Andere redenering

e) $Y(s) = G_2 \left(H_2 Y + U_1 \right)$

met $U_1 = G_1 \left[H_1 U_1 + \left(U - H_3 Y \right) \right]$

$\rightarrow U_1 = \dfrac{G_1 U - G_1 H_3 Y}{1 - H_1 G_1}$

$$Y(s) = G_2 H_2 Y + G_2 G_1 \left(\frac{U - H_3 Y}{1 - H_1 G_1} \right)$$

$$\left(1 - G_2 H_2 + \frac{G_1 G_2 H_3}{1 - H_1 G_1} \right) Y = \frac{G_1 G_2 U}{1 - H_1 G_1}$$

$$Y = \frac{\dfrac{G_1 G_2}{1 - H_1 G_1}}{1 - G_2 H_2 + \dfrac{G_1 G_2 H_3}{1 - H_1 G_1}} U \quad \checkmark$$

c) $Y = G_2 \left[U + H_1 Z \right] + G_1 \left[U + H_1 Z \right]$

$= \left(G_1 + G_2 \right) \left(U + H_1 Z \right)$

met $Z = G_1 \left(U + H_1 Z \right)$

$\rightarrow Z = \dfrac{G_1 U}{1 - H_1 G_1}$

$$Y = \left(G_1 + G_2 \right) \left(U + \frac{H_1 G_1 U}{1 - H_1 G_1} \right) = \frac{G_1 + G_2}{1 - H_1 G_1} U \quad \checkmark$$

TOPIC 4: Toestandsruimtemodel

$\boxed{\text{OEFENING 1}}$ ✓

* $\bar{u} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$ $\bar{x} = \begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix}$ y = uitgang over L_1

* Weerstand: $v(t) = R.i(t)$
  Capaciteit: $v(t) = \frac{1}{C} \int_0^t i(t)dt$
  Spoel: $v(t) = L \frac{di(t)}{dt}$

* Wetten van Kirchoff:
  1. $e_1 = v + L_1 \frac{di_1(t)}{dt} + Ri_1(t)$
  2. $e_2 = v + L_2 \frac{di_2(t)}{dt}$
  3. Midden: $v = \frac{1}{C} \int (i_1(t) + i_2(t))dt$

* 1 $\frac{di_1(t)}{dt} = \frac{e_1}{L_1} - \frac{v}{L_1} - \frac{R}{L_1} i_1(t)$

  2 $\frac{di_2(t)}{dt} = \frac{e_2}{L_2} - \frac{v}{L_2}$

  3 $\frac{dv}{dt} = \frac{1}{C} i_1(t) + \frac{1}{C} i_2(t)$

toestandsruimtemodel ✓

$$\Rightarrow \frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix} = \begin{bmatrix} -R/L_1 & 0 & -1/L_1 \\ 0 & 0 & -1/L_2 \\ 1/C & 1/C & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix} + \begin{bmatrix} 1/L_1 & 0 \\ 0 & 1/L_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$ ✓

* Uitgang: $y = L \frac{d i_1(t)}{dt}$

$= e_1 - v - R i_1(t)$

$$y = \begin{bmatrix} -R & 0 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix} + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad \checkmark$$

$\boxed{\text{OEFENING 2}}$

niet

$\boxed{\text{OEFENING 3}}$ ✓

* SISO:
$$\frac{d\bar{x}}{dt} = \dot{\bar{x}} = A\,\bar{x} + \bar{b}\,.u$$
$$y = \bar{c}^T\,\bar{x} + d\,.u$$

$$s\,\bar{X}(s) - \bar{x}_0 = A\,\bar{X}(s) + \bar{b}\,.U(s)$$
$$(sI_n - A)\,\bar{X}(s) = \bar{b}\,.U(s) + \bar{x}_0$$
$$\bar{X}(s) = (sI_n - A)^{-1}\,\bar{b}\,U(s) + (sI_n - A)^{-1}\,\bar{x}_0$$

$$Y(s) = \bar{c}^T\,\bar{X}(s) + d\,.U(s)$$

$$= \underbrace{\left[\bar{c}^T (sI_n - A)^{-1}\,\bar{b} + d\right]}_{H(s)} U(s) + \bar{c}^T (sI_n - A)^{-1} \underbrace{\bar{x}_0}_{\text{initiële toestand vector}} \quad \checkmark$$

* **MIMO** $\frac{d\bar{x}}{dt} = \dot{\bar{x}} = A\,\bar{x} + B\,\bar{u}$

$\bar{y} = C\,.\,\bar{x} + D\,.\,\bar{u}$

$$s\,\bar{X}(s) - \bar{x}_0 = A\,\bar{X}(s) + B\,\bar{U}(s)$$
$$(sI_n - A)\,\bar{X}(s) = B\,\bar{U}(s) + \bar{x}_0$$
$$\bar{X}(s) = (sI_n - A)^{-1} B\,\bar{U}(s) + (sI_n - A)^{-1}\bar{x}_0$$

$$\bar{Y}(s) = C\,\bar{X}(s) + D\,\bar{U}(s)$$
$$= \left[C\,(sI_n - A)^{-1} B + D\right]\bar{U}(s) + C\,(sI_n - A)^{-1}\bar{x}_0 \checkmark$$

OEFENING 4

* $$(sI_n - A) = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 2 & 4 & s+3 \end{bmatrix}$$

$$(sI_n - A)^{-1} = \frac{\begin{bmatrix} s(s+3)+4 & -2 & -2s \\ +(s+3) & s(s+3) & -4s-2 \\ 1 & s & s^2 \end{bmatrix}^T}{s^2(s+3) + 2 + 4s}$$

$$= \frac{\begin{bmatrix} s^2+3s+4 & s+3 & 1 \\ -2 & s(s+3) & s \\ -2s & -2(2s+1) & s^2 \end{bmatrix}}{s^3 + 3s^2 + 4s + 2}$$

$$\begin{array}{c|cccc} & 1 & 3 & 4 & 2 \\ -1 & & -1 & -2 & -2 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

$D < 0$

$$\bar{X}(s) = \frac{\begin{bmatrix} s(s+3)+4 & s+3 & 1 \\ -2 & s(s+3) & s \\ -2s & -2(2s+1) & s^2 \end{bmatrix}}{s^3+3s^2+4s+2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \bar{U}(s)$$

$$+ (sI_n - A)^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \text{met } \bar{U}(s) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} s(s+3)+4-1 \\ -2-s \\ -2s-s^2 \end{bmatrix} + \begin{bmatrix} s(s+3)+4 \\ -2 \\ -2s \end{bmatrix}}{s^3+3s^2+4s+2}$$

$$= \frac{\begin{bmatrix} 2s(s+3)+7 \\ -4-s \\ -4s-s^2 \end{bmatrix}}{s^3+3s^2+4s+2}$$

$$\bar{Y}(s) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \frac{\begin{bmatrix} 2s(s+3)+7 \\ -4-s \\ -4s-s^2 \end{bmatrix}}{s^3+3s^2+4s+2} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} -4-s+4s+s^2 \\ 2s(s+3)+7-8-2s-4s-s^2 \end{bmatrix}}{s^3+3s^2+4s+2} = \frac{\begin{bmatrix} +s^2+3s-4 \\ s^2-1 \end{bmatrix}}{(s+1)(s^2+2s+2)}$$

$$* \; y_1(t) = \mathcal{L}^{-1}\left(\frac{s^2+3s-4}{(s+1)(s^2+2s+2)}\right)$$

$$\frac{s^2+3s-4}{(s+1)(s^2+2s+2)} = \frac{A(s^2+2s+2)+(Bs+C)(s+1)}{(s+1)(s^2+2s+2)}$$

$$\begin{cases} s=-1 \rightarrow (1-2+2)A = 1-3-4 \rightarrow A=-6 \\ A+B=1 \rightarrow B=7 \\ 2A+C=-4 \rightarrow C=8 \end{cases}$$

$$y_1(t) = \mathcal{L}^{-1}\left(\frac{-6}{s+1}\right) + \mathcal{L}^{-1}\left(\frac{7s}{(s+1)^2+1}\right) + \mathcal{L}^{-1}\left(\frac{8}{(s+1)^2+1}\right)$$

$$= -6e^{-t} + 7e^{-t}\cos t + 8e^{-t}\sin t$$

$$= e^{-t}(7\cos t + \cancel{8}\sin t) - 6e^{-t} \quad ; \; t \geq 0$$

$$* \; y_2(t) = \mathcal{L}^{-1}\left(\frac{s^2-1}{(s+1)(s^2+2s+2)}\right)$$

$$\frac{s^2-1}{(s+1)(s^2+2s+2)} = \frac{A(s^2+2s+2)+(Bs+C)(s+1)}{(s+1)(s^2+2s+2)}$$

$$\begin{cases} s=-1 \rightarrow (1-2+2)A = 0 \rightarrow A=0 \\ A+B=1 \rightarrow B=1 \\ 2A+C=-1 \rightarrow C=-1 \end{cases}$$

$$y_2(t) = \mathcal{L}^{-1}\left(\frac{s}{(s+1)^2+1}\right) - \mathcal{L}^{-1}\left(\frac{1}{(s+1)^2+1}\right)$$

$$= e^{-t}\cos t - e^{-t}\sin t$$

$$= e^{-t}(\cos t - 2\sin t) \quad ; \; t \geq 0$$

OEFENING 5 ✓

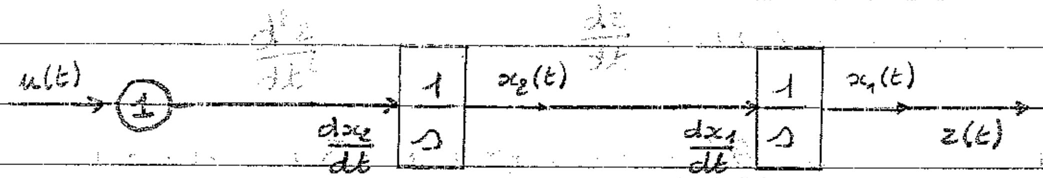
$$H(s) = \frac{s-1}{s^2+3s+2} \qquad Z(s) = \frac{1}{s^2+3s+2}$$

* $Z(s) = \dfrac{Y(s)}{s-1}$ $\qquad$ $Z(s) = \dfrac{U(s)}{s^2+3s+2}$

$Y(s) = sZ(s) - Z(s)$? $\qquad$ $U(s) = s^2 Z(s) + 3sZ(s) + 2Z(s)$

$y(t) = \dfrac{dz}{dt} - z \quad (2)$ $\qquad$ $u(t) = \dfrac{d^2z}{dt^2} + 3\dfrac{dz}{dt} + 2z$

*✓ <u>Serie:</u> $u(t) = \dfrac{d^2z}{dt^2} + \dfrac{3dz}{dt} + 2z \quad (1)$



$x_2(t) = \dfrac{dx_1}{dt} \quad (*) \qquad z(t) = x_1(t)$

uit (*) en (*)

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \quad ✓$$

z vervangen door x

$$z(t) = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + 0\,u$$

uit (1): $\dfrac{dx_2}{dt} + 3x_2 + 2x_1 = u \quad (*)$

uit (2): $y(t) = x_2 - x_1$

$$y = \begin{pmatrix} -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + 0\,u \quad ✓$$

Oefening 5 topic 4

gegeven:

$$H(s) = \frac{(s-1)}{(s^2+3s+2)}$$

gevraagd: toestandsruimtemodel, minimale realisatie met sommatoren en integratoren, stabiliteit

oplossing

$$Z(s) = \frac{Y(s)}{(s-1)}$$

$$\Leftrightarrow s\,Z(s) - Z(s) = Y(s)$$

$$\Rightarrow y(t) = \frac{dz}{dt} - z$$

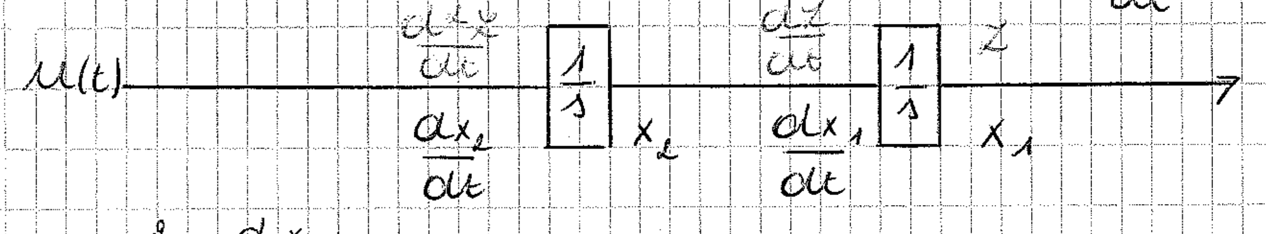
$$Z(s) = \frac{U(s)}{s^2+3s+2}$$

$$U(s) = s^2 Z(s) + 3s\,Z(s) + 2Z(s)$$

$$\Rightarrow u(t) = \frac{d^2z}{dt} + 3\frac{dz}{dt} + 2z \quad *$$

Noemer transferfunctie v graad 2

$\Rightarrow$ 2 integratoren   Serie: $u(t) = \frac{d^2z}{dt} + 3\frac{dz}{dt} + 2z$



$$x^2 = \frac{dx_1}{dt}$$

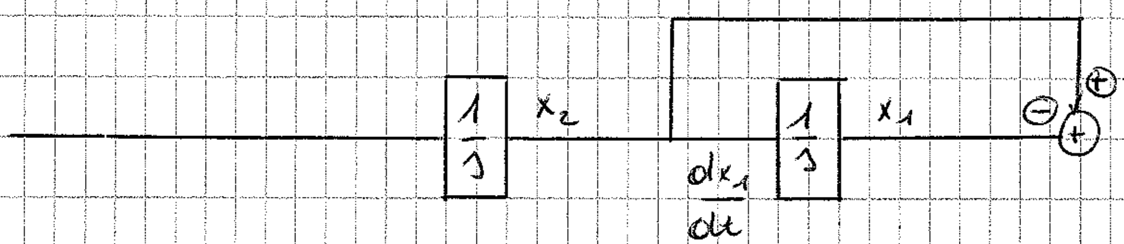
$$\frac{dx_2}{dt} = -3x_2 - 2x_1 + u \quad \text{uit } *$$

$$\Rightarrow \frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

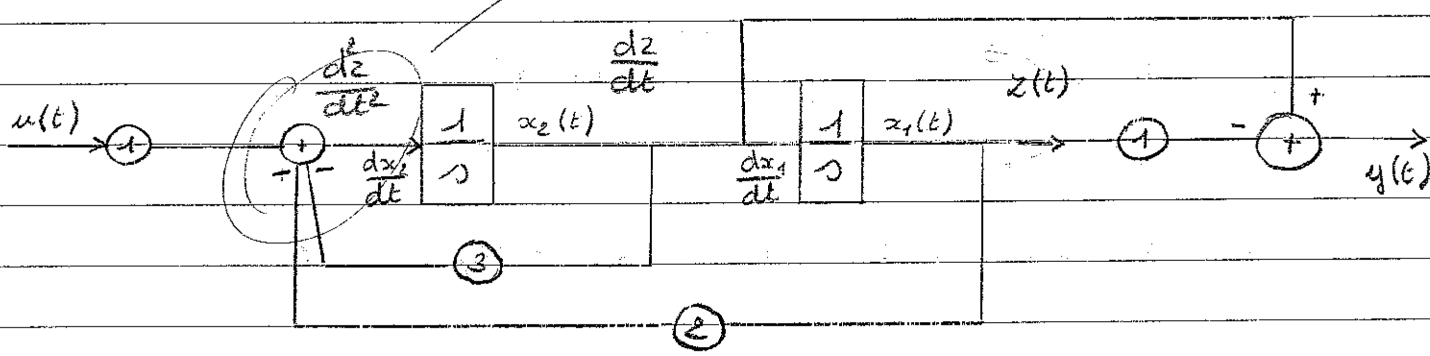
$$y(t) = x_2 - x_1$$

$$y(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} u$$

Prosa tekening



Volledige tekening: waarom -



+ **Parallel:** Polen v/d transferfunctie zoeken
→ elke partieelbreuk = deel van model

$$H(s) = \frac{s-1}{s^2+3s+2} = \frac{A(s+2)+B(s+1)}{(s+1)(s+2)}$$

$s=-2 \rightarrow -B=-3 \rightarrow B=3$

$s=-1 \rightarrow A=-2$

$$= \frac{-2}{s+1} + \frac{3}{s+2}$$

$$Y(s) = \underbrace{\frac{-2}{s+1}U(s)}_{X_1(s)} + \underbrace{\frac{3}{s+2}U(s)}_{X_2(s)} \rightarrow Y(s) = X_1(s)+X_2(s)$$

$$X_1(s) = \frac{-2}{s+1}U(s) \qquad X_2(s) = \frac{3}{s+2}U(s)$$

$$sX_1(s) = -2U(s) - X_1(s) \qquad sX_2(s) = 3U(s) - 2X_2(s)$$

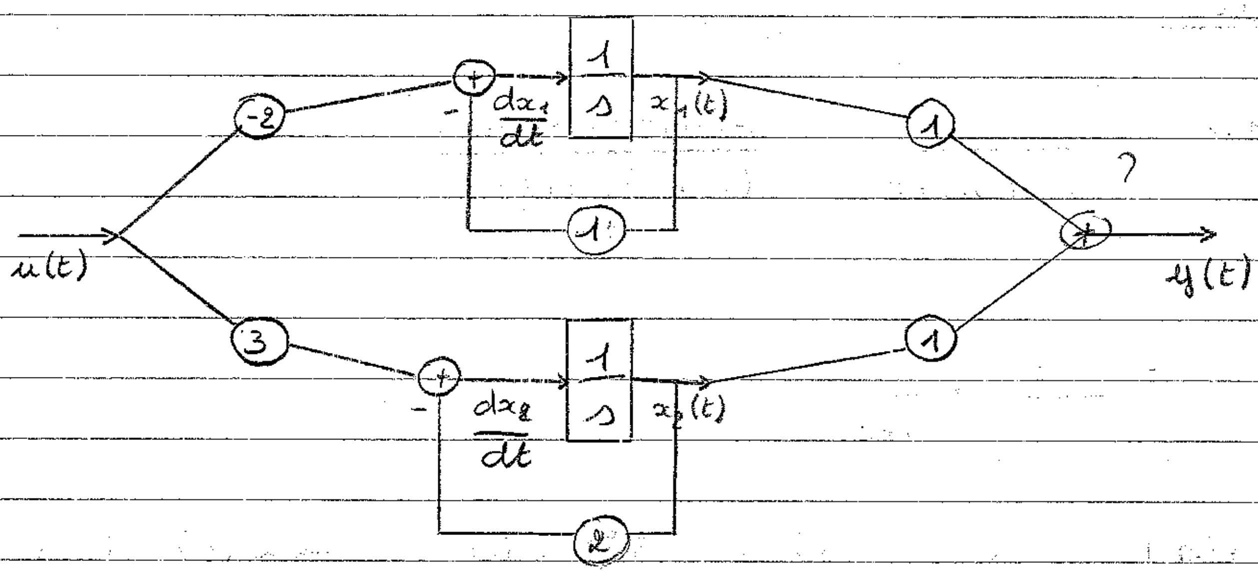
$$\frac{dx_1}{dt} = -2u - x_1 \qquad \frac{dx_2}{dt} = 3u - 2x_2$$

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} u \checkmark$$

$$y = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + 0\, u \checkmark$$

↳ komt van $y(t) = x_1 + x_2$ (3)

Tekening:



* <u>Stabiliteit:</u>

- EWn van systeemmatrix A

$$|s I_n - A| = \begin{vmatrix} s+1 & 0 \\ 0 & s+2 \end{vmatrix} = (s+1)(s+2) = 0$$

$\{\lambda_i(A)\} = \{-1, -2\}$ reële delen neg → inwendig stab

$\Rightarrow$ inwendige stabiliteit

- Polen van $H(s)$: $\{s_p(H(s))\} = \{-1, -2\}$ ~~uitg~~ reële deel nulpunten noemer neg → I/O stabiel

$\Rightarrow$ I/O stabiliteit

- Opm: Inwendige stabiliteit $\Rightarrow$ I/O stabiliteit

→ reeel deel 0 marginaal stabiel

* Extra: verifiëren dat $H(s) = \bar{c}^{\tau}(sI-A)^{-1}\bar{b} + d$

Parallel: $(sI-A)^{-1} = \begin{pmatrix} s+1 & 0 \\ 0 & s+2 \end{pmatrix}^{-1} = \dfrac{\begin{pmatrix} s+2 & 0 \\ 0 & s+1 \end{pmatrix}}{(s+1)(s+2)}$

$$H(s) = \begin{pmatrix} 1 & 1 \end{pmatrix} \frac{\begin{pmatrix} s+2 & 0 \\ 0 & s+1 \end{pmatrix}}{(s+1)(s+2)} \begin{pmatrix} -2 \\ 3 \end{pmatrix} + 0$$

$$= \begin{pmatrix} 1 & 1 \end{pmatrix} \frac{\begin{pmatrix} -2(s+2) \\ 3(s+1) \end{pmatrix}}{(s+1)(s+2)} = \frac{-2(s+2)+3(s+1)}{(s+1)(s+2)}$$

$$= \frac{s-1}{(s+1)(s+2)} \checkmark$$

$\boxed{\cancel{\text{OEFENING 6}}}$

$$\cancel{H(s) = \frac{(s-3)(s-1)}{(s+3)(s+1)}}$$

Serie: $(sI-A)^{-1} = \begin{pmatrix} s & -1 \\ 2 & s+3 \end{pmatrix}^{-1} = \dfrac{\begin{pmatrix} s+3 & -2 \\ +1 & s \end{pmatrix}^{\tau}}{s(s+3)+2}$

$$H(s) = \begin{pmatrix} -1 & 1 \end{pmatrix} \frac{\begin{pmatrix} s+3 & +1 \\ -2 & s \end{pmatrix}}{s^2+3s+2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 0$$

$$= \frac{\begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ s \end{pmatrix}}{s^2+3s+2}$$

$$= \frac{-1+s}{s^2+3s+2} \checkmark$$

puntje 4 oef 6 niet !

$\boxed{\text{OEFENING 6}}$ ✓

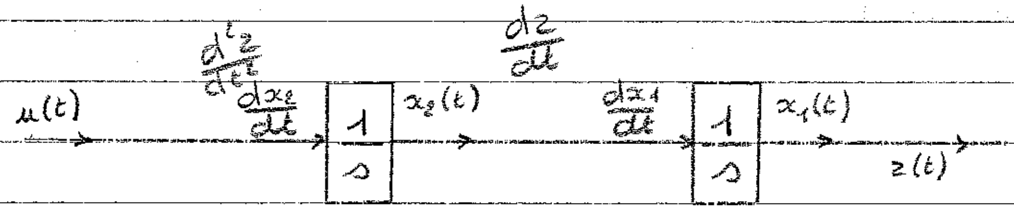
$$H(s) = \frac{(s-3)(s-1)}{(s+3)(s+1)} = \frac{s^2-4s+3}{s^2+4s+3}$$

* $Z(s) = \dfrac{Y(s)}{(s-3)(s-1)}$ $\qquad$ $Z(s) = \dfrac{U(s)}{(s+3)(s+1)}$

$Y(s) = s^2 Z(s) - 4s Z(s) + 3Z(s)$ $\qquad$ $U(s) = s^2 Z(s) + 4s Z(s) + 3Z(s)$

$y(t) = \dfrac{d^2z}{dt^2} - 4\dfrac{dz}{dt} + 3z$ $\qquad$ $u(t) = \dfrac{d^2z}{dt^2} + 4\dfrac{dz}{dt} + 3z$

*



$x_2 = \dfrac{dx_1}{dt}$ $\qquad$ $x_1 = z(t)$

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \quad ✓$$

$$\frac{dx_2}{dt} + 4x_2 + 3x_1 = u$$

$$y = \begin{pmatrix} 0 & -8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + 1\, u \quad ✓$$

$$y = \frac{dx_2}{dt} - 4x_2 + 3x_1 = -4x_2 - \cancel{3x_1} + u - 4x_2 + \cancel{3x_1}$$

* Impulsresponsie:

$Y(s) = H(s) \cdot U(s)$ met $u(t) = \delta(t)$

$= H(s)$ $\rightarrow U(s) = 1$

$y(t) = h(t) \cdot u(t)$

$= h(t) \; \delta(t)$

$$\begin{array}{r|l} s^2 - 4s + 3 & s^2 + 4s + 3 \\ \hline -s^2 \mp 4s \mp 3 & 1 \\ \hline -8s & \end{array}$$

$$H(s) = \frac{s^2 - 4s + 3}{(s+3)(s+1)} = 1 + \frac{-8s}{(s+1)(s+3)}$$

$\hookrightarrow A(s+3) + B(s+1)$

$s = -1 \rightarrow 2B = 8 \rightarrow B = 4$

$s = -3 \rightarrow -2A = 24 \rightarrow A = -12$

$$= \frac{-12}{s+3} + \frac{4}{s+1} + 1$$

$h(t) = -12e^{-3t} + 4e^{-t} + \delta(t)$, $t \geq 0$ ✓

* Inwendigstabiel: $\{\lambda_i(A)\} = \{ \; .. \; \} < 0$

$$|sI - A| = \begin{vmatrix} s & -\cancel{1} \\ 3 & s+4 \end{vmatrix} = s(s+4) + \cancel{3} = s^2 + 4s + \cancel{3}$$

$D = 16 - 4 \cdot \cancel{3} = \cancel{8}\; 4$

$\frac{-4 \pm 2}{2}$ $\rightarrow$ -1, -3

$$\cancel{s = \frac{-4 \pm \sqrt{-8}}{2} = \frac{-4 \pm j2\sqrt{2}}{2} = -2 \pm j\sqrt{2}}$$

$\rightarrow$ reële gedeelte < 0 $\Rightarrow$ inwendig stabiel

polen van $H(s) < 0$ $\Rightarrow$ I/O stabiel ✓

~~OEFENING 7~~

$$H(s) = C\,(sI - A)^{-1}\,B + D$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \frac{\begin{bmatrix} s(s+3)+4 & s+3 & 1 \\ -2 & s(s+3) & s \\ -2s & -2(2s+1) & s^2 \end{bmatrix}}{s^3+3s^2+4s+2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \frac{\begin{bmatrix} s(s+3)+4-1 & s+3+1 \\ -2-s & s(s+3)+s \\ -2s-s^2 & 2(2s+1)+s^2 \end{bmatrix}}{s^3+3s^2+4s+2}$$

$$= \frac{\begin{bmatrix} -2-s+2s+s^2 & s(s+3)+s+2(2s+1)-s^2 \\ s(s+3)+3-4-2s-2s-s^2 & s+4+2s(s+3)+2s-2(2s+1)+s^2 \end{bmatrix}}{s^3+3s^2+4s+2}$$

$$= \frac{\begin{bmatrix} s^2+s-2 & 8s+2 \\ -s-1 & 3s^2+5s+2 \end{bmatrix}}{s^3+3s^2+4s+2} \qquad s^3+3s^2+4s+2 = (s+1)(s^2+2s+2)$$

OEFENING 7

* $H(s) = \dfrac{(s-3)(s-1)}{(s+3)(s+1)}$

$$H_1(s) = \frac{H(s)}{1+H(s)} = \frac{(s-3)(s-1)}{(s+3)(s+1)+(s-3)(s-1)}$$

* $H(s)$ is I/O stabiel : polen < 0

$H_1(s)$: polen zoeken

$(s+3)(s+1)+(s-3)(s-1)=0$

$s^2+4s+3+s^2-4s+3=0$

$2s^2+6=0$

$s^2=-3$

$s = \pm i\sqrt{3}$

→ zuiver imaginaire polen

⇝ marginaal I/O stabiel

⇝ onstabiel

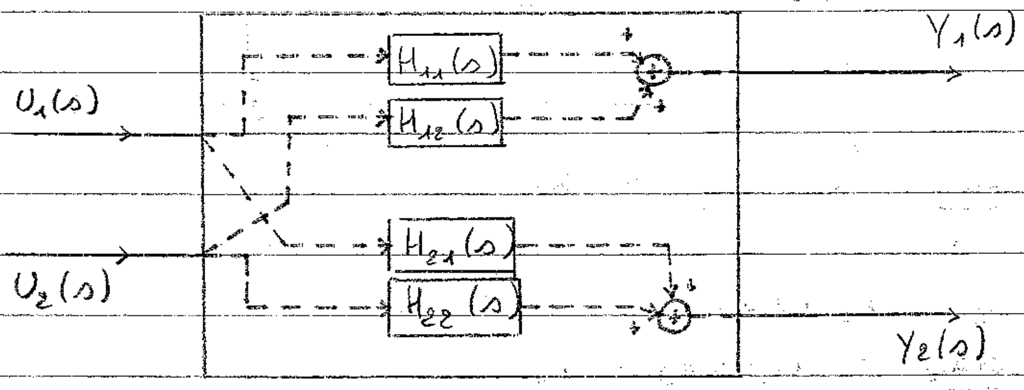
Topic 5: MIMO systeem

OEFENING 1 ✓

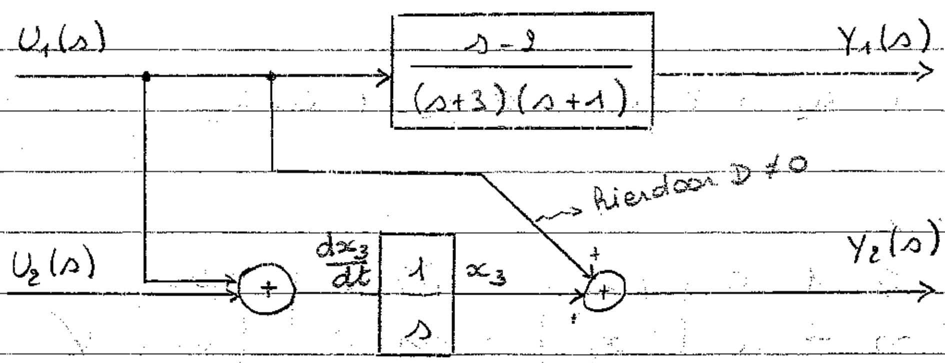
$$\begin{pmatrix} Y_1(s) \\ Y_2(s) \end{pmatrix} = \begin{pmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{pmatrix} \begin{pmatrix} U_1(s) \\ U_2(s) \end{pmatrix}$$

↳ Hieruit weet je direct hoeveel in- en uitgangen er zijn

Meest algemeen geval:



→ altijd proberen vereenvoudigen (minimaal # integratoren)



$$Y_1(s) = H_{11}(s)\,U_1(s) = \frac{s-2}{(s+3)(s+1)}\,U_1(s)$$

$$Y_2(s) = \frac{1+s}{s}\,U_1(s) + \frac{1}{s}\,U_2(s)$$

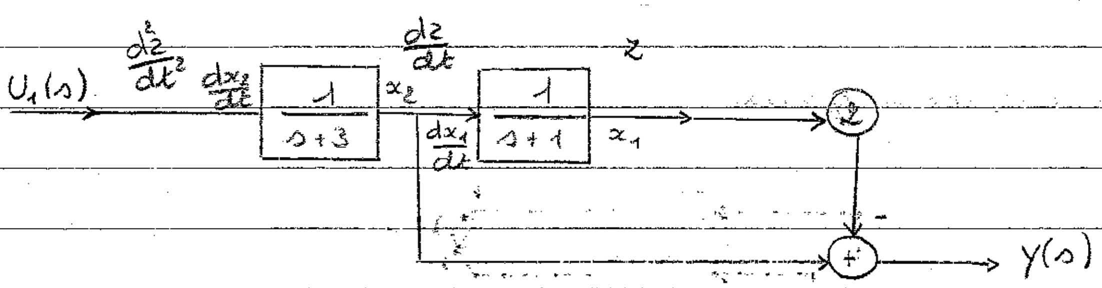
$$= \frac{1}{s}\big(U_1(s) + U_2(s)\big) + U_1(s)$$

Cruciale stap voor minimaal # integrato

* $H_{11}(s) = \dfrac{s-2}{(s+3)(s+1)}$

$$Z(s) = \frac{Y(s)}{s-2} \qquad\qquad Z(s) = \frac{U_1(s)}{s^2+4s+3}$$

$$y(t) = \frac{dz}{dt} - 2z \qquad\qquad u_1(t) = \frac{d^2z}{dt^2} + 4\frac{dz}{dt} + 3z$$



$$\frac{dx_1}{dt} = x_2 \qquad\qquad z = x_1$$

$$y_1 = x_2 - 2x_1$$

$$\frac{dx_2}{dt} = u_1 - 4x_2 - 3x_1$$

* $Y_2(s) = \dfrac{1}{s}\left(U_1(s) + U_2(s)\right) + U_1(s)$

$$\left.\begin{aligned} \frac{dx_3}{dt} &= u_1 + u_2 + 0\,x_1 + 0\,x_2 + 0\cdot x_3 \\ y_2(t) &= x_3 + u_1 \end{aligned}\right\} \text{doit figurer}$$

$$\Rightarrow \quad \frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -3 & -4 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \checkmark$$

$$\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \checkmark$$

* Impulsresponsie

$$h(t) = \mathcal{L}^{-1}\left(H(s)\right]$$

$$H_1(s) = \frac{A(s+1) + B(s+3)}{(s+3)(s+1)} = \frac{(s-2)}{(s+3)(s+1)}$$

$$\begin{cases} s=-1 \rightarrow 2B = -3 \rightarrow B = -3/2 \\ s=-3 \rightarrow -2A = -5 \rightarrow A = 5/2 \end{cases}$$

$$= \frac{5}{2}\frac{1}{s+3} - \frac{3}{2}\frac{1}{s+1}$$

$$h(t) = \begin{bmatrix} \frac{5}{2}e^{-3t} - \frac{3}{2}e^{-t} & 0 \\ 1 + \delta(t) & 1 \end{bmatrix}, \quad t \geq 0 \quad \checkmark$$

* I/O onstabiel (polen nt strikt negatief)

inwendig onstabiel want

$$|sI - A| = \begin{vmatrix} s & -1 & 0 \\ 3 & s+4 & 0 \\ 0 & 0 & s \end{vmatrix} = s^2(s+4)$$

→ EWn nt strikt negatief ✓

~~want~~

$\boxed{\text{OEFENING 2}}$ ✓

1. $$\frac{d}{dt}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} -R_1/L_1 & R_1/L_1 \\ R_1/L_2 & -\frac{(R_1+R_2)}{L_2} \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 1/L_1 \\ 0 \end{pmatrix} u \quad ✓$$

$$i_2 = y = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + 0\,u \quad ✓$$

① $$L_1 \frac{di_1}{dt} + R_1(i_1 - i_2) = u$$

$$\frac{di_1}{dt} = -\frac{R_1}{L_1} i_1 + \frac{R_1}{L_1} i_2 + \frac{u}{L_1}$$

② $$L_2 \frac{di_2}{dt} + R_1(i_2 - i_1) + R_2 i_2 = 0$$

$$\frac{di_2}{dt} = \frac{R_1}{L_2} i_1 - \frac{R_1+R_2}{L_2} i_2$$

2 $$\frac{d}{dt}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} -10 & 10 \\ 5 & -15 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 10 \\ 0 \end{pmatrix} u$$

EWn van A:

$$\begin{vmatrix} s+10 & -10 \\ -5 & s+15 \end{vmatrix} = (s+10)(s+15) - 50 = s^2 + 25s + 100$$

$$D = 625 - 400 = 225 = 15^2$$

$$s_1 = \frac{-25+15}{2} = -5$$

$$s_2 = \frac{-25-15}{2} = -20$$

=> inwendige stabiliteit

=> I/O stabiliteit ✓

3. $e^{At} = ?$

$$\mathcal{L}(e^{At}) = \frac{1}{sI-A} = (sI-A)^{-1} = \frac{\begin{pmatrix} s+15 & 10 \\ 5 & s+10 \end{pmatrix}}{(s+5)(s+20)}$$

$$e^{At} = \mathcal{L}^{-1}\left[\frac{\begin{pmatrix} s+15 & 10 \\ 5 & s+10 \end{pmatrix}}{(s+5)(s+20)}\right] = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$* \; a = \mathcal{L}^{-1}\left(\frac{s+15}{(s+5)(s+20)}\right) = \mathcal{L}^{-1}\left(\frac{A(s+20)+B(s+5)}{(s+5)(s+20)}\right)$$

$$\begin{cases} s=-5 \rightarrow 15A = 10 \rightarrow A = 2/3 \\ s=-20 \rightarrow -15B = -5 \rightarrow B = 1/3 \end{cases}$$

$$= \frac{1}{3}\mathcal{L}^{-1}\left(\frac{2}{s+5} + \frac{1}{s+20}\right)$$

$$* \; b = \mathcal{L}^{-1}\left(\frac{10}{(s+5)(s+20)}\right) = \mathcal{L}^{-1}\left(\frac{A(s+20)+B(s+5)}{(s+5)(s+20)}\right)$$

$$\begin{cases} s=-5 \rightarrow 15A = 10 \rightarrow A = 2/3 \\ s=-20 \rightarrow -15B = 10 \rightarrow B = -2/3 \end{cases}$$

$$= \frac{1}{3}\mathcal{L}^{-1}\left(\frac{2}{s+5} - \frac{2}{s+20}\right)$$

$$* \; c = \mathcal{L}^{-1}\left(\frac{5}{(s+5)(s+20)}\right) \qquad \begin{cases} s=-5 \rightarrow 15A = 5 \rightarrow A = 1/3 \\ s=-20 \rightarrow -15B = 5 \rightarrow B = -1/3 \end{cases}$$

$$= \frac{1}{3}\mathcal{L}^{-1}\left(\frac{1}{s+5} - \frac{1}{s+20}\right)$$

$$* \; d = \mathcal{L}^{-1}\left(\frac{s+10}{(s+5)(s+20)}\right) \qquad \begin{cases} s=-5 \rightarrow 15A = 5 \rightarrow A = 1/3 \\ s=-20 \rightarrow -15B = -10 \rightarrow B = 2/3 \end{cases}$$

$$\Rightarrow e^{At} = \frac{1}{3}\begin{pmatrix} 2e^{-5t} + e^{-20t} & 2e^{-5t} - 2e^{-20t} \\ e^{-5t} - e^{-20t} & e^{-5t} + 2e^{-20t} \end{pmatrix} \checkmark$$

4. $H(s) = \bar{c}^{T} (sI_n - A)^{-1} \bar{b} + d$

$$(sI - A)^{-1} = \begin{pmatrix} s+10 & -10 \\ -5 & s+15 \end{pmatrix}^{-1} = \frac{\begin{pmatrix} s+15 & +5 \\ 10 & s+10 \end{pmatrix}^{T}}{s^2+25s+100}$$

$$H(s) = \begin{pmatrix} 0 & 1 \end{pmatrix} \frac{\begin{pmatrix} s+15 & 10 \\ 5 & s+10 \end{pmatrix}}{s^2+25s+100} \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

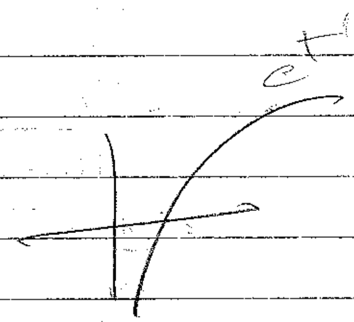
$$= \frac{50}{s^2+25s+100} = \frac{50}{(s+5)(s+20)} \quad \checkmark$$

5. $Y(s) = H(s)\,\frac{10}{s}$

$$= \frac{500}{s(s+5)(s+20)} = \frac{A(s+5)(s+20) + Bs(s+20) + Cs(s+5)}{s(s+5)(s+20)}$$

$$\begin{cases} s=0 \rightarrow 100A = 500 \rightarrow A = 5 \\ s=-5 \rightarrow -75B = 500 \rightarrow B = -20/3 \\ s=-20 \rightarrow 300C = 500 \rightarrow C = 5/3 \end{cases}$$

$$= \frac{5}{s} - \frac{20}{3}\frac{1}{s+5} + \frac{5}{3}\frac{1}{s+20}$$



$$y(t) = i_2(t) = 5 - \frac{20}{3} e^{-5t} + \frac{5}{3} e^{-20t}$$

regimewaarde van y $(t \rightarrow \infty)$ want in opgave staat uiteindelijk: $\lim_{t \to \infty} y(t) = 5$ ✓

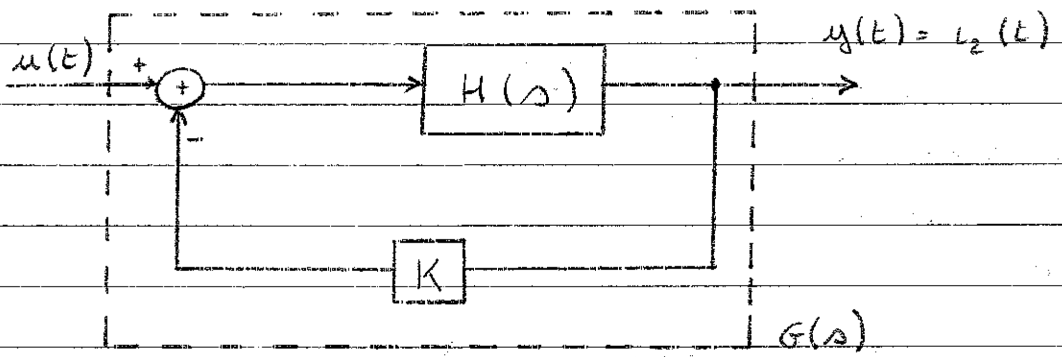
6 Tijdsconstantes van het equivalent 2^e ordesysteem?

2^e ordesysteem: $\dfrac{K}{(1+s\tau_1)(1+s\tau_2)}$

$\rightarrow \tau_1 = \dfrac{1}{5} = 0{,}2$ ✓

$\tau_2 = \dfrac{1}{20} = 0{,}05$ ✓

7.



$I_2(s) = H(s)\left[U(s) - K\,I_2(s)\right]$

$I_2(s) = \underbrace{\dfrac{H(s)}{1+K\,H(s)}}_{G(s)}\;U(s)$ → nieuwe transferfunctie G(s)

H(s) invullen

$G(s) = \dfrac{50}{(s+5)(s+20)+50K} = \dfrac{50}{s^2+25s+100+50K}$

hulpmiddel D berekenen

$D = 625 - 4(100+50K) = 225 - 200K$

$s_{1,2} = \dfrac{-25 \pm \sqrt{225-200K}}{2}$

Canonische vorm van 2 1ste orde-systemen:

opschrijven op formule blad

$$\frac{K}{(1+s\tau_1)(1+s\tau_2)} = \frac{\frac{K}{\tau_1 \tau_2}}{\left(\frac{1}{\tau_1}+s\right)\left(\frac{1}{\tau_2}+s\right)}$$

$$\tau_{1,2} = \frac{2}{25 \pm \sqrt{225-200K}} < 0{,}1$$

$$\tau_{1,2} < 0{,}1$$

* $2 < 0{,}1\left(25 + \sqrt{225-200K}\right)$

$2 < 2{,}5 + 0{,}1\sqrt{225-200K}$

$-5 < \sqrt{225-200K}$

$-200 < -200K$

$1 < K$

* $2 < 0{,}1\left(25 - \sqrt{225-200K}\right)$

$2 < 2{,}5 - 0{,}1\sqrt{225-200K}$

$-5 < -\sqrt{225-200K}$

$25 < -225 + 200K$

$250 < 200K$

$1{,}25 < K$

9

$\Rightarrow \quad 1 < K < 1{,}125$