Chapter 12
BERNOULLI AND ENERGY EQUATIONS

Mechanical Energy and Pump Efficiency

12-1C The *mechanical energy* is the form of energy that can be converted to mechanical work completely and directly by a mechanical device such as a propeller. It differs from thermal energy in that thermal energy cannot be converted to work directly and completely. The forms of mechanical energy of a fluid stream are kinetic, potential, and flow energies.

12-2C *Mechanical efficiency* is defined as the ratio of the mechanical energy output to the mechanical energy input. A mechanical efficiency of 100% for a hydraulic turbine means that the entire mechanical energy of the fluid is converted to mechanical (shaft) work.

12-3C The *combined pump-motor efficiency* of a pump/motor system is defined as the ratio of the increase in the mechanical energy of the fluid to the electrical power consumption of the motor,

$$\eta_{\text{pump-motor}} = \eta_{\text{pump}}\eta_{\text{motor}} = \frac{\dot{E}_{\text{mech,out}} - \dot{E}_{\text{mech,in}}}{\dot{W}_{\text{elect,in}}} = \frac{\Delta\dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}} = \frac{\dot{W}_{\text{pump}}}{\dot{W}_{\text{elect,in}}}$$

The combined pump-motor efficiency cannot be greater than either of the pump or motor efficiency since both pump and motor efficiencies are less than 1, and the product of two numbers that are less than one is less than either of the numbers.

12-4C The turbine efficiency, generator efficiency, and *combined turbine-generator efficiency* are defined as follows:

$$\eta_{\text{turbine}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy extracted from the fluid}} = \frac{\dot{W}_{\text{shaft,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|}$$

$$\eta_{\text{generator}} = \frac{\text{Electrical power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{shaft,in}}}$$

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\eta_{\text{generaor}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{E}_{\text{mech,in}} - \dot{E}_{\text{mech,out}}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|}$$

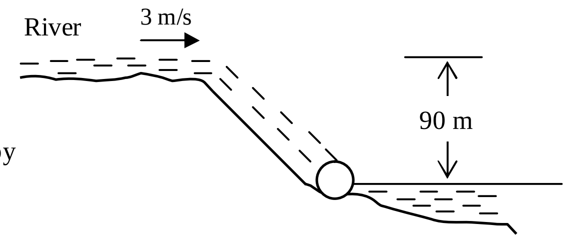
12-5 A river is flowing at a specified velocity, flow rate, and elevation. The total mechanical energy of the river water per unit mass, and the power generation potential of the entire river are to be determined. √

Assumptions **1** The elevation given is the elevation of the free surface of the river. **2** The velocity given is the average velocity. **3** The mechanical energy of water at the turbine exit is negligible.

Properties We take the density of water to be ρ = 1000 kg/m^3.

Analysis Noting that the sum of the flow energy and the potential energy is constant for a given fluid body, we can take the elevation of the entire river water to be the elevation of the free surface, and ignore the flow energy. Then the total mechanical energy of the river water per unit mass becomes

$$
\begin{aligned}
e_{\text{mech}} &= pe + ke = gh + \frac{\mathbf{V}^2}{2} \\
&= \left((9.81\,\text{m/s}^2)(90\,\text{m}) + \frac{(3\,\text{m/s})^2}{2} \right)\left(\frac{1\,\text{kJ/kg}}{1000\,\text{m}^2/\text{s}^2} \right) \\
&= 0.887\,\text{kJ/kg}
\end{aligned}
$$



The power generation potential of the river water is obtained by multiplying the total mechanical energy by the mass flow rate,

$$\dot{m} = \rho \dot{V} = (1000\,\text{kg/m}^3)(500\,\text{m}^3/\text{s}) = 500{,}000\,\text{kg/s}$$

$$\dot{W}_{\text{max}} = \dot{E}_{\text{mech}} = \dot{m} e_{\text{mech}} = (500{,}000\,\text{kg/s})(0.887\,\text{kg/s}) = 444{,}000\,\text{kW} = \mathbf{444\ MW}$$

Therefore, 444 MW of power can be generated from this river as it discharges into the lake if its power potential can be recovered completely.

Discussion Note that the kinetic energy of water is negligible compared to the potential energy, and it can be ignored in the analysis. Also, the power output of an actual turbine will be less than 444 MW because of losses and inefficiencies.

12-6 A hydraulic turbine-generator is generating electricity from the water of a large reservoir. The combined turbine-generator efficiency and the turbine efficiency are to be determined.

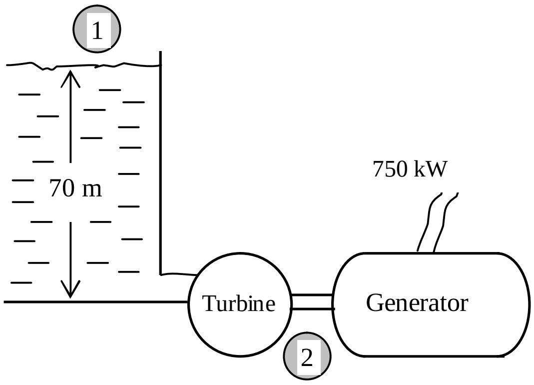
Assumptions **1** The elevation of the reservoir remains constant. **2** The mechanical energy of water at the turbine exit is negligible.

Analysis We take the free surface of the reservoir to be point 1 and the turbine exit to be point 2. We also take the turbine exit as the reference level ($z_2 = 0$), and thus the potential energy at points 1 and 2 are $\mathrm{pe}_1 = gz_1$ and $\mathrm{pe}_2 = 0$. The flow energy P/ρ at both points is zero since both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$). Further, the kinetic energy at both points is zero ($\mathrm{ke}_1 = \mathrm{ke}_2 = 0$) since the water at point 1 is essentially motionless, and the kinetic energy of water at turbine exit is assumed to be negligible. The potential energy of water at point 1 is

$$pe_1 = gz_1 = (9.81\,\mathrm{m/s^2})(70\,\mathrm{m})\left(\frac{1\,\mathrm{kJ/kg}}{1000\,\mathrm{m^2/s^2}}\right) = 0.687\,\mathrm{kJ/kg}$$

Then the rate at which the mechanical energy of the fluid is supplied to the turbine become

$$\left|\Delta\dot{E}_{\mathrm{mech,fluid}}\right| = \dot{m}(e_{\mathrm{mech,in}} - e_{\mathrm{mech,out}}) = \dot{m}(pe_1 - 0) = \dot{m}pe_1$$
$$= (1500\,\mathrm{kg/s})(0.687\,\mathrm{kJ/kg})$$
$$= 1031\,\mathrm{kW}$$



The combined turbine-generator and the turbine efficiency are determined from their definitions,

$$\eta_{\mathrm{turbine\text{-}gen}} = \frac{\dot{W}_{\mathrm{elect,out}}}{|\Delta\dot{E}_{\mathrm{mech,fluid}}|} = \frac{750\,\mathrm{kW}}{1031\,\mathrm{kW}} = 0.727 \quad \text{or} \quad \mathbf{72.7\%}$$

$$\eta_{\mathrm{turbine}} = \frac{\dot{W}_{\mathrm{shaft,out}}}{|\Delta\dot{E}_{\mathrm{mech,fluid}}|} = \frac{800\,\mathrm{kW}}{1031\,\mathrm{kW}} = 0.776 \quad \text{or} \quad \mathbf{77.6\%}$$

Therefore, the reservoir supplies 1031 kW of mechanical energy to the turbine, which converts 800 kW of it to shaft work that drives the generator, which generates 750 kW of electric power.

Discussion This problem can also be solved by taking point 1 to be at the turbine inlet, and using flow energy instead of potential energy. It would give the same result since the flow energy at the turbine inlet is equal to the potential energy at the free surface of the reservoir.

12-7 Wind is blowing steadily at a certain velocity. The mechanical energy of air per unit mass, the power generation potential, and the actual electric power generation are to be determined. √EES

Assumptions **1** The wind is blowing steadily at a constant uniform velocity. **2** The efficiency of the wind turbine is independent of the wind speed.

Properties The density of air is given to be ρ = 1.25 kg/m^3.

Analysis Kinetic energy is the only form of mechanical energy the wind possesses, and it can be converted to work entirely. Therefore, the power potential of the wind is its kinetic energy, which is $\mathbf{V}^2/2$ per unit mass, and $\dot{m}\mathbf{V}^2/2$ for a given mass flow rate:

$$e_{mech} = ke = \frac{\mathbf{V}^2}{2} = \frac{(12\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 0.072\text{ kJ/kg}$$

$$\dot{m} = \rho \mathbf{V} A = \rho \mathbf{V}\frac{\pi D^2}{4} = (1.25\text{ kg/m}^3)(12\text{ m/s})\frac{\pi(50\text{ m})^2}{4} = 29{,}450\text{ kg/s}$$

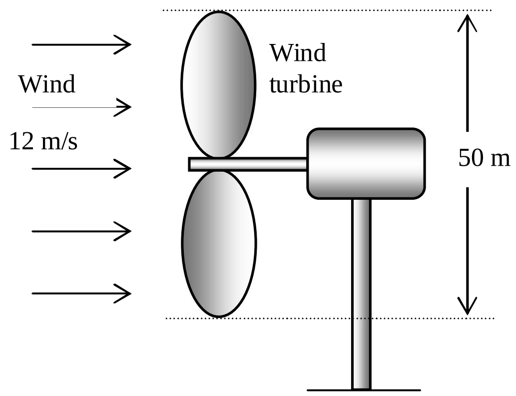
$$\dot{W}_{max} = \dot{E}_{mech} = \dot{m}e_{mech} = (29{,}450\text{ kg/s})(0.072\text{ kJ/kg}) = \mathbf{2121\ kW}$$

The actual electric power generation is determined by multiplying the power generation potential by the efficiency,

$$\dot{W}_{elect} = \eta_{\text{wind turbine}}\dot{W}_{max} = (0.30)(2121\text{ kW}) = \mathbf{636\ kW}$$

Therefore, 636 kW of actual power can be generated by this wind turbine at the stated conditions.

Discussion The power generation of a wind turbine is proportional to the cube of the wind velocity, and thus the power generation will change strongly with the wind conditions.



12-8 Problem 12-7 is reconsidered. The effect of wind velocity and the blade span diameter on wind power generation as the velocity varies from 5 m/s to 20 m/s in increments of 5 m/s, and the diameter varies from 20 m to 80 m in increments of 20 m is to be investigated.

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D1=20 "m"
D2=40 "m"
D3=60 "m"
D4=80 "m"
Eta=0.30
rho=1.25 "kg/m3"
m1_dot=rho*V*(pi*D1^2/4); W1_Elect=Eta*m1_dot*(V^2/2)/1000 "kW"
m2_dot=rho*V*(pi*D2^2/4); W2_Elect=Eta*m2_dot*(V^2/2)/1000 "kW"
m3_dot=rho*V*(pi*D3^2/4); W3_Elect=Eta*m3_dot*(V^2/2)/1000 "kW"
m4_dot=rho*V*(pi*D4^2/4); W4_Elect=Eta*m4_dot*(V^2/2)/1000 "kW"
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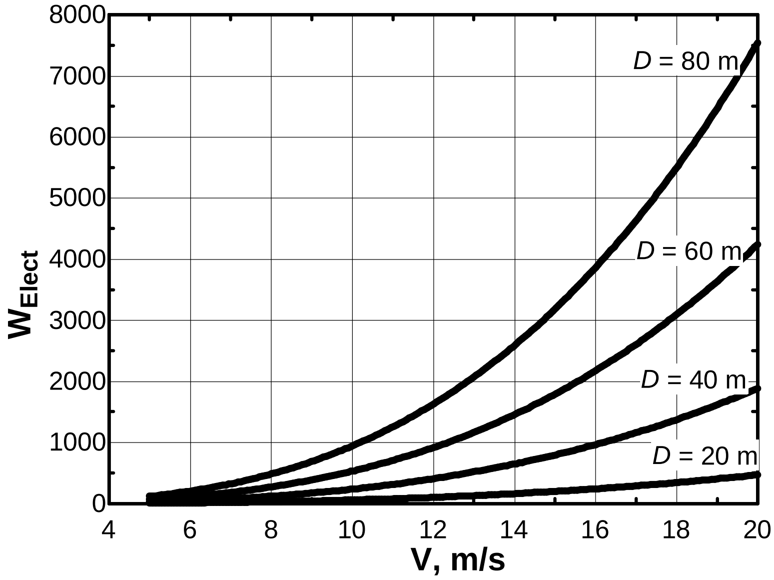
D, m	V, m/s	m, kg/s	W_{elect}, kW
20	5	1,963	7
	10	3,927	59
	15	5,890	199
	20	7,854	471
40	5	7,854	29
	10	15,708	236
	15	23,562	795
	20	31,416	1885
60	5	17,671	66
	10	35,343	530
	15	53,014	1789
	20	70,686	4241
80	5	31,416	118
	10	62,832	942
	15	94,248	3181
	20	125,664	7540

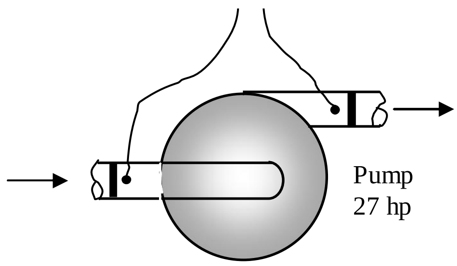


12-9E A differential thermocouple indicates that the temperature of water rises a certain amount as it flows through a pump at a specified rate. The mechanical efficiency of the pump is to be determined.

Assumptions **1** The pump is adiabatic so that there is no heat transfer with the surroundings, and the temperature rise of water is completely due to frictional heating. **2** Water is an incompressible substance.

Properties We take the density of water to be $\rho = 62.4$ lbm/ft^3 and its specific heat to be $C = 1.0$ Btu/lbm·°F (Table A-3E).

Analysis The increase in the temperature of water is due to the conversion of mechanical energy to thermal energy, and the amount of mechanical energy converted to thermal energy is equal to the increase in the internal energy of water,



$$\dot{m} = \rho \dot{V} = (62.4\ \text{lbm/ft}^3)(1.5\ \text{ft}^3/\text{s}) = 93.6\ \text{lbm/s}$$

$$\dot{E}_{\text{mech, loss}} = \Delta \dot{U} = \dot{m} C \Delta T$$

$$= (93.6\ \text{lbm/s})(1.0\ \text{Btu/lbm}\cdot°\text{F})(0.072°\text{F})\left(\frac{1\ \text{hp}}{0.7068\ \text{Btu/s}}\right) = 9.53\ \text{hp}$$

The mechanical efficiency of the pump is determined from the general definition of mechanical efficiency,

$$\eta_{\text{pump}} = 1 - \frac{\dot{E}_{\text{mech, loss}}}{\dot{W}_{\text{mech, in}}} = 1 - \frac{9.53\ \text{hp}}{27\ \text{hp}} = 0.647 \quad \text{or} \quad \mathbf{64.7\%}$$

Discussion Note that despite the conversion of more than one-third of the mechanical power input into thermal energy, the temperature of water rises by only a small fraction of a degree. Therefore, the temperature rise of a fluid due to frictional heating is usually negligible in heat transfer analysis.

12-10 Water is pumped from a lake to a storage tank at a specified rate. The overall efficiency of the pump-motor unit and the pressure difference between the inlet and the exit of the pump are to be determined. √

Assumptions **1** The elevations of the tank and the lake remain constant. **2** Frictional losses in the pipes are negligible. **3** The changes in kinetic energy are negligible. **4** The elevation difference across the pump is negligible.

Properties We take the density of water to be ρ = 1000 kg/m³.

Analysis (*a*) We take the free surface of the lake to be point 1 and the free surfaces of the storage tank to be point 2. We also take the lake surface as the reference level (z_1 = 0), and thus the potential energy at points 1 and 2 are $pe_1 = 0$ and $pe_2 = gz_2$. The flow energy at both points is zero since both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$). Further, the kinetic energy at both points is zero ($ke_1 = ke_2 = 0$) since the water at both locations is essentially stationary. The mass flow rate of water and its potential energy at point 2 are

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(0.070\ \text{m}^3/\text{s}) = 70\ \text{kg/s}$$

$$pe_1 = gz_1 = (9.81\ \text{m/s}^2)(20\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.196\ \text{kJ/kg}$$

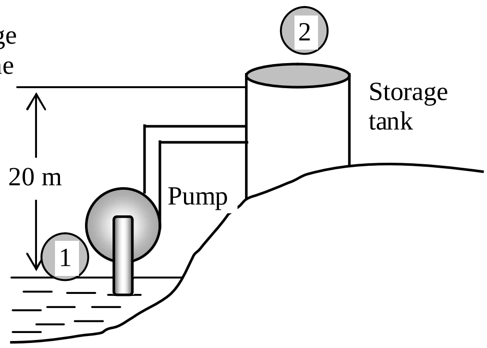
Then the rate of increase of the mechanical energy of water becomes

$$\Delta\dot{E}_{\text{mech, fluid}} = \dot{m}(e_{\text{mech, out}} - e_{\text{mech, in}}) = \dot{m}(pe_2 - 0) = \dot{m}pe_2 = (70\ \text{kg/s})(0.196\ \text{kJ/kg}) = 13.7\ \text{kW}$$

The overall efficiency of the combined pump-motor unit is determined from its definition,

$$\eta_{\text{pump-motor}} = \frac{\Delta\dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{elect,in}}} = \frac{13.7\ \text{kW}}{20.4\ \text{kW}} = 0.672 \quad \text{or} \quad \mathbf{67.2\%}$$

(*b*) Now we consider the pump. The change in the mechanical energy of water as it flows through the pump consists of the change in the flow energy only since the elevation difference across the pump and the change in the kinetic energy are negligible. Also, this change must be equal to the useful mechanical energy supplied by the pump, which is 13.7 kW:



$$\Delta\dot{E}_{\text{mech, fluid}} = \dot{m}(e_{\text{mech, out}} - e_{\text{mech, in}}) = \dot{m}\frac{P_2 - P_1}{\rho} = \dot{V}\Delta P$$

Solving for ΔP and substituting,

$$\Delta P = \frac{\Delta\dot{E}_{\text{mech, fluid}}}{\dot{V}} = \frac{13.7\ \text{kJ/s}}{0.070\ \text{m}^3/\text{s}}\left(\frac{1\ \text{kPa}\cdot\text{m}^3}{1\ \text{kJ}}\right) = \mathbf{196\ kPa}$$

Therefore, the pump must boost the pressure of water by 196 kPa in order to raise its elevation by 20 m.

Discussion Note that only two-thirds of the electric energy consumed by the pump-motor is converted to the mechanical energy of water; the remaining one-third is wasted because of the inefficiencies of the pump and the motor.

Bernoulli Equation

12-11C The acceleration of a fluid particle along a streamline is called *streamwise acceleration*, and it is due to a change in speed along a streamline. *Normal acceleration* (or centrifugal acceleration), on the other hand, is the acceleration of a fluid particle in the direction normal to the streamline, and it is due to a change in direction.

12-12C The Bernoulli equation can be expressed in three different ways as follows:

(a) energies: $\dfrac{P}{\rho}+\dfrac{\mathbf{V}^2}{2}+gz=\text{constant}$

(b) pressures: $P+\rho\dfrac{\mathbf{V}^2}{2}+\rho gz=\text{constant}$

(c) heads: $\dfrac{P}{\rho g}+\dfrac{\mathbf{V}^2}{2g}+z=H=\text{constant}$

12-13C The three major assumptions used in the derivation of the Bernoulli equation are that the flow is steady, frictionless, and incompressible.

12-14C The *static pressure P* is the actual pressure of the fluid. The *dynamic pressure* $\rho\mathbf{V}^2/2$ is the pressure rise when the fluid in motion is brought to a stop. The *hydrostatic pressure* ρgz is not pressure in a real sense since its value depends on the reference level selected, and it accounts for the effects of fluid weight on pressure.

The sum of static, dynamic, and hydrostatic pressures is constant when flow is steady, frictionless, and incompressible.

12-15C The sum of the static and dynamic pressures is called the stagnation pressure, and it is expressed as $P_{\text{stag}}=P+\rho\mathbf{V}^2/2$. The *stagnation pressure* can be measured by a pitot tube whose inlet is normal to flow.

12-16C The *pressure head* $P/\rho g$ is the height of a fluid column that produces the static pressure *P*. The *velocity head* $\mathbf{V}^2/2$ is the elevation needed for a fluid to reach the velocity **V** during frictionless free fall. The *elevation head z* is the height of a fluid relative to a reference level.

12-17C The line that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the *hydraulic grade line*. The line that represents the total head of the fluid, $P/\rho g + \mathbf{V}^2/2g + z$, is called the *energy line*. For stationary bodies such as reservoirs or lakes, the EL and HGL coincide with the free surface of the liquid.

12-18C For open channel flow, the hydraulic grade line (HGL) coincides with the free surface of the liquid. At the exit of a pipe discharging to the atmosphere, it coincides with the center of the pipe.

12-19C With no losses and a 100% efficient nozzle, the water stream could reach to the water level in the tank, or 20 meters. In reality, friction losses in the hose, nozzle inefficiencies, orifice losses and air drag would prevent attainment of the maximum theoretical height.

12-20C The lower density liquid can go over a higher wall, provided that cavitation pressure is not reached. Therefore, oil can go over a higher wall.

12-21C Siphoning works because of the elevation and thus pressure difference between the inlet and exit of a tube. The pressure at the tube exit and at the free surface of a liquid is the atmospheric pressure. When the tube exit is below the free surface of the liquid, the elevation head difference drives the flow through the tube. At sea level, 1 atm pressure can support about 10.3 m of cold water (cold water has a low vapor pressure). Therefore, siphoning cold water over a 7 m wall is feasible.

12-22C At sea level, a person can siphon water over a wall as high as 10.3 m. At the top of a high mountain where the pressure is about half of the atmospheric pressure at sea level, a person can siphon water over a wall that is only half as high. An atmospheric pressure of 58.5 kPa is insufficient to support a 8.5 meter high siphon.

12-25C By Bernoulli's Equation, the smaller pipe section is consistent with higher velocity and concomitant lower pressure. Thus Manometer A is correct. The fluid levels in a manometer is independent of the flow direction, and reversing the flow direction will have no effect on the manometer.

12-24C The arrangement *B* measures the total head and static head at the same location, and thus it is more accurate. The static probe in arrangement *A* will indicate *D*/2 less water head, and thus the difference between the static and stagnation pressures (the dynamic pressure) will be larger. Consequently, arrangement *A* will indicate a higher velocity. In the case of air, the static pressure difference corresponding to the elevation head of *D*/2 is negligible, and thus both arrangements will indicate the same velocity.

12-25 A water pipe bursts as a result of freezing, and water shoots up into the air a certain height. The gage pressure of water in the pipe is to be determined. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The water pressure in the pipe at the burst section is equal to the water main pressure. **3** Friction between the water and air is negligible. **4** The irreversibilities that may occur at the burst section of the pipe due to abrupt expansion are negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($\mathbf{V}_1 \cong 0$) and we take the burst section of the pipe as the reference level ($z_1 = 0$). At the top of the water trajectory $\mathbf{V}_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli Equation simplifies to

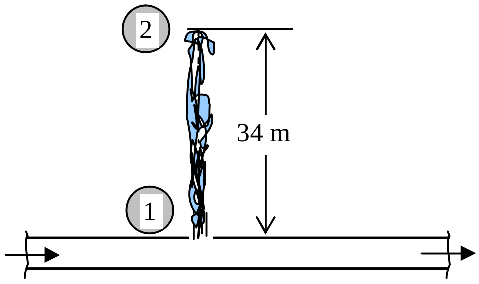
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+z_2 \quad\rightarrow\quad \frac{P_1-P_{atm}}{\rho g}=z_2 \quad\rightarrow\quad \frac{P_{1,\text{gage}}}{\rho g}=z_2$$

Solving for $P_{1,\text{gage}}$ and substituting,

$$P_{1,\text{gage}}=\rho g z_2=(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(34\ \text{m})\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)=\mathbf{334\ kPa}$$

Therefore, the pressure in the main must be at least 334 kPa above the atmospheric pressure.

Discussion The result obtained by the Bernoulli equation represents a limit, and should be interpreted accordingly. It tells us that the water pressure (gage) cannot possibly be less than 334 kPa (giving us a lower limit), and in all likelihood, the pressure will be much higher.



12-26 The velocity of an aircraft is to be measured by a pitot tube. For a given differential pressure reading, the velocity of the aircraft is to be determined. √

Assumptions **1** The air flow over the aircraft is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Standard atmospheric conditions exist. **3** The wind effects are negligible.

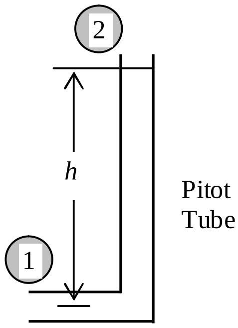
Properties The density of the atmosphere at an elevation of 3000 m is ρ = 0.909 kg/m³ (Table A-24).

Analysis We take point 1 at the entrance of the tube whose opening is parallel to flow, and point 2 at the entrance of the tube whose entrance is normal to flow. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{\mathbf{V}_1^2}{2g}=\frac{P_2-P_1}{\rho g} \quad\rightarrow\quad \frac{\mathbf{V}_1^2}{2}=\frac{P_{stag}-P_1}{\rho}$$

Solving for $\mathbf{V}_1$ and substituting,

$$\mathbf{V}_1=\sqrt{\frac{2(P_{stag}-P_1)}{\rho}}=\sqrt{\frac{2(3000\ \text{N/m}^2)}{0.909\ \text{kg/m}^3}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)}=\mathbf{81.2\ m/s}=\mathbf{292\ km/h}$$



since 1 Pa = 1 N/m² and 1 m/s = 3.6 km/h.

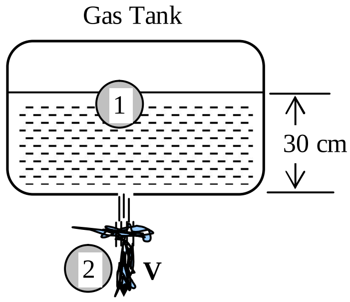
Discussion Note that the velocity of an aircraft can be determined by simply measuring the differential pressure on a pitot tube.

12-27 The bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. For a given height of gasoline, the initial velocity of the gasoline out of the hole is to be determined. Also, the variation of velocity with time and the effect of the tightness of the lid on flow rate are to be discussed. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The air space in the tank is at atmospheric pressure. **3** The splashing of the gasoline in the tank during travel is not considered.

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere) $\mathbf{V}_1 \cong 0$ (the tank is large relative to the outlet), and $z_1 = 0.3$ m and $z_2 = 0$ (we take the reference level at the hole. Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli Equation simplifies to

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{\mathbf{V}_2^2}{2g}$$



Solving for $\mathbf{V}_2$ and substituting,

$$\mathbf{V}_2 = \sqrt{2gz_1} = \sqrt{2(9.81\,\text{m/s}^2)(0.3\,\text{m})} = \mathbf{2.43\ m/s}$$

Therefore, the gasoline will initially leave the tank with a velocity of 2.43 m/s.

Discussion The Bernoulli equation applies along a streamline, and streamlines generally do not make sharp turns. The velocity will be less than 2.43 m/s since the hole is probably sharp-edged and it will cause some head loss.

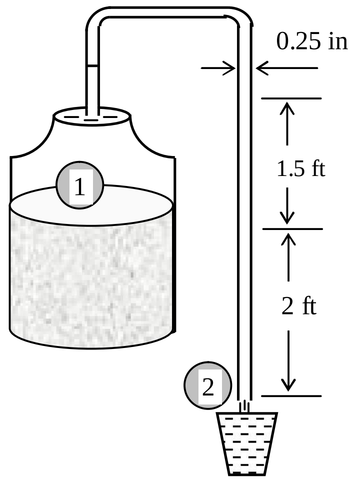
As the gasoline level is reduced, the velocity will decrease since velocity is proportional to the square root of liquid height. If the lid is tightly closed and no air can replace the lost gasoline volume, the pressure above the gasoline level will be reduced, and the velocity will be decreased.

12-28E The drinking water needs of an office are met by large water bottles with a plastic hose inserted in it. The minimum filling time of an 8-oz glass is to be determined when the bottle is full and when it is near empty. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** All losses are neglected to obtain the minimum filling time.

Analysis We take point 1 to be at the free surface of water in the bottle and point 2 at the exit of the tube so that $P_1 = P_2 = P_{atm}$ (the bottle is open to the atmosphere and water discharges into the atmosphere), $\mathbf{V}_1 \cong 0$ (the bottle is large relative to the tube diameter), and $z_2 = 0$ (we take point 2 as the reference level). Then the Bernoulli Equation simplifies to

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{2gz_1}$$



Substituting, the discharge velocity of water and the filling time are determined as follows:

(*a*) *Full bottle* (z_1 = 3.5 ft):

$$\mathbf{V}_2 = \sqrt{2(32.2\text{ ft/s}^2)(3.5\text{ ft})} = 15.0\text{ ft/s}$$

$$A = \pi D^2/4 = \pi(0.25/12\text{ ft})^2/4 = 3.41\times10^{-4}\text{ ft}^2$$

$$\Delta t = \frac{V}{\dot{V}} = \frac{V}{A\mathbf{V}} = \frac{0.00835\text{ ft}^3}{(3.41\times10^{-4}\text{ ft}^2)(15\text{ ft/s})} = \mathbf{1.6\ s}$$

(*b*) *Empty bottle* (z_1 = 2 ft):

$$\mathbf{V}_2 = \sqrt{2(32.2\text{ ft/s}^2)(2\text{ ft})} = 11.3\text{ ft/s}$$

$$\Delta t = \frac{V}{\dot{V}} = \frac{V}{A\mathbf{V}} = \frac{0.00835\text{ ft}^3}{(3.41\times10^{-4}\text{ ft}^2)(11.3\text{ ft/s})} = \mathbf{2.2\ s}$$

Discussion The siphoning time is determined assuming frictionless flow, and thus this is the *minimum time* required. In reality, the time will be longer because of friction between water and the tube surface.

12-29 The static and stagnation pressures in a horizontal pipe are measured. The velocity at the center of the pipe is to be determined. √

Assumptions The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Analysis We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the entrance of the pitot tube (the stagnation point). This is a steady flow with straight and parallel streamlines, and thus the static pressure at any point is equal to the hydrostatic pressure at that point. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{\mathbf{V}_1^2}{2g}=\frac{P_2-P_1}{\rho g}$$

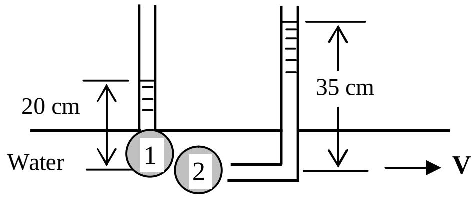
Substituting the P_1 and P_2 expressions give

$$\frac{\mathbf{V}_1^2}{2g}=\frac{P_2-P_1}{\rho g}=\frac{\rho g(h_{\text{pitot}}+R)-\rho g(h_{\text{piezo}}+R)}{\rho g}=\frac{\rho g(h_{\text{pitot}}-h_{\text{piezo}})}{\rho g}=h_{\text{pitot}}-h_{\text{piezo}}$$

Solving for $\mathbf{V}_1$ and substituting,

$$\mathbf{V}_1=\sqrt{2g(h_{\text{pitot}}-h_{\text{piezo}})}=\sqrt{2(9.81\,\text{m/s}^2)[(0.35-0.20)\,\text{m}]}=\mathbf{1.72\ m/s}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the pitot tube.



12-30 A water tank of diameter D_o and height H open to the atmosphere is initially filled with water. An orifice of diameter D with a smooth entrance (no losses) at the bottom drains to the atmosphere. Relations are to be developed for the time required for the tank to empty completely and half-way. √

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

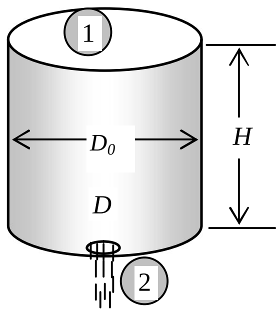
Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice. We take the reference level at the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{\text{atm}}$) and that the fluid velocity at the free surface is very low $(\mathbf{V}_1 \cong 0)$, the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\to\quad z_1=\frac{\mathbf{V}_2^2}{2g} \quad\to\quad \mathbf{V}_2=\sqrt{2gz_1}$$

For generality, we express the water height in the tank at any time t by z, and the discharge velocity by $\mathbf{V}_2=\sqrt{2gz}$. Note that water surface in the tank moves down as the tank drains, and thus z is a variable whose value changes from H at the beginning to 0 when the tank is emptied completely.

We denote the diameter of the orifice by D, and the diameter of the tank by D_o. The flow rate of water from the tank is obtained by multiplying the discharge velocity by the orifice cross-sectional area,

$$\dot{V}=A_{\text{orifice}}\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{2gz}$$



Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{2gz}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV=A_{\text{tank}}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{2gz}\,dt=-\frac{\pi D_0^2}{4}dz \quad\to\quad dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1}{2gz}}\,dz=-\frac{D_0^2}{D^2\sqrt{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_i = H$ to $t = t_f$ when $z = z_f$ gives

$$\int_{t=0}^{t_f}dt=-\frac{D_0^2}{D^2\sqrt{2g}}\int_{z=z_1}^{z_f}z^{-1/2}dz \quad\to\quad t_f=-\frac{D_0^2}{D^2\sqrt{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{z_f}=\frac{2D_0^2}{D^2\sqrt{2g}}\left(\sqrt{z_i}-\sqrt{z_f}\right)=\frac{D_0^2}{D^2}\left(\sqrt{\frac{2z_i}{g}}-\sqrt{\frac{2z_f}{g}}\right)$$

Then the discharging time for the two cases becomes as follows:

(*a*) *The tank empties halfway*: $z_i = H$ and $z_f = H/2$: $\quad t_f=\dfrac{D_0^2}{D^2}\left(\sqrt{\dfrac{2H}{g}}-\sqrt{\dfrac{H}{g}}\right)$

(*b*) *The tank empties completely*: $z_i = H$ and $z_f = 0$: $\quad t_f=\dfrac{D_0^2}{D^2}\sqrt{\dfrac{2H}{g}}$

Discussion Note that the discharging time is inversely proportional to the square of the orifice diameter. Therefore, the discharging time can be reduced to one-fourth by doubling the diameter of the orifice.

12-31 Water discharges to the atmosphere from the orifice at the bottom of a pressurized tank. Assuming frictionless flow, the discharge rate of water from the tank is to be determined. √EES

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Properties We take the density of water to be 1000 kg/m^3 (Table A-3).

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$) and water discharges into the atmosphere (and thus $P_2 = P_{atm}$), the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{\mathbf{V}_2^2}{2g}=\frac{P_1-P_2}{\rho g}+z_1$$

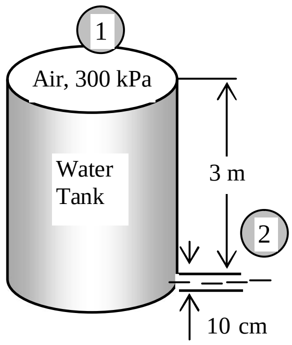
Solving for $\mathbf{V}_2$ and substituting, the discharge velocity is determined to

$$\mathbf{V}_2=\sqrt{\frac{2(P_1-P_2)}{\rho}+2gz_1}=\sqrt{\frac{2(300-100)\text{ kPa}}{1000\text{ kg/m}^3}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)+2(9.81\text{ m/s}^2)(3\text{ m})}$$

$$=21.4\text{ m/s}$$

Then the initial rate of discharge of water becomes

$$\dot{V}=A_{orifice}\mathbf{V}_2=\frac{\pi D^2}{4}\mathbf{V}_2=\frac{\pi(0.10\text{ m})^2}{4}(21.4\text{ m/s})=\mathbf{0.168\ m^3/s}$$



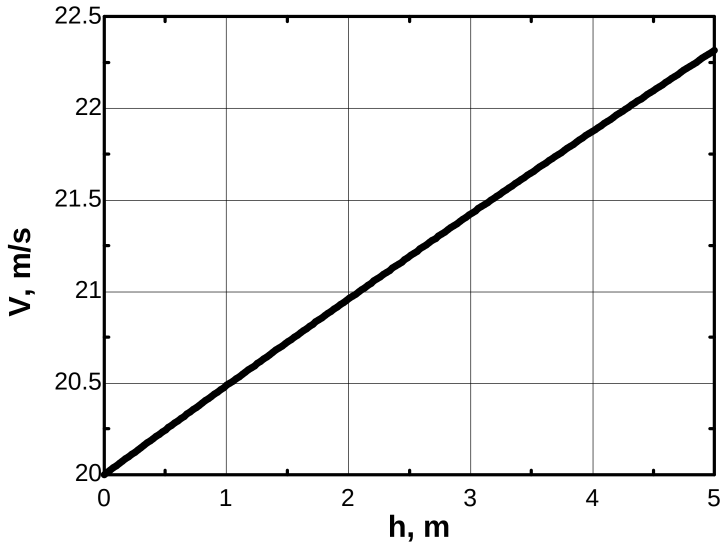
Discussion Note that this is the maximum flow rate since the frictional effects are ignored. Also, the velocity and the flow rate will decrease as the water level in the tank decreases.

12-32 Problem 12-31 is reconsidered. The effect of water height in the tank on the discharge velocity as the water height varies from 0 to 5 m in increments of 0.5 m is to be investigated.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"
d=0.10 "m"

P1=300 "kPa"
P_atm=100 "kPa"
V=SQRT(2*(P1-P_atm)*1000/rho+2*g*h)
Ac=pi*D^2/4
 V_dot=Ac*V
```

h, m	*V*, m/s	$\dot{V}$, m^3/s
0.00	20.0	0.157
0.50	20.2	0.159
1.00	20.5	0.161
1.50	20.7	0.163
2.00	21.0	0.165
2.50	21.2	0.166
3.00	21.4	0.168
3.50	21.6	0.170
4.00	21.9	0.172
4.50	22.1	0.174
5.00	22.3	0.175



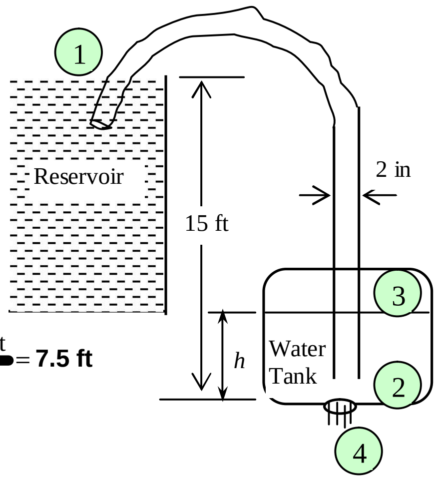
12-33E A siphon pumps water from a large reservoir to a lower tank which is initially empty. Water leaves the tank through an orifice. The height the water will rise in the tank at equilibrium is to be determined. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Both the tank and the reservoir are open to the atmosphere. **3** The water level of the reservoir remains constant.

Analysis We take the reference level to be at the bottom of the tank, and the water height in the tank at any time to be h. We take point 1 to be at the free surface of reservoir, point 2 at the exit of the siphon, which is placed at the bottom of the tank, and point 3 at the free surface of the tank, and point 4 at the exit of the orifice at the bottom of the tank. Then z_1 = 20 ft, $z_2 = z_4 = 0$, $z_3 = h$, $P_1 = P_3 = P_4 = P_{\text{atm}}$ (the reservoir is open to the atmosphere and water discharges into the atmosphere) $P_2 = P_{\text{atm}}+\rho g h$ (the hydrostatic pressure at the bottom of the tank where the siphon discharges), and $\mathbf{V}_1 \cong \mathbf{V}_3 \cong 0$ (the free surfaces of reservoir and the tank are large relative to the tube diameter). Then the Bernoulli Equation between 1-2 and 3-4 simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_{atm}}{\rho g}+z_1=\frac{P_{atm}+\rho g h}{\rho g}+\frac{\mathbf{V}_2^2}{2g} \quad\rightarrow\quad \mathbf{V}_2=\sqrt{2gz_1-2gh}=\sqrt{2g(z_1-h)}$$

$$\frac{P_3}{\rho g}+\frac{\mathbf{V}_3^2}{2g}+z_3=\frac{P_4}{\rho g}+\frac{\mathbf{V}_4^2}{2g}+z_4 \quad\rightarrow\quad h=\frac{\mathbf{V}_4^2}{2g} \quad\rightarrow\quad \mathbf{V}_4=\sqrt{2gh}$$



Noting that the diameters of the tube and the orifice are the same, the flow rates of water into and out of the tank will be the same when the water velocities in the tube and the orifice are equal since

$$\dot{V}_2=\dot{V}_4 \quad\rightarrow\quad A\mathbf{V}_2=A\mathbf{V}_4 \quad\rightarrow\quad \mathbf{V}_2=\mathbf{V}_4$$

Setting the two velocities equal to each other gives

$$\mathbf{V}_2=\mathbf{V}_4 \quad\rightarrow\quad \sqrt{2g(z_1-h)}=\sqrt{2gh} \quad\rightarrow\quad z_1-h=h \quad\rightarrow\quad h=\frac{z_1}{2}=\frac{15\text{ ft}}{2}=\mathbf{7.5\ ft}$$

Therefore, the water level in the tank will stabilize when the water level rises to 7.5 ft.

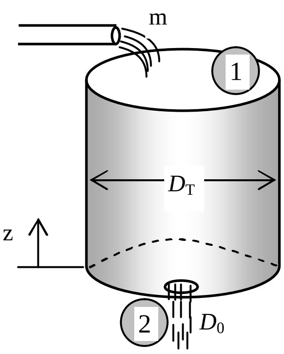
Discussion This result is obtained assuming negligible friction. The result would be somewhat different if the friction in the pipe and orifice were considered.

12-34 Water enters an empty tank steadily at a specified rate. An orifice at the bottom allows water to escape. The maximum water level in the tank is to be determined, and a relation for water height z as a function of time is to be obtained.

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow through the orifice is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of orifice. We take the reference level at the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$) (it becomes zero when the water in the tank reaches its maximum level), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{2gz_1}$$



Then the mass flow rate through the orifice for a water height of z becomes

$$\dot{m}_{out} = \rho \dot{V}_{out} = \rho A_{orifice} \mathbf{V}_2 = \rho \frac{\pi D_0^2}{4} \sqrt{2gz} \quad \rightarrow \quad z = \frac{1}{2g} \left(\frac{4\dot{m}_{out}}{\rho \pi D_0^2} \right)^2$$

Setting $z = h_{max}$ and $\dot{m}_{out} = \dot{m}_{in}$ (the incoming flow rate) gives the desired relation for the maximum height the water will reach in the tank,

$$h_{max} = \frac{1}{2g} \left(\frac{4\dot{m}_{in}}{\rho \pi D_0^2} \right)^2$$

(*b*) The amount of water that flows through the orifice and the increase in the amount of water in the tank during a differential time interval dt are

$$dm_{out} = \dot{m}_{out} dt = \rho \frac{\pi D_0^2}{4} \sqrt{2gz}\, dt$$

$$dm_{tank} = \rho A_{tank} dz = \rho \frac{\pi D_T^2}{4} dz$$

The amount of water that enters the tank during dt is $dm_{in} = \dot{m}_{in} dt$ (Recall that $\dot{m}_{in}$ = constant). Substituting them into the conservation of mass relation $dm_{tank} = dm_{in} - dm_{out}$ gives

$$dm_{tank} = \dot{m}_{in} dt - \dot{m}_{out} dt \quad \rightarrow \quad \rho \frac{\pi D_T^2}{4} dz = \left(\dot{m}_{in} - \rho \frac{\pi D_0^2}{4} \sqrt{2gz} \right) dt$$

Separating the variables, and integrating it from $z = 0$ at $t = 0$ to $z = z$ at time $t = t$ gives

$$\frac{\frac{1}{4}\rho \pi D_T^2 dz}{\dot{m}_{in} - \frac{1}{4}\rho \pi D_0^2 \sqrt{2gz}} = dt \quad \rightarrow \quad \int_{z=0}^{z} \frac{\frac{1}{4}\rho \pi D_T^2 dz}{\dot{m}_{in} - \frac{1}{4}\rho \pi D_0^2 \sqrt{2gz}} = \int_{t=0}^{t} dt = t$$

Performing the integration, the desired relation between the water height z and time t is obtained to be

$$\frac{\frac{1}{2}\rho \pi D_T^2}{(\frac{1}{4}\rho \pi D_0^2 \sqrt{2g})^2} \left(\frac{1}{4}\rho \pi D_0^2 \sqrt{2gz} - \dot{m}_{in} \ln \frac{\dot{m}_{in} - \frac{1}{4}\rho \pi D_0^2 \sqrt{2gz}}{\dot{m}_{in}} \right) = t$$

Discussion Note that this relation is implicit in z, and thus we can't obtain a relation in the form $z = f(t)$. Substituting a z value in the left side gives the time it takes for the fluid level in the tank to reach that level.

12-35E Water flows through a horizontal pipe that consists of two sections at a specified rate. The differential height of a mercury manometer placed between the two pipe sections is to be determined. √

Assumptions **1** The flow through the pipe is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The losses in the reducing section are negligible.

Properties The densities of mercury and water are $\rho_{Hg} = 847$ lbm/ft^3 and $\rho_w = 62.4$ lbm/ft^3 (Table A-3E).

Analysis We take points 1 and 2 along the centerline of the pipe over the two tubes of the manometer. Noting that $z_1 = z_2$, the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad P_1-P_2=\frac{\rho_w(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2} \qquad (1)$$

We let the differential height of the mercury manometer be h and the distance between the centerline and the mercury level in the tube where mercury is raised be s. Then the pressure difference $P_2 - P_1$ can also be expressed as

$$P_1+\rho_w g(s+h)=P_2+\rho_w gs+\rho_{Hg}gh \quad\rightarrow\quad P_1-P_2=(\rho_{Hg}-\rho_w)gh \qquad (2)$$

Combining Eqs. (1) and (2) and solving for h,

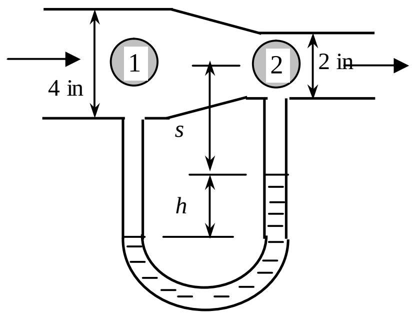
$$\frac{\rho_w(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2}=(\rho_{Hg}-\rho_w)gh \quad\rightarrow\quad h=\frac{\rho_w(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2g(\rho_{Hg}-\rho_w)}=\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2g(\rho_{Hg}/\rho_w-1)}$$

Calculating the velocities and substituting,

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{1\text{ gal/s}}{\pi(4/12\text{ ft})^2/4}\left(\frac{0.13368\text{ ft}^3}{1\text{ gal}}\right)=1.53\text{ ft/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{1\text{ gal/s}}{\pi(2/12\text{ ft})^2/4}\left(\frac{0.13368\text{ ft}^3}{1\text{ gal}}\right)=6.13\text{ ft/s}$$

$$h=\frac{(6.13\text{ ft/s})^2-(1.53\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)(847/62.4-1)}=0.0435\text{ ft}=\mathbf{0.52\text{ in}}$$



Therefore, the differential height of the mercury column will be 0.52 in.

12-36 An airplane is flying at a certain altitude at a given speed. The pressure on the stagnation point on the nose of the plane is to be determined, and the approach to be used at high velocities is to be discussed. √

Assumptions **1**The air flow over the aircraft is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Standard atmospheric conditions exist. **3** The wind effects are negligible.

Properties The density of the atmospheric air at an elevation of 12,000 m is ρ = 0.312 kg/m^3 (Table A-24).

Analysis We take point 1 well ahead of the plane at the level of the nose, and point 2 at the nose where the flow comes to a stop. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

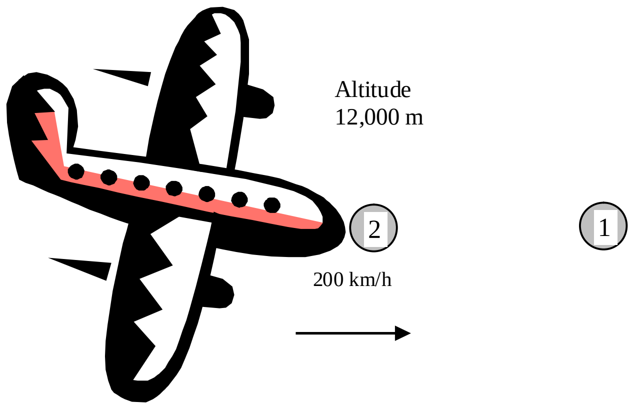
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{\mathbf{V}_1^2}{2g}=\frac{P_2-P_1}{\rho g} \quad\rightarrow\quad \frac{\mathbf{V}_1^2}{2}=\frac{P_{stag}-P_{\text{atm}}}{\rho}=\frac{P_{\text{stag, gage}}}{\rho}$$

Solving for $P_{\text{stag, gage}}$ and substituting,

$$P_{\text{stag, gage}}=\frac{\rho\mathbf{V}_1^2}{2}=\frac{(0.312\text{ kg/m}^3)(200/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)=482\text{ N/m}^2=\mathbf{482\ Pa}$$

since 1 Pa = 1 N/m^2 and 1 m/s = 3.6 km/h.

Discussion A flight velocity of 1050 km/h = 292 m/s corresponds to a Mach number much greater than 0.3 (the speed of sound is about 340 m/s at room conditions, and lower at higher altitudes, and thus a Mach number of 292/340 = 0.86). Therefore, the flow can no longer be assumed to be incompressible, and the Bernoulli equation given above cannot be used. This problem can be solved using the modified Bernoulli equation that accounts for the effects of compressibility, assuming isentropic flow.



12-37 A pitot tube is inserted into the duct of an air heating system parallel to flow, and the differential height of the water column is measured. The flow velocity and the pressure rise at the tip of the pitot tube are to be determined. √

Assumptions **1**The flow through the duct is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Air is an ideal gas.

Properties We take the density of water to be ρ = 1000 kg/m^3 (Table A-3). The gas constant of air is R = 0.287 kPa·m^3/kg·K (Table A-1).

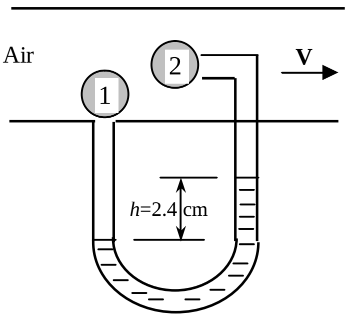
Analysis We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the pitot tube, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the pitot tube. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}=\frac{P_2}{\rho g} \quad\rightarrow\quad \mathbf{V}_1=\sqrt{\frac{2(P_2-P_1)}{\rho_{air}}}$$

where the pressure rise at the tip of the pitot tube is

$$P_2-P_1=\rho_w gh=(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.024\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$=235\,\text{N/m}^2=\mathbf{235\ Pa}$$



Also, $\rho_{air}=\dfrac{P}{RT}=\dfrac{98\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(45+273\,\text{K})}=1.074\,\text{kg/m}^3$

Substituting,

$$\mathbf{V}_1=\sqrt{\frac{2(235\,\text{N/m}^2)}{1.074\,\text{kg/m}^3}\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right)}=\mathbf{20.9\ m/s}$$

Discussion Note that the flow velocity in a pipe or duct can be measured easily by a pitot tube by inserting the probe into the pipe or duct parallel to flow, and reading the differential pressure height. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.

12-38 The water in an above the ground swimming pool is to be emptied by unplugging the orifice of a horizontal pipe attached to the bottom of the pool. The maximum discharge rate of water is to be determined. √EES

Assumptions **1** The orifice has a smooth entrance, and all frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Analysis We take point 1 at the free surface of the pool, and point 2 at the exit of pipe. We take the reference level at the pipe exit ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the Bernoulli equation between these two points simplifies to

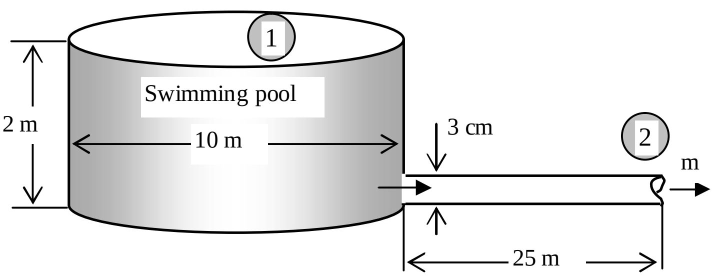
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad z_1=\frac{\mathbf{V}_2^2}{2g} \quad\rightarrow\quad \mathbf{V}_2=\sqrt{2gz_1}$$

The maximum discharge rate occurs when the water height in the pool is a maximum, which is the case at the beginning and thus $z_1 = h$. Substituting, the maximum flow velocity and discharge rate become

$$\mathbf{V}_{2,\max}=\sqrt{2gh}=\sqrt{2(9.81\text{ m/s}^2)(2\text{ m})}=6.26\text{ m/s}$$

$$\dot{V}_{\max}=A_{pipe}\mathbf{V}_{2,\max}=\frac{\pi D^2}{4}\mathbf{V}_{2,\max}=\frac{\pi(0.03\text{ m})^2}{4}(6.26\text{ m/s})=0.00443\text{ m}^3\text{/s}=\mathbf{4.43\ L/s}$$

Discussion The result above is obtained by disregarding all frictional effects. The actual flow rate will be less because of frictional effects during flow and the resulting pressure drop. Also, the flow rate will gradually decrease as the water level in the pipe decreases.



12-39 The water in an above the ground swimming pool is to be emptied by unplugging the orifice of a horizontal pipe attached to the bottom of the pool. The time it will take to empty the tank is to be determined. √EES

Assumptions **1** The orifice has a smooth entrance, and all frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Analysis We take point 1 at the free surface of water in the pool, and point 2 at the exit of pipe. We take the reference level at the pipe exit ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad z_1=\frac{\mathbf{V}_2^2}{2g} \quad\rightarrow\quad \mathbf{V}_2=\sqrt{2gz_1}$$

For generality, we express the water height in the pool at any time t by z, and the discharge velocity by $\mathbf{V}_2=\sqrt{2gz}$. Note that water surface in the pool moves down as the pool drains, and thus z is a variable whose value changes from h at the beginning to 0 when the pool is emptied completely.

We denote the diameter of the orifice by D, and the diameter of the pool by D_o. The flow rate of water from the pool is obtained by multiplying the discharge velocity by the orifice cross-sectional area,

$$\dot{V} = A_{\text{orifice}}\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{2gz}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{2gz}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the pool,

$$dV=A_{\text{tank}}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the pool during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{2gz}\,dt=-\frac{\pi D_0^2}{4}dz \quad\rightarrow\quad dt=-\frac{D_0^2}{D^2}\frac{1}{\sqrt{2gz}}dz=-\frac{D_0^2}{D^2\sqrt{2g}}z^{-\frac{1}{2}}dz$$

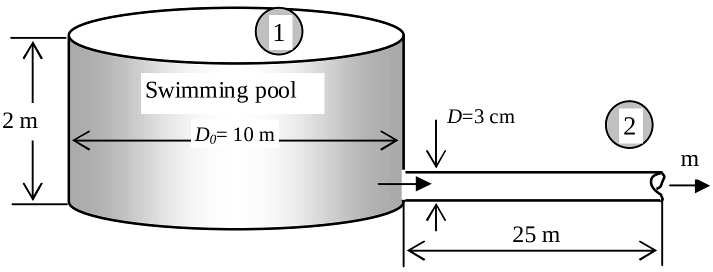
The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = h$ to $t = t_f$ when $z = 0$ (completely drained pool) gives

$$\int_{t=0}^{t_f}dt=-\frac{D_0^2}{D^2\sqrt{2g}}\int_{z=z_1}^{0}z^{-1/2}dz \quad\rightarrow\quad t_f=-\frac{D_0^2}{D^2\sqrt{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0}=\frac{2D_0^2}{D^2\sqrt{2g}}\sqrt{h}=\frac{D_0^2}{D^2}\sqrt{\frac{2h}{g}}$$

Substituting, the draining time of the pool will be

$$t_f=\frac{(10\text{ m})^2}{(0.03\text{ m})^2}\sqrt{\frac{2(2\text{ m})}{9.81\text{ m/s}^2}}=70{,}900\text{ s}=19.7\text{ h}$$

Discussion This is the minimum discharging time since it is obtained by neglecting all friction; the actual discharging time will be longer. Note that the discharging time is inversely proportional to the square of the orifice diameter. Therefore, the discharging time can be reduced to one-fourth by doubling the diameter of the orifice.

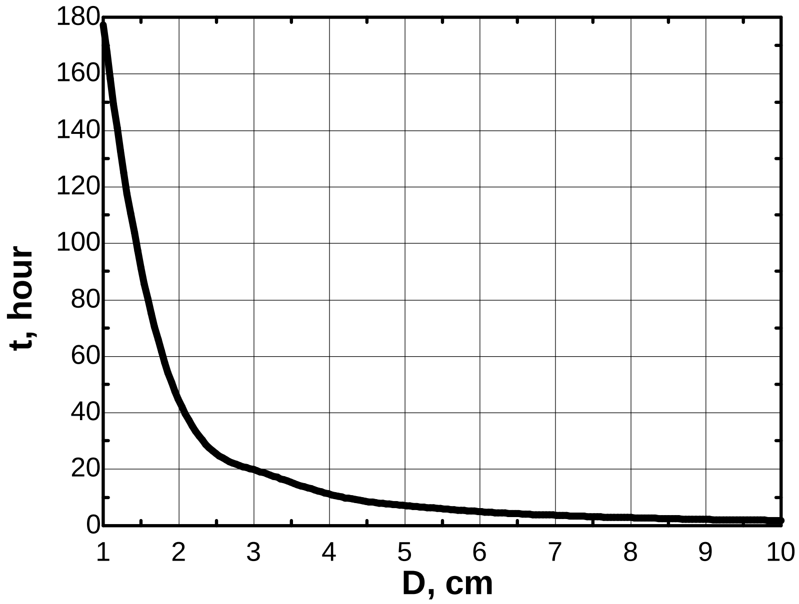


12-40 Problem 12-39 is reconsidered. The effect of the discharge pipe diameter on the time required to empty the pool completely as the diameter varies from 1 to 10 cm in increments of 1 cm is to be investigated.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"
h=2 "m"
D=d_pipe/100 "m"
D_pool=10 "m"

V_initial=SQRT(2*g*h) "m/s"
Ac=pi*D^2/4
V_dot=Ac*V_initial*1000 "m3/s"
t=(D_pool/D)^2*SQRT(2*h/g)/3600 "hour"
```

Pipe diameter *D*, m	Discharge time *t*, h
1	177.4
2	44.3
3	19.7
4	11.1
5	7.1
6	4.9
7	3.6
8	2.8
9	2.2
10	1.8



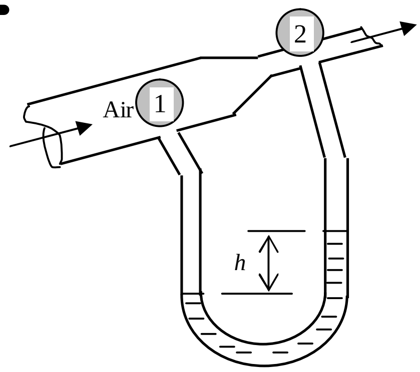
12-41 Air flows upward at a specified rate through an inclined pipe whose diameter is reduced through a reducer. The differential height between fluid levels of the two arms of a water manometer attached across the reducer is to be determined. √

Assumptions **1**The flow through the duct is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Air is an ideal gas. **3** The effect of air column on the pressure change is negligible because of its low density. **3** The air flow is parallel to the entrance of each arm of the manometer, and thus no dynamic effects are involved.

Properties We take the density of water to be ρ = 1000 kg/m^3 (Table A-3). The gas constant of air is R = 0.287 kPa·m^3/kg·K (Table A-1).

Analysis We take points 1 and 2 at the lower and upper connection points, respectively, of the two arms of the manometer, and take the lower connection point as the reference level. Noting that the effect of elevation on the pressure change of a gas is negligible, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad P_1-P_2=\rho_{\text{air}}\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2}$$



where $\rho_{air}=\dfrac{P}{RT}=\dfrac{110\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(50+273\,\text{K})}=1.19\,\text{kg/m}^3$

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.045\,\text{m}^3/\text{s}}{\pi(0.06\,\text{m})^2/4}=15.9\,\text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.045\,\text{m}^3/\text{s}}{\pi(0.04\,\text{m})^2/4}=35.8\,\text{m/s}$$

Substituting,

$$P_2-P_1=(1.19\,\text{kg/m}^3)\frac{(35.8\,\text{m/s})^2-(15.9\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)=612\,\text{N/m}^2=612\,\text{Pa}$$

The differential height of water in the manometer corresponding to this pressure change is determined from $\Delta P=\rho_{water}gh$ to be

$$h=\frac{P_1-P_2}{\rho_{water}g}=\frac{612\,\text{N/m}^2}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right)=0.0624\,\text{m}=\mathbf{6.24\ cm}$$

Discussion When the effect of air column on pressure change is considered, the pressure chance becomes

$$\begin{aligned}P_1-P_2&=\frac{\rho_{air}(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2}+g(z_2-z_1)\\&=(1.19\,\text{kg/m}^3)\left[\frac{(35.8\,\text{m/s})^2-(15.9\,\text{m/s})^2}{2}+(9.81\,\text{m/s}^2)(0.2\,\text{m})\right]\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\\&=(612+2)\,\text{N/m}^2=614\,\text{N/m}^2=614\,\text{Pa}\end{aligned}$$

This difference between the two results (612 and 614 Pa) is less than 1%. Therefore, the effect of air column on pressure change is, indeed, negligible as assumed. In other words, the pressure change of air in the duct is almost entirely due to velocity change, and the effect of elevation change is negligible.

12-42E Air is flowing through a venturi meter with known diameters and measured pressures. A relation for the flow rate is to be obtained, and its numerical value is to be determined.

Assumptions **1**The flow through the venturi is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The effect of air column on the pressure change is negligible because of its low density, and thus the pressure can be assumed to be uniform at a given cross-section of the venturi meter (independent of elevation change). **3** The flow is horizontal (this assumption is usually unnecessary for gas flow.).

Properties The density of air is given to be ρ = 0.075 lbm/ft^3.

Analysis We take point 1 at the main flow section and point 2 at the throat along the centerline of the venturi meter. Noting that $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

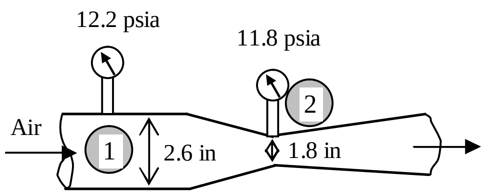
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad P_1-P_2=\rho\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2} \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad\rightarrow\quad A_1\mathbf{V}_1=A_2\mathbf{V}_2=\dot{V} \quad\rightarrow\quad \mathbf{V}_1=\frac{\dot{V}}{A_1} \quad\text{and}\quad \mathbf{V}_2=\frac{\dot{V}}{A_2} \qquad (2)$$

Substituting into Eq. (1),

$$P_1-P_2=\rho\frac{(\dot{V}/A_2)^2-(\dot{V}/A_1)^2}{2}=\frac{\rho\dot{V}^2}{2A_2^2}\left(1-\frac{A_2^2}{A_1^2}\right)$$



Solving for $\dot{V}$ gives the desired relation for the flow rate,

$$\dot{V}=A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}} \qquad (3)$$

The flow rate for the given case can be determined by substituting the given values into this relation to be

$$\dot{V}=\frac{\pi D_2^2}{4}\sqrt{\frac{2(P_1-P_2)}{\rho[1-(D_2/D_1)^4]}}=\frac{\pi(1.8/12\text{ ft})^2}{4}\sqrt{\frac{2(12.2-11.8)\text{ psi}}{(0.075\text{ lbm/ft}^3)[1\text{-}(1.8/2.6)^4]}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)}$$

$$=\mathbf{4.48\ ft^3/s}$$

Discussion Venturi meters are commonly used as flow meters to measure the flow rate of gases and liquids by simply measuring the pressure difference P_1 - P_2 by a manometer or pressure transducers. The actual flow rate will be less than the value obtained from Eq. (3) because of the friction losses along the wall surfaces in actual flow. But this difference can be as little as 1% in a well-designed venturi meter. The effects of deviation from the idealized Bernoulli flow can be accounted for by expressing Eq. (3) as

$$\dot{V}=C_cA_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}}$$

where C_c is the *venturi discharge coefficient* whose value is less than 1 (it is as large as 0.99 for well-designed venturi meters in certain ranges of flow). For Re > 10^5, the value of venturi discharge coefficient is usually greater than 0.96.

12-43 The gage pressure in the water mains of a city at a particular location is given. It is to be determined if this main can serve water to neighborhoods that are at a given elevation relative to this location. √

Assumptions Water is incompressible and thus its density is constant.

Properties We take the density of water to be ρ = 1000 kg/m^3 (Table A-3).

Analysis Noting that the gage pressure at a dept of h in a fluid is given by $P_{gage}=\rho_{water}gh$, the height of a fluid column corresponding to a gage pressure of 400 kPa is determined to be

$$h=\frac{P_{gage}}{\rho_{water}g}=\frac{400{,}000\text{ N/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)=40.8\text{ m}$$

which is less than 50 m. Therefore, this main **cannot** serve water to neighborhoods that are 50 m above this location.

Discussion Note that *h* must be much greater than 50 m for water to have enough pressure to serve the water needs of the neighborhood.

12-44 A hand-held bicycle pump with a liquid reservoir is used as an atomizer by forcing air at a high velocity through a small hole. The minimum speed that the piston must be moved in the cylinder to initiate the atomizing effect is to be determined. √

Assumptions **1**The flows of air and water are steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Air is an ideal gas. **3** The liquid reservoir is open to the atmosphere. **4** The device is held horizontal. **5** The water velocity through the tube is low.

Properties We take the density of water to be ρ = 1000 kg/m^3 (Table A-3). The gas constant of air is R = 0.287 kPa·m^3/kg·K (Table A-1).

Analysis We take point 1 at the exit of the hole, point 2 in air far from the hole on a horizontal line, point 3 at the exit of the tube in air stream (so that points 1 and 3 coincide), and point 4 at the free surface of the liquid in the reservoir ($P_2 = P_4 = P_{\text{atm}}$ and $P_1 = P_3$). We also take the level of the hole to be the reference level (so that $z_1 = z_2 = z_3 = 0$ and $z_4 = -h$). Noting that $\mathbf{V}_2 \cong \mathbf{V}_3 \cong \mathbf{V}_4 \cong 0$, the Bernoulli equation for the air and water streams becomes

Water (3-4): $$\frac{P_3}{\rho g}+\frac{\mathbf{V}_3^2}{2g}+z_3=\frac{P_4}{\rho g}+\frac{\mathbf{V}_4^2}{2g}+z_4 \;\rightarrow\; \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+(-h) \;\rightarrow\; P_1-P_{atm}=-\rho_{\text{water}}gh \qquad (1)$$

Air (1-2): $$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \;\rightarrow\; \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}=\frac{P_{atm}}{\rho g} \;\rightarrow\; \mathbf{V}_1=\sqrt{\frac{2(P_{atm}-P_1)}{\rho_{\text{air}}}} \qquad (2)$$

where

$$\rho_{air}=\frac{P}{RT}=\frac{95\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273\,\text{K})}=1.13\,\text{kg/m}^3$$

Combining Eqs. (1) and (2) and substituting the numerical values,

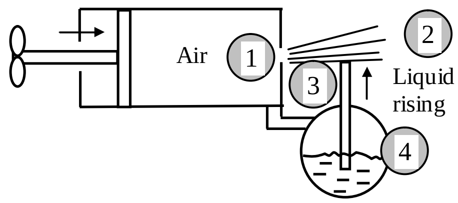
$$\mathbf{V}_1=\sqrt{\frac{2(P_{atm}-P_1)}{\rho_{\text{air}}}}=\;\mathbf{V}_1=\sqrt{\frac{2\rho_{\text{water}}gh}{\rho_{\text{air}}}}=\sqrt{\frac{2(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.1\,\text{m})}{1.13\,\text{kg/m}^3}}=41.7\,\text{m/s}$$

Taking the flow of air to be steady and incompressible, the conservation of mass for air can be expressed as

$$\dot{V}_{\text{piston}}=\dot{V}_{\text{hole}} \;\rightarrow\; \mathbf{V}_{\text{piston}}A_{\text{piston}}=\mathbf{V}_{\text{hole}}A_{\text{hole}} \;\rightarrow\; \mathbf{V}_{\text{piston}}=\frac{A_{\text{hole}}}{A_{\text{piston}}}\mathbf{V}_{\text{hole}}=\frac{\pi D_{\text{hole}}^2/4}{\pi D_{\text{piston}}^2/4}\mathbf{V}_1$$

Simplifying and substituting, the piston velocity is determined to be

$$\mathbf{V}_{\text{piston}}=\left(\frac{D_{\text{hole}}}{D_{\text{piston}}}\right)^2\mathbf{V}_1=\left(\frac{0.3\,\text{cm}}{5\,\text{cm}}\right)^2(41.7\,\text{m/s})=\mathbf{0.15\,m/s}$$



Discussion In reality, the piston velocity must be higher to overcome the losses. Also, a lower piston velocity will do the job if the diameter of the hole is reduced.

12-45 The water height in an airtight pressurized tank is given. A hose pointing straight up is connected to the bottom of the tank. The maximum height to which the water stream could rise is to be determined. √

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The friction between the water and air is negligible.

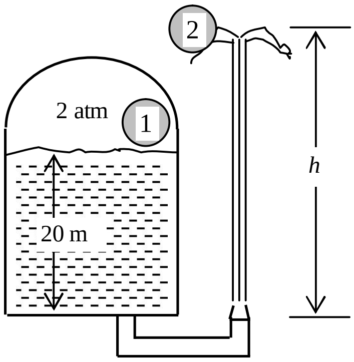
Properties We take the density of water to be 1000 kg/m^3.

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory. Also, we take the reference level at the bottom of the tank. At the top of the water trajectory $\mathbf{V}_2$ = 0, and atmospheric pressure pertains. Noting that z_1 = 20 m, $P_{1,\text{gage}}$ = 2 atm, $P_2 = P_{\text{atm}}$, and that the fluid velocity at the free surface of the tank is very low ($\mathbf{V}_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+z_1=\frac{P_{atm}}{\rho g}+z_2 \quad\rightarrow\quad z_2=\frac{P_1-P_{atm}}{\rho g}+z_1=\frac{P_{1,\text{gage}}}{\rho g}+z_1$$

Substituting,

$$z_2=\frac{2\text{ atm}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{101{,}325\text{ N/m}^2}{1\text{ atm}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)+20=\mathbf{40.7\text{ m}}$$



Therefore, the water jet can rise as high as 40.7 m into the sky from the ground.

Discussion The result obtained by the Bernoulli equation represents the upper limit, and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.7 m (giving us an upper limit), and in all likelihood, the rise will be much less because of frictional losses.

12-46 A pitot tube equipped with a water manometer is held parallel to air flow, and the differential height of the water column is measured. The flow velocity of air is to be determined. √

Assumptions **1**The flow of air is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The effect of air column on the pressure change is negligible because of its low density, and thus the air column in the manometer can be ignored.

Properties We take the density of water to be ρ = 1000 kg/m³. The density of air is given to be 1.25 kg/m³.

Analysis We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the pitot tube, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the pitot tube. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}=\frac{P_2}{\rho g} \quad\rightarrow\quad \mathbf{V}_1=\sqrt{\frac{2(P_2-P_1)}{\rho_{air}}} \qquad (1)$$

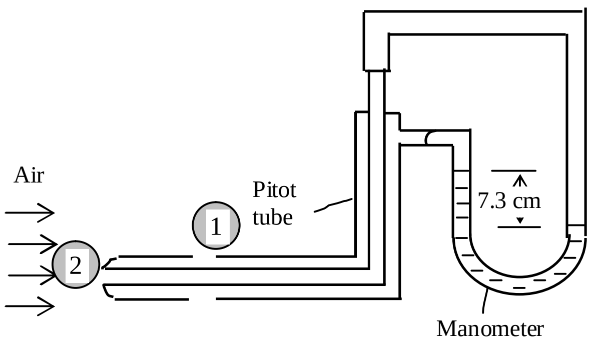
The pressure rise at the tip of the pitot tube is simply the pressure change indicated by the differential water column of the manometer,

$$P_2 - P_1 = \rho_{\text{water}}\, gh \qquad (2)$$

Combining Eqs. (1) and (2) and substituting, the flow velocity is determined to be

$$\mathbf{V}_1=\sqrt{\frac{2\rho_{\text{water}}\,gh}{\rho_{\text{air}}}}=\sqrt{\frac{2(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.073\,\text{m})}{1.25\,\text{kg/m}^3}}=\mathbf{33.8\,m/s}$$

Discussion Note that flow velocity in a pipe or duct can be measured easily by a pitot tube by inserting the probe into the pipe or duct parallel to flow, and reading the differential height. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.



12-47E A pitot tube equipped with a differential pressure gage is used to measure the air velocity in a duct. For a given differential pressure reading, the flow velocity of air is to be determined. √

Assumptions The flow of air is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Properties The gas constant of air is R = 0.3704 psia·ft^3/lbm·R.

Analysis We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the pitot tube, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the pitot tube. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}=\frac{P_2}{\rho g} \quad \rightarrow \quad \mathbf{V}_1=\sqrt{\frac{2(P_2-P_1)}{\rho}}$$

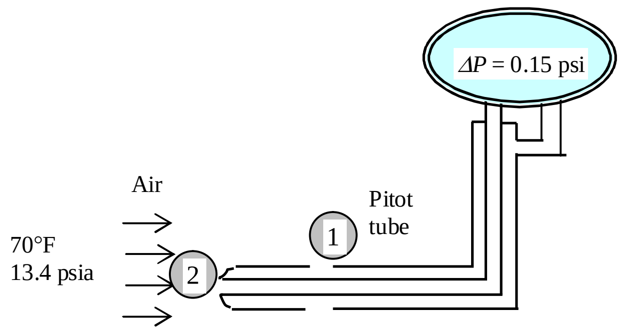
where

$$\rho=\frac{P}{RT}=\frac{13.4\text{ psia}}{(0.3704\text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(70+460\text{ R})}=0.0683\text{ lbm/ft}^3$$

Substituting the given values, the flow velocity is determined to be

$$\mathbf{V}_1=\sqrt{\frac{2(0.15\text{ psi})}{0.0683\text{lbm/ft}^3}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)}=\mathbf{143\ ft/s}$$

Discussion Note that flow velocity in a pipe or duct can be measured easily by a pitot tube by inserting the probe into the pipe or duct parallel to flow, and reading the pressure differential. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.



12-48 In a power plant, water enters the nozzles of a hydraulic turbine at a specified pressure. The maximum velocity water can be accelerated to by the nozzles is to be determined. √

Assumptions **1**The flow of water is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Water enters the nozzle with a low velocity.

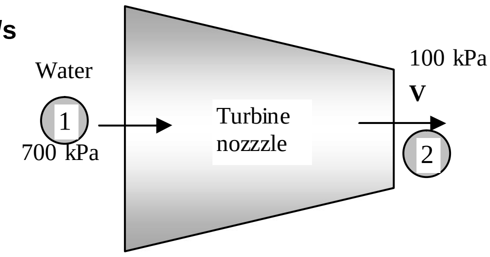
Properties We take the density of water to be ρ = 1000 kg/m³.

Analysis We take points 1 and 2 at the inlet and exit of the nozzle, respectively. Noting that $\mathbf{V}_1 \cong 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+\frac{\mathbf{V}_2^2}{2g} \quad\rightarrow\quad \mathbf{V}_2=\sqrt{\frac{2(P_1-P_{atm})}{\rho}}$$

Substituting the given values, the nozzle exit velocity is determined to be

$$\mathbf{V}_1=\sqrt{\frac{2(700-100)\text{ kPa}}{1000\text{ kg/m}^3}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}=\mathbf{34.6\text{ m/s}}$$



Discussion This is the maximum nozzle exit velocity, and the actual velocity will be less because of friction between water and the walls of the nozzle.

Energy Equation

12-49C It is *impossible* for the fluid temperature to decrease during steady, incompressible, adiabatic flow since this would require the entropy of an adiabatic system to decrease, which would be a violation of the 2nd law of thermodynamics.

12-50C Yes, the *frictional effects* are negligible if the fluid temperature remains constant during steady, incompressible flow since any irreversibility such as friction would cause the entropy and thus temperature of the fluid to increase during adiabatic flow.

12-51C *Head loss* is the loss of mechanical energy expressed as an equivalent column height of fluid, i.e., head. It is related to the mechanical energy loss by $h_L = \dfrac{e_{\text{mech, loss}}}{g} = \dfrac{\dot{E}_{\text{mech, loss}}}{\dot{m}g}$.

12-52C Pump head is the useful power input to the pump expressed as an equivalent column height of fluid. It is related to the useful pumping power input by $h_{\text{pump}} = \dfrac{w_{\text{pump, u}}}{g} = \dfrac{\dot{W}_{\text{pump, u}}}{\dot{m}g}$

12-53C The *kinetic energy correction factor* is a correction factor to account for the fact that kinetic energy using average velocity is not the same as the actual kinetic energy using the actual velocity profile. Its effect is usually negligible (the square of a sum is not equal to the sum of the squares of its components).

12-54C By Bernoulli Equation, the maximum theoretical height to which the water stream could rise is the tank water level, which is 20 meters above the ground. Since the water rises above the tank level, the tank cover must be airtight, containing pressurized air above the water surface. Otherwise, a pump would have to pressurize the water somewhere in the hose.

12-55 Underground water is pumped to a pool at a given elevation. The maximum flow rate and the pressures at the inlet and outlet of the pump are to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the inlet and the outlet of the pump is negligible. **3** We assume the frictional effects to be negligible since the *maximum* flow rate is to be determined, $\dot{E}_{\text{mech, loss}} = 0$.

Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 (Table A-3).

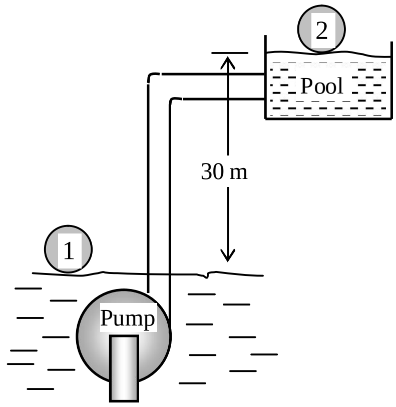
Analysis (*a*) The pump-motor draws 3-kW of power, and is 70% efficient. Then the useful mechanical (shaft) power it delivers to the fluid is

$$\dot{W}_{\text{pump, u}} = \eta_{\text{pump-motor}} \dot{W}_{\text{electric}} = (0.70)(3\text{ kW}) = 2.1\text{ kW}$$

We take point 1 at the free surface of underground water, which is also taken as the reference level ($z_1 = 0$), and point 2 at the free surface of the pool. Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$), the velocities are negligible at both points ($\mathbf{V}_1 = \mathbf{V}_2 = 0$), and frictional losses are disregarded. Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}}$$

$$\rightarrow \quad \dot{W}_{\text{pump, u}} = \dot{m}gz_2$$



Then the mass and volume flow rates of water become

$$\dot{m} = \frac{\dot{W}_{\text{pump, u}}}{gz_2} = \frac{2.1\text{ kJ/s}}{(9.81\text{ m/s}^2)(30\text{ m})}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ}}\right) = 7.14\text{ kg/s}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{7.14\text{ kg/s}}{1000\text{ kg/m}^3} = \mathbf{7.14\times10^{-3}\ m^3/s}$$

(*b*) We take points 3 and 4 at the inlet and the exit of the pump, respectively, where the flow velocities are

$$\mathbf{V}_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi D_3^2/4} = \frac{7.14\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.05\text{ m})^2/4} = 3.64\text{ m/s}, \quad \mathbf{V}_4 = \frac{\dot{V}}{A_4} = \frac{\dot{V}}{\pi D_4^2/4} = \frac{7.14\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.07\text{ m})^2/4} = 1.86\text{ m/s}$$

We take the pump as the control volume. Noting that $z_3 = z_4$ and $\dot{E}_{\text{mech, loss}} = 0$ (since we consider the useful pump power only), the energy equation for this control volume reduces to

$$\dot{m}\left(\frac{P_3}{\rho} + \frac{\mathbf{V}_3^2}{2} + gz_3\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_4}{\rho} + \frac{\mathbf{V}_4^2}{2} + gz_4\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}} \rightarrow P_4 - P_3 = \frac{\rho(\mathbf{V}_3^2 - \mathbf{V}_4^2)}{2} + \frac{\dot{W}_{\text{pump, u}}}{\dot{V}}$$

Substituting,

$$P_4 - P_3 = \frac{(1000\text{ kg/m}^3)[(3.64\text{ m/s})^2 - (1.86\text{ m/s})^2]}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) + \frac{2.1\text{ kJ/s}}{7.14\times10^{-3}\text{ m}^3/\text{s}}\left(\frac{1\text{ kN}\cdot\text{m}}{1\text{ kJ}}\right)$$

$$= (4.9 + 294.1)\text{ kN/m}^2 = \mathbf{299\ kPa}$$

Discussion In an actual system, the flow rate of water will be less because of friction in pipes. Also, the effect of flow velocities on the pressure change across the pump is negligible in this case (under 2%) and can be ignored.

12-56 Underground water is pumped to a pool at a given elevation. For a given head loss, the flow rate and the pressures at the inlet and outlet of the pump are to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the inlet and the outlet of the pump is negligible.

Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 (Table A-3).

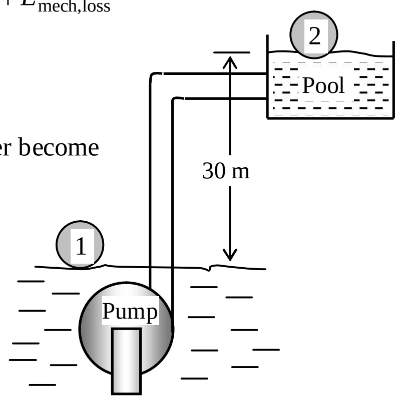
Analysis (*a*) The pump-motor draws 3-kW of power, and is 70% efficient. Then the useful mechanical (shaft) power it delivers to the fluid is

$$\dot{W}_{\text{pump, u}} = \eta_{\text{pump-motor}} \dot{W}_{\text{electric}} = (0.70)(3\,\text{kW}) = 2.1\,\text{kW}$$

We take point 1 at the free surface of underground water, which is also taken as the reference level ($z_1 = 0$), and point 2 at the free surface of the pool. Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$), and the velocities are negligible at both points ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}}$$

$$\rightarrow \quad \dot{W}_{\text{pump, u}} = \dot{m}gz_2 + \dot{E}_{\text{mech, loss}}$$



Noting that $\dot{E}_{\text{mech, loss}} = \dot{m}gh_L$, the mass and volume flow rates of water become

$$\dot{m} = \frac{\dot{W}_{\text{pump, u}}}{gz_2 + gh_L} = \frac{\dot{W}_{\text{pump, u}}}{g(z_2 + h_L)}$$

$$= \frac{2.1\,\text{kJ/s}}{(9.81\,\text{m/s}^2)(30+5\,\text{m})}\left(\frac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ}}\right) = 6.12\,\text{kg/s}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{6.12\,\text{kg/s}}{1000\,\text{kg/m}^3} = \mathbf{6.12\times10^{-3}\ m^3/s}$$

(*b*) We take points 3 and 4 at the inlet and the exit of the pump, respectively, where the flow velocities are

$$\mathbf{V}_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi D_3^2/4} = \frac{6.12\times10^{-3}\,\text{m}^3/\text{s}}{\pi(0.05\,\text{m})^2/4} = 3.14\,\text{m/s}, \quad \mathbf{V}_4 = \frac{\dot{V}}{A_4} = \frac{\dot{V}}{\pi D_4^2/4} = \frac{6.12\times10^{-3}\,\text{m}^3/\text{s}}{\pi(0.07\,\text{m})^2/4} = 1.60\,\text{m/s}$$

We take the pump as the control volume. Noting that $z_3 = z_4$ and $\dot{E}_{\text{mech, loss}} = 0$ (since we consider the pump and useful pump power only), the energy equation for this control volume reduces to

$$\dot{m}\left(\frac{P_3}{\rho} + \frac{\mathbf{V}_3^2}{2} + gz_3\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_4}{\rho} + \frac{\mathbf{V}_4^2}{2} + gz_4\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \rightarrow P_4 - P_3 = \frac{\rho(\mathbf{V}_3^2 - \mathbf{V}_4^2)}{2} + \frac{\dot{W}_{\text{pump,u}}}{\dot{V}}$$

Substituting,

$$P_4 - P_3 = \frac{(1000\,\text{kg/m}^3)[(3.14\,\text{m/s})^2 - (1.60\,\text{m/s})^2]}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) + \frac{2.1\,\text{kJ/s}}{6.12\times10^{-3}\,\text{m}^3/\text{s}}\left(\frac{1\,\text{kN}\cdot\text{m}}{1\,\text{kJ}}\right)$$

$$= (4+343)\,\text{kN/m}^2 = \mathbf{347\ kPa}$$

Discussion Note that frictional losses in pipes causes the flow rate of water to decrease. Also, the effect of flow velocities on the pressure change across the pump is negligible in this case (about 1%) and can be ignored.

12-57E In a hydroelectric power plant, the elevation difference, the power generation, and the overall turbine-generator efficiency are given. The minimum flow rate required is to be determined.√

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water levels at the reservoir and the discharge site remain constant. **3** We assume the flow to be *frictionless* since the *minimum* flow rate is to be determined, $\dot{E}_{\text{mech, loss}} = 0$.

Properties We take the density of water to be ρ = 62.4 lbm/ft^3.

Analysis We take point 1 at the free surface of the reservoir and point 2 at the free surface of the discharge water stream, which is also taken as the reference level (z_2 = 0). Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$), the velocities are negligible at both points ($\mathbf{V}_1 = \mathbf{V}_2 = 0$), and frictional losses are disregarded. Then the energy equation in terms of heads for steady incompressible flow through a control volume between these two points that includes the turbine and the pipes reduces to

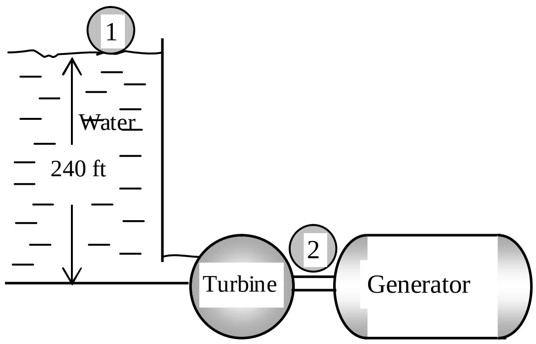
$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 + h_{\text{pump}} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 + h_{\text{turbine}} + h_L \quad \rightarrow \quad h_{\text{turbine}} = z_1$$

Substituting and noting that $\dot{W}_{\text{turbine, elect}} = \eta_{\text{turbine-gen}} \dot{m} g h_{\text{turbine}}$, the turbine head and the mass and volume flow rates of water are determined to be

$$h_{\text{turbine}} = z_1 = 240 \text{ ft}$$

$$\dot{m} = \frac{\dot{W}_{\text{turbine,elect}}}{\eta_{\text{turbine-gen}} g h_{\text{turbine}}} = \frac{100 \text{ kW}}{0.83(32.2 \text{ ft/s}^2)(240 \text{ ft})} \left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}} \right) \left(\frac{0.9478 \text{ Btu/s}}{1 \text{ kW}} \right) = \mathbf{370 \text{ lbm/s}}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{370 \text{ lbm/s}}{62.4 \text{ lbm/ft}^3} = \mathbf{5.93 \text{ ft}^3/\text{s}}$$



Therefore, the flow rate of water must be at least 5.93 ft^3/s to generate the desired electric power while overcoming friction losses in pipes.

Discussion In an actual system, the flow rate of water will be more because of frictional losses in pipes.

12-58E In a hydroelectric power plant, the elevation difference, the head loss, the power generation, and the overall turbine-generator efficiency are given. The flow rate required is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water levels at the reservoir and the discharge site remain constant.

Properties We take the density of water to be ρ = 62.4 lbm/ft^3.

Analysis We take point 1 at the free surface of the reservoir and point 2 at the free surface of the discharge water stream, which is also taken as the reference level ($z_2 = 0$). Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$), the velocities are negligible at both points ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation in terms of heads for steady incompressible flow through a control volume between these two points that includes the turbine and the pipes reduces to

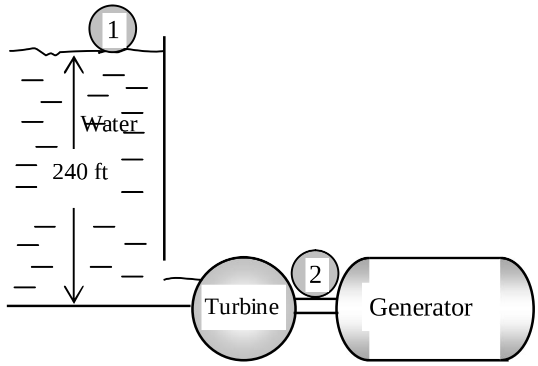
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{turbine}}=z_1-h_L$$

Substituting and noting that $\dot{W}_{\text{turbine, elect}}=\eta_{\text{turbine-gen}}\dot{m}gh_{\text{turbine}}$, the turbine head and the mass and volume flow rates of water are determined to be

$$h_{\text{turbine}}=z_1-h_L=240-36=204\ \text{ft}$$

$$\dot{m}=\frac{\dot{W}_{\text{turbine,elect}}}{\eta_{\text{turbine-gen}}\,gh_{\text{turbine}}}=\frac{100\ \text{kW}}{0.83(32.2\ \text{ft/s}^2)(204\ \text{ft})}\left(\frac{25{,}037\ \text{ft}^2/\text{s}^2}{1\ \text{Btu/lbm}}\right)\left(\frac{0.9478\ \text{Btu/s}}{1\ \text{kW}}\right)=\mathbf{435\ lbm/s}$$

$$\dot{V}=\frac{\dot{m}}{\rho}=\frac{435\ \text{lbm/s}}{62.4\ \text{lbm/ft}^3}=\mathbf{6.98\ ft^3/s}$$



Therefore, the flow rate of water must be at least 6.98 ft^3/s to generate the desired electric power while overcoming friction losses in pipes.

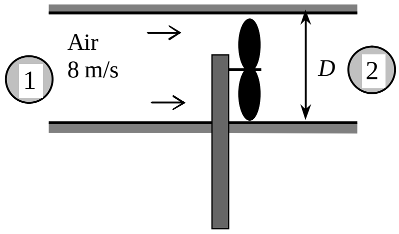
Discussion Note that the effect of frictional losses in pipes is to increase the required flow rate of water to generate a specified amount of electric power.

12-59 A fan is to ventilate a bathroom by replacing the entire volume of air once every 10 minutes while air velocity remains below a specified value. The wattage of the fan-motor unit, the diameter of the fan casing, and the pressure difference across the fan are to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** Frictional losses along the flow (other than those due to the fan-motor inefficiency) are negligible. **3** The fan unit is horizontal so that z = constant along the flow (or, the elevation effects are negligible because of the low density of air).

Properties The density of air is given to be 1.25 kg/m^3.

Analysis (a) The volume of air in the bathroom is $V = 2\text{ m} \times 3\text{ m} \times 3\text{ m} = 18$ m^3. Then the volume and mass flow rates of air through the casing must be



$$\dot{V} = \frac{V}{\Delta t} = \frac{18\text{ m}^3}{10 \times 60\text{ s}} = 0.03\text{ m}^3/\text{s}$$

$$\dot{m} = \rho \dot{V} = (1.25\text{ kg/m}^3)(0.03\text{ m}^3/\text{s}) = 0.0375\text{ kg/s}$$

We take points 1 and 2 on the inlet and exit sides of the fan, respectively. Point 1 is sufficiently far from the fan so that $P_1 = P_{\text{atm}}$ and the flow velocity is negligible ($\mathbf{V}_1 = 0$). Also, $P_2 = P_{\text{atm}}$. Then the energy equation for this control volume between the points 1 and 2 reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2} = (0.0375\text{ kg/s})\frac{(8\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = 1.2\text{ W}$$

and $$\dot{W}_{\text{fan, elect}} = \frac{\dot{W}_{\text{fan, u}}}{\eta_{\text{fan-motor}}} = \frac{1.2\text{ W}}{0.5} = \mathbf{2.4\text{ W}}$$

Therefore, the electric power rating of the fan/motor unit must be 2.4 W.

(b) For air mean velocity to remain below the specified value, the diameter of the fan casing should be

$$\dot{V} = A_2\mathbf{V}_2 = (\pi D_2^2/4)\mathbf{V}_2 \quad \rightarrow \quad D_2 = \sqrt{\frac{4\dot{V}}{\pi \mathbf{V}_2}} = \sqrt{\frac{4(0.03\text{ m}^3/\text{s})}{\pi(8\text{ m/s})}} = 0.069\text{ m} = \mathbf{6.9\text{ cm}}$$

(c) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. Noting that $z_3 = z_4$ and $\mathbf{V}_3 = \mathbf{V}_4$ since the fan is a narrow cross-section and neglecting flow loses (other than the loses of the fan unit, which is accounted for by the efficiency), the energy equation for the fan section reduces to

$$\dot{m}\frac{P_3}{\rho} + \dot{W}_{\text{fan, u}} = \dot{m}\frac{P_4}{\rho} \quad \rightarrow \quad P_4 - P_3 = \frac{\dot{W}_{\text{fan, u}}}{\dot{m}/\rho} = \frac{\dot{W}_{\text{fan, u}}}{\dot{V}}$$

Substituting, $$P_4 - P_3 = \frac{1.2\text{ W}}{0.03\text{ m}^3/\text{s}}\left(\frac{1\text{ N}\cdot\text{m/s}}{1\text{ W}}\right) = 40\text{ N/m}^2 = \mathbf{40\text{ Pa}}$$

Therefore, the fan will raise the pressure of air by 40 Pa before discharging it.

Discussion Note that only half of the electric energy y consumed by the fan-motor unit is converted to the mechanical energy of air while the remaining half is converted to heat because of imperfections.

12-60 Water is pumped from a large lake to a higher reservoir. The head loss of the piping system is given. The mechanical efficiency of the pump is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the lake and the reservoir is constant.

Properties We take the density of water to be ρ = 1000 kg/m³ (Table A-3).

Analysis We choose points 1 and 2 at the free surfaces of the lake and the reservoir, respectively, and take the surface of the lake as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the velocities at both locations are negligible ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

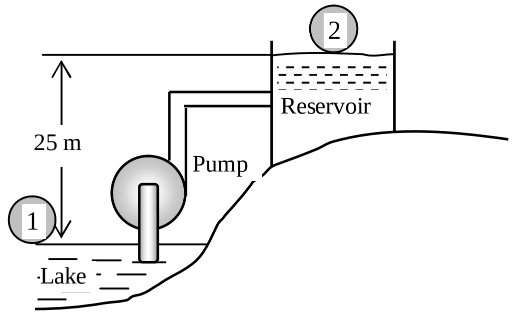
$$\dot{m}\left(\frac{P_1}{\rho}+\frac{\mathbf{V}_1^2}{2}+gz_1\right)+\dot{W}_{\text{pump,u}}=\dot{m}\left(\frac{P_2}{\rho}+\frac{\mathbf{V}_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \rightarrow \dot{W}_{\text{pump, u}}=\dot{m}gz_2+\dot{E}_{\text{mech, loss}}$$

Noting that $\dot{E}_{\text{mech, loss}} = \dot{m}gh_L$, the useful pump power is

$$\dot{W}_{\text{pump,u}} = \dot{m}gz_2 + \dot{m}gh_L = \rho\dot{V}g(z_2 + h_L)$$

$$= (1000\,\text{kg/m}^3)(0.025\,\text{m}^3/\text{s})[(25+7)\ \text{m}]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 7.85\,\text{kNm/s} = 7.85\,\text{kW}$$



Then the mechanical efficiency of the pump becomes

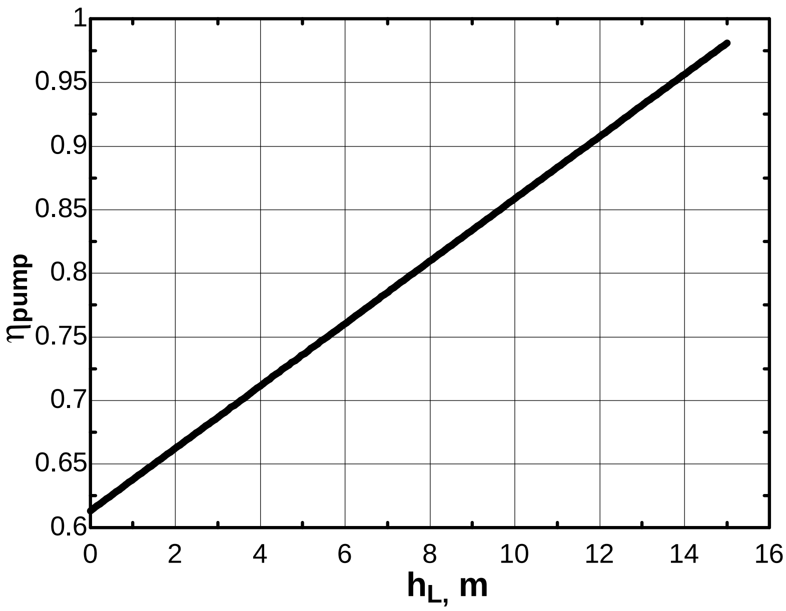
$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{shaft}}} = \frac{7.85\,\text{kW}}{10\,\text{kW}} = 0.785 = \mathbf{78.5\%}$$

Discussion A more practical measure of performance of the pump is the overall efficiency, which can be obtained by multiplying the pump efficiency by the motor efficiency.

12-61 Problem 12-60 is reconsidered. The effect of head loss on mechanical efficiency of the pump. as the head loss varies 0 to 20 m in increments of 2 m is to be investigated.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"
z2=25 "m"
W_shaft=10 "kW"
V_dot=0.025 "m3/s"
W_pump_u=rho*V_dot*g*(z2+h_L)/1000 "kW"
Eta_pump=W_pump_u/W_shaft
```

Head Loss, h_L, m	Pumping power $W_{pump, u}$	Efficiency η_{pump}
0	6.13	0.613
1	6.38	0.638
2	6.62	0.662
3	6.87	0.687
4	7.11	0.711
5	7.36	0.736
6	7.60	0.760
7	7.85	0.785
8	8.09	0.809
9	8.34	0.834
10	8.58	0.858
11	8.83	0.883
12	9.07	0.907
13	9.32	0.932
14	9.56	0.956
15	9.81	0.981



Note that the useful pumping power is used to raise the fluid and to overcome head losses. For a given power input, the pump that overcomes more head loss is more efficient.

12-62 A pump with a specified shaft power and efficiency is used to raise water to a higher elevation. The maximum flow rate of water is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the reservoirs is constant. **3** We assume the flow to be *frictionless* since the *maximum* flow rate is to be determined, $\dot{E}_{\text{mech,loss}} = 0$.

Properties We take the density of water to be ρ = 1000 kg/m^3 (Table A-3).

Analysis We choose points 1 and 2 at the free surfaces of the lower and upper reservoirs, respectively, and take the surface of the lower reservoir as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$) and the velocities at both locations are negligible ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho}+\frac{\mathbf{V}_1^2}{2}+gz_1\right)+\dot{W}_{\text{pump,u}}=\dot{m}\left(\frac{P_2}{\rho}+\frac{\mathbf{V}_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \quad\rightarrow\quad \dot{W}_{\text{pump,u}}=\dot{m}gz_2=\rho\dot{V}gz_2$$

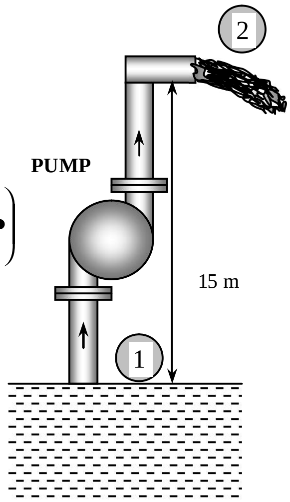
where the useful pumping power is

$$\dot{W}_{\text{pump, u}}=\eta_{\text{pump}}\dot{W}_{\text{pump, shaft}}=(0.82)(7\text{ hp})=5.74\text{ hp}$$

Substituting, the volume flow rate of water is determined to be

$$\dot{V}=\frac{\dot{W}_{\text{pump, u}}}{\rho g z_2}=\frac{5.74\text{ hp}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(15\text{ m})}\left(\frac{745.7\text{ W}}{1\text{ hp}}\right)\left(\frac{1\text{ N}\cdot\text{m/s}}{1\text{ W}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)$$

$$=\mathbf{0.0291\ m^3/s}$$



Discussion This is the maximum flow rate since the frictional effects are ignored. In an actual system, the flow rate of water will be less because of friction in pipes.

12-63 Water flows at a specified rate in a horizontal pipe whose diameter is decreased by a reducer. The pressures are measured before and after the reducer. The head loss in the reducer is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The pipe is horizontal.

Properties We take the density of water to be ρ = 1000 kg/m³ (Table A-3).

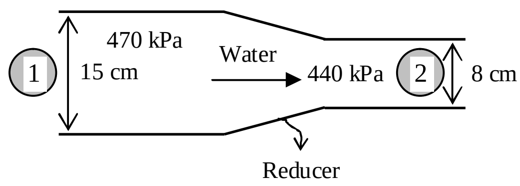
Analysis We take points 1 and 2 along the centerline of the pipe before and after the reducer, respectively. Noting that $z_1 = z_2$, the energy equation for steady incompressible flow through a control volume between these two points reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_L=\frac{P_1-P_2}{\rho g}+\frac{\mathbf{V}_1^2-\mathbf{V}_2^2}{2g}$$

where

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.035\text{ m}^3/\text{s}}{\pi(0.15\text{ m})^2/4}=1.98\text{ m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.035\text{ m}^3/\text{s}}{\pi(0.08\text{ m})^2/4}=6.96\text{ m/s}$$



Substituting, the head loss in the reducer is determined to be

$$h_L=\frac{(470-440)\text{ kPa}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1\text{ kN/m}^2}{1\text{ kPa}}\right)\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)+\frac{(1.98\text{ m/s})^2-(6.96\text{ m/s})^2}{2(9.81\text{ m/s}^2)}$$
$$=3.06-2.27=\mathbf{0.79\text{ m}}$$

Discussion Note that the 0.79 m of the head loss is due to frictional effects and 2.27 m is due to the increase in velocity. This head loss corresponds to a power potential loss of

$$\dot{E}_{\text{mech,loss}}=\rho\dot{V}gh_L=(1000\text{ kg/m}^3)(0.035\text{ m}^3/\text{s})(9.81\text{ m/s}^2)(0.79\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right)=\mathbf{271\text{ W}}$$

12-64 A hose connected to the bottom of a tank is equipped with a nozzle at the end pointing straight up. The water is pressurized by a pump, and the height of the water jet is measured. The minimum pressure rise supplied by the pump is to be determined.

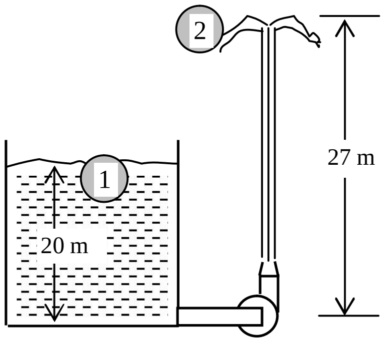
Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** Friction between the water and air as well as friction in the hose is negligible. **3** The water surface is open to the atmosphere.

Properties We take the density of water to be ρ = 1000 kg/m^3 (Table A-3).

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory where $\mathbf{V}_2 = 0$ and $P_1 = P_2 = P_{atm}$. Also, we take the reference level at the bottom of the tank. Noting that z_1 = 20 m and z_2 = 27 m, h_L =0 (to get the minimum value for required pressure rise), and that the fluid velocity at the free surface of the tank is very low ($\mathbf{V}_1 \cong 0$), the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the water stream reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{pump}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{turbine}+h_L$$

$$\rightarrow \qquad h_{pump}=z_2-z_1$$



Substituting,

$$h_{pump}=27-20=7\text{ m}$$

A water column height of 7 m corresponds to a pressure rise of

$$\Delta P_{pump,\,min}=\rho g h_{pump}=(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(7\text{ m})\left(\frac{1\text{ N}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$=68.7\text{ kN/m}^2=\mathbf{68.7\ kPa}$$

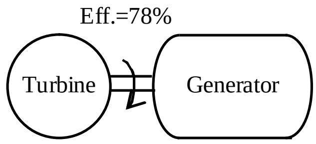
Therefore, the pump must supply a minimum pressure rise of 68.7 kPa.

Discussion The result obtained above represents the minimum value, and should be interpreted accordingly. In reality, a larger pressure rise will need to be supplied to overcome friction.

12-65 The available head of a hydraulic turbine and its overall efficiency are given. The electric power output of this turbine is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The available head remains constant.

Properties We take the density of water to be ρ = 1000 kg/m^3 (Table A-3).



Analysis When the turbine head is available, the corresponding power output is determined from

$$\dot{W}_{\text{turbine}} = \eta_{\text{turbine}} \dot{m} g h_{\text{turbine}} = \eta_{\text{turbine}} \rho \dot{V} g h_{\text{turbine}}$$

Substituting,

$$\dot{W}_{\text{turbine}} = 0.78(1000\ \text{kg/m}^3)(0.25\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(85\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right) = \mathbf{163\ kW}$$

Discussion Note that the power output of a hydraulic turbine is proportional to the available turbine head and the flow rate.

12-66 An entrepreneur is to build a large reservoir above the lake level, and pump water from the lake to the reservoir at night using cheap power, and let the water flow from the reservoir back to the lake during the day, producing power. The potential revenue this system can generate per year is to be determined.

Assumptions **1** The flow in each direction is steady, one-dimensional, and incompressible. **2** The elevation difference between the lake and the reservoir can be taken to be constant, and the elevation change of reservoir during charging and discharging is disregarded. **3** The given unit prices remain constant. **4** The system operates every day of the year for 10 hours in each mode.

Properties We take the density of water to be ρ = 1000 kg/m^3 (Table A-3).

Analysis We choose points 1 and 2 at the free surfaces of the lake and the reservoir, respectively, and take the surface of the lake as the reference level. Both points are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the velocities at both locations are negligible ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation in terms of heads for steady incompressible flow through a control volume between these two points that includes the pump (or the turbine) and the pipes reduces to

Pump mode: $$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \;\rightarrow\; h_{\text{pump}}=z_2+h_L=40+4=44\text{ m}$$

Turbine mode: (switch points 1 and 2 so that 1 is on inlet side) $\rightarrow$ $h_{\text{turbine}}=z_1-h_L=40-4=36\text{ m}$

The pump and turbine power corresponding to these heads are

$$\dot{W}_{\text{pump, elect}}=\frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\rho\dot{V}gh_{\text{pump}}}{\eta_{\text{pump-motor}}}$$
$$=\frac{(1000\text{ kg/m}^3)(2\text{ m}^3/\text{s})(9.81\text{ m/s}^2)(44\text{ m})}{0.75}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right)=1151\text{ kW}$$

$$\dot{W}_{\text{turbine}}=\eta_{\text{turbine-gen}}\dot{m}gh_{\text{turbine}}=\eta_{\text{turbine-gen}}\rho\dot{V}gh_{\text{turbine}}$$
$$=0.75(1000\text{ kg/m}^3)(2\text{ m}^3/\text{s})(9.81\text{ m/s}^2)(36\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right)=530\text{ kW}$$

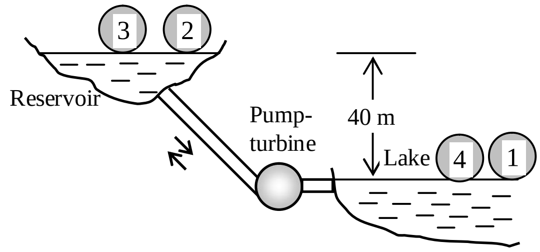
Then the power cost of the pump, the revenue generated by the turbine, and the net income (revenue minus cost) per year become

$$\text{Cost}=\dot{W}_{\text{pump, elect}}\Delta t\times\text{Unit price}=(1151\text{ kW})(365\times 24\text{ h/year})(\$0.03/\text{kWh})=\$302{,}500/\text{year}$$

$$\text{Reveue}=\dot{W}_{\text{turbine}}\Delta t\times\text{Unit price}=(530\text{ kW})(365\times 24\text{ h/year})(\$0.08/\text{kWh})=\$371{,}400/\text{year}$$

Net income = Revenue – Cost = 371,400 – 302,500 = **$68,900/year**

Discussion It appears that this pump-turbine system has a potential annual income of about $70,000. A decision on such a system will depend on the initial cost of the system, its life, the operating and maintenance costs, the interest rate, and the length of the contract period, among other things.



12-67 Water flows through a horizontal pipe at a specified rate. The pressure drop across a valve in the pipe is measured. The corresponding head loss and the power needed to overcome it are to be determined. √

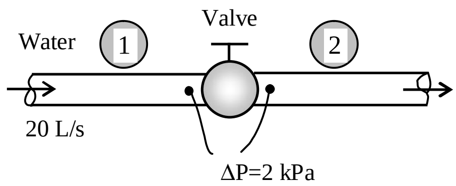
Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The pipe is given to be horizontal (otherwise the elevation difference across the valve is negligible). **3** The mean flow velocities at the inlet and the exit of the valve are equal since the pipe diameter is constant.

Properties We take the density of water to be ρ = 1000 kg/m^3 (Table A-3).

Analysis We take the valve as the control volume, and points 1 and 2 at the inlet and exit of the valve, respectively. Noting that $z_1 = z_2$ and $\mathbf{V}_1 = \mathbf{V}_2$, the energy equation for steady incompressible flow through this control volume reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_L=\frac{P_1-P_2}{\rho g}$$

Substituting,



$$h_L=\frac{2\text{ kN/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)=\mathbf{0.204\text{ m}}$$

The pumping power needed to overcome this head loss is

$$\dot{W}_{\text{pump}}=\dot{E}_{\text{mech, loss}}=\dot{m}gh_L=\rho\dot{V}gh_L$$

$$=(1000\text{ kg/m}^3)(0.020\text{ m}^3/\text{s})(9.81\text{ m/s}^2)(0.204\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right)=\mathbf{40\text{ W}}$$

Therefore, this valve would cause a head loss of 0.204 m, and it would take 40 W of useful pumping power to overcome it.

Discussion The required useful pumping power could also be determined from

$$\dot{W}_{\text{pump}}=\dot{V}\Delta P=(0.020\text{ m}^3/\text{s})(2000\text{ Pa})\left(\frac{1\text{ W}}{1\text{ Pa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{40\text{ W}}$$

12-68E A hose connected to the bottom of a pressurized tank is equipped with a nozzle at the end pointing straight up. The minimum tank air pressure (gage) corresponding to a given height of water jet is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** Friction between water and air as well as friction in the hose is negligible. **3** The water surface is open to the atmosphere.

Properties We take the density of water to be ρ = 62.4 lbm/ft^3 (Table A-3).

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory where $\mathbf{V}_2 = 0$ and $P_1 = P_2 = P_{atm}$. Also, we take the reference level at the bottom of the tank. Noting that z_1 = 66 ft and z_2 = 90 ft, $h_L = 0$ (to get the minimum value for the required air pressure), and that the fluid velocity at the free surface of the tank is very low ($\mathbf{V}_1 \cong 0$), the energy equation for steady incompressible flow through a control volume between these two points reduces to

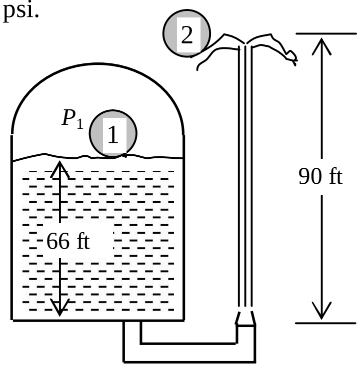
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad \frac{P_1-P_{atm}}{\rho g}=z_2-z_1 \quad\rightarrow\quad \frac{P_{1,gage}}{\rho g}=z_2-z_1$$

Rearranging and substituting, the gage pressure of pressurized air in the tank is determined to be

$$P_{1,\text{ gage}}=\rho g(z_2-z_1)=(62.4\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(90-66\text{ ft})\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ psi}}{144\text{ lbf/ft}^2}\right)=10.4\text{ psi}$$

Therefore, the gage air pressure on top of the water tank must be at least 10.4 psi.

Discussion The result obtained above represents the minimum value, and should be interpreted accordingly. In reality, a larger pressure will be needed to overcome friction.



12-69 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere. The initial discharge velocity from the tank is to be determined. √

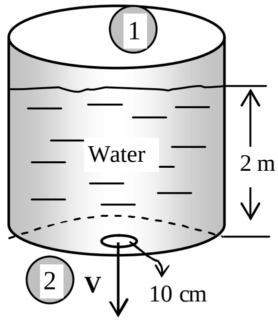
Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The tank is open to the atmosphere.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the orifice. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the tank is very low ($\mathbf{V}_1 \cong 0$), the energy equation between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad z_1=\frac{\mathbf{V}_2^2}{2g}+z_2+h_L$$

Solving for $\mathbf{V}_2$ and substituting,

$$\mathbf{V}_2=\sqrt{2g(z_1-z_2-h_L)}=\sqrt{2(9.81\,\text{m/s}^2)(2-0.3\,\text{m})}=\mathbf{5.78\ m/s}$$



Discussion This is the velocity that will prevail at the beginning. The mean flow velocity will decrease as the water level in the tank decreases.

12-70 Water enters a hydraulic turbine-generator system with a known flow rate, pressure drop, and efficiency. The net electric power output is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** All losses in the turbine are accounted for by turbine efficiency and thus $h_L = 0$. **3** The elevation difference across the turbine is negligible.

Properties We take the density of water to be 1000 kg/m^3 and the density of mercury to be 13,560 kg/m^3 (Table A-3).

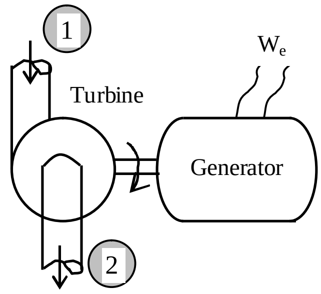
Analysis We choose points 1 and 2 at the inlet and the exit of the turbine, respectively. Noting that the elevation effects are negligible, the energy equation in terms of heads for the turbine reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{turbine}}=\frac{P_1-P_2}{\rho_{\text{water}}\, g}+\frac{\mathbf{V}_1^2-\mathbf{V}_2^2}{2g} \qquad (1)$$

where

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.6\ \text{m}^3/\text{s}}{\pi(0.30\ \text{m})^2/4}=8.49\ \text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.6\ \text{m}^3/\text{s}}{\pi(0.25\ \text{m})^2/4}=12.2\ \text{m/s}$$



The pressure drop corresponding to a differential height of 1.2 m in the mercury manometer is

$$\begin{aligned}P_1-P_2&=(\rho_{\text{Hg}}-\rho_{\text{water}})gh\\&=[(13{,}560-1000)\ \text{kg/m}^3](9.81\ \text{m/s}^2)(1.2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\\&=148\ \text{kN/m}^2=148\ \text{kPa}\end{aligned}$$

Substituting into Eq. (1), the turbine head is determined to be

$$h_{\text{turbine}}=\frac{148\ \text{kN/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right)+\frac{(8.49\ \text{m/s})^2-(12.2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=15.1-3.9=11.2\ \text{m}$$

Then the net electric power output of this hydroelectric turbine becomes

$$\begin{aligned}\dot{W}_{\text{turbine}}&=\eta_{\text{turbine-gen}}\dot{m}gh_{\text{turbine}}=\eta_{\text{turbine-gen}}\rho\dot{V}gh_{\text{turbine}}\\&=0.83(1000\ \text{kg/m}^3)(0.6\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(11.2\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right)=\mathbf{55\ kW}\end{aligned}$$

Discussion It appears that this hydroelectric turbine will generate 55 kW of electric power under given conditions. Note that almost half of the available pressure head is discarded as kinetic energy. This demonstrates the need for a larger turbine exit area and better recovery. For example, the power output can be increased to 74 kW by redesigning the turbine and making the exit diameter of the pipe equal to the inlet diameter, $D_2 = D_1$. Further, if a much larger exit diameter is used and the exit velocity is reduced to a very low level, the power generation can increase to as much as 92 kW.

12-71 The velocity profile for turbulent flow in a circular pipe is given. The kinetic energy correction factor for this flow is to be determined.

Analysis The velocity profile is given by $u(r) = u_{\max}(1 - r/R)^{1/n}$ with n = 7 The kinetic energy correction factor is then expressed as

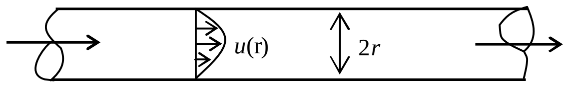
$$\alpha = \frac{1}{A}\int_A \left(\frac{u(r)}{\mathbf{V}_m}\right)^3 dA = \frac{1}{A\mathbf{V}_m^3}\int_A u(r)^3\,dA = \frac{1}{\pi R^2 \mathbf{V}_m^3}\int_{r=0}^{R} u_{\max}^3 \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} (2\pi r)dr = \frac{2u_{\max}^3}{R^2\mathbf{V}_m^3}\int_{r=0}^{R}\left(1 - \frac{r}{R}\right)^{\frac{3}{n}} rdr$$

where the mean velocity is

$$\mathbf{V}_m = \frac{1}{A}\int_A u(r)dA = \frac{1}{\pi R^2}\int_{r=0}^{R} u_{\max}\left(1 - \frac{r}{R}\right)^{1/n}(2\pi r)dr = \frac{2u_{\max}}{R^2}\int_{r=0}^{R}\left(1 - \frac{r}{R}\right)^{1/n} rdr$$

From integral tables,

$$\int (a+bx)^n x dx = \frac{(a+bx)^{n+2}}{b^2(n+2)} - \frac{a(a+bx)^{n+1}}{b^2(n+1)}$$



Then,

$$\int_{r=0}^{R} u(r) rdr = \int_{r=0}^{R}\left(1 - \frac{r}{R}\right)^{1/n} rdr = \left.\frac{(1 - r/R)^{\frac{1}{n}+2}}{\frac{1}{R^2}(\frac{1}{n}+2)} - \frac{(1 - r/R)^{\frac{1}{n}+1}}{\frac{1}{R^2}(\frac{1}{n}+1)}\right|_{r=0}^{R} = \frac{n^2R^2}{(n+1)(2n+1)}$$

$$\int_{r=0}^{R} u(r)^3 rdr = \int_{r=0}^{R}\left(1 - \frac{r}{R}\right)^{3/n} rdr = \left.\frac{(1 - r/R)^{\frac{3}{n}+2}}{\frac{1}{R^2}(\frac{3}{n}+2)} - \frac{(1 - r/R)^{\frac{3}{n}+1}}{\frac{1}{R^2}(\frac{3}{n}+1)}\right|_{r=0}^{R} = \frac{n^2R^2}{(n+3)(2n+3)}$$

Substituting,

$$\mathbf{V}_m = \frac{2u_{\max}}{R^2}\frac{n^2R^2}{(n+1)(2n+1)} = \frac{2n^2u_{\max}}{(n+1)(2n+1)} = 0.8167u_{\max}$$

and

$$\alpha = \frac{2u_{\max}^3}{R^2}\left(\frac{2n^2u_{\max}}{(n+1)(2n+1)}\right)^{-3}\frac{n^2R^2}{(n+3)(2n+3)} = \frac{(n+1)^3(2n+1)^3}{4n^4(n+3)(2n+3)} = \frac{(7+1)^3(2\times7+1)^3}{4\times7^4(7+3)(2\times7+3)} = \mathbf{1.06}$$

Discussion Note that ignoring the kinetic energy correction factor results in an error of just 6% in this case in the kinetic energy term (which may be small itself). Considering that the uncertainties in some terms are usually more that 6%, we can usually ignore this correction factor in analysis.

12-72 A pump is pumping oil at a specified rate. The pressure rise of oil in the pump is measured, and the motor efficiency is specified. The mechanical efficiency of the pump is to be determined.

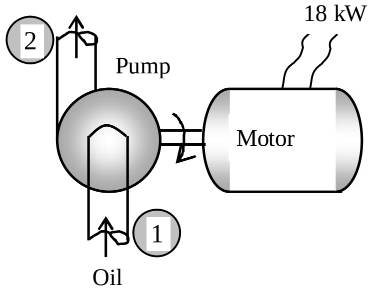
Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference across the pump is negligible. **3** All the losses in the pump are accounted for by the pump efficiency and thus $h_L = 0$.

Properties The density of oil is given to be $\rho = 860\ \text{kg/m}^3$.

Analysis We take points 1 and 2 at the inlet and the exit of the pump, respectively. Noting that $z_1 = z_2$, the energy equation for the pump reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{pump}}=\frac{P_2-P_1}{\rho g}+\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2g}$$

where



$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.1\ \text{m}^3/\text{s}}{\pi(0.08\ \text{m})^2/4}=19.9\ \text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.1\ \text{m}^3/\text{s}}{\pi(0.12\ \text{m})^2/4}=8.84\ \text{m/s}$$

Substituting, the useful pump head and the corresponding useful pumping power are determined to be

$$h_{\text{pump}}=\frac{400{,}000\ \text{N/m}^2}{(860\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)+\frac{(8.84\ \text{m/s})^2-(19.9\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=47.4-16.2=31.2\ \text{m}$$

$$\dot{W}_{\text{pump, u}}=\rho\dot{V}gh_{\text{pump}}=(860\ \text{kg/m}^3)(0.1\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(31.2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right)=26.3\ \text{kW}$$

Then the shaft pumping power and the mechanical efficiency of the pump become

$$\dot{W}_{\text{pump,shaft}}=\eta_{\text{motor}}\dot{W}_{\text{electric}}=(0.90)(35\ \text{kW})=31.5\ \text{kW}$$

$$\eta_{\text{pump}}=\frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump, shaft}}}=\frac{26.3\ \text{kW}}{31.5\ \text{kW}}=0.835=83.5\%$$

Discussion The overall efficiency of this pump/motor unit is the product of the mechanical and motor efficiencies, which is $0.9\times0.835 = 0.75$.

12-73E Water is pumped from a lake to a nearby pool by a pump with specified power and efficiency. The head loss of the piping system and the mechanical power used to overcome it are to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the lake and the free surface of the pool is constant. **3** All the losses in the pump are accounted for by the pump efficiency and thus h_L represents the losses in piping.

Properties We take the density of water to be ρ = 62.4 lbm/ft^3 (Table A-3E).

Analysis The useful pumping power and the corresponding useful pumping head are

$$\dot{W}_{\text{pump, u}} = \eta_{\text{pump}} \dot{W}_{\text{pump}} = (0.73)(12 \text{ hp}) = 8.76 \text{ hp}$$

$$h_{\text{pump}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{m}g} = \frac{\dot{W}_{\text{pump, u}}}{\rho \dot{V} g}$$

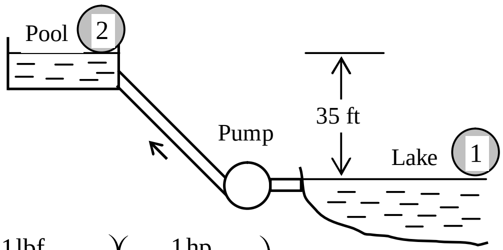
$$= \frac{8.76 \text{ hp}}{(62.4 \text{ lbm/ft}^3)(1.2 \text{ ft}^3\text{/s})(32.2 \text{ ft/s}^2)} \left(\frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right) \left(\frac{550 \text{ lbf} \cdot \text{ft/s}}{1 \text{ hp}} \right) = 64.3 \text{ ft}$$

We choose points 1 and 2 at the free surfaces of the lake and the pool, respectively. Both points are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$) and the velocities at both locations are negligible ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 + h_{\text{pump}} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 + h_{\text{turbine}} + h_L \quad \rightarrow \quad h_L = h_{\text{pump}} + z_1 - z_2$$

Substituting, the head loss is determined to be

$$h_L = h_{\text{pump, u}} - (z_2 - z_1) = 64.3 - 35 = \mathbf{29.3\ ft}$$



Then the power used to overcome it becomes

$$\dot{E}_{\text{mech, loss}} = \rho \dot{V} g h_L$$

$$= (62.4 \text{ lbm/ft}^3)(1.2 \text{ ft}^3\text{/s})(32.2 \text{ ft/s}^2)(29.3 \text{ ft}) \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \left(\frac{1 \text{ hp}}{550 \text{ lbf} \cdot \text{ft/s}} \right)$$

$$= \mathbf{4.0\ hp}$$

Discussion Note that the pump must raise the water an additional height of 29.3 ft to overcome the frictional losses in pipes, which requires an additional useful pumping power of about 4 hp.

12-74 A fireboat is fighting fires by drawing sea water and discharging it through a nozzle. The head loss of the system and the elevation of the nozzle are given. The shaft power input to the pump and the water discharge velocity are to be determined. √

Assumptions The flow is steady, one-dimensional, and incompressible.

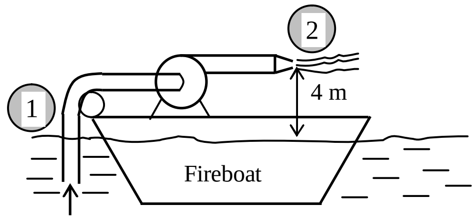
Properties The density of sea water is given to be ρ =1030 kg/m³.

Analysis We take point 1 at the free surface of the sea and point 2 at the nozzle exit. Noting that $P_1 = P_2 = P_{atm}$ and $\mathbf{V}_1 \cong 0$ (point 1 is at the free surface; not at the pipe inlet), the energy equation for the control volume between 1 and 2 that includes the pump and the piping system reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{pump}}=z_2-z_1+\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the water discharge velocity is

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.1\text{ m}^3/\text{s}}{\pi(0.05\text{ m})^2/4}=\mathbf{50.9\ m/s}$$



Substituting, the useful pump head and the corresponding useful pump power are determined to be

$$h_{\text{pump}}=(4\text{ m})+\frac{(50.9\text{ m/s})^2}{2(9.81\text{ m/s}^2)}+(3\text{ m})=138.5\text{ m}$$

$$\dot{W}_{\text{pump,u}}=\rho\dot{V}gh_{\text{pump}}=(1030\text{ kg/m}^3)(0.1\text{m}^3/\text{s})(9.81\text{m/s}^2)(138.5\text{m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right)$$
$$=140\text{ kW}$$

Then the required shaft power input to the pump becomes

$$\dot{W}_{\text{pump, shaft}}=\frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump}}}=\frac{140\text{ kW}}{0.70}=\mathbf{200\ kW}$$

Discussion Note that the pump power is used primarily to increase the kinetic energy of water.

Review Problems

12-75 Water discharges from the orifice at the bottom of a pressurized tank. The time it will take for half of the water in the tank to be discharged and the water level after 10 s are to be determined.

Assumptions **1** The flow is uniform and incompressible, and the frictional effects are negligible. **2** The tank air pressure above the water level is maintained constant.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice. We take the positive direction of z to be upwards with reference level at the orifice ($z_2 = 0$). Fluid at point 2 is open to the atmosphere (and thus $P_2 = P_{\rm atm}$) and the velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$). Then,

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+z_1=\frac{P_{atm}}{\rho g}+\frac{\mathbf{V}_2^2}{2g} \quad\rightarrow\quad \mathbf{V}_2=\sqrt{2gz_1+2P_{1,\rm gage}/\rho}$$

or, $\mathbf{V}_2=\sqrt{2gz+2P_{1,\rm gage}/\rho}$ where z is the water height in the tank at any time t. Water surface moves down as the tank drains, and the value of z changes from H initially to 0 when the tank is emptied completely.

We denote the diameter of the orifice by D, and the diameter of the tank by D_o. The flow rate of water from the tank is obtained by multiplying the discharge velocity by the orifice cross-sectional area,

$$\dot{V}=A_{\rm orifice}\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{2gz+2P_{1,\rm gage}/\rho}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{2gz+2P_{1,\rm gage}/\rho}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

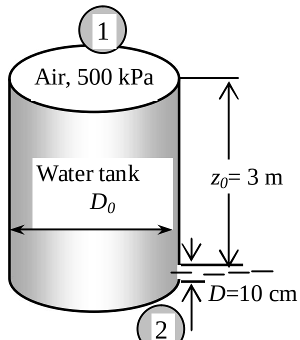
$$dV=A_{\rm tank}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{2gz+2P_{1,\rm gage}/\rho}\,dt=-\frac{\pi D_0^2}{4}dz \quad\rightarrow\quad dt=-\frac{D_0^2}{D^2}\frac{1}{\sqrt{2gz+2P_{1,\rm gage}/\rho}}dz$$

The last relation can be integrated since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_0$ to $t = t$ when $z = z$ gives

$$\sqrt{\frac{2z_0}{g}+\frac{2P_{1,gage}}{\rho g^2}}-\sqrt{\frac{2z}{g}+\frac{2P_{1,gage}}{\rho g^2}}=\frac{D_0^2}{D^2}t$$



where $$\frac{2P_{1,\rm gage}}{\rho g^2}=\frac{2(450-100)\ \text{kN/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)^2}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right)=7.274\ \text{s}^2$$

The time for half of the water in the tank to be discharged ($z = z_0/2$) is

$$\sqrt{\frac{2(3\ \text{m})}{9.81\ \text{m/s}^2}+7.274\ \text{s}^2}-\sqrt{\frac{2(1.5\ \text{m})}{9.81\ \text{m/s}^2}+7.274\ \text{s}^2}=\frac{(0.1\ \text{m})^2}{(2\ \text{m})^2}t \quad\rightarrow\quad t=\mathbf{22.0\ s}$$

(b) Water level after 10s is $$\sqrt{\frac{2(3\ \text{m})}{9.81\ \text{m/s}^2}+7.274\ \text{s}^2}-\sqrt{\frac{2z}{9.81\ \text{m/s}^2}+7.274\ \text{s}^2}=\frac{(0.1\ \text{m})^2}{(2\ \text{m})^2}(10\ \text{s}) \rightarrow z=\mathbf{2.31\ m}$$

Discussion Note that the discharging time is inversely proportional to the square of the orifice diameter. Therefore, the discharging time can be reduced to one-fourth by doubling the diameter of the orifice.

12-76 Air flows through a pipe that consists of two sections at a specified rate. The differential height of a water manometer placed between the two pipe sections is to be determined. √

Assumptions **1**The flow through the pipe is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The losses in the reducing section are negligible. **3** The pressure difference across an air column is negligible because of the low density of air, and thus the air column in the manometer can be ignored.

Properties The density of air is given to be $\rho_{air} = 1.20$ kg/m^3. We take the density of water to be $\rho_w = 1000$ kg/m^3.

Analysis We take points 1 and 2 along the centerline of the pipe over the two tubes of the manometer. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases), the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 \quad \rightarrow \quad P_1 - P_2 = \frac{\rho_{air}(\mathbf{V}_2^2 - \mathbf{V}_1^2)}{2} \qquad (1)$$

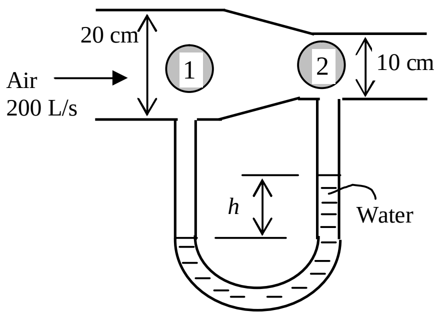
We let the differential height of the water manometer be h. Then the pressure difference $P_2 - P_1$ can also be expressed as

$$P_1 - P_2 = \rho_w g h \qquad (2)$$

Combining Eqs. (1) and (2) and solving for h,

$$\frac{\rho_{air}(\mathbf{V}_2^2 - \mathbf{V}_1^2)}{2} = \rho_w g h \quad \rightarrow \quad h = \frac{\rho_{air}(\mathbf{V}_2^2 - \mathbf{V}_1^2)}{2g\rho_w} = \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g\rho_w / \rho_{air}}$$

Calculating the velocities and substituting,



$$\mathbf{V}_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{0.2\ \text{m}^3/\text{s}}{\pi (0.2\ \text{m})^2/4} = 6.37\ \text{m/s}$$

$$\mathbf{V}_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.2\ \text{m}^3/\text{s}}{\pi (0.1\ \text{m})^2/4} = 25.5\ \text{m/s}$$

$$h = \frac{(25.5\ \text{m/s})^2 - (6.37\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)(1000/1.20)} = 0.037\ \text{m} = \mathbf{3.7\ cm}$$

Therefore, the differential height of the water column will be 3.7 cm.

Discussion Note that the differential height of the manometer is inversely proportional to the density of the manometer fluid. Therefore, heavy fluids such as mercury are used when measuring large pressure differences.

12-77 Air flows through a horizontal duct of variable cross-section. For a given differential height of a water manometer placed between the two pipe sections, the downstream velocity of air is to be determined, and an error analysis is to be conducted. √

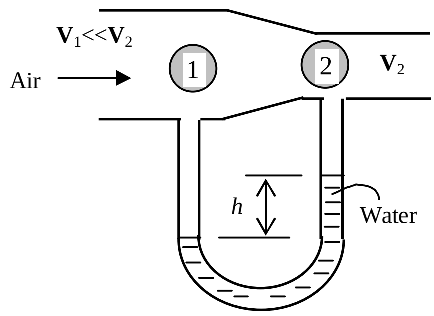
Assumptions **1**The flow through the duct is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The losses in this section of the duct are negligible. **3** The pressure difference across an air column is negligible because of the low density of air, and thus the air column in the manometer can be ignored.

Properties The gas constant of air is R = 0.287 kPa·m^3/kg·K (Table A-1). We take the density of water to be ρ_w = 1000 kg/m^3.

Analysis We take points 1 and 2 along the centerline of the duct over the two tubes of the manometer. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases) and $\mathbf{V}_1 \cong 0$, the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g} \quad\rightarrow\quad \mathbf{V}_2=\sqrt{\frac{2(P_1-P_2)}{\rho_{air}}} \qquad (1)$$

where $\quad P_1 - P_2 = \rho_w gh$



and $$\rho_{air}=\frac{P}{RT}=\frac{100\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{ K})}=1.17\text{ kg/m}^3$$

Substituting into (1), the downstream velocity of air $\mathbf{V}_2$ is determined to be

$$\mathbf{V}_2=\sqrt{\frac{2\rho_w gh}{\rho_{air}}}=\sqrt{\frac{2(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.08\text{ m})}{1.17\text{ kg/m}^3}}=\mathbf{36.6\ m/s} \qquad (2)$$

Therefore, the velocity of air increases from a low level in the first section to 36.6 m/s in the second section.

Error Analysis We observe from Eq. (2) that the velocity is proportional to the square root of the differential height of the manometer fluid. That is, $\mathbf{V}_2 = k\sqrt{h}$.

Taking the differential: $\quad d\mathbf{V}_2 = \frac{1}{2}k\frac{dh}{\sqrt{h}}$

Dividing by $\mathbf{V}_2$: $\quad \frac{d\mathbf{V}_2}{\mathbf{V}_2}=\frac{1}{2}k\frac{dh}{\sqrt{h}}\frac{1}{k\sqrt{h}} \quad\rightarrow\quad \frac{d\mathbf{V}_2}{\mathbf{V}_2}=\frac{dh}{2h}=\frac{\pm 2\text{ mm}}{2\times 80\text{ mm}}=\pm\mathbf{0.0}13$

Therefore, the uncertainty in the velocity corresponding to an uncertainty of 2 mm in the differential height of water is 1.3%, which corresponds to 0.013×(36.6 m/s) = 0.5m/s. Then the discharge velocity can be expressed as

$$\mathbf{V}_2 = 36.6 \pm 0.5 \text{ m/s}$$

12-78 A tap is opened on the wall of a very large tank that contains air. The maximum flow rate of air through the tap is to be determined, and the effect of a larger diameter lead section is to be assessed. √

Assumptions The flow through the tap is steady, frictionless, incompressible, and irrotational (so that the flow rate is maximum, and the Bernoulli equation is applicable).

Properties The gas constant of air is $R = 0.287$ kPa·m^3/kg·K (Table A-1).

Analysis The density of air in the tank is

$$\rho_{air} = \frac{P}{RT} = \frac{102 \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293 \text{ K})} = 1.21 \text{ kg/m}^3$$

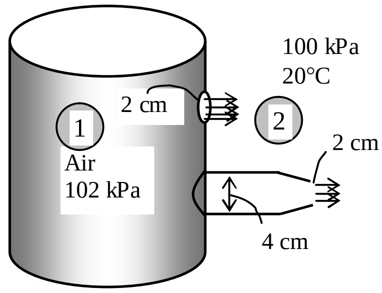
We take point 1 in the tank, and point 2 at the exit of the tap along the same horizontal line. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases) and $\mathbf{V}_1 \cong 0$, the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_{air}}}$$

Substituting, the discharge velocity and the flow rate becomes

$$\mathbf{V}_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_{air}}} = \sqrt{\frac{2(102-100) \text{ kN/m}^2}{1.21 \text{ kg/m}^3}\left(\frac{1000 \text{ kg}\cdot\text{m/s}^2}{1 \text{ kN}}\right)} = 57.5 \text{ m/s}$$

$$\dot{V} = A\mathbf{V}_2 = \frac{\pi D_2^2}{4}\mathbf{V}_2 = \frac{\pi(0.02 \text{ m})^2}{4}(57.5 \text{ m/s}) = \mathbf{0.0181 \text{ m}^3\text{/s}}$$



This is the *maximum* flow rate since it is determined by assuming frictionless flow. The actual flow rate will be less.

Adding a 2-m long larger diameter lead section will have **no effect** on the flow rate since the flow is frictionless (by using the Bernoulli equation, it can be shown that the velocity in this section increases, but the pressure decreases, and there is a smaller pressure difference to drive the flow through the tab, with zero net effect on the discharge rate).

Discussion If the pressure in the tank were 300 kPa, the flow is no longer incompressible, and thus the problem in that case should be analyzed using compressible flow theory.

12-79 Water is flowing through a venturi meter with known diameters and measured pressures. The flow rate of water is to be determined for the case of frictionless flow.

Assumptions **1** The flow through the venturi is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The flow is horizontal so that elevation along the centerline is constant. **3** The pressure is uniform at a given cross-section of the venturi meter (or the elevation effects on pressure measurement are negligible).

Properties We take the density of water to be $\rho = 1000$ kg/m^3.

Analysis We take point 1 at the main flow section and point 2 at the throat along the centerline of the venturi meter. Noting that $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

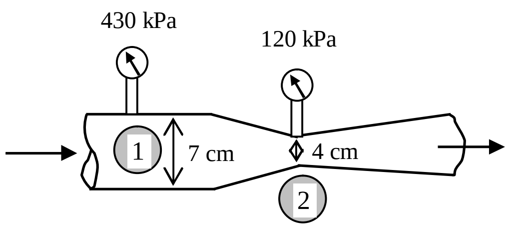
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad P_1-P_2=\rho\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2} \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad\rightarrow\quad A_1\mathbf{V}_1=A_2\mathbf{V}_2=\dot{V} \quad\rightarrow\quad \mathbf{V}_1=\frac{\dot{V}}{A_1} \quad \text{and} \quad \mathbf{V}_2=\frac{\dot{V}}{A_2} \qquad (2)$$

Substituting into Eq. (1),

$$P_1-P_2=\rho\frac{(\dot{V}/A_2)^2-(\dot{V}/A_1)^2}{2}=\frac{\rho\dot{V}^2}{2A_2^2}\left(1-\frac{A_2^2}{A_1^2}\right)$$



Solving for $\dot{V}$ gives the desired relation for the flow rate,

$$\dot{V}=A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}} \qquad (3)$$

The flow rate for the given case can be determined by substituting the given values into this relation to be

$$\dot{V}=\frac{\pi D_2^2}{4}\sqrt{\frac{2(P_1-P_2)}{\rho[1-(D_2/D_1)^4]}}=\frac{\pi(0.04\text{ m})^2}{4}\sqrt{\frac{2(430-120)\text{ kN/m}^2}{(1000\text{ kg/m}^3)[1\text{-}(4/7)^4]}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)}=\mathbf{0.538\ m^3/s}$$

Discussion Venturi meters are commonly used as flow meters to measure the flow rate of gases and liquids by simply measuring the pressure difference P_1 - P_2 by a manometer or pressure transducers. The actual flow rate will be less than the value obtained from Eq. (3) because of the friction losses along the wall surfaces in actual flow. But this difference can be as little as 1% in a well-designed venturi meter. The effects of deviation from the idealized Bernoulli flow can be accounted for by expressing Eq. (3) as

$$\dot{V}=C_c A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}}$$

where C_c is the *venturi discharge coefficient* whose value is less than 1 (it is as large as 0.99 for well-designed venturi meters in certain ranges of flow). For Re > 10^5, the value of venturi discharge coefficient is usually greater than 0.96.

12-80E A hose is connected to the bottom of a water tank open to the atmosphere. The hose is equipped with a pump and a nozzle at the end. The maximum height to which the water stream could rise is to be determined.

Assumptions **1** The flow is frictionless and incompressible. **2** The friction between the water and air is negligible. **3** We take the head loss to be zero ($h_L = 0$) to determine the maximum rise of water jet.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take point 1 at the free surface of the tank, and point 2 at the top of the water trajectory where $\mathbf{V}_2 = 0$. We take the reference level at the bottom of the tank. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

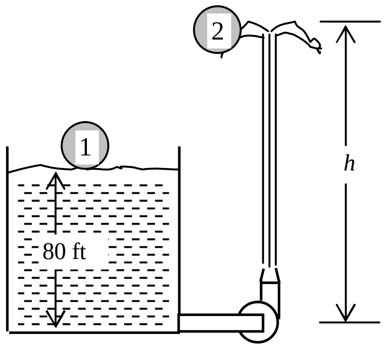
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \to \quad z_1+h_{\text{pump}}=z_2$$

where the pump head is

$$h_{\text{pump}}=\frac{\Delta P_{\text{pump}}}{\rho g}=\frac{10\text{ psi}}{(62.4\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)=23.1\text{ ft}$$

Substituting, the maximum height rise of water jet from the ground level is determined to be

$$z_2=z_1+h_{\text{pump}}=80+23.2=\mathbf{103.2\ ft}$$



Discussion The actual rise of water will be less because of the frictional effects between the water and the hose walls and between the water jet and air.

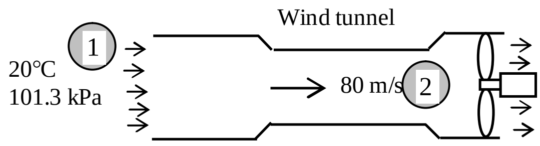
12-81 A wind tunnel draws atmospheric air by a large fan. For a given air velocity, the pressure in the tunnel is to be determined. √

Assumptions **1**The flow through the pipe is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** Air is and ideal gas.

Properties The gas constant of air is R = 0.287 kPa·m^3/kg·K (Table A-1).

Analysis We take point 1 in atmospheric air before it enters the wind tunnel (and thus $P_1 = P_{atm}$ and $\mathbf{V}_1 \cong 0$), and point 2 in the wind tunnel. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases), the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad P_2=P_1-\frac{\rho\mathbf{V}_2^2}{2} \qquad (1)$$



where

$$\rho=\frac{P}{RT}=\frac{101.3\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\,\text{K})}=1.205\,\text{kg/m}^3$$

Substituting, the pressure in the wind tunnel is determined to be

$$P_2=(101.3\,\text{kPa})-(1.205\,\text{kg/m}^3)\frac{(80\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1000\,\text{N/m}^2}\right)=\mathbf{97.4\,kPa}$$

Discussion Note that the velocity in a wind tunnel increases at the expense of pressure. In reality, the pressure will be even lower because of losses.

12-82 Water flows through the enlargement section of a horizontal pipe at a specified rate. For a given head loss, the pressure change across the enlargement section is to be determined. √

Assumptions **1** The flow through the pipe is steady, one-dimensional, and incompressible. **2** The pipe is horizontal.

Properties We take the density of water to be ρ = 1000 kg/m³.

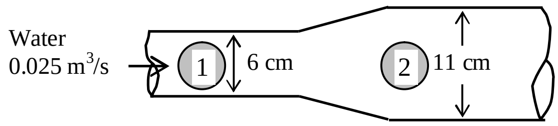
Analysis We take points 1 and 2 at the inlet and exit of the enlargement section along the centerline of the pipe. Noting that $z_1 = z_2$, the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad P_2-P_1=\rho\frac{\mathbf{V}_1^2-\mathbf{V}_2^2}{2}-\rho g h_L$$

where the inlet and exit velocities are

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.025\ \text{m}^3/\text{s}}{\pi(0.06\ \text{m})^2/4}=8.84\ \text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.025\ \text{m}^3/\text{s}}{\pi(0.11\ \text{m})^2/4}=2.63\ \text{m/s}$$



Substituting, the change in static pressure across the enlargement section is determined to b e

$$P_2-P_1=(1000\ \text{kg/m}^3)\left(\frac{(8.84\ \text{m/s})^2-(2.63\ \text{m/s})^2}{2}-(9.81\ \text{m/s}^2)(0.45\ \text{m})\right)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)=\mathbf{31.2\ kPa}$$

Therefore, the water pressure increases by 31.2 kPa across the enlargement section.

Discussion Note that the pressure increases despite the head loss in the enlargement section. This is due to dynamic pressure being converted to static pressure. But the total pressure (static + dynamic) decreases by 0.45 m (or 4.41 kPa) as a result of frictional effects.

12-83 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe with a specified head loss. The initial discharge velocity is to be determined. √EES

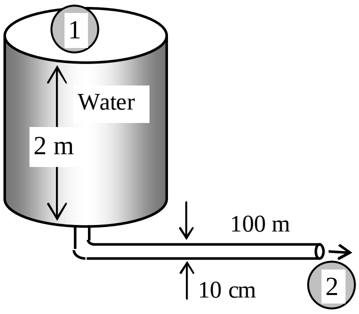
Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** There are no pumps or turbines in the system.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the orifice (z_2 = 0. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the head loss is given to be h_L = 1.5 m. Solving for $\mathbf{V}_2$ and substituting, the discharge velocity of water is determined to be

$$\mathbf{V}_2=\sqrt{2g(z_1-h_L)}=\sqrt{2(9.81\,\text{m/s}^2)(2-1.5)\text{ m}}=\mathbf{3.13\ m/s}$$



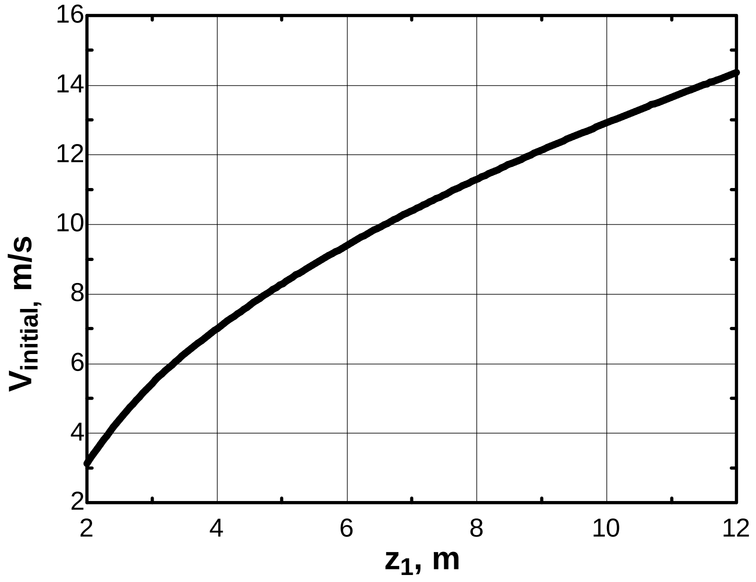
Discussion Note that this is the discharge velocity at the beginning, and the velocity will decrease as the water level in the tank drops. The head loss in that case will change since it depends on velocity.

12-84 Problem 12-83 is reconsidered. The effect of the tank height on the initial discharge velocity of water from the completely filled tank as the tank height varies from 2 to 20 m in increments of 2 m at constant heat loss is to be investigated.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"

h_L=1.5 "m"
D=0.10 "m"
V_initial=SQRT(2*g*(z1-h_L)) "m/s"
```

Tank height, $z1$, m	Head Loss, h_L, m	Initial velocity $V_{initial}$, m/s
2	1.5	3.13
3	1.5	5.42
4	1.5	7.00
5	1.5	8.29
6	1.5	9.40
7	1.5	10.39
8	1.5	11.29
9	1.5	12.13
10	1.5	12.91
11	1.5	13.65
12	1.5	14.35



12-85 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe equipped with a pump with a specified head loss. The required pump head to assure a certain velocity is to be determined.

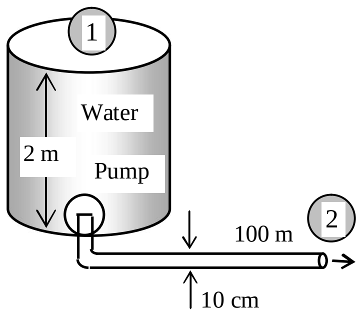
Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad z_1+h_{\text{pump}}=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the head loss is given to be $h_L = 1.5$ m. Solving for $h_{\text{pump, u}}$ and substituting, the required pump head is determined to be

$$h_{\text{pump}}=\frac{\mathbf{V}_2^2}{2g}-z_1+h_L=\frac{(6\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}-(2\ \text{m})+(1.5\ \text{m})=\mathbf{1.15\ m}$$



Discussion Note that this is the required useful pump head at the beginning, and it will need to be increased as the water level in the tank drops to make up for the lost elevation head to maintain the constant discharge velocity.

12-86 … 12-89 Design and Essay Problems

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