Examenvragen hoofdstuk 8 van de laatste drie jaren

Werner Peeters

1. Bereken, en schrijf *bij elke stap* de gedane rij-en kolombewerkingen!

$$\begin{vmatrix} 1 & -3 & 8 & 2 \\ 6 & 0 & 4 & -11 \\ -4 & 8 & 1 & 1 \\ 7 & 0 & 0 & 3 \end{vmatrix}$$

2. Los op met de regel van Cramer:

$$\begin{cases} x+y+z=6 \\ w+y+z=5 \\ w+x+z=8 \\ w+x+y=-1 \end{cases}$$

3. Gegeven $A = \begin{pmatrix} \frac{7}{4} & -1 & \frac{17}{4} \\ -\frac{5}{4} & 1 & -\frac{15}{4} \\ \frac{3}{2} & -1 & \frac{9}{2} \end{pmatrix}$. Bereken A^{-1}.

4. Los op met de methode van Gauss-Jordan, en schrijf *bij elke stap* de gedane rijbewerkingen

$$\begin{cases} w+2x-4y+7z=33 \\ 2w-3x+y-5z=28 \\ -2w+17x-19y+13z=48 \end{cases}$$

5. Voor welke waarde (n) van λ is de determinant

$$\begin{vmatrix} \lambda+8 & 10-4\lambda & -\lambda-2 & 2\lambda \\ 3 & -\lambda-1 & -2\lambda-2 & 0 \\ 5-3\lambda & 6-2\lambda & -1 & 2\lambda \\ 6 & 9-3\lambda & -1 & 2\lambda \end{vmatrix}$$

nul? Hint: gebruik de meest economische rij–en kolombewerkingen.

6. Los op met de methode van Cramer (en géén andere!)

$$\begin{cases} -2x+y+z=8 \\ 3x+16y-4z=3 \end{cases}$$

7. Schrijf het element $\overrightarrow{v}\,(2,17,-1) \in \mathbb{R}^3$ ten opzichte van de basis $\overrightarrow{e_1}\,(1,3,0)$, $\overrightarrow{e_2}\,(2,-1,1)$ en $\overrightarrow{e_3}\,(1,0,1)$

8. Los op met de methode van Gauss-Jordan:

$$\begin{cases} x + 2y - z = 6 \\ x + 3y - 3z = 3 \\ 2x + 6y + z = 13 \end{cases}$$

9. Bereken $\begin{vmatrix} 1 & 6 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 1 & -2 & 5 & 1 \\ 0 & 0 & 3 & 7 \end{vmatrix}$ door goede rij-en kolombewerkingen.

10. Los op met de methode van Gauss-Jordan:

$$\begin{cases} w + 3x + 5y - 7z = 23 \\ w + 2x + y - 10z = -4 \end{cases}$$

11. Los op met een methode naar keuze:

$$\begin{cases} 2x + 3y + 4z = 16 \\ 5w + 6y + 7z = 39 \\ 8w + 9x + 10z = 69 \\ 11w + 12x + 13y = 106 \end{cases}$$

12. Voor welke waarde(n) van λ zijn de vectoren $\overrightarrow{u}\left(\lambda + 3, 3\lambda^2 - 3, 0\right)$, $\overrightarrow{v}\left(1, \lambda^2 - 5, -2\lambda - 4\right)$ en $\overrightarrow{w}\,(0, 2, \lambda + 2)$ lineair afhankelijk?

13. Bereken

$$\begin{pmatrix} 2 & 0 & 5 & 1 \\ -3 & 0 & 6 & 2 \\ -1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 0 & -3 & -1 \\ 5 & 2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

14. Los op met de methode van Gauss–Jordan:

$$\begin{cases} 2x + y - 3z = 9 \\ 2y - z = 7 \\ 4x + 4y - 7z = 25 \\ 2x - 3y - z = -5 \end{cases}$$

15. Ga na wanneer voor welke waarde (n) van k de volgende determinant nul is.

$$\begin{vmatrix} 2 & 0 & 1 & -1 \\ 1 & -2 & k & 2 \\ k & 2 & 0 & -3 \\ 1 & 5 & 2 & 1 \end{vmatrix}$$

16. Los op met de methode van Cramer:

$$\begin{cases} 13x - 3y + z = 1 \\ 3x + 2z = -3 \\ 2x - y - z = 2 \end{cases}$$

Oplossingen:

Opmerking: Meer dan bij eender welk ander hoofdstuk zijn er uiteraard oneindig veel mogelijkheden om de juiste oplossing te vinden. De onderstaande oplossingen zijn dan ook maar *een* modeloplossing; elke andere oplossing die tot dezelfde of een equivalente uitkomst leidt is uiteraard ook juist.

1. Bereken, en schrijf *bij elke stap* de gedane rij-en kolombewerkingen!

$$\begin{vmatrix} 1 & -3 & 8 & 2 \\ 6 & 0 & 4 & -11 \\ -4 & 8 & 1 & 1 \\ 7 & 0 & 0 & 3 \end{vmatrix}$$

$$\overset{3K_1-7K_2}{=} \frac{1}{3} \begin{vmatrix} -11 & -3 & 8 & 2 \\ 95 & 0 & 4 & -11 \\ -19 & 8 & 1 & 1 \\ 0 & 0 & 0 & 3 \end{vmatrix} = \begin{vmatrix} -11 & -3 & 8 \\ 95 & 0 & 4 \\ -19 & 8 & 1 \end{vmatrix} = 6945$$

$$\overset{8R_1+3R_2}{=} \frac{1}{8} \begin{vmatrix} -145 & 0 & 67 \\ 95 & 0 & 4 \\ -19 & 8 & 1 \end{vmatrix} = - \begin{vmatrix} -145 & 67 \\ 95 & 4 \end{vmatrix} = 6945$$

2. Los op met de regel van Cramer:

$$\begin{cases} x+y+z=6 \\ w+y+z=5 \\ w+x+z=8 \\ w+x+y=-1 \end{cases}$$

$$D = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -3$$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 & 1 \\ 5 & 0 & 1 & 1 \\ 8 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 0 & 6 & 1 & 1 \\ 1 & 5 & 1 & 1 \\ 1 & 8 & 0 & 1 \\ 1 & -1 & 1 & 0 \end{vmatrix} = -3$$

$$D_3 = \begin{vmatrix} 0 & 1 & 6 & 1 \\ 1 & 0 & 5 & 1 \\ 1 & 1 & 8 & 1 \\ 1 & 1 & -1 & 0 \end{vmatrix} = 6$$

$$D_4 = \begin{vmatrix} 0 & 1 & 1 & 6 \\ 1 & 0 & 1 & 5 \\ 1 & 1 & 0 & 8 \\ 1 & 1 & 1 & -1 \end{vmatrix} = -21$$

$$\Rightarrow (w,x,y,z) = \frac{-1}{3}(0,-3,6,-21) = (0,1,-2,7)$$

3. Gegeven $A = \begin{pmatrix} \frac{7}{4} & -1 & \frac{17}{4} \\ -\frac{5}{4} & 1 & -\frac{15}{4} \\ \frac{3}{2} & -1 & \frac{9}{2} \end{pmatrix}$. Bereken A^{-1}.

$$\det A = \begin{vmatrix} \frac{7}{4} & -1 & \frac{17}{4} \\ -\frac{5}{4} & 1 & -\frac{15}{4} \\ \frac{3}{2} & -1 & \frac{9}{2} \end{vmatrix} \overset{R_1+R_2, R_3+R_2}{=} \begin{vmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{5}{4} & 1 & -\frac{15}{4} \\ \frac{1}{4} & 0 & \frac{3}{4} \end{vmatrix} =^{R_1+R_2, R_3+R_2} \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{vmatrix} = \frac{3}{8} - \frac{1}{8} = \frac{1}{4}$$

$$A^\tau = \begin{pmatrix} \frac{7}{4} & -\frac{5}{4} & \frac{3}{2} \\ -1 & 1 & -1 \\ \frac{17}{4} & -\frac{15}{4} & \frac{9}{2} \end{pmatrix}$$

$$\Rightarrow A^{ad} = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{2} \\ 0 & \frac{3}{2} & \frac{5}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

$$\Rightarrow A^{-1} = 4 \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{2} \\ 0 & \frac{3}{2} & \frac{5}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 3 & 1 & -2 \\ 0 & 6 & 5 \\ -1 & 1 & 2 \end{pmatrix}$$

4. Los op met de methode van Gauss-Jordan, en schrijf *bij elke stap* de gedane rijbewerkingen

$$\begin{cases} w + 2x - 4y + 7z = 33 \\ 2w - 3x + y - 5z = 28 \\ -2w + 17x - 19y + 13z = 48 \end{cases}$$

$$\text{rg} \left(\begin{array}{cccc|c} 1 & 2 & -4 & 7 & 33 \\ 2 & -3 & 1 & -5 & 28 \\ -2 & 17 & -19 & 13 & 48 \end{array} \right) \overset{R_2-2R_1, R_3+2R_1}{=} \text{rg} \left(\begin{array}{cccc|c} 1 & 2 & -4 & 7 & 33 \\ 0 & -7 & 9 & -19 & -38 \\ 0 & 21 & -27 & 27 & 114 \end{array} \right)$$

$$\overset{R_3+3R_2}{=} \text{rg} \left(\begin{array}{cccc|c} 1 & 2 & -4 & 7 & 33 \\ 0 & -7 & 9 & -19 & -38 \\ 0 & 0 & 0 & -30 & 0 \end{array} \right) = 3$$

Kies $y = \lambda \Rightarrow$

$$... = \text{rg} \left(\begin{array}{ccc|cc} 1 & 2 & -7 & 4 & 33 \\ 0 & -7 & -19 & -9 & -38 \\ 0 & 0 & -30 & 0 & 0 \end{array} \right)$$

$$\overset{R_3/(-30)}{=} \text{rg} \left(\begin{array}{ccc|cc} 1 & 2 & -7 & 4 & 33 \\ 0 & -7 & -19 & -9 & -38 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right)$$

$$\overset{R_1+7R_3, R_2+19R_3}{=} \text{rg} \left(\begin{array}{ccc|cc} 1 & 2 & 0 & 4 & 33 \\ 0 & -7 & 0 & -9 & -38 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right)$$

$$\overset{7R_1+2R_2}{=} \text{rg} \left(\begin{array}{ccc|cc} 7 & 0 & 0 & 10 & 155 \\ 0 & -7 & 0 & -9 & -38 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right)$$

$$\overset{R_1/7, R_2/(-7)}{=} \text{rg} \left(\begin{array}{ccc|cc} 1 & 0 & 0 & \frac{10}{7} & \frac{155}{7} \\ 0 & 1 & 0 & \frac{9}{7} & \frac{38}{7} \\ 0 & 0 & 1 & 0 & 0 \end{array} \right)$$

$$\Rightarrow (w, x, y, z) = \left(\frac{155}{7}, \frac{38}{7}, 0, 0 \right) + \lambda \left(10, 9, 7, 0 \right)$$

5. Voor welke waarde (n) van λ is de determinant

$$\begin{vmatrix} \lambda+8 & 10-4\lambda & -\lambda-2 & 2\lambda \\ 3 & -\lambda-1 & -2\lambda-2 & 0 \\ 5-3\lambda & 6-2\lambda & -1 & 2\lambda \\ 6 & 9-3\lambda & -1 & 2\lambda \end{vmatrix}$$

nul? Hint: gebruik de meest economische rij–en kolombewerkingen.

$$\begin{vmatrix} \lambda+8 & 10-4\lambda & -\lambda-2 & 2\lambda \\ 3 & -\lambda-1 & -2\lambda-2 & 0 \\ 5-3\lambda & 6-2\lambda & -1 & 2\lambda \\ 6 & 9-3\lambda & -1 & 2\lambda \end{vmatrix} \overset{K_4/(2\lambda)}{=} 2\lambda \begin{vmatrix} \lambda+8 & 10-4\lambda & -\lambda-2 & 1 \\ 3 & -\lambda-1 & -2\lambda-2 & 0 \\ 5-3\lambda & 6-2\lambda & -1 & 1 \\ 6 & 9-3\lambda & -1 & 1 \end{vmatrix}$$

$$\overset{\substack{R_3-R_1 \\ R_4-R_1}}{=} 2\lambda \begin{vmatrix} \lambda+8 & 10-4\lambda & -\lambda-2 & 1 \\ 3 & -\lambda-1 & -2\lambda-2 & 0 \\ -4\lambda-3 & 2\lambda-4 & \lambda+1 & 0 \\ -\lambda-2 & \lambda-1 & \lambda+1 & 0 \end{vmatrix} = -2\lambda \begin{vmatrix} 3 & -\lambda-1 & -2\lambda-2 \\ -4\lambda-3 & 2\lambda-4 & \lambda+1 \\ -\lambda-2 & \lambda-1 & \lambda+1 \end{vmatrix}$$

$$\overset{K_3/(\lambda+1)}{=} -2\lambda(\lambda+1) \begin{vmatrix} 3 & -\lambda-1 & -2 \\ -4\lambda-3 & 2\lambda-4 & 1 \\ -\lambda-2 & \lambda-1 & 1 \end{vmatrix} \overset{\substack{R_1+2R_3 \\ R_2-R_3}}{=} -2\lambda(\lambda+1) \begin{vmatrix} -2\lambda-1 & \lambda-3 & 0 \\ -3\lambda-1 & \lambda-3 & 0 \\ -\lambda-2 & \lambda-1 & 1 \end{vmatrix}$$

$$= -2\lambda(\lambda+1) \begin{vmatrix} -2\lambda-1 & \lambda-3 \\ -3\lambda-1 & \lambda-3 \end{vmatrix} \overset{K_2/(\lambda-3)}{=} -2\lambda(\lambda+1)(\lambda-3) \begin{vmatrix} -2\lambda-1 & 1 \\ -3\lambda-1 & 1 \end{vmatrix}$$

$= -2\lambda(\lambda+1)(\lambda-3)\lambda = -2\lambda^2(\lambda+1)(\lambda-3)$

$\Rightarrow \lambda \in \{0, 3, -1\}$

6. Los op met de methode van Cramer (en géén andere!)

$$\begin{cases} -2x+y+z=8 \\ 3x+16y-4z=3 \end{cases}$$

$D = \begin{vmatrix} -2 & 1 \\ 3 & 16 \end{vmatrix} = -35 \neq 0$

Stel $z=\lambda \Rightarrow \begin{cases} -2x+y=8-\lambda \\ 3x+16y=3+4\lambda \end{cases}$

$D_x = \begin{vmatrix} 8-\lambda & 1 \\ 3+4\lambda & 16 \end{vmatrix} = 125-20\lambda$

$D_y = \begin{vmatrix} -2 & 8-\lambda \\ 3 & 3+4\lambda \end{vmatrix} = -5\lambda-30$

$\Rightarrow (x,y,z) = \left(\frac{125-20\lambda}{-35}, \frac{-5\lambda-30}{-35}, 1\right) = \left(\frac{4}{7}\lambda - \frac{25}{7}, \frac{1}{7}\lambda + \frac{6}{7}, 1\right) = (4,1,7)\lambda + \left(-\frac{25}{7}, \frac{6}{7}, 0\right)$

7. Schrijf het element $\overrightarrow{v}\,(2,17,-1) \in \mathbb{R}^3$ ten opzichte van de basis $\overrightarrow{e_1}\,(1,3,0)$, $\overrightarrow{e_2}\,(2,-1,1)$ en $\overrightarrow{e_3}\,(1,0,1)$

$(2,17,-1) = \alpha(1,3,0) + \beta(2,-1,1) + \gamma(1,0,1)$

$\begin{cases} \alpha+2\beta+\gamma=2 \\ 3\alpha-\beta=17 \\ \beta+\gamma=-1 \end{cases} \Rightarrow \begin{cases} \alpha=5 \\ \beta=-2 \\ \gamma=1 \end{cases}$

8. Los op met de methode van Gauss-Jordan:

$$\begin{cases} x+2y-z=6 \\ x+3y-3z=3 \\ 2x+6y+z=13 \end{cases}$$

$$\text{rg}\, A^+ = \text{rg}\left(\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 1 & 3 & -3 & 3 \\ 2 & 6 & 1 & 13 \end{array}\right)$$

$$\overset{\substack{R_2 - R_1 \\ R_3 - 2R_1}}{=} \text{rg}\left(\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 0 & 1 & -2 & -3 \\ 0 & 2 & 3 & 1 \end{array}\right)$$

$$\overset{\substack{R_1 - 2R_2 \\ R_3 - 2R_2}}{=} \text{rg}\left(\begin{array}{ccc|c} 1 & 0 & 3 & 12 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 7 & 7 \end{array}\right)$$

$$\overset{R_3/7}{=} \text{rg}\left(\begin{array}{ccc|c} 1 & 0 & 3 & 12 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\overset{\substack{R_1 - 3R_3 \\ R_2 + 2R_3}}{=} \text{rg}\left(\begin{array}{ccc|c} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\Rightarrow \begin{cases} x = 9 \\ y = -1 \\ z = 1 \end{cases}$$

9. Bereken $\begin{vmatrix} 1 & 6 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 1 & -2 & 5 & 1 \\ 0 & 0 & 3 & 7 \end{vmatrix}$ door goede rij-en kolombewerkingen.

$$\begin{vmatrix} 1 & 6 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 1 & -2 & 5 & 1 \\ 0 & 0 & 3 & 7 \end{vmatrix} \overset{R_3 - R_1}{=} \begin{vmatrix} 1 & 6 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & -8 & 2 & -1 \\ 0 & 0 & 3 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ -8 & 2 & -1 \\ 0 & 3 & 7 \end{vmatrix} \overset{K_2 - K_1}{=} \begin{vmatrix} 1 & 0 & 0 \\ -8 & 10 & -1 \\ 0 & 3 & 7 \end{vmatrix} = \begin{vmatrix} 10 & -1 \\ 3 & 7 \end{vmatrix} = 73$$

10. Los op met de methode van Gauss-Jordan:

$$\begin{cases} w + 3x + 5y - 7z = 23 \\ w + 2x + y - 10z = -4 \end{cases}$$

$$\text{rg}\, A^+ = \text{rg}\left(\begin{array}{cccc|c} 1 & 3 & 5 & -7 & 23 \\ 1 & 2 & 1 & -10 & -4 \end{array}\right)$$

$$\overset{R_2 - R_1}{=} \text{rg}\left(\begin{array}{cccc|c} 1 & 3 & 5 & -7 & 23 \\ 0 & -1 & -4 & -3 & -27 \end{array}\right)$$

$$\overset{R_2/(-1)}{=} \text{rg}\left(\begin{array}{cccc|c} 1 & 3 & 5 & -7 & 23 \\ 0 & 1 & 4 & 3 & 27 \end{array}\right)$$

$$\overset{R_2 - 3R_2}{=} \text{rg}\left(\begin{array}{cccc|c} 1 & 0 & -7 & -16 & -58 \\ 0 & 1 & 4 & 3 & 27 \end{array}\right)$$

Stel $y = \lambda$ en $z = \mu$

$$= \text{rg}\left(\begin{array}{cc|ccc} 1 & 0 & 7 & 16 & -58 \\ 0 & 1 & -4 & -3 & 27 \end{array}\right)$$

$$\Rightarrow (w, x, y, z) = (7\lambda + 16\mu - 58, -4\lambda - 3\mu + 27, \lambda, \mu) = (-58, 27, 0, 0) + \lambda\,(7, -4, 1, 0) + \mu\,(16, -3, 0, 1)$$

11. Los op met een methode naar keuze:

$$\begin{cases} 2x+3y+4z=16 \\ 5w+6y+7z=39 \\ 8w+9x+10z=69 \\ 11w+12x+13y=106 \end{cases}$$

$$\text{rg}\left(\begin{array}{cccc|c} 0 & 2 & 3 & 4 & 16 \\ 5 & 0 & 6 & 7 & 39 \\ 8 & 9 & 0 & 10 & 69 \\ 11 & 12 & 13 & 0 & 106 \end{array}\right) \overset{\substack{5R_3-8R_2 \\ 5R_4-11R_2}}{=} \text{rg}\left(\begin{array}{cccc|c} 0 & 2 & 3 & 4 & 16 \\ 5 & 0 & 6 & 7 & 39 \\ 0 & 45 & -48 & -6 & 33 \\ 0 & 60 & -1 & -77 & 101 \end{array}\right)$$

$$\overset{R_3:3}{=} \text{rg}\left(\begin{array}{cccc|c} 0 & 2 & 3 & 4 & 16 \\ 5 & 0 & 6 & 7 & 39 \\ 0 & 15 & -16 & -2 & 11 \\ 0 & 60 & -1 & -77 & 101 \end{array}\right) \overset{R_1 \leftrightarrow R_2}{=} \text{rg}\left(\begin{array}{cccc|c} 5 & 0 & 6 & 7 & 39 \\ 0 & 2 & 3 & 4 & 16 \\ 0 & 15 & -16 & -2 & 11 \\ 0 & 60 & -1 & -77 & 101 \end{array}\right)$$

$$\overset{\substack{2R_3-15R_2 \\ R_4-30R_2}}{=} \text{rg}\left(\begin{array}{cccc|c} 5 & 0 & 6 & 7 & 39 \\ 0 & 2 & 3 & 4 & 16 \\ 0 & 0 & -77 & -64 & -218 \\ 0 & 0 & -182 & -394 & -758 \end{array}\right) \overset{\substack{R_2:(-1) \\ R_3:(-2)}}{=} \text{rg}\left(\begin{array}{cccc|c} 5 & 0 & 6 & 7 & 39 \\ 0 & 2 & 3 & 4 & 16 \\ 0 & 0 & 77 & 64 & 218 \\ 0 & 0 & 91 & 197 & 379 \end{array}\right)$$

$$\overset{77R_4-91R_3}{=} \text{rg}\left(\begin{array}{cccc|c} 5 & 0 & 6 & 7 & 39 \\ 0 & 2 & 3 & 4 & 16 \\ 0 & 0 & 77 & 64 & 218 \\ 0 & 0 & 0 & 9345 & 9345 \end{array}\right) = 4$$

$$\overset{R_4:9345}{=} \text{rg}\left(\begin{array}{cccc|c} 5 & 0 & 6 & 7 & 39 \\ 0 & 2 & 3 & 4 & 16 \\ 0 & 0 & 77 & 64 & 218 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \overset{\substack{R_1-7R_4 \\ R_2-4R_4 \\ R_3-64R_4}}{=} \text{rg}\left(\begin{array}{cccc|c} 5 & 0 & 6 & 0 & 32 \\ 0 & 2 & 3 & 0 & 12 \\ 0 & 0 & 77 & 0 & 154 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\overset{R_4:77}{=} \text{rg}\left(\begin{array}{cccc|c} 5 & 0 & 6 & 0 & 32 \\ 0 & 2 & 3 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \overset{\substack{R_1-6R_3 \\ R_2-3R_3}}{=} \text{rg}\left(\begin{array}{cccc|c} 5 & 0 & 0 & 0 & 20 \\ 0 & 2 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\overset{\substack{R_1:5 \\ R_2:2}}{=} \text{rg}\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow (w,x,y,z) = (4,3,2,1)$$

12. Voor welke waarde(n) van λ zijn de vectoren $\overrightarrow{u}\left(\lambda+3, 3\lambda^2-3, 0\right)$, $\overrightarrow{v}\left(1, \lambda^2-5, -2\lambda-4\right)$ en $\overrightarrow{w}\left(0, 2, \lambda+2\right)$ lineair afhankelijk?

$$\text{Eis: } \begin{vmatrix} \lambda+3 & 1 & 0 \\ 3\lambda^2-3 & \lambda^2-5 & 2 \\ 0 & -2\lambda-4 & \lambda+2 \end{vmatrix} = 0 \overset{K_1-(\lambda+3)K_2}{\Leftrightarrow} \begin{vmatrix} 0 & 1 & 0 \\ -\lambda^3+5\lambda+12 & \lambda^2-5 & 2 \\ 2\lambda^2+10\lambda+12 & -2\lambda-4 & \lambda+2 \end{vmatrix} = 0$$

$$\Leftrightarrow \begin{vmatrix} -\lambda^3+5\lambda+12 & 2 \\ 2\lambda^2+10\lambda+12 & \lambda+2 \end{vmatrix} = 0$$

$$\Leftrightarrow \left(-\lambda^3+5\lambda+12\right)(\lambda+2) - 2\left(2\lambda^2+10\lambda+12\right) = 0$$

$$\Leftrightarrow -\lambda^4-2\lambda^3+\lambda^2+2\lambda = 0$$

$$\Leftrightarrow -\lambda(\lambda-1)(\lambda+2)(\lambda+1) = 0$$

$$\Rightarrow \lambda \in \{0, 1, -1, -2\}$$

13. Bereken

$$\begin{pmatrix} 2 & 0 & 5 & 1 \\ -3 & 0 & 6 & 2 \\ -1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 0 & -3 & -1 \\ 5 & 2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 5 & 1 \\ -3 & 0 & 6 & 2 \\ -1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 0 & -3 & -1 \\ 5 & 2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 & 0 & -10 & -2 \\ 7 & 7 & -8 & -7 \\ 1 & 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix}$$

14. Los op met de methode van Gauss–Jordan:

$$\begin{cases} 2x + y - 3z = 9 \\ 2y - z = 7 \\ 4x + 4y - 7z = 25 \\ 2x - 3y - z = -5 \end{cases}$$

$$\operatorname{rg} A^+ = \operatorname{rg}\left(\begin{array}{ccc|c} 2 & 1 & -3 & 9 \\ 0 & 2 & -1 & 7 \\ 4 & 4 & -7 & 25 \\ 2 & -3 & -1 & -5 \end{array}\right) \overset{\substack{R_3 - 2R_1 \\ R_4 - R_1}}{=} \operatorname{rg}\left(\begin{array}{ccc|c} 2 & 1 & -3 & 9 \\ 0 & 2 & -1 & 7 \\ 0 & 2 & -1 & 7 \\ 0 & -4 & 2 & -14 \end{array}\right) = \operatorname{rg}\left(\begin{array}{ccc|c} 2 & 1 & -3 & 9 \\ 0 & 2 & -1 & 7 \end{array}\right) = 2$$

$\Rightarrow$ Kies $z = \lambda$ dan is

$$\operatorname{rg} A^+ = \operatorname{rg}\left(\begin{array}{cc|cc} 2 & 1 & 3 & 9 \\ 0 & 2 & 1 & 7 \end{array}\right) \overset{R_2/2}{=} \operatorname{rg}\left(\begin{array}{cc|cc} 2 & 1 & 3 & 9 \\ 0 & 1 & \frac{1}{2} & \frac{7}{2} \end{array}\right)$$

$$\overset{R_1 - R_2}{=} \operatorname{rg}\left(\begin{array}{cc|cc} 2 & 0 & \frac{5}{2} & \frac{11}{2} \\ 0 & 1 & \frac{1}{2} & \frac{7}{2} \end{array}\right) \overset{R_1/2}{=} \operatorname{rg}\left(\begin{array}{cc|cc} 1 & 0 & \frac{5}{4} & \frac{11}{4} \\ 0 & 1 & \frac{1}{2} & \frac{7}{2} \end{array}\right)$$

$$\Rightarrow (x, y, z) = \left(\frac{11}{4}, \frac{7}{2}, 0\right) + \lambda\left(\frac{5}{4}, \frac{1}{2}, 1\right)$$

15. Ga na wanneer voor welke waarde (n) van k de volgende determinant nul is.

$$\begin{vmatrix} 2 & 0 & 1 & -1 \\ 1 & -2 & k & 2 \\ k & 2 & 0 & -3 \\ 1 & 5 & 2 & 1 \end{vmatrix}$$

$$\overset{\substack{K_1 - 2K_3 \\ K_4 + K_3}}{=} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 1-2k & -2 & k & 2+k \\ k & 2 & 0 & -3 \\ -3 & 5 & 2 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 1-2k & -2 & 2+k \\ k & 2 & -3 \\ -3 & 5 & 3 \end{vmatrix}$$

$$\overset{\substack{R_2 + R_1 \\ R_3 + \frac{5}{2}R_1}}{=} \begin{vmatrix} 1-2k & -2 & 2+k \\ 1-k & 0 & -1+k \\ -\frac{1}{2} - 5k & 0 & 8 + \frac{5}{2}k \end{vmatrix}$$

$$= 2\begin{vmatrix} 1-k & -1+k \\ -\frac{1}{2} - 5k & 8 + \frac{5}{2}k \end{vmatrix}$$

$= 2\left((1-k)\left(8+\frac{5}{2}k\right)-\left(-\frac{1}{2}-5k\right)(-1+k)\right)$
$= 5k^2-20k+15 = 5(k-1)(k-3) = 0$
$\Rightarrow k \in \{1,3\}$

16. Los op met de methode van Cramer:

$$\begin{cases} 13x-3y+z=1 \\ 3x+2z=-3 \\ 2x-y-z=2 \end{cases}$$

$D = \begin{vmatrix} 13 & -3 & 1 \\ 3 & 0 & 2 \\ 2 & -1 & -1 \end{vmatrix} = 2$
$D_x = \begin{vmatrix} 1 & -3 & 1 \\ -3 & 0 & 2 \\ 2 & -1 & -1 \end{vmatrix} = 2$
$D_y = \begin{vmatrix} 13 & 1 & 1 \\ 3 & -3 & 2 \\ 2 & 2 & -1 \end{vmatrix} = 6$
$D_z = \begin{vmatrix} 13 & -3 & 1 \\ 3 & 0 & -3 \\ 2 & -1 & 2 \end{vmatrix} = -6$
$\Rightarrow (x,y,z) = (1,3,-3)$