Chapter 10
INTRODUCTION TO FLUID MECHANICS

Classification of Fluid Flows

10-1C The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is *external flow*. The flow in a pipe or duct is *internal flow* if the fluid is completely bounded by solid surfaces. The flow of liquids in a pipe is called *open-channel flow* if the pipe is partially filled with the liquid and there is a free surface, such as the flow of water in rivers and irrigation ditches.

10-2C A fluid flow during which the density of the fluid remains nearly constant is called *incompressible flow*. A fluid whose density is practically independent of pressure (such as a liquid) is called an incompressible fluid. The flow of compressible fluid (such as air) is not necessarily compressible since the density of a compressible fluid may still remain constant during flow.

10-3C A fluid in direct contact with a solid surface sticks to the surface and there is no slip. This is known as the *no-slip condition*, and it is due to the viscosity of the fluid.

10-4C In forced flow, the fluid is forced to flow over a surface or in a tube by external means such as a pump or a fan. In natural flow, any fluid motion is caused by natural means such as the buoyancy effect that manifests itself as the rise of the warmer fluid and the fall of the cooler fluid. The flow caused by winds is natural flow for the earth, but it is forced flow for bodies subjected to the winds since for the body it makes no difference whether the air motion is caused by a fan or by the winds.

10-5C The pressure of a vapor, whether it exists alone or in a mixture with other gases, is called the **vapor pressure** P_v. During phase change processes between the liquid and vapor phases of a pure substance, the saturation pressure and the vapor pressure are equivalent since the vapor is pure.

10-6C Yes. The saturation temperature of a pure substance depends on pressure. The higher the pressure, the higher the saturation or boiling temperature.

10-7C If the pressure of a substance is increased during a boiling process, the temperature will also increase since the boiling (or saturation) temperature of a pure substance depends on pressure and increases with it.

10-8C During liquid flow, vaporization may occur at locations where the pressure drops below the vapor pressure. The vapor bubbles collapse as they are swept away from the low pressure regions, generating highly destructive, extremely high pressure waves. This phenomenon which is a common cause for drop in performance and even the erosion of impeller blades is called **cavitation**.

10-9 The minimum pressure in a piping system to avoid cavitation is to be determined.

Properties The vapor pressure of water at 40°C is 7.38 kPa (Table 10-1).

Analysis To avoid cavitation, the pressure anywhere in flow should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\min} = P_{sat\,@\,40°C} = \mathbf{7.38\ kPa}$$

Therefore, the pressure should be maintained above 7.38 kPa everywhere in flow.

Discussion Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

10-10 The minimum pressure in a pump is given. It is to be determined if there is a danger of cavitation.

Properties The vapor pressure of water at 20°C is 1.71 kPa (Table 10-1).

Analysis To avoid cavitation, the pressure everywhere in the flow should remain above the vapor (or saturation) pressure at the given temperature, which is

$$P_v = P_{sat\,@\,20°C} = 1.71\,\text{kPa}$$

The minimum pressure in the pump is 2 kPa, which is greater than the vapor pressure. Therefore, there is **no danger** of cavitation in the pump.

Discussion Note that the vapor pressure increases with increasing temperature, and thus there may be a danger of cavitation at higher fluid temperatures.

10-11E The minimum pressure in a pump is given. It is to be determined if there is a danger of cavitation.

Properties The vapor pressure of water at 70°F is 0.3632 psia (Table A-4E).

Analysis To avoid cavitation, the pressure everywhere in the flow should remain above the vapor (or saturation) pressure at the given temperature, which is

$$P_v = P_{sat\,@\,70°F} = 0.3632\,\text{psia}$$

The minimum pressure in the pump is 0.1 psia, which is less than the vapor pressure. Therefore, **there is danger** of cavitation in the pump.

Discussion Note that the vapor pressure increases with increasing temperature, and the danger of cavitation increases at higher fluid temperatures.

10-12 The minimum pressure in a pump to avoid cavitation is to be determined.

Properties The vapor pressure of water at 25°C is 3.17 kPa (Table 10-1).

Analysis To avoid cavitation, the pressure anywhere in the system should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\min} = P_{sat\,@\,25°C} = \mathbf{3.17\ kPa}$$

Therefore, the lowest pressure that can exist in the pump is 3.17 kPa.

Discussion Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

Viscosity

10-13C Viscosity is a measure of the "stickiness" or "resistance to deformation" of a fluid. It is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids, and by the molecular collisions in gases. Liquids have higher dynamic viscosities than gases.

10-14C The fluids whose shear stress is proportional to the velocity gradient are called *Newtonian fluids*. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids.

10-15C When two identical small glass balls dropped into two identical containers, one filled with water and the other with oil, the ball **dropped in water** will reach the bottom of the container first because of the much lower viscosity of water relative to oil.

10-16C (*a*) The dynamic viscosity of liquids decreases with temperature. (*b*) The dynamic viscosity of gases increases with temperature.

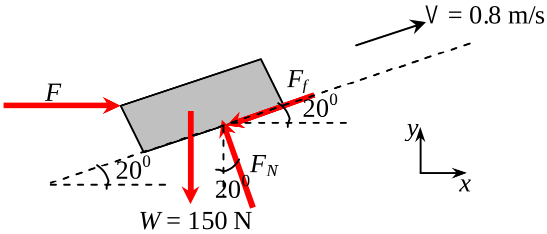
10-17C For *liquids*, the kinematic viscosity is practically independent of pressure. For *gases*, the kinematic viscosity is inversely proportional to density and thus pressure since the density of a gas is proportional to its pressure.

10-18 A block is moved at a constant velocity on an inclined surface. The force that needs to be applied in the horizontal direction when the block is dry, and the percent reduction in the required force when an oil film is applied on the surface are to be determined.

Assumptions **1** The inclined surface is plane. **2** The friction coefficient and the oil film thickness are uniform. **3** The weight of the oil layer is negligible.

Properties The absolute viscosity of oil is given to be μ = 0.012 Pa·s = 0.012 N·s/m^2.

Analysis (*a*) The velocity of the block is constant, and thus its acceleration and the net force acting on it are zero. Free body diagram of the block is given. Then the force balance gives



$$\sum F_x = 0: \quad F - F_f \cos 20° - F_N \sin 20° = 0 \quad (1)$$

$$\sum F_y = 0: \quad F_N \cos 20° - F_f \sin 20° - W = 0 \quad (2)$$

Friction force: $F_f = fF_N$ (3)

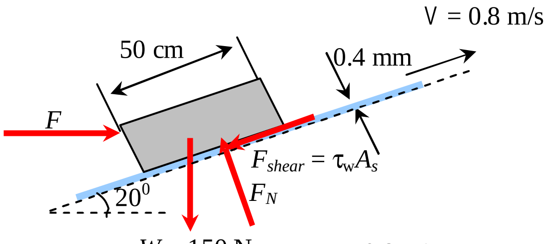
Substituting Eq. (3) into Eq. (2) and solving for F_N gives

$$F_N = \frac{W}{\cos 20° - f \sin 20°} = \frac{150 \text{ N}}{\cos 20° - 0.27 \sin 20°} = 177.0 \text{ N}$$

Then from Eq. (1):

$$F = F_f \cos 20° + F_N \sin 20° = (0.27 \times 177 \text{ N}) \cos 20° + (177 \text{ N}) \sin 20° = \mathbf{105\ N}$$

(*b*) In this case, the friction force will be replaced the shear force applied on the bottom surface of the block by oil. Because of the no-slip condition, the oil film will stick no the inclined surface at the bottom and the lower surface of the block at the top. Then the shear force can be expressed as



$$F_{shear} = \tau_w A_s = \mu A_s \frac{\mathbf{V}}{h} = (0.012 \text{ N}\cdot\text{s/m}^2)(0.5 \times 0.2 \text{ m}^2) \frac{0.8 \text{ m/s}}{4 \times 10^{-4} \text{ m}} = 2.4 \text{ N}$$

Replacing the friction force by the shear force in part (*a*), the required horizontal force is determined to be

$$F = F_{shear} \cos 20° + F_N \sin 20° = (2.4 \text{ N}) \cos 20° + (177 \text{ N}) \sin 20° = 62.8 \text{ N}$$

Then,

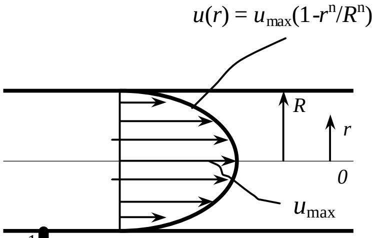
$$\text{Percentage reduction in required force} = \frac{105 - 62.8}{105} 100 = \mathbf{40.2\%}$$

Discussion Note that the force required to push the block on the inclined surface reduces significantly by oiling the surface.

10-19 The velocity profile of a fluid flowing though a circular pipe is given. The friction drag force exerted on the pipe by the fluid in the flow direction per unit length of the pipe is to be determined.

Assumptions The viscosity of the fluid is constant.

Analysis The wall shear stress is determined from its definition to be



$$\tau_w = -\mu \left.\frac{du}{dr}\right|_{r=R} = -\mu u_{max} \frac{d}{dr}\left(1-\frac{r^n}{R^n}\right)_{r=R} = -\mu u_{max} \left.\frac{-nr^{n-1}}{R^n}\right|_{r=R} = \frac{n\mu u_{max}}{R}$$

Note that the quantity du/dr is negative in pipe flow, and the negative sign is added to the τ_w relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or, $du/dr = -du/dy$ since $y = R - r$). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F = \tau_w A_w = \frac{n\mu u_{max}}{R}(2\pi R)L = 2n\pi\mu u_{max} L$$

Therefore, the drag force per unit length of the pipe is

$$F/L = 2n\pi\mu u_{max}\ .$$

Discussion Note that the drag force acting on the pipe in this case is independent of the pipe diameter.

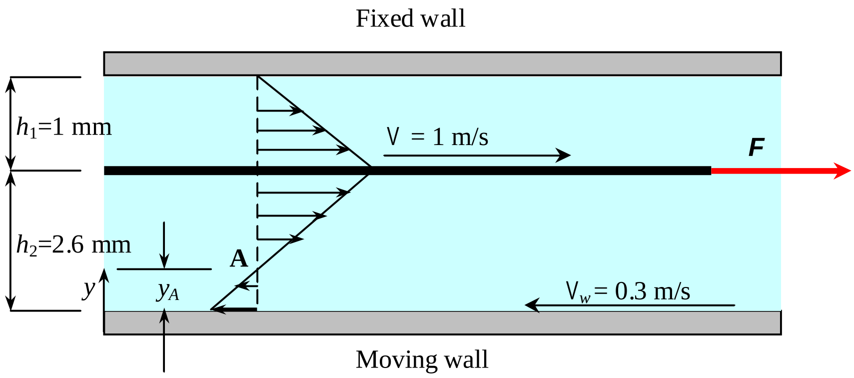
10-20 A thin flat plate is pulled horizontally through an oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity. The location in oil where the velocity is zero and the force that needs to be applied on the plate are to be determined.

Assumptions **1** The thickness of the plate is negligible. **2** The velocity profile in each oil layer is linear.

Properties The absolute viscosity of oil is given to be μ = 0.027 Pa·s = 0.027 N·s/m^2.

Analysis (*a*) The velocity profile in each oil layer relative to the fixed wall is as shown in the figure below. The point of zero velocity is indicated by point *A*, and its distance from the lower plate is determined from geometric considerations (the similarity of the two triangles in the lower oil layer) to be

$$\frac{2.6 - y_A}{y_A} = \frac{1}{0.3} \quad \rightarrow \quad y_A = \mathbf{0.60\ mm}$$



(*b*) The magnitudes of shear forces acting on the upper and lower surfaces of the plate are

$$F_{\text{shear, upper}} = \tau_{w,\text{ upper}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V - 0}{h_1} = (0.027\ \text{N}\cdot\text{s/m}^2)(0.2\times 0.2\ \text{m}^2)\frac{1\ \text{m/s}}{2.6\times 10^{-3}\ \text{m}} = 1.08\ \text{N}$$

$$F_{\text{shear, lower}} = \tau_{w,\text{ lower}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V - V_w}{h_2} = (0.027\ \text{N}\cdot\text{s/m}^2)(0.2\times 0.2\ \text{m}^2)\frac{[1-(-0.3)]\ \text{m/s}}{2.6\times 10^{-3}\ \text{m}} = 0.54\ \text{N}$$

Noting that both shear forces are in the opposite direction of motion of the plate, the force *F* is determined from a force balance on the plate to be

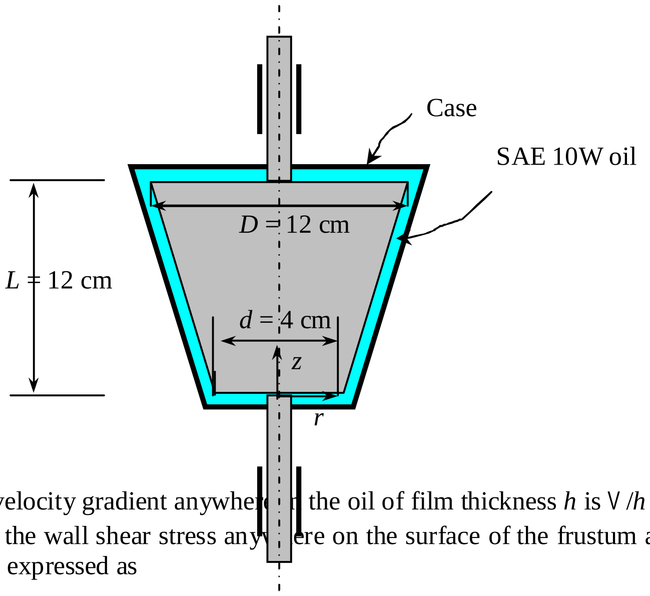
$$F = F_{\text{shear, upper}} + F_{\text{shear, lower}} = 1.08 + 0.54 = \mathbf{1.62\ N}$$

Discussion Note that wall shear is a friction force between a solid and a liquid, and it acts in the opposite direction of motion.

10-21 A frustum shaped body is rotating at a constant angular speed in an oil container. The power required to maintain this motion, and the reduction in the required power input when the oil temperature rises are to be determined.

Assumptions The thickness of the oil layer remains constant.

Properties The absolute viscosity of oil is given to be μ = 0.1 Pa·s = 0.1 N·s/m² at 20°C and 0.0078 Pa·s at 80°C.



Analysis The velocity gradient anywhere in the oil of film thickness h is V/h where $V = \omega r$ is the tangential velocity. Then the wall shear stress anywhere on the surface of the frustum at a distance r from the axis of rotation can be expressed as

$$\tau_w = \mu\frac{du}{dr} = \mu\frac{V}{h} = \mu\frac{\omega r}{h}$$

Then the shear force acting on a differential area dA on the surface, the torque it generates, and the shaft power associated with it can be expressed as

$$dF = \tau_w dA = \mu\frac{\omega r}{h}dA$$

$$d\mathrm{T} = r dF = \mu\frac{\omega r^2}{h}dA$$

$$\mathrm{T} = \frac{\mu\omega}{h}\int_A r^2 dA$$

$$\dot{W}_{\text{sh}} = \omega\mathrm{T} = \frac{\mu\omega^2}{h}\int_A r^2 dA$$

__Top surface:__ For the top surface, $dA = 2\pi r dr$. Substituting and integrating,

$$\dot{W}_{\text{sh, top}} = \frac{\mu\omega^2}{h}\int_{r=0}^{D/2} r^2(2\pi r)dr = \frac{2\pi\mu\omega^2}{h}\int_{r=0}^{D/2} r^3 dr = \frac{2\pi\mu\omega^2}{h}\left.\frac{r^4}{4}\right|_{r=0}^{D/2} = \frac{\pi\mu\omega^2 D^4}{32h}$$

__Bottom surface__: A relation for the bottom surface is obtained by replacing D by d,

$$\dot{W}_{\text{sh, bottom}} = \frac{\pi\mu\omega^2 d^4}{32h}$$

__Side surface:__ The differential area for the side surface can be expressed as $dA = 2\pi r dz$. From geometric considerations, the variation of radius with axial distance can be expressed as

$$r = \frac{d}{2} + \frac{D-d}{2L}z.$$

Differentiating gives $dr = \frac{D-d}{2L}dz$ or $dz = \frac{2L}{D-d}dr$. Therefore, $dA = 2\pi dz = \frac{4\pi L}{D-d}rdr$.

Substituting and integrating,

$$\dot{W}_{\text{sh, top}} = \frac{\mu\omega^2}{h}\int_{r=0}^{D/2} r^2 \frac{4\pi L}{D-d} rdr = \frac{4\pi\mu\omega^2 L}{h(D-d)}\int_{r=d/2}^{D/2} r^3 dr = \frac{4\pi\mu\omega^2 L}{h(D-d)} \left. \frac{r^4}{4} \right|_{r=d/2}^{D/2} = \frac{\pi\mu\omega^2 L(D^2-d^2)}{16h(D-d)}$$

Then the total power required becomes

$$\dot{W}_{\text{sh, total}} = \dot{W}_{\text{sh, top}} + \dot{W}_{\text{sh, bottom}} + \dot{W}_{\text{sh, side}} = \frac{\pi\mu\omega^2 D^4}{32h}\left[1 + (d/D)^4 + \frac{2L[1-(d/D)^4)]}{D-d}\right]$$

where d/D = 4/12 = 1/3. Substituting,

$$\dot{W}_{\text{sh, total}} = \frac{\pi(0.1\,\text{N}\cdot\text{s/m}^2)(200\,/\text{s})^2(0.12\,\text{m})^4}{32(0.0012\,\text{m})}\left[1+(1/3)^4 + \frac{2(0.12\,\text{m})[1-(1/3)^4)]}{(0.12-0.4)\text{ m}}\right]\left(\frac{1\,\text{W}}{1\,\text{Nm/s}}\right) = \mathbf{270\ W}$$

Noting that power is proportional to viscosity, the power required at 80°C is

$$\dot{W}_{\text{sh, total, 80°C}} = \frac{\mu_{80°C}}{\mu_{20°C}}\dot{W}_{\text{sh, total, 20°C}} = \frac{0.0078\text{ N}\cdot\text{s/m}^2}{0.0078\text{ N}\cdot\text{s/m}^2}(270\text{ W}) = 21.0\text{ W}$$

Therefore, the reduction in the requires power input at 80°C is

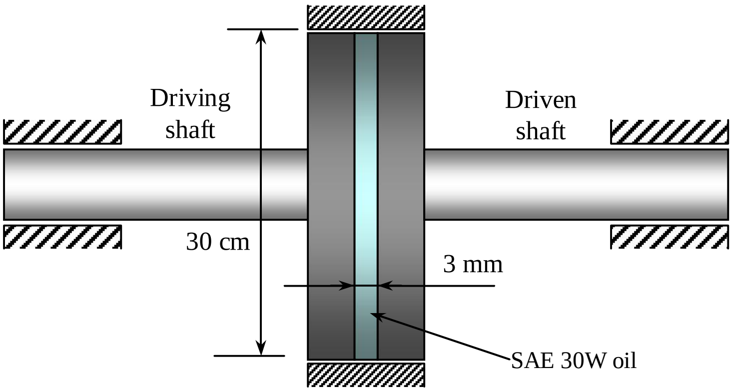
$$\text{Reduction} = \dot{W}_{\text{sh, total, 20°C}} - \dot{W}_{\text{sh, total, 80°C}} = 270 - 21 = \mathbf{249\ W} \quad (92\%)$$

Discussion Note that the power required to overcome shear forces in a viscous fluid greatly depends on temperature.

10-22 A clutch system is used to transmit torque through an oil film between two identical disks. For specified rotational speeds, the torque transmited is to be determined.

Assumptions **1** The thickness of the oil film is uniform. **2** The rotational speeds of disks remains constant.

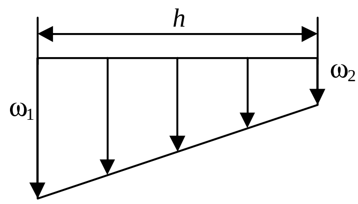
Properties The absolute viscosity of oil is given to be μ = 0.38 N·s/m^2.



Analysis The disks are rotting in the same direction at different angular speeds of ω_1 and of ω_2 . Therefore, we can assume one of the disks to be stationary and the other to be rotating at an angular speed of $\omega_1 - \omega_2$. The velocity gradient anywhere in the oil of film thickness h is V /h where V $= (\omega_1 - \omega_2)r$ is the tangential velocity. Then the wall shear stress anywhere on the surface of the faster disk at a distance r from the axis of rotation can be expressed as

$$\tau_w = \mu\frac{du}{dr} = \mu\frac{V}{h} = \mu\frac{(\omega_1 - \omega_2)r}{h}$$

Then the shear force acting on a differential area dA on the surface and the torque generation associated with it can be expressed as



$$dF = \tau_w dA = \mu\frac{(\omega_1 - \omega_2)r}{h}(2\pi r)dr$$

$$d\mathrm{T} = rdF = \mu\frac{(\omega_1 - \omega_2)r^2}{h}(2\pi r)dr = \frac{2\pi\mu(\omega_1 - \omega_2)}{h}r^3 dr$$

Integrating,

$$\mathrm{T} = \frac{2\pi\mu(\omega_1 - \omega_2)}{h}\int_{r=0}^{D/2} r^3 dr = \frac{2\pi\mu(\omega_1 - \omega_2)}{h}\left.\frac{r^4}{4}\right|_{r=0}^{D/2} = \frac{\pi\mu(\omega_1 - \omega_2)D^4}{32h}$$

Noting that $\omega = 2\pi\dot{n}$, the relative angular speed is

$$\omega_1 - \omega_2 = 2\pi(\dot{n}_1 - \dot{n}_2) = (2\pi \text{ rad/rev})[(1450 - 1398)\text{ rev/min}]\left(\frac{1\text{ min}}{60\text{ s}}\right) = 5.445 \text{ rad/s},$$

Substituting, the torque transmitted is determined to be

$$\mathrm{T} = \frac{\pi(0.38\text{ N}\cdot\text{s/m}^2)(5.445\text{ /s})(0.30\text{ m})^4}{32(0.003\text{ m})} = \mathbf{0.55\ N\cdot m}$$

Discussion Note that the torque transmitted is proportional to the fourth power of disk diameter, and is inversely proportional to the thickness of the oil film.

10-23 Prob. 10-22 is reconsidered. Using EES software, the effect of oil film thickness on the torque transmitted is investigated. Film thickness varied from 0.1 mm to 10 mm, and the results are tabulated and plotted. The relation used is, from Prob. 10-22, $T = \frac{\pi\mu(\omega_1 - \omega_2)D^4}{32h}$.

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mu=0.38
n1=1450 "rpm"
w1=2*pi*n1/60 "rad/s"
n2=1398 "rpm"
w2=2*pi*n2/60 "rad/s"
D=0.3 "m"
Tq=pi*mu*(w1-w2)*(D^4)/(32*h)
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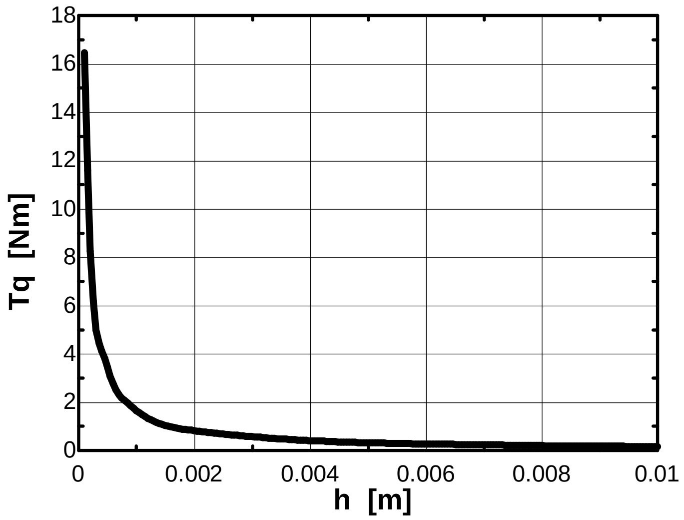
Film thickness *h*, mm	**Torque transmitted T, Nm**
0.1	16.46
0.2	8.23
0.4	4.11
0.6	2.74
0.8	2.06
1	1.65
2	0.82
4	0.41
6	0.27
8	0.21
10	0.16

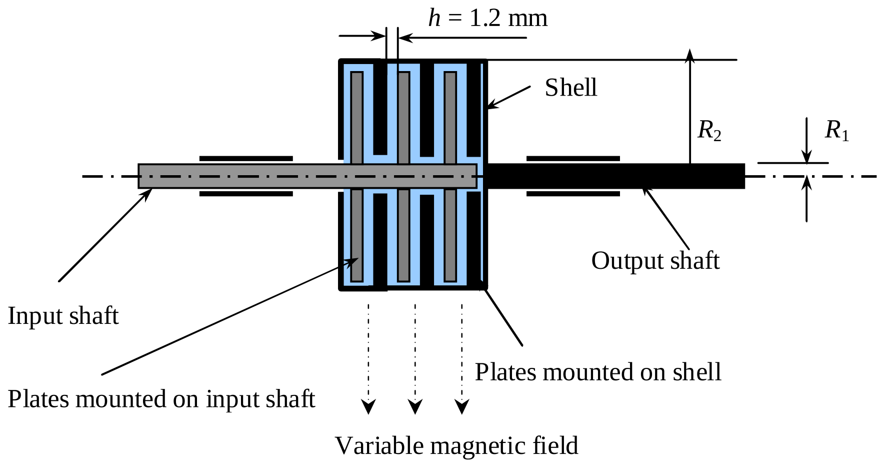


Conclusion Torque transmitted is inversely proportional to oil film thickness, and the film thickness should be as small as possible to maximize the transmitted torque.

10-24 A multi-disk Electro-rheological "ER" clutch with a fluid in which shear stress is expressed as $\tau = \tau_y + \mu(du/dy)$ is considered. A relationship for the torque transmitted by the clutch is to be obtained, and the numerical value of the torque is to be calculated.

Assumptions **1** The thickness of the oil layer between the disks is constant. **2** The Bingham plastic model for shear stress expressed as $\tau = \tau_y + \mu(du/dy)$ is valid.

Properties The constants in shear stress relation are given to be μ = 0.1 Pa·s and τ_y=2.5 kPa.



Analysis (*a*) The velocity gradient anywhere in the oil of film thickness *h* is V/h where $V = \omega r$ is the tangential velocity relative to plates mounted on the shell. Then the wall shear stress anywhere on the surface of a plate mounted on the input shaft at a distance *r* from the axis of rotation can be expressed as

$$\tau_w = \tau_y + \mu\frac{du}{dr} = \tau_y + \mu\frac{V}{h} = \tau_y + \mu\frac{\omega r}{h}$$

Then the shear force acting on a differential area *dA* on the surface of a disk and the torque generation associated with it can be expressed as

$$dF = \tau_w dA = \left(\tau_y + \mu\frac{\omega r}{h}\right)(2\pi r)dr$$

$$d\mathrm{T} = r dF = r\left(\tau_y + \mu\frac{\omega r}{h}\right)(2\pi r)dr = 2\pi\left(\tau_y r^2 + \mu\frac{\omega r^3}{h}\right)dr$$

Integrating,

$$\mathrm{T} = 2\pi\int_{r=R_1}^{R_2}\left(\tau_y r^2 + \mu\frac{\omega r^3}{h}\right)dr = 2\pi\left[\tau_y\frac{r^3}{3} + \frac{\mu\omega r^4}{4h}\right]_{r=R_1}^{R_2} = 2\pi\left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{\mu\omega}{4h}(R_2^4 - R_1^4)\right]$$

This is the torque transmitted by one surface of a plate mounted on the input shaft. Then the torque transmitted by both surfaces of N plates attached to input shaft in the clutch becomes

$$\mathrm{T} = 4\pi N\left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{\mu\omega}{4h}(R_2^4 - R_1^4)\right]$$

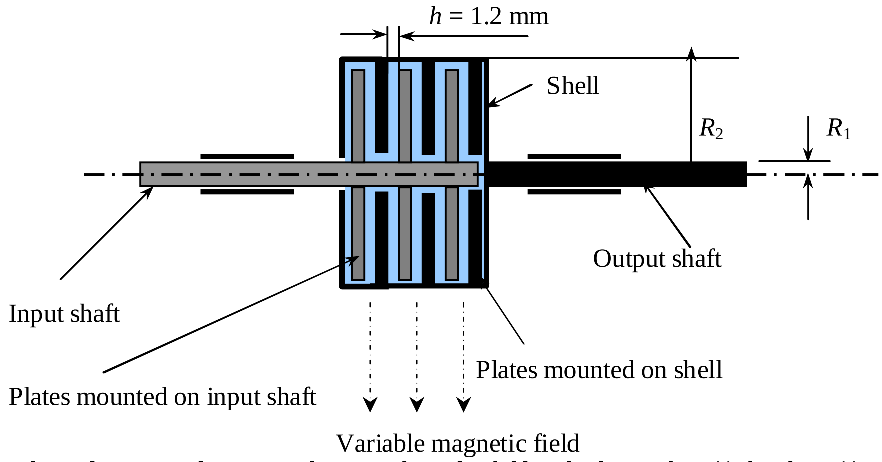
(*b*) Noting that $\omega = 2\pi\dot{n} = 2\pi(2400\text{ rev/min}) = 15{,}080\text{ rad/min} = 251.3\text{ rad/s}$ and substituting,

$$\mathrm{T} = (4\pi)(11)\left[\frac{2500\text{ N/m}^2}{3}[(0.20\text{ m})^3 - (0.05\text{ m})^3] + \frac{(0.1\text{ N}\cdot\text{s/m}^2)(251.3\text{/s})}{4(0.0012\text{ m})}[(0.20\text{ m})^4 - (0.05\text{ m})^4]\right] = \mathbf{2060\ N\cdot m}$$

10-25 A multi-disk called magnetorheological "MR" clutch with a fluid in which the shear stress is expressed as $\tau = \tau_y + K(du/dy)^m$ is considered. A relationship for the torque transmitted by the clutch is to be obtained, and the numerical value of the torque is to be calculated.

Assumptions **1** The thickness of the oil layer between the disks is constant. **2** The Herschel-Bulkley model for shear stress expressed as $\tau = \tau_y + K(du/dy)^m$ is valid.

Properties The constants in shear stress relation are given to be τ_y = 900 Pa, K = 58 Pa·s^m , and m = 0.82.



Analysis (a) The velocity gradient anywhere in the oil of film thickness h is V/h where $V = \omega r$ is the tangential velocity relative to plates mounted on the shell. Then the wall shear stress anywhere on the surface of a plate mounted on the input shaft at a distance r from the axis of rotation can be expressed as

$$\tau_w = \tau_y + K\left(\frac{du}{dr}\right)^m = \tau_y + K\left(\frac{V}{h}\right)^m = \tau_y + K\left(\frac{\omega r}{h}\right)^m$$

Then the shear force acting on a differential area dA on the surface of a disk and the torque generation associated with it can be expressed as

$$dF = \tau_w dA = \left(\tau_y + K\left(\frac{\omega r}{h}\right)^m\right)(2\pi r)dr$$

$$d\mathrm{T} = rdF = r\left(\tau_y + K\left(\frac{\omega r}{h}\right)^m\right)(2\pi r)dr = 2\pi\left(\tau_y r^2 + K\frac{\omega^m r^{m+2}}{h^m}\right)dr$$

Integrating,

$$\mathrm{T} = 2\pi\int_{R_1}^{R_2}\left(\tau_y r^2 + K\frac{\omega^m r^{m+2}}{h^m}\right)dr = 2\pi\left[\tau_y\frac{r^3}{3} + \frac{K\omega^m r^{m+3}}{(m+3)h^m}\right]_{R_1}^{R_2} = 2\pi\left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{K\omega^m}{(m+3)h^m}(R_2^{m+3} - R_1^{m+3})\right]$$

This is the torque transmitted by one surface of a plate mounted on the input shaft. Then the torque transmitted by both surfaces of N plates attached to input shaft in the clutch becomes

$$\mathrm{T} = 4\pi N\left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{K\omega^m}{(m+3)h^m}(R_2^{m+3} - R_1^{m+3})\right]$$

(b) Noting that $\omega = 2\pi\dot{n} = 2\pi(2400 \text{ rev/min}) = 15{,}080 \text{ rad/min} = 251.3 \text{ rad/s}$ and substituting,

$$\mathrm{T} = (4\pi)(11)\left[\frac{900 \text{ N/m}^2}{3}[(0.20\text{ m})^3 - (0.05\text{ m})^3] + \frac{(58\text{ N}\cdot\text{s}^{0.82}/\text{m}^2)(251.3\text{ /s})^{0.82}}{(0.82+3)(0.0012\text{ m})^{0.82}}[(0.20\text{ m})^{3.82} - (0.05\text{ m})^{3.82}]\right]$$

$$= \mathbf{103.4\ kN\cdot m}$$

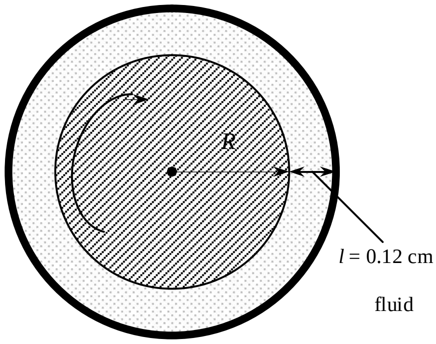
10-26 The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in oil. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is Newtonian.

Analysis Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathbf{T}l}{4\pi^2 R^3 \dot{n} L} = \frac{(0.8\ \text{N}\cdot\text{m})(0.012\ \text{m})}{4\pi^2 (0.075\ \text{m})^3 (200/60\ \text{s}^{-1})(0.75\ \text{m})} = \mathbf{0.0231\ N\cdot s/m^2}$$

Discussion This is the viscosity value at the temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.



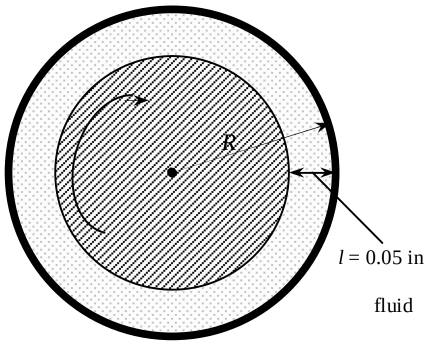
10-27E The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in the fluid. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is Newtonian.

Analysis Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathbf{T}l}{4\pi^2 R^3 \dot{n} L} = \frac{(1.2\ \text{lbf}\cdot\text{ft})(0.05/12\ \text{ft})}{4\pi^2 (5.6/12\ \text{ft})^3 (250/60\ \text{s}^{-1})(3\ \text{ft})} = \mathbf{0.000682\ lbf\cdot s/ft^2}$$

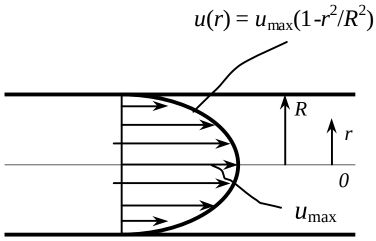
Discussion This is the viscosity value at temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.



10-28 The velocity profile for laminar one-dimensional flow through a circular pipe is given. A relation for friction drag force exerted on the pipe and its numerical value for water are to be determined.

Assumptions **1** The flow through the circular pipe is one-dimensional. **2** The fluid is Newtonian.

Properties The viscosity of water at 20°C is given to be 0.0010 kg/m·s.



Analysis The velocity profile is given by $u(r) = u_{max}\left(1-\frac{r^2}{R^2}\right)$

where R is the radius of the pipe, r is the radial distance from the center of the pipe, and u_{max} is the maximum flow velocity, which occurs at the center, $r = 0$. The shear stress at the pipe surface can be expressed as

$$\tau_w = -\mu \left.\frac{du}{dr}\right|_{r=R} = -\mu u_{max} \frac{d}{dr}\left(1-\frac{r^2}{R^2}\right)_{r=R} = -\mu u_{max} \left.\frac{-2r}{R^2}\right|_{r=R} = \frac{2\mu u_{max}}{R}$$

Note that the quantity du/dr is negative in pipe flow, and the negative sign is added to the τ_w relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or, $du/dr = - du/dy$ since $y = R - r$). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F_D = \tau_w A_s = \frac{2\mu u_{max}}{R}(2\pi RL) = 4\pi\mu L u_{max}$$

Substituting,

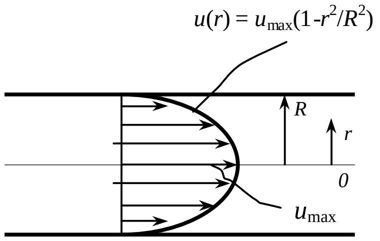
$$F_D = 4\pi\mu L u_{max} = 4\pi(0.0010\ \text{kg/m}\cdot\text{s})(15\ \text{m})(3\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{0.565\ N}$$

Discussion In the entrance region and during turbulent flow, the velocity gradient is greater near the wall, and thus the drag force in such cases will be greater.

10-29 The velocity profile for laminar one-dimensional flow through a circular pipe is given. A relation for friction drag force exerted on the pipe and its numerical value for water are to be determined.

Assumptions **1** The flow through the circular pipe is one-dimensional. **2** The fluid is Newtonian.

Properties The viscosity of water at 20°C is given to be 0.0010 kg/m·s.



Analysis The velocity profile is given by $u(r) = u_{max}\left(1 - \frac{r^2}{R^2}\right)$

where R is the radius of the pipe, r is the radial distance from the center of the pipe, and u_{max} is the maximum flow velocity, which occurs at the center, r = 0. The shear stress at the pipe surface can be expressed as

$$\tau_w = -\mu \left.\frac{du}{dr}\right|_{r=R} = -\mu u_{max} \frac{d}{dr}\left(1 - \frac{r^2}{R^2}\right)_{r=R} = -\mu u_{max} \left.\frac{-2r}{R^2}\right|_{r=R} = \frac{2\mu u_{max}}{R}$$

Note that the quantity du/dr is negative in pipe flow, and the negative sign is added to the τ_w relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or, $du/dr = - du/dy$ since $y = R - r$). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F_D = \tau_w A_s = \frac{2\mu u_{max}}{R}(2\pi RL) = 4\pi\mu L u_{max}$$

Substituting,

$$F_D = 4\pi\mu L u_{max} = 4\pi(0.0010 \text{ kg/m}\cdot\text{s})(15 \text{ m})(5 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{0.942\ N}$$

Discussion In the entrance region and during turbulent flow, the velocity gradient is greater near the wall, and thus the drag force in such cases will be larger.

Surface Tension and Capillary Effect

10-30C The magnitude of the pulling force at the surface of a liquid per unit length is called ***surface tension*** σ_s. It is caused by the attractive forces between the molecules. The surface tension is also surface energy since it represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.

10-31C The pressure inside a soap bubble is greater than the pressure outside, as evidenced by the stretch of the soap film.

10-32C The *capillary effect* is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. It is caused by the net effect of the *cohesive forces* (the forces between like molecules, like water) and *adhesive forces* (the forces between disalike molecules, like water and glass). The capillary effect is proportional to the cosine of the *contact angle*, which is the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.

10-33C The liquid level in the tube will drop since the contact angle is greater than 90°, and cos 110° < 0.

10-34C The capillary rise is inversely proportional to the diameter of the tube, and thus it is greater in the smaller-diameter tube.

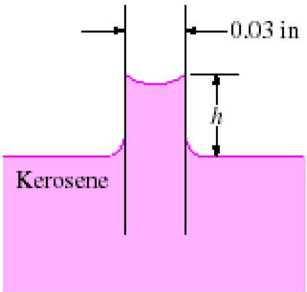
10-35E A slender glass tube is inserted into kerosene. The capillary rise of kerosene in the tube is to be determined.

Assumptions **1** There are no impurities in the kerosene, and no contamination on the surfaces of the glass tube. **2** The kerosene is open to the atmospheric air.

Properties The surface tension of kerosene-glass at 68°F (20°C) is $\sigma_s = 0.028\times0.06852 = 0.00192$ lbf/ft (Table 10-3). The density of kerosene at 68°F is $\rho = 51.2$ lbm/ft^3 (Table A-3E). The contact angle of kerosene with the glass surface is given to be 26°.

Analysis Substituting the numerical values, the capillary rise is determined to be

$$h = \frac{2\sigma_s \cos\phi}{\rho g R} = \frac{2(0.00192\,\text{lbf/ft})(\cos 26°)}{(51.2\,\text{lbm/ft}^3)(32.2\,\text{ft/s}^2)(0.015/12\,\text{ft})}\left(\frac{32.2\,\text{lbm}\cdot\text{ft/s}^2}{1\,\text{lbf}}\right)$$
$$= 0.0539\,\text{ft}$$
$$= 0.65\,\text{in}$$



Discussion The capillary rise in this case more than half of an inch, and thus it is clearly noticeable.

10-36 A glass tube is inserted into a liquid, and the capillary rise is measured. The surface tension of the liquid is to be determined.

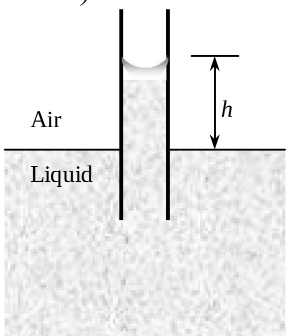
Assumptions **1** There are no impurities in the liquid, and no contamination on the surfaces of the glass tube. **2** The liquid is open to the atmospheric air.

Properties The density of the liquid is given to be 960 kg/m^3. The contact angle is given to be 15°.

Analysis Substituting the numerical values, the surface tension is determined from the capillary rise relation to be

$$\sigma_s = \frac{\rho g R h}{2\cos\phi} = \frac{(960\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.0019/2\text{ m})(0.005\text{ m})}{2(\cos 15°)}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{0.0232\ N/m}$$

Discussion The surface tension depends on temperature. Therefore, the value determined is valid at the temperature of the liquid.



10-37 The diameter of a soap bubble is given. The gage pressure inside the bubble is to be determined.

Assumptions The soap bubble is in atmospheric air.

Properties The surface tension of soap water at 20°C is σ_s = 0.025 N/m (Table 10-3).

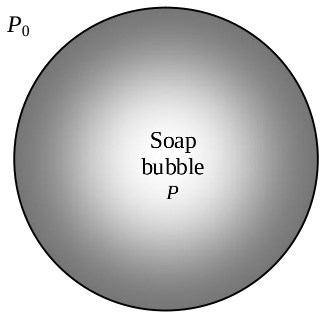
Analysis The pressure difference between the inside and the outside of a bubble is given by

$$\Delta P_{\text{bubble}} = P_i - P_0 = \frac{4\sigma_s}{R}$$

In the open atmosphere $P_0 = P_{\text{atm}}$, and thus ΔP_{bubble} is equivalent to the gage pressure. Substituting,

$$P_{i,gage} = \Delta P_{\text{bubble}} = \frac{4(0.025\text{ N/m})}{0.002/2\text{ m}} = 100\text{ N/m}^2 = 100\text{ Pa}$$

$$P_{i,gage} = \Delta P_{\text{bubble}} = \frac{4(0.025\text{ N/m})}{0.05/2\text{ m}} = 4\text{ N/m}^2 = 4\text{ Pa}$$



Discussion Note that the gage pressure in a soap bubble is inversely proportional to the radius. Therefore, the excess pressure is larger in smaller bubbles.

10-38 Nutrients dissolved in water are carried to upper parts of plants. The height that the water solution will rise in a tree as a result of the capillary effect is to be determined.

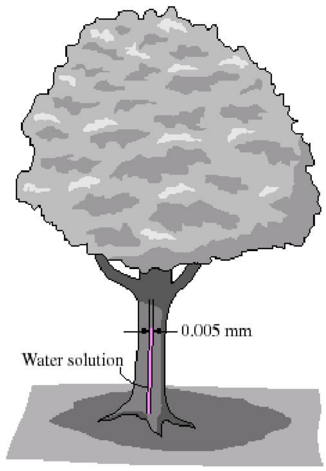
Assumptions **1** The solution can be treated as water with a contact angle of 15°. **2** The diameter of the tube is constant. **3** The temperature of the water solution is 20°C.

Properties The surface tension of water at 20°C is σ_s = 0.073 N/m (Table 10-3). The density of water solution can be taken to be 1000 kg/m^3. The contact angle is given to be 15°.

Analysis Substituting the numerical values, the capillary rise is determined to be

$$h = \frac{2\sigma_s \cos\phi}{\rho g R} = \frac{2(0.073\ \text{N/m})(\cos 15°)}{(100\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.5\times 10^{-6}\ \text{m})}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 5.75\ \text{m}$$

Discussion Other effects such as the chemical potential difference also cause the fluid to rise in trees.



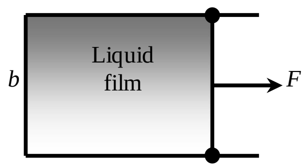
10-39 The force acting on the movable wire of a liquid film suspended on a U-shaped wire frame is measured. The surface tension of the liquid in the air is to be determined.

Assumptions **1** There are no impurities in the liquid, and no contamination on the surfaces of the wire frame. **2** The liquid is open to the atmospheric air.

Analysis Substituting the numerical values, the surface tension is determined from the surface tension force relation to be

$$\sigma_s = \frac{F}{2b} = \frac{0.012\ \text{N}}{2(0.08\ \text{m})} = 0.075\ \mathbf{N/m}$$

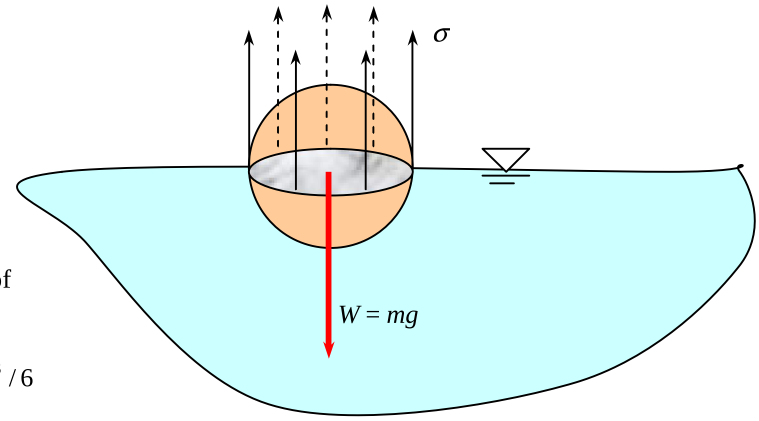
Discussion The surface tension depends on temperature. Therefore, the value determined is valid at the temperature of the liquid.



10-40 A steel ball floats on water due to the surface tension effect. The maximum diameter of the ball is to be determined, and the calculations are to be repeated for aluminum.

Assumptions **1** The water is pure, and its temperature is constant. **2** The ball is dropped on water slowly so that the inertial effects are negligible. **3** The contact angle is taken to be 0° for maximum diameter.

Properties The surface tension of water at 20°C is σ_s = 0.073 N/m (Table 10-3). The contact angle is taken to be 0°. The densities of steel and aluminum are given to be ρ_{steel} = 7800 kg/m^3 and ρ_{Al} = 2700 kg/m^3.



Analysis The surface tension force and the weigh of the ball can be expressed as

$$F_s = \pi D\sigma_s \quad \text{and} \quad W = mg = \rho g V = \rho g \pi D^3 / 6$$

When the ball floats, the net force acting on the ball in the vertical direction is zero. Therefore, setting $F_s = W$ and solving for diameter D gives

$$D = \sqrt{\frac{6\sigma_s}{\rho g}}$$

Substitititing the known quantities, the maximum diameters for the steel and aluminum balls become

$$D_{steel} = \sqrt{\frac{6\sigma_s}{\rho g}} = \sqrt{\frac{6(0.073\ \text{N/m})}{(7800\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = 2.4\times10^{-3}\ \text{m} = 2.4\ \text{mm}$$

$$D_{steel} = \sqrt{\frac{6\sigma_s}{\rho g}} = \sqrt{\frac{6(0.073\ \text{N/m})}{(2700\ \text{kg/m}^3)(9.81\text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = 4.1\times10^{-3}\ \text{m} = 4.1\,\text{mm}$$

Discussion Note that the ball diameter is inversely proportional to the square root of density, and thus for a given material, the smaller balls are more likely to float.

Review Problems

10-41E The minimum pressure on the suction side of a water pump is given. The maximum water temperature to avoid the danger of cavitation is to be determined.

Properties The saturation temperature of water at 0.95 psia is 100°F (Table A-4E).

Analysis To avoid cavitation at a specified pressure, the fluid temperature everywhere in the flow should remain below the saturation temperature at the given pressure, which is

$$T_{\max} = T_{sat\,@\,0.95\,psia} = \mathbf{100°F}$$

Therefore, the T < 100°F to avoid cavitation.

Discussion Note that saturation temperature increases with pressure, and thus cavitation may occur at higher pressure locations at higher fluid temperatures.

10-42 Air in a partially filled closed water tank is evacuated. The absolute pressure in the evacuated space is to be determined.

Properties The saturation pressure of water at 60°C is 19.94 kPa (Table A-4).

Analysis When air is completely evacuated, the vacated space is filled with water vapor, and the tank contains a saturated water-vapor mixture at the given pressure. Since we have a two-phase mixture of a pure substance at a specified temperature, the vapor pressure must be the saturation pressure at this temperature. That is,

$$P_v = P_{sat\,@\,60°C} = \mathbf{19.94\ kPa}$$

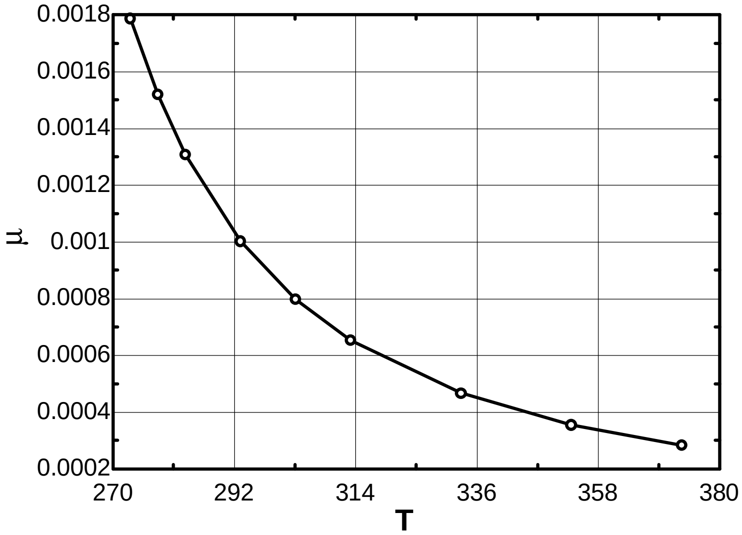
Discussion If there is any air left in the contained, the vapor pressure will be less. In that case the sum of the component pressures of vapor and air would equal 19.94 kPa.

10-43 The variation of the dynamic viscosity of water with absolute temperature is given. Using tabular data, a relation is to be obtained for viscosity as a 4th degree polynomial. The result is to be compared to Andrade's equation in the form of $\mu = D \cdot e^{B/T}$.

Properties The viscosity data are given in tabular form as

T(K)	μ (Pa.s)
273.15	1.787×10^{-3}
278.15	1.519×10^{-3}
283.15	1.307×10^{-3}
293.15	1.002×10^{-3}
303.15	7.975×10^{-4}
313.15	6.529×10^{-4}
333.15	4.665×10^{-4}
353.15	3.547×10^{-4}
373.15	2.828×10^{-4}

Analysis Using EES, (1) Define a trivial function a=mu+T in equation window, (2) select new parametric table from Tables, and type the data in a two-column table, (3) select Plot and plot the data, and (4) select plot and click on "curve fit" to get curve fit window. Then specify polynomial and enter/edit equation. The results are:



μ = 0.489291758 - 0.00568904387T + 0.0000249152104T^2 - 4.86155745$\times10^{-8}T^3$ + 3.56198079$\times10^{-11}T^4$
μ = 0.000001475*EXP(1926.5/T) [used initial guess of a0=1.8$\times10^{-6}$ and a1=1800 in mu=a0*exp(a1/T)]

At T = 323.15 K, the polynomial and exponential curve fits give

Polynomial: μ(323.15 K) = 0.0005529 Pa·s (1.1% error, relative to 0.0005468 Pa·s)
Exponential: μ(323.15 K) = 0.0005726 Pa·s (4.7% error, relative to 0.0005468 Pa·s)

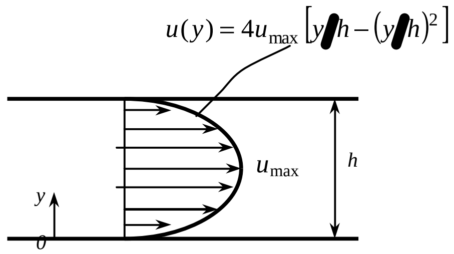
Discussion This problem can also be solved using an Excel worksheet, with the following results:
Polynomial: A = 0.4893, B = -0.005689, C = 0.00002492, D = -0.000000048612, and E = 0.00000000003562

Andrade's equation: $\mu = 1.807952E-6 * e^{1864.06/T}$

10-44 The velocity profile for laminar one-dimensional flow between two parallel plates is given. A relation for friction drag force exerted on the plates per unit area of the plates is to be obtained.

Assumptions **1** The flow between the plates is one-dimensional. **2** The fluid is Newtonian.

Analysis The velocity profile is given by $u(y) = 4u_{\max}\left[y/h - (y/h)^2\right]$



where h is the distance between the two plates, y is the vertical distance from the bottom plate, and $u_{\max}$ is the maximum flow velocity that occurs at midplane. The shear stress at the bottom surface can be expressed as

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0} = 4\mu u_{\max} \frac{d}{dy}\left(\frac{y}{h} - \frac{y^2}{h^2}\right)_{y=0} = 4\mu u_{\max}\left.\left(\frac{1}{h} - \frac{2y}{h^2}\right)\right|_{y=0} = \frac{4\mu u_{\max}}{h}$$

Because of symmetry, the wall shear stress is identical at both bottom and top plates. Then the friction drag force exerted by the fluid on the inner surface of the plates becomes

$$F_D = 2\tau_w A_{plate} = \frac{8\mu u_{\max}}{h} A_{\text{plate}}$$

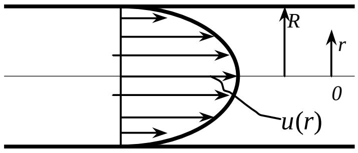
Therefore, the friction drag per unit plate area is

$$F_D / A_{\text{plate}} = \frac{8\mu u_{\max}}{h}$$

Discussion Note that the friction drag force acting on the plates is inversely proportional to the distance between plates.

10-45 The laminar flow of a Bingham plastic fluid in a horizontal pipe of radius R is considered. The shear stress at the pipe wall and the friction drag force acting on a pipe section of length L are to be determined.

Assumptions **1** The fluid is a Bingham plastic with $\tau = \tau_y + \mu(du/dr)$ where τ_y is the yield stress. **2** The flow through the pipe is one-dimensional.



Analysis The velocity profile is given by $u(r) = \dfrac{\Delta P}{4\mu L}(r^2 - R^2) + \dfrac{\tau_y}{\mu}(r - R)$ where $\Delta p/L$ is the pressure drop along the pipe per unit length, μ is the dynamic viscosity, r is the radial distance from the centerline. Its gradient at the pipe wall ($r = R$) is

$$\left.\frac{du}{dr}\right|_{r=R} = \left.\frac{d}{dr}\left(\frac{\Delta P}{4\mu L}(r^2 - R^2) + \frac{\tau_y}{\mu}(r - R)\right)\right|_{r=R} = \left(2r\frac{\Delta P}{4\mu L} + \frac{\tau_y}{\mu}\right)_{r=R} = \frac{1}{\mu}\left(\frac{\Delta P}{2L}R + \tau_y\right)$$

Substituing into $\tau = \tau_y + \mu(du/dr)$, the wall shear stress at the pipe surface becomes

$$\tau_w = \tau_y + \mu\left.\frac{du}{dr}\right|_{r=R} = \tau_y + \frac{\Delta P}{2L}R + \tau_y = 2\tau_y + \frac{\Delta P}{2L}R$$

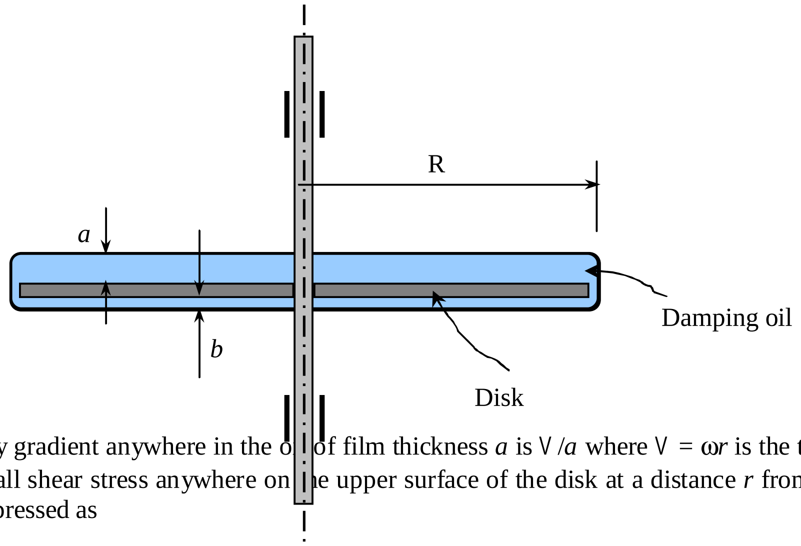
Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F_D = \tau_w A_s = \left(2\tau_y + \frac{\Delta P}{2L}R\right)(2\pi RL) = 2\pi RL\left(2\tau_y + \frac{\Delta P}{2L}R\right) = 4\pi RL\,\tau_y + \pi R^2 \Delta P$$

Discussion Note that the total friction drag is proportional to yield shear stress and the pressure drop.

10-46 A circular disk immersed in oil is used as a damper, as shown in the figure. It is to be shown that the damping torque is $\mathrm{T}_{\text{damping}} = C\omega$ where $C = 0.5\pi\mu(1/a + 1/b)R^4$.

Assumptions **1** The thickness of the oil layer on each side remains constant. **2** The velocity profiles are linear on both sides of the disk. **3** The tip effects are negligible. **4** The effect of the shaft is negligible.



Analysis The velocity gradient anywhere in the oil of film thickness a is V/a where $V = \omega r$ is the tangential velocity. Then the wall shear stress anywhere on the upper surface of the disk at a distance r from the axis of rotation can be expressed as

$$\tau_w = \mu\frac{du}{dr} = \mu\frac{V}{a} = \mu\frac{\omega r}{a}$$

Then the shear force acting on a differential area dA on the surface and the torque it generates can be expressed as

$$dF = \tau_w dA = \mu\frac{\omega r}{a}dA$$

$$d\mathrm{T} = r\,dF = \mu\frac{\omega r^2}{a}dA$$

Noting that $dA = 2\pi r\,dr$ and integrating, the torque on the top surface is determined to be

$$\mathrm{T}_{\text{top}} = \frac{\mu\omega}{a}\int_A r^2 dA = \frac{\mu\omega}{a}\int_{r=0}^{R} r^2(2\pi r)dr = \frac{2\pi\mu\omega}{a}\int_{r=0}^{R} r^3 dr = \frac{2\pi\mu\omega}{a}\left.\frac{r^4}{4}\right|_{r=0}^{R} = \frac{\pi\mu\omega R^4}{2a}$$

The torque on the bottom surface is obtained by replaying a by b,

$$\mathrm{T}_{\text{bottom}} = \frac{\pi\mu\omega R^4}{2b}$$

The total torque acting on the disk is the sum of the torques acting on the top and bottom surfaces,

$$\mathrm{T}_{\text{damping, total}} = \mathrm{T}_{\text{bottom}} + \mathrm{T}_{\text{top}} = \frac{\pi\mu\omega R^4}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

or,

$$\mathrm{T}_{\text{damping, total}} = C\omega \quad \text{where } C = \frac{\pi\mu R^4}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

This completes the proof.

Discussion Note that the damping torque (and thus damping power) is inversely proportional to the thickness of oil films on either side, and it is proportional to the 4$^{\text{th}}$ power of the radius of the damper disk.

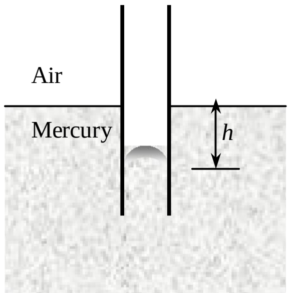
10-47E A glass tube is inserted into mercury. The capillary drop of mercury in the tube is to be determined.

Assumptions **1** There are no impurities in mercury, and no contamination on the surfaces of the glass tube. **2** The mercury is open to the atmospheric air.

Properties The surface tension of mercury-glass in atmospheric air at 68°F (20°C) is $\sigma_s = 0.440 \times 0.06852 = 0.03015$ lbf/ft (Table 9-2). The density of mercury is $\rho = 847$ lbm/ft^3 at 77°F (Table A-3E), but we can also use this value at 68°F. The contact angle is given to be 140°.

Analysis Substituting the numerical values, the capillary drop is determined to be

$$h = \frac{2\sigma_s \cos\phi}{\rho g R} = \frac{2(0.03015\,\text{lbf/ft})(\cos 140°)}{(847\,\text{lbm/ft}^3)(32.2\,\text{ft/s}^2)(0.45/12\,\text{ft})}\left(\frac{32.2\,\text{lbm}\cdot\text{ft/s}^2}{1\,\text{lbf}}\right)$$
$$= -0.00145\,\text{ft}$$
$$= -0.0175\,\text{in}$$



Discussion The negative sign indicates capillary drop instead of rise. The drop is very small in this case because of the large diameter of the tube.

10-48 A relation is to be derived for the capillary rise of a liquid between two large parallel plates a distance *t* apart inserted into a liquid vertically. The contact is given to be φ..

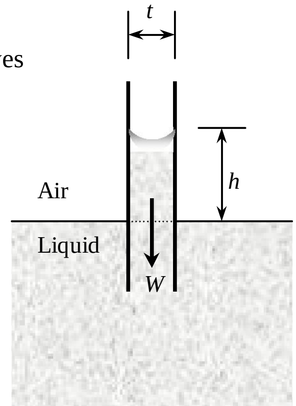
Assumptions There are no impurities in the liquid, and no contamination on the surfaces of the plates.

Analysis The magnitude of the capillary rise between two large parallel plates can be determined from a force balance on the rectangular liquid column of height *h* and width *w* between the plates. The bottom of the liquid column is at the same level as the free surface of the liquid reservoir, and thus the pressure there must be atmospheric pressure. This will balance the atmospheric pressure acting from the top surface, and thus these two effects will cancel each other. The weight of the liquid column is

$$W = mg = \rho g V = \rho g (w \times t \times h)$$

Equating the vertical component of the surface tension force to the weight gives

$$W = F_{surface} \quad \rightarrow \quad \rho g (w \times t \times h) = 2 w \sigma_s \cos\phi$$



Canceling *w* and solving for *h* gives the capillary rise to be

Capillary rise: $$h = \frac{2\sigma_s \cos\phi}{\rho g t}$$

Discussion The relation above is also valid for non-wetting liquids (such as mercury in glass), and gives the capillary drop.

10-49 A journal bearing is lubricated with oil whose viscosity is known. The torques needed to overcome the bearing friction during start-up and steady operation are to be determined.

Assumptions **1** The gap is uniform, and is completely filled with oil. **2** The end effects on the sides of the bearing are negligible. **3** The fluid is Newtonian.

Properties The viscosity of oil is given to be 0.1 kg/m·s at 20°C, and 0.008 kg/m·s at 80°C.

Analysis The radius of the shaft is R = 0.04 m. Substituting the given values, the torque is determined to be

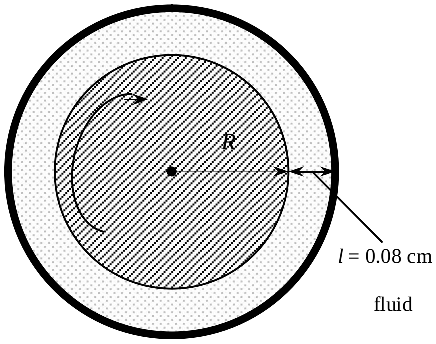
At start up at 20°C:

$$\mathbf{T} = \mu \frac{4\pi^2 R^3 \dot{n} L}{l} = (0.1\,\text{kg/m}\cdot\text{s})\frac{4\pi^2 (0.04\,\text{m})^3 (500/60\,\text{s}^{-1})(0.30\,\text{m})}{0.00008\,\text{m}} = \mathbf{0.79\,N\cdot m}$$

During steady operation at 80°C:

$$\mathbf{T} = \mu \frac{4\pi^2 R^3 \dot{n} L}{l} = (0.008\,\text{kg/m}\cdot\text{s})\frac{4\pi^2 (0.04\,\text{m})^3 (500/60\,\text{s}^{-1})(0.30\,\text{m})}{0.00008\,\text{m}} = \mathbf{0.063\,N\cdot m}$$

Discussion Note that the torque needed to overcome friction reduces considerably due to the decrease in the viscosity of oil at higher temperature.



10-50 … 10-52 Design and Essay Problems

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