case turns out to be to identify the key variables and functional forms using dimensional analysis, and then to use experimental data to determine the numerical values of any constants.

The thickness of the viscous sublayer is very small (typically, much less than 1 mm), but this thin layer next to the wall plays a dominant role on flow characteristics because of the large velocity gradients it involves. The wall dampens any eddy motion, and thus the flow in this layer is essentially laminar and the shear stress consists of laminar shear stress which is proportional to the fluid viscosity. Considering that velocity changes from zero to nearly the core region value across a layer no thicker than a hair (almost like a step function), we would expect the velocity profile in this layer to be very nearly linear, and experiments confirm that. Then the velocity gradient in the viscous sublayer remains constant at $du/dy = u/y$, and the wall shear stress can be expressed as

$$\tau_w = \mu\frac{u}{y} = \rho\upsilon\frac{u}{y} \qquad \text{or} \qquad \frac{\tau_w}{\rho} = \frac{\upsilon u}{y} \tag{14–40}$$

where y is the distance from the wall (note that $y = R - r$ for a circular pipe). The quantity τ_w/ρ is frequently encountered in the analysis of turbulent velocity profiles. The square root of τ_w/ρ has the dimensions of velocity, and thus it is convenient to view it as a fictitious velocity called the **friction velocity** expressed as $u_* = \sqrt{\tau_w/\rho}$. Substituting this into Eq. 14–40, the velocity profile in the viscous sublayer can be expressed in dimensionless form as

Viscous sublayer:
$$\frac{u}{u_*} = \frac{yu_*}{\upsilon} \tag{14–41}$$

This equation is known as the **law of the wall,** and it is found to correlate experimental data for smooth surfaces well for $0 \le yu_*/\upsilon \le 5$. Therefore, the thickness of the viscous sublayer is roughly

Thickness of viscous sublayer:
$$y = \delta_{\text{sublayer}} = \frac{5\upsilon}{u_*} = \frac{25\upsilon}{u_\delta} \tag{14–42}$$

where u_δ is the flow velocity at the edge of the viscous sublayer, which is closely related to the mean velocity in a pipe. Thus we conclude that *the thickness of the viscous sublayer is proportional to the kinematic viscosity, and inversely proportional to the mean flow velocity*. In other words, the viscous sublayer is suppressed and it gets thinner as the velocity (and thus the Reynolds number) increases. Consequently, the velocity profile becomes nearly flat and thus the velocity distribution nearly uniform at very high Reynolds numbers.

The quantity υ/u_* has dimensions of length and is called **viscous length,** and it is used to nondimensionalize the distance y from the surface. In boundary layer analysis, it is convenient to work with normalized distance and normalized velocity defined as

Normalized variables:
$$y^+ = \frac{yu_*}{\upsilon} \quad \text{and} \quad u^+ = \frac{u}{u_*} \tag{14–43}$$

Then the law of the wall simply becomes

Normalized law of the wall:
$$y^+ = u^+ \tag{14–44}$$

Note that the friction velocity u_* is used to normalize both y and u, and y^+ resembles the Reynolds number expression.

In the overlap layer, the experimental data for velocity are observed always to line up on a straight line when plotted against the logarithm of distance from the wall. Dimensional analysis indicates and the experiments confirm that the velocity in the overlap layer is proportional to the logarithm of distance, and the velocity profile can be expressed as

The logarithmic law:
$$\frac{u}{u_*} = \frac{1}{k}\ln\frac{yu_*}{\upsilon} + B \tag{14–45}$$

where k and B are constants whose values are determined experimentally to be about 0.40 and 5.0, respectively. Eq. 14–45 is known as the **logarithmic law.** Substituting the values of the constant, the velocity profile is determined to be

Overlap layer:
$$\frac{u}{u_*} = 2.5\ln\frac{yu_*}{\upsilon} + 5.0 \tag{14–46}$$

It turns out that the logarithmic law in Eq. 14–46 represents experimental data well for the entire flow region except for the regions very close to the wall and near the pipe center, as shown in Fig. 14–25, and thus it is viewed as a *universal velocity profile* for turbulent flow in pipes or over surfaces. Note from the figure that the logarithmic law velocity profile is quite accurate for $y^+ > 30$, but neither velocity profile is accurate in the buffer layer, i.e., the region $5 < y^+ < 30$. Also, the viscous sublayer appears much larger in the figure than it is since we used a logarithmic scale for distance from the wall.

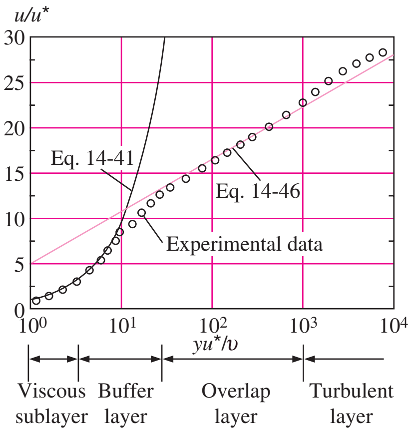


FIGURE 14–25
Comparison of the law of the wall and the logarithmic-law velocity profiles with experimental data for fully developed turbulent flow in a pipe.

A good approximation for the outer turbulent layer of pipe flow can be obtained by evaluating the constant B in Eq. 14–45 from the requirement that maximum velocity in a pipe occurs at the centerline where $r = 0$. Solving for B from Eq. 14–45 by setting $y = R - r = R$ and $u = u_{max}$, and substituting it back into Eq. 14–45 together with $k = 0.4$ gives

Outer turbulent layer:
$$\frac{u_{max} - u}{u_*} = 2.5 \ln \frac{R}{R - r} \qquad \textbf{(14–47)}$$

The deviation of velocity from the centerline value $u_{max} - u$ is called the **velocity defect,** and Eq. 14–47 is called the **velocity defect law.** This relation shows that the normalized velocity profile in the core region of turbulent flow in a pipe depends on the distance from the centerline, and is independent of the viscosity of the fluid. This is not surprising since the eddy motion is dominant in this region, and the effect of fluid viscosity is negligible.

Numerous other empirical velocity profiles exist for turbulent pipe flow. Among those, the simplest and the best known is the **power-law velocity profile** expressed as

Power-law velocity profile:
$$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{1/n} \quad \text{or} \quad \frac{u}{u_{max}} = \left(1 - \frac{r}{R}\right)^{1/n} \qquad \textbf{(14–48)}$$

where the exponent n is a constant whose value depends on the Reynolds number. The value of n increases with increasing Reynolds number. The value $n = 7$ approximates many flows in practice well, giving rise to the term *one-seventh power law velocity profile*.

Various power-law velocity profiles are shown in Fig. 14–26 for $n = 6$, 8, and 10 together with the velocity profile for fully developed laminar flow for comparison. Note that the turbulent velocity profile is fuller than the laminar one, and it becomes more flat as n (and thus the Reynolds number) increases. Also note that the power-law profile cannot be used to calculate wall shear stress since it gives a velocity gradient of infinity there, and it fails to give zero slope at the centerline. But these regions of discrepancy constitute a small portion of flow, and the power-law profile gives highly accurate results for mass, momentum, and energy flow rates through a pipe.

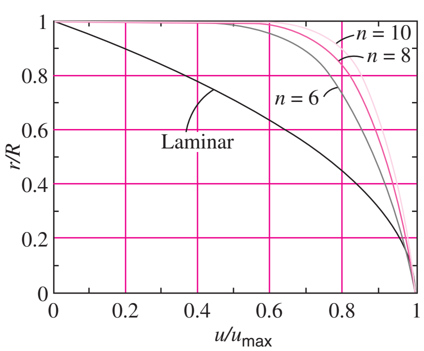


FIGURE 14–26
Power-law velocity profiles for fully developed turbulent flow in a pipe for different exponents, and its comparison with the laminar velocity profile.

Despite the small thickness of the viscous sublayer (usually much less than 1 percent of the pipe diameter), the characteristics of the flow in this layer are very important since they set the stage for flow in the rest of the pipe. Any irregularity or roughness on the surface disturbs this layer and affects the flow. Therefore, unlike laminar flow, the friction factor in turbulent flow is a strong function of surface roughness.

It should be kept in mind that roughness is a relative concept, and it has significance when its height ε is comparable to the thickness of the laminar sublayer (which is a function of the Reynolds number). All materials appear "rough" under a microscope with sufficient magnification. In fluid mechanics, a surface is characterized as being rough when the hills of roughness protrude out of the laminar sublayer. A surface is said to be smooth when the sublayer submerges the roughness elements. Glass and plastic surfaces are considered to be hydrodynamically smooth.

The Moody Chart

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness** ε/D, which is the ratio of the mean height of roughness of the pipe to the pipe diameter. The functional form of this dependence cannot be obtained from a theoretical analysis, and all available results are obtained from painstaking experiments using artificially roughened surfaces (usually by gluing sand grains of a known size on the inner surfaces of the pipes). Most such experiments were conducted by Prandtl's student J. Nikuradse in 1933, followed by the works of others. The friction factor was calculated from the measurements of the flow rate and the pressure drop.

The experimental results obtained are presented in tabular, graphical, and functional forms obtained by curve-fitting experimental data. In 1939, C. F. Colebrook combined all the data for transition and turbulent flow in smooth as well as rough pipes into the following implicit relation known as the **Colebrook equation:**

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)} \qquad \textbf{(14–49)}$$

In 1944, L. F. Moody plotted this formula into the now famous **Moody chart,** given in the Appendix (Fig. A–32). It presents the friction factors for pipe flow as a function of the Reynolds number and $\varepsilon//D$ over a wide range. It is probably one of the most widely accepted and used charts in engineering. Although it is developed for circular pipes, it can also be used for noncircular pipes by replacing the diameter by the hydraulic diameter.

Commercially available pipes differ from those used in the experiments in that the roughness of pipes in the market is not uniform, and it is difficult to give a precise description of it. Equivalent roughness values for some commercial pipes are given in Table 14–2 as well as on the Moody chart. But it should be kept in mind that these values are for new pipes, and the relative roughness of pipes may increase with use as a result of corrosion, scale buildup, and precipitation. As a result, the friction factor may increase by a factor of 5 to 10. Actual operating conditions must be considered in the design of piping systems. Also, the Moody chart and its equivalent Colebrook equation involve several uncertainties (the roughness size, experimental error, curve fitting of data, etc.), and thus the results obtained should not be treated as "exact." It is usually considered to be accurate to $\pm$15 percent over the entire range in the figure.

The Colebrook equation is implicit in f, and thus the determination of the friction factor requires some iteration unless an equation solver such as EES is used. An approximate explicit relation for f was given by S. E. Haaland in 1983 as

$$\frac{1}{\sqrt{f}} \approx -1.8 \log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right] \qquad \textbf{(14–50)}$$

TABLE 14–2

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

*The uncertainty in these values can be as much as $\pm$60 percent.

The results obtained from this relation are within 2 percent of those obtained from the Colebrook equation. If more accurate results are desired, Eq. 14–50 can be used as a good *first guess* in a Newton iteration when using a programmable calculator or a spreadsheet to solve for f with Eq. 14–49.

We make the following observations from the Moody chart:

- For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness.
- The friction factor is a minimum for a smooth pipe (but still not zero because of the no-slip condition) and increases with roughness. The Colebrook equation in this case ($\varepsilon = 0$) reduces to $1/\sqrt{f} = 2.0 \log(\text{Re}\sqrt{f}) - 0.8$ (Fig. 14–27).
- The transition region from the laminar to turbulent regime ($2300 < \text{Re} < 4{,}000$) is indicated by the shaded area in the Moody chart (Figs. 14–28 and A–32). The flow in this region may be laminar or turbulent, depending on flow disturbances, or it may alternate between laminar and turbulent, and thus the friction factor may also alternate between the values for laminar and turbulent flow. The data in this range are the least reliable. At small relative roughnesses, the friction factor increases in the transition region and approaches the value for smooth pipes.
- At very large Reynolds numbers (to the right of the dashed line on the chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number (Fig. 14–28). The flow in that region is called *fully rough flow,* or *completely* (or *fully*) *turbulent flow.* This is because the thickness of the laminar sublayer decreases with increasing Reynolds number, and it becomes so thin that the surface roughness protrudes into the flow. The viscous effects in this case are produced in the main flow primarily by the protruding roughness elements, and the contribution of the laminar sublayer is negligible. The Colebrook equation in the completely turbulent zone ($\text{Re} \to \infty$) reduces to $1/\sqrt{f} = -2.0 \log[(\varepsilon/D)/3.7]$ which is explicit in f.

In calculations, we should make sure that we use the internal diameter of the pipe, which may be different than the nominal diameter. For example, the

Relative Roughness, ε/D	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for Re = 10^6, and are calculated from Colebrook equation.

FIGURE 14–27
The friction factor is minimum for a smooth pipe and increases with roughness.

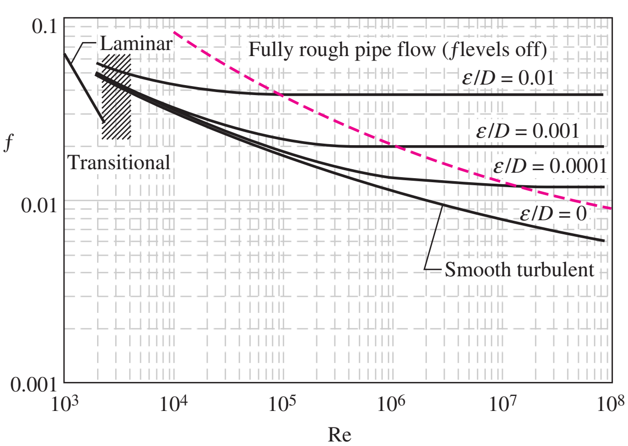


FIGURE 14–28
At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number.

internal diameter of a steel pipe whose nominal diameter is 1 in is 1.049 in (Table 14–3).

Types of Fluid Flow Problems

In the design and analysis of piping systems that involve the use of the Moody chart (or the Colebrook equation), we usually encounter three types of problems (the fluid and the roughness of the pipe are assumed to be specified in all cases) (Fig. 14–29):

1. Determining the **pressure drop** (or head loss) when the pipe length and diameter are given for a specified flow rate (or velocity).
2. Determining the **flow rate** when the pipe length and diameter are given for a specified pressure drop (or head loss).
3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss).

Problems of the *first type* are straightforward and can be solved directly by using the Moody chart. Problems of the *second type* and *third type* are commonly encountered in engineering design (in the selection of pipe diameter, for example, that minimizes the sum of the construction and pumping costs), but the use of the Moody chart with such problems requires an iterative approach unless an equation solver is used.

In problems of the *third type*, the diameter is not known and thus the Reynolds number and the relative roughness cannot be calculated. Therefore, we start calculations by assuming a pipe diameter. The pressure drop calculated for the assumed diameter is then compared to the specified pressure drop, and calculations are repeated with another pipe diameter in an iterative fashion until convergence.

In problems of the *second type*, the diameter is given but the flow rate is unknown. A good guess for the friction factor in that case is obtained from the completely turbulent flow region for the given roughness. This is true for large Reynolds numbers, which is often the case in practice. Once the flow rate is obtained, the friction factor can be corrected using the Moody chart or the Colebrook equation, and the process is repeated until the solution converges. (Typically only a few iterations are required for convergence to three or four digits of precision.)

To avoid tedious iterations in head loss, flow rate, and diameter calculations, Swamee and Jain (Ref. 14) proposed the following explicit relations in 1976 that are accurate to within 2 percent of the Moody chart:

$$h_L = 1.07\frac{\dot{V}^2 L}{gD^5}\left\{\ln\left[\frac{\varepsilon}{3.7\,D} + 4.62\left(\frac{\upsilon D}{\dot{V}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{matrix} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \text{Re} < 3 \times 10^8 \end{matrix} \qquad \textbf{(14–51)}$$

$$\dot{V} = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5}\ln\left[\frac{\varepsilon}{3.7\,D} + \left(\frac{3.17\upsilon^2 L}{gD^3 h_L}\right)^{0.5}\right] \qquad \text{Re} > 2000 \qquad \textbf{(14–52)}$$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \upsilon\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{matrix} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \text{Re} < 3 \times 10^8 \end{matrix} \qquad \textbf{(14–53)}$$

Note that all quantities are dimensional and the units simplify to the desired unit (for example, to m or ft in the last relation) when consistent units are used. Noting that the Moody chart is accurate to within 5 percent of experimental

TABLE 14–3

Standard sizes for Schedule 40 steel pipes

Nominal Size, in	Actual Inside Diameter, in
1/8	0.269
1/4	0.364
3/8	0.493
1/2	0.622
3/4	0.824
1	1.049
1 1/2	1.610
2	2.067
2 1/2	2.469
3	3.068
5	5.047
10	10.02

Problem type	Given	Find
1	$L, D, \dot{V}$	ΔP (or h_L)
2	$L, D, \Delta P$	$\dot{V}$
3	$L, \Delta P, \dot{V}$	D

FIGURE 14–29
The three types of problems encountered in pipe flow.

data, we should have no reservation in using these approximate relations in the design of piping systems.

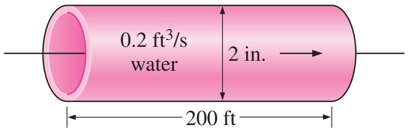


FIGURE 14–30
Schematic for Example 14–3.

EXAMPLE 14–3 Determining the Head Loss in a Water Pipe

Water at 60°F (ρ = 62.36 lbm/ft^3 and μ = 2.713 lbm/ft · h) is flowing steadily in a 2-in-diameter horizontal pipe made of stainless steel at a rate of 0.2 ft^3/s (Fig. 14–30). Determine the pressure drop, the head loss, and the required pumping power input for flow over a 200-ft-long section of the pipe.

SOLUTION The flow rate through a specified water pipe is given. The pressure drop, the head loss, and the pumping power requirements are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as a pump or a turbine.

Properties The density and dynamic viscosity of water are given to be ρ = 62.36 lbm/ft^3 and μ = 2.713 lbm/ft · h, respectively.

Analysis We recognize this as a problem of the first type, since flow rate, pipe length, and pipe diameter are known. First we calculate the mean velocity and the Reynolds number to determine the flow regime:

$$\mathcal{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2\ \text{ft}^3/\text{s}}{\pi(2/12\ \text{ft})^2/4} = 9.17\ \text{ft/s}$$

$$\text{Re} = \frac{\rho \mathcal{V} D}{\mu} = \frac{(62.36\ \text{lbm/ft}^3)(9.17\ \text{ft/s})(2/12\ \text{ft})}{2.713\ \text{lbm/ft}\cdot\text{h}}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 126{,}400$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is calculated using Table 14–2

$$\varepsilon/D = \frac{0.000007\ \text{ft}}{2/12\ \text{ft}} = 0.000042$$

The friction factor corresponding to this relative roughness and the Reynolds number can simply be determined from the Moody chart. To avoid any reading error, we determine *f* from the Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.000042}{3.7} + \frac{2.51}{126{,}400\sqrt{f}}\right)$$

Using an equation solver or an iterative scheme, the friction factor is determined to be f = 0.0174. Then the pressure drop (which is equivalent to pressure loss in this case), head loss, and the required power input become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho \mathcal{V}^2}{2} = 0.0174\ \frac{200\ \text{ft}}{2/12\ \text{ft}}\ \frac{(62.36\ \text{lbm/ft}^3)(9.17\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)^2$$

$$= \mathbf{1700\ lbf/ft^2} = \mathbf{11.8\ psi}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{\mathcal{V}^2}{2g} = 0.0174\ \frac{200\ \text{ft}}{2/12\ \text{ft}}\ \frac{(9.17\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = \mathbf{27.3\ ft}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.2\ \text{ft}^3/\text{s})(1700\ \text{lbf/ft}^2)\left(\frac{1\ \text{W}}{0.737\ \text{lbf}\cdot\text{ft/s}}\right) = \mathbf{461\ W}$$

Therefore, power input in the amount of 461 W is needed to overcome the frictional losses in the pipe.

Discussion It is common practice to write our final answers to three significant digits, even though we know that the results are accurate to at most two significant digits because of inherent inaccuracies in the Colebrook equation, as discussed previously. The friction factor also could be determined easily from the explicit Haaland relation (Eq. 14–50). It would give $f = 0.0172$, which is sufficiently close to 0.0174. Also, the friction factor corresponding to $\varepsilon = 0$ in this case is 0.0171, which indicates that stainless steel pipes can be assumed to be smooth with negligible error.

EXAMPLE 14–4 Determining the Diameter of an Air Duct

Heated air at 1 atm and 35°C is to be transported in a 150-m-long circular plastic duct at a rate of 0.35 m^3/s (Fig. 14–31). If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.

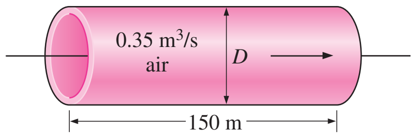


FIGURE 14–31
Schematic for Example 14–4.

SOLUTION The flow rate and the head loss in an air duct are given. The diameter of the duct is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The duct involves no components such as bends, valves, and connectors. **4** Air is an ideal gas. **5** The duct is smooth since it is made of plastic. **6** The flow is turbulent (to be verified).

Properties The density, dynamic viscosity, and kinematic viscosity of air at 35°C are $\rho = 1.145$ kg/m^3, $\mu = 1.895 \times 10^{-5}$ kg/m · s, and $\upsilon = 1.655 \times 10^{-5}$ m^2/s (Table A–22).

Analysis This is a problem of the third type since it involves the determination of diameter for specified flow rate and head loss. We can solve this problem by three different approaches: (1) an iterative approach by assuming a pipe diameter, calculating the head loss, comparing the result to the specified head loss, and repeating calculations until the calculated head loss matches the specified value; (2) writing all the relevant equations (leaving the diameter as an unknown) and solving them simultaneously using an equation solver; and (3) using the third Swamee–Jain formula. Below we demonstrate the use of the last two approaches.

The average velocity, the Reynolds number, the friction factor, and the head loss relations can be expressed as (D is in m, and $\mathcal{V}$ is in m/s, Re and f are dimensionless)

$$\mathcal{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.35 \text{ m}^3/\text{s}}{\pi D^2/4}$$

$$\text{Re} = \frac{\mathcal{V} D}{\upsilon} = \frac{\mathcal{V} D}{1.655 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0 \log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{\mathcal{V}^2}{2g} \quad \rightarrow \quad 20 = f\frac{150 \text{ m}}{D}\frac{\mathcal{V}^2}{2(9.81 \text{ m/s}^2)}$$

The roughness is approximately zero for a plastic pipe (Table 14–2). Therefore, this is a set of four equations in four unknowns, and solving them with an equation solver such as EES gives

$$D = \mathbf{0.267\ m}, \qquad f = 0.0180, \qquad \mathcal{V} = 6.24 \text{ m/s}, \qquad \text{and} \qquad \text{Re} = \mathbf{100{,}800}$$

Therefore, the diameter of the duct should be more than 26.7 cm if the head loss is not to exceed 20 m. Note that Re > 4000, and thus the turbulent flow assumption is verified.

The diameter can also be determined directly from the third Swamee–Jain formula to be

$$\begin{aligned} D &= 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \upsilon\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \\ &= 0.66\left[0 + (1.655 \times 10^{-5}\text{ m}^2\text{/s})(0.35\text{ m}^3\text{/s})^{9.4}\left(\frac{150\text{ m}}{(9.81\text{ m/s}^2)(20\text{ m})}\right)^{5.2}\right]^{0.04} \\ &= 0.271\text{ m} \end{aligned}$$

Discussion Note that the difference between the two results is less than 2 percent. Therefore, the simple Swamee–Jain relation can be used with confidence. Finally, the first (iterative) approach requires an initial guess for D. If we use the Swamee and Jain result as our initial guess, the diameter converges to D = 0.267 m in short order.

EXAMPLE 14–5 Determining the Flow Rate of Air in a Duct

Reconsider Example 14–4. Now the duct length is doubled while its diameter is maintained constant. If the total head loss is to remain constant, determine the drop in the flow rate through the duct.

SOLUTION The diameter and the head loss in an air duct are given. The flow rate is to be determined.

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and head loss. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

The average velocity, Reynolds number, friction factor, and the head loss relations can be expressed as (D is in m, and $\mathcal{V}$ is in m/s, Re and f are dimensionless)

$$\mathcal{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad\rightarrow\quad \mathcal{V} = \frac{\dot{V}}{\pi(0.267\text{ m})^2/4}$$

$$\text{Re} = \frac{\mathcal{V}D}{\upsilon} \quad\rightarrow\quad \text{Re} = \frac{\mathcal{V}(0.267\text{ m})}{1.655 \times 10^{-5}\text{ m}^2\text{/s}}$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad\rightarrow\quad \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{\mathcal{V}^2}{2g} \quad\rightarrow\quad 20 = f\frac{300\text{ m}}{0.267\text{ m}}\frac{\mathcal{V}^2}{2(9.81\text{ m/s}^2)}$$

This is a set of four equations in four unknowns, and solving them with an equation solver such as EES gives

$$\dot{V} = 0.24 \text{ m}^3/\text{s}, \qquad f = 0.0195, \qquad \mathcal{V} = 4.23 \text{ m/s}, \qquad \text{and} \qquad \text{Re} = 68{,}300$$

Then the drop in the flow rate becomes

$$\dot{V}_{\text{drop}} = \dot{V}_{\text{old}} - \dot{V}_{\text{new}} = 0.35 - 0.24 = \mathbf{0.11\ m^3/s} \qquad \text{(a drop of 31\%)}$$

Therefore, for a specified head loss (or available head or fan pumping power), the flow rate drops by about 31% from 0.35 to 0.24 m^3/s when the duct length doubles.

Alternative Solution If a computer is not available (as in an exam situation), another option is to set up a *manual iteration loop*. We have found that the best convergence is usually realized by first guessing the friction factor f, then solving for the velocity $\mathcal{V}$. The equation for $\mathcal{V}$ as a function of f is

Mean velocity through the pipe: $$\mathcal{V} = \sqrt{\frac{2gh_L}{fL/D}}$$

Now that $\mathcal{V}$ is known, the Reynolds number can be calculated, from which a *corrected* friction factor is obtained from the Moody chart or the Colebrook equation. We repeat the calculations with the corrected value of f until convergence. We guess $f = 0.04$ for illustration:

Iteration	f (guess)	$\mathcal{V}$ (m/s)	Re	Corrected f
1	0.04	2.955	4.724×10^4	0.0212
2	0.0212	4.059	6.489×10^4	0.01973
3	0.01973	4.207	6.727×10^4	0.01957
4	0.01957	4.224	6.754×10^4	0.01956
5	0.01956	4.225	6.756×10^4	0.01956

Notice that the iteration has converged to three digits in only three iterations, and to four digits in only five iterations. The final results are identical to those obtained with EES, yet do not require a computer.

Discussion The new flow rate can also be determined directly from the second Swamee–Jain formula to be

$$\dot{V} = -0.965\left(\frac{gD^5h_L}{L}\right)^{0.5} \ln\left[\frac{\varepsilon}{3.7\,D} + \left(\frac{3.17\nu^2 L}{gD^3h_L}\right)^{0.5}\right]$$

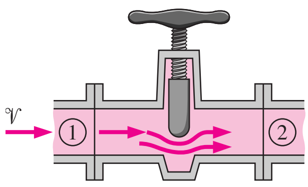
$$= -0.965\left(\frac{(9.81\text{ m/s}^2)(0.267\text{ m})^5(20\text{ m})}{300\text{ m}}\right)^{0.5} \ln\left[0 + \left(\frac{3.17(1.655\times10^{-5}\text{ m}^2/\text{s})^2(300\text{ m})}{(9.81\text{ m/s}^2)(0.267\text{ m})^3(20\text{ m})}\right)^{0.5}\right]$$

$$= 0.24\text{ m}^3/\text{s}$$

Note that the result from the Swamee–Jain relation is the same (to two significant digits) as that obtained with the Colebrook equation using EES or using our manual iteration technique. Therefore, the simple Swamee–Jain relation can be used with confidence.

14–6 ▪ MINOR LOSSES

The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions in addition to the pipes. These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce. In a typical system with long pipes, these losses are minor compared to the total head loss in the pipes (the *major losses*) and are called **minor losses.** Although this is generally true, in some cases the minor losses may be greater than the major losses. This is the case in systems with several turns and valves in a short distance. The head loss introduced by a completely open valve, for example, may be negligible. But a partially closed valve may cause the largest head loss in the system, as evidenced by the drop in the flow rate. Flow through valves and fittings is very complex, and a theoretical analysis is generally not plausible. Therefore, minor losses are determined experimentally, usually by the manufacturers of the components.



$$\Delta P_L = \Delta P = P_1 - P_2$$

$$K_L = \frac{\Delta P_L}{\frac{1}{2}\rho\mathcal{V}^2}$$

FIGURE 14–32
The loss coefficient of a component (such as the gate valve shown) is determined by measuring the pressure loss it causes and dividing it by the dynamic pressure in the pipe.

Minor losses are usually expressed in terms of the **loss coefficient** K_L, defined as (Fig. 14–32)

Loss coefficient:
$$K_L = \frac{h_L}{\mathcal{V}^2/(2g)} \tag{14–54}$$

When the inlet diameter equals outlet diameter, the loss coefficient of a component can also be determined by measuring the pressure loss across the component and dividing it by the dynamic pressure, $K_L = \Delta P_L/(0.5\,\rho\mathcal{V}^2)$. When the loss coefficient for a component is available, the head loss for that component is determined from

Minor loss:
$$h_L = K_L\frac{\mathcal{V}^2}{2g} \tag{14–55}$$

The loss coefficient, in general depends on the geometry of the component and the Reynolds number, just like the friction factor. However, it is usually assumed to be independent of the Reynolds number. This is a reasonable approximation since most flows in practice have large Reynolds numbers and the loss coefficients (including the friction factor) tend to be independent of the Reynolds number at large Reynolds numbers.

Minor losses are also expressed in terms of the **equivalent length** L_{equiv}, defined as (Fig. 14–33)

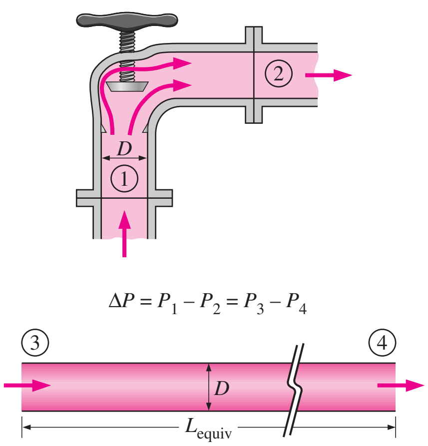


FIGURE 14–33
The head loss caused by a component (such as the angle valve shown) is equivalent to the head loss caused by a section of the pipe whose length is the equivalent length.

Equivalent length:
$$h_L = K_L\frac{\mathcal{V}^2}{2g} = f\frac{L_{equiv}}{D}\frac{\mathcal{V}^2}{2g} \quad \rightarrow \quad L_{equiv} = \frac{D}{f}K_L \tag{14–56}$$

where f is the friction factor and D is the diameter of the pipe that contains the component. The head loss caused by the component is equivalent to the head loss caused by a section of the pipe whose length is L_{equiv}. Therefore, the contribution of a component to the head loss can be accounted for by simply adding L_{equiv} to the total pipe length.

Both approaches are used in practice, but the use of loss coefficients is more common. Therefore, we will also use that approach in this book. Once all the loss coefficients are available, the total head loss in a piping system can be determined from

Total head loss (general):
$$h_{L,\text{ total}} = h_{L,\text{ major}} + h_{L,\text{ minor}} = \sum_i f_i \frac{L_i}{D_i}\frac{\mathcal{V}_i^2}{2g} + \sum_j K_{L,j}\frac{\mathcal{V}_j^2}{2g} \tag{14–57}$$

where i represents each pipe section with constant diameter and j represents each component that causes a minor loss. If the entire piping system being analyzed has a constant diameter, the last relation reduces to

Total head loss (D = constant):
$$h_{L,\text{ total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{\mathcal{V}^2}{2g} \tag{14–58}$$

where $\mathcal{V}$ is the average flow velocity through the entire system (note that $\mathcal{V}$ = constant since D = constant).

Representative loss coefficients K_L are given in Table 14–4 for inlets, exits, bends, sudden and gradual area changes, and valves. There is considerable uncertainty in these values since the loss coefficients, in general, vary with the pipe diameter, the surface roughness, the Reynolds number, and the details of the design. The loss coefficients of two seemingly identical valves by two different manufacturers, for example, can differ by a factor of 2 or more. Therefore, the particular manufacturer's data should be consulted in the final design of piping systems rather than relying on the representative values in handbooks.

The head loss at the inlet of a pipe is a strong function of geometry. It is almost negligible for well-rounded inlets ($K_L = 0.03$ for $r/D > 0.2$), but increases to about 0.50 for sharp-edged inlets (Fig. 14–34). That is, a sharp-edged inlet causes half of the velocity head to be lost as the fluid enters the pipe. This is because the fluid cannot make sharp 90° turns easily, especially at high velocities. As a result, the flow separates at the corners, and the flow is constricted into the *vena contracta* region formed in the midsection of the pipe (Fig. 14–35). Therefore, a sharp-edged inlet acts like a flow constriction. The velocity increases in the vena contracta region (and the pressure decreases) because of the reduced effective flow area, and then decreases as the flow fills the entire cross section of the pipe. There would be negligible loss if the pressure were increased in accordance with Bernoulli's equation (the velocity head would simply be converted into pressure head). However, this deceleration process is far from being ideal and the viscous dissipation caused by intense mixing and the turbulent eddies convert part of the kinetic energy into frictional heating, as evidenced by a slight rise in fluid temperature. The end result is a drop in velocity without much pressure recovery, and the inlet loss is a measure of this irreversible pressure drop.

Even slight rounding of the edges can result in significant reduction of K_L, as shown in Fig. 14–36. The loss coefficient rises sharply (to about $K_L = 0.8$) when the pipe protrudes into the reservoir since some fluid near the edge in this case is forced to make a 180° turn. The loss coefficient for a submerged exit is $K_L = 1$ (actually, K_L = the kinetic energy correction factor, which is nearly 1) since the fluid loses its entire kinetic energy and the velocity head through mixing and comes to rest when it discharges into a reservoir

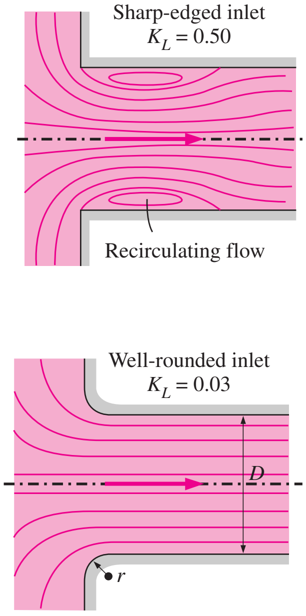


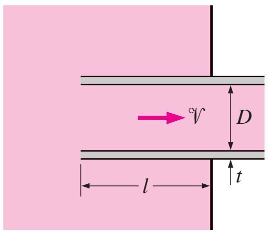
FIGURE 14–34
The head loss at the inlet of a pipe is almost negligible for well-rounded inlets ($K_L = 0.03$ for $r/D > 0.2$) but increases to about 0.50 for sharp-edged inlets.

TABLE 14–4

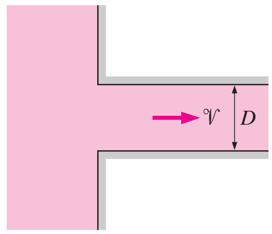
Loss coefficients K_L of various pipe components for turbulent flow
(for use in the relation $h_L = K_L \mathcal{V}^2/(2g)$ where $\mathcal{V}$ is the mean velocity in the pipe that contains the component)*

Pipe Entrance

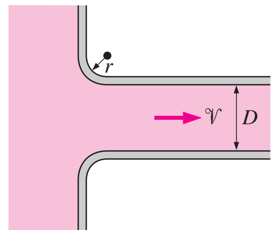
Reentrant: $K_L = 0.80$
($t \ll D$ and $l \sim 0.1D$)



Sharp-edged: $K_L = 0.50$

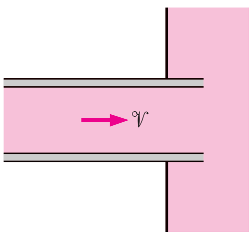


Well-rounded ($r/D > 0.2$): $K_L = 0.03$
Slightly rounded ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 14–36)

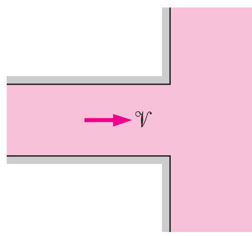


Pipe Exit

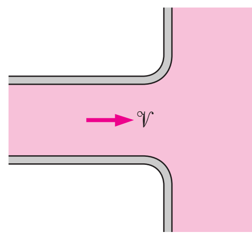
Reentrant: $K_L = 1.0$



Sharp-edged: $K_L = 1.0$

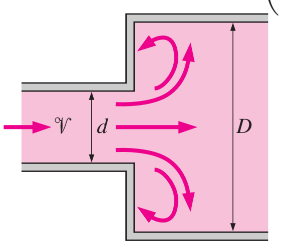


Rounded: $K_L = 1.0$

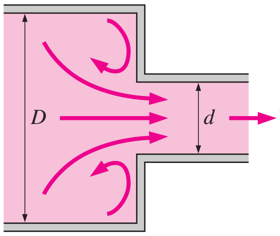


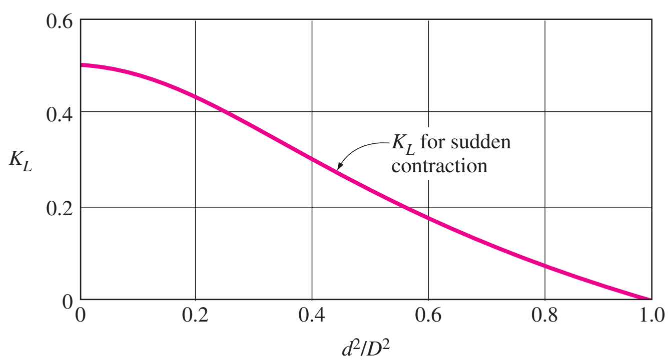
Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Sudden expansion: $K_L = \left(1 - \frac{d^2}{D^2}\right)^2$



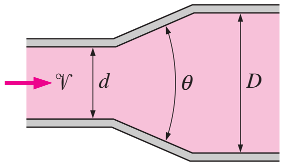
Sudden contraction: See chart.





Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion:
$K_L = 0.02$ for $\theta = 20°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$



Contraction (for $\theta = 20°$):
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

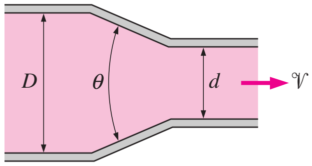
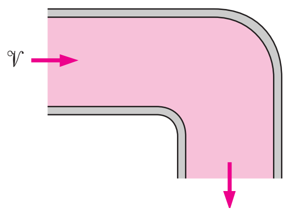
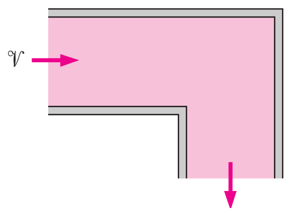
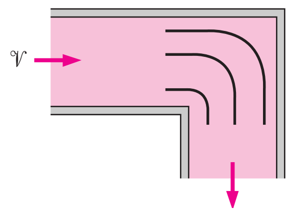
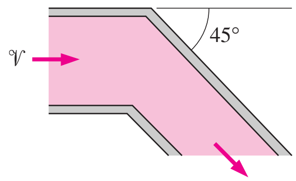
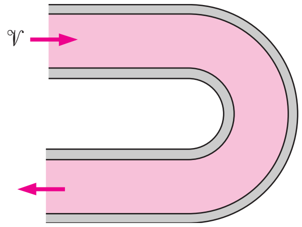
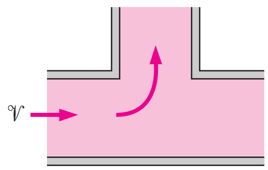
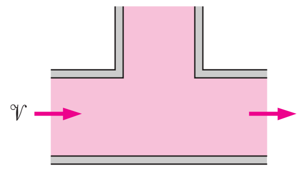
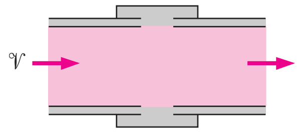


TABLE 14–4 *(Concluded)*

Bends and Branches *90° smooth bend:* Flanged: $K_L = 0.3$ Threaded: $K_L = 0.9$ 	*90° miter bend* (without vanes): $K_L = 1.1$ 	*90° miter bend* (with vanes): $K_L = 0.2$ 	*45° threaded elbow:* $K_L = 0.4$ 
180° return bend: Flanged: $K_L = 0.2$ Threaded: $K_L = 1.5$ 	*Tee* (branch flow): Flanged: $K_L = 1.0$ Threaded: $K_L = 2.0$ 	*Tee* (line flow): Flanged: $K_L = 0.2$ Threaded: $K_L = 0.9$ 	*Threaded union:* $K_L = 0.08$ 
Valves *Globe valve*, fully open: $K_L = 10$ *Angle valve*, fully open: $K_L = 5$ *Ball valve*, fully open: $K_L = 0.05$ *Swing check valve:* $K_L = 2$		*Gate valve*, fully open: $K_L = 0.2$ $\frac{1}{4}$ closed: $K_L = 0.3$ $\frac{1}{2}$ closed: $K_L = 2.1$ $\frac{3}{4}$ closed: $K_L = 17$	

*These are representative values for loss coefficients. Actual values strongly depend on the design and manufacture of the components and may differ from the given values considerably (especially for valves). Actual manufacturer's data should be used in the final design.

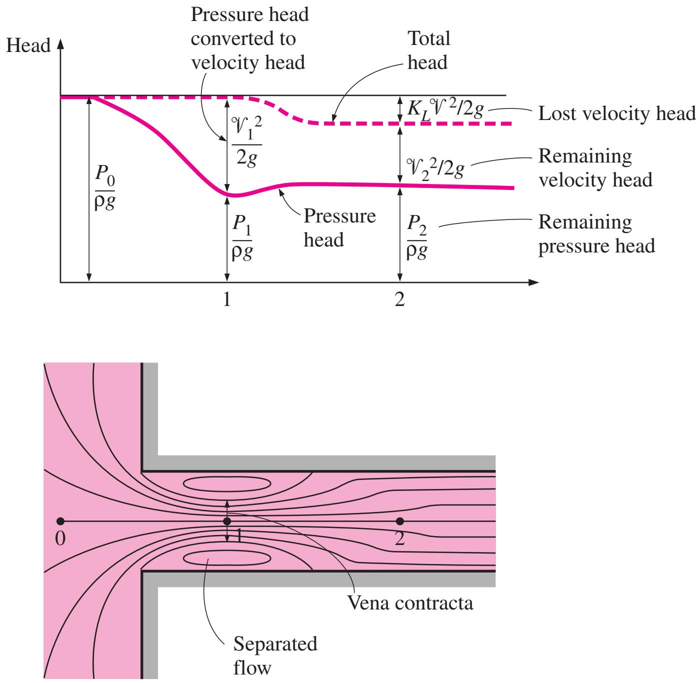


FIGURE 14–35
Graphical representation of flow contraction and the associated head loss at a sharp-edged pipe inlet.

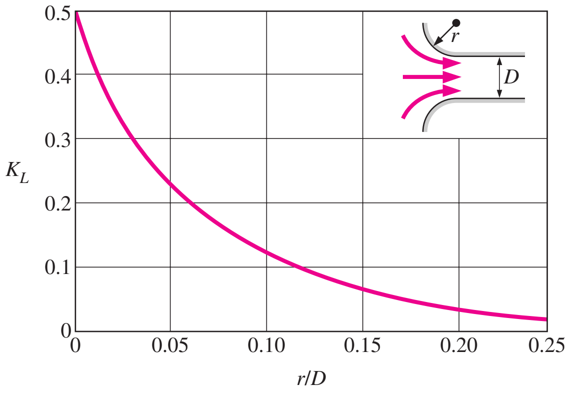


FIGURE 14–36
The effect of rounding of a pipe inlet on the loss coefficient (from *ASHRAE Handbook of Fundamentals*).

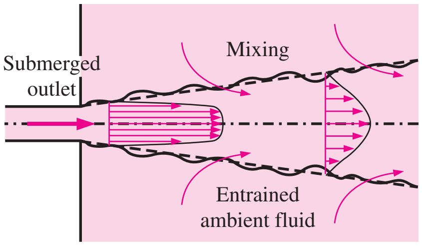


FIGURE 14–37
All of the kinetic energy of the flow is "lost" (turned into thermal energy) through friction as the jet decelerates and mixes with ambient fluid downstream of a submerged outlet.

regardless of the shape of the exit (Fig. 14–37). Therefore, there is no need to round the pipe exits.

Piping systems often involve *sudden* or *gradual* expansion or contraction sections to accommodate changes in flow rates or properties such as density and velocity. The losses are usually much greater in the case of *sudden* expansion and contraction (or wide-angle expansion) because of flow separation. By combining the conservation of mass, momentum, and energy equations, the loss coefficient for the case of **sudden expansion** is determined to be

$$K_L = \left(1 - \frac{A_{\text{small}}}{A_{\text{large}}}\right)^2 \qquad \text{(sudden expansion)} \tag{14–59}$$

where A_{small} and A_{large} are the cross-sectional areas of the small and large pipes, respectively. Note that $K_L = 0$ when there is no area change ($A_{\text{small}} = A_{\text{large}}$) and $K_L = 1$ when a pipe discharges into a reservoir ($A_{\text{large}} \gg A_{\text{small}}$), as expected. No such relation exists for a sudden contraction, and the K_L values in that case can be read from the chart in Table 14–4. The losses due to expansion and contraction can be reduced significantly by installing conical gradual area changers (nozzles and diffusers) between the small and large pipes. The K_L values for representative cases of gradual expansion and contraction are given in Table 14–4. Note that in head loss calculations, the velocity in the *small pipe* is to be used. Losses during expansion are usually much higher than the losses during contraction because of flow separation.

Piping systems also involve changes in direction without a change in diameter, and such flow sections are called *bends* or *elbows*. The losses in these devices are due to flow separation (just like a car being thrown off the road when it enters a turn too fast) on the inner side and the swirling secondary flows caused by different path lengths. The losses during changes of direction can be minimized by making the turn "easy" on the fluid by using circular arcs (like the 90° elbow) instead of sharp turns (like the miter bends) (Fig. 14–38). But the use of sharp turns (and thus suffering a penalty in loss coefficient) may be necessary when the turning space is limited. In such cases, the losses can be minimized by utilizing properly placed guide vanes to help the flow turn in an orderly manner without being thrown off the course. The loss coefficients for some elbows and miter bends as well as tees are given in Table 14–4. These coefficients do not include the frictional

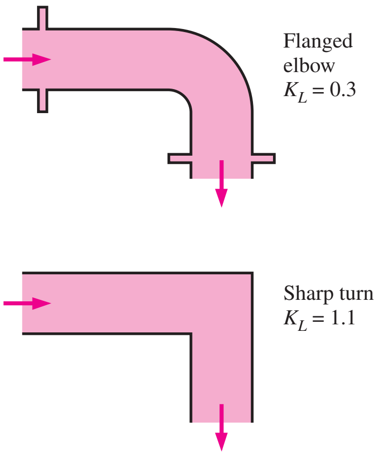


FIGURE 14–38
The losses during changes of direction can be minimized by making the turn "easy" on the fluid by using circular arcs instead of sharp turns.

losses along the pipe bend. Such losses should be calculated as in straight pipes (using the length of the centerline as the pipe length) and added to other losses.

Valves are commonly used in piping systems to control the flow rates by simply altering the head loss until the desired flow rate is achieved. For valves it is desirable to have a very low loss coefficient when they are fully open so that they cause minimal head loss during full-load operation. Several different valve designs, each with its own advantages and disadvantages, are in common use today. The *gate valve* slides up and down like a gate, the *globe valve* closes a hole placed in the valve, the *angle valve* is a globe valve with a 90° turn, and the *check valve* allows the fluid to flow only in one direction like a diode in an electric circuit. Table 14–4 lists the representative loss coefficients of the popular designs. Note that the loss coefficient increases drastically as a valve is closed (Fig. 14–39). Also, the deviation in the loss coefficients for different manufacturers is greatest for valves because of their complex geometries.

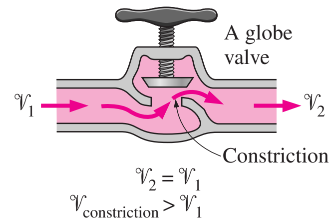


FIGURE 14–39
The large head loss in a partially closed valve is due to irreversible deceleration, flow separation, and mixing of high-velocity fluid coming from the narrow valve passage.

EXAMPLE 14–6 Head Loss and Pressure Rise during Gradual Expansion

A 6-cm-diameter horizontal water pipe expands gradually to a 9-cm-diameter pipe (Fig. 14–40). The walls of the expansion section are angled 30° from the horizontal. The mean velocity and pressure of water before the expansion section are 7 m/s and 150 kPa, respectively. Determine the head loss in the expansion section, and the pressure in the larger-diameter pipe.

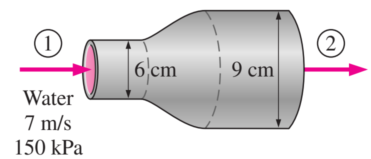


FIGURE 14–40
Schematic for Example 14–6.

SOLUTION A horizontal water pipe expands gradually into a larger-diameter pipe. The head loss and pressure after the expansion are to be determined.
Assumptions The flow is steady and incompressible.
Properties We take the density of water to be $\rho = 1000$ kg/m³. The loss coefficient for gradual expansion of $\theta = 60°$ total included angle is $K_L = 0.07$ (Table 14–4).
Analysis Noting that the density of water remains constant, the downstream velocity of water is determined from conservation of mass to be

$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho\mathcal{V}_1 A_1 = \rho\mathcal{V}_2 A_2 \quad \rightarrow \quad \mathcal{V}_2 = \frac{A_1}{A_2}\mathcal{V}_1 = \frac{D_1^2}{D_2^2}\mathcal{V}_1$$

$$\mathcal{V}_2 = \frac{(0.06\text{ m})^2}{(0.09\text{ m})^2}(7\text{ m/s}) = 3.11\text{ m/s}$$

Then the head loss in the expansion section becomes

$$h_L = K_L\frac{\mathcal{V}_1^2}{2g} = (0.07)\frac{(7\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{0.175\text{ m}}$$

Noting that $z_1 = z_2$ and there are no pumps or turbines involved, the energy equation for the expansion section can be expressed in terms of heads as

$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} + \cancel{z_1} + \cancelto{0}{h_{\text{pump}}} = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + \cancel{z_2} + \cancelto{0}{h_{\text{turbine}}} + h_L \rightarrow \frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + h_L$$

Solving for P_2 and substituting,

$$P_2 = P_1 + \rho\left\{\frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2} - gh_L\right\} = (150\text{ kPa}) + (1000\text{ kg/m}^3)$$

$$\times\left\{\frac{(7\text{ m/s})^2 - (3.11\text{ m/s})^2}{2} - (9.81\text{ m/s}^2)(0.175\text{ m})\right\}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)$$

$$= \mathbf{168\text{ kPa}}$$

Therefore, despite the head (and pressure) loss, the pressure increases from 150 kPa to 168 kPa after the expansion. This is due to the conversion of dynamic pressure to static pressure when the mean flow velocity is decreased in the larger pipe.

Discussion It is common knowledge that higher pressure upstream is necessary to cause flow, and it may come as a surprise to you that the downstream pressure has *increased* after the expansion, despite the loss. This is because the flow is driven by the sum of the three heads that comprise the total head (namely, the pressure head, velocity head, and elevation head). During flow expansion, the higher velocity head upstream is converted to pressure head downstream, and this increase outweighs the non-recoverable head loss. Also, you may be tempted to solve this problem using the Bernoulli equation. Such a solution would ignore the head (and the associated pressure) loss, and result in a higher pressure for the fluid downstream.

FIGURE 14–41
A piping network in an industrial facility. (Courtesy of UMDE Engineering, Contracting, and Trading.)

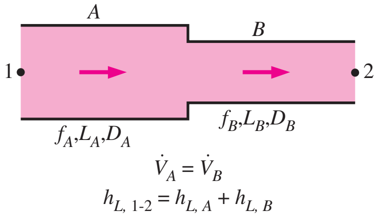


FIGURE 14–42
For pipes *in series,* the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in individual pipes.

14–7 ▪ PIPING NETWORKS AND PUMP SELECTION

Most piping systems encountered in practice such as the water distribution systems in cities or commercial or residential establishments involve numerous parallel and series connections as well as several sources (supply of fluid into the system) and loads (discharges of fluid from the system) (Fig. 14–41). A piping project may involve the design of a new system or the expansion of an existing system. The engineering objective in such projects is to design a piping system that will deliver the specified flow rates at specified pressures reliably at minimum total (initial plus operating and maintenance) cost. Once the layout of the system is prepared, the determination of the pipe diameters and the pressures throughout the system, while remaining within the budget constraints, typically requires solving the system repeatedly until the optimal solution is reached. Computer modeling and analysis of such systems make this tedious task a simple chore.

Piping systems typically involve several pipes connected to each other in series or in parallel, as shown in Figs. 14–42 and 14–43. When the pipes are connected **in series,** the flow rate through the entire system remains constant regardless of the diameters of the individual pipes in the system. This is a natural consequence of the conservation of mass principle for steady incompressible flow. The total head loss in this case is equal to the sum of the head losses in individual pipes in the system, including the minor losses. The expansion or contraction losses at connections are considered to belong to the smaller-diameter pipe since the expansion and contraction loss coefficients are defined on the basis of the mean velocity in the smaller-diameter pipe.

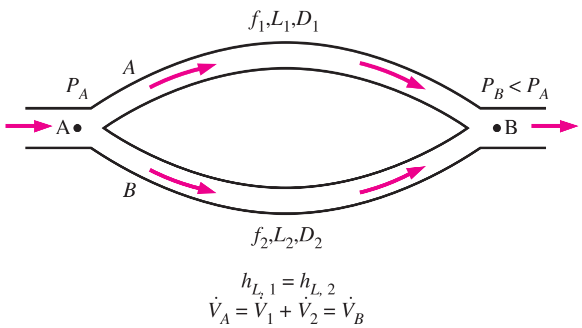


FIGURE 14–43
For pipes *in parallel,* the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.

For a pipe that branches out into two (or more) **parallel pipes** and then rejoins at a junction downstream, the total flow rate is the sum of the flow rates in the individual pipes. The pressure drop (or head loss) in each individual pipe connected in parallel must be the same since $\Delta P = P_A - P_B$ and the junction pressures P_A and P_B are the same for all of the individual pipes. For a system of two parallel pipes 1 and 2 between junctions A and B, this can be expressed as

$$h_{L,1} = h_{L,2} \qquad \rightarrow \qquad f_1 \frac{L_1}{D_1} \frac{\mathcal{V}_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{\mathcal{V}_2^2}{2g}$$

Then the ratio of the mean velocities and the flow rates in the two parallel pipes become

$$\frac{\mathcal{V}_1}{\mathcal{V}_2} = \left(\frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2}\right)^{0.5} \qquad \text{and} \qquad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1}\,\mathcal{V}_1}{A_{c,2}\,\mathcal{V}_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2}\right)^{0.5}$$

Therefore, the relative flow rates in parallel pipes are established from the requirement that the head loss in each pipe be the same. This result can be extended to any number of pipes connected in parallel. The result is also valid for pipes for which the minor losses are significant if the equivalent lengths for components that contribute to minor losses are added to the pipe length. Note that the flow rate in one of the parallel branches is proportional to the 2.5th power of the diameter and is inversely proportional to the square root of its length and friction factor.

The analysis of piping networks, no matter how complex they are, is based on two simple principles:

1. *Conservation of mass throughout the system must be satisfied.* This is done by requiring the total flow into a junction to be equal to the total flow out of the junction for all junctions in the system. Also, the flow rate must remain constant in pipes connected in series regardless of the changes in diameters.
2. *Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions.* This is because pressure is a point function and it cannot have two values at a specified point. In practice this rule is used by requiring that the algebraic sum of head losses in a loop (for all loops) be equal to zero. (A head loss is taken to be positive for flow in the clockwise direction and negative for flow in the counterclockwise direction.)

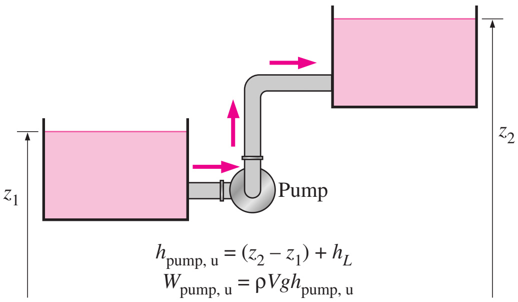


FIGURE 14–44
When a pump moves a fluid from one reservoir to another, the useful pump head requirement is equal to the elevation difference between the two reservoirs plus the head loss.

Therefore, the analysis of piping networks is very similar to the analysis of electric circuits, with flow rate corresponding to electric current and pressure corresponding to electric potential. However, the situation is much more complex here since, unlike the electric resistance, the "flow resistance" is a highly nonlinear function. Therefore, the analysis of piping networks requires the solution of a system of nonlinear equations simultaneously. The analysis of such systems is beyond the scope of this introductory text.

Energy Equation Revisited

When a piping system involves a pump and/or turbine, the steady-flow energy equation on a unit mass basis can be expressed as (see Section 12–4)

$$\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2} + gz_2 + w_{\text{turbine}} + gh_L \qquad \textbf{(14–60)}$$

It can also be expressed in terms of heads as

$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \qquad \textbf{(14–61)}$$

where $h_{\text{pump, u}} = w_{\text{pump, u}}/g$ is the useful pump head delivered to the fluid, $h_{\text{turbine, e}} = w_{\text{turbine, e}}/g$ is the turbine head extracted from the fluid,, and h_L is the total head loss in piping (including the minor losses if they are significant) between points 1 and 2. The pump head is zero if the piping system does not involve a pump or a fan, the turbine head is zero if the system does not involve a turbine, and both are zero if the system does not involve any mechanical work-producing or work-consuming devices.

Many practical piping systems involve a pump to move a fluid from one reservoir to another. Taking points 1 and 2 to be at the *free surfaces* of the reservoirs, the energy equation in this case reduces for the useful pump head required to (Fig. 14–44)

$$h_{\text{pump, u}} = (z_2 - z_1) + h_L \qquad \textbf{(14–62)}$$

since the velocities at free surfaces are negligible and the pressures are atmospheric pressure. Therefore, the useful pump head is equal to the elevation difference between the two reservoirs plus the head loss. If the head loss is negligible compared to $z_2 - z_1$, the useful pump head is simply equal to the elevation difference between the two reservoirs. In the case of $z_1 > z_2$ (the first reservoir being at a higher elevation than the second one) with no pump, the flow is driven by gravity at a flow rate that causes a head loss equal to the elevation difference. A similar argument can be given for the turbine head for a hydroelectric power plant by replacing $h_{\text{pump, u}}$ in Eq. 14–62 by $-h_{\text{turbine, e}}$.

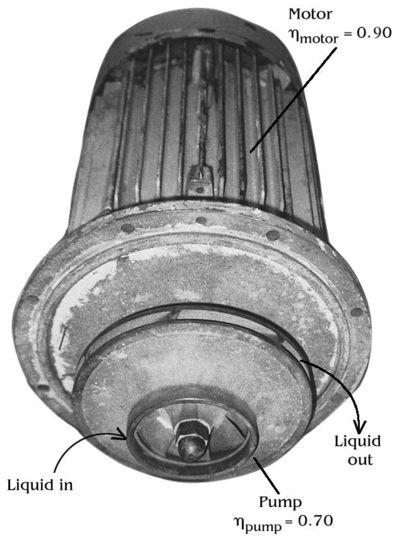
Once the useful pump head is known, the *mechanical power that needs to be delivered by the pump to the fluid* and the *electric power consumed by the motor of the pump* for a specified flow rate are determined from

$$\dot{W}_{\text{pump, shaft}} = \frac{\rho \dot{V} g h_{\text{pump, }u}}{\eta_{\text{pump}}} \quad \text{and} \quad \dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump, }u}}{\eta_{\text{pump}-\text{motor}}} \qquad \text{(14–63)}$$

where $\eta_{\text{pump-motor}}$ is the *efficiency of the pump-motor combination,* which is the product of the pump and the motor efficiencies (Fig. 14–45). The pump-motor efficiency is defined as the ratio of the net mechanical energy delivered to the fluid by the pump to the electric energy consumed by the motor of the pump, and it usually ranges between 50 and 85 percent.

The head loss of a piping system increases (usually quadratically) with the flow rate. A plot of required useful pump head $h_{\text{pump, u}}$ as a function of flow rate is called the **system** (or **demand**) **curve.** The head produced by a pump is not a constant either. Both the pump head and the pump efficiency vary with the flow rate, and pump manufacturers supply this variation in tabular or graphical form, as shown in Fig. 14–46. These experimentally determined $h_{\text{pump, u}}$ and $\eta_{\text{pump, u}}$ versus $\dot{V}$ curves are called **characteristic** (or **supply**) **curves.** Note that the flow rate of a pump increases as the required head decreases. The intersection point of the pump head curve with the vertical axis represents the *maximum head* the pump can provide, while the intersection point with the horizontal axis indicates the *maximum flow rate* the pump can supply.

The *efficiency* of a pump is sufficiently high for a certain range of head and flow rate combination. Therefore, a pump that can supply the required head and flow rate is not necessarily a good choice for a piping system unless the efficiency of the pump at those conditions is sufficiently high. The pump installed in a piping system will operate at the point where the *system curve* and the *characteristic curve* intersect. This point of intersection is called the **operating point,** as shown in Fig. 14–46. The useful head produced by the pump at this point matches the head requirements of the system at that flow rate.



$$\eta_{\text{pump-motor}} = \eta_{\text{pump}}\eta_{\text{motor}} = 0.70 \times 0.90 = 0.63$$

FIGURE 14–45
The efficiency of the pump-motor combination is the product of the pump and the motor efficiencies.
(© Yunus Çengel)

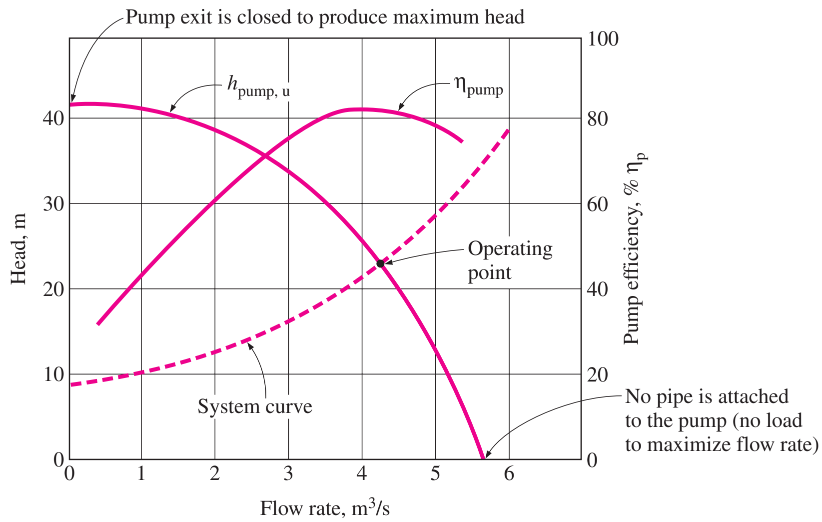


FIGURE 14–46
Characteristic pump curves for centrifugal pumps, the system curve for a piping system, and the operating point.

Also, the efficiency of the pump during operation is the value corresponding to that flow rate.

EXAMPLE 14–7 Pumping Water through Two Parallel Pipes

Water at 20°C is to be pumped from a reservoir (z_A = 5 m) to another reservoir at a higher elevation (z_B = 13 m) through two 36-m-long pipes connected in parallel, as shown in Fig. 14–47. The pipes are made of commercial steel, and the diameters of the two pipes are 4 cm and 8 cm. Water is to be pumped by a 70 percent efficient motor-pump combination that draws 8 kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rate through each of the parallel pipes.

SOLUTION The pumping power input to a piping system with two parallel pipes is given. The flow rates are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** The minor losses and the head loss in pipes other than the parallel pipes are said to be negligible. **5** Flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and μ = 1.002 × 10^{-3} kg/m · s (Table A–15). The roughness of commercial steel pipe is ε = 0.000045 m (Table 14–2 or Fig. A-32).

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Therefore, we would normally use a trial-and-error approach here. However, nowadays equation solvers such as EES are widely available, and thus below we will simply set up the equations to be solved by an equation solver. The useful head supplied by the pump to the fluid is determined from

$$\dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump, u}}}{\eta_{\text{pump-motor}}} \quad \rightarrow \quad 8000\text{ W} = \frac{(998\text{ kg/m}^3)\dot{V}(9.81\text{ m/s}^2)h_{\text{pump, u}}}{0.70} \qquad \textbf{(1)}$$

We choose points A and B at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_A = P_B = P_{\text{atm}}$)

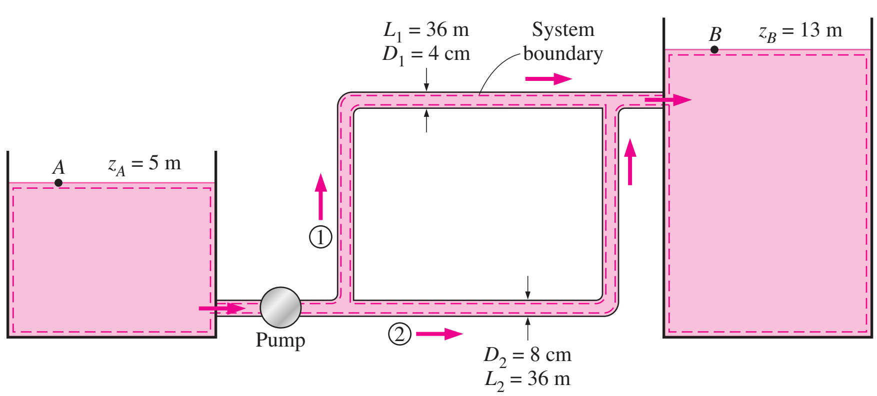


FIGURE 14–47
The piping system discussed in Example 14–7.

and that the fluid velocities at both points are zero ($\mathcal{V}_A = \mathcal{V}_B = 0$), the energy equation between these two points simplifies to

$$\cancel{\frac{P_A}{\rho g}} + \cancelto{0}{\frac{\mathcal{V}_A^2}{2g}} + z_A + h_{\text{pump, u}} = \cancel{\frac{P_B}{\rho g}} + \cancelto{0}{\frac{\mathcal{V}_B^2}{2g}} + z_B + h_L \quad \rightarrow \quad h_{\text{pump, u}} = (z_B - z_A) + h_L$$

or

$$h_{\text{pump, u}} = (13 - 5) + h_L \tag{2}$$

where

$$h_L = h_{L,1} = h_{L,2} \qquad \textbf{(3)(4)}$$

We designate the 4-cm-diameter pipe by 1 and the 8-cm-diameter pipe by 2. The average velocity, the Reynolds number, the friction factor, and the head loss in each pipe are expressed as

$$\mathcal{V}_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad \mathcal{V}_1 = \frac{\dot{V}_1}{\pi(0.04\text{ m})^2/4} \tag{5}$$

$$\mathcal{V}_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad \mathcal{V}_2 = \frac{\dot{V}_2}{\pi(0.08\text{ m})^2/4} \tag{6}$$

$$\text{Re}_1 = \frac{\rho \mathcal{V}_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(998\text{ kg/m}^3)\mathcal{V}_1(0.04\text{ m})}{1.002 \times 10^{-3}\text{ kg/m}\cdot\text{s}} \tag{7}$$

$$\text{Re}_2 = \frac{\rho \mathcal{V}_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(998\text{ kg/m}^3)\mathcal{V}_2(0.08\text{ m})}{1.002 \times 10^{-3}\text{ kg/m}\cdot\text{s}} \tag{8}$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{0.000045}{3.7 \times 0.04} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \tag{9}$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{0.000045}{3.7 \times 0.08} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \tag{10}$$

$$h_{L,1} = f_1\frac{L_1}{D_1}\frac{\mathcal{V}_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1\frac{36\text{ m}}{0.04\text{ m}}\frac{\mathcal{V}_1^2}{2(9.81\text{ m/s}^2)} \tag{11}$$

$$h_{L,2} = f_2\frac{L_2}{D_2}\frac{\mathcal{V}_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2\frac{36\text{ m}}{0.08\text{ m}}\frac{\mathcal{V}_2^2}{2(9.81\text{ m/s}^2)} \tag{12}$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \tag{13}$$

This is a system of 13 equations in 13 unknowns, and their simultaneous solution by an equation solver gives

$\dot{V} = \mathbf{0.0300\ m^3/s},\quad \dot{V}_1 = \mathbf{0.00415\ m^3/s},\quad \dot{V}_2 = \mathbf{0.0259\ m^3/s}$

$\mathcal{V}_1 = 3.30\text{ m/s},\quad \mathcal{V}_2 = 5.15\text{ m/s},\quad h_L = h_{L,1} = h_{L,2} = 11.1\text{ m},\quad h_{\text{pump}} = 19.1\text{ m}$

$\text{Re}_1 = 131{,}600,\quad \text{Re}_2 = 410{,}000,\quad f_1 = 0.0221,\quad f_2 = 0.0182$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion The two parallel pipes are identical, except the diameter of the first pipe is half the diameter of the second one. But only 14 percent of the water flows through the first pipe. This shows the strong dependence of the flow rate (and the head loss) on diameter. Also, it can be shown that if the free surfaces of the two reservoirs were at the same elevation (and thus $z_A = z_B$), the flow rate would increase by 20 percent from 0.0300 to 0.0361 m^3/s. Alternately, if the reservoirs were as given but the irreversible head losses were negligible, the flow rate would become 0.0715 m^3/s (an increase of 138 percent).

EXAMPLE 14–8 Gravity-Driven Water Flow in a Pipe

Water at 10°C flows from a large reservoir to a smaller one through a 5-cm-diameter cast iron piping system, as shown in Fig. 14–48. Determine the elevation z_1 for a flow rate of 6 L/s.

SOLUTION The flow rate through a piping system connecting two reservoirs is given. The elevation of the source is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The elevations of the reservoirs remain constant. **3** There are no pumps or turbines in the line.

Properties The density and dynamic viscosity of water at 10°C are $\rho = 999.7$ kg/m^3 and $\mu = 1.307 \times 10^{-3}$ kg/m · s (Table A-15). The roughness of cast iron pipe is $\varepsilon = 0.00026$ m (Fig. A-32).

Analysis The piping system involves 89 m of piping, a sharp-edged entrance ($K_L = 0.5$), two standard flanged elbows ($K_L = 0.3$ each), a fully open gate valve ($K_L = 0.2$), and a submerged exit ($K_L = 1.0$). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are zero ($\mathcal{V}_1 = \mathcal{V}_2 = 0$), the energy equation for a control volume between these two points simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \cancelto{0}{\frac{\mathcal{V}_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \cancelto{0}{\frac{\mathcal{V}_2^2}{2g}} + z_2 + h_L \quad \rightarrow \quad z_1 = z_2 + h_L$$

where

$$h_L = h_{L,\text{ total}} = h_{L,\text{ major}} + h_{L,\text{ minor}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{\mathcal{V}^2}{2g}$$

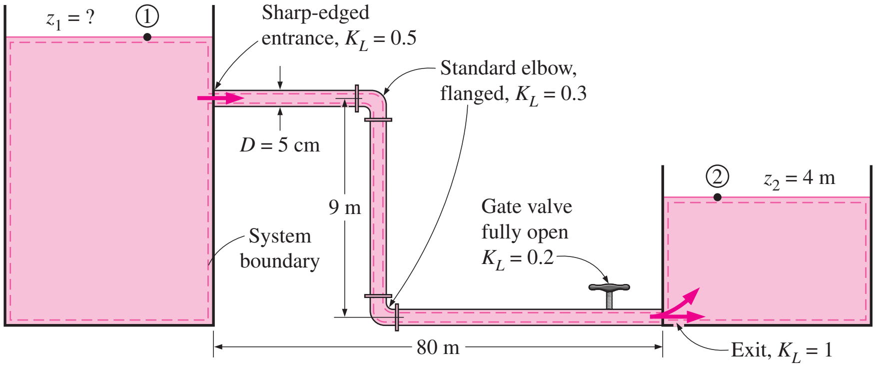


FIGURE 14–48
The piping system discussed in Example 14–8.

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$\mathcal{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006 \text{ m}^3/\text{s}}{\pi(0.05 \text{ m})^2/4} = 3.06 \text{ m/s}$$

$$\text{Re} = \frac{\rho \mathcal{V} D}{\mu} = \frac{(999.7 \text{ kg/m}^3)(3.06 \text{ m/s})(0.05 \text{ m})}{1.307 \times 10^{-3} \text{ kg/m} \cdot \text{s}} = 117{,}000$$

The flow is turbulent since Re > 4000. Noting that $\varepsilon/D = 0.00026/0.05 = 0.0052$, the friction factor can be determined from the Colebrook equation (or the Moody chart),

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0.0052}{3.7} + \frac{2.51}{117{,}000\sqrt{f}}\right)$$

It gives $f = 0.0315$. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{ entrance}} + 2K_{L,\text{ elbow}} + K_{L,\text{ valve}} + K_{L,\text{ exit}} = 0.5 + 2 \times 0.3 + 0.2 + 1.0 = 2.3$$

Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{\mathcal{V}^2}{2g} = \left(0.0315\frac{89 \text{ m}}{0.05 \text{ m}} + 2.3\right)\frac{(3.06 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 27.9 \text{ m}$$

$$z_1 = z_2 + h_L = 4 + 27.9 = \mathbf{31.9 \text{ m}}$$

Therefore, the free surface of the first reservoir must be 31.9 m above the ground level to ensure water flow between the two reservoirs at the specified rate.

Discussion Note that $fL/D = 56.1$ in this case, which is about 24 times the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in about 4 percent error.

It can be shown that the total head loss would be 35.9 m (instead of 27.9 m) if the valve were three-fourths closed, and it would drop to 24.8 m if the pipe between the two reservoirs were straight at the ground level (thus eliminating the elbows and the vertical section of the pipe). The head loss could be reduced further (from 24.8 to 24.6 m) by rounding the entrance. The head loss can be reduced from 27.9 to 16.0 m by replacing the cast iron pipes by smooth pipes such as those made of plastic.

EXAMPLE 14–9 Effect of Flushing on Flow Rate from a Shower

The bathroom plumbing of a building consists of 1.5-cm-diameter copper pipes with threaded connectors, as shown in Fig. 14–49. (*a*) If the gage pressure at the inlet of the system is 200 kPa during a shower and the toilet reservoir is full, determine the flow rate of water through the shower head. (*b*) Determine the effect of flushing of the toilet on the flow rate through the shower head. Take the loss coefficients of the shower head and the reservoir to be 12 and 14, respectively.

SOLUTION The plumbing system of a bathroom is given. The flow rate through the shower and the effect of flushing the toilet on the flow rate are to be determined.

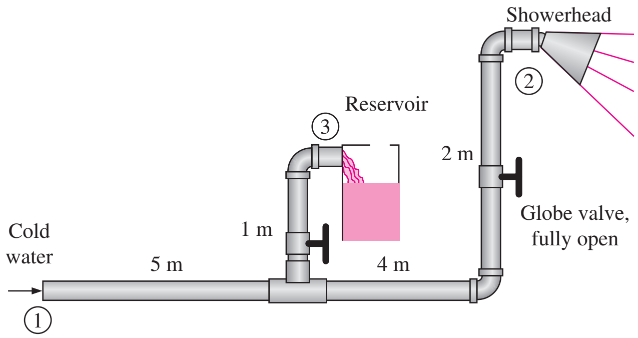


FIGURE 14–49
Schematic for Example 14–9.

Assumptions **1** The flow is steady and incompressible. **2** The flow is turbulent and fully developed. **3** The reservoir is open to the atmosphere. **4** The velocity heads are negligible.

Properties The properties of water at 20°C are $\rho = 998\ \text{kg/m}^3$, $\mu = 1.002 \times 10^{-3}\ \text{kg/m} \cdot \text{s}$, and $\nu = \mu/\rho = 1.004 \times 10^{-6}\ \text{m}^2/\text{s}$ (Table A–15). The roughness of copper pipes is $\varepsilon = 1.5 \times 10^{-6}$ m (Fig. A–32).

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and pressure drop. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

(*a*) The piping system of the shower alone involves 11 m of piping, a tee with line flow ($K_L = 0.9$), two standard elbows ($K_L = 0.9$ each), a fully open globe valve ($K_L = 10$), and a shower head ($K_L = 12$). Therefore, $\Sigma K_L = 0.9 + 2 \times 0.9 + 10 + 12 = 24.7$. Noting that the shower head is open to the atmosphere, and the velocity heads are negligible, the energy equation between points 1 and 2 simplifies to

$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \rightarrow \frac{P_{1,g}}{\rho g} = (z_2 - z_1) + h_L$$

Therefore, the head loss is

$$h_L = \frac{200{,}000\ \text{N/m}^2}{(998\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} - 2\ \text{m} = 18.4\ \text{m}$$

Also,

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{\mathcal{V}^2}{2g} \quad \rightarrow \quad h_L = \left(f\frac{11\ \text{m}}{0.015\ \text{m}} + 24.7\right)\frac{\mathcal{V}^2}{2(9.81\ \text{m/s}^2)}$$

since the diameter of the piping system is constant. The average velocity in the pipe, the Reynolds number, and the friction factor are

$$\mathcal{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad \mathcal{V} = \frac{\dot{V}}{\pi(0.015\ \text{m})^2/4}$$

$$\text{Re} = \frac{\mathcal{V}D}{\upsilon} \quad \rightarrow \quad \text{Re} = \frac{\mathcal{V}(0.015\ \text{m})}{1.004 \times 10^{-6}\ \text{m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

This is a set of four equations with four unknowns, and solving them with an equation solver such as EES gives

$$\dot{V} = 0.00053 \text{ m}^3/\text{s}, \quad f = 0.0218, \quad \mathcal{V} = 2.98 \text{ m/s}, \quad \text{and} \quad \text{Re} = 44{,}550$$

Therefore, the flow rate of water through the showerhead is **0.53 L/s**.

(*b*) When the toilet is flushed, the float moves and opens the valve. The discharged water starts to refill the reservoir, resulting in parallel flow after the tee connection. The head loss and minor loss coefficient for the shower branch were determined in (*a*) to be $h_{L,\,2} = 18.4$ m and $K_{L,\,2} = 24.7$ The corresponding quantities for the reservoir branch can be determined similarly to be

$$h_{L,\,3} = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 1 \text{ m} = 19.4 \text{ m}$$

$$K_{L,\,3} = 2 + 10 + 0.9 + 14 = 26.9$$

The relevant equations in this case are:

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$h_{L,\,2} = f_1 \frac{5 \text{ m}}{0.015 \text{ m}} \frac{\mathcal{V}_1^2}{2(9.81 \text{ m/s}^2)} + \left(f_2 \frac{6 \text{ m}}{0.015 \text{ m}} + 24.7\right) \frac{\mathcal{V}_2^2}{2(9.81 \text{ m/s}^2)} = 18.4$$

$$h_{L,\,3} = f_1 \frac{5 \text{ m}}{0.015 \text{ m}} \frac{\mathcal{V}_1^2}{2(9.81 \text{ m/s}^2)} + \left(f_3 \frac{1 \text{ m}}{0.015 \text{ m}} + 26.9\right) \frac{\mathcal{V}_3^2}{2(9.81 \text{ m/s}^2)} = 19.4$$

$$\mathcal{V}_1 = \frac{\dot{V}_1}{\pi(0.015 \text{ m})^2/4}, \quad \mathcal{V}_2 = \frac{\dot{V}_2}{\pi(0.015 \text{ m})^2/4}, \quad \mathcal{V}_3 = \frac{\dot{V}_3}{\pi(0.015 \text{ m})^2/4}$$

$$\text{Re}_1 = \frac{\mathcal{V}_1(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}, \quad \text{Re}_2 = \frac{\mathcal{V}_2(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}, \quad \text{Re}_3 = \frac{\mathcal{V}_3(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right),$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}_3\sqrt{f_3}}\right)$$

Solving these 12 equations in 12 unknowns simultaneously using an equation solver, the flow rates are determined to be

$$\dot{V}_1 = 0.00090 \text{ m}^3/\text{s}, \quad \dot{V}_2 = 0.00042 \text{ m}^3/\text{s}, \quad \text{and} \quad \dot{V}_3 = 0.00048 \text{ m}^3/\text{s}$$

Therefore, the flushing of the toilet reduces the flow rate through the shower by 21% from 0.53 L/s to 0.42 L/s (Fig. 14–50).

Discussion If the velocity heads were considered, the flow rate through the shower would be 0.43 L/s instead of 0.42 L/s. Therefore, the assumption of negligible velocity heads is reasonable in this case.

Note that a leak in a piping system will cause the same effect, and thus an unexplained drop in flow rate at an end point may signal a leak in the system.

FIGURE 14–50
Flow rate through a shower may be affected significantly by the flushing of a nearby toilet.

SUMMARY

In *internal flow*, a pipe is completely filled with a fluid. *Laminar flow* is characterized by smooth streamlines and highly ordered motion, and *turbulent flow* is characterized by velocity fluctuations and highly disordered motion. The *Reynolds number* is defined as

$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{\mathcal{V}_m D}{\upsilon} = \frac{\rho \mathcal{V}_m D}{\mu}$$

Under most practical conditions, the flow in a pipe is laminar at Re $<$ 2300, turbulent at Re $>$ 4000, and transitional in between.

The region of the flow in which the effects of the viscous shearing forces are felt is called the *velocity boundary layer*. The region from the pipe inlet to the point at which the boundary layer merges at the centerline is called the *hydrodynamic entrance region*, and the length of this region is called the *hydrodynamic entry length* L_h. It is given by

$$L_{h,\,\text{laminar}} \approx 0.05\ \text{Re}\ D \quad \text{and} \quad L_{h,\,\text{turbulent}} \approx 10\ D$$

The friction coefficient in the fully developed flow region remains constant. The *maximum* and *mean* velocities in fully developed laminar flow in a circular pipe are

$$u_{\max} = 2\mathcal{V}_m \qquad \text{and} \qquad \mathcal{V}_m = \frac{\Delta P D^2}{32\mu L}$$

The *volume flow rate* and the *pressure drop* for laminar flow in a horizontal pipe are

$$\dot{V} = \mathcal{V}_m A_c = \frac{\Delta P \pi D^4}{128\mu L} \qquad \text{and} \qquad \Delta P = \frac{32\mu L \mathcal{V}_m}{D^2}$$

The above results for horizontal pipes can also be used for inclined pipes provided that ΔP is replaced by $\Delta P - \rho g L \sin\theta$,

$$\mathcal{V}_m = \frac{(\Delta P - \rho g L \sin\theta)D^2}{32\mu L} \quad \text{and} \quad \dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

The *pressure loss* and *head loss* for all types of internal flows (laminar or turbulent, in circular or noncircular pipes, smooth or rough surfaces) are expressed as

$$\Delta P_L = f\frac{L}{D}\frac{\rho \mathcal{V}_m^2}{2} \qquad \text{and} \qquad h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{\mathcal{V}_m^2}{2g}$$

where $\rho \mathcal{V}_m^2/2$ is the *dynamic pressure* and the dimensionless quantity f is the *friction factor*. For fully developed laminar flow in a circular pipe, the friction factor is $f = 64/\text{Re}$.

For non-circular pipes, the diameter in the above relations is replaced by the *hydraulic diameter* defined as $D_h = 4A_c/p$, where A_c is the cross-sectional area of the pipe and p is its perimeter.

In fully developed turbulent flow, the friction factor depends on the Reynolds number and the *relative roughness* ε/D. The friction factor in turbulent flow is given by the *Colebrook equation*, expressed as

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

The plot of this formula is known as the Moody chart. The design and analysis of piping systems involve the determination of the head loss, flow rate, or the pipe diameter. Tedious iterations in these calculations can be avoided by the approximate Swamee and Jain formulas expressed as

$$h_L = 1.07\frac{\dot{V}^2 L}{gD^5}\left\{\ln\left[\frac{\varepsilon}{3.7\,D} + 4.62\left(\frac{\upsilon D}{\dot{V}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{matrix} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \text{Re} < 3 \times 10^8 \end{matrix}$$

$$\dot{V} = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5} \ln\left[\frac{\varepsilon}{3.7\,D} + \left(\frac{3.17\upsilon^2 L}{gD^3 h_L}\right)^{0.5}\right] \qquad \text{Re} > 2000$$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \upsilon \dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{matrix} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \text{Re} < 3 \times 10^8 \end{matrix}$$

The losses that occur in the piping components such as the fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions are called *minor losses*. The minor losses are usually expressed in terms of the *loss coefficient* K_L. The head loss for a component is determined from

$$h_L = K_L \frac{\mathcal{V}^2}{2g}$$

When all the loss coefficients are available, the total head loss in a piping system is determined from

$$h_{L,\,\text{total}} = h_{L,\,\text{major}} + h_{L,\,\text{minor}} = \sum f_i \frac{L_i}{D_i}\frac{\mathcal{V}_i^2}{2g} + \sum K_{L,j}\frac{\mathcal{V}_j^2}{2g}$$

If the entire piping system has a constant diameter, the total head loss reduces to

$$h_{L,\,\text{total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{\mathcal{V}^2}{2g}$$

The analysis of a piping system is based on two simple principles: (1) The conservation of mass throughout the system must be satisfied and (2) the pressure drop between two points must be the same for all paths between the two points. When the pipes are connected *in series*, the flow rate through the entire system remains constant regardless of the diameters of the individual pipes. For a pipe that branches out into two (or more) *parallel pipes* and then rejoins at a junction downstream, the total flow rate is the sum of the flow rates in the individual pipes.

When a piping system involves a pump and/or turbine, the steady-flow energy equation is expressed as

$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

When the useful pump head $h_{\text{pump, u}}$ is known, the mechanical power that needs to the supplied by the pump to the fluid and the electric power consumed by the motor of the pump for a specified flow rate are determined from

$$\dot{W}_{\text{pump, shaft}} = \frac{\rho \dot{V} g h_{\text{pump, u}}}{\eta_{\text{pump}}} \quad \text{and} \quad \dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump, u}}}{\eta_{\text{pump-motor}}}$$

where $\eta_{\text{pump-motor}}$ is the *efficiency of the pump-motor combination,* which is the product of the pump and the motor efficiencies.

The plot of the head loss versus the flow rate $\dot{V}$ is called the *system curve.* The head produced by a pump is not a constant either. The $h_{\text{pump, u}}$ and η_{pump} versus $\dot{V}$ curves of pumps are called the *characteristic curves.* A pump installed in a piping system operates at the *operating point,* which is the point of intersection of the system curve and the characteristic curve.

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PROBLEMS*

Laminar and Turbulent Flow

14–1C Why are liquids usually transported in circular pipes?

14–2C What is the physical significance of the Reynolds number? How is it defined for (*a*) flow in a circular pipe of inner diameter D and (*b*) flow in a rectangular duct of cross section $a \times b$?

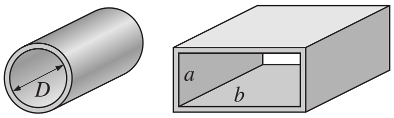

FIGURE P14–2C

14–3C Consider a person walking first in air and then in water at same speed. For which motion will the Reynolds number be higher?

14–4C Show that the Reynolds number for flow in a circular pipe of diameter D can be expressed as $\mathrm{Re} = 4\dot{m}/(\pi D\mu)$.

14–5C Which fluid at room temperature requires a larger pump to flow at a specified velocity in a given pipe: water or engine oil? Why?

14–6C What is the generally accepted value of the Reynolds number above which the flow in smooth pipes is turbulent?

14–7C Consider the flow of air and water in pipes of the same diameter, at the same temperature, and at the same mean velocity. Which flow is more likely to be turbulent? Why?

14–8C What is hydraulic diameter? How is it defined? What is it equal to for a circular pipe of diameter D?

14–9C How is the hydrodynamic entry length defined for flow in a pipe? Is the entry length longer in laminar or turbulent flow?

14–10C Consider laminar flow in a circular pipe. Will the wall shear stress τ_w be higher near the inlet of the pipe or near the exit? Why? What would your response be if the flow were turbulent?

14–11C How does surface roughness affect the pressure drop in a pipe if the flow is turbulent? What would your response be if the flow were laminar?

14–12C How does the wall shear stress τ_w vary along the flow direction in the fully developed region in (*a*) laminar flow and (*b*) turbulent flow?

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with a CD-EES icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

Fully Developed Flow in Pipes

14–13C What fluid property is responsible for the development of the velocity boundary layer? For what kinds of fluids will there be no velocity boundary layer in a pipe?

14–14C In the fully developed region of flow in a circular pipe, will the velocity profile change in the flow direction?

14–15C How is the friction factor for flow in a pipe related to the pressure loss? How is the pressure loss related to the pumping power requirement for a given mass flow rate?

14–16C Someone claims that the shear stress at the center of a circular pipe during fully developed laminar flow is zero. Do you agree with this claim? Explain.

14–17C Someone claims that in fully developed turbulent flow in a pipe, the shear stress is a maximum at the pipe surface. Do you agree with this claim? Explain.

14–18C Consider fully developed flow in a circular pipe with negligible entrance effects. If the length of the pipe is doubled, the head loss will (*a*) double, (*b*) more than double, (*c*) less than double, (*d*) reduce by half, or (*e*) remain constant.

14–19C Someone claims that the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross-sectional area, and dividing the result by 2. Do you agree? Explain.

14–20C Someone claims that the average velocity in a circular pipe in fully developed laminar flow can be determined by simply measuring the velocity at $R/2$ (midway between the wall surface and the centerline). Do you agree? Explain.

14–21C Consider fully developed laminar flow in a circular pipe. If the diameter of the pipe is reduced by half while the flow rate and the pipe length are held constant, the head loss will (*a*) double, (*b*) triple, (*c*) quadruple, (*d*) increase by a factor of 8, (*e*) increase by a factor of 16.

14–22C What is the physical mechanism that causes the friction factor to be higher in turbulent flow?

14–23C What is turbulent viscosity? What is it caused by?

14–24C The head loss for a certain circular pipe is given by $h_L = 0.0826\, fL\,\dfrac{\dot{V}^2}{D^5}$, where f is the friction factor (dimensionless), L is the pipe length, $\dot{V}$ is the volumetric flow rate, and D is the pipe diameter. Determine if the 0.0826 is a dimensional or dimensionless constant. Is this equation dimensionally homogeneous as it stands?

14–25C Consider fully developed laminar flow in a circular pipe. If the viscosity of the fluid is reduced by half by heating while the flow rate is held constant, how will the head loss change?

14–26C How is head loss related to pressure loss? For a given fluid, explain how you would convert head loss to pressure loss.

14–27C Consider laminar flow of air in a circular pipe with perfectly smooth surfaces. Do you think the friction factor for this flow will be zero? Explain.

14–28C Explain why the friction factor is independent of the Reynolds number at very large Reynolds numbers.

14–29E Oil at 80°F (ρ = 56.8 lbm/ft^3 and μ = 0.0278 lbm/ft · s) is flowing steadily in a 0.5-in-diameter, 120-ft-long pipe. During the flow, the pressure at the pipe inlet and exit is measured to be 120 psi and 14 psi, respectively. Determine the flow rate of oil through the pipe assuming the pipe is (*a*) horizontal, (*b*) inclined 20° upward, and (*c*) inclined 20° downward.

14–30 Oil with a density of 850 kg/m^3 and kinematic viscosity of 0.00062 m^2/s is being discharged by a 5-mm-diameter, 40-m-long horizontal pipe from a storage tank open to the atmosphere. The height of the liquid level above the center of the pipe is 3 m. Disregarding the minor losses, determine the flow rate of oil through the pipe.

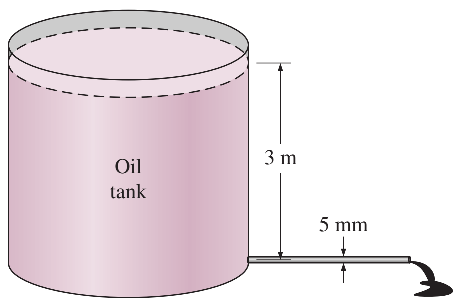

FIGURE P14–30

14–31 Water at 10°C (ρ = 999.7 kg/m^3 and μ = 1.307 × 10^{-3} kg/m · s) is flowing steadily in a 0.20-cm-diameter, 15-m-long pipe at an average velocity of 1.2 m/s. Determine (*a*) the pressure drop, (*b*) the head loss, and (*c*) the pumping power requirement to overcome this pressure drop.
Answers: (*a*) 188 kPa, (*b*) 19.2 m, (*c*) 0.71 W

14–32 Water at 15°C (ρ = 999.1 kg/m^3 and μ = 1.138 × 10^{-3} kg/m · s) is flowing steadily in a 30-m-long and 4-cm-diameter horizontal pipe made of stainless steel at a rate of 8 L/s. Determine (*a*) the pressure drop, (*b*) the head loss, and (*c*) the pumping power requirement to overcome this pressure drop.

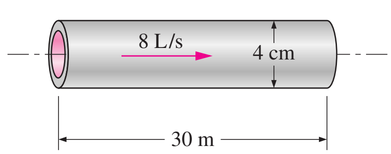

FIGURE P14–32

14–33E Heated air at 1 atm and 100°F is to be transported in a 400-ft-long circular plastic duct at a rate of 12 ft^3/s. If the head loss in the pipe is not to exceed 50 ft, determine the minimum diameter of the duct.

14–34 In fully developed laminar flow in a circular pipe, the velocity at R/2 (midway between the wall surface and the centerline) is measured to be 6 m/s. Determine the velocity at the center of the pipe. *Answer:* 8 m/s

14–35 The velocity profile in fully developed laminar flow in a circular pipe of inner radius R = 2 cm, in m/s, is given by $u(r) = 4(1 - r^2/R^2)$. Determine the mean and maximum velocities in the pipe and the volume flow rate.

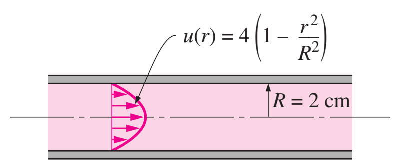

FIGURE P14–35

14–36 Repeat Prob. 14–35 for a pipe of inner radius 7 cm.

14–37 Consider an air solar collector that is 1 m wide and 5 m long and has a constant spacing of 3 cm between the glass cover and the collector plate. Air flows at an average temperature of 45°C at a rate of 0.15 m^3/s through the 1-m-wide edge of the collector along the 5-m-long passageway. Disregarding the entrance and roughness effects, determine the pressure drop in the collector. *Answer:* 29 Pa

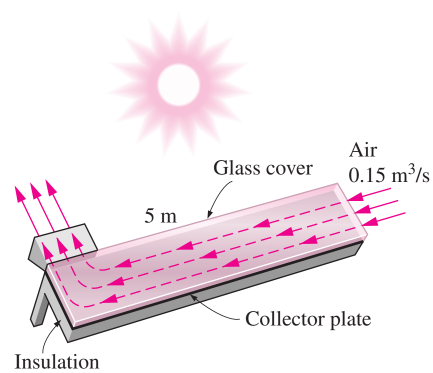

FIGURE P14–37

14–38 Consider the flow of oil with ρ = 894 kg/m^3 and μ = 2.33 kg/m · s in a 40-cm-diameter pipeline at an average velocity of 0.5 m/s. A 300-m-long section of the pipeline passes through the icy waters of a lake. Disregarding the entrance effects, determine the pumping power required to overcome the pressure losses and to maintain the flow of oil in the pipe.

14–39 Consider laminar flow of a fluid through a square channel with smooth surfaces. Now the mean velocity of the fluid is doubled. Determine the change in the head loss of the fluid. Assume the flow regime remains unchanged.

14–40 Repeat Prob. 14–39 for turbulent flow in smooth pipes for which the friction factor is given as $f = 0.184\ \text{Re}^{-0.2}$. What would your answer be for fully turbulent flow in a rough pipe?

14–41 Air enters a 7-m-long section of a rectangular duct of cross section 15 cm × 20 cm made of commercial steel at 1 atm and 35°C at an average velocity of 7 m/s. Disregarding the entrance effects, determine the fan power needed to overcome the pressure losses in this section of the duct. *Answer:* 4.9 W

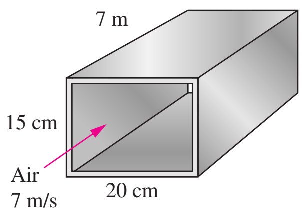

FIGURE P14–41

14–42E Water at 60°F passes through 0.75-in-internal-diameter copper tubes at a rate of 1.2 lbm/s. Determine the pumping power per ft of pipe length required to maintain this flow at the specified rate.

14–43 Oil with ρ = 876 kg/m^3 and μ = 0.24 kg/m · s is flowing through a 1.5-cm-diameter pipe that discharges into the atmosphere at 88 kPa. The absolute pressure 15 m before the exit is measured to be 135 kPa. Determine the flow rate of oil through the pipe if the pipe is (*a*) horizontal, (*b*) inclined 8° upward from the horizontal, and (*c*) inclined 8° downward from the horizontal.

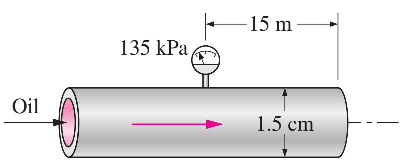

FIGURE P14–43

14–44 Glycerin at 40°C with ρ = 1252 kg/m^3 and μ = 0.27 kg/m · s is flowing through a 2-cm-diameter, 25-m-long pipe that discharges into the atmosphere at 100 kPa. The flow rate through the pipe is 0.035 L/s. (*a*) Determine the absolute pressure 25 m before the pipe exit. (*b*) At what angle θ must the pipe be inclined downwards from the horizontal for the pressure in the entire pipe to be atmospheric pressure and the flow rate to be maintained the same?

14–45 In an air heating system, heated air at 40°C and 105 kPa absolute is distributed through a 0.2 m × 0.3 m rectangular duct made of commercial steel duct at a rate of 0.5 m^3/s. Determine the pressure drop and head loss through a 40-m-long section of the duct. *Answers:* 128 Pa, 93.8 m

14–46 Glycerin at 40°C with ρ = 1252 kg/m^3 and μ = 0.27 kg/m · s is flowing through a 5-cm-diameter horizontal smooth pipe with a mean velocity of 3.5 m/s. Determine the pressure drop per 10 m of the pipe.

14–47 Reconsider Prob. 14–46. Using EES (or other) software, investigate the effect of the pipe diameter on the pressure drop for the same constant flow rate. Let the pipe diameter vary from 1 cm to 10 cm in increments of 1 cm. Tabulate and plot the results, and draw conclusions.

14–48E Air at 1 atm and 60°F is flowing through a 1 ft × 1 ft square duct made of commercial steel at a rate of 1200 cfm. Determine the pressure drop and head loss per ft of the duct.

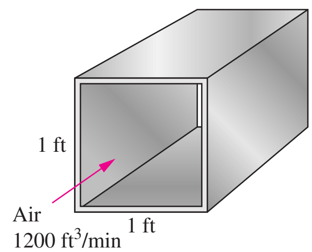

FIGURE P14–48E

14–49 Liquid ammonia at −20°C is flowing through a 30-m-long section of a 5-mm-diameter copper tube at a rate of 0.15 kg/s. Determine the pressure drop, the head loss, and the pumping power required to overcome the frictional losses in the tube. *Answers:* 4792 kPa, 743 m, 1.08 kW

14–50 Shell-and-tube heat exchangers with hundreds of tubes housed in a shell are commonly used in practice for heat transfer between two fluids. Such a heat exchanger used in an active solar hot-water system transfers heat from a water-antifreeze solution flowing through the shell and the solar collector to fresh water flowing through the tubes at an average temperature of 60°C at a rate of 15 L/s. The heat exchanger contains 80 brass tubes 1 cm in inner diameter and 1.5 m in length. Disregarding inlet, exit, and header losses, determine the pressure drop across a single tube and the pumping power required by the tube-side fluid of the heat exchanger.

After operating a long time, 1-mm-thick scale builds up on the inner surfaces with an equivalent roughness of 0.4 mm. For the same pumping power input, determine the percent reduction in the flow rate of water through the tubes.

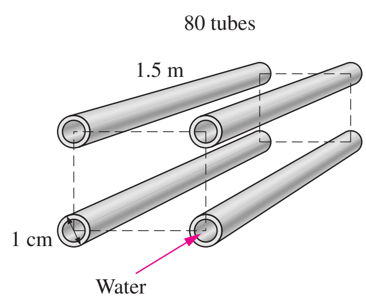

FIGURE P14–50

Minor Losses

14–51C What is minor loss in pipe flow? How is the minor loss coefficient K_L defined?

14–52C Define equivalent length for minor loss in pipe flow. How is it related to the minor loss coefficient?

14–53C The effect of rounding of a pipe inlet on the loss coefficient is (*a*) negligible, (*b*) somewhat significant, (*c*) very significant.

14–54C The effect of rounding of a pipe exit on the loss coefficient is (*a*) negligible, (*b*) somewhat significant, (*c*) very significant.

14–55C Which has a greater minor loss coefficient during pipe flow: gradual expansion or gradual contraction? Why?

14–56C A piping system involves sharp turns, and thus large minor head losses. One way of reducing the head loss is to replace the sharp turns by circular elbows. What is another way?

14–57C During a retrofitting project of a fluid-flow system to reduce the pumping power, it is proposed to install vanes into the miter elbows or to replace the sharp turns in 90° miter elbows by smooth curved bends. Which approach will result in a greater reduction in pumping power requirements?

14–58 Water is to be withdrawn from a 3-m-high water reservoir by drilling a 1.5-cm-diameter hole at the bottom surface. Determine the flow rate of water through the hole if (*a*) the entrance of the hole is well-rounded and (*b*) the entrance is sharp-edged.

14–59 Consider flow from a water reservoir through a circular hole of diameter D at the side wall at a vertical distance H from the free surface. The flow rate through an actual hole with a sharp-edged entrance ($K_L = 0.5$) will be considerably less than the flow rate calculated assuming "frictionless" flow and thus zero loss for the hole. Obtain a relation for the "equivalent diameter" of the sharp-edged hole for use in frictionless flow relations.

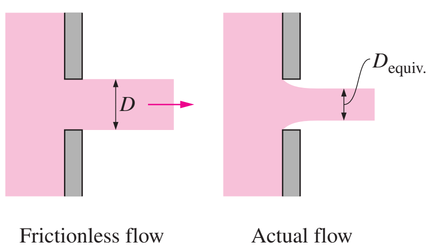

FIGURE P14–59

14–60 Repeat Prob. 14–59 for a slightly rounded entrance ($K_L = 0.12$).

14–61 A horizontal pipe has an abrupt expansion from $D_1 =$ 8 cm to $D_2 = 16$ cm. The water velocity in the smaller section is 10 m/s, and the flow is turbulent. The pressure in the smaller section is $P_1 = 300$ kPa. Determine the downstream pressure P_2, and estimate the error that would have occurred if Bernoulli's equation had been used. *Answers:* 319 kPa, 28 kPa

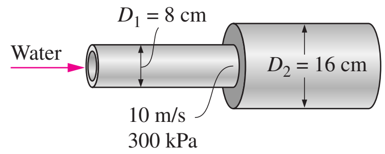

FIGURE P14–61

Piping Systems and Pump Selection

14–62C A piping system involves two pipes of different diameters (but of identical length, material, and roughness) connected in series. How would you compare the (*a*) flow rates and (*b*) pressure drops in these two pipes?

14–63C A piping system involves two pipes of different diameters (but of identical length, material, and roughness) connected in parallel. How would you compare the (*a*) flow rates and (*b*) pressure drops in these two pipes?

14–64C A piping system involves two pipes of identical diameters but of different lengths connected in parallel. How would you compare the pressure drops in these two pipes?

14–65C Water is pumped from a large lower reservoir to a higher reservoir. Someone claims that if the head loss is negligible, the required pump head is equal to the elevation difference between the free surfaces of the two reservoirs. Do you agree?

14–66C A piping system equipped with a pump is operating steadily. Explain how the operating point (the flow rate and the head loss) is established.

14–67C For a piping system, define the system curve, the characteristic curve, and the operating point on a head versus flow rate chart.

14–68 Water at 20°C is to be pumped from a reservoir ($z_A = 2$ m) to another reservoir at a higher elevation ($z_B = 9$ m) through two 25-m-long plastic pipes connected in parallel. The diameters of the two pipes are 3 cm and

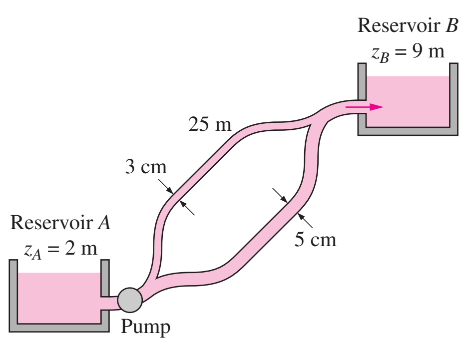

FIGURE P14–68

5 cm. Water is to be pumped by a 68 percent efficient motor/pump unit that draws 7 kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rates through each of the parallel pipes.

14–69E Water at 70°F flows by gravity from a large reservoir at a high elevation to a smaller one through a 120-ft-long, 2-in-diameter cast iron piping system that involves four standard flanged elbows, a well-rounded entrance, a sharp-edged exit, and a fully open gate valve. Taking the free surface of the lower reservoir as the reference level, determine the elevation z_1 of the higher reservoir for a flow rate of 10 ft^3/min.
Answer: 23.1 ft

14–70 A 3-m-diameter tank is initially filled with water 2 m above the center of a sharp-edged 10-cm-diameter orifice. The tank water surface is open to the atmosphere, and the orifice drains to the atmosphere. Calculate (*a*) the initial velocity from the tank and (*b*) the time required to empty the tank. Does the loss coefficient of the orifice cause a significant increase in the draining time of the tank?

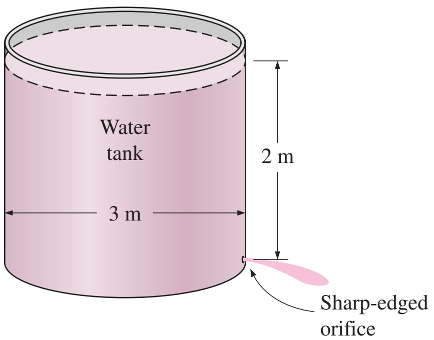

FIGURE P14–70

14–71 A 3-m-diameter tank is initially filled with water 2 m above the center of a sharp-edged 10-cm-diameter orifice. The tank water surface is open to the atmosphere, and the orifice drains to the atmosphere through a 100-m-long pipe. The friction coefficient of the pipe can be taken to be 0.015. Determine (*a*) the initial velocity from the tank and (*b*) the time required to empty the tank.

14–72 Reconsider Prob. 14–71. In order to drain the tank faster, a pump is installed near the tank exit. Determine how much pump power input is necessary to establish an average water velocity of 4 m/s when the tank is full at $z = 2$ m. Also, assuming the discharge velocity to remain constant, estimate the time required to drain the tank.

Someone suggested that it makes no difference whether the pump is located at the beginning or at the end of the pipe, and that the performance will be the same in either case, but another person argued that placing the pump near the end of the pipe may cause cavitation. The water temperature is 30°C, so the water vapor pressure is $P_v = 4.246$ kPa $= 0.43$ m-H_2O, and the system is located at sea level. Investigate if there is the possibility of cavitation and if we should be concerned about the location of the pump.

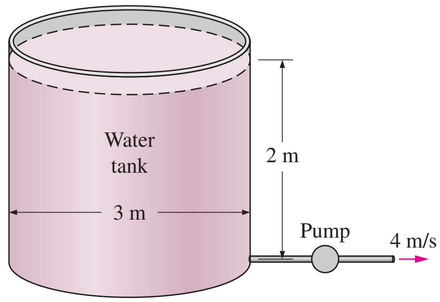

FIGURE P14–72

14–73 Oil at 20°C is flowing through a vertical glass funnel that consists of a 15-cm-high cylindrical reservoir and a 1-cm-diameter, 25-cm-high pipe. The funnel is always maintained full by the addition of oil from a tank. Assuming the entrance effects to be negligible, determine the flow rate of oil through the funnel and calculate the "funnel effectiveness," which can be defined as the ratio of the actual flow rate through the funnel to the maximum flow rate for the "frictionless" case.
Answers: 4.09×10^{-6} m^3/s, 1.86 percent

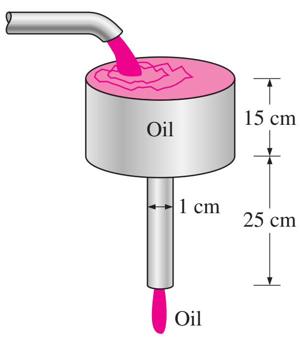

FIGURE P14–73

14–74 Repeat Prob. 14–73 assuming (*a*) the diameter of the pipe is doubled and (*b*) the length of the pipe is doubled.

14–75 Water at 15°C is drained from a large reservoir using two horizontal plastic pipes connected in series. The first pipe is 20 m long and has a 10-cm diameter while the second pipe is

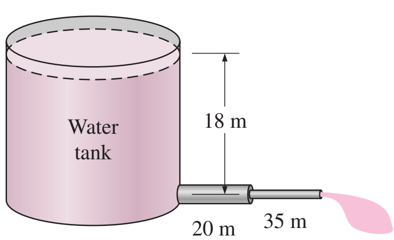

FIGURE P14–75

35 m long and has a 4-cm diameter. The water level in the reservoir is 18 m above the centerline of the pipe. The pipe entrance is sharp-edged, and the contraction between the two pipes is sudden. Determine the discharge rate of water from the reservoir.

14–76E A farmer is to pump water at 70°F from a river to a water storage tank nearby using a total of 125 ft-long, 5-in-diameter plastic pipes with three flanged 90° smooth bends. The water velocity near the river surface is 6 ft/s, and the pipe inlet is placed in the river normal to the flow direction of water to take advantage of the dynamic pressure. The elevation difference between the free surface of the tank and the river is 12 ft. For a flow rate of 1.5 ft^3/s and an overall pump efficiency of 70 percent, determine the required electric power input to the pump.

14–77E Reconsider Prob. 14–76E. Using EES (or other) software, investigate the effect of the pipe diameter on the required electric power input to the pump. Let the pipe diameter vary from 1 to 10 in, in increments of 1 in. Tabulate and plot the results, and draw conclusions.

14–78 A water tank filled with solar-heated water is to be used for showers in a field using gravity-driven flow. The system involves 20 m of 1.5-cm-diameter galvanized iron piping with four miter bends (90°) without vanes and a wide-open globe valve. If water is to flow at a rate of 0.7 L/s through the shower head, determine how high the water level in the tank must be from the exit level of the shower. Disregard the losses at the entrance and at the shower head, and take the water temperature to be 40°C.

14–79 Two water reservoirs *A* and *B* are connected to each other through a 40-m-long, 2-cm-diameter cast iron pipe with a sharp-edged entrance. The pipe also involves a swing check valve and a fully open gate valve. The water level in both reservoirs is the same, but reservoir *A* is pressurized by compressed air while reservoir *B* is open to the atmosphere at 88 kPa. If the initial flow rate through the pipe is 1.2 L/s, determine the absolute air pressure on top of reservoir *A*. Take the water temperature to be 10°C. *Answer:* 733 kPa

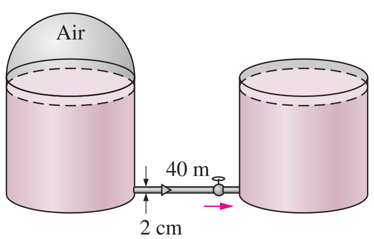

FIGURE P14–79

14–80 A tanker is to be filled with fuel oil with ρ = 920 kg/m^3 and μ = 0.045 kg/m · s from an underground reservoir using a 20-m-long, 5-cm-diameter plastic hose with a slightly rounded entrance and two 90° smooth bends. The elevation difference between the oil level in the reservoir and the top of the tanker where the hose is discharged is 5 m. The capacity of the tanker is 18 m^3, and the filling time is 30 min. Assuming an overall pump efficiency of 82 percent, determine the required power input to the pump.

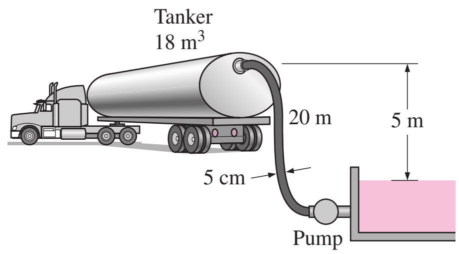

FIGURE P14–80

14–81 Two pipes of identical length and material are connected in parallel. The diameter of pipe *A* is twice the diameter of pipe *B*. Assuming the friction factor to be the same in both cases and disregarding minor losses, determine the ratio of the flow rates in the two pipes.

14–82 A certain part of cast iron piping of a water distribution system involves a parallel section. Both parallel pipes have a diameter of 30 cm, and the flow is fully turbulent. One of the branches (pipe *A*) is 1000 m long while the other branch (pipe *B*) is 3000 m long. If the flow rate through pipe *A* is 0.4 m^3/s, determine the flow rate through pipe *B*. Disregard minor losses and assume the water temperature to be 15°C. Show that the flow is fully turbulent, and thus the friction factor is independent of Reynolds number. *Answer:* 0.231 m^3/s

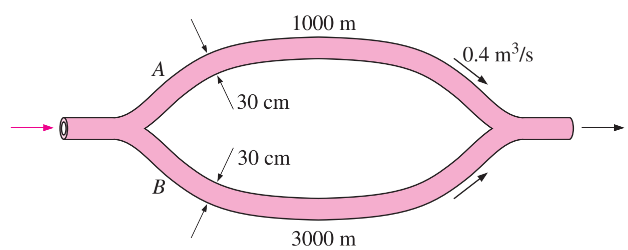

FIGURE P14–82

14–83 Repeat Prob. 14–82 assuming pipe *A* has a halfway-closed gate valve (K_L = 2.1) while pipe *B* has a fully open globe valve (K_L = 10), and the other minor losses are negligible. Assume the flow to be fully turbulent.

14–84 A geothermal district heating system involves the transport of geothermal water at 110°C from a geothermal well to a city at about the same elevation for a distance of 12 km at a rate of 1.5 m^3/s in 60-cm-diameter stainless steel pipes. The fluid pressures at the wellhead and the arrival point in the city are to be the same. The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. (*a*) Assuming the pump-motor efficiency to be 74 percent, determine the electric power consumption of the system for pumping. Would you recommend the use of a single large pump or several smaller pumps of the same total pumping power scattered along the pipeline? Explain. (*b*) Determine the daily cost of power consumption of the system if the unit cost of electricity is \$0.06/kWh. (*c*) The temperature of geothermal water is estimated to drop 0.5°C during this long flow. Determine if the frictional heating during flow can make up for this drop in temperature.

14–85 Repeat Prob. 14–84 for cast iron pipes of the same diameter.

14–86E A clothes drier discharges air at 1 atm and 120°F at a rate of 1.2 ft^3/s when its 5-in-diameter, well-rounded vent with negligible loss is not connected to any duct. Determine the flow rate when the vent is connected to a 15-ft-long, 5-in-diameter duct made of galvanized iron, with three 90° flanged smooth bends. Take the friction factor of the duct to be 0.019, and assume the fan power input to remain constant.

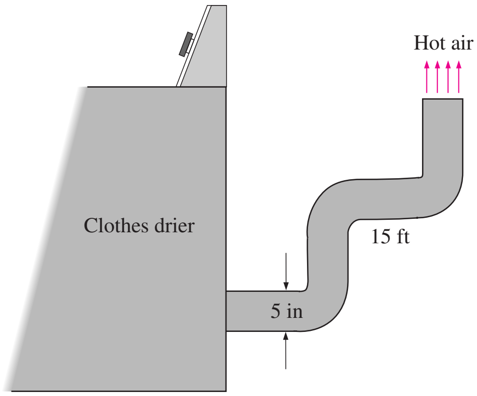

FIGURE P14–86E

14–87 In large buildings, hot water in a water tank is circulated through a loop so that the user doesn't have to wait for all the water in long piping to drain before hot water starts coming out. A certain recirculating loop involves 40-m-long, 1.2-cm-diameter cast iron pipes with six 90° threaded smooth bends and two fully open gate valves. If the mean flow velocity through the loop is 2.5 m/s, determine the required power input for the recirculating pump. Take the average water temperature to be 60°C and the efficiency of the pump to be 70 percent. *Answer:* 0.217 kW

14–88 EES Reconsider Prob. 14–87. Using EES (or other) software, investigate the effect of the mean flow velocity on the power input to the recirculating pump. Let the velocity vary from 0 m/s to 3 m/s in increments of 0.3 m/s. Tabulate and plot the results.

14–89 Repeat Prob. 14–87 for plastic pipes.

Review Problems

14–90 The compressed air requirements of a manufacturing facility are met by a 150-hp compressor that draws in air from the outside through an 8-m-long, 20-cm-diameter duct made of thin galvanized iron sheets. The compressor takes in air at a rate of 0.27 m^3/s at the outdoor conditions of 15°C and 95 kPa. Disregarding any minor losses, determine the useful power used by the compressor to overcome the frictional losses in this duct. *Answer:* 9.66 W

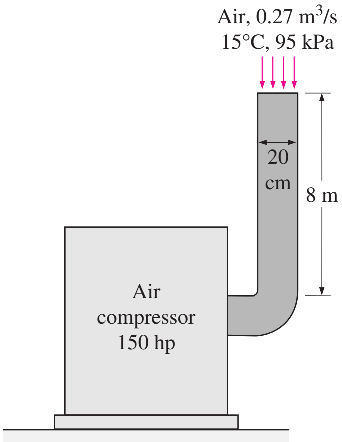

FIGURE P14–90

14–91 A house built on a riverside is to be cooled in summer by utilizing the cool water of the river. A 15-m-long section of a circular stainless steel duct of 20-cm diameter passes through the water. Air flows through the underwater section of the duct at 3 m/s at an average temperature of 15°C. For an overall fan efficiency of 62 percent, determine the fan power needed to overcome the flow resistance in this section of the duct.

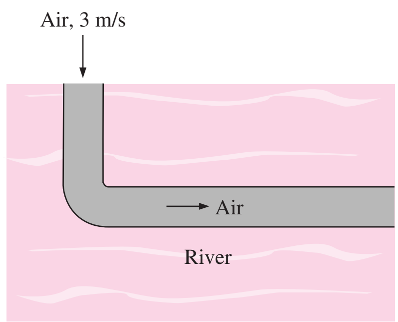

FIGURE P14–91

14–92 The velocity profile in fully developed laminar flow in a circular pipe, in m/s, is given by $u(r) = 6(1 - 100r^2)$, where r is the radial distance from the centerline of the pipe in m. Determine (*a*) the radius of the pipe, (*b*) the mean velocity through the pipe, and (*c*) the maximum velocity in the pipe.

14–93E The velocity profile in fully developed laminar flow of water at 40°F in a 80-ft-long horizontal circular pipe, in ft/s, is given by $u(r) = 0.8(1 - 625r^2)$ where r is the radial distance from the centerline of the pipe in ft. Determine (*a*) the volume flow rate of water through the pipe, (*b*) the pressure drop across the pipe, and (*c*) the useful pumping power required to overcome this pressure drop.

14–94E Repeat Prob. 14–93E assuming the pipe is inclined 12° from the horizontal and the flow is uphill.

14–95 Consider flow from a reservoir through a horizontal pipe of length *L* and diameter *D* that penetrates into the side wall at a vertical distance *H* from the free surface. The flow rate through an actual pipe with a reentrant section ($K_L = 0.8$) will be considerably less than the flow rate through the hole calculated assuming "frictionless" flow and thus zero loss. Obtain a relation for the "equivalent diameter" of the reentrant pipe for use in relations for frictionless flow through a hole and determine its value for a pipe friction factor, length, and diameter of 0.018, 10 m, and 0.04 m, respectively. Assume the friction factor of the pipe to remain constant.

14–96 Water is to be withdrawn from a 5-m-high water reservoir by drilling a well-rounded 3-cm-diameter hole with negligible loss at the bottom surface and attaching a horizontal 90° bend of negligible length. Determine the flow rate of water through the bend if (*a*) the bend is a flanged smooth bend and (*b*) the bend is a miter bend without vanes.
Answers: (*a*) 0.00614 m^3/s, (*b*) 0.00483 m^3/s

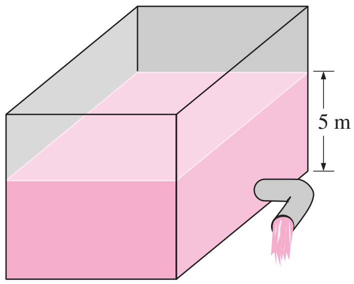

FIGURE P14–96

14–97 In a geothermal district heating system, 10,000 kg/s of hot water must be delivered a distance of 10 km in a horizontal pipe. The minor losses are negligible, and the only significant energy loss will arise from pipe friction. The friction factor can be taken to be 0.015. Specifying a larger diameter pipe would reduce water velocity, velocity head, pipe friction, and thus power consumption. But a larger pipe also would cost more money initially to purchase and install. Otherwise stated, there is an optimum pipe diameter that will minimize the sum of pipe cost and future electric power cost.

Assume the system will run 24 h/day, every day, for 30 years. During this time the cost of electricity will remain constant at \$0.06/kWh. Assume system performance stays constant over the decades (this may not be true, especially if highly mineralized water is passed through the pipeline—scale may form). The pump has an overall efficiency of 80 percent. The cost to purchase, install, and insulate a 10-km pipe depends on the diameter *D* and is given by *Cost* = $\$10^6 D^2$, where *D* is in m. Assuming zero inflation and interest rate for simplicity and zero salvage value and zero maintenance cost, determine the optimum pipe diameter.

14–98 Water at 15°C is to be discharged from a reservoir at a rate of 18 L/s using two horizontal cast iron pipes connected in series and a pump between them. The first pipe is 20 m long and has a 6-cm diameter, while the second pipe is 35 m long and has a 4-cm diameter. The water level in the reservoir is 30 m above the centerline of the pipe. The pipe entrance is sharp-edged, and losses associated with the connection of the pump are negligible. Determine the required pumping head and the minimum pumping power to maintain the indicated flow rate.

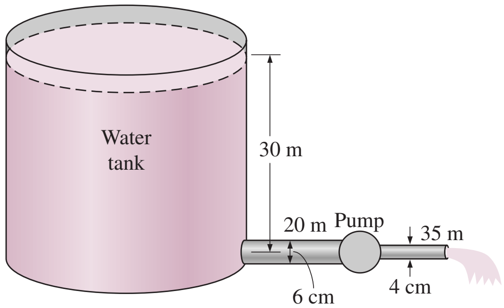

FIGURE P14–98

14–99 Reconsider Prob. 14–98. Using EES (or other) software, investigate the effect of the second pipe diameter on the required pumping head to maintain the indicated flow rate. Let the diameter vary from 1 cm to 10 cm in increments of 1 cm. Tabulate and plot the results.

14–100 Two pipes of identical diameter and material are connected in parallel. The length of pipe *A* is twice the length of pipe *B*. Assuming the flow is fully turbulent in both pipes and thus the friction factor is independent of the Reynolds number and disregarding minor losses, determine the ratio of the flow rates in the two pipes. *Answer:* 0.707

14–101 A pipeline that transports oil at 40°C at a rate of 3 m^3/s branches out into two parallel pipes made of commercial steel that reconnect downstream. Pipe *A* is 500 m long and has a diameter of 30 cm while pipe *B* is 800 m

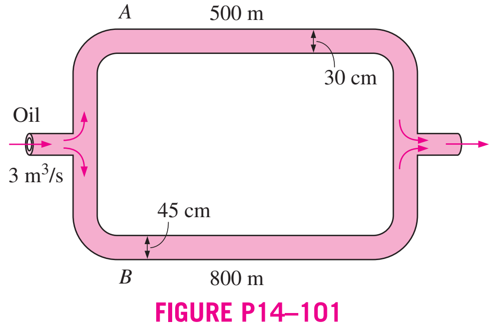

FIGURE P14–101

long and has a diameter of 45 cm. The minor losses are considered to be negligible. Determine the flow rate through each of the parallel pipes.

14–102 Repeat Prob. 14–101 for hot-water flow of a district heating system at 100°C.

14–103E A water fountain is to be installed at a remote location by attaching a cast iron pipe directly to a water main through which water is flowing at 70°F and 60 psig. The entrance to the pipe is sharp-edged, and the 50-ft-long piping system involves three 90° miter bends without vanes, a fully open gate valve, and an angle valve with a loss coefficient of 5 when fully open. If the system is to provide water at a rate of 20 gal/min and the elevation difference between the pipe and the fountain is negligible, determine the minimum diameter of the piping system. *Answer:* 0.76 in

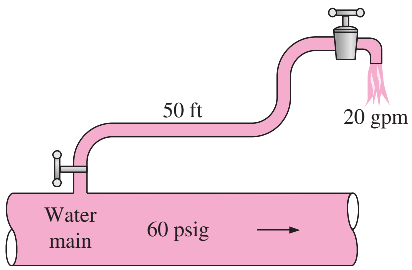

FIGURE P14–103E

14–104E Repeat Prob. 14–103E for plastic pipes.

14–105 In a hydroelectric power plant, water at 20°C is supplied to the turbine at a rate of 0.8 m^3/s through a 200-m-long, 0.35-m-diameter cast iron pipe. The elevation difference between the free surface of the reservoir and the turbine discharge is 70 m, and the combined turbine-generator efficiency is 84 percent. Disregarding the minor losses because of the large length-to-diameter ratio, determine the electric power output of this plant.

14–106 In Prob. 14–105, the pipe diameter is tripled in order to reduce the pipe losses. Determine the percent increase in the net power output as a result of this modification.

14–107E The drinking water needs of an office are met by large water bottles. One end of a 0.35-in-diameter, 6-ft-long plastic hose is inserted into the bottle placed on a high stand, while the other end with an on/off valve is maintained 3 ft below the bottom of the bottle. If the water level in the bottle is 1 ft when it is full, determine how long it will take to fill an 8-oz glass (= 0.00835 ft^3) (*a*) when the bottle is first opened and (*b*) when the bottle is almost empty. Take the total minor loss coefficient, including the on/off valve, to be 2.8 when it is fully open. Assume the water temperature to be the same as the room temperature of 70°F. *Answers:* (*a*) 2.4 s, (*b*) 2.8 s

14–108E Reconsider Prob. 14–107E. Using EES (or other) software, investigate the effect of the hose diameter on the time required to fill a glass when the bottle is full. Let the diameter vary from 0.2 to 2 in, in increments of 0.2 in. Tabulate and plot the results.

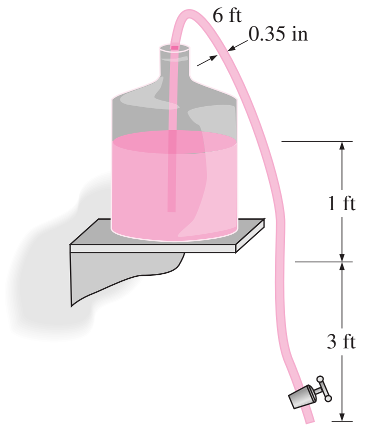

FIGURE P14–107E

14–109E Reconsider Prob. 14–107E. The office worker who set up the siphoning system purchased a 12-ft-long reel of the plastic tube and wanted to use the whole thing to avoid cutting it in pieces, thinking that it is the elevation difference that makes siphoning work, and the length of the tube is not important. So he used the entire 12-ft-long tube. Assuming there are no additional turns or constrictions in the tube (being very optimistic), determine the time it takes to fill a glass of water for both cases.

14–110 A circular water pipe has an abrupt expansion from diameter D_1 = 15 cm to D_2 = 20 cm. The pressure and the mean water velocity in the smaller pipe are P_1 = 120 kPa and 10 m/s, and the flow is turbulent. By applying the continuity, momentum, and energy equations, show that the loss coefficient for sudden expansion is $K_L = (1 - D_1^2/D_2^2)^2$, and calculate K_L and P_2 for the given case.

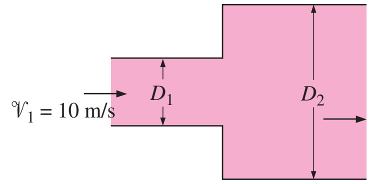

FIGURE P14–110

14–111 The water at 20°C in a 10-m-diameter, 2-m-high above-the-ground swimming pool is to be emptied by unplugging a 3-cm-diameter, 25-m-long horizontal plastic pipe attached to the bottom of the pool. Determine the initial rate of discharge of water through the pipe and the time it will take to

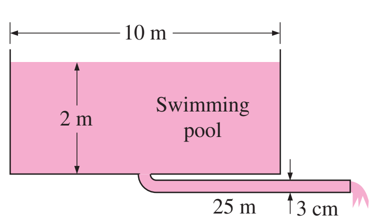

FIGURE P14–111

empty the swimming pool completely assuming the entrance to the pipe is well-rounded with negligible loss. Take the friction factor of the pipe to be 0.022. Using the initial discharge velocity, check if this is a reasonable value for the friction factor.

14–112 Reconsider Prob. 14–111. Using EES (or other) software, investigate the effect of the discharge pipe diameter on the time required to empty the pool completely. Let the diameter vary from 1 to 10 cm, in increments of 1 cm. Tabulate and plot the results.

14–113 Repeat Prob. 14–111 for a sharp-edged entrance to the pipe with $K_L = 0.5$.

14–114 A system that consists of two interconnected cylindrical tanks with $D_1 = 30$ cm and $D_2 = 12$ cm is to be used to determine the discharge coefficient of a short $D_0 = 5$ mm diameter orifice. At the beginning ($t = 0$ s), the fluid heights in the tanks are $h_1 = 50$ cm and $h_2 = 15$ cm, as shown in the figure. If it takes 170 s for the fluid levels in the two tanks to equalize and the flow to stop, determine the discharge coefficient of the orifice. Disregard any other losses associated with this flow.

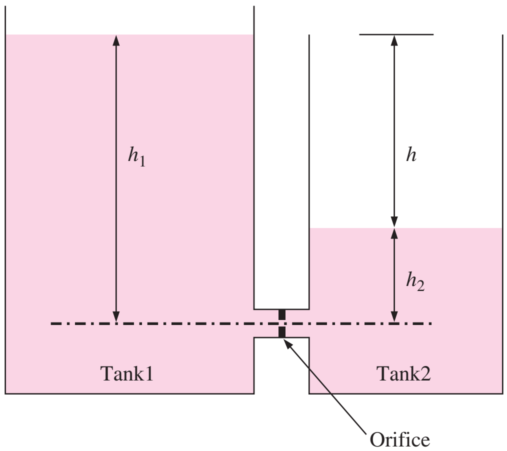

FIGURE P14–114

14–115 A highly viscous liquid discharges from a large container through a small diameter tube in laminar flow. Disregarding entrance effects and velocity heads, obtain a relation for the variation of fluid depth in the tank with time.

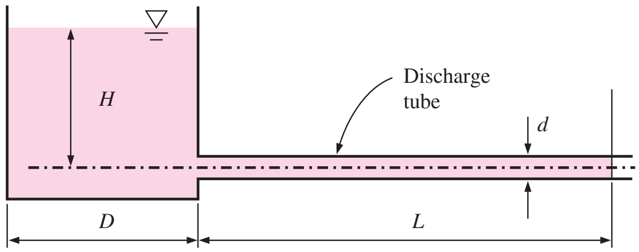

FIGURE P14–115

14–116 A student is to determine the kinematic viscosity of an oil using the system shown in the previous problem. The initial fluid height in the tank is $H = 40$ cm, the tube diameter is $d = 6$ mm, the tube length is $L = 0.65$ m, and the tank diameter is $D = 0.63$ m. The student observes that it takes 2842 s for the fluid level in the tank to drop to 36 cm. Find the fluid viscosity .

Design and Essay Problems

14–117 Electronic boxes such as computers are commonly cooled by a fan. Write an essay on forced air cooling of electronic boxes and on the selection of the fan for electronic devices.

14–118 Design an experiment to measure the viscosity of liquids using a vertical funnel with a cylindrical reservoir of height h and a narrow flow section of diameter D and length L. Making appropriate assumptions, obtain a relation for viscosity in terms of easily measurable quantities such as density and volume flow rate. Is there a need for the use of a correction factor?

14–119 A pump is to be selected for a waterfall in a garden. The water collects in a pond at the bottom, and the elevation difference between the free surface of the pond and the location where the water is discharged is 3 m. The flow rate of water is to be at least 8 L/s. Select an appropriate motor-pump unit for this job and identify three manufacturers with product model numbers and prices. Make a selection and explain why you selected that particular product. Also estimate the cost of annual power consumption of this unit assuming continuous operation.