Chapter 15
FLOW OVER BODIES: DRAG AND LIFT

Drag, Lift, and Drag Coefficients of Common Geometries

15-1C The flow over a body is said to be *two-dimensional* when the body is too long and of constant cross-section, and the flow is normal to the body (such as the wind blowing over a long pipe perpendicular to its axis). There is no significant flow along the axis of the body. The flow along a body that possesses symmetry along an axis in the flow direction is said to be *axisymmetric* (such as a bullet piercing through air). Flow over a body that cannot be modeled as two-dimensional or axisymmetric is *three-dimensional*. The flow over a car is three-dimensional.

15-2C The velocity of the fluid relative to the immersed solid body sufficiently far away from a body is called the *free-stream velocity*, $\mathbf{V}_\infty$. The *upstream* (or *approach*) *velocity* $\mathbf{V}$ is the velocity of the approaching fluid far ahead of the body. These two velocities are equal if the flow is uniform and the body is small relative to the scale of the free-stream flow.

15-3C A body is said to be *streamlined* if a conscious effort is made to align its shape with the anticipated streamlines in the flow. Otherwise, a body tends to block the flow, and is said to be *blunt*. A tennis ball is a blunt body (unless the velocity is very low and we have "creeping flow").

15-4C The formation of vapor cavities in regions of low pressure in liquid flow is called *cavitation*. It occurs at locations such as the constrictions in a valve or the tips of impeller blades where the pressure drops below vapor pressure due to high velocities. We try to avoid cavitation since it reduces performance, generates annoying vibrations and noise, and causes erosion, surface pitting, fatigue failure, and the eventual destruction of the components or machinery.

15-5C The force a flowing fluid exerts on a body in the flow direction is called *drag*. Drag is caused by friction between the fluid and the solid surface, and the pressure difference between the front and back of the body. We try to minimize drag in order to reduce fuel consumption in vehicles, improve safety and durability of structures subjected to high winds, and to reduce noise and vibration.

15-6C The force a flowing fluid exerts on a body in the normal direction to flow that tend to move the body in that direction is called *lift*. It is caused by the components of the pressure and wall shear forces in the normal direction to flow. The wall shear also contributes to lift (unless the body is very slim), but its contribution is usually small.

15-7C When the drag force F_D, the upstream velocity $\mathbf{V}$, and the fluid density ρ are measured during flow over a body, the drag coefficient can be determined from

$$C_D = \frac{F_D}{\frac{1}{2}\rho \mathbf{V}^2 A}$$

where A is ordinarily the *frontal area* (the area projected on a plane normal to the direction of flow) of the body.

15-8C When the lift force F_L, the upstream velocity **V**, and the fluid density ρ are measured during flow over a body, the lift coefficient can be determined from

$$C_L = \frac{F_L}{\frac{1}{2}\rho \mathbf{V}^2 A}$$

where A is ordinarily the *planform area*, which is the area that would be seen by a person looking at the body from above in a direction normal to the body.

15-9C The *frontal area* of a body is the area seen by a person when looking from upstream. The frontal area is appropriate to use in drag and lift calculations for blunt bodies such as cars, cylinders, and spheres.

15-10C The *planform area* of a body is the area that would be seen by a person looking at the body from above in a direction normal to flow. The planform area is appropriate to use in drag and lift calculations for slender bodies such as flat plate and airfoils when the frontal area is very small.

15-11C The maximum velocity a free falling body can attain is called the *terminal velocity*. It is determined by setting the weight of the body equal to the drag and buoyancy forces, $W = F_D + F_B$.

15-12C The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag* $F_{D,\text{ friction}}$ since it is caused by frictional effects, and the part that is due directly to pressure P and depends strongly on the shape of the body is called the *pressure drag* $F_{D,\text{ pressure}}$. For slender bodies such as airfoils, the friction drag is usually more significant.

15-13C The friction drag coefficient is independent of surface roughness in *laminar flow*, but is a strong function of surface roughness in *turbulent flow* due to surface roughness elements protruding further into the highly viscous laminar sublayer.

15-14C (*a*) In general, the drag coefficient decreases with the Reynolds number at low and moderate Reynolds numbers. (*b*) The drag coefficient is nearly independent of the Reynolds number at high Reynolds numbers (Re > 10^4).

15-15C As a result of attaching fairings to the front and back of a cylindrical body at high Reynolds numbers, (*a*) friction drag increases, (*b*) pressure drag decreases, and (*c*) total drag decreases.

15-16C As a result of streamlining, (*a*) friction drag increases, (*b*) pressure drag decreases, and (*c*) total drag decreases at high Reynolds numbers (the general case), but increases at very low Reynolds numbers since the friction drag dominates at low Reynolds numbers.

15-17C At sufficiently high velocities, the fluid stream detaches itself from the surface of the body. This is called *separation*. It is caused by a fluid flowing over a curved surface at a high velocity (or technically, by adverse pressure gradient). Separation increases the drag coefficient drastically.

15-18C For a moving body to follow another moving body closely by staying close behind is called *drafting*. It reduces the pressure drag and thus the drag coefficient for the drafted body by taking advantage of the low pressure wake region of the moving body in front.

15-19C The car that is contoured to resemble an ellipse has a smaller drag coefficient and thus smaller air resistance, and it is more likely to be more fuel efficient than a car with sharp corners.

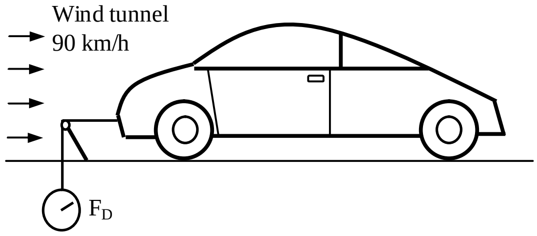
15-20C The bicyclist who leans down and brings his body closer to his knees will go faster since the frontal area and thus the drag force will be less in that position. The drag coefficient will also go down somewhat, but this is a secondary effect.

15-21 The drag force acting on a car is measured in a wind tunnel. The drag coefficient of the car at the test conditions is to be determined. √

Assumptions **1** The flow of air is steady and incompressible. **2** The cross-section of the tunnel is large enough to simulate free flow over the car. **3** The bottom of the tunnel is also moving at the speed of air to approximate actual driving conditions or this effect is negligible. **4** Air is an ideal gas.

Properties The density of air at 1 atm and 25°C is ρ = 1.164 kg/m^3 (Table A-22).

Analysis The drag force acting on a body and the drag coefficient are given by



$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} \quad \text{and} \quad C_D = \frac{2F_D}{\rho A \mathbf{V}^2}$$

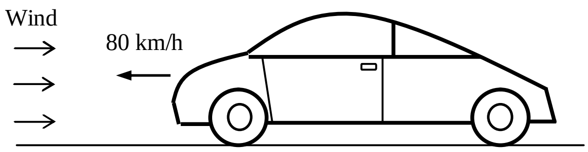
where A is the frontal area. Substituting and noting that 1 m/s = 3.6 km/h, the drag coefficient of the car is determined to be

$$C_D = \frac{2 \times (350\text{ N})}{(1.164\text{ kg/m}^3)(1.40 \times 1.65\text{ m}^2)(90/3.6\text{ m/s})^2}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = \mathbf{0.42}$$

Discussion Note that the drag coefficient depends on the design conditions, and its value will be different at different conditions. Therefore, the published drag coefficients of different vehicles can be compared meaningfully only if they are determined under identical conditions. This shows the importance of developing standard testing procedures in industry.

15-22 A car is moving at a constant velocity. The upstream velocity to be used in fluid flow analysis is to be determined for the cases of calm air, wind blowing against the direction of motion of the car, and wind blowing in the same direction of motion of the car. √

Analysis In fluid flow analysis, the velocity used is the relative velocity between the fluid and the solid body. Therefore:



(*a*) Calm air: $\mathbf{V} = \mathbf{V}_{car}$ = **80 km/h**

(*b*) Wind blowing against the direction of motion:
$\mathbf{V} = \mathbf{V}_{car} + \mathbf{V}_{wind}$ = 80 + 30 = **110 km/h**

(*c*) Wind blowing in the same direction of motion:
$\mathbf{V} = \mathbf{V}_{car} - \mathbf{V}_{wind}$ = 80 - 50 = **30 km/h**

Discussion Note that the wind and car velocities are added when they are in opposite directions, and subtracted when they are in the same direction.

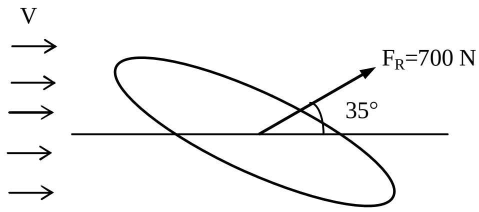
15-23 The resultant of the pressure and wall shear forces acting on a body is given. The drag and the lift forces acting on the body are to be determined.

Analysis The drag and lift forces are determined by decomposing the resultant force into its components in the flow direction and the normal direction to flow,

Drag force: $F_D = F_R \cos\theta = (700\text{ N})\cos 35° = 573\text{ N}$

Lift force: $F_L = F_R \sin\theta = (700\text{ N})\sin 35° = 402\text{ N}$

Discussion Note that the greater the angle between the resultant force and the flow direction, the greater the lift.



15-24 The total drag force acting on a spherical body is measured, and the pressure drag acting on the body is calculated by integrating the pressure distribution. The friction drag coefficient is to be determined.

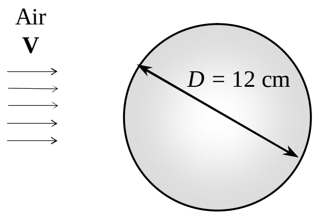
Assumptions **1** The flow of air is steady and incompressible. **2** The surface of the sphere is smooth. **3** The flow over the sphere is turbulent (to be verified).

Properties The density and kinematic viscosity of air at 1 atm and 5°C are ρ = 1.269 kg/m^3 and ν = 1.382×10^{-5} m^2/s (Table A-22). The drag coefficient of sphere in turbulent flow is C_D = 0.2, and its frontal area is $A = \pi D^2/4$ (Table 15-2).

Analysis The total drag force is the sum of the friction and pressure drag forces. Therefore,

$$F_{D,\text{friction}} = F_D - F_{D,\text{pressure}} = 5.2 - 4.9 = 0.3\text{ N}$$

where $F_D = C_D A \dfrac{\rho \mathbf{V}^2}{2}$ and $F_{D,\text{friction}} = C_{D,\text{friction}} A \dfrac{\rho \mathbf{V}^2}{2}$



Taking the ratio of the two relations above gives

$$C_{D,\text{friction}} = \frac{F_{D,\text{friction}}}{F_D} C_D = \frac{0.3\text{ N}}{5.2\text{ N}}(0.2) = \mathbf{0.0115}$$

Now we need to verify that the flow is turbulent. This is done by calculating the flow velocity from the drag force relation, and then the Reynolds number:

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} \;\rightarrow\; \mathbf{V} = \sqrt{\frac{2F_D}{\rho C_D A}} = \sqrt{\frac{2(5.2\text{ N})}{(1.269\text{ kg/m}^3)(0.2)[\pi(0.12\text{ m})^2/4]}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)} = 42.6\text{ m/s}$$

$$\text{Re} = \frac{\mathbf{V}D}{\nu} = \frac{(42.6\text{ m/s})(0.12\text{ m})}{1.382\times10^{-5}\text{ m}^2/s} = 3.70\times10^5$$

which is greater than 2×10^5. Therefore, the flow is turbulent as assumed.

Discussion Note that knowing the flow regime is important in the solution of this problem since the total drag coefficient for a sphere is 0.5 in laminar flow and 0.2 in turbulent flow.

15-25E The frontal area of a car is reduced by redesigning. The amount of fuel and money saved per year as a result are to be determined. √EES

Assumptions **1** The car is driven 12,000 miles a year at an average speed of 55 km/h. **2** The effect of reduction of the frontal area on the drag coefficient is negligible.

Properties The densities of air and gasoline are given to be 0.075 lbm/ft^3 and 50 lbm/ft^3, respectively. The heating value of gasoline is given to be 20,000 Btu/lbm. The drag coefficient is C_D = 0.3 for a passenger car (Table 15-2).

Analysis The drag force acting on a body is determined from

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2}$$

where A is the frontal area of the body. The drag force acting on the car before redesigning is

$$F_D = 0.3(18\text{ ft}^2)\frac{(0.075\text{ lbm/ft}^3)(55\text{ mph})^2}{2}\left(\frac{1.4667\text{ ft/s}}{1\text{ mph}}\right)^2\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 40.9\text{ lbf}$$

Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 12,000 miles are

$$W_{\text{drag}} = F_D L = (40.9\text{ lbf})(12{,}000\text{ miles/year})\left(\frac{5280\text{ ft}}{1\text{ mile}}\right)\left(\frac{1\text{ Btu}}{778.169\text{ lbf}\cdot\text{ft}}\right) = 3.33\times10^6\text{ Btu/year}$$

$$E_{in} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{3.33\times10^6\text{ Btu/year}}{0.32} = 1.041\times10^7\text{ Btu/year}$$

Then the amount and costs of the fuel that supplies this much energy are

$$\text{Amont of fuel} = \frac{m_{\text{fuel}}}{\rho_{\text{fuel}}} = \frac{E_{\text{in}}/\text{HV}}{\rho_{\text{fuel}}} = \frac{(1.041\times10^7\text{ Btu/year})/(20{,}000\text{ Btu/lbm})}{50\text{ lbm/ft}^3} = 10.41\text{ ft}^3\text{/year}$$

$$\text{Cost} = (\text{Amount of fuel})(\text{Unit cost}) = (10.41\text{ ft}^3\text{/year})(\$2.20\text{/gal})\left(\frac{7.4804\text{ gal}}{1\text{ ft}^3}\right) = \$171.3\text{/year}$$

That is, the car uses 10.41 ft^3 = 77.9 gallons of gasoline at a cost of \$171.3 per year to overcome the drag.

The drag force and the work done to overcome it are directly proportional to the frontal area. Then the percent reduction in the fuel consumption due to reducing frontal area is equal to the percent reduction in the frontal area:

$$\text{Reduction ratio} = \frac{A - A_{\text{new}}}{A} = \frac{18-15}{18} = 0.167$$

$$\text{Amount reduction} = (\text{Reduction ratio})(\text{Amount})$$
$$= 0.167(77.9\text{ gal/year}) = \mathbf{13.0\text{ gal/year}}$$

$$\text{Cost reduction} = (\text{Reduction ratio})(\text{Cost}) = 0.167(\$171.3\text{/year}) = \mathbf{\$28.6\text{/year}}$$

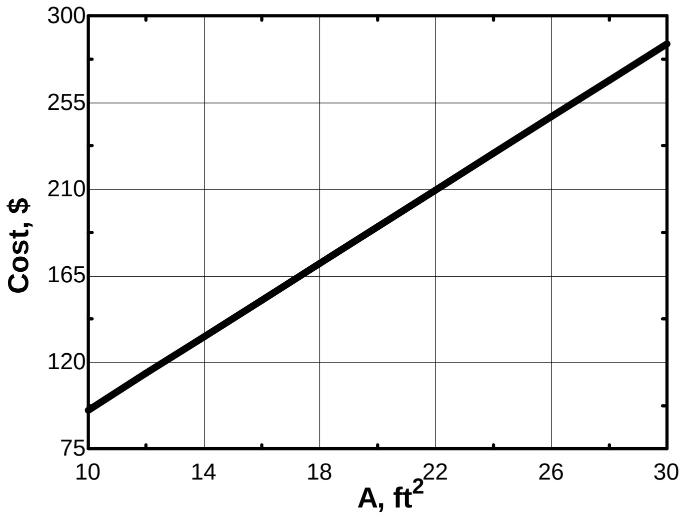
Therefore, reducing the frontal area reduces the fuel consumption due to drag by 16.7%.

Discussion Note from this example that significant reductions in drag and fuel consumption can be achieved by reducing the frontal area of a vehicle.

15-26E Problem 15-25E is reconsidered. The effect of frontal area on the annual fuel consumption of the car as the frontal area varied from 10 to 30 ft^2 in increments of 2 ft^2 are to be investigated.

```
CD=0.3
rho=0.075 "lbm/ft3"
V=55*1.4667 "ft/s"
Eff=0.32
Price=2.20 "$/gal"
efuel=20000 "Btu/lbm"
rho_gas=50 "lbm/ft3"
L=12000*5280 "ft"
FD=CD*A*(rho*V^2)/2/32.2 "lbf"
Wdrag=FD*L/778.169 "Btu"
Ein=Wdrag/Eff
m=Ein/efuel "lbm"
Vol=(m/rho_gas)*7.4804 "gal"
Cost=Vol*Price
```

A, ft^2	F_{drag} lbf	Amount, gal	Cost, $
10	22.74	43.27	95.2
12	27.28	51.93	114.2
14	31.83	60.58	133.3
16	36.38	69.24	152.3
18	40.92	77.89	171.4
20	45.47	86.55	190.4
22	50.02	95.2	209.4
24	54.57	103.9	228.5
26	59.11	112.5	247.5
28	63.66	121.2	266.6
30	68.21	129.8	285.6

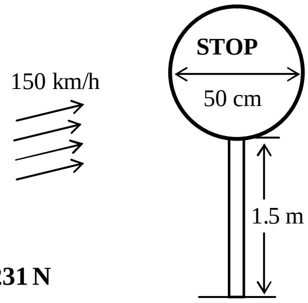


15-27 A circular stop sign is subjected to high winds. The drag force acting on the sign and the bending moment at the bottom of its pole are to be determined. √EES

Assumptions **1** The flow of air is steady and incompressible. **2** The drag force on the pole is negligible. **3** The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties The drag coefficient for a thin circular disk is C_D = 1.1 (Table 15-2). The density of air at 100 kPa and 10°C = 283 K is

$$\rho = \frac{P}{RT} = \frac{100\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg.K})(283\text{ K})} = 1.231\text{ kg/m}^3$$



Analysis The frontal area of a circular plate subjected to normal flow is $A = \pi D^2/4$. Then the drag force acting on the stop sign is

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2}$$

$$= (1.1)[\pi(0.5\text{ m})^2/4]\frac{(1.231\text{ kg/m}^3)(150/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{231\ N}$$

Noting that the resultant force passes through the center of the stop sign, the bending moment at the bottom of the pole becomes

$$M_{bottom} = F_D \times L = (231\text{ N})(1.5+0.25)\text{ m} = \mathbf{404\ Nm}$$

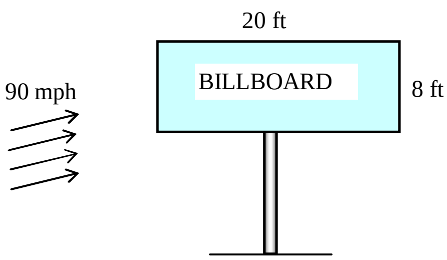
Discussion Note that the drag force is equivalent to the weight of over 23 kg of mass. Therefore, the pole must be strong enough to withstand the weight of 23 kg hanged at one of its end when it is held from the other end horizontally.

15-28E A rectangular billboard is subjected to high winds. The drag force acting on the billboard is to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The drag force on the supporting poles are negligible. **3** The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties The drag coefficient for a thin rectangular plate for normal flow is C_D = 2.0 (Table 15-2). The density of air at 14.3 psia and 40°F = 500 R is

$$\rho = \frac{P}{RT} = \frac{14.3\text{ psia}}{(0.3704\text{ psia}\cdot\text{ft}^3/\text{lbm.R})(500\text{ R})} = 0.0772\text{ lbm/ft}^3$$



Analysis The drag force acting on the billboard is determined from

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2}$$

$$= (2.0)(8\times 20\text{ ft}^2)\frac{(0.0772\text{ lbm/ft}^3)(90\times 1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{6684\ lbf}$$

Discussion Note that the drag force is equivalent to the weight of 6684 lbm of mass. Therefore, the support bars must be strong enough to withstand the weight of 6684 lbm hanged at one of their ends when they are held from the other end horizontally.

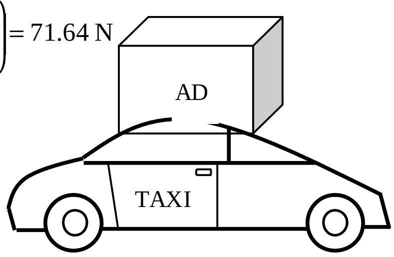
15-29 An advertisement sign in the form of a rectangular block that has the same frontal area from all four sides is mounted on top of a taxicab. The increase in the annual fuel cost due to this sign is to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The car is driven 60,000 km a year at an average speed of 50 km/h. **3** The overall efficiency of the engine is 28%. **4** The effect of the sign and the taxicab on the drag coefficient of each other is negligible (no interference), and the edge effects of the sign are negligible (a crude approximation). **5** The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties The densities of air and gasoline are given to be 1.25 kg/m^3 and 0.75 kg/L, respectively. The heating value of gasoline is given to be 42,000 kJ/kg. The drag coefficient for a square rod for normal flow is C_D = 2.2 (Table 15-1).

Analysis Noting that 1 m/s = 3.6 km/h, the drag force acting on the sign is

$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} = (2.2)(0.9\times 0.3\,\text{m}^2)\frac{(1.25\,\text{kg/m}^3)(50/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 71.64\,\text{N}$$



Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 60,000 km are

$$W_{\text{drag}} = F_D \times L = (71.64\text{ N})(60{,}000\text{ km/year}) = 4.30\times 10^6\ \text{kJ/year}$$

$$E_{in} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{4.30\times 10^6\ \text{kJ/year}}{0.28} = 1.54\times 10^7\ \text{kJ/year}$$

Then the amount and cost of the fuel that supplies this much energy are

$$\text{Amont of fuel} = \frac{m_{\text{fuel}}}{\rho_{\text{fuel}}} = \frac{E_{\text{in}}/\text{HV}}{\rho_{\text{fuel}}} = \frac{(1.54\times 10^7\ \text{kJ/year})/(42{,}000\,\text{kJ/kg})}{0.75\,\text{kg/L}} = \mathbf{488\,L/year}$$

$$\text{Cost} = (\text{Amount of fuel})(\text{Unit cost}) = (488\,\text{L/year})(\$0.50/\text{L}) = \mathbf{\$244/year}$$

That is, the taxicab will use 488 L of gasoline at a cost of $244 per year to overcome the drag generated by the advertisement sign.

Discussion Note that the advertisement sign increases the fuel cost of the taxicab significantly. The taxicab operator may end up losing money by installing the sign if he/she is not aware of the major increase in the fuel cost, and negotiate accordingly.

15-30 The water needs of a recreational vehicle (RV) are to be met by installing a cylindrical tank on top of the vehicle. The additional power requirements of the RV at a specified speed for two orientations of the tank are to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The effect of the tank and the RV on the drag coefficient of each other is negligible (no interference). **3** The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties The drag coefficient for a cylinder corresponding to L/D = 2/0.5 = 4 is C_D = 0.9 when the circular surfaces of the tank face the front and back, and C_D = 0.8 when the circular surfaces of the tank face the sides of the RV (Table 15-2). The density of air at the specified conditions is

$$\rho = \frac{P}{RT} = \frac{87\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg.K})(295\ \text{K})} = 1.028\ \text{kg/m}^3$$

Analysis (a) The drag force acting on the tank when the circular surfaces face the front and back is

$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} = (0.9)[\pi(0.5\ \text{m})^2/4]\frac{(1.028\ \text{kg/m}^3)(95\ \text{km/h})^2}{2}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)^2\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 63.3\ \text{N}$$

Noting that power is force times velocity, the amount of additional power needed to overcome this drag force is

$$\dot{W}_{\text{drag}} = F_D \times \mathbf{V} = (63.3\ \text{N})(95/3.6\ \text{m/s})\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right) = \mathbf{1.67\ kW}$$

(b) The drag force acting on the tank when the circular surfaces face the sides of the RV is

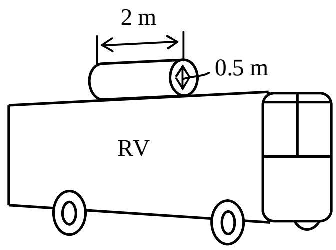
$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} = (0.8)[0.5\times 2\ \text{m}^2]\frac{(1.028\ \text{kg/m}^3)(95\ \text{km/h})^2}{2}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)^2\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 286\ \text{N}$$

Then the additional power needed to overcome this drag force is

$$\dot{W}_{\text{drag}} = F_D \times \mathbf{V} = (286\ \text{N})(95/3.6\ \text{m/s})\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right) = \mathbf{7.55\ kW}$$

Therefore, the additional power needed to overcome the drag caused by the tank is 1.67 kW and 7.55 W for the two orientations indicated.

Discussion Note that the additional power requirement is the lowest when the tank is installed such that its circular surfaces face the front and back of the RV. This is because the frontal area of the tank is minimum in this orientation.

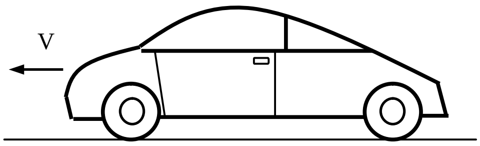


15-31E A person who normally drives at 55 mph now starts driving at 75 mph. The percentage increase in fuel consumption of the car is to be determined.

Assumptions **1** The power generated by the car engine is used entirely to overcome aerodynamic drag, and thus the fuel consumption is proportional to the drag force on a level road. **2** The drag coefficient remains the same.

Analysis The drag force is proportional to the square of the velocity, and power is force times velocity. Therefore,

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} \quad \text{and} \quad \dot{W}_{\text{drag}} = F_D \mathbf{V} = C_D A \frac{\rho \mathbf{V}^3}{2}$$



Then the ratio of the drag force at $\mathbf{V}_2$= 70 mph to the drag force at $\mathbf{V}_1$ = 55 mph becomes

$$\frac{F_{D2}}{F_{D1}} = \frac{\mathbf{V}_2^2}{\mathbf{V}_1^2} = \frac{75^3}{55^3} = \mathbf{1.86}$$

Therefore, the power to overcome the drag force and thus fuel consumption per unit time more than doubles as a result of increasing the velocity from 55 to 75 mph.

Discussion This increase appears to be large. This is because all the engine power is assumed to be used entirely to overcome drag. Still, the simple analysis above shows the strong dependence of the fuel consumption on the cruising speed of a vehicle.

A better measure of fuel consumption is the amount of fuel used per unit distance (rather than per unit time). A car cruising at 55 mph will travel a distance of 55 miles in 1 hour. But a car cruising at 75 mph will travel the same distance at 55/75 = 0.733 h or 73.3% of the time. Therefore, for a given distance, the increase in fuel consumption is 1.86×0.733 = 1.36 – an increase of **36%.**

15-32 A plastic sphere is dropped into water. The terminal velocity of the sphere in water is to be determined.

Assumptions **1** The fluid flow over the sphere is laminar (to be verified). **2** The drag coefficient remains constant.

Properties The density of plastic sphere is 1150 kg/m^3. The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s, respectively (Table A-15). The drag coefficient for a sphere in laminar flow is C_D = 0.5 (Table 15-2).

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the fluid,

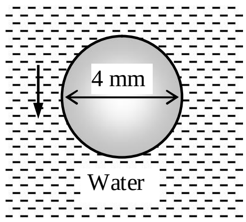
$$F_D = W - F_B \qquad \text{where} \qquad F_D = C_D A\frac{\rho_f \mathbf{V}^2}{2}, \qquad W = \rho_s gV, \quad \text{and } F_B = \rho_f gV$$

Here $A = \pi D^2/4$ is the frontal area and $V = \pi D^3/6$ is the volume of the sphere. Substituting and simplifying,

$$C_D A\frac{\rho_f \mathbf{V}^2}{2} = \rho_s gV - \rho_f gV \;\rightarrow\; C_D \frac{\pi D^2}{4}\frac{\rho_f \mathbf{V}^2}{2} = (\rho_s - \rho_f)g\frac{\pi D^3}{6} \;\rightarrow\; C_D \frac{\mathbf{V}^2}{8} = \left(\frac{\rho_s}{\rho_f} - 1\right)\frac{gD}{6}$$

Solving for **V** and substituting,

$$\mathbf{V} = \sqrt{\frac{4gD(\rho_s/\rho_f - 1)}{3C_D}} = \sqrt{\frac{4(9.81\text{ m/s}^2)(0.004\text{ m})(1150/998-1)}{3\times 0.5}} = 0.126\text{ m/s}$$



The Reynolds number is

$$\text{Re} = \frac{\rho \mathbf{V}D}{\mu} = \frac{(998\text{ kg/m}^3)(0.126\text{ m/s})(4\times10^{-3}\text{ m})}{1.002\times10^{-3}\text{ kg}\cdot\text{m/s}} = 503$$

which is less than 2×10^5. Therefore, the flow is laminar as assumed.

Discussion This problem can be solved more accurately using a trial-and-error approach by using C_D data from Fig. 15-33 (the C_D value corresponding to Re = 503 is about 0.6. Repeating the calculations for this value gives 0.115 m/s for the terminal velocity.

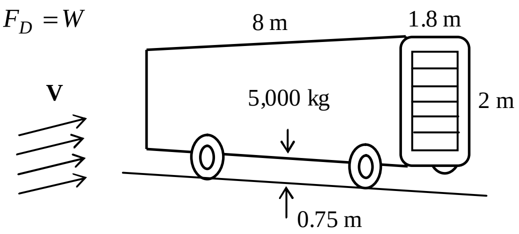
15-33 A semi truck is exposed to winds from its side surface. The wind velocity that will tip the truck over to its side is to be determined.

Assumptions **1** The flow of air in the wind is steady and incompressible. **2** The edge effects on the semi truck are negligible (a crude approximation), and the resultant drag force acts through the center of the side surface. **3** The flow is turbulent so that the tabulated value of the drag coefficient can be used. **4** The distance between the wheels on the same axle is also 2 m. **5** The semi truck is loaded uniformly so that its weight acts through its center.

Properties The density of air is given to be ρ = 1.10 kg/m^3. The drag coefficient for a body of rectangular cross-section corresponding to L/D = 2/2 = 1 is C_D = 2.2 when the wind is normal to the side surface (Table 15-2).

Analysis When the truck is first tipped, the wheels on the wind-loaded side of the truck will be off the ground, and thus all the reaction forces from the ground will act on wheels on the other side. Taking the moment about an axis passing through these wheels and setting it equal to zero gives the required drag force to be

$$\sum M_{\text{wheels}} = 0 \;\rightarrow\; F_D \times (1\,\text{m}) - W \times (1\,\text{m}) = 0 \;\rightarrow\; F_D = W$$



Substituting, the required drag force is determined to be

$$F_D = mg = (5000\,\text{kg})(9.81\,\text{m/s}^2)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 49{,}050\,\text{N}$$

The wind velocity that will cause this drag force is determined to be

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} \;\rightarrow\; 49{,}050\,\text{N} = (2.2)(2\times 8\,\text{m}^2)\frac{(1.10\,\text{kg/m}^3)\mathbf{V}^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) \;\rightarrow\; \mathbf{V} = 50.3\,\text{m/s}$$

which is equivalent to a wind velocity of **V** = 50.3×3.6 = **181 km/h.**

Discussion This is very high velocity, and it can be verified easily by calculating the Reynolds number that the flow is turbulent as assumed.

15-34 A bicyclist is riding his bicycle downhill on a road with a specified slope without pedaling or breaking. The terminal velocity of the bicyclist is to be determined for the upright and racing positions.

Assumptions **1** The rolling resistance and bearing friction are negligible. **2** The drag coefficient remains constant. **3** The buoyancy of air is negligible.

Properties The density of air is given to be ρ = 1.25 kg/m³. The frontal area and the drag coefficient are given to be 0.45 m² and 1.1 in the upright position, and 0.4 m² and 0.9 on the racing position.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the component of the total weight (bicyclist + bicycle) in the flow direction,

$$F_D = W_{\text{total}} \sin\theta \qquad \to \qquad C_D A \frac{\rho \mathbf{V}^2}{2} = m_{\text{total}}\, g \sin\theta$$

Solving for **V** gives $$\mathbf{V} = \sqrt{\frac{2 g m_{\text{total}} \sin\theta}{C_D A \rho}}$$

The terminal velocities for both positions are obtained by substituting the given values:

Upright position: $$\mathbf{V} = \sqrt{\frac{2(9.81\ \text{m/s}^2)(80+15\ \text{kg})\sin 12^\circ}{1.1(0.45\ \text{m}^2)(1.25\ \text{kg/m}^3)}} = 25.0\ \text{m/s} = 90\ \text{km/h}$$

Racing position: $$\mathbf{V} = \sqrt{\frac{2(9.81\ \text{m/s}^2)(80+15\ \text{kg})\sin 12^\circ}{0.9(0.4\ \text{m}^2)(1.25\ \text{kg/m}^3)}} = 29.3\ \text{m/s} = 106\ \text{km/h}$$

Discussion Note that the position of the bicyclist has a significant effect on the drag force, and thus the terminal velocity. So it is no surprise that the bicyclists maintain the racing position during a race.

15-35 The pivot of a wind turbine with two hollow hemispherical cups is stuck as a result of some malfunction. For a given wind speed, the maximum torque applied on the pivot is to be determined. √EES

Assumptions **1** The flow of air in the wind is steady and incompressible. **2** The air flow is turbulent so that the tabulated values of the drag coefficients can be used.

Properties The density of air is given to be ρ = 1.25 kg/m^3. The drag coefficient for a hemispherical cup is 0.4 and 1.2 when the hemispherical and plain surfaces are exposed to wind flow, respectively.

Analysis The maximum torque occurs when the cups are normal to the wind since the length of the moment arm is maximum in this case. Noting that the frontal area is $\pi D^2/4$ for both cups, the drag force acting on each cup is determined to be

Convex side:

$$F_{D1} = C_{D1} A \frac{\rho \mathbf{V}^2}{2} = (0.4)[\pi(0.08\,\text{m})^2/4]\frac{(1.25\,\text{kg/m}^3)(15\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{0.282\,N}$$

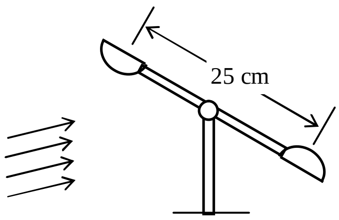
Concave side:

$$F_{D2} = C_{D2} A \frac{\rho \mathbf{V}^2}{2} = (1.2)[\pi(0.08\,\text{m})^2/4]\frac{(1.25\,\text{kg/m}^3)(15\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{0.847\,N}$$

The moment arm for both forces is 12.5 cm since the distance between the centers of the two cups is given to be 25 cm. Taking the moment about the pivot, the net torque applied on the pivot is determined to be

$$M_{\max} = F_{D2}L - F_{D1}L = (F_{D2} - F_{D1})L = (0.847 - 0.282\,\text{N})(0.125\,\text{m}) = 0.0707\,\text{Nm}$$

Discussion Note that the torque varies between zero when both cups are aligned with the wind to the maximum value calculated above.

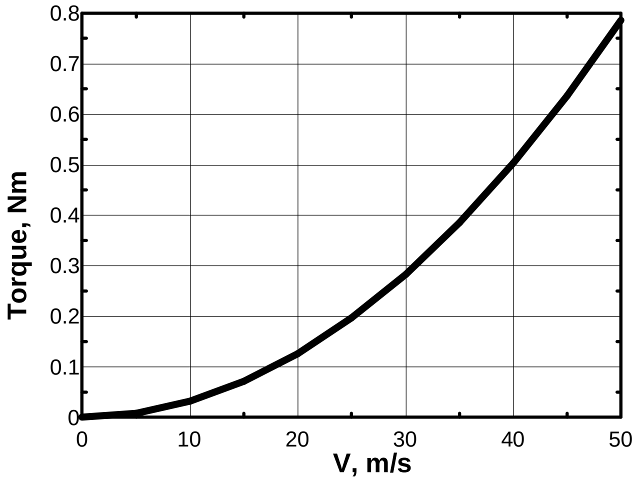


15-36 Prob. 15-35 is reconsidered. The effect of wind speed on the torque applied on the pivot as the wind speed varies from 0 to 50 m/s in increments of 5 m/s is to be investigated.

```
CD1=0.40 "Curved bottom"
CD2=1.2 "Plain frontal area"

"rho=density(Air, T=T, P=P)" "kg/m^3"
rho=1.25 "kg/m3"
D=0.08 "m"
L=0.25 "m"
A=pi*D^2/4 "m^2"
FD1=CD1*A*(rho*V^2)/2 "N"
FD2=CD2*A*(rho*V^2)/2 "N"
FD_net=FD2-FD1
Torque=(FD2-FD1)*L/2
```

V, m/s	$F_{\text{drag, net}}$, N	Torque, Nm
0	0.00	0.000
5	0.06	0.008
10	0.25	0.031
15	0.57	0.071
20	1.01	0.126
25	1.57	0.196
30	2.26	0.283
35	3.08	0.385
40	4.02	0.503
45	5.09	0.636
50	6.28	0.785



15-37E A spherical tank completely submerged in fresh water is being towed by a ship at a specified velocity. The required towing power is to be determined.

Assumptions **1** The flow is turbulent so that the tabulated value of the drag coefficient can be used. **2** The drag of the towing bar is negligible.

Properties The drag coefficient for a sphere is C_D = 0.2 in turbulent flow (it is 0.5 for laminar flow). We take the density of water to be 62.4 lbm/ft^3.

Analysis The frontal area of a sphere is $A = \pi D^2/4$. Then the drag force acting on the spherical tank is

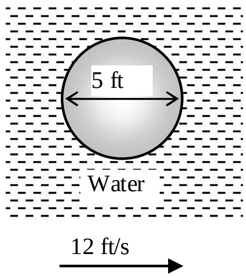
$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} = (0.2)[\pi (5\text{ ft})^2/4]\frac{(62.4\text{ lbm/ft}^3)(12\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 548\text{ lbf}$$

Noting that power is force times velocity, the power needed to overcome this drag force during towing is

$$\dot{W}_{\text{Towing}} = \dot{W}_{\text{drag}} = F_D \times \mathbf{V} = (548\text{ lbf})(12\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 8.92\text{ kW} = 12.0\text{ hp}$$

Therefore, the additional power needed to tow the tank is **12.0 hp**.

Discussion Note that the towing power is proportional the cube of the velocity. Therefore, the towing power can be reduced to one-eight (which is 1.5 hp) by reducing the towing velocity by half to 6 ft/s. But the towing time will double this time for a given distance.



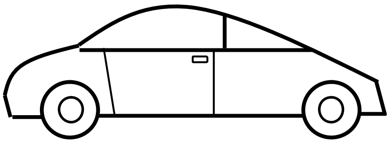
15-38 The power delivered to the wheels of a car is used to overcome aerodynamic drag and rolling resistance. For a given power, the speed at which the rolling resistance is equal to the aerodynamic drag and the maximum speed of the car are to be determined. √EES

Assumptions **1** The air flow is steady and incompressible. **2** The bearing friction is negligible. **3** The drag and rolling resistance coefficients of the car are constant. **4** The car moves horizontally on a level road.

Properties The density of air is given to be ρ = 1.20 kg/m^3. The drag and rolling resistance coefficients are given to be C_D = 0.32 and C_{RR} = 0.04, respectively.

Analysis (*a*) The rolling resistance of the car is

$$F_{RR} = C_{RR}W = 0.04(950\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 372.8\text{ N}$$



The velocity at which the rolling resistance equals the aerodynamic drag force is determined by setting these two forces equal to each other,

$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} \quad\rightarrow\quad 372.8\text{ N} = (0.32)(1.8\text{ m}^2)\frac{(1.20\text{ kg/m}^3)\mathbf{V}^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) \quad\rightarrow\quad \mathbf{V} = 32.8\text{ m/s}$$

(or 118 km/h)

(*b*) Power is force times speed, and thus the power needed to overcome drag and rolling resistance is the product of the sum of the drag force and the rolling resistance and the velocity of the car,

$$\dot{W}_{\text{total}} = \dot{W}_{\text{drag}} + \dot{W}_{\text{RR}} = (F_D + F_{RR})\mathbf{V} = C_D A\frac{\rho \mathbf{V}^3}{2} + F_{RR}\mathbf{V}$$

Substituting the known quantities, the maximum speed corresponding to a wheel power of 80 kW is determined to be

$$(0.32)(1.8\text{ m}^2)\frac{(1.20\text{ kg/m}^3)\mathbf{V}^3}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) + 372.8\mathbf{V} = 80{,}000\text{ W}\left(\frac{1\text{ N}\cdot\text{m/s}}{1\text{ W}}\right)$$

or,

$$0.3456\mathbf{V}^3 + 372.8\mathbf{V} = 80{,}000$$

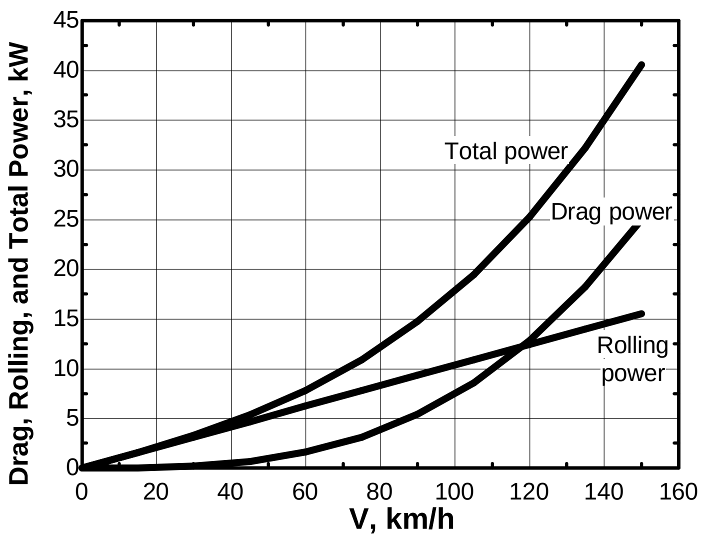
whose solution is **V** = 55.56 m/s **= 200 km/h**.

Discussion Note that a net power input of 80 kW is needed to overcome the rolling resistance and the aerodynamic drag at a velocity of 200 km/h. About 75% of this power is used to overcome drag and the remaining 25% to overcome the rolling resistance. At much higher velocities, the fraction of drag becomes even higher as it is proportional to the cube of car velocity.

15-39 Problem 13-38 is reconsidered. The effect of car speed on the required power to overcome (*a*) rolling resistance, (*b*) the aerodynamic drag, and (*c*) their combined effect as the car speed varies from 0 to 150 km/h in increments of 15 km/h is to be investigated.

```
rho=1.20 "kg/m3"
C_roll=0.04
m=950 "kg"
g=9.81 "m/s2"
V=Vel/3.6 "m/s"
W=m*g
F_roll=C_roll*W
A=1.8 "m2"
C_D=0.32
F_D=C_D*A*(rho*V^2)/2 "N"
Power_RR=F_roll*V/1000 "W"
Power_Drag=F_D*V/1000 "W"
Power_Total=Power_RR+Power_Drag
```

V, m/s	W_{drag} kW	$W_{rolling}$, kW	W_{total}, kW
0	0.00	0.00	0.00
15	0.03	1.55	1.58
30	0.20	3.11	3.31
45	0.68	4.66	5.33
60	1.60	6.21	7.81
75	3.13	7.77	10.89
90	5.40	9.32	14.72
105	8.58	10.87	19.45
120	12.80	12.43	25.23
135	18.23	13.98	32.20
150	25.00	15.53	40.53



15-40 A submarine is treated as an ellipsoid at a specified length and diameter. The powers required for this submarine to cruise horizontally in seawater and to tow it in air are to be determined.

Assumptions **1** The submarine can be treated as an ellipsoid. **2** The flow is turbulent. **3** The drag of the towing rope is negligible. **4** The motion of submarine is steady and horizontal.

Properties The drag coefficient for an ellipsoid with L/D = 25/5 = 5 is C_D = 0.1 in turbulent flow (Table 15-2). The density of sea water is given to be 1025 kg/m^3. The density of air is given to be 1.30 kg/m^3.

Analysis Noting that 1 m/s = 3.6 km/h, the velocity of the submarine is equivalent to $\mathbf{V}$ = 40/3.6 = 11.1 m/s. The frontal area of an ellipsoid is $A = \pi D^2/4$. Then the drag force acting on the submarine becomes

In water: $$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} = (0.1)[\pi(5\text{ m})^2/4]\frac{(1025\text{ kg/m}^3)(11.1\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 124.2\text{ kN}$$

In air: $$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} = (0.1)[\pi(5\text{ m})^2/4]\frac{(1.30\text{ kg/m}^3)(11.1\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 0.158\text{ kN}$$

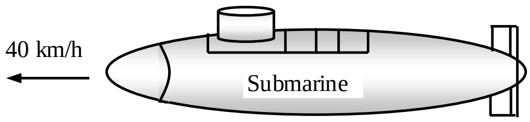
Noting that power is force times velocity, the power needed to overcome this drag force is

In water: $$\dot{W}_{\text{drag}} = F_D \mathbf{V} = (124.2\text{ kN})(11.1\text{ m/s})\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{1380\text{ kW}}$$

In air: $$\dot{W}_{\text{drag}} = F_D \mathbf{V} = (0.158\text{ kN})(11.1\text{ m/s})\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{1.75\text{ kW}}$$

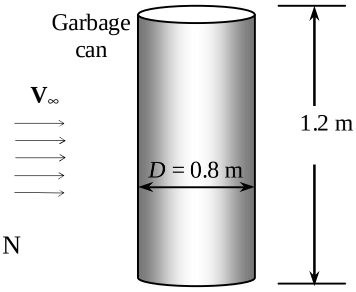
Therefore, the power required for this submarine to cruise horizontally in seawater is 1380 kW and the power required to tow this submarine in air at the same velocity is 1.75 kW.

Discussion Note that the power required to move the submarine in water is about 800 times the power required to move it in air. This is due to the higher density of water compared to air (sea water is about 800 times denser than air). Also, the drag coefficient is C_D = 0.3 for laminar flow. Therefore, the power requirements would be 3 times larger in laminar flow.



15-41 A garbage can is found in the morning tipped over due to high winds the night before. The wind velocity during the night when the can was tipped over is to be determined.

Assumptions **1** The flow of air in the wind is steady and incompressible. **2** The ground effect on the wind and the drag coefficient is negligible (a crude approximation) so that the resultant drag force acts through the center of the side surface. **3** The garbage can is loaded uniformly so that its weight acts through its center.



Properties The density of air is given to be ρ = 1.25 kg/m^3, and the average density of the garbage inside the can is given to be 150 kg/m^3. The drag coefficient of the garbage can is given to be 0.7.

Analysis The volume of the garbage can and the weight of the garbage are

$$V = [\pi D^2/4]H = [\pi(0.80\text{ m})^2/4](1.2\text{ m}) = 0.6032\text{ m}^2$$

$$W = mg = \rho g V = (150\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.6032\text{ m}^3)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 887.6\text{ N}$$

When the garbage can is first tipped, the edge on the wind-loaded side of the can will be off the ground, and thus all the reaction forces from the ground will act on the other side. Taking the moment about an axis passing through the contact point and setting it equal to zero gives the required drag force to be

$$\sum M_{\text{contact}} = 0 \;\rightarrow\; F_D \times (H/2) - W \times (D/2) = 0 \quad\rightarrow\quad F_D = \frac{WD}{H} = \frac{(887.6\text{ N})(0.80\text{ m})}{1.2\text{ m}} = 591.7\text{ N}$$

Noting that the frontal area is DH, the wind velocity that will cause this drag force is determined to be

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} \;\rightarrow\; 591.7\text{ N} = (0.7)[1.2\times 0.80\text{ m}^2]\frac{(1.25\text{ kg/m}^3)\mathbf{V}^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) \;\rightarrow\; \mathbf{V} = 37.5\text{ m/s}$$

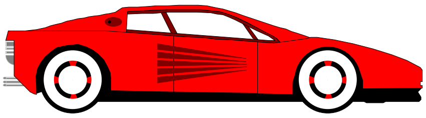
which is equivalent to a wind velocity of **V** = 37.5×3.6 **= 135 km/h.**

Discussion The analysis above shows that under the stated assumptions, the wind velocity at some moment exceeded 135 km/h. But we cannot tell how high the wind velocity has been. Such analysis and predictions are commonly used in forensic engineering.

15-42E The drag coefficient of a sports car increases when the sunroof is open, and it requires more power to overcome aerodynamic drag. The additional power consumption of the car when the sunroof is opened is to be determined at two different velocities.

Assumptions **1** The car moves steadily at a constant velocity on a straight path. **2** The effect of velocity on the drag coefficient is negligible.

Properties The density of air is given to be 0.075 lbm/ft^3. The drag coefficient of the car is given to be C_D = 0.32 when the sunroof is closed, and C_D = 0.41 when it is open.



Analysis (*a*) Noting that 1 mph = 1.4667 ft/s and that power is force times velocity, the drag force acting on the car and the power needed to overcome it at 35 mph are:

Open sunroof: $$F_{D1} = 0.32(18\text{ ft}^2)\frac{(0.075\text{ lbm/ft}^3)(35\times1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 17.7\text{ lbf}$$

$$\dot{W}_{\text{drag1}} = F_{D1}\mathbf{V} = (17.7\text{ lbf})(35\times1.4667\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 1.23\text{ kW}$$

Closed sunroof: $$F_{D2} = 0.41(18\text{ ft}^2)\frac{(0.075\text{ lbm/ft}^3)(35\times1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 22.7\text{ lbf}$$

$$\dot{W}_{\text{drag2}} = F_{D2}\mathbf{V} = (22.7\text{ lbf})(35\times1.4667\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 1.58\text{ kW}$$

Therefore, the additional power required for this car when the sunroof is open is

$$\dot{W}_{\text{extra}} = \dot{W}_{\text{drag2}} - \dot{W}_{\text{drag2}} = 1.58 - 1.23 = \mathbf{0.35\text{ kW}} \quad \text{(at 35 mph)}$$

(*b*) We now repeat the calculations for 70 mph:

Open sunroof: $$F_{D1} = 0.32(18\text{ ft}^2)\frac{(0.075\text{ lbm/ft}^3)(70\times1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 70.7\text{ lbf}$$

$$\dot{W}_{\text{drag1}} = F_{D1}\mathbf{V} = (70.7\text{ lbf})(70\times1.4667\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 9.82\text{ kW}$$

Closed sunroof: $$F_{D2} = 0.41(18\text{ ft}^2)\frac{(0.075\text{ lbm/ft}^3)(70\times1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 90.6\text{ lbf}$$

$$\dot{W}_{\text{drag2}} = F_{D2}\mathbf{V} = (90.6\text{ lbf})(70\times1.4667\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 12.6\text{ kW}$$

Therefore, the additional power required for this car when the sunroof is open is

$$\dot{W}_{\text{extra}} = \dot{W}_{\text{drag2}} - \dot{W}_{\text{drag2}} = 12.6 - 9.82 = \mathbf{2.78\text{ kW}} \quad \text{(at 70 mph)}$$

Discussion Note that the additional drag caused by open sunroof is 0.35 kW at 35 mph, and 2.78 kW at 70 mph, which is an increase of 8 folds when the velocity is doubled. This is expected since the power consumption to overcome drag is proportional to the cube of velocity.

Flow over Flat Plates

15-43C The fluid viscosity is responsible for the development of the velocity boundary layer. For the idealized inviscid fluids (fluids with zero viscosity), there will be no velocity boundary layer.

15-44C The friction coefficient represents the resistance to fluid flow over a flat plate. It is proportional to the drag force acting on the plate. The drag coefficient for a flat surface is equivalent to the mean friction coefficient.

15-45C The friction coefficient will change with position in laminar flow over a flat plate (it will decrease along the plate in the flow direction).

15-46C The average friction coefficient in flow over a flat plate is determined by integrating the local friction coefficient over the entire plate, and then dividing it by the length of the plate. Or, it can be determined experimentally by measuring the drag force, and dividing it by the dynamic pressure.

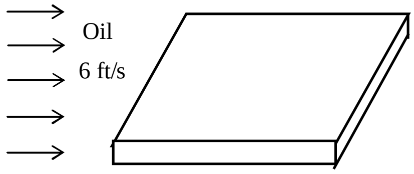
15-47E Light oil flows over a flat plate. The total drag force per unit width of the plate is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5 \times 10^5$. **3** The surface of the plate is smooth.

Properties The density and kinematic viscosity of light oil at 75°F are $\rho = 55.3$ lbm/ft^3 and $\nu = 7.751\times10^{-3}$ ft^2/s (Table A-19E).

Analysis Noting that L = 15 ft, the Reynolds number at the end of the plate is

$$\mathrm{Re}_L = \frac{\mathbf{V}L}{\nu} = \frac{(6\text{ ft/s})(15\text{ ft})}{7.751\times10^{-3}\text{ ft}^2/\text{s}} = 1.161\times10^4$$



which is less than the critical Reynolds number. Thus we have *laminar flow* over the entire plate, and the average friction coefficient is determined from

$$C_f = 1.328\mathrm{Re}_L^{-0.5} = 1.328\times(1.161\times10^4)^{-0.5} = 0.01232$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the top surface of the plate per unit width becomes

$$F_D = C_f A\frac{\rho\mathbf{V}^2}{2} = 0.01232\times(15\times1\text{ ft}^2)\frac{(56.8\text{ lbm/ft}^3)(6\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{5.87\text{ lbf}}$$

The total drag force acting on the entire plate can be determined by multiplying the value obtained above by the width of the plate.

Discussion The force per unit width corresponds to the weight of a mass of 5.87 lbm. Therefore, a person who applies an equal and opposite force to the plate to keep it from moving will feel like he or she is using as much force as is necessary to hold a 5.87 lbm mass from dropping.

15-48 Air flows over a plane surface at high elevation. The drag force acting on the top surface of the plate is to be determined for flow along the two sides of the plate. √

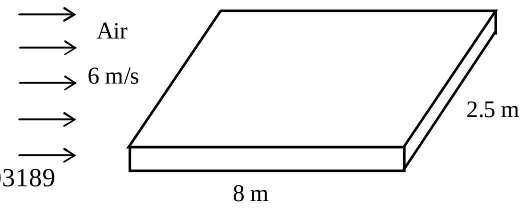
Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

Properties The dynamic viscosity is independent of pressure, and for air at 25°C it is $\mu = 1.849\times10^{-5}$ kg/m⋅s (Table A-22). The air density at 25°C = 298 K and 83.4 kPa is

$$\rho = \frac{P}{RT} = \frac{83.4\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\ \text{K})} = 0.9751\ \text{kg/m}^3$$

Analysis (a) If the air flows parallel to the 8 m side, the Reynolds number becomes

$$\mathrm{Re}_L = \frac{\rho \mathbf{V}_\infty L}{\mu} = \frac{(0.9751\ \text{kg/m}^3)(6\ \text{m/s})(8\ \text{m})}{1.849\times10^{-5}\ \text{kg/m}\cdot\text{s}} = 2.531\times10^6$$



which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} = \frac{0.074}{(2.531\times10^6)^{1/5}} - \frac{1742}{2.531\times10^6} = 0.003189$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the top surface of the plate becomes

$$F_D = C_f A \frac{\rho \mathbf{V}^2}{2} = 0.003189\times(8\times2.5\ \text{m}^2)\frac{(0.9751\ \text{kg/m}^3)(6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{1.12\ N}$$

(b) If the air flows parallel to the 2.5 m side, the Reynolds number is

$$\mathrm{Re}_L = \frac{\rho \mathbf{V}_\infty L}{\mu} = \frac{(0.9751\ \text{kg/m}^3)(6\ \text{m/s})(2.5\ \text{m})}{1.849\times10^{-5}\ \text{kg/m}\cdot\text{s}} = 7.910\times10^5$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} = \frac{0.074}{(7.910\times10^5)^{1/5}} - \frac{1742}{7.910\times10^5} = 0.002691$$

Then the drag force acting on the top surface of the plate becomes

$$F_D = C_f A \frac{\rho \mathbf{V}^2}{2} = 0.002691\times(8\times2.5\ \text{m}^2)\frac{(0.9751\ \text{kg/m}^3)(6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{0.94\ N}$$

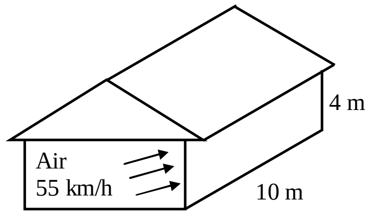
Discussion Note that the drag force is proportional to density, which is proportional to the pressure. Therefore, the altitude has a major influence on the drag force acting on a surface. Commercial airplanes take advantage of this phenomenon and cruise at high altitudes where the air density is much lower to save fuel.

15-49 Wind is blowing parallel to the side wall of a house. The drag force acting on the wall is to be determined for two different wind velocities. √EES

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The wall surface is smooth (the actual wall surface is usually very rough). **5** The wind blows parallel to the wall.

Properties The density and kinematic viscosity of air at 1 atm and 5°C are ρ = 1.269 kg/m^3 and ν = 1.382×10^{-5} m^2/s (Table A-22).

Analysis The Reynolds number is



$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(55/3.6\ \mathrm{m/s})(10\ \mathrm{m})}{1.382\times10^{-5}\ \mathrm{m^2/s}} = 1.105\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} = \frac{0.074}{(1.105\times10^7)^{1/5}} - \frac{1742}{1.105\times10^7} = 0.002730$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the wall surface is

$$F_D = C_f A\frac{\rho \mathbf{V}^2}{2} = 0.00273\times(10\times4\ \mathrm{m^2})\frac{(1.269\ \mathrm{kg/m^3})(55/3.6\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg\cdot m/s^2}}\right) = \mathbf{16.2\ N}$$

(b) When the wind velocity is doubled to 110 km/h, the Reynolds number becomes

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(110/3.6\ \mathrm{m/s})(10\ \mathrm{m})}{1.382\times10^{-5}\ \mathrm{m^2/s}} = 2.211\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient and the drag force become

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} = \frac{0.074}{(2.211\times10^7)^{1/5}} - \frac{1742}{2.211\times10^7} = 0.002435$$

$$F_D = C_f A\frac{\rho \mathbf{V}^2}{2} = 0.002435\times(10\times4\ \mathrm{m^2})\frac{(1.269\ \mathrm{kg/m^3})(110/3.6\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg\cdot m/s^2}}\right) = \mathbf{57.7\ N}$$

Discussion Note that the actual drag will probably be much higher since the wall surfaces are typically very rough. Also, we can solve this problem using the turbulent flow relation (instead of the combined laminar-turbulent flow relation) without much loss in accuracy. Finally, the drag force nearly quadruples when the velocity is doubled. This is expected since the drag force is proportional to the square of the velocity, and the effect of velocity on the friction coefficient is small.

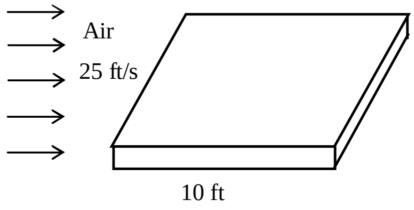
15-50E Air flows over a flat plate. The local friction coefficients at intervals of 1 ft is to be determined and plotted against the distance from the leading edge.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

Properties The density and kinematic viscosity of air at 1 atm and 70°F are ρ = 0.07489 lbm/ft^3 and ν = 0.5913 ft^2/h = 1.643×10^{-4} ft^2/s (Table A-22E).

Analysis For the first 1 ft interval, the Reynolds number is

$$\mathrm{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(25\text{ ft/s})(1\text{ ft})}{1.643\times10^{-4}\text{ ft}^2/\text{s}} = 1.522\times10^5$$



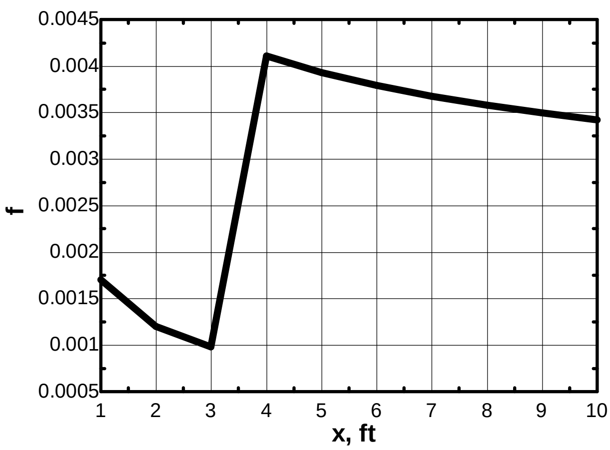
which is less than the critical value of 5×10^5. Therefore, the flow is laminar. The local friction coefficient is

$$C_{f,x} = \frac{0.664}{\mathrm{Re}^{0.5}} = \frac{0.664}{(1.522\times10^5)^{0.5}} = 0.001702$$

We repeat calculations for all 1-ft intervals. The results are

```
rho=0.07489 "lbm/ft3"
nu=0.5913/3600 "ft2/s"
V=25
"Local Re and C_f"
Re=x*V/nu
"f=0.664/Re^0.5"
f=0.059/Re^0.2
```

x, in	Re	C_f
1	1.522E+05	0.001702
2	3.044E+05	0.001203
3	4.566E+05	0.000983
4	6.088E+05	0.004111
5	7.610E+05	0.003932
6	9.132E+05	0.003791
7	1.065E+06	0.003676
8	1.218E+06	0.003579
9	1.370E+06	0.003496
10	1.522E+06	0.003423



Discussion Note that the Reynolds number exceeds the critical value for x > 3 ft, and thus the flow is turbulent over most of the plate. For x > 3 ft, we used $C_f = 0.074/\mathrm{Re}_L^{1/5}$-1742/Re$_L$ for friction coefficient. Note that C_f decreases with Re in both laminar and turbulent flows.

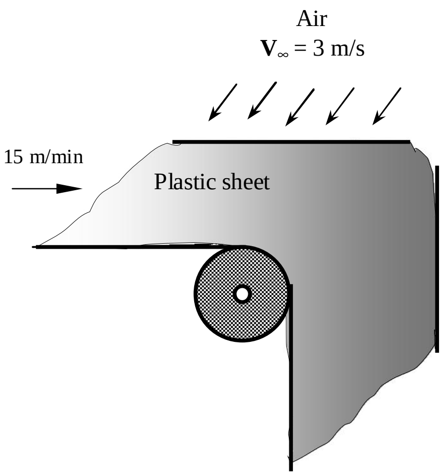
15-51 Air flows on both sides of a continuous sheet of plastic. The drag force air exerts on the plastic sheet in the direction of flow is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** Both surfaces of the plastic sheet are smooth. **5** The plastic sheet does not vibrate and thus it does not induce turbulence in air flow.

Properties The density and kinematic viscosity of air at 1 atm and 60°C are ρ = 1.059 kg/m^3 and ν = 1.896×10^{-5} m^2/s (Table A-22).

Analysis The length of the cooling section is

$$L = \mathbf{V}_{\text{sheet}}\Delta t = [(15/60)\text{ m/s}](2\text{ s}) = 0.5\text{ m}$$



The Reynolds number is

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(3\text{ m/s})(1.2\text{ m})}{1.896\times10^{-5}\text{ m}^2/\text{s}} = 1.899\times10^5$$

which is less than the critical Reynolds number. Thus the flow is laminar. The area on both sides of the sheet exposed to air flow is

$$A = 2wL = 2(1.2\text{ m})(0.5\text{ m}) = 1.2\text{ m}^2$$

Then the friction coefficient and the drag force become

$$C_f = \frac{1.328}{\mathrm{Re}_L^{0.5}} = \frac{1.328}{(1.899\times10^5)^{0.5}} = 0.003048$$

$$F_D = C_f A\frac{\rho\mathbf{V}_\infty^2}{2} = (0.003048)(1.2\text{ m}^2)\frac{(1.059\text{ kg/m}^3)(3\text{ m/s})^2}{2} = \mathbf{0.0174\text{ N}}$$

Discussion Note that the Reynolds number remains under the critical value, and thus the flow remains laminar over the entire plate. In reality, the flow may be turbulent because of the motion of the plastic sheet.

15-52 A train is cruising at a specified velocity. The drag force acting on the top surface of a passenger car of the train is to be determined.

Assumptions **1** The air flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The top surface of the train is smooth (in reality it can be rough). **5** The air is calm (no significant winds).

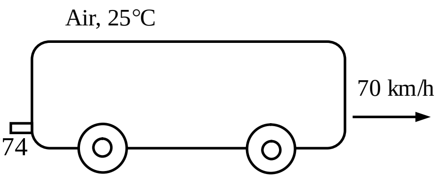
Properties The density and kinematic viscosity of air at 1 atm and 25°C are $\rho = 1.184$ kg/m^3 and $\nu = 1.562\times10^{-5}$ m^2/s (Table A-22).

Analysis The Reynolds number is

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(70/3.6\ \mathrm{m/s})(8\ \mathrm{m})}{1.562\times10^{-5}\ \mathrm{m^2/s}} = 9.959\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} = \frac{0.074}{(9.959\times10^6)^{1/5}} - \frac{1742}{9.959\times10^6} = 0.002774$$



Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the surface becomes

$$F_D = C_f A\frac{\rho\mathbf{V}^2}{2} = 0.002774\times(8\times3.2\ \mathrm{m^2})\frac{(1.184\ \mathrm{kg/m^3})(70/3.6\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg\cdot m/s^2}}\right) = \mathbf{15.9\ N}$$

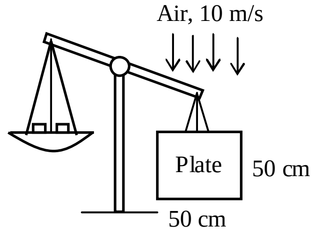
Discussion Note that we can solve this problem using the turbulent flow relation (instead of the combined laminar-turbulent flow relation) without much loss in accuracy since the Reynolds number is much greater than the critical value. Also, the actual drag force will probably be greater because of the surface roughness effects.

15-53 The weight of a thin flat plate exposed to air flow on both sides is balanced by a counterweight. The mass of the counterweight that needs to be added in order to balance the plate is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surfaces of the plate are smooth.

Properties The density and kinematic viscosity of air at 1 atm and 25°C are ρ = 1.184 kg/m^3 and ν = 1.562×10^{-5} m^2/s (Table A-22).

Analysis The Reynolds number is



$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(10\ \mathrm{m/s})(0.5\ \mathrm{m})}{1.562\times10^{-5}\ \mathrm{m^2/s}} = 3.201\times10^5$$

which is less than the critical Reynolds number of 5×10^5. Therefore the flow is laminar. The average friction coefficient, drag force and the corresponding mass are

$$C_f = \frac{1.328}{\mathrm{Re}_L^{0.5}} = \frac{1.328}{(3.201\times10^5)^{0.5}} = 0.002347$$

$$F_D = C_f A \frac{\rho \mathbf{V}_\infty^2}{2} = (0.002347)[(2\times0.5\times0.5)\ \mathrm{m^2}]\frac{(1.184\ \mathrm{kg/m^3})(10\ \mathrm{m/s})^2}{2} = 0.0695\ \mathrm{kg\cdot m/s^2} = 0.0695\ \mathrm{N}$$

The mass whose weight is 0.0695 N is

$$m = \frac{F_D}{g} = \frac{0.0695\ \mathrm{kg.m/s^2}}{9.81\ \mathrm{m/s^2}} = \mathbf{0.0071\ kg = 7.1\ g}$$

Therefore, the mass of the counterweight must be 7.1 g to counteract the drag force acting on the plate.

Discussion Note that the apparatus described in this problem provides a convenient mechanism to measure drag force and thus drag coefficient.

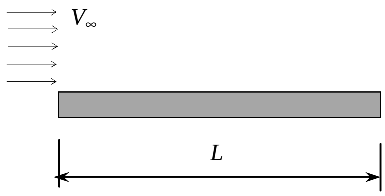
15-54 Laminar flow of a fluid over a flat plate is considered. The change in the drag force is to be determined when the free-stream velocity of the fluid is doubled. √

Analysis For the laminar flow of a fluid over a flat plate the drag force is given by

$$F_{D1} = C_f A \frac{\rho \mathbf{V}_\infty^{\;2}}{2} \quad \text{where} \quad C_f = \frac{1.328}{\mathrm{Re}^{0.5}}$$

Therefore

$$F_{D1} = \frac{1.328}{\mathrm{Re}^{0.5}} A \frac{\rho \mathbf{V}_\infty^{\;2}}{2}$$



Substituting Reynolds number relation, we get

$$F_{D1} = \frac{1.328}{\left(\dfrac{\mathbf{V}_\infty L}{\upsilon}\right)^{0.5}} A \frac{\rho \mathbf{V}_\infty^{\;2}}{2} = 0.664 \mathbf{V}_\infty^{\;3/2} A \frac{\upsilon^{0.5}}{L^{0.5}}$$

When the free-stream velocity of the fluid is doubled, the new value of the drag force on the plate becomes

$$F_{D2} = \frac{1.328}{\left(\dfrac{(2\mathbf{V}_\infty)L}{\upsilon}\right)^{0.5}} A \frac{\rho (2\mathbf{V}_\infty)^2}{2} = 0.664 (2\mathbf{V}_\infty)^{3/2} A \frac{\upsilon^{0.5}}{L^{0.5}}$$

The ratio of drag forces corresponding to $\mathbf{V}_\infty$ and $2\mathbf{V}_\infty$ is

$$\frac{F_{D2}}{F_{D1}} = \frac{(2V_\infty)^{3/2}}{V_\infty^{\;3/2}} = \mathbf{2^{3/2}} = \mathbf{2.83}$$

Discussion Note that the drag force increases almost three times in laminar flow when the fluid velocity is doubled.

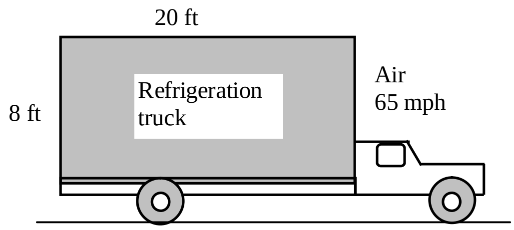
15-55E A refrigeration truck is traveling at a specified velocity. The drag force acting on the top and side surfaces of the truck and the power needed to overcome it are to be determined. √

Assumptions **1** The process is steady and incompressible. **2** The airflow over the entire outer surface is turbulent because of constant agitation. **3** Air is an ideal gas. **4** The top and side surfaces of the truck are smooth (in reality they can be rough). **5** The air is calm (no significant winds).

Properties The density and kinematic viscosity of air at 1 atm and 80°F are $\rho = 0.07350$ lbm/ft^3 and $\nu = 0.6110$ ft^2/s = 1.697×10^{-4} ft^2/s (Table A-22E).

Analysis The Reynolds number is

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{[65\times1.4667\ \mathrm{ft/s}](20\ \mathrm{ft})}{1.697\times10^{-4}\ \mathrm{ft^2/s}} = 1.124\times10^7$$



The air flow over the entire outer surface is assumed to be turbulent. Then the friction coefficient becomes

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(1.124\times10^7)^{1/5}} = 0.002878$$

The area of the top and side surfaces of the truck is

$$A = A_{\mathrm{top}} + 2A_{\mathrm{side}} = 9\times20+2\times8\times20 = 500\ \mathrm{ft}^2$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a plane surface, the drag force acting on these surfaces becomes

$$F_D = C_f A \frac{\rho \mathbf{V}^2}{2} = 0.002878\times(500\ \mathrm{ft}^2)\frac{(0.07350\ \mathrm{lbm/ft^3})(65\times1.4667\ \mathrm{ft/s})^2}{2}\left(\frac{1\ \mathrm{lbf}}{32.2\ \mathrm{lbm\cdot ft/s^2}}\right) = \mathbf{14.9\ lbf}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

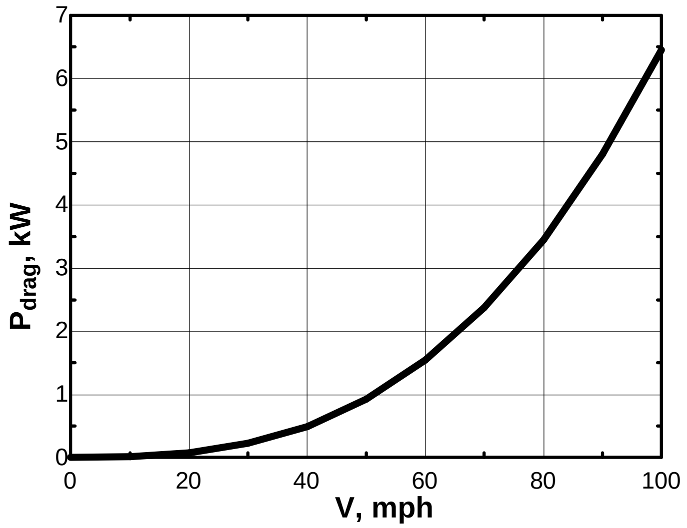
$$\dot{W}_{\mathrm{drag}} = F_D \mathbf{V} = (14.9\ \mathrm{lbf})(65\times1.4667\ \mathrm{ft/s})\left(\frac{1\ \mathrm{kW}}{737.56\ \mathrm{lbf\cdot ft/s}}\right) = \mathbf{1.93\ kW}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

15-56E Prob. 15-55E is reconsidered. The effect of truck speed on the total drag force acting on the top and side surfaces, and the power required to overcome as the truck speed varies from 0 to 100 mph in increments of 10 mph is to be investigated.

```
rho=0.07350 "lbm/ft3"
nu=0.6110/3600 "ft2/s"
V=Vel*1.4667 "ft/s"
L=20 "ft"
W=2*8+9
A=L*W
Re=L*V/nu
Cf=0.074/Re^0.2
g=32.2 "ft/s2"
F=Cf*A*(rho*V^2)/2/32.2 "lbf"
Pdrag=F*V/737.56 "kW"
```

V, mph	Re	F_{drag}, lbf	P_{drag}, kW
0	0	0.00	0.000
10	1.728E+06	0.51	0.010
20	3.457E+06	1.79	0.071
30	5.185E+06	3.71	0.221
40	6.913E+06	6.23	0.496
50	8.642E+06	9.31	0.926
60	1.037E+07	12.93	1.542
70	1.209E+07	17.06	2.375
80	1.382E+07	21.69	3.451
90	1.555E+07	26.82	4.799
100	1.728E+07	32.42	6.446



15-57 Air is flowing over a long flat plate with a specified velocity. The distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

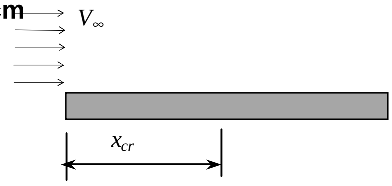
Properties The density and kinematic viscosity of air at 1 atm and 25°C are $\rho = 1.184$ kg/m^3 and $\nu = 1.562\times10^{-5}$ m^2/s (Table A-22).

Analysis The critical Reynolds number is given to be $\mathrm{Re}_{cr} = 5\times10^5$. The distance from the leading edge of the plate where the flow becomes turbulent is the distance x_{cr} where the Reynolds number becomes equal to the critical Reynolds number,

$$\mathrm{Re}_{cr} = \frac{\mathbf{V}_\infty x_{cr}}{\upsilon} \quad\rightarrow\quad x_{cr} = \frac{\nu\,\mathrm{Re}_{cr}}{\mathbf{V}_\infty} = \frac{(1.562\times10^{-5}\ \mathrm{m^2/s})(5\times10^5)}{8\ \mathrm{m/s}} = \mathbf{0.976\ m}$$

The thickness of the boundary layer at that location is obtained by substituting this value of x into the laminar boundary layer thickness relation,

$$\delta_{v,x} = \frac{4.91x}{\mathrm{Re}_x^{1/2}} \quad\rightarrow\quad \delta_{v,cr} = \frac{4.91x_{cr}}{\mathrm{Re}_{cr}^{1/2}} = \frac{4.91(0.976\ \mathrm{m})}{(5\times10^5)^{1/2}} = 0.00678\ \mathrm{m} = \mathbf{0.678\ cm}$$



Discussion When the flow becomes turbulent, the boundary layer thickness starts to increase, and the value of its thickness can be determined from the boundary layer thickness relation for turbulent flow.

15-58 Water is flowing over a long flat plate with a specified velocity. The distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** The surface of the plate is smooth.

Properties The density and dynamic viscosity of water at 1 atm and 25°C are ρ = 997 kg/m^3 and μ = 0.891×10^{-3} kg/m⋅s (Table A-15).

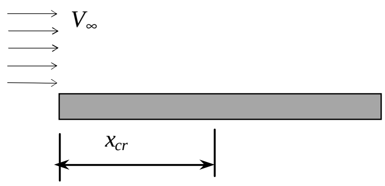
Analysis The critical Reynolds number is given to be $\mathrm{Re}_{cr} = 5\times10^5$. The distance from the leading edge of the plate where the flow becomes turbulent is the distance x_{cr} where the Reynolds number becomes equal to the critical Reynolds number,

$$\mathrm{Re}_{cr} = \frac{\rho \mathbf{V}_\infty x_{cr}}{\mu} \quad \rightarrow \quad x_{cr} = \frac{\mu \,\mathrm{Re}_{cr}}{\rho \mathbf{V}_\infty} = \frac{(0.891\times10^{-3}\ \mathrm{kg/m\cdot s})(5\times10^5)}{(997\,\mathrm{kg/m^3})(8\,\mathrm{m/s})} = \mathbf{0.056\ m}$$

The thickness of the boundary layer at that location is obtained by substituting this value of x into the laminar boundary layer thickness relation,

$$\delta_{v,x} = \frac{5x}{\mathrm{Re}_x^{1/2}} \quad \rightarrow \quad \delta_{v,cr} = \frac{4.91 x_{cr}}{\mathrm{Re}_{cr}^{1/2}} = \frac{4.91(0.056\,\mathrm{m})}{(5\times10^5)^{1/2}} = 0.00039\,\mathrm{m} = \mathbf{0.39\ mm}$$

Therefore, the flow becomes turbulent after about 5.6 cm from the leading edge of the plate, and the thickness of the boundary layer at that location is 0.39 mm.



Discussion When the flow becomes turbulent, the boundary layer thickness starts to increase, and the value of its thickness can be determined from the boundary layer thickness relation for turbulent flow.

Flow across Cylinders and Spheres

15-59C Turbulence moves the fluid separation point further back on the rear of the body, reducing the size of the wake, and thus the magnitude of the pressure drag (which is the dominant mode of drag). As a result, the drag coefficient suddenly drops. In general, turbulence increases the drag coefficient for flat surfaces, but the drag coefficient usually remains constant at high Reynolds numbers when the flow is turbulent.

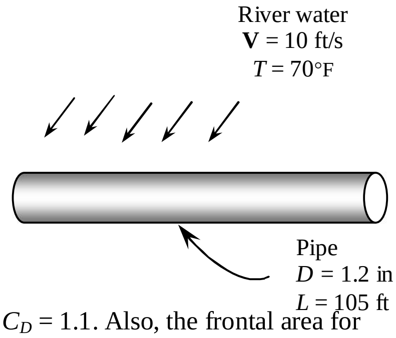
15-60C Friction drag is due to the shear stress at the surface whereas the pressure drag is due to the pressure differential between the front and back sides of the body when a wake is formed in the rear.

15-61C Flow separation in flow over a cylinder is delayed in turbulent flow because of the extra mixing due to random fluctuations and the transverse motion.

15-62E A pipe is crossing a river while remaining completely immersed in water. The drag force exerted on the pipe by the river is to be determined. √

Assumptions **1** The outer surface of the pipe is smooth so that Fig. 15-33 can be used to determine the drag coefficient. **2** Water flow in the river is steady. **3** The turbulence in water flow in the river is not considered. **4** The direction of water flow is normal to the pipe.

Properties The density and dynamic viscosity of water at 70°F are ρ = 62.30 lbm/ft^3 and μ = 2.36 lbm/ft·h = 6.556×10^{-4} lbm/ft·s (Table A-15E).



Analysis Noting that D = 1.2 in = 0.1 ft, the Reynolds number for flow over the pipe is

$$\text{Re} = \frac{\mathbf{V}D}{\nu} = \frac{\rho \mathbf{V}D}{\mu} = \frac{(62.30\ \text{lbm/ft}^3)(10\ \text{ft/s})(0.1\ \text{ft})}{6.556\times10^{-4}\ \text{lbm/ft}\cdot\text{s}} = 9.50\times10^4$$

The drag coefficient corresponding to this value is, from Fig. 15-33, C_D = 1.1. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force acting on the cylinder becomes

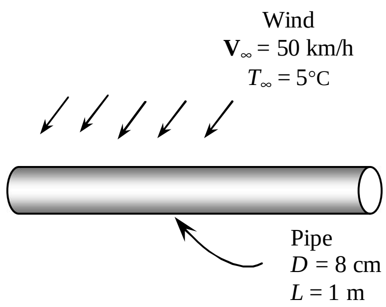
$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} = 1.1\times(105\times0.1\ \text{ft}^2)\frac{(62.30\ \text{lbm/ft}^3)(10\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{1320\ lbf}$$

Discussion Note that this force is equivalent to the weight of a 1320 lbm mass. Therefore, the drag force the river exerts on the pipe is equivalent to hanging a mass of 1320 lbm on the pipe supported at its ends 70 ft apart. The necessary precautions should be taken if the pipe cannot support this force. Also, the fluctuations in water flow may reduce the drag coefficients by inducing turbulence and delaying flow separation.

15-63 A pipe is exposed to high winds. The drag force exerted on the pipe by the winds is to be determined.

Assumptions **1** The outer surface of the pipe is smooth so that Fig. 15-33 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible. **3** The turbulence in the wind is not considered. **4**The direction of wind is normal to the pipe.

Properties The density and kinematic viscosity of air at 1 atm and 5°C are ρ = 1.269 kg/m^3 and ν = 1.382×10^{-5} m^2/s (Table A-22).



Analysis Noting that D = 0.08 m and 1 m/s = 3.6 km/h, the Reynolds number for flow over the pipe is

$$\mathrm{Re} = \frac{\mathbf{V}D}{\nu} = \frac{(50/3.6\ \mathrm{m/s})(0.08\ \mathrm{m})}{1.382\times10^{-5}\ \mathrm{m^2/s}} = 0.8040\times10^5$$

The drag coefficient corresponding to this value is, from Fig. 15-33, C_D = 1.0. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force becomes

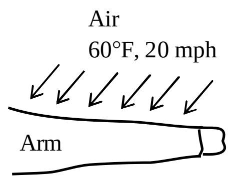
$$F_D = C_D A\frac{\rho\mathbf{V}^2}{2} = 1.0(1\times0.08\ \mathrm{m}^2)\frac{(1.269\ \mathrm{kg/m^3})(50/3.6\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg\cdot m/s^2}}\right) = \mathbf{9.79\ N}\ \text{(per m length)}$$

Discussion Note that the drag force acting on a unit length of the pipe is equivalent to the weight of 1 kg mass. The total drag force acting on the entire pipe can be obtained by multiplying the value obtained by the pipe length. It should be kept in mind that wind turbulence may reduce the drag coefficients by inducing turbulence and delaying flow separation.

15-64E A person extends his uncovered arms into the windy air outside. The drag force exerted on both arms by the wind is to be determined. √

Assumptions **1** The surfaces of the arms are smooth so that Fig. 15-33 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible. **3** The turbulence in the wind is not considered. **4**The direction of wind is normal to the arms. **5** The arms can be treated as 2-ft-long and 3-in.-diameter cylinders with negligible end effects.

Properties The density and kinematic viscosity of air at 1 atm and 60°F are ρ = 0.07633 lbm/ft^3 and ν = 0.5718 ft^2/h = 1.588×10^{-4} ft^2/s (Table A-22E).



Analysis Noting that D = 3 in = 0.25 ft and 1 mph = 1.4667 ft/s, the Reynolds number for flow over the arm is

$$\mathrm{Re} = \frac{\mathbf{V}D}{\nu} = \frac{(20\times1.4667\ \mathrm{ft/s})(0.25\ \mathrm{ft})}{1.588\times10^{-4}\ \mathrm{ft^2/s}} = 4.618\times10^4$$

The drag coefficient corresponding to this value is, from Fig. 15-33, C_D = 1.0. Also, the frontal area for flow past a cylinder is $A = LD$. Then the total drag force acting on *both* arms becomes

$$F_D = C_D A\frac{\rho\mathbf{V}^2}{2} = 1.0\times(2\times2\times0.25\ \mathrm{ft}^2)\frac{(0.07633\ \mathrm{lbm/ft^3})(20\times1.4667\ \mathrm{ft/s})^2}{2}\left(\frac{1\ \mathrm{lbf}}{32.2\ \mathrm{lbm\cdot ft/s^2}}\right) = \mathbf{1.02\ lbf}$$

Discussion Note that this force is equivalent to the weight of 1 lbm mass. Therefore, the drag force the wind exerts on the arms of this person is equivalent to hanging 0.5 lbm of mass on each arm. Also, it should be kept in mind that the wind turbulence and the surface roughness may affect the calculated result significantly.

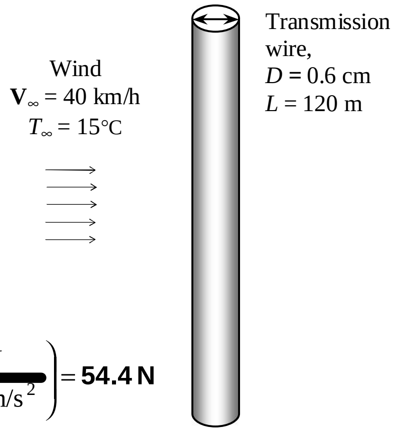
15-65 Wind is blowing across the wire of a transmission line. The drag force exerted on the wire by the wind is to be determined. √

Assumptions **1** The wire surfaces are smooth so that Fig. 15-33 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible. **3** The turbulence in the wind is not considered. **4**The direction of wind is normal to the wire.

Properties The density and kinematic viscosity of air at 1 atm and 15°C are ρ = 1.225 kg/m^3 and ν = 1.470×10^{-5} m^2/s (Table A-22).

Analysis Noting that D = 0.006 m and 1 m/s = 3.6 km/h, the Reynolds number for the flow is

$$\mathrm{Re} = \frac{\mathbf{V}D}{\nu} = \frac{(40/3.6\ \mathrm{m/s})(0.006\ \mathrm{m})}{1.470\times10^{-5}\ \mathrm{m^2/s}} = 4.535\times10^3$$



The drag coefficient corresponding to this value is, from Fig. 15-33, C_D = 1.0. Also, the frontal area for flow past a cylinder is A = LD. Then the drag force becomes

$$F_D = C_D A\frac{\rho\mathbf{V}^2}{2} = 1.0(120\times0.006\ \mathrm{m^2})\frac{(1.225\ \mathrm{kg/m^3})(40/3.6\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg\cdot m/s^2}}\right) = \mathbf{54.4\ N}$$

Therefore, the drag force acting on the wire is 54.4 N, which is equivalent to the weight of about 5.4 kg mass hanging on the wire.

Discussion It should be kept in mind that wind turbulence may reduce the drag coefficients by inducing turbulence and delaying flow separation.

15-66 A spherical hail is falling freely in the atmosphere. The terminal velocity of the hail in air is to be determined. √

Assumptions **1** The surface of the hail is smooth so that Fig. 15-33 can be used to determine the drag coefficient. **2** The variation of the air properties with altitude is negligible. **3** The buoyancy force applied by air to hail is negligible since $\rho_{air} << \rho_{hail}$ (besides, the uncertainty in the density of hail is greater than the density of air). **4** Air flow over the hail is steady and incompressible when terminal velocity is established. **5** The atmosphere is calm (no winds or drafts).

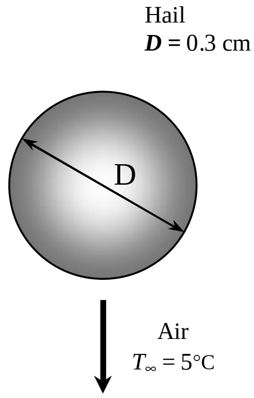
Properties The density and kinematic viscosity of air at 1 atm and 5°C are ρ = 1.269 kg/m^3 and ν = 1.382×10^{-5} m^2/s (Table A-22). The density of hail is given to be 910 kg/m^3.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_D = W - F_B \qquad \text{where} \qquad F_D = C_D A \frac{\rho_f \mathbf{V}^2}{2}, \qquad W = mg = \rho_s g V = \rho_s g(\pi D^3/6), \quad \text{and} \;\; F_B \cong 0$$

and $A = \pi D^2/4$ is the frontal area. Substituting and simplifying,

$$C_D A \frac{\rho_f \mathbf{V}^2}{2} = W \;\;\rightarrow\;\; C_D \frac{\pi D^2}{4} \frac{\rho_f \mathbf{V}^2}{2} = \rho_s g \frac{\pi D^3}{6} \;\;\rightarrow\;\; C_D \rho_f \mathbf{V}^2 = \rho_s g \frac{4D}{3}$$



Solving for **V** and substituting,

$$\mathbf{V} = \sqrt{\frac{4 g \rho_s D}{3 C_D \rho_f}} = \sqrt{\frac{4(9.81\,\text{m/s}^2)(910\,\text{kg/m}^3)(0.008\,\text{m})}{3 C_D (1.269\,\text{kg/m}^3)}} \;\;\rightarrow\;\; \mathbf{V} = \frac{8.662}{\sqrt{C_D}} \qquad (1)$$

The drag coefficient C_D is to be determined from Fig. 15-33, but it requires the Reynolds number which cannot be calculated since we do not know velocity. Therefore, the solution requires a trial-error approach. First we express the Reynolds number as

$$\text{Re} = \frac{\mathbf{V}D}{\nu} = \frac{\mathbf{V}(0.008\,\text{m})}{1.382\times10^{-5}\,\text{m}^2/\text{s}} \;\;\rightarrow\;\; \text{Re} = 578.9\,\mathbf{V} \qquad (2)$$

Now we choose a velocity in m/s, calculate the Re from Eq. (2), read the corresponding C_D from Fig. 15-33, and calculate **V** from Eq. (1). Repeat calculations until the assumed velocity matches the calculated velocity. With this approach the terminal velocity is determined to be

V = 13.7 m/s

The corresponding Re and C_D values are Re = 7930 and C_D = 0.40. Therefore, the velocity of hail will remain constant when it reaches the terminal velocity of 13.7 m/s = 49 km/h.

Discussion The simple analysis above gives us a reasonable value for the terminal velocity. A more accurate answer can be obtained by a more detailed (and complex) analysis by considering the variation of air properties with altitude, and by considering the uncertainty in the drag coefficient (a hail is not necessarily spherical and smooth).

15-67 A spherical dust particle is suspended in the air at a fixed point as a result of an updraft air motion. The magnitude of the updraft velocity is to be determined using Stokes law.

Assumptions **1** The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). **2** The updraft is steady and incompressible. **3** The buoyancy force applied by air to the dust particle is negligible since $\rho_{air} << \rho_{dust}$ (besides, the uncertainty in the density of dust is greater than the density of air). (We will solve the problem without utilizing this assumption for generality).

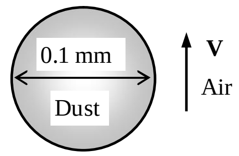
Properties The density of dust is given to be ρ_s = 2.1 g/cm^3 = 2100 kg/m^3. The density and dynamic viscosity of air at 1 atm and 25°C are ρ_f = 1.184 kg/m^3 and μ = 1.849×10^{-5} kg/m·s (Table A-22).

Analysis The terminal velocity of a free falling object is reached (or the suspension of an object in a flow stream is established) when the drag force equals the weight of the solid object less the buoyancy force applied by the surrounding fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu \mathbf{V} D \text{ (Stoke's law)}, \quad W = \rho_s gV, \quad \text{and } F_B = \rho_f gV$$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting,

$$3\pi\mu\mathbf{V}D = \rho_s gV - \rho_f gV \quad \rightarrow \quad 3\pi\mu\mathbf{V}D = (\rho_s - \rho_f)g\frac{\pi D^3}{6}$$



Solving for the velocity **V** and substituting the numerical values, the updraft velocity is determined to be

$$\mathbf{V} = \frac{gD^2(\rho_s - \rho_f)}{18\mu} = \frac{(9.81\,\text{m/s}^2)(0.0001\text{m})^2(2100\text{-}1.184)\,\text{kg/m}^3}{18(1.849\times10^{-5}\ \text{kg/m}\cdot\text{s})} = \mathbf{0.618\ m/s}$$

The Reynolds number in this case is

$$\text{Re} = \frac{\rho\mathbf{V}D}{\mu} = \frac{(1.184\,\text{kg/m}^3)(0.618\,\text{m/s})(0.0001\,\text{m})}{1.849\times10^{-5}\ \text{kg}\cdot\text{m/s}} = 4.0$$

which is in the order of 1. Therefore, the creeping flow idealization and thus Stokes law is applicable, and the value calculated is valid.

Discussion Flow separation starts at about Re = 10. Therefore, Stokes law can be used for Reynolds numbers upto this value, but this should be done with care.

15-68 Dust particles that are unsettled during high winds rise to a specified height, and start falling back when things calm down. The time it takes for the dust particles to fall back to the ground and their velocity are to be determined using Stokes law.

Assumptions **1** The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). **2** The atmosphere is calm during fall back (no winds or drafts). **3** The initial transient period during which the dust particle accelerates to its terminal velocity is negligible. **4** The buoyancy force applied by air to the dust particle is negligible since $\rho_{air} << \rho_{dust}$ (besides, the uncertainty in the density of dust is greater than the density of air). (We will solve this problem without utilizing this assumption for generality).

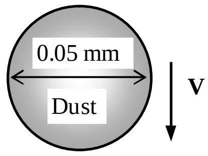
Properties The density of dust is given to be ρ_s = 1.8 g/cm^3 = 1800 kg/m^3. The density and dynamic viscosity of air at 1 atm and 15°C are ρ_f = 1.225 kg/m^3 and μ = 1.802×10^{-5} kg/m⋅s (Table A-22).

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the surrounding fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu \mathbf{V} D \ \text{(Stoke's law)}, \quad W = \rho_s gV, \quad \text{and } F_B = \rho_f gV$$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting,

$$3\pi\mu \mathbf{V} D = \rho_s gV - \rho_f gV \quad \rightarrow \quad 3\pi\mu \mathbf{V} D = (\rho_s - \rho_f) g \frac{\pi D^3}{6}$$



Solving for the velocity **V** and substituting the numerical values, the terminal velocity is determined to be

$$\mathbf{V} = \frac{gD^2(\rho_s - \rho_f)}{18\mu} = \frac{(9.81\,\text{m/s}^2)(5\times10^{-5}\ \text{m})^2(1800\text{-}1.225)\,\text{kg/m}^3}{18(1.802\times10^{-5}\ \text{kg/m}\cdot\text{s})} = \mathbf{0.136\ m/s}$$

Then the time it takes for the dust particle to travel 350 m at this velocity becomes

$$\Delta t = \frac{L}{\mathbf{V}} = \frac{350\,\text{m}}{0.136\,\text{m/s}} = 2573\,\text{s} = \mathbf{42.9\ min}$$

The Reynolds number is

$$\text{Re} = \frac{\rho \mathbf{V} D}{\mu} = \frac{(1.225\,\text{kg/m}^3)(0.136\,\text{m/s})(5\times10^{-5}\ \text{m})}{1.802\times10^{-5}\ \text{kg}\cdot\text{m/s}} = 0.45$$

which is in the order of 1. Therefore, the creeping flow idealization and thus Stokes law is applicable.

Discussion Note that the dust particle reaches a terminal velocity of 0.136 m/s, and it takes about an hour to fall back to the ground. The presence of drafts in air may significantly increase the settling time.

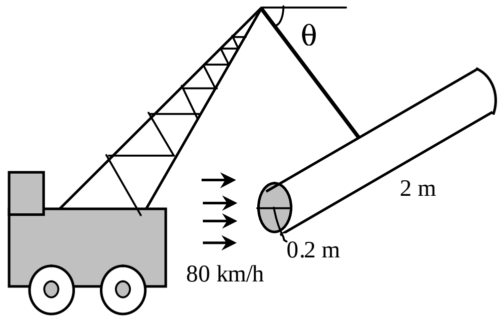
15-69 A cylindrical log suspended by a crane is subjected to normal winds. The angular displacement of the log and the tension on the cable are to be determined.

Assumptions **1** The surfaces of the log are smooth so that Fig. 15-33 can be used to determine the drag coefficient (not a realistic assumption). **2** Air flow in the wind is steady and incompressible. **3** The turbulence in the wind is not considered. **4**The direction of wind is normal to the log, which always remains horizontal. **5** The end effects of the log are negligible. **6** The weight of the cable and the drag acting on it are negligible. **7** Air is an ideal gas.

Properties The dynamic viscosity of air at 5°C (independent of pressure) is $\mu = 1.754\times10^{-5}$ kg/m·s (Table A-22). Then the density and kinematic viscosity of air are calculated to be

$$\rho = \frac{P}{RT} = \frac{88\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(278\text{ K})} = 1.103\text{ kg/m}^3$$

$$\nu = \frac{\mu}{\rho} = \frac{1.754\times10^{-5}\text{ kg/m}\cdot\text{s}}{1.103\text{ kg/m}^3} = 1.590\times10^{-5}\text{ m}^2/\text{s}$$



Analysis Noting that D = 0.2 m and 1 m/s = 3.6 km/h, the Reynolds number is

$$\text{Re} = \frac{\mathbf{V}D}{\nu} = \frac{(40/3.6\text{ m/s})(0.2\text{ m})}{1.590\times10^{-5}\text{ m}^2/\text{s}} = 1.398\times10^5$$

The drag coefficient corresponding to this value is, from Fig. 15-33, C_D = 1.2. Also, the frontal area for flow past a cylinder is $A = LD$. Then the total drag force acting on the log becomes

$$F_D = C_D A\frac{\rho\mathbf{V}^2}{2} = 1.2(2\times0.2\text{ m}^2)\frac{(1.103\text{ kg/m}^3)(40/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 32.8\text{ N}$$

The weight of the log is

$$W = mg = \rho g V = \rho g\frac{\pi D^2 L}{4} = (513\text{ kg/m}^3)(9.81\text{ m/s}^2)\frac{\pi(0.2\text{ m})^2(2\text{ m})}{4}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 316\text{ N}$$

Then the resultant force acting on the log and the angle it makes with the horizontal become

$$F_{\text{log}} = R = \sqrt{W^2 + F_D^2} = \sqrt{32.8^2 + 316^2} = \mathbf{318\text{ N}}$$

$$\tan\theta = \frac{W}{F_D} = \frac{316}{32.8} = 9.63 \quad\rightarrow\quad \theta = \mathbf{84}°$$

Drawing a free body diagram of the log and doing a force balance will show that the magnitude of the tension on the cable must be equal to the resultant force acting on the log. Therefore, the tension on the cable is 318 N and the cable makes 84° with the horizontal.

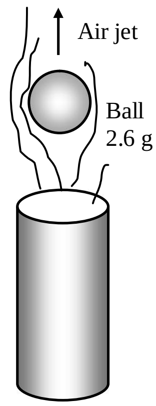
Discussion Note that the wind in this case has rotated the cable by 6° from its vertical position, and increased the tension action on it somewhat. At very high wind speeds, the increase in the cable tension can be very significant, and wind loading must always be considered in bodies exposed to high winds.

15-70 A ping-pong ball is suspended in air by an upward air jet. The velocity of the air jet is to be determined, and the phenomenon that the ball returns to the center of the air jet after a disturbance is to be explained.

Assumptions **1** The surface of the ping-pong ball is smooth so that Fig. 15-33 can be used to determine the drag coefficient. **2** Air flow over the ball is steady and incompressible.

Properties The density and kinematic viscosity of air at 1 atm and 25°C are ρ = 1.184 kg/m^3 and ν = 1.562×10^{-5} m^2/s (Table A-22).

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the fluid,



$$F_D = W - F_B \quad \text{where} \quad F_D = C_D A \frac{\rho_f \mathbf{V}^2}{2}, \quad W = mg, \quad \text{and} \quad F_B = \rho_f g V$$

Here $A = \pi D^2/4$ is the frontal area and $V = \pi D^3/6$ is the volume of the sphere. Also,

$$W = mg = (0.0026\,\text{kg})(9.81\,\text{m/s}^2) = 0.0255\,\text{kg}\cdot\text{m/s}^2 = 0.0255\,\text{N}$$

$$F_B = \rho_f g \frac{\pi D^3}{6} = (1.184\,\text{kg/m}^3)(9.81\,\text{m/s}^2)\frac{\pi(0.038\,\text{m})^3}{6} = 0.000334\,\text{kg}\cdot\text{m/s}^2 = 0.000334\,\text{N}$$

Substituting and solving for **V**,

$$C_D \frac{\pi D^2}{4}\frac{\rho_f \mathbf{V}^2}{2} = W - F_B \;\rightarrow\; \mathbf{V} = \sqrt{\frac{8(W-F_B)}{\pi D^2 C_D \rho_f}} = \sqrt{\frac{8(0.0255-0.000334)\,\text{kg}\cdot\text{m/s}^2)}{\pi(0.038\,\text{m})^2 C_D (1.184\,\text{kg/m}^3)}} \;\rightarrow\; \mathbf{V} = \frac{6.122}{\sqrt{C_D}} \quad (1)$$

The drag coefficient C_D is to be determined from Fig. 15-33, but it requires the Reynolds number which cannot be calculated since we do not know velocity. Therefore, the solution requires a trial-error approach. First we express the Reynolds number as

$$\text{Re} = \frac{\mathbf{V}D}{\nu} = \frac{\mathbf{V}(0.038\,\text{m})}{1.562\times10^{-5}\,\text{m}^2/\text{s}} \quad \rightarrow \quad \text{Re} = 2433\mathbf{V} \qquad (2)$$

Now we choose a velocity in m/s, calculate the Re from Eq. (2), read the corresponding C_D from Fig. 15-33, and calculate **V** from Eq. (1). Repeat calculations until the assumed velocity matches the calculated velocity. With this approach the velocity of the fluid jet is determined to be

V = 9.3 m/s

The corresponding Re and C_D values are Re = 22,600 and C_D = 0.43. Therefore, the ping-pong ball will remain suspended in the air jet when the air velocity reaches 9.3 m/s = 33.5 km/h.

Discussion **1** If the ball is pushed to the side by a finger, the ball will come back to the center of the jet (instead of falling off) due to the Bernoulli effect. In the core of the jet the velocity is higher, and thus the pressure is lower relative to a location away from the jet.

2 Note that this simple apparatus can be used to determine the drag coefficients of certain object by simply measuring the air velocity, which is easy to do.

3 This problem can also be solved roughly by taking C_D = 0.5 from Table 15-3 for a sphere in laminar flow, and then verifying that the flow is laminar.

Lift

15-71C The contribution of viscous effects to lift is usually negligible for airfoils since the wall shear is parallel to the surfaces of such devices and thus nearly normal to the direction of lift.

15-72C When air flows past a symmetrical airfoil at zero angle of attack, (*a*) the lift will be zero, but (*b*) the drag acting on the airfoil will be nonzero.

15-73C When air flows past a nonsymmetrical airfoil at zero angle of attack, both the (*a*) lift and (*b*) drag acting on the airfoil will be nonzero.

15-74C When air flows past a symmetrical airfoil at an angle of attack of 5°, both the (*a*) lift and (*b*) drag acting on the airfoil will be nonzero.

15-75C The decrease of lift with an increase in the angle of attack is called *stall*. When the flow separates over nearly the entire upper half of the airfoil, the lift is reduced dramatically (the separation point is near the leading edge). Stall is caused by flow separation and the formation of a wide wake region over the top surface of the airfoil. The commercial aircraft are not allowed to fly at velocities near the stall velocity for safety reasons. Airfoils stall at high angles of attack (flow cannot negotiate the curve around the leading edge). If a plane stalls, it loses mush of its lift, and it can crash.

15-76C Both the lift and the drag of an airfoil increase with an increase in the angle of attack, but in general lift increases at a much higher rate than does the drag.

15-77C Flaps are used at the leading and trailing edges of the wings of large aircraft during takeoff and landing to alter the shape of the wings to maximize lift and to enable the aircraft to land or takeoff at low speeds. An aircraft can takeoff or land without flaps, but it can do so at very high velocities, which is undesirable during takeoff and landing.

15-78C Flaps increase both the lift and the drag of the wings. But the increase in drag during takeoff and landing is not much of a concern because of the relatively short time periods involved. This is the penalty we pay willingly to takeoff and land at safe speeds.

15-79C The effect of wing tip vortices is to increase drag (induced drag) and to decrease lift. This effect is also due to the downwash, which causes an effectively smaller angle of attack.

15-80C Induced drag is the additional drag caused by the tip vortices. The tip vortices have a lot of kinetic energy, all of which is wasted and is ultimately dissipated as heat in the air downstream. The induced drag can be reduced by using long and narrow wings.

15-81C When air is flowing past a spherical ball, the lift exerted on the ball will be zero if the ball is not spinning, and it will be nonzero if the ball is spinning about an axis normal to the free stream velocity (no lift will be generated if the ball is spinning about an axis parallel to the free stream velocity).

15-82 A tennis ball is hit with a backspin. It is to be determined if the ball will fall or rise after being hit.

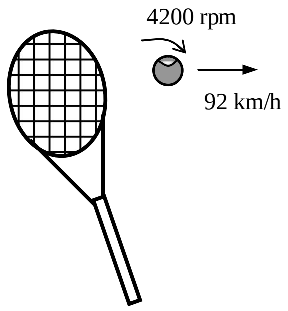
Assumptions **1** The outer surface of the ball is smooth enough for Fig. 15-52 to be applicable. **2** The ball is hit horizontally so that it starts its motion horizontally.

Properties The density and kinematic viscosity of air at 1 atm and 25°C are ρ = 1.184 kg/m^3 and ν = 1.562$\times$10^{-5} m^2/s (Table A-22).

Analysis The ball is hit horizontally, and thus it would normally fall under the effect of gravity without the spin. The backspin will generate a lift, and the ball will rise if the lift is greater than the weight of the ball. The lift can be determined from

$$F_L = C_L A \frac{\rho \mathbf{V}^2}{2}$$

where A is the frontal area of the ball, which is $A = \pi D^2/4$. The regular and angular velocities of the ball are



$$\mathbf{V} = (92\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 25.56\text{ m/s}$$

$$\omega = (4200\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 440\text{ rad/s}$$

Then,

$$\frac{\omega D}{2\mathbf{V}} = \frac{(440\text{ rad/s})(0.064\text{ m})}{2(25.56\text{ m/s})} = 0.551\text{ rad}$$

From Fig. 15-52, the lift coefficient corresponding to this value is C_L = 0.11. Then the lift acting on the ball is

$$F_L = (0.11)\frac{\pi(0.064\text{ m})^2}{4}\frac{(1.184\text{ kg/m}^3)(25.56\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 0.27\text{ N}$$

The weight of the ball is

$$W = mg = (0.057\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 0.56\text{ N}$$

which is more than the lift. Therefore, the ball will **drop** under the combined effect of gravity and lift due to spinning after hitting, with a net force of 0.56 - 0.27 = 0.29 N.

Discussion The Reynolds number for this problem is

$$\text{Re}_L = \frac{\mathbf{V}D}{\upsilon} = \frac{(25.56\text{m/s})(0.064\text{ m})}{1.562\times10^{-5}\text{ m}^2/\text{s}} = 1.05\times10^5$$

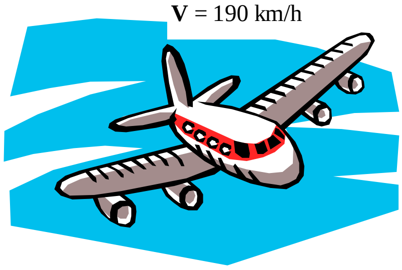
which is close enough to 6$\times$10^4 for which Fig. 15-52 is prepared. Therefore, the result should be close enough to the actual answer.

15-83 The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the weight of the aircraft is increased by 20% as a result of overloading is to be determined. √

Assumptions **1** The atmospheric conditions (and thus the properties of air) remain the same. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \frac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$



We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and area remain constant, the ratio of the velocities of the overloaded and fully loaded aircraft becomes

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\sqrt{2W_2 / \rho C_L A}}{\sqrt{2W_1 / \rho C_L A}} = \frac{\sqrt{W_2}}{\sqrt{W_1}} \quad \rightarrow \quad \mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{W_2}{W_1}}$$

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

$$\mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{1.2W_1}{W_1}} = (190\text{ km/h})\sqrt{1.2} = \mathbf{208\ km/h}$$

Discussion A similar analysis can be performed for the effect of the variations in density, lift coefficient, and planform area on the takeoff velocity.

15-84 The takeoff speed and takeoff time of an aircraft at sea level are given. The required takeoff speed, takeoff time, and the additional runway length required at a higher elevation are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane and the planform area remain constant. **3** The acceleration of the aircraft during takeoff remains constant.

Properties The density of standard air is ρ_1 = 1.225 kg/m^3 at sea level, and ρ_2 = 1.048 kg/m^3 at 1600 m altitude (Table A-24).

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \frac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$

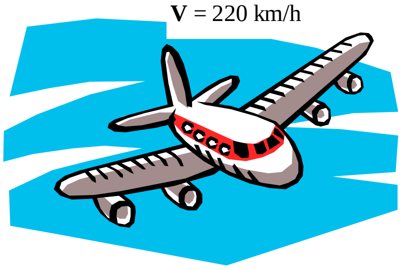
We note that the takeoff speed is inversely proportional to the square root of air density. When the weight, lift coefficient, and area remain constant, the ratio of the speeds of the aircraft at high altitude and at sea level becomes

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\sqrt{2W/\rho_2 C_L A}}{\sqrt{2W/\rho_1 C_L A}} = \sqrt{\frac{\rho_1}{\rho_2}} \quad \rightarrow \quad \mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{\rho_1}{\rho_2}} = (220\text{ km/h})\sqrt{\frac{1.225}{1.048}} = \mathbf{238\ km/h}$$

Therefore, the takeoff velocity of the aircraft at higher altitude is 238 km/h.

(*b*) The acceleration of the aircraft at sea level is

$$a = \frac{\Delta \mathbf{V}}{\Delta t} = \frac{220\text{ km/h - 0}}{15\text{ s}}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 4.074\text{ m/s}^2$$



which is assumed to be constant both at sea level and the higher altitude. Then the takeoff time at the higher altitude becomes

$$a = \frac{\Delta \mathbf{V}}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{\Delta \mathbf{V}}{a} = \frac{238\text{ km/h - 0}}{4.074\text{ m/s}^2}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = \mathbf{16.2\ s}$$

(*c*) The additional runway length is determined by calculating the distance traveled during takeoff for both cases, and taking their difference:

$$L_1 = \frac{1}{2} a t_1^2 = \frac{1}{2}(4.074\text{ m/s}^2)(15\text{ s})^2 = 458\text{ m}$$

$$L_2 = \frac{1}{2} a t_2^2 = \frac{1}{2}(4.074\text{ m/s}^2)(16.2\text{ s})^2 = 535\text{ m}$$

$$\Delta L = L_2 - L_1 = 535 - 458 = \mathbf{97\ m}$$

Discussion Note that altitude has a significant effect on the length of the runways, and it should be a major consideration on the design of airports. It is interesting that a 1.2 second increase in takeoff time increases the required runway length by about 100 m.

15-85E The rate of fuel consumption of an aircraft while flying at a low altitude is given. The rate of fuel consumption at a higher altitude is to be determined for the same flight velocity.

Assumptions **1** Standard atmospheric conditions exist. **2** The settings of the plane during takeoff are maintained the same so that the drag coefficient of the plane and the planform area remain constant. **3** The velocity of the aircraft and the propulsive efficiency remain constant. **4** The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties The density of standard air is ρ_1 = 0.05648 lbm/ft^3 at 10,000 ft, and ρ_2 = 0.02866 lbm/ft^3 at 30,000 ft altitude (Table A-24).

Analysis When an aircraft cruises steadily (zero acceleration) at a constant altitude, the net force acting on the aircraft is zero, and thus the thrust provided by the engines must be equal to the drag force. Also, power is force times velocity (distance per unit time), and thus the propulsive power required to overcome drag is equal to the thrust times the cruising velocity. Therefore,

$$\dot{W}_{\text{propulsive}} = \text{Thrust} \times \mathbf{V} = F_D \mathbf{V} = C_D A \frac{\rho \mathbf{V}^2}{2} \mathbf{V} = C_D A \frac{\rho \mathbf{V}^3}{2}$$

The propulsive power is also equal to the product of the rate of fuel energy supplied (which is the rate of fuel consumption times the heating value of the fuel, $\dot{m}_{\text{fuel}}\text{HV}$) and the propulsive efficiency. Then,

$$\dot{W}_{\text{prop}} = \eta_{\text{prop}} \dot{m}_{\text{fuel}} \text{HV} \quad \rightarrow \quad C_D A \frac{\rho \mathbf{V}^3}{2} = \eta_{\text{prop}} \dot{m}_{\text{fuel}} \text{HV}$$

We note that the rate of fuel consumption is proportional to the density of air. When the drag coefficient, the wing area, the velocity, and the propulsive efficiency remain constant, the ratio of the rates of fuel consumptions of the aircraft at high and low altitudes becomes

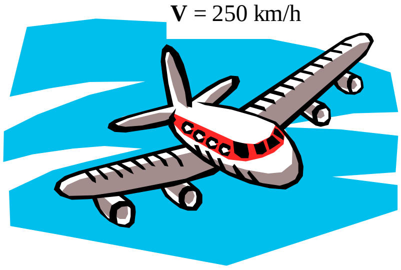
$$\frac{\dot{m}_{\text{fuel, 2}}}{\dot{m}_{\text{fuel, 1}}} = \frac{C_D A \rho_2 \mathbf{V}^3 / 2\eta_{\text{prop}} \ \text{HV}}{C_D A \rho_1 \mathbf{V}^3 / 2\eta_{\text{prop}} \ \text{HV}} = \frac{\rho_2}{\rho_1} \quad \rightarrow \quad \dot{m}_{\text{fuel, 2}} = \dot{m}_{\text{fuel, 1}} \frac{\rho_2}{\rho_1} = (5 \text{ gal/min}) \frac{0.02866}{0.05648} = \mathbf{2.54 \ gal/min}$$

Discussion Note the fuel consumption drops by half when the aircraft flies at 30,000 ft instead of 10,000 ft altitude. Therefore, large passenger planes routinely fly at high altitudes (usually between 30,000 and 40,000 ft) to save fuel. This is especially the case for long flights.

15-86 The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the aircraft has 100 empty seats is to be determined. √EES

Assumptions **1** The atmospheric conditions (and thus the properties of air) remain the same. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same. **3** A passenger with luggage has an average mass of 140 kg.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,



$$W = F_L \quad \rightarrow \quad W = \frac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$

We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and wing area remain constant, the ratio of the velocities of the under-loaded and fully loaded aircraft becomes

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\sqrt{2W_2 / \rho C_L A}}{\sqrt{2W_1 / \rho C_L A}} = \sqrt{\frac{W_2}{W_1}} = \sqrt{\frac{m_2 g}{m_1 g}} = \sqrt{\frac{m_2}{m_1}} \quad \rightarrow \quad \mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{m_2}{m_1}}$$

where

$$m_2 = m_1 - m_{\text{unused capacity}} = 400{,}000 \text{ kg} - (140 \text{ kg/passanger}) \times (100 \text{ passengers}) = 386{,}000 \text{ kg}$$

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

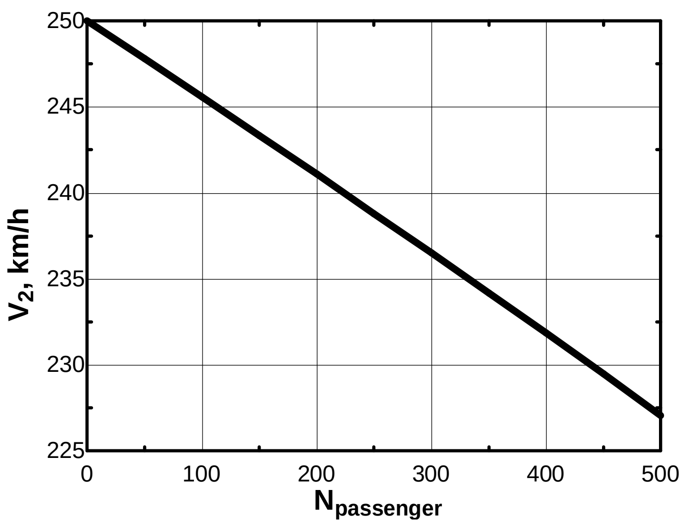
$$\mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{m_2}{m_1}} = (250 \text{ km/h}) \sqrt{\frac{386{,}000}{400{,}000}} = \mathbf{246\ km/h}$$

Discussion Note that the effect of empty seats on the takeoff velocity of the aircraft is small. This is because the most weight of the aircraft is due to its empty weight (the aircraft itself rather than the passengers themselves and their luggage.)

15-87 Problem 15-86 is reconsidered. The effect of passenger count on the takeoff speed of the aircraft as the number of passengers varies from 0 to 500 in increments of 50 is to be investigated.

```
m_passenger=140 "kg"
m1=400000 "kg"
m2=m1-N_passenger*m_passenger
V1=250 "km/h"
V2=V1*SQRT(m2/m1)
```

Passenger count	$m_{\text{airplane},1}$, kg	$m_{\text{airplane},1}$, kg	V_{takeoff}, m/s
0	400000	400000	250.0
50	400000	393000	247.8
100	400000	386000	245.6
150	400000	379000	243.3
200	400000	372000	241.1
250	400000	365000	238.8
300	400000	358000	236.5
350	400000	351000	234.2
400	400000	344000	231.8
450	400000	337000	229.5
500	400000	330000	227.1

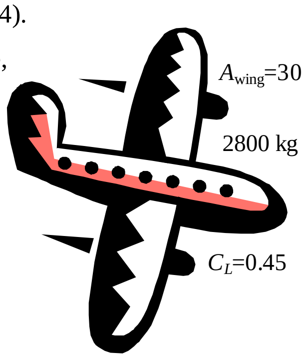


15-88 The wing area, lift coefficient at takeoff settings, the cruising drag coefficient, and total mass of a small aircraft are given. The takeoff speed, the wing loading, and the required power to maintain a constant cruising speed are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered.

Properties The density of standard air at sea level is $\rho = 1.225$ kg/m^3 (Table A-24).

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,



$$W = F_L \quad \rightarrow \quad W = \frac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$

Substituting, the takeoff speed is determined to be

$$\mathbf{V}_{\text{takeoff}} = \sqrt{\frac{2mg}{\rho C_{L,\text{takeoff}} A}} = \sqrt{\frac{2(2800 \text{ kg})(9.81 \text{ m/s}^2)}{(1.225 \text{ kg/m}^3)(0.45)(30 \text{ m}^2)}}$$
$$= 57.6 \text{ m/s} = \mathbf{207\ km/h}$$

(*b*) Wing loading is the average lift per unit planform area, which is equivalent to the ratio of the lift to the planform area of the wings since the lift generated during steady cruising is equal to the weight of the aircraft. Therefore,

$$F_{\text{loading}} = \frac{F_L}{A} = \frac{W}{A} = \frac{(2800 \text{ kg})(9.81 \text{ m/s}^2)}{30 \text{ m}^2} = \mathbf{916\ N/m^2}$$

(*c*) When the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force, which is

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} = (0.0035)(30 \text{ m}^2) \frac{(1.225 \text{ kg/m}^3)(300/3.6 \text{ m/s})^2}{2} \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 4.466 \text{ kN}$$

Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Thrust} \times \text{Velocity} = F_D \mathbf{V} = (4.466 \text{ kN})(300/3.6 \text{ m/s}) \left(\frac{1 \text{ kW}}{1 \text{ kN} \cdot \text{m/s}} \right) = \mathbf{372\ kW}$$

Therefore, the engines must supply 372 kW of propulsive power to overcome the drag during cruising.

Discussion The power determined above is the power to overcome the drag that acts on the wings only, and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc). Therefore, the total power required during cruising will be greater. The required rate of energy input can be determined by dividing the propulsive power by the propulsive efficiency.

15-89 The total mass, wing area, cruising speed, and propulsive power of a small aircraft are given. The lift and drag coefficients of this airplane while cruising are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered. **3** The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties The density of standard air at an altitude of 4000 m is $\rho = 0.819$ kg/m^3 (Table A-24).

Analysis Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity. Also, when the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. Then,

$$\dot{W}_{\text{prop}} = \text{Thrust} \times \text{Velocity} = F_D \mathbf{V} \quad \rightarrow \quad F_D = \frac{\dot{W}_{\text{prop}}}{\mathbf{V}} = \frac{190\text{ kW}}{280/3.6\text{ m/s}}\left(\frac{1000\text{ N}\cdot\text{m/s}}{1\text{ kW}}\right) = 2443\text{ N}$$

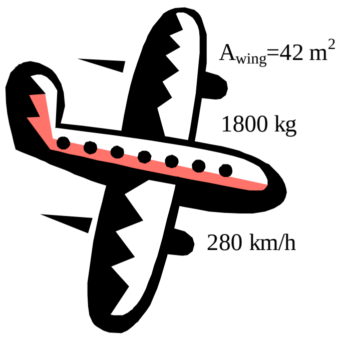
Then the drag coefficient becomes

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} \quad \rightarrow \quad C_D = \frac{2F_D}{\rho A \mathbf{V}^2} = \frac{2(2443\text{ N})}{(0.819\text{ kg/m}^3)(42\text{ m}^2)(280/3.6\text{ m/s})^2}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = \mathbf{0.0235}$$

An aircraft cruises at constant altitude when lift equals the total weight. Therefore,

$$W = F_L = \frac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad C_L = \frac{2W}{\rho \mathbf{V}^2 A} = \frac{2(1800\text{ kg})(9.81\text{ m/s}^2)}{(0.819\text{ kg/m}^3)(42\text{ m}^2)(280/3.6\text{ m/s})^2} = \mathbf{0.17}$$

Therefore, the drag and lift coefficients of this aircraft during cruising are 0.0235 and 0.17, respectively, with a C_L/C_D ratio of 7.2.



Discussion The drag and lift coefficient determined are for cruising conditions. The values of these coefficient can be very different during takeoff because of the angle of attack and the wing geometry.

15-90 An airfoil has a given lift-to drag ratio at 0° angle of attack. The angle of attack that will raise this ratio to 80 is to be determined.

Analysis The ratio C_L/C_D for the given airfoil is plotted against the angle of attack in Fig. 15-42. The angle of attack corresponding to $C_L/C_D = 80$ is θ = **3°**.

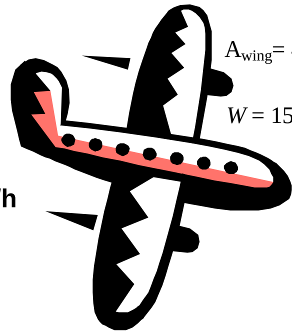
Discussion Note that different airfoils have different C_L/C_D vs. θ charts.

15-91 The wings of a light plane resemble the NACA 23012 airfoil with no flaps. Using data for that airfoil, the takeoff speed at a specified angle of attack and the stall speed are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered.

Properties The density of standard air at sea level is $\rho = 1.225$ kg/m^3 (Table A-24). At an angle of attack of 5°, the lift and drag coefficients are read from Fig. 15-44 to be $C_L = 0.6$ and $C_D = 0.015$. The maximum lift coefficient is $C_{L,\max} = 1.52$ and it occurs at an angle of attack of 15°.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,



$$W = F_L \quad \rightarrow \quad W = \frac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$

Substituting, the takeoff speed is determined to be

$$\mathbf{V}_{\text{takeoff}} = \sqrt{\frac{2(15{,}000\ \text{N})}{(1.225\ \text{kg/m}^3)(0.6)(46\ \text{m}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = 29.8\ \text{m/s} = \mathbf{107\ km/h}$$

since 1 m/s = 3.6 km/h. The stall velocity (the minimum takeoff velocity corresponding the stall conditions) is determined by using the maximum lift coefficient in the above equation,

$$\mathbf{V}_{\min} = \sqrt{\frac{2W}{\rho C_{L,\max} A}} = \sqrt{\frac{2(15{,}000\ \text{N})}{(1.225\ \text{kg/m}^3)(1.52)(46\ \text{m}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = 18.7\ \text{m/s} = \mathbf{67.4\ km/h}$$

Discussion The "safe" minimum velocity to avoid the stall region is obtained by multiplying the stall velocity by 1.2:

$$\mathbf{V}_{\text{min, safe}} = 1.2\mathbf{V}_{\min} = 1.2 \times (18.7\ \text{m/s}) = 22.4\ \text{m/s} = 80.8\ \text{km/h}$$

Note that the takeoff velocity decreased from 107 km/h at an angle of attack of 5° to 80.8 km/s under stall conditions with a safety margin.

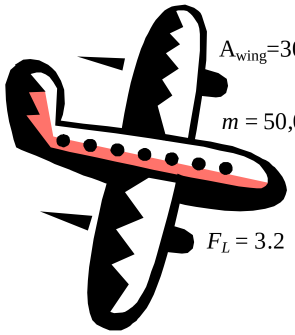
15-92 The mass, wing area, the maximum (stall) lift coefficient, the cruising speed and the cruising drag coefficient of an airplane are given. The safe takeoff speed at sea level and the thrust that the engines must deliver during cruising are to be determined.

Assumptions **1** Standard atmospheric conditions exist **2** The drag and lift produced by parts of the plane other than the wings are not considered. **3** The takeoff speed is 20% over the stall speed. **4** The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties The density of standard air is ρ_1 = 1.225 kg/m^3 at sea level, and ρ_2 = 0.312 kg/m^3 at 12,000 m altitude (Table A-24). The cruising drag coefficient is given to be C_D = 0.03. The maximum lift coefficient is given to be $C_{L,\max}$ = 3.2.

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \frac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}} = \sqrt{\frac{2mg}{\rho C_L A}}$$



The stall velocity (the minimum takeoff velocity corresponding the stall conditions) is determined by using the maximum lift coefficient in the above equation,

$$\mathbf{V}_{\min} = \sqrt{\frac{2mg}{\rho_1 C_{L,\max} A}} = \sqrt{\frac{2(50{,}000\text{ kg})(9.81\text{ m/s}^2)}{(1.225\text{ kg/m}^3)(3.2)(300\text{ m}^2)}} = 28.9\text{ m/s} = 104\text{ km/h}$$

since 1 m/s = 3.6 km/h. Then the "safe" minimum velocity to avoid the stall region becomes

$$\mathbf{V}_{\min,\text{safe}} = 1.2\mathbf{V}_{\min} = 1.2 \times (28.9\text{ m/s}) = 34.7\text{ m/s} = \mathbf{125\ km/h}$$

(*b*) When the aircraft cruises steadily at a constant altitude, the net force acting on the aircraft is zero, and thus the thrust provided by the engines must be equal to the drag force, which is

$$F_D = C_D A \frac{\rho_2 \mathbf{V}^2}{2} = (0.03)(300\text{ m}^2)\frac{(0.312\text{ kg/m}^3)(700/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 53.08\text{ kN}$$

Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Ttrust} \times \text{Velocity} = F_D \mathbf{V} = (53.08\text{ kN})(700/3.6\text{ m/s})\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{10{,}300\ kW}$$

Therefore, the engines must supply 10,300 kW of propulsive power to overcome drag during cruising.

Discussion The power determined above is the power to overcome the drag that acts on the wings only, and does not include the drag that act on the remaining parts of the aircraft (the fuselage, the tail, etc). Therefore, the total power required during cruising will be greater. The required rate of energy input can be determined by dividing the propulsive power by the propulsive efficiency.

15-93E A spinning ball is dropped into a water stream. The lift and drag forces acting on the ball are to be determined.

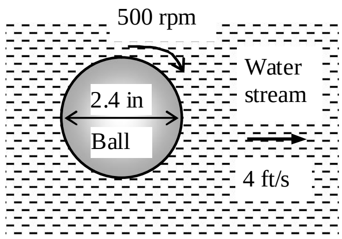
Assumptions **1** The outer surface of the ball is smooth enough for Fig. 15-52 to be applicable. **2** The ball is completely immersed in water.

Properties The density and dynamic viscosity of water at 60°F are ρ =62.36 lbm/ft^3 and μ = 2.713 lbm/ft·h =7.536×10^{-4} lbm/ft·s (Table A-15E).

Analysis The drag and lift forces can be determined from

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} \quad \text{and} \quad F_L = C_L A \frac{\rho \mathbf{V}^2}{2}$$

where A is the frontal area of the ball, which is $A = \pi D^2 / 4$, and D = 2.4/12 = 0.2 ft. The Reynolds number and the angular velocity of the ball are



$$\mathrm{Re} = \frac{\rho \mathbf{V} D}{\mu} = \frac{(62.36\ \mathrm{lbm/ft^3})(4\ \mathrm{ft/s})(0.2\ \mathrm{ft})}{7.536\times10^{-4}\ \mathrm{ft^2/s}} = 6.62\times10^4$$

$$\omega = (500\ \mathrm{rev/min})\left(\frac{2\pi\ \mathrm{rad}}{1\ \mathrm{rev}}\right)\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right) = 52.4\ \mathrm{rad/s}$$

and

$$\frac{\omega D}{2\mathbf{V}} = \frac{(52.4\ \mathrm{rad/s})(0.2\ \mathrm{ft})}{2(4\ \mathrm{ft/s})} = 1.31\ \mathrm{rad}$$

From Fig. 15-52, the drag and lift coefficients corresponding to this value are C_D = 0.56 and C_L = 0.35. Then the drag and the lift acting on the ball are

$$F_D = (0.56)\frac{\pi(0.2\ \mathrm{ft})^2}{4}\frac{(62.36\ \mathrm{lbm/ft^3})(4\ \mathrm{ft/s})^2}{2}\left(\frac{1\ \mathrm{lbf}}{32.2\ \mathrm{lbm\cdot ft/s^2}}\right) = \mathbf{0.27\ lbf}$$

$$F_L = (0.35)\frac{\pi(0.2\ \mathrm{ft})^2}{4}\frac{(62.36\ \mathrm{lbm/ft^3})(4\ \mathrm{ft/s})^2}{2}\left(\frac{1\ \mathrm{lbf}}{32.2\ \mathrm{lbm\cdot ft/s^2}}\right) = \mathbf{0.17\ lbf}$$

Discussion The Reynolds number for this problem is 6.62×10^4 which is close enough to 6×10^4 for which Fig. 15-52 is prepared. Therefore, the result should be close enough to the actual answer.

Review Problems

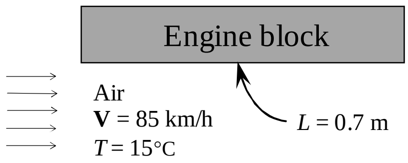
15-94 An automotive engine is approximated as a rectangular block. The drag force acting on the bottom surface of the engine is to be determined. √

Assumptions **1** The air flow is steady and incompressible. **2** Air is an ideal gas. **3** The atmospheric air is calm (no significant winds). **3** The air flow is turbulent over the entire surface because of the constant agitation of the engine block. **4** The bottom surface of the engine is a flat surface, and it is smooth (in reality it is quite rough because of the dirt collected on it).

Properties The density and kinematic viscosity of air at 1 atm and 15°C are ρ = 1.225 kg/m^3 and ν = 1.470×10^{-5} m^2/s (Table A-22).

Analysis The Reynolds number at the end of the engine block is

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(85/3.6\ \mathrm{m/s})(0.7\ \mathrm{m})}{1.470\times10^{-5}\ \mathrm{m^2/s}} = 1.124\times10^6$$



The flow is assumed to be turbulent over the entire surface. Then the average friction coefficient and the drag force acting on the surface becomes

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(1.124\times10^6)^{1/5}} = 0.004561$$

$$F_D = C_f A \frac{\rho \mathbf{V}_\infty^2}{2} = (0.004561)[0.6\times0.7\ \mathrm{m}^2]\frac{(1.225\ \mathrm{kg/m^3})(85/3.6\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg.m/s^2}}\right) = \mathbf{0.65\ N}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

15-95 A fluid flows over a 2.5-m long flat plate. The thickness of the boundary layer at intervals of 0.25 m is to be determined and plotted against the distance from the leading edge for air, water, and oil.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

Properties The kinematic viscosity of the three fluids at 1 atm and 20°C are: $\nu = 1.516\times10^{-5}$ m²/s for air (Table A-22), $\nu = \mu/\rho = (1.002\times10^{-3}$ kg/m·s)/(998 kg/m³) $= 1.004\times10^{-6}$ m²/s for water (Table A-15), and $\nu = 9.429\times10^{-4}$ m²/s for oil (Table A-19).

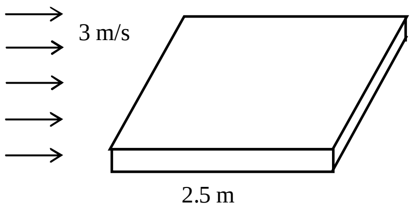
Analysis The thickness of the boundary layer along the flow for laminar and turbulent flows is given by

Laminar flow: $\delta_x = \dfrac{4.91x}{Re_x^{1/2}}$, Turbulent flow: $\delta_x = \dfrac{0.38x}{Re_x^{1/5}}$

(*a*) AIR: The Reynolds number and the boundary layer thickness at the end of the first 0.25 m interval are

$$Re_x = \frac{V_\infty x}{\upsilon} = \frac{(3\text{ m/s})(0.25\text{ m})}{1.516\times10^{-5}\text{ m}^2/\text{s}} = 0.495\times10^5,$$

$$\delta_x = \frac{5x}{Re_x^{1/2}} = \frac{4.91\times(0.25\text{ m})}{(0.495\times10^5)^{0.5}} = 5.52\times10^{-3}\text{ m}$$



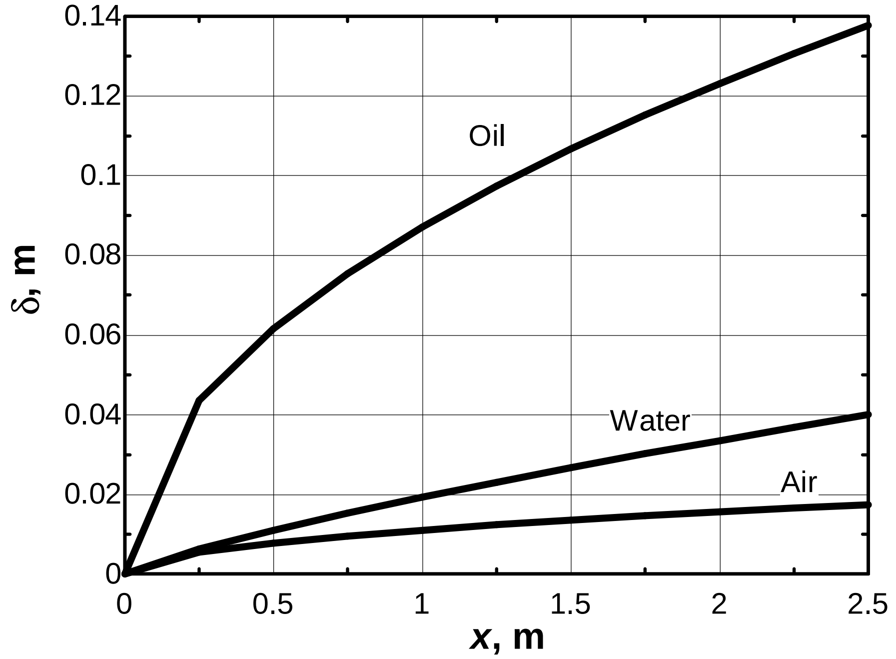
We repeat calculations for all 0.25 m intervals. The results are:

```
V=3 "m/s"
nu1=1.516E-5 "m2/s, Air"
Re1=x*V/nu1
delta1=4.91*x*Re1^(-0.5) "m, laminar flow"

nu2=1.004E-6 "m2/s, water"
Re2=x*V/nu2
delta2=0.38*x*Re2^(-0.2) "m, turbulent flow"

nu3=9.429E-4 "m2/s, oil"
Re3=x*V/nu3
delta3=4.91*x*Re3^(-0.5) "m, laminar flow"
```

x, cm	Air		Water		Oil	
	Re	δ_x	Re	δ_x	Re	δ_x
0.00	0.000E+00	0.0000	0.000E+00	0.0000	0.000E+00	0.0000
0.25	4.947E+04	0.0055	7.470E+05	0.0064	7.954E+02	0.0435
0.50	9.894E+04	0.0078	1.494E+06	0.0111	1.591E+03	0.0616
0.75	1.484E+05	0.0096	2.241E+06	0.0153	2.386E+03	0.0754
1.00	1.979E+05	0.0110	2.988E+06	0.0193	3.182E+03	0.0870
1.25	2.474E+05	0.0123	3.735E+06	0.0230	3.977E+03	0.0973
1.50	2.968E+05	0.0135	4.482E+06	0.0266	4.773E+03	0.1066
1.75	3.463E+05	0.0146	5.229E+06	0.0301	5.568E+03	0.1152
2.00	3.958E+05	0.0156	5.976E+06	0.0335	6.363E+03	0.1231
2.25	4.453E+05	0.0166	6.723E+06	0.0369	7.159E+03	0.1306
2.50	4.947E+05	0.0175	7.470E+06	0.0401	7.954E+03	0.1376



Discussion Note that the flow is laminar for (a) and (c), and turbulent for (b). Also note that the thickness of the boundary layer is very small for air and water, but it is very large for oil. This is due to the high viscosity of oil.

15-96E The passenger compartment of a minivan is modeled as a rectangular box. The drag force acting on the top and the two side surfaces and the power needed to overcome it are to be determined. √

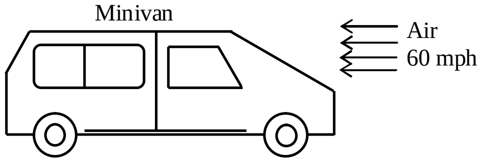
Assumptions **1** The air flow is steady and incompressible. **2** The air flow over the exterior surfaces is turbulent because of constant agitation. **3** Air is an ideal gas. **4** The top and side surfaces of the minivan are flat and smooth (in reality they can be rough). **5** The atmospheric air is calm (no significant winds).

Properties The density and kinematic viscosity of air at 1 atm and 80°F are $\rho = 0.07350$ lbm/ft^3 and $\nu = 0.6110$ ft^2/h $= 1.697\times10^{-4}$ ft^2/s (Table A-22E).

Analysis The Reynolds number at the end of the top and side surfaces is

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{[60\times1.4667\ \mathrm{ft/s}](11\ \mathrm{ft})}{1.697\times10^{-4}\ \mathrm{ft^2/s}} = 5.704\times10^6$$

The air flow over the entire outer surface is assumed to be turbulent. Then the friction coefficient becomes



$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(5.704\times10^6)^{1/5}} = 0.00330$$

The area of the top and side surfaces of the minivan is

$$A = A_{\mathrm{top}} + 2A_{\mathrm{side}} = 6\times11+2\times3.2\times11 = 136.4\ \mathrm{ft}^2$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a plane surface, the drag force acting on these surfaces becomes

$$F_D = C_f A \frac{\rho \mathbf{V}^2}{2} = 0.00330\times(136.4\ \mathrm{ft}^2)\frac{(0.074\ \mathrm{lbm/ft^3})(60\times1.4667\ \mathrm{ft/s})^2}{2}\left(\frac{1\ \mathrm{lbf}}{32.2\ \mathrm{lbm\cdot ft/s^2}}\right) = \mathbf{4.0\ lbf}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

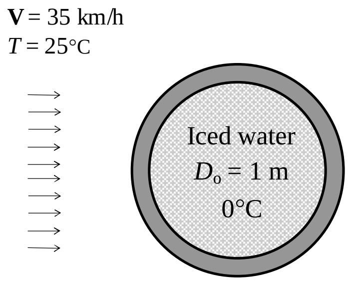
$$\dot{W}_{\mathrm{drag}} = F_D\mathbf{V} = (4.0\ \mathrm{lbf})(60\times1.4667\ \mathrm{ft/s})\left(\frac{1\ \mathrm{kW}}{737.56\ \mathrm{lbf\cdot ft/s}}\right) = \mathbf{0.48\ kW}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

15-97 A large spherical tank located outdoors is subjected to winds. The drag force exerted on the tank by the winds is to be determined.

Assumptions **1** The outer surfaces of the tank are smooth so that Fig. 15-32 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible, and flow around the tank is uniform. **3** Turbulence in the wind is not considered. **4** The effect of any support bars on flow and drag is negligible.

Properties The density and kinematic viscosity of air at 1 atm and 25°C are ρ = 1.184 kg/m^3 and ν = 1.562×10^{-5} m^2/s (Table A-22).



Analysis Noting that D = 1 m and 1 m/s = 3.6 km/h, the Reynolds number for the flow is

$$\mathrm{Re} = \frac{\mathbf{V}D}{\nu} = \frac{(35/3.6\ \mathrm{m/s})(1\ \mathrm{m})}{1.562\times10^{-5}\ \mathrm{m^2/s}} = 6.224\times10^5$$

The drag coefficient for a sphere corresponding to this value is, from Fig. 15-33, C_D = 0.08. Also, the frontal area for flow past a sphere is $A = \pi D^2/4$. Then the drag force becomes

$$F_D = C_D A\frac{\rho\mathbf{V}^2}{2} = 0.09[\pi(1\ \mathrm{m}^2)/4]\frac{(1.184\ \mathrm{kg/m^3})(35/3.6\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg\cdot m/s^2}}\right) = \mathbf{3.5\ N}$$

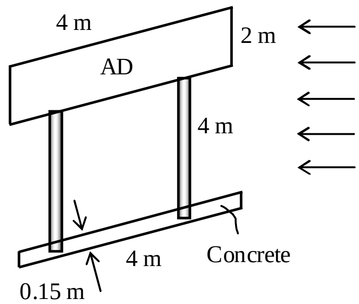
Discussion Note that the drag coefficient is very low in this case since the flow is turbulent (Re > 2×10^5). Also, it should be kept in mind that wind turbulence may affect the drag coefficient.

15-98 A rectangular advertisement panel attached to a rectangular concrete block by two poles is to withstand high winds. For a given maximum wind speed, the maximum drag force on the panel and the poles, and the minimum length L of the concrete block for the panel to resist the winds are to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The wind is normal to the panel (to check for the worst case). **3** The flow is turbulent so that the tabulated value of the drag coefficients can be used.

Properties In turbulent flow, the drag coefficient is C_D = 0.3 for a circular rod, and C_D = 2.0 for a thin rectangular plate (Table 15-2). The densities of air and concrete block are given to be ρ = 1.30 kg/m^3 and ρ_c = 2300 kg/m^3.

Analysis (a) The drag force acting on the panel is



$$F_{D,\text{panel}} = C_D A \frac{\rho \mathbf{V}^2}{2}$$
$$= (2.0)(2\times 4\ \text{m}^2)\frac{(1.30\ \text{kg/m}^3)(150/3.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= \mathbf{18{,}000\ N}$$

(b) The drag force acting on each pole is

$$F_{D,\text{pole}} = C_D A \frac{\rho \mathbf{V}^2}{2}$$
$$= (0.3)(0.05\times 4\ \text{m}^2)\frac{(1.30\ \text{kg/m}^3)(150/3.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= \mathbf{68\ N}$$

Therefore, the drag force acting on both poles is 68 × 2 = **136 N**. Note that the drag force acting on poles is negligible compared to the drag force acting on the panel.

(c) The weight of the concrete block is

$$W = mg = \rho g V = (2300\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(\text{L}\times 4\ \text{m}\times 0.15\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 13{,}540L\ \ \text{N}$$

Note that the resultant drag force on the panel passes through its center, the drag force on the pole passes through the center of the pole, and the weight of the panel passes through the center of the block. When the concrete block is first tipped, the wind-loaded side of the block will be lifted off the ground, and thus the entire reaction force from the ground will act on the other side. Taking the moment about this side and setting it equal to zero gives

$$\sum M = 0 \quad\rightarrow\quad F_{D,\text{panel}}\times(1+4+0.15) + F_{D,\text{pole}}\times(2+0.15) - W\times(L/2) = 0$$

Substituting and solving for L gives

$$18{,}000\times 5.15 + 136\times 2.15 - 13{,}540L\times L/2 = 0 \quad\rightarrow\quad \text{L} = 3.70\ \text{m}$$

Therefore, the minimum length of the concrete block must be L = **3.70**.

Discussion This length appears to be large and impractical. It can be reduced to a more reasonable value by (a) increasing the height of the concrete block, (b) reducing the length of the poles (and thus the tipping moment), or (c) by attaching the concrete block to the ground (through long nails, for example).

15-99 The bottom surface of a plastic boat is approximated as a flat surface. The friction drag exerted on the bottom surface of the boat by water and the power needed to overcome it are to be determined. √EES

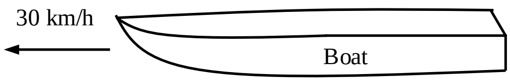
Assumptions **1** The flow is steady and incompressible. **2** The water is calm (no significant currents or waves). **3** The water flow is turbulent over the entire surface because of the constant agitation of the boat. **4** The bottom surface of the boat is a flat surface, and it is smooth.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and μ = 1.138×10^{-3} kg/m·s (Table A-15).

Analysis The Reynolds number at the end of the bottom surface of the boat is

$$\mathrm{Re}_L = \frac{\rho \mathbf{V} L}{\mu} = \frac{(999.1\,\mathrm{kg/m^3})(30/3.6\ \mathrm{m/s})(2\,\mathrm{m})}{1.138\times10^{-3}\ \mathrm{kg/m\cdot s}} = 1.463\times10^7$$

The flow is assumed to be turbulent over the entire surface. Then the average friction coefficient and the drag force acting on the surface becomes



$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(1.463\times10^7)^{1/5}} = 0.00273$$

$$F_D = C_f A \frac{\rho \mathbf{V}^2}{2} = (0.00273)[1.5\times 2\ \mathrm{m}^2]\frac{(999.1\mathrm{kg/m^3})(30/3.6\,\mathrm{m/s})^2}{2}\left(\frac{1\,\mathrm{N}}{1\,\mathrm{kg.m/s^2}}\right) = \mathbf{284.1N}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

$$\dot{W}_{\mathrm{drag}} = F_D \mathbf{V} = (284.1\,\mathrm{N})(30/3.6\ \mathrm{m/s})\left(\frac{1\,\mathrm{kW}}{1000\,\mathrm{N\cdot m/s}}\right) = \mathbf{2.37\ kW}$$

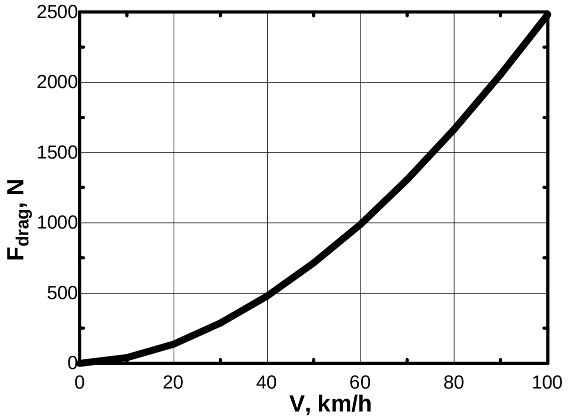
Discussion Note that the calculated drag force (and the power required to overcome it) is relatively small. This is not surprising since the drag force for blunt bodies (including those partially immersed in a liquid) is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

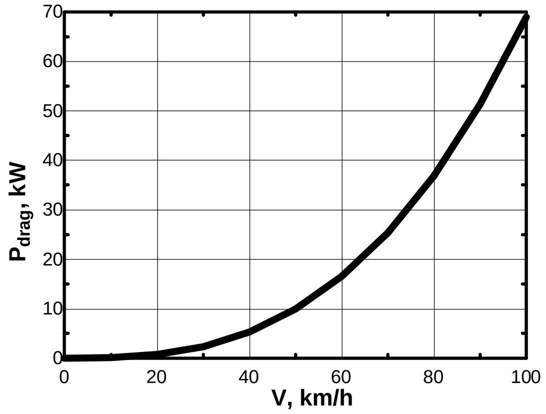
15-100 Problem 15-99 is reconsidered. The effect of boat speed on the drag force acting on the bottom surface of the boat and the power needed to overcome as the boat speed varies from 0 to 100 km/h in increments of 10 km/h is to be investigated.

```
rho=999.1 "kg/m3"
mu=1.138E-3 "m2/s"
V=Vel/3.6 "m/s"
L=2 "m"
W=1.5 "m"
A=L*W

Re=rho*L*V/mu
Cf=0.074/Re^0.2
g=9.81 "m/s2"
F=Cf*A*(rho*V^2)/2 "N"
P_drag=F*V/1000 "kW"
```

V, km/h	Re	C_f	F_{drag}, N	$P_{drag,}$ kW
0	0	0	0	0.0
10	4.877E+06	0.00340	39	0.1
20	9.755E+06	0.00296	137	0.8
30	1.463E+07	0.00273	284	2.4
40	1.951E+07	0.00258	477	5.3
50	2.439E+07	0.00246	713	9.9
60	2.926E+07	0.00238	989	16.5
70	3.414E+07	0.00230	1306	25.4
80	3.902E+07	0.00224	1661	36.9
90	4.390E+07	0.00219	2053	51.3
100	4.877E+07	0.00215	2481	68.9





15-101E Cruising conditions of a passenger plane are given. The minimum safe landing and takeoff speeds with and without flaps, the angle of attack during cruising, and the power required are to be determined. √

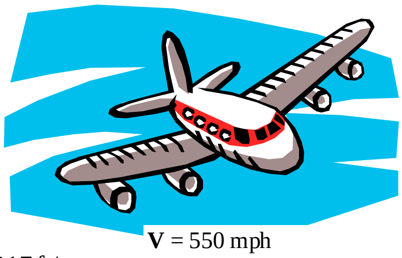
Assumptions **1** The drag and lift produced by parts of the plane other than the wings are not considered. **2** The wings are assumed to be two-dimensional airfoil sections, and the tip effects are neglected. **4** The lift and drag characteristics of the wings can be approximated by NACA 23012 so that Fig. 15-44 is applicable.

Properties The densities of air are 0.075 lbm/ft^3 on the ground and 0.0208 lbm/ft^3 at cruising altitude. The maximum lift coefficients of the wings are 3.48 and 1.52 with and without flaps, respectively (Fig. 15-44).

Analysis (*a*) The weight and cruising speed of the airplane are

$$W = mg = (150{,}000\text{ lbm})(32.2\text{ ft/s}^2)\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 150{,}000\text{ lbf}$$

$$\mathbf{V} = (550\text{ mph})\left(\frac{1.4667\text{ ft/s}}{1\text{ mph}}\right) = 806.7\text{ ft/s}$$



$\mathbf{V}$ = 550 mph
m = 150,000 lbm
A_{wing} = 1800 m^2

Minimum velocity corresponding the stall conditions with and without flaps are

$$\mathbf{V}_{\min 1} = \sqrt{\frac{2W}{\rho C_{L,\max 1} A}} = \sqrt{\frac{2(150{,}000\text{ lbf})}{(0.075\text{ lbm/ft}^3)(1.52)(1800\text{ ft}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)} = 217\text{ ft/s}$$

$$\mathbf{V}_{\min 2} = \sqrt{\frac{2W}{\rho C_{L,\max 2} A}} == \sqrt{\frac{2(150{,}000\text{ lbf})}{(0.075\text{ lbm/ft}^3)(3.48)(1800\text{ ft}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)} = 143\text{ ft/s}$$

The "safe" minimum velocities to avoid the stall region are obtained by multiplying these values by 1.2:

Without flaps: $\mathbf{V}_{\min 1,\text{safe}} = 1.2\mathbf{V}_{\min 1} = 1.2\times(217\text{ ft/s}) = 260\text{ ft/s} = \mathbf{178\text{ mph}}$

With flaps: $\mathbf{V}_{\min 2,\text{safe}} = 1.2\mathbf{V}_{\min 2} = 1.2\times(143\text{ ft/s}) = 172\text{ ft/s} = \mathbf{117\text{ mph}}$

since 1 mph = 1.4667 ft/s. Note that the use of flaps allows the plane to takeoff and land at considerably lower velocities, and thus at a shorter runway.

(*b*) When an aircraft is cruising steadily at a constant altitude, the lift must be equal to the weight of the aircraft, $F_L = W$. Then the lift coefficient is determined to be

$$C_L = \frac{F_L}{\frac{1}{2}\rho\mathbf{V}^2 A} = \frac{150{,}000\text{ lbf}}{\frac{1}{2}(0.0208\text{ lbm/ft}^3)(806.7\text{ ft/s})^2(1800\text{ ft}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right) = 0.40$$

For the case of no flaps, the angle of attack corresponding to this value of C_L is determined from Fig. 15-44 to be about α = **3.5°**.

(*c*) When aircraft cruises steadily, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. The drag coefficient corresponding to the cruising lift coefficient of 0.40 is C_D = 0.015 (Fig. 15-44). Then the drag force acting on the wings becomes

$$F_D = C_D A\frac{\rho\mathbf{V}^2}{2} = (0.015)(1800\text{ ft}^2)\frac{(0.0208\text{ lbm/ft}^3)(806.7\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 5675\text{ lbf}$$

Noting that power is force times velocity (distance per unit time), the power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Thrust}\times\text{Velocity} = F_D\mathbf{V} = (5675\text{ lbf})(806.7\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{6200\text{ kW}}$$

Discussion Note that the engines must supply 6200 kW of power to overcome the drag during cruising. This is the power required to overcome the drag that acts on the wings only, and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc).

15-102 A smooth ball is moving at a specified velocity. The increase in the drag coefficient when the ball spins is to be determined.

Assumptions **1** The outer surface of the ball is smooth so that Figs. 15-33 and 15-52 can be used to determine the drag coefficient. **2** The air is calm (no winds or drafts).

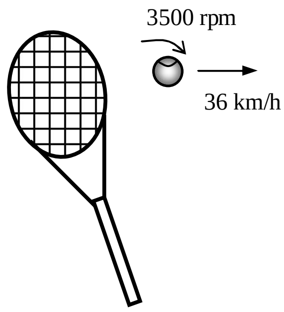
Properties The density and kinematic viscosity of air at 1 atm and 25°C are ρ = 1.184 kg/m^3 and ν = 1.562×10^{-5} m^2/s (Table A-22).

Analysis Noting that D = 0.08 m and 1 m/s = 3.6 km/h, the regular and angular velocities of the ball and the Reynolds number are

$$\mathbf{V} = (36\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 10\text{ m/s}$$

$$\omega = (3500\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 367\text{ rad/s}$$

$$\text{Re} = \frac{\mathbf{V}D}{\nu} = \frac{(10\text{ m/s})(0.08\text{ m})}{1.562\times10^{-5}\text{ m}^2/\text{s}} = 5.122\times10^4$$



and

$$\frac{\omega D}{2\mathbf{V}} = \frac{(367\text{ rad/s})(0.08\text{ m})}{2(10\text{ m/s})} = 1.468\text{ rad}$$

Then the drag coefficients for the ball with and without spin are determined from Figs. 15-33 and 15-52 to be:

Without spin:	C_D = 0.48	(Fig. 15-33)
With spin:	C_D = 0.58	(Fig. 15-52)

Then the increase in the drag coefficient due to spinning becomes

$$\text{Increase in } C_D = \frac{C_{D,\text{spin}} - C_{D,\text{ no spin}}}{C_{D,\text{ no spin}}} = \frac{0.58 - 0.48}{0.48} = 0.21 \text{ (or } \mathbf{21\%}\text{)}$$

Therefore, the drag coefficient increases 21% in this case because of spinning.

Discussion Note that the Reynolds number for this problem is 5.122×10^4 which is close enough to 6×10^4 for which Fig. 15-52 is prepared. Therefore, the result obtained should be fairly accurate.

15-103 The total weight of a paratrooper and its parachute is given. The terminal velocity of the paratrooper in air is to be determined.

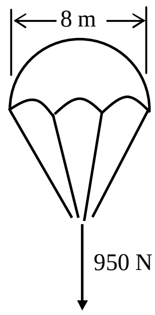
Assumptions **1** The air flow over the parachute is turbulent so that the tabulated value of the drag coefficient can be used. **2** The variation of the air properties with altitude is negligible. **3** The buoyancy force applied by air to the person (and the parachute) is negligible because of the small volume occupied and the low air density.

Properties The density of air is given to be 1.20 kg/m^3. The drag coefficient of a parachute is C_D = 1.3 (Table 15-2).

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_D = W - F_B \quad \text{where} \quad F_D = C_D A \frac{\rho_f \mathbf{V}^2}{2}, \quad W = mg = 950\text{ N}, \quad \text{and } F_B \cong 0$$

where $A = \pi D^2/4$ is the frontal area. Substituting and simplifying,



$$C_D A \frac{\rho_f \mathbf{V}^2}{2} = W \quad \rightarrow \quad C_D \frac{\pi D^2}{4} \frac{\rho_f \mathbf{V}^2}{2} = W$$

Solving for **V** and substituting,

$$\mathbf{V} = \sqrt{\frac{8W}{C_D \pi D^2 \rho_f}} = \sqrt{\frac{8(950\text{ N})}{1.3\pi(8\text{ m})^2(1.20\text{ kg/m}^3)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)} = \mathbf{4.9\ m/s}$$

Therefore, the velocity of the paratrooper will remain constant when it reaches the terminal velocity of 4.9 m/s = 18 km/h.

Discussion The simple analysis above gives us a rough value for the terminal velocity. A more accurate answer can be obtained by a more detailed (and complex) analysis by considering the variation of air density with altitude, and by considering the uncertainty in the drag coefficient.

15-104 A fairing is installed to the front of a rig to reduce the drag coefficient. The maximum speed of the rig after the fairing is installed is to be determined.

Assumptions **1** The rig moves steadily at a constant velocity on a straight path in calm weather. **2** The bearing friction resistance is constant. **3** The effect of velocity on the drag and rolling resistance coefficients is negligible. **4** The buoyancy of air is negligible. **5** The power produced by the engine is used to overcome rolling resistance, bearing friction, and aerodynamic drag.

Properties The density of air is given to be 1.25 kg/m^3. The drag coefficient of the rig is given to be C_D = 0.96, and decreases to C_D = 0.76 when a fairing is installed. The rolling resistance coefficient is C_{RR} = 0.05.

Analysis The bearing friction resistance is given to be F_{bearing} = 350 N. The rolling resistance is

$$F_{RR} = C_{RR}W = 0.05(17{,}000\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 8339\text{ N}$$

The maximum drag occurs at maximum velocity, and its value before the fairing is installed is

$$F_{D1} = C_D A\frac{\rho \mathbf{V}_1^2}{2} = (0.96)(9.2\text{ m}^2)\frac{(1.25\text{ kg/m}^3)(110/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 5154\text{ N}$$

Power is force times velocity, and thus the power needed to overcome bearing friction, drag, and rolling resistance is the product of the sum of these forces and the velocity of the rig,

$$\begin{aligned}\dot{W}_{\text{total}} &= \dot{W}_{\text{bearing}} + \dot{W}_{\text{drag}} + \dot{W}_{\text{RR}} = (F_{\text{bearing}} + F_D + F_{RR})\mathbf{V} \\ &= (350 + 8339 + 5154)(110/3.6\text{ m/s})\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right) \\ &= 423\text{ kW}\end{aligned}$$

The maximum velocity the rig can attain at the same power of 423 kW after the fairing is installed is determined by setting the sum of the bearing friction, rolling resistance, and the drag force equal to 423 kW,

$$\dot{W}_{\text{total}} = \dot{W}_{\text{bearing}} + \dot{W}_{\text{drag2}} + \dot{W}_{\text{RR}} = (F_{\text{bearing}} + F_{D2} + F_{RR})\mathbf{V} = \left(350 + C_{D2}A\frac{\rho \mathbf{V}_2^2}{2} + 5154\right)\mathbf{V}_2$$

Substituting the known quantities,

$$(423\text{ kW})\left(\frac{1000\text{ N}\cdot\text{m/s}}{1\text{ kW}}\right) = \left(350\text{ N} + (0.76)(9.2\text{ m}^2)\frac{(1.25\text{ kg/m}^3)\mathbf{V}_2^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) + 5154\text{ N}\right)\mathbf{V}_2$$

or,

$$423{,}000 = 5504\mathbf{V}_2 + 4.37\mathbf{V}_2^3$$

Solving it with an equation solver gives $\mathbf{V_2}$ **= 36.9 m/s = 133 km/h**.

Discussion Note that the maximum velocity of the rig increases from 110 km/h to 133 km/h as a result of reducing its drag coefficient from 0.96 to 0.76 while holding the bearing friction and the rolling resistance constant.

15-105 A spherical object is dropped into a fluid, and its terminal velocity is measured. The viscosity of the fluid is to be determined.

Assumptions **1** The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). **2** The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body. **3** The tube is long enough to assure that the velocity measured is the terminal velocity.

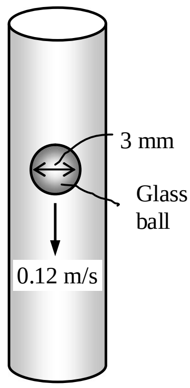
Properties The density of glass ball is given to be ρ_s = 2500 kg/m^3. The density of the fluid is given to be ρ_f = 875 kg/m^3.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu \mathbf{V} D \text{ (Stoke's law)}, \quad W = \rho_s gV, \quad \text{and } F_B = \rho_f gV$$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting and simplifying,

$$3\pi\mu\mathbf{V}D = \rho_s gV - \rho_f gV \quad \rightarrow \quad 3\pi\mu\mathbf{V}D = (\rho_s - \rho_f)g\frac{\pi D^3}{6}$$



Solving for μ and substituting, the dynamic viscosity of the fluid is determined to be

$$\mu = \frac{gD^2(\rho_s - \rho_f)}{18\mathbf{V}} = \frac{(9.81\,\text{m/s}^2)(0.003\,\text{m})^2(2500\text{-}875)\,\text{kg/m}^3}{18(0.12\,\text{m/s})} = \mathbf{0.0666\ kg/m \cdot s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho\mathbf{V}D}{\mu} = \frac{(875\text{ kg/m}^3)(0.12\text{ m/s})(0.003\text{ m})}{0.185\text{ kg}\cdot\text{m/s}} = 1.01$$

which is at the order of 1. Therefore, the creeping flow idealization is applicable, and the value calculated is valid.

Discussion Flow separation starts at about Re = 10. Therefore, Stokes law can be used for Reynolds numbers upto this value, but this should be done with care.

15-106 Spherical aluminum balls are dropped into glycerin, and their terminal velocities are measured. The velocities are to be compared to those predicted by Stoke's law, and the error involved is to be determined.

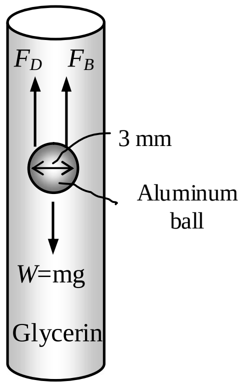
Assumptions **1** The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). **2** The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body. **3** The tube is long enough to assure that the velocity measured is terminal velocity.

Properties The density of aluminum balls is given to be ρ_s = 2700 kg/m^3. The density and viscosity of glycerin are given to be ρ_f = 1274 kg/m^3 and μ = 1 kg/m·s.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu \mathcal{V} D \text{ (Stoke's law)}, \quad W = \rho_s g V, \quad \text{and } F_B = \rho_f g V$$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting and simplifying,



$$3\pi\mu \mathcal{V} D = \rho_s g V - \rho_f g V \quad \rightarrow \quad 3\pi\mu \mathcal{V} D = (\rho_s - \rho_f) g \frac{\pi D^3}{6}$$

Solving for the terminal velocity $\mathcal{V}$ of the ball gives

$$\mathcal{V} = \frac{g D^2 (\rho_s - \rho_f)}{18\mu}$$

(a) D = 2 mm and $\mathcal{V}$ = 3.2 mm/s

$$\mathcal{V} = \frac{(9.81 \text{ m/s}^2)(0.002 \text{ m})^2 (2700\text{-}1274) \text{ kg/m}^3}{18(1 \text{ kg/ m}\cdot\text{s})} = \mathbf{0.00311 \text{ m/s}} = \mathbf{3.11 \text{ mm/s}}$$

$$\text{Error} = \frac{\mathcal{V}_{\text{experimental}} - \mathcal{V}_{\text{Stokes}}}{\mathcal{V}_{\text{experimental}}} = \frac{3.2 - 3.11}{3.2} = \mathbf{0.029 \quad \text{or } 2.9\%}$$

(b) D = 4 mm and $\mathcal{V}$ = 12.8 mm/s

$$\mathcal{V} = \frac{(9.81 \text{ m/s}^2)(0.004 \text{ m})^2 (2700\text{-}1274) \text{ kg/m}^3}{18(1 \text{ kg/ m}\cdot\text{s})} = \mathbf{0.0124 \text{ m/s}} = \mathbf{12.4 \text{ mm/s}}$$

$$\text{Error} = \frac{\mathcal{V}_{\text{experimental}} - \mathcal{V}_{\text{Stokes}}}{\mathcal{V}_{\text{experimental}}} = \frac{12.8 - 12.4}{12.8} = \mathbf{0.029 \quad \text{or } 2.9\%}$$

(c) D = 10 mm and $\mathcal{V}$ = 60.4 mm/s

$$\mathcal{V} = \frac{(9.81 \text{ m/s}^2)(0.010 \text{ m})^2 (2700\text{-}1274) \text{ kg/m}^3}{18(1 \text{ kg/ m}\cdot\text{s})} = \mathbf{0.0777 \text{ m/s}} = \mathbf{77.7 \text{ mm/s}}$$

$$\text{Error} = \frac{\mathcal{V}_{\text{experimental}} - \mathcal{V}_{\text{Stokes}}}{\mathcal{V}_{\text{experimental}}} = \frac{60.4 - 77.7}{60.4} = -\mathbf{0.287 \quad \text{or } -28.7\%}$$

There is a good agreement for the first two diameters. However the error for third one is large. The Reynolds number for each case is

$$(a)\ \text{Re} = \frac{\rho \mathcal{V} D}{\mu} = \frac{(1274 \text{ kg/m}^3)(0.0032 \text{ m/s})(0.002 \text{ m})}{1 \text{ kg}\cdot\text{m/s}} = 0.008, \ (b)\ \text{Re} = 0.065, \ \text{and} \ \ (c)\ \text{Re} = 0.770.$$

We observe that Re << 1 for the first two cases, and thus the creeping flow idealization is applicable. But this is not the case for the third case.

Discussion If we used the general form of the equation (see next problem) we would obtain $\mathcal{V}$ = 59.7 mm/s for part (c), which is very close to the experimental result (60.4 mm/s).

15-107 Spherical aluminum balls are dropped into glycerin, and their terminal velocities are measured. The velocities predicted by general form of Stoke's law, and the error involved are to be determined.

Assumptions **1** The Reynolds number is low (of order 1) so that Stokes law is applicable (to be verified). **2** The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body. **3** The tube is long enough to assure that the velocity measured is terminal velocity.

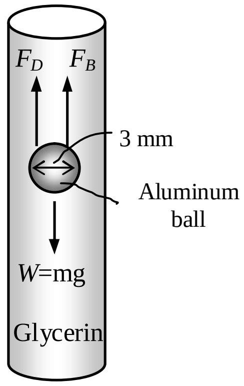
Properties The density of aluminum balls is given to be ρ_s = 2700 kg/m^3. The density and viscosity of glycerin are given to be ρ_f = 1274 kg/m^3 and μ = 1 kg/m·s.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu D\mathrm{V} + (9\pi/16)\rho_s \mathrm{V}^2 D^2, \quad W = \rho_s gV, \quad \text{and} \quad F_B = \rho_f gV$$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting and simplifying,

$$3\pi\mu\mathrm{V}D + (9\pi/16)\rho_s \mathrm{V}^2 D^2 = (\rho_s - \rho_f)g\frac{\pi D^3}{6}$$



Solving for the terminal velocity V of the ball gives

$$\mathrm{V} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{where} \quad a = \frac{9\pi}{16}\rho_s D^2, \quad b = 3\pi\mu D, \text{ and } c = -(\rho_s - \rho_f)g\frac{\pi D^3}{6}$$

(*a*) D = 2 mm and V = 3.2 mm/s: a = 0.01909, b = 0.01885, c = -0.0000586

$$\mathrm{V} = \frac{-0.01885 + \sqrt{(0.01885)^2 - 4\times 0.01909 \times (-0.0000586)}}{2\times 0.01909} = \mathbf{0.00310\ m/s = 3.10\ mm/s}$$

$$\text{Error} = \frac{\mathrm{V}_{\text{experimental}} - \mathrm{V}_{\text{Stokes}}}{\mathrm{V}_{\text{experimental}}} = \frac{3.2 - 3.10}{3.2} = \mathbf{0.032 \quad or\ 3.2\%}$$

(*b*) D = 4 mm and V = 12.8 mm/s: a = 0.07634, b = 0.0377, c = -0.0004688

$$\mathrm{V} = \frac{-0.0377 + \sqrt{(0.0377)^2 - 4\times 0.07634 \times (-0.0004688)}}{2\times 0.07634} = \mathbf{0.0121 m/s = 12.1 mm/s}$$

$$\text{Error} = \frac{\mathrm{V}_{\text{experimental}} - \mathrm{V}_{\text{Stokes}}}{\mathrm{V}_{\text{experimental}}} = \frac{12.8 - 12.1}{12.8} = \mathbf{0.052 \quad or\ 5.2\%}$$

(*c*) D = 10 mm and V = 60.4 mm/s: a = 0.4771, b = 0.09425, c = -0.007325

$$\mathrm{V} = \frac{-0.09425 + \sqrt{(0.09425)^2 - 4\times 0.3771 \times (-0.007325)}}{2\times 0.4771} = \mathbf{0.0597\ m/s = 59.7\ mm/s}$$

$$\text{Error} = \frac{\mathrm{V}_{\text{experimental}} - \mathrm{V}_{\text{Stokes}}}{\mathrm{V}_{\text{experimental}}} = \frac{60.4 - 59.7}{60.4} = \mathbf{0.012 \quad or\ 1.2\%}$$

The Reynolds number for the three cases are

$$(a)\ \text{Re} = \frac{\rho\mathrm{V}D}{\mu} = \frac{(1274\ \text{kg/m}^3)(0.0032\ \text{m/s})(0.002\ \text{m})}{1\ \text{kg}\cdot\text{m/s}} = 0.008, \ (b)\ \text{Re} = 0.065, \text{ and } (c)\ \text{Re} = 0.770.$$

Discussion There is a good agreement for the third case (case *c*), but the general Stoke's law increased the error for the first two cases (cases *a* and *b*) from 2.9% and 2.9% to 3.2% and 5.2%, respectively. Therefore, the basic form of Stoke's law should be preferred when the Reynolds number is much lower than 1.

15-108 A spherical aluminum ball is dropped into oil. A relation is to be obtained for the variation of velocity with time and the terminal velocity of the ball. The variation of velocity with time is to be plotted, and the time it takes to reach 99% of terminal velocity is to be determined.

Assumptions **1** The Reynolds number is low (<< 1) so that Stokes law is applicable. **2** The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body.

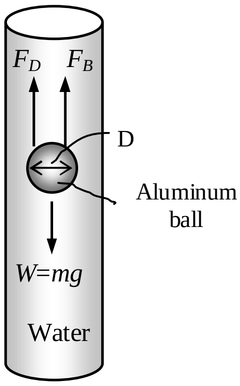
Properties The density of aluminum balls is given to be ρ_s = 2700 kg/m^3. The density and viscosity of oil are given to be ρ_f = 876 kg/m^3 and μ = 0.2177 kg/m·s.

Analysis The free body diagram is shown in the figure. The net force acting downward on the ball is the weight of the ball less the weight of the ball and the buoyancy force applied by the fluid,

$$F_{net} = W - F_D - F_B \quad \text{where} \quad F_D = 3\pi\mu D\mathrm{V} \quad , \quad W = m_s g = \rho_s gV, \quad \text{and} \quad F_B = \rho_f gV$$

where F_D the drag force, F_B the buoyancy force, and W is the weight. Also, $V = \pi D^3/6$ is the volume, m_s is the mass, D is the diameter, and V the velocity of the ball. Applying Newton's second law in the vertical direction,

$$F_{net} = ma \quad \rightarrow \quad m_s g - F_D - F_B = m\frac{d\mathrm{V}}{dt}$$



where. Substituting the drag and buoyancy force relations,

$$\rho_s \frac{\pi D^3}{6} g - 3\pi\mu D\mathrm{V} - \rho_f g \frac{\pi D^3}{6} = \rho_s \frac{\pi D^3}{6}\frac{d\mathrm{V}}{dt}$$

or, $$g\left(1 - \frac{\rho_f}{\rho s}\right) - \frac{18\mu}{\rho_s D^2}\mathrm{V} = \frac{d\mathrm{V}}{dt} \quad \rightarrow \quad a - b\mathrm{V} = \frac{d\mathrm{V}}{dt}$$

where $a = g(1 - \rho_f / \rho_s)$ and $b = 18\mu/(\rho_s D^2)$. It can be rearranged as

$$\frac{d\mathrm{V}}{a - b\mathrm{V}} = dt$$

Integrating from t = 0 where V = 0 to $t = t$ where $\mathrm{V} = \mathrm{V}$ gives

$$\int_0^{\mathrm{V}} \frac{d\mathrm{V}}{a - b\mathrm{V}} = \int_0^t dt \quad \rightarrow \quad -\frac{\ln(a - b\mathrm{V})}{b}\Bigg|_0^{\mathrm{V}} = t\Big|_0^t \quad \rightarrow \quad \ln\left(\frac{a - b\mathrm{V}}{a}\right) = -bt$$

Solving for V gives the desired relation for the variation of velocity of the ball with time,

$$\mathrm{V} = \frac{a}{b}\left(1 - e^{-bt}\right) \quad \text{or} \quad \mathrm{V} = \frac{(\rho_s - \rho_f)gD^2}{18\mu}\left(1 - e^{-\frac{18\mu}{\rho_s D^2}t}\right) \qquad (1)$$

Note that as $t \rightarrow \infty$, it gives the terminal velocity as $\mathrm{V}_{\text{terminal}} = \dfrac{a}{b} = \dfrac{(\rho_s - \rho_f)gD^2}{18\mu}$ (2)

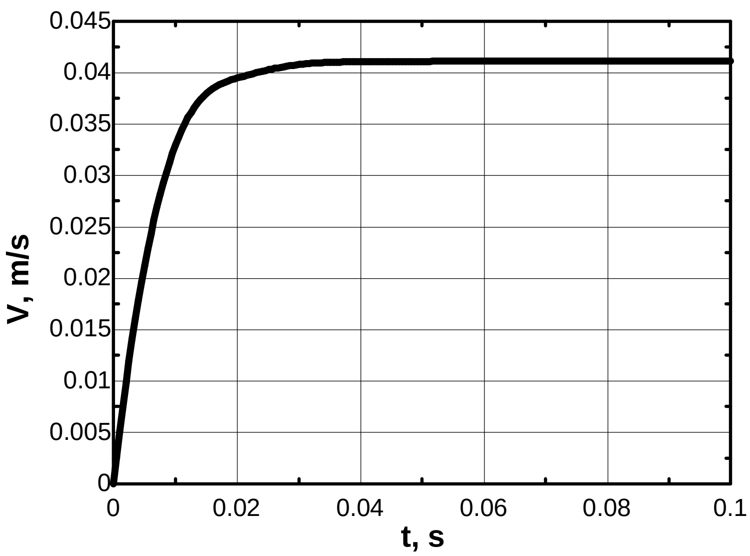
The time it takes to reach 99% of terminal velocity can to be determined by replacing V in Eq. (1) by $0.99\mathrm{V}_{\text{terminal}} = 0.99a/b$. It gives e^{-bt} = 0.01 or

$$t_{99\%} = -\frac{\ln(0.01)}{b} = -\frac{\ln(0.01)\rho_s D^2}{18\mu} \qquad (3)$$

Given values: D = 0.003 m, ρ_f = 876 kg/m^3, μ = 0.2177 kg/m·s, g = 9.81 m/s^2.

Calculation results: Re = 0.50, a = 6.627, b = 161.3, $t_{99\%}$ = **0.029 s**, and $V_{terminal}$= a/b = **0.04 m/s**.

t, s	V, m/s
0.00	0.000
0.01	0.033
0.02	0.039
0.03	0.041
0.04	0.041
0.05	0.041
0.06	0.041
0.07	0.041
0.08	0.041
0.09	0.041
0.10	0.041



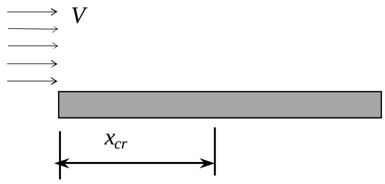
15-109 Engine oil flows over a long flat plate. The distance from the leading edge x_{cr} where the flow becomes turbulent is to be determined, and thickness of the boundary layer over a distance of $2x_{cr}$ is to be plotted.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** The surface of the plate is smooth.

Properties The kinematic viscosity of engine oil at 40°C is $\nu = 2.485\times10^{-4}$ m²/s (Table A-19).

Analysis The thickness of the boundary layer along the flow for laminar and turbulent flows is given by

$$\text{Laminar flow:} \quad \delta_x = \frac{4.91x}{\mathrm{Re}_x^{1/2}}, \qquad \text{Turbulent flow:} \quad \delta_x = \frac{0.38x}{\mathrm{Re}_x^{1/5}}$$



The distance from the leading edge x_{cr} where the flow turns turbulent is determined by setting Reynolds number equal to the critical Reynolds number,

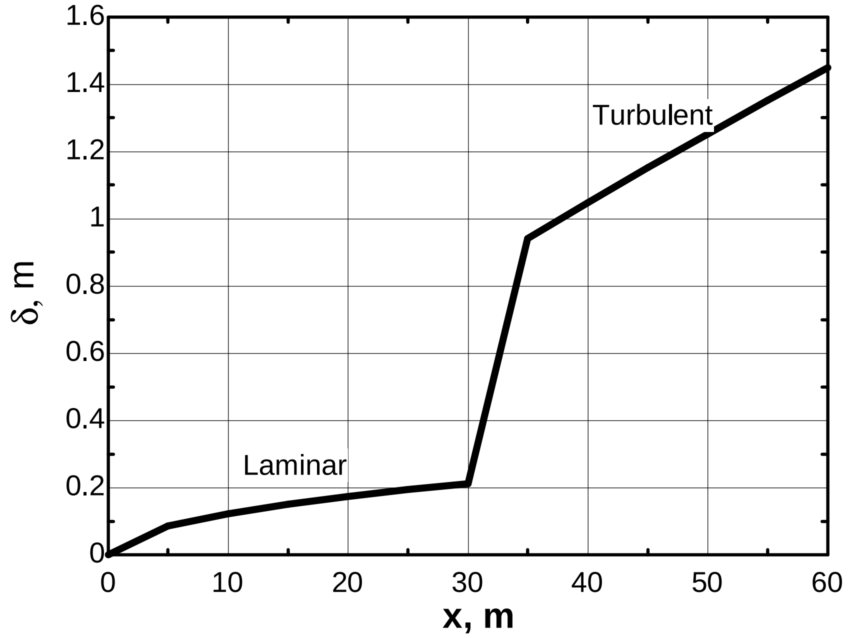
$$\mathrm{Re}_{cr} = \frac{\mathbf{V}x_{cr}}{\upsilon} \rightarrow \quad x_{cr} = \frac{\mathrm{Re}_{cr}\,\upsilon}{\mathbf{V}} = \frac{(5\times10^5)(2.485\times10^{-4}\ \mathrm{m^2/s})}{4\ \mathrm{m/s}} = \mathbf{31.1\,m},$$

Therefore, we should consider flow over 2×31.1 = 62.2 m long section of the plate, and use the laminar relation for the first half, and the turbulent relation for the second part to determine the boundary layer thickness. For example, the Reynolds number and the boundary layer thickness at a distance 2 m from the leading edge of the plate are

$$\mathrm{Re}_x = \frac{\mathbf{V}x}{\upsilon} = \frac{(4\ \mathrm{m/s})(2\ \mathrm{m})}{2.485\times10^{-4}\ \mathrm{m^2/s}} = 32{,}190, \qquad \delta_x = \frac{4.91x}{\mathrm{Re}_x^{1/2}} = \frac{4.91\times(2\ \mathrm{m})}{(32{,}190)^{0.5}} = 0.0547\ \mathrm{m}$$

Calculating the boundary layer thickness and plotting give

x, m	Re	$\delta_{x,\ \text{laminar}}$	$\delta_{x,\ \text{turbulent}}$
0.00	0	0.000	-
5.00	8.05E+04	0.087	-
10.00	1.61E+05	0.122	-
15.00	2.41E+05	0.150	-
20.00	3.22E+05	0.173	-
25.00	4.02E+05	0.194	-
30.00	4.83E+05	0.212	-
35.00	5.63E+05	-	0.941
40.00	6.44E+05	-	1.047
45.00	7.24E+05	-	1.151
50.00	8.05E+05	-	1.252
55.00	8.85E+05	-	1.351
60.00	9.66E+05	-	1.449



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