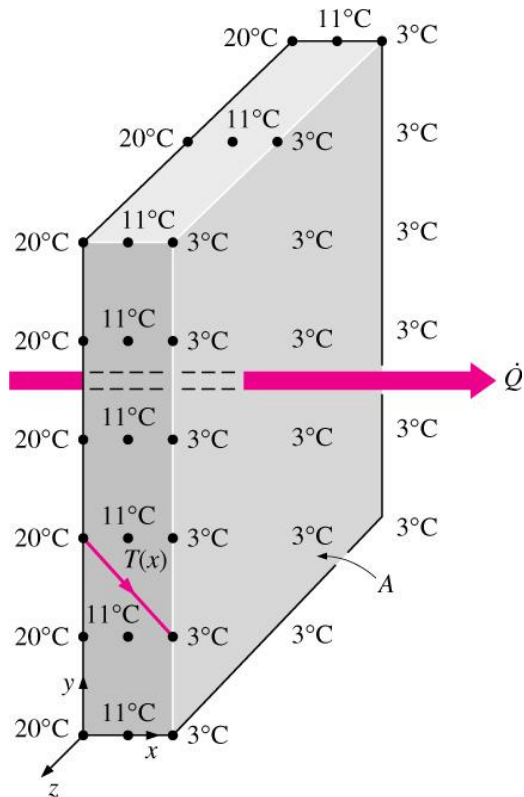


# Hfdst 9: Stationaire warmtegeleiding

1. Stationaire geleiding in een vlakke wand
2. Thermische contactweerstand
3. Algemene thermische weerstandsnetwerken
4. Geleiding in cilinders en sferen
5. Kritische straal voor isolatie
6. Koelvinnen
7. Warmteoverdracht in algemene configuraties

# 1. Stationaire geleiding in een vlakke wand

- Geleiding in een vlakke wand:
  - 1-dimensie: T verandert enkel in 1 richting
  - Stationair (geen opslag van energie):  $dE_{\text{wall}}/dt = 0$



$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt}$$

$$\dot{Q}_{\text{cond, wall}} = -kA \frac{dT}{dx} \quad (\text{W})$$

$$\int_{x=0}^L \dot{Q}_{\text{cond, wall}} dx = - \int_{T=T_1}^{T_2} kA dT$$

$$\dot{Q}_{\text{cond, wall}} = kA \frac{T_1 - T_2}{L} \quad (\text{W})$$

- Thermische weerstand

- Thermische weerstand  $R_{wand}$  van een wand tegen geleiding = geleidingsweerstand.
- Hangt van de geometrie en thermische eigenschappen van het medium af.

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \quad (\text{W})$$

met  $R_{\text{wall}} = \frac{L}{kA} \quad (^\circ\text{C/W})$

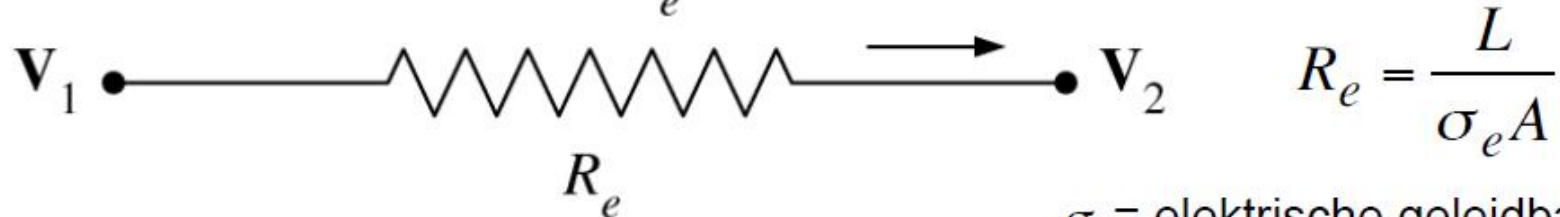
- Elektrisch equivalent van de warmtestroom

$$\dot{Q} = \frac{T_1 - T_2}{R}$$



(a) Heat flow

$$I = \frac{V_1 - V_2}{R_e}$$

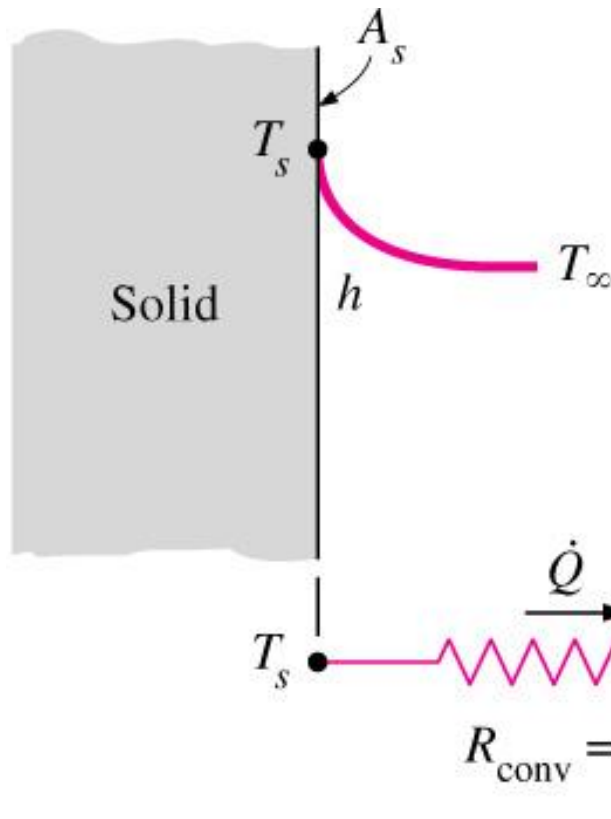


$$R_e = \frac{L}{\sigma_e A}$$

$\sigma_e$  = elektrische geleidbaarheid

(b) Electric current flow

## • Convectieweerstand



$$\dot{Q}_{\text{conv}} = hA_s (T_s - T_\infty) = \frac{T_s - T_\infty}{R_{\text{conv}}} \quad (\text{W})$$

$$R_{\text{conv}} = \frac{1}{hA_s} \quad (^\circ\text{C/W})$$

Als de warmteoverdrachtscoëfficiënt zeer groot is ( $h \rightarrow \infty$ ), dan wordt de convectieweerstand nul en  $T_s \approx T_\infty$ .

M.a.w. er is *geen weerstand voor convectie* en de warmteoverdracht wordt niet afgeremd.

Deze situatie wordt in de praktijk benaderd op oppervlakken waar **koken** of **condensatie** plaatsgrijpt.

- Wand omgeven door gas: straling niet verwaarlozen

$$\dot{Q}_{\text{rad}} = \varepsilon \sigma A_s (T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}} A_s (T_s - T_{\text{surr}}) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}}$$

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_s} \quad (\text{K/W})$$

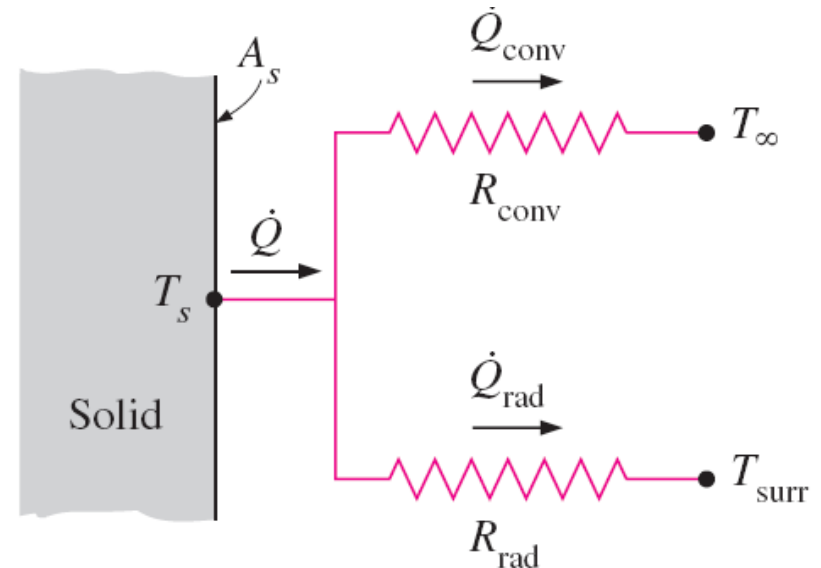
**Stralingsweerstand aan de oppervlakte:**  
*Thermische weerstand van het opp. tegen straling*

$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s (T_s - T_{\text{surr}})} = \varepsilon \sigma (T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \quad (\text{W/m}^2 \cdot \text{K})$$

Stralingswarmteoverdrachtscoëfficiënt  $h_{\text{rad}}$

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}}$$

Gecombineerde  
warmteoverdrachtscoëff.



$$\dot{Q} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \quad 6$$

- Netwerk van thermische weerstanden:  
wand met externe convectie

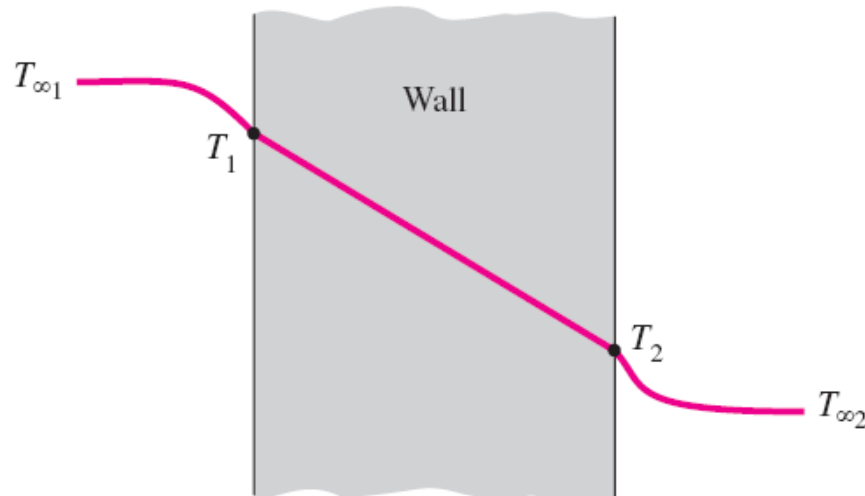
$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA \frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

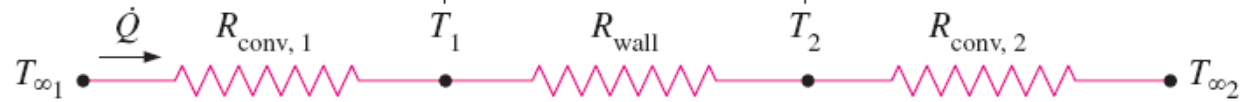
$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv}, 2}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \quad (\text{W})$$

$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{wall}} + R_{\text{conv}, 2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad (^\circ\text{C/W})$$

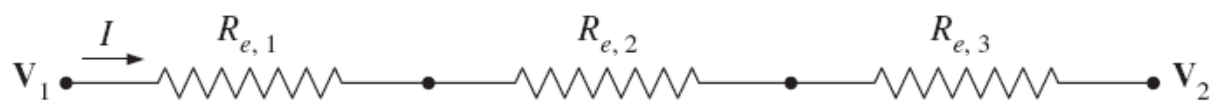


$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv}, 1} + R_{\text{wall}} + R_{\text{conv}, 2}}$$



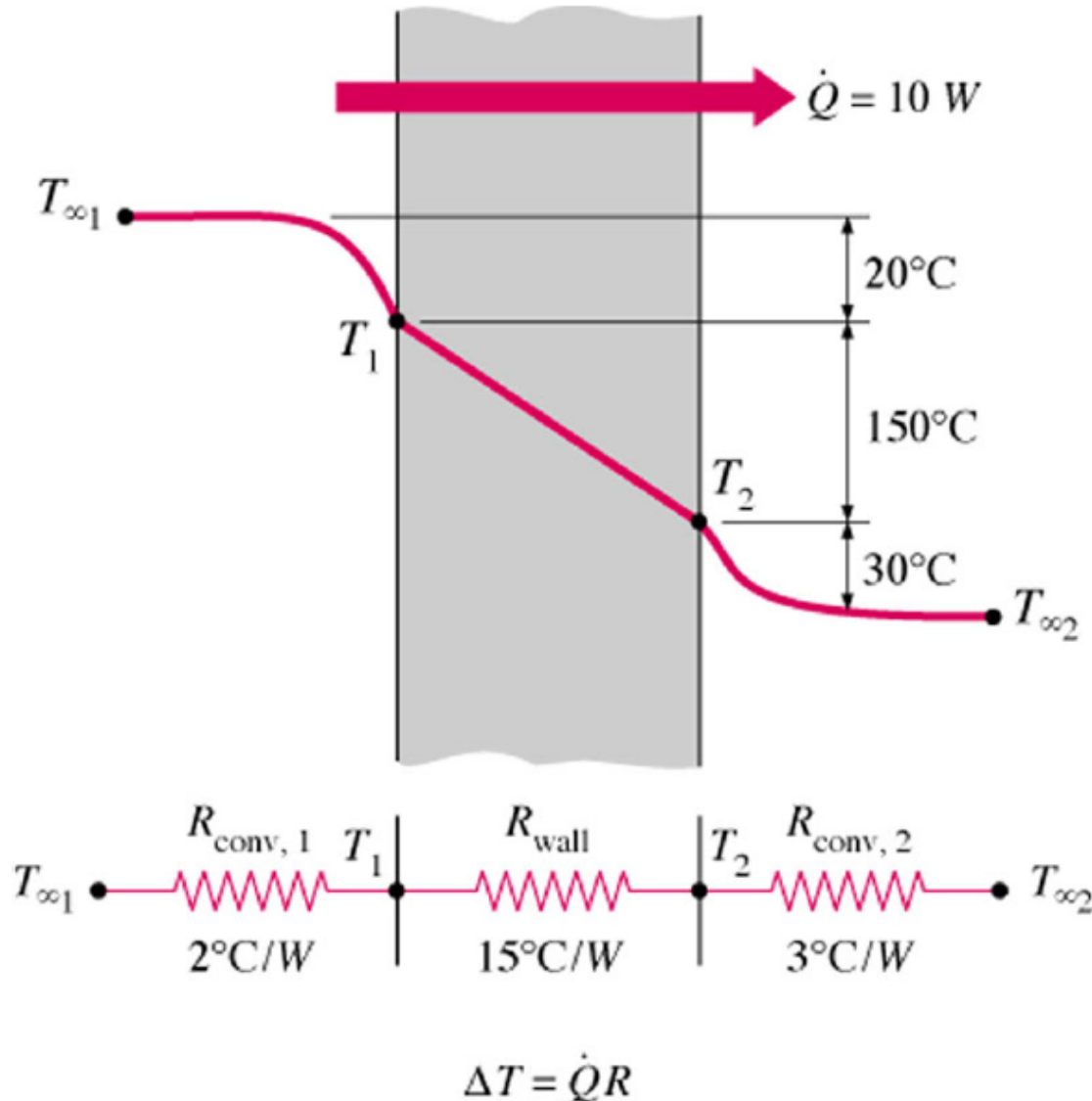
Thermal network

$$I = \frac{V_1 - V_2}{R_{e, 1} + R_{e, 2} + R_{e, 3}}$$



Electrical analogy

- **Voorbeeld:** netwerk van thermische weerstanden met aan de wand externe convectie



Als  $\dot{Q}$  gekend is, dan kan de oppervlaktetemperatuur  $T_1$  bepaald worden met:

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$

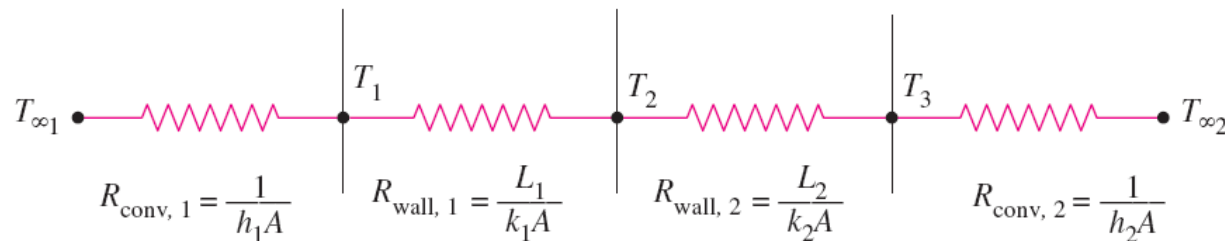
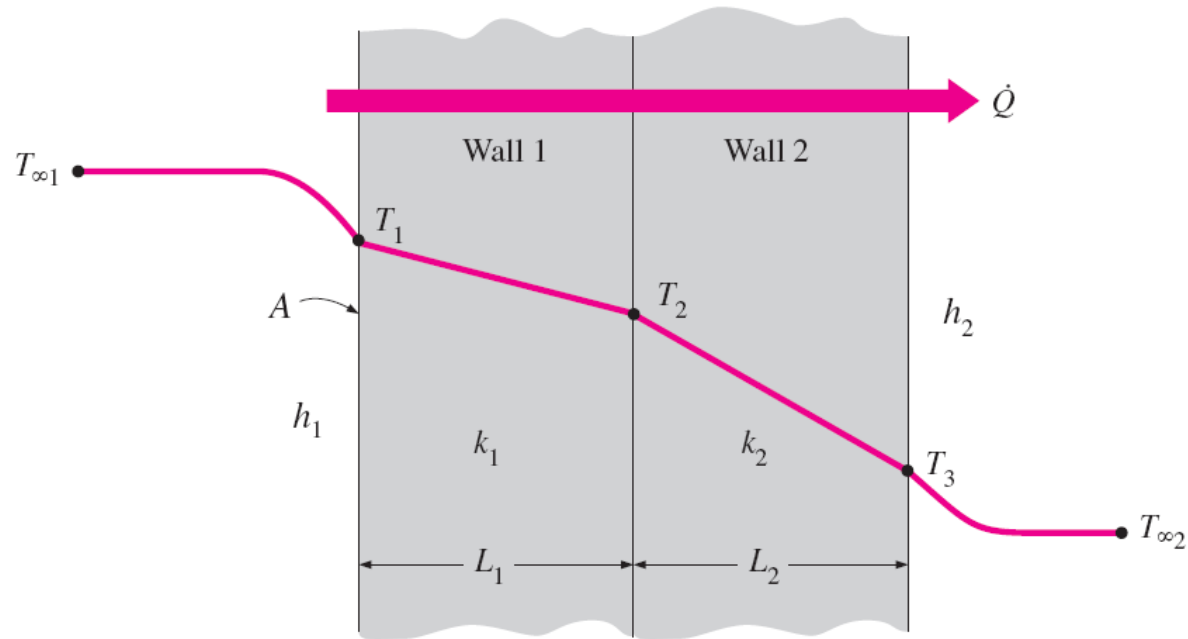
$$\dot{Q} = UA \Delta T \quad (\text{W})$$

$$UA = \frac{1}{R_{\text{totaal}}}$$

$U$  de totale  
warmteoverdrachts-  
coëfficiënt



- Netwerk van thermische weerstanden:  
**samengestelde wand**



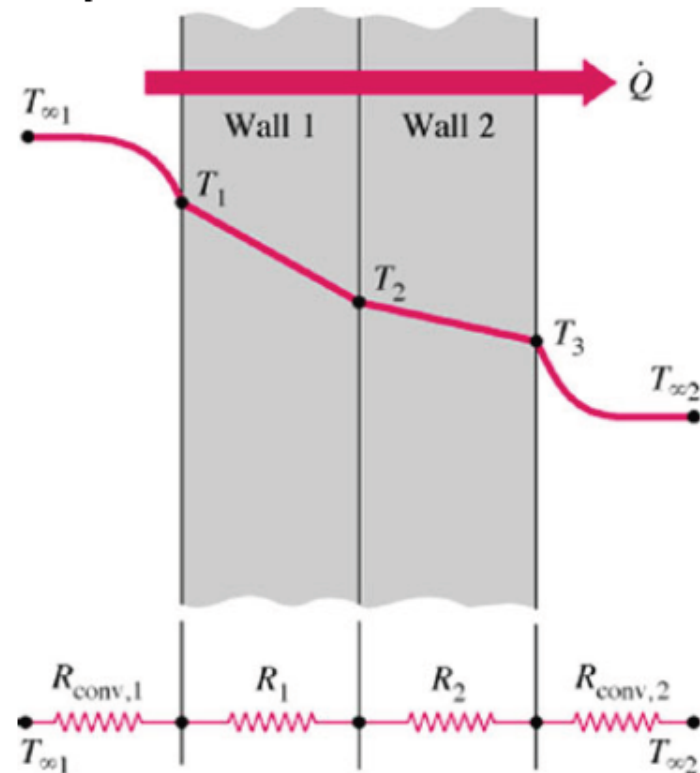
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv}, 1} + R_{\text{wall}, 1} + R_{\text{wall}, 2} + R_{\text{conv}, 2} \\ &= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \end{aligned}$$

- **Netwerk van thermische weerstanden:** *samengestelde wand*
- Bepalen van intermediaire temperaturen

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total}, i-j}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv}, 1} + R_{\text{wall}, 1}} = \frac{T_{\infty 1} - T_2}{\frac{1}{h_1 A} + \frac{L_1}{k_1 A}}$$

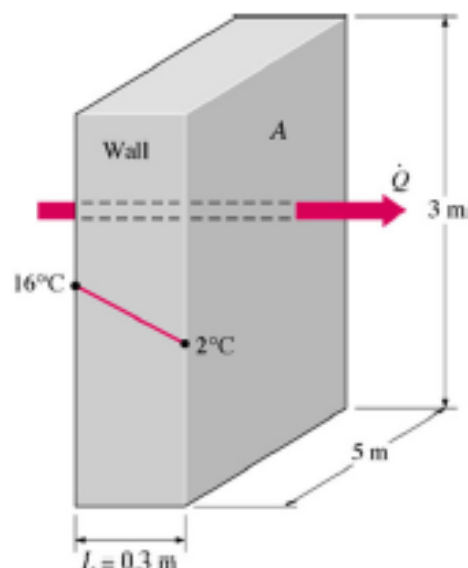


$$\text{To find } T_1: \dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}}$$

$$\text{To find } T_2: \dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv}, 1} + R_1}$$

$$\text{To find } T_3: \dot{Q} = \frac{T_3 - T_{\infty 2}}{R_{\text{conv}, 2}}$$

- Voorbeeld 1: Warmteverlies door een wand



$$k = 0.9 \text{ W/m } ^\circ\text{C}$$

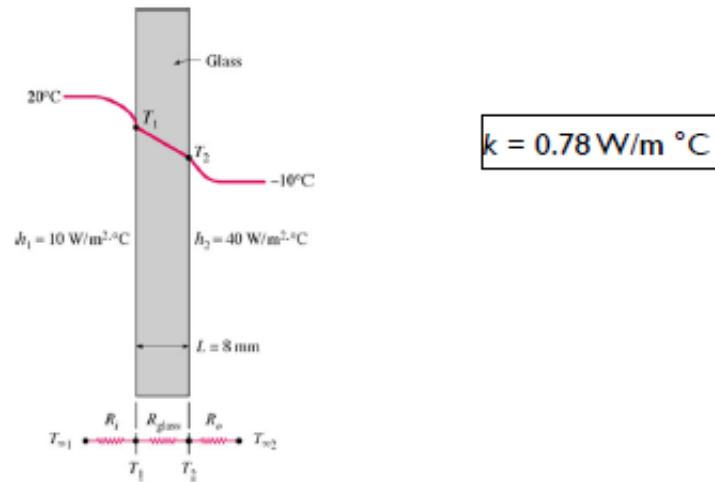
Bepaal het warmtedebiet door de wand dat verloren gaat.

$$\dot{Q} = kA \frac{T_1 - T_2}{L} = (0.9 \text{ W/m} \cdot ^\circ\text{C})(15 \text{ m}^2) \frac{(16 - 2)^\circ\text{C}}{0.3 \text{ m}} = \mathbf{630 \text{ W}}$$

$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}} \quad R_{\text{wall}} = \frac{L}{kA} = \frac{0.3 \text{ m}}{(0.9 \text{ W/m} \cdot ^\circ\text{C})(15 \text{ m}^2)} = 0.02222^\circ\text{C/W}$$

$$\dot{Q} = \frac{(16 - 2)^\circ\text{C}}{0.02222^\circ\text{C/W}} = 630 \text{ W}$$

• Voorbeeld 2: Warmteverlies door een enkele glazen wand



Bepaal: (a) het warmtedebit door het glasraam, en (b) de temperatuur  $T_1$ .

$$(a) \quad R_i = R_{\text{conv}, 1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m} \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.00855^\circ\text{C/W}$$

$$R_o = R_{\text{conv}, 2} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.02083^\circ\text{C/W}$$

$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{glass}} + R_{\text{conv}, 2} = 0.08333 + 0.00855 + 0.02083 = 0.1127^\circ\text{C/W}$$

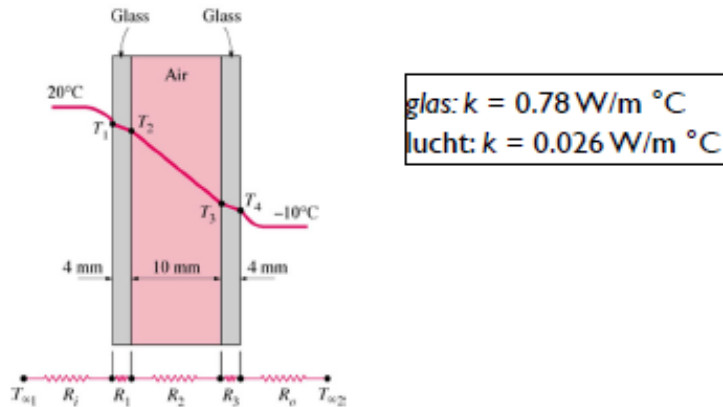
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.1127^\circ\text{C/W}} = 266 \text{ W}$$

$$(b) \quad \dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} \quad \longrightarrow \quad T_1 = T_{\infty 1} - \dot{Q} R_{\text{conv}, 1}$$

$$= 20^\circ\text{C} - (266 \text{ W})(0.08333^\circ\text{C/W})$$

$$= -2.2^\circ\text{C}$$

- Voorbeeld 3: Warmteverlies door een dubbele glazen wand



Bepaal: (a) het warmtedebiet door het glasraam, en (b) de temperatuur  $T_1$ .

(a)  $R_i = R_{\text{conv}, 1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.08333^\circ\text{C/W}$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004 \text{ m}}{(0.78 \text{ W/m} \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.00427^\circ\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01 \text{ m}}{(0.026 \text{ W/m} \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.3205^\circ\text{C/W}$$

$$R_o = R_{\text{conv}, 2} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.02083^\circ\text{C/W}$$

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv}, 1} + R_{\text{glass}, 1} + R_{\text{air}} + R_{\text{glass}, 2} + R_{\text{conv}, 2} \\ &= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 \\ &= 0.4332^\circ\text{C/W} \end{aligned}$$

(b)  $\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.4332^\circ\text{C/W}} = \mathbf{69.2 \text{ W}}$

$$T_1 = T_{\infty 1} - \dot{Q} R_{\text{conv}, 1} = 20^\circ\text{C} - (69.2 \text{ W})(0.08333^\circ\text{C/W}) = \mathbf{14.2^\circ\text{C}}$$

## 2. Thermische contactweerstand

- Warmteoverdracht door vaste contactpunten en door luchtopeningen:

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}}$$

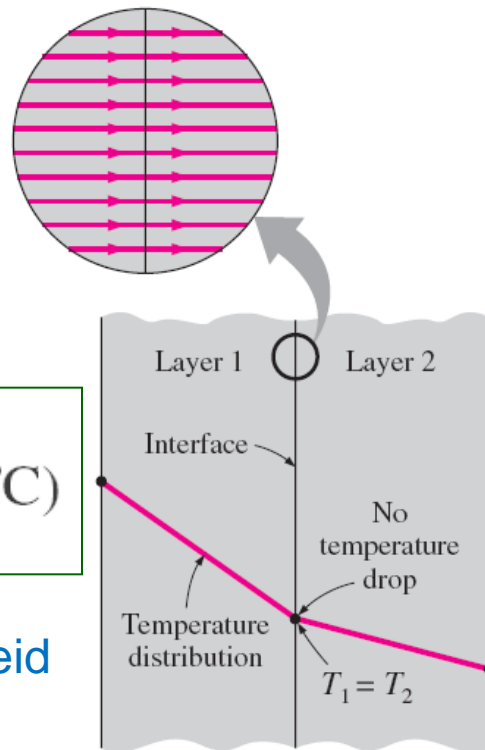
$$\dot{Q} = h_c A \Delta T_{\text{interface}}$$

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \quad (\text{W/m}^2 \cdot ^\circ\text{C})$$

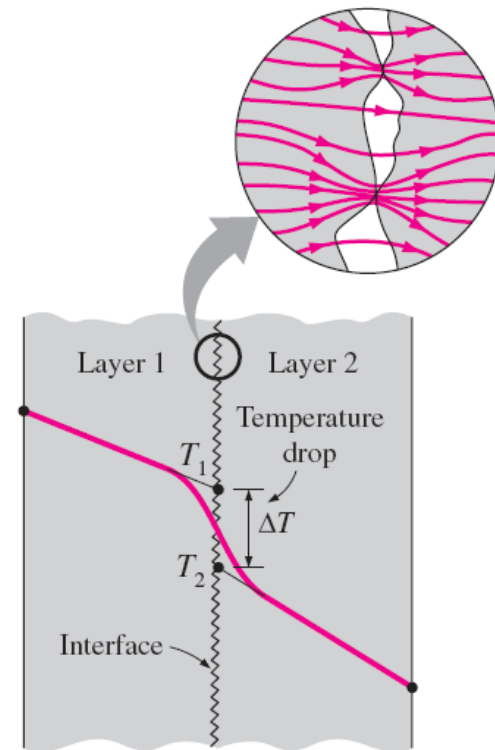
$h_c$  : thermische contactgeleidbaarheid

$R_c$  : thermische contactweerstand

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \quad (\text{m}^2 \cdot ^\circ\text{C/W})$$



(a) Ideal (perfect) thermal contact

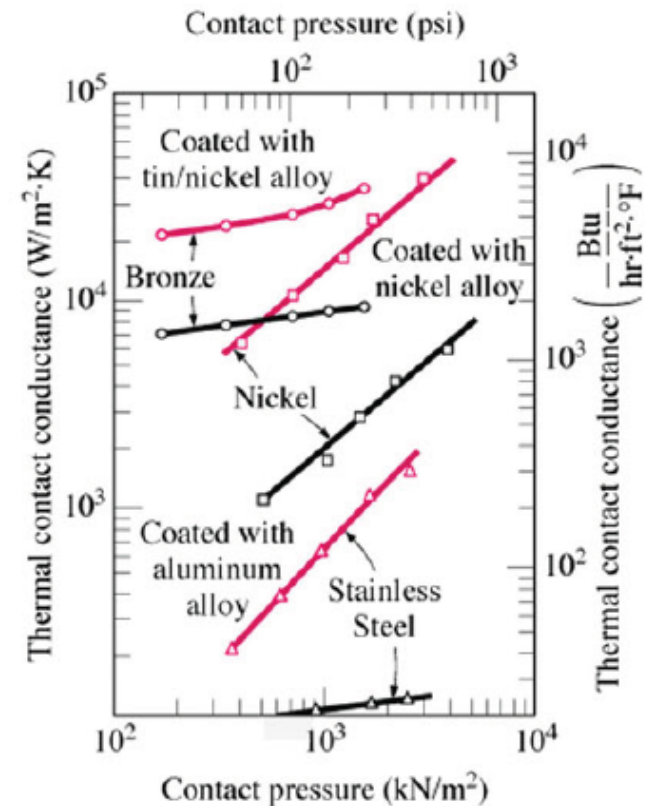


(b) Actual (imperfect) thermal contact

- Verminderen van de thermische contactweerstand
  - Gebruik van een *thermisch geleidende vloeistof* (thermisch vet).
  - *Zachte metalen* (tin, zilver, koper, nikkel, aluminium) *folie* tussen de 2 oppervlakken steken.
  - *Beter geleidend gas* zoals helium of waterstof gebruiken.

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of  $10\ \mu\text{m}$  and interface pressure of 1 atm (from Fried, 1969).

Fluid at the interface	Contact conductance, $h_c$ , $\text{W/m}^2 \cdot \text{K}$
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700



Metallische coating:



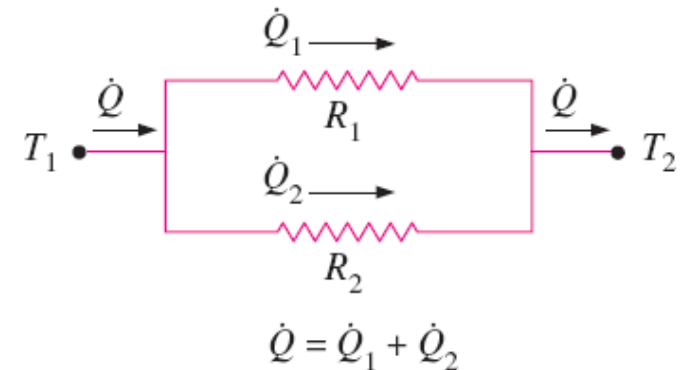
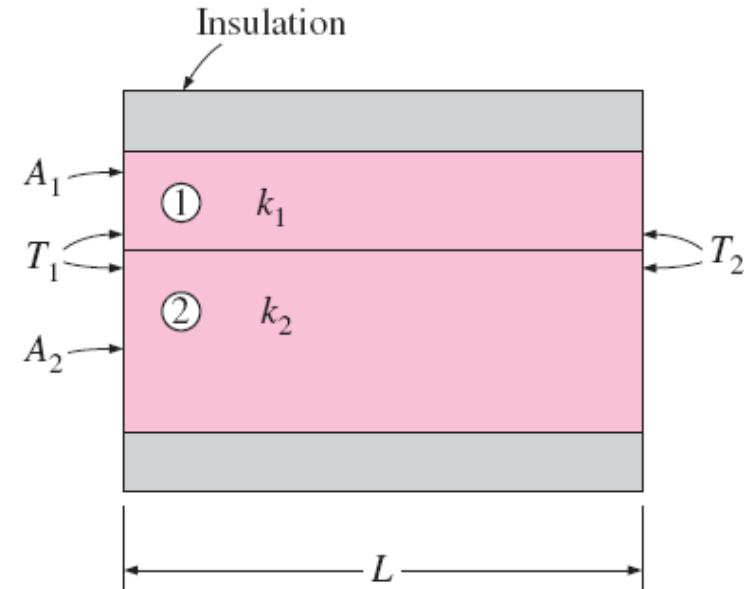
# 3. Algemene thermische weerstandsnetwerken

- Parallele lagen

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}}$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$





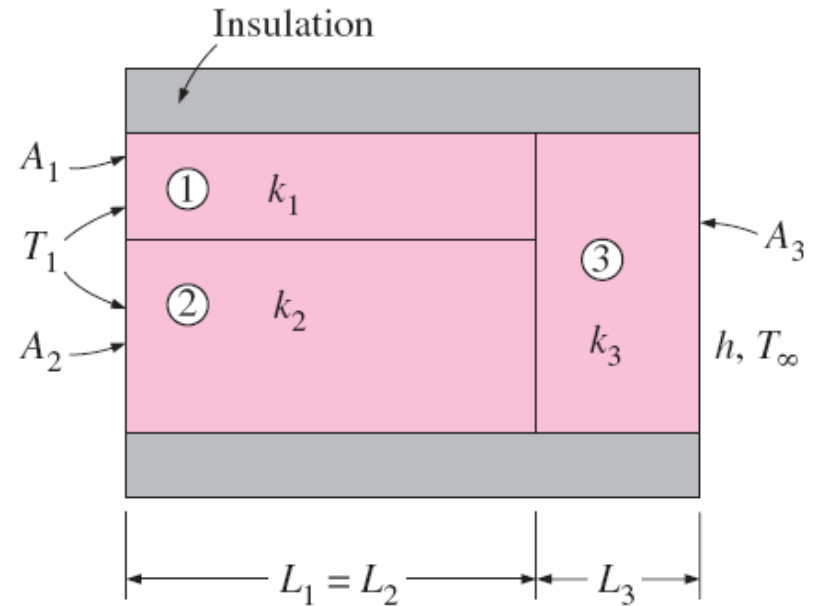
# • Combinatie serie en parallel

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}}$$

$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}}$$

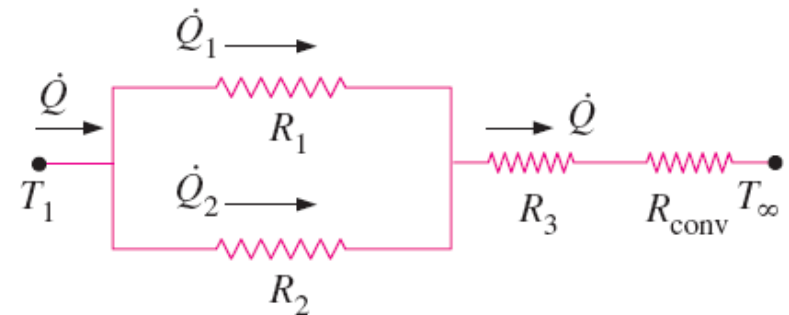
$$R_1 = \frac{L_1}{k_1 A_1} \quad R_2 = \frac{L_2}{k_2 A_2}$$

$$R_3 = \frac{L_3}{k_3 A_3} \quad R_{\text{conv}} = \frac{1}{h A_3}$$

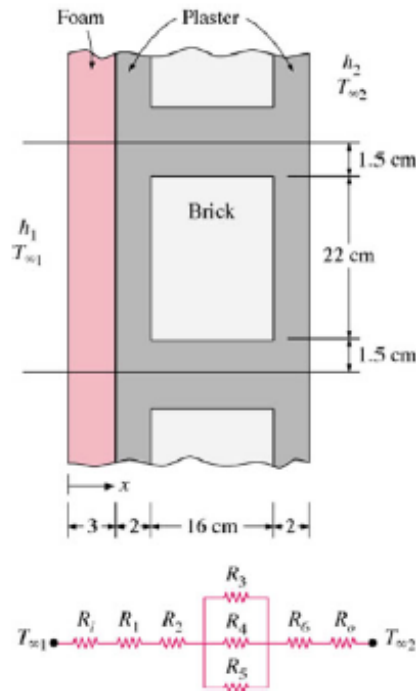


## Twee methodes:

- (1) Elke muur loodrecht op de x-as is *isothermisch* (i.e.,  $T$  varieert enkel in de x-richting)
- (2) Elke muur parallel met de x-as is *adiabatisch* (i.e., warmtetransport gebeurt enkel in de x-richting)



- Voorbeeld:** warmteverlies door een komposietwand



$$R_i = R_{\text{conv}, 1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot ^\circ\text{C})(0.25 \times 1 \text{ m}^2)} = 0.4^\circ\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03 \text{ m}}{(0.026 \text{ W/m} \cdot ^\circ\text{C})(0.25 \times 1 \text{ m}^2)} = 4.6^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.22 \text{ W/m} \cdot ^\circ\text{C})(0.25 \times 1 \text{ m}^2)} = 0.36^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.22 \text{ W/m} \cdot ^\circ\text{C})(0.015 \times 1 \text{ m}^2)} = 48.48^\circ\text{C/W}$$

$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.72 \text{ W/m} \cdot ^\circ\text{C})(0.22 \times 1 \text{ m}^2)} = 1.01^\circ\text{C/W}$$

$$R_o = R_{\text{conv}, 2} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2 \cdot ^\circ\text{C})(0.25 \times 1 \text{ m}^2)} = 0.16^\circ\text{C/W}$$

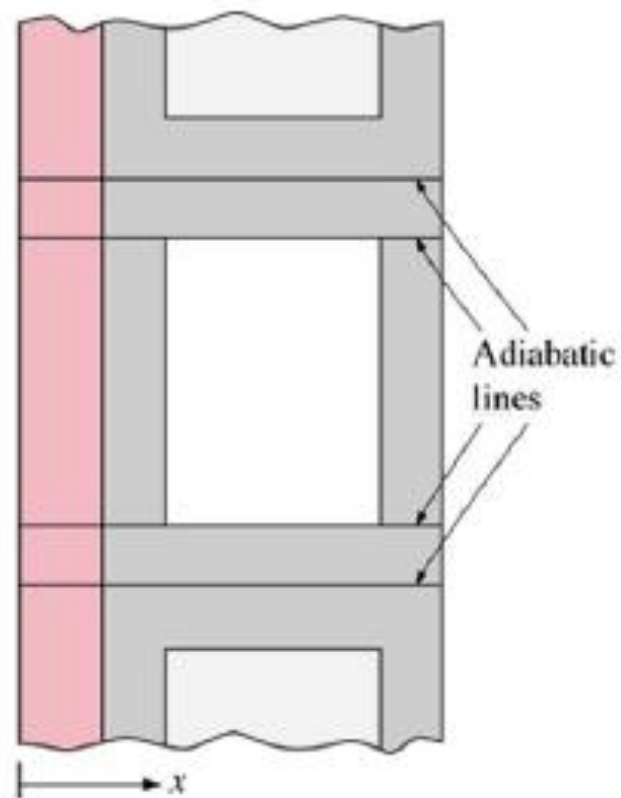
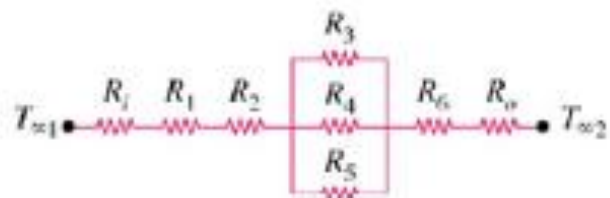
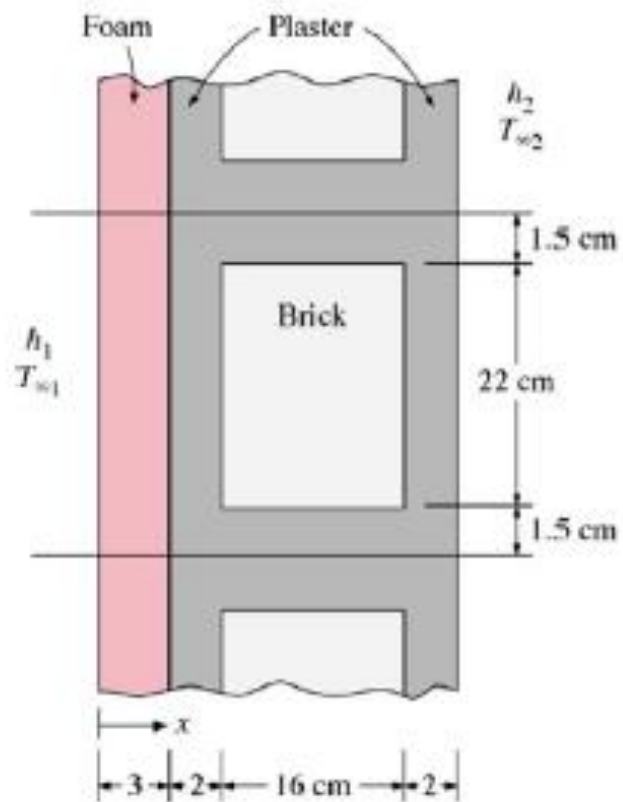
$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03 \text{ W/}^\circ\text{C}$$

$$R_{\text{mid}} = 0.97^\circ\text{C/W}$$

$$\begin{aligned} R_{\text{total}} &= R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o \\ &= 0.4 + 4.6 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.85^\circ\text{C/W} \end{aligned}$$

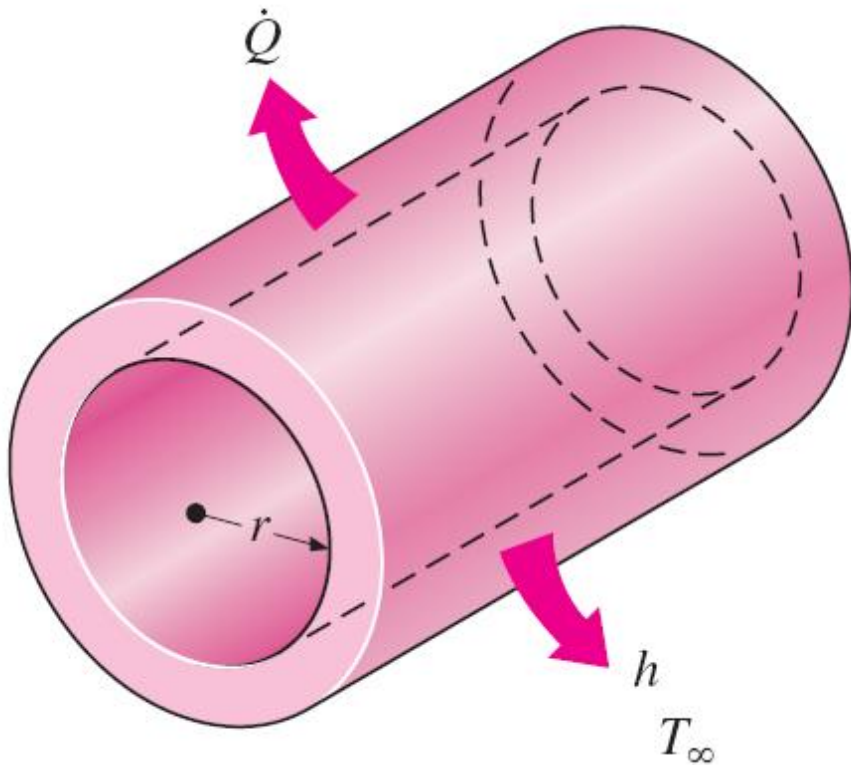
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{6.85^\circ\text{C/W}} = 4.38 \text{ W} \quad (\text{per } 0.25 \text{ m}^2 \text{ surface area})$$

$$\dot{Q}_{\text{total}} = (17.5 \text{ W/m}^2)(15 \text{ m}^2) = \mathbf{263 \text{ W}}$$



# 4. Geleiding in cilinders en sferen

- **Cilinder**

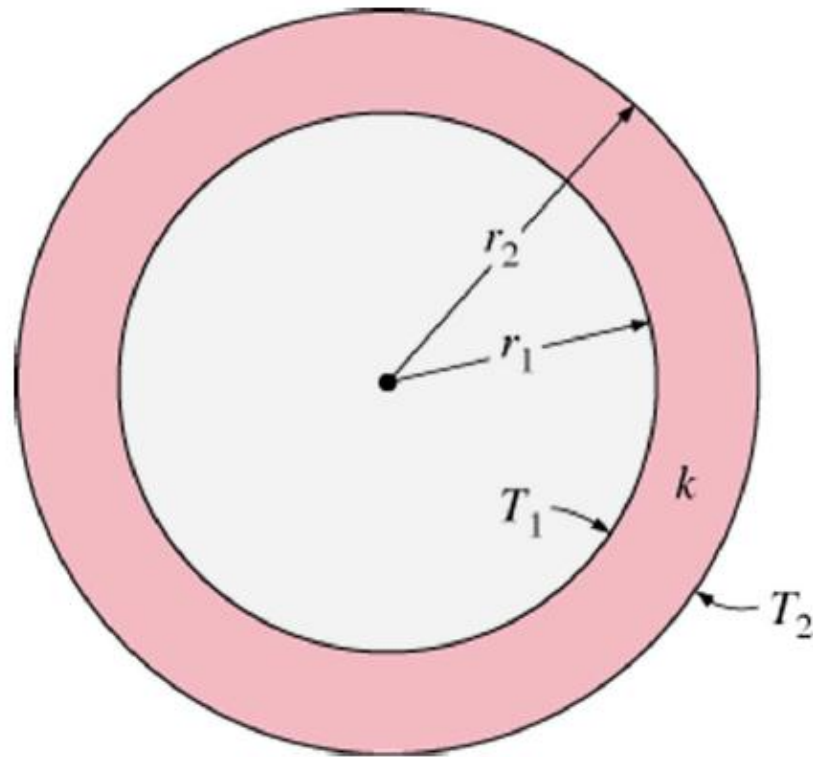


Warmteoverdracht door een buis kan gemodelleerd worden als **stationair** en **1-dimensionaal**.

De temperatuur hangt enkel van 1 richting af (de radiale  $r$ -richting) en kan uitgedrukt worden als  **$T = T(r)$** .

Deze situatie wordt in de praktijk benaderd in lange cilindrische buizen en sferische containers.

## Cilindrische wand:



$$\dot{Q}_{\text{cond, cyl}} = -kA \frac{dT}{dr} \quad (\text{W})$$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A} dr = - \int_{T=T_1}^{T_2} k dT$$

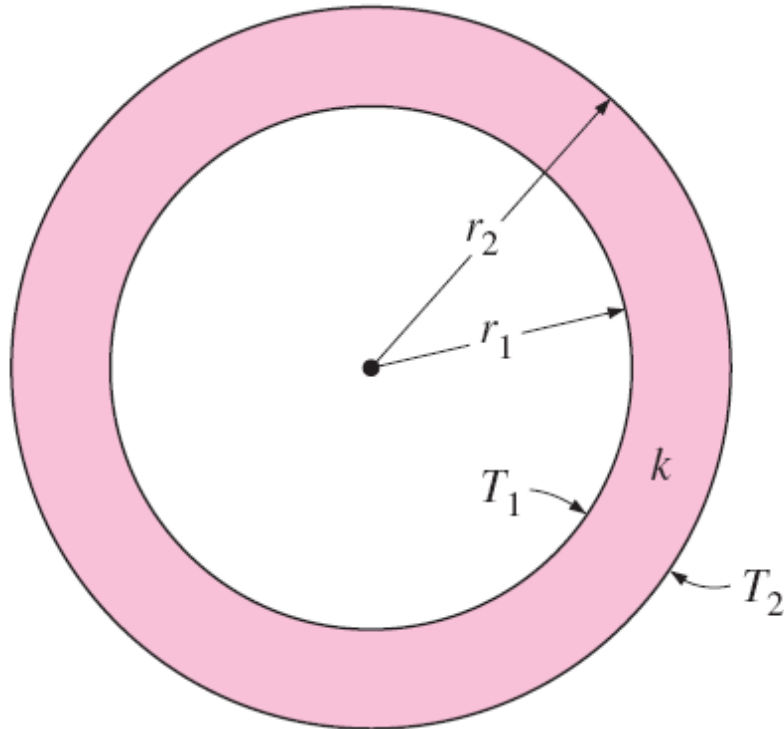
$$A = 2\pi rL$$

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk \frac{T_1 - T_2}{\ln(r_2/r_1)} \quad (\text{W})$$

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \quad (\text{W})$$

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}}$$

- **Bolvormige wand**

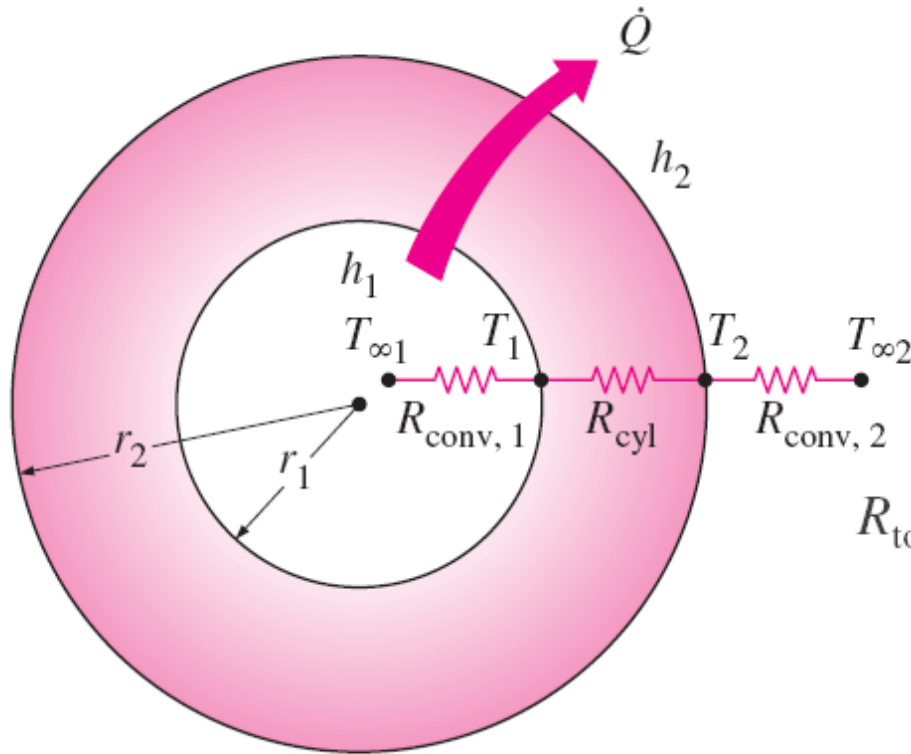


$$A = 4 \pi r^2$$

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}}$$

$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

- Geleiding en convectie in cilindrische of bolvormige lagen



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

*Cilindrische lagen:*

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv}, 1} + R_{\text{cyl}} + R_{\text{conv}, 2} \\ &= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{(2\pi r_2 L)h_2} \end{aligned}$$

*Bolvormige lagen:*

$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{cyl}} + R_{\text{conv}, 2}$$

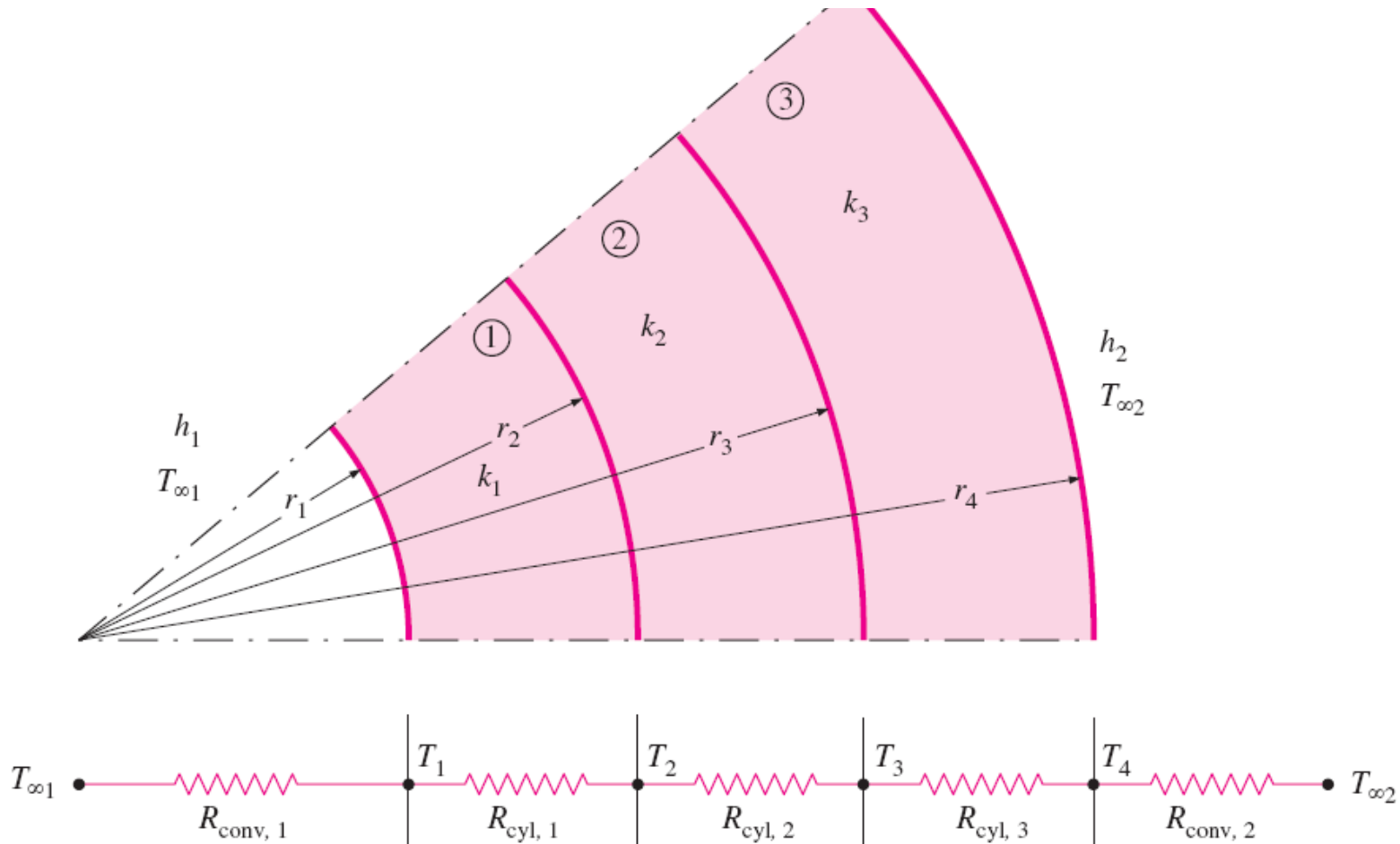
$$\begin{aligned} R_{\text{total}} &= R_{\text{conv}, 1} + R_{\text{sph}} + R_{\text{conv}, 2} \\ &= \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2} \end{aligned}$$

- Meerdere cilindrische of bolvormige lagen

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

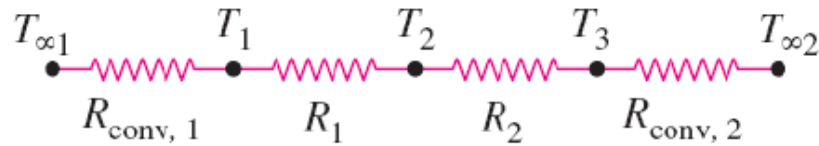
$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{cyl}, 1} + R_{\text{cyl}, 2} + R_{\text{cyl}, 3} + R_{\text{conv}, 2}$$

$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$





# • Berekenen van intermediaire temperaturen



De verhouding  $\Delta T/R$  over elke laag is gelijk aan  $\dot{Q}$  en blijft constant in 1D stationaire geleiding.

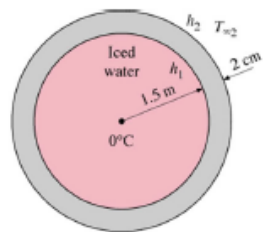
$$\begin{aligned}\dot{Q} &= \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} \\ &= \frac{T_{\infty 1} - T_2}{R_{\text{conv}, 1} + R_1} \\ &= \frac{T_1 - T_3}{R_1 + R_2} \\ &= \frac{T_2 - T_3}{R_2} \\ &= \frac{T_2 - T_{\infty 2}}{R_2 + R_{\text{conv}, 2}} \\ &= \dots\end{aligned}$$

Bvb. bepalen van  $T_2$ :

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv}, 1} + R_{\text{cyl}, 1}} = \frac{T_{\infty 1} - T_2}{\frac{1}{h_1(2\pi r_1 L)} + \frac{\ln(r_2/r_1)}{2\pi L k_1}}$$

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{\text{conv}, 2}} = \frac{T_2 - T_{\infty 2}}{\frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_o(2\pi r_4 L)}}$$

# • Voorbeeld 1: bolvormige container



Roestvrij stalen wand:  $k = 15 \text{ W/m} \cdot ^\circ\text{C}$

$T_{\infty 1} = 0^\circ\text{C}$

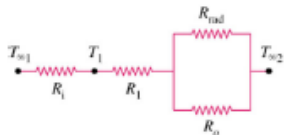
$T_{\infty 2} = 22^\circ\text{C}$

Temperatuur van de wanden van de kamer =  $22^\circ\text{C}$

Buitenoppervlak van de tank is zwart

$h_1 = 80 \text{ W/m}^2 \cdot ^\circ\text{C}$

$h_2 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$



Bepaal (a) het warmtedebiet dat overgedragen wordt naar het ijswater, (b) de hoeveelheid ijs die smelt gedurende 25 uur.

(a)  $A_1 = \pi D_1^2 = \pi(3 \text{ m})^2 = 28.3 \text{ m}^2$

$A_2 = \pi D_2^2 = \pi(3.04 \text{ m})^2 = 29.0 \text{ m}^2$

$h_{\text{rad}} = \varepsilon \sigma (T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2}) \Rightarrow \text{vgl. 17-11}$

$h_{\text{rad}} = (1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(295 \text{ K})^2 + (278 \text{ K})^2][(295 + 278) \text{ K}]$   
 $= 5.34 \text{ W/m}^2 \cdot \text{K} = 5.34 \text{ W/m}^2 \cdot ^\circ\text{C}$

$T_2$  is niet gekend  $\Rightarrow$  schat de waarde, controleer na de berekening en itereer indien nodig

$R_i = R_{\text{conv}, 1} = \frac{1}{h_1 A_1} = \frac{1}{(80 \text{ W/m}^2 \cdot ^\circ\text{C})(28.3 \text{ m}^2)} = 0.000442^\circ\text{C/W}$

$R_1 = R_{\text{sphere}} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.50) \text{ m}}{4\pi (15 \text{ W/m} \cdot ^\circ\text{C})(1.52 \text{ m})(1.50 \text{ m})}$   
 $= 0.000047^\circ\text{C/W}$

$R_o = R_{\text{conv}, 2} = \frac{1}{h_2 A_2} = \frac{1}{(10 \text{ W/m}^2 \cdot ^\circ\text{C})(29.0 \text{ m}^2)} = 0.00345^\circ\text{C/W}$

$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_2} = \frac{1}{(5.34 \text{ W/m}^2 \cdot ^\circ\text{C})(29.0 \text{ m}^2)} = 0.00646^\circ\text{C/W}$

$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_o} + \frac{1}{R_{\text{rad}}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7 \text{ W/}^\circ\text{C}$

$R_{\text{equiv}} = 0.00225^\circ\text{C/W}$   $R_{\text{total}} = R_i + R_1 + R_{\text{equiv}} = 0.000442 + 0.000047 + 0.00225 = 0.00274^\circ\text{C/W}$

$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{\text{total}}} = \frac{(22 - 0)^\circ\text{C}}{0.00274^\circ\text{C/W}} = 8029 \text{ W}$  (or  $\dot{Q} = 8.027 \text{ kJ/s}$ )

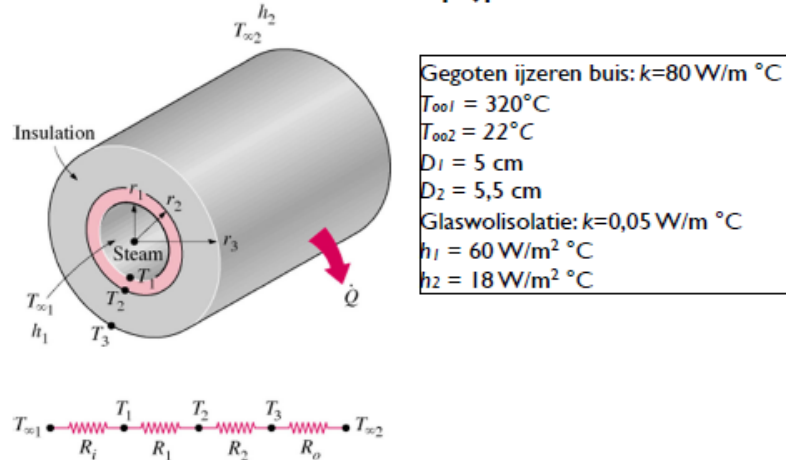
$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{\text{equiv}}} \longrightarrow T_2 = T_{\infty 2} - \dot{Q} R_{\text{equiv}}$   
 $= 22^\circ\text{C} - (8029 \text{ W})(0.00225^\circ\text{C/W}) = 4^\circ\text{C}$

$Q = \dot{Q} \Delta t = (8.029 \text{ kJ/s})(24 \times 3600 \text{ s}) = 673,700 \text{ kJ}$

$m_{\text{ice}} = \frac{Q}{h_{\text{if}}} = \frac{673,700 \text{ kJ}}{333.7 \text{ kJ/kg}} = 2079 \text{ kg}$

$T_2$  bepalen  $\Rightarrow$  iteratie is niet nodig omdat  $4^\circ\text{C}$  ligt bij de gekozen waarde van  $5^\circ\text{C}$  ligt

- Voorbeeld 2: stoompijp



Bepaal: (a) het warmtedebiet per eenheidslengte, en (b) de temperatuursdaling over de buiswand en de isolatie.

(a)

$$A_1 = 2\pi r_1 L = 2\pi(0.025 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2$$

$$A_3 = 2\pi r_3 L = 2\pi(0.0575 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2$$

$$R_i = R_{\text{conv}, 1} = \frac{1}{h_1 A} = \frac{1}{(60 \text{ W/m}^2 \cdot ^\circ\text{C})(0.157 \text{ m}^2)} = 0.106^\circ\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80 \text{ W/m} \cdot ^\circ\text{C})(1 \text{ m})} = 0.0002^\circ\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05 \text{ W/m} \cdot ^\circ\text{C})(1 \text{ m})} = 2.35^\circ\text{C/W}$$

$$R_o = R_{\text{conv}, 2} = \frac{1}{h_2 A_3} = \frac{1}{(18 \text{ W/m}^2 \cdot ^\circ\text{C})(0.361 \text{ m}^2)} = 0.154^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)^\circ\text{C}}{2.61^\circ\text{C/W}} = \mathbf{121 \text{ W}} \quad (\text{per m pipe length})$$

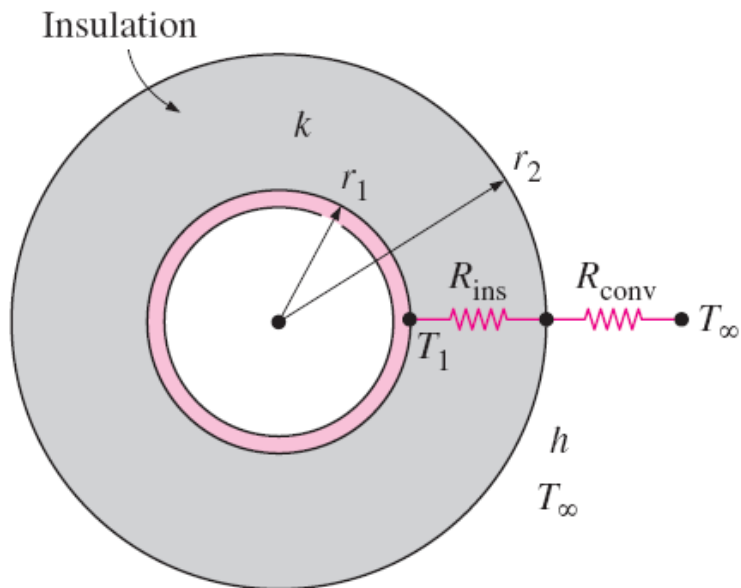
(b)

$$\Delta T_{\text{pipe}} = \dot{Q} R_{\text{pipe}} = (121 \text{ W})(0.0002^\circ\text{C/W}) = \mathbf{0.02^\circ\text{C}}$$

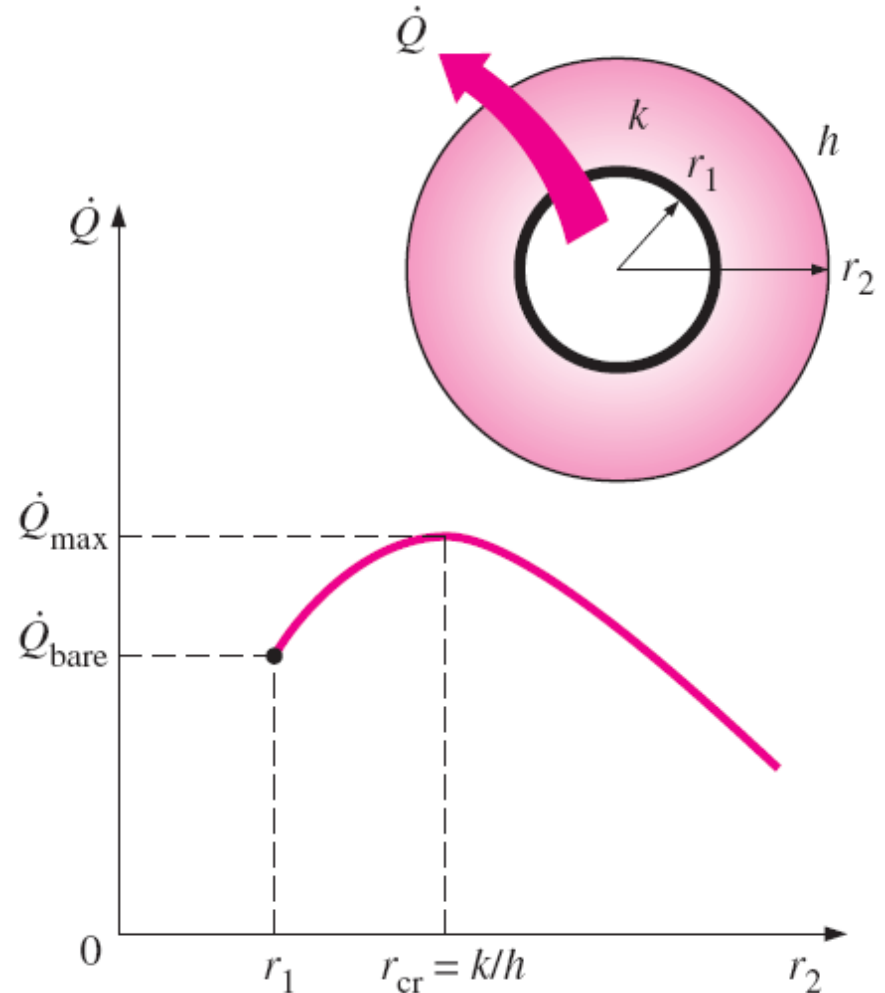
$$\Delta T_{\text{insulation}} = \dot{Q} R_{\text{insulation}} = (121 \text{ W})(2.35^\circ\text{C/W}) = \mathbf{284^\circ\text{C}}$$

# 5. Kritische straal voor isolatie

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{h(2\pi r_2 L)}}$$



$$r_{\text{cr, cylinder}} = \frac{k}{h} \quad (\text{m})$$

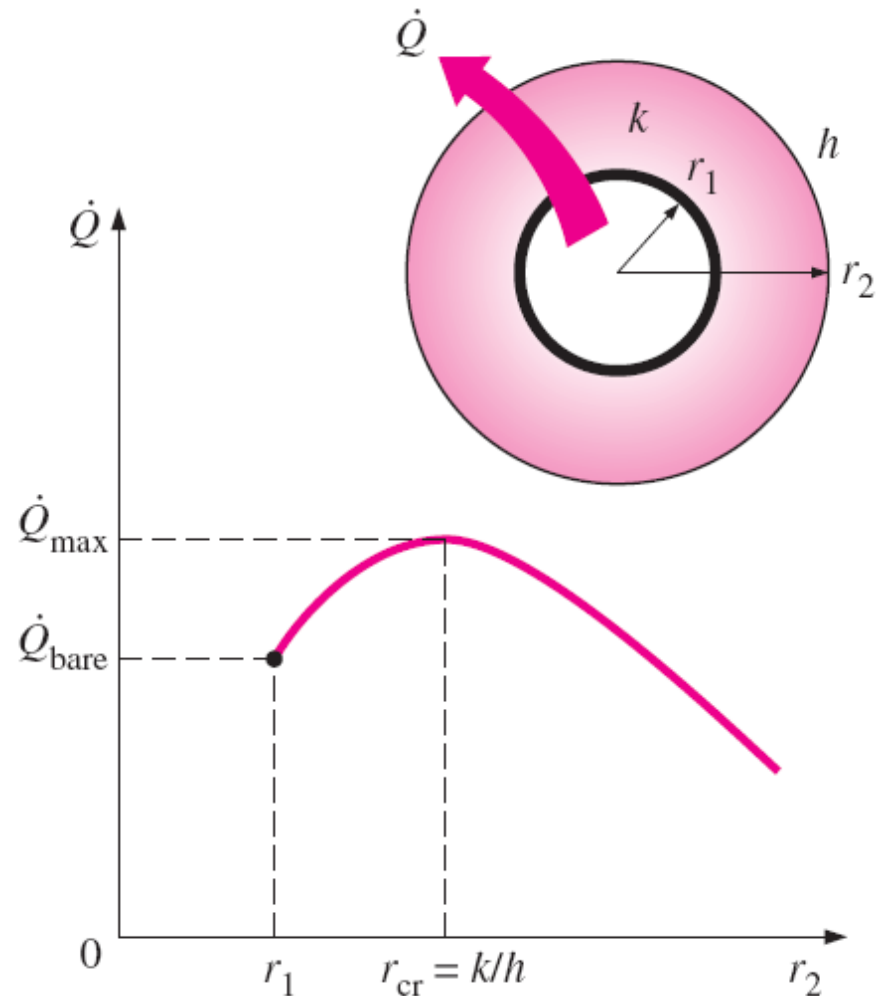


$$r_{\text{cr, cylinder}} = \frac{k}{h} \quad (\text{m})$$

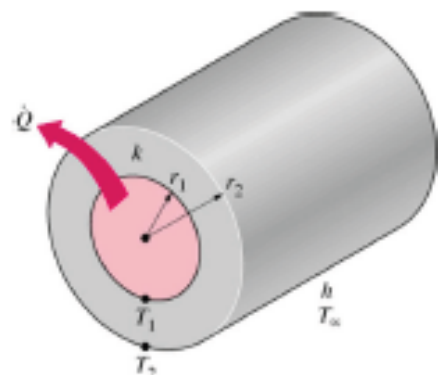
$$r_{\text{cr, sphere}} = \frac{2k}{h}$$

De grootste waarde van de kritische straal die we kunnen tegenkomen is

$$r_{\text{cr, max}} = \frac{k_{\text{max, insulation}}}{h_{\text{min}}} \approx \frac{0.05 \text{ W/m} \cdot ^\circ\text{C}}{5 \text{ W/m}^2 \cdot ^\circ\text{C}} = 0.01 \text{ m} = 1 \text{ cm}$$



- Voorbeeld: geïsoleerde elektrische draad



$r_1 = 1,5 \text{ mm}$   
 $r_2 = 3,5 \text{ mm}$   
 Plastieken omhulsel:  $k = 0,15 \text{ W/m} \cdot ^\circ\text{C}$   
 Stroom door de draad =  $10 \text{ A}$   
 Potentiaalverschil over de draad =  $8 \text{ V}$   
 $T_{\infty} = 30^\circ\text{C}$   
 $h = 12 \text{ W/m}^2 \cdot ^\circ\text{C}$

(a) Bepaal de temperatuur op de interface draad-plastiek ( $T_1$ ) en (b) bepaal of een verdubbeling van de dikte van het plastieken omhulsel de interfacetemperatuur zal doen dalen of stijgen.

a)  $\dot{Q} = \dot{W}_e = VI = (8 \text{ V})(10 \text{ A}) = 80 \text{ W}$

$$A_2 = (2\pi r_2)L = 2\pi(0.0035 \text{ m})(5 \text{ m}) = 0.110 \text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_2} = \frac{1}{(12 \text{ W/m}^2 \cdot ^\circ\text{C})(0.110 \text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15 \text{ W/m} \cdot ^\circ\text{C})(5 \text{ m})} = 0.18^\circ\text{C/W}$$

$$R_{\text{total}} = R_{\text{plastic}} + R_{\text{conv}} = 0.76 + 0.18 = 0.94^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_1 - T_{\infty}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty} + \dot{Q}R_{\text{total}} = 30^\circ\text{C} + (80 \text{ W})(0.94^\circ\text{C/W}) = 105^\circ\text{C}$$

(b)

$$r_{cr} = \frac{k}{h} = \frac{0.15 \text{ W/m} \cdot ^\circ\text{C}}{12 \text{ W/m}^2 \cdot ^\circ\text{C}} = 0.0125 \text{ m} = 12.5 \text{ mm}$$

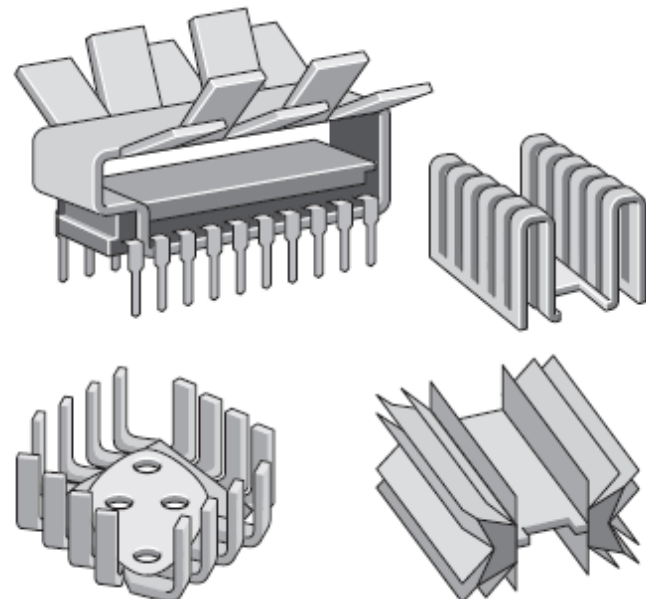
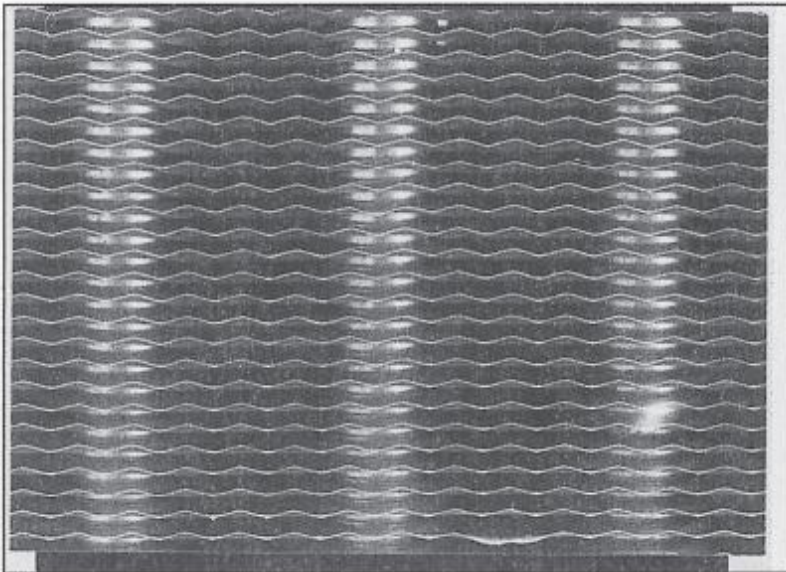
$$\text{Dikte} = (3,5 - 1,5)2 = 4 \text{ mm}$$

$r_2 = 1,4 + 4 = 5,5 \text{ mm} < r_{cr} \Rightarrow$  verhoging van het warmtdebiet

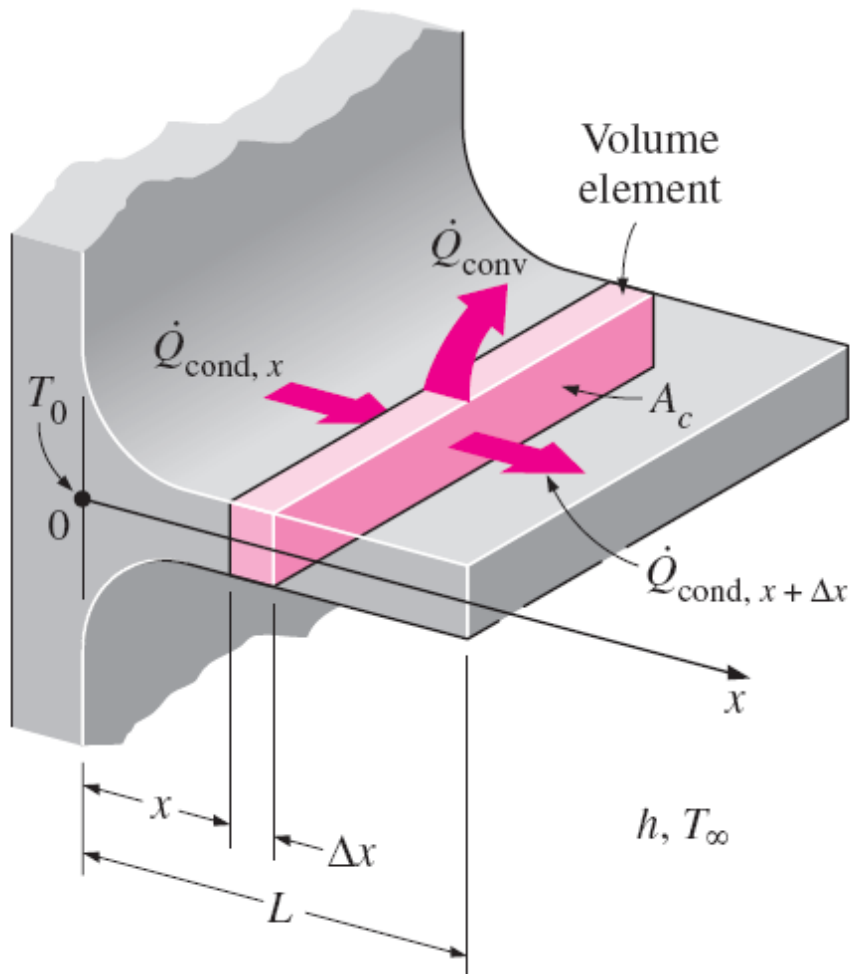
# 6. Koelvinnen

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_{\infty})$$

- Verhogen van warmteoverdracht door  $A_s$  te vergroten
- Veronderstel  $h$  constant en uniform over het oppervlak
- Veronderstel  $k$  constant
- Veronderstel stationaire werking en geen warmtegeneratie



- Vinvergelijking



$$\dot{Q}_{\text{cond}, x} = \dot{Q}_{\text{cond}, x + \Delta x} + \dot{Q}_{\text{conv}}$$

$$\dot{Q}_{\text{conv}} = h(p \Delta x)(T - T_\infty)$$

$$\frac{\dot{Q}_{\text{cond}, x + \Delta x} - \dot{Q}_{\text{cond}, x}}{\Delta x} + hp(T - T_\infty) = 0$$

$$\Delta x \rightarrow 0$$

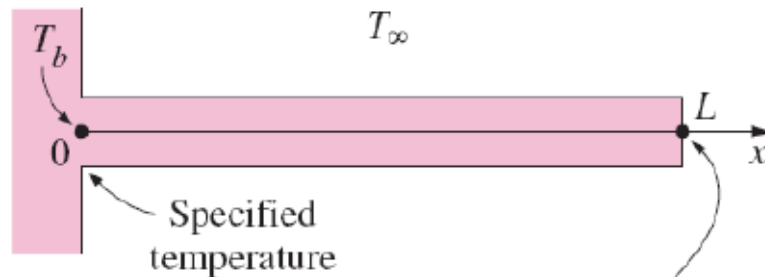
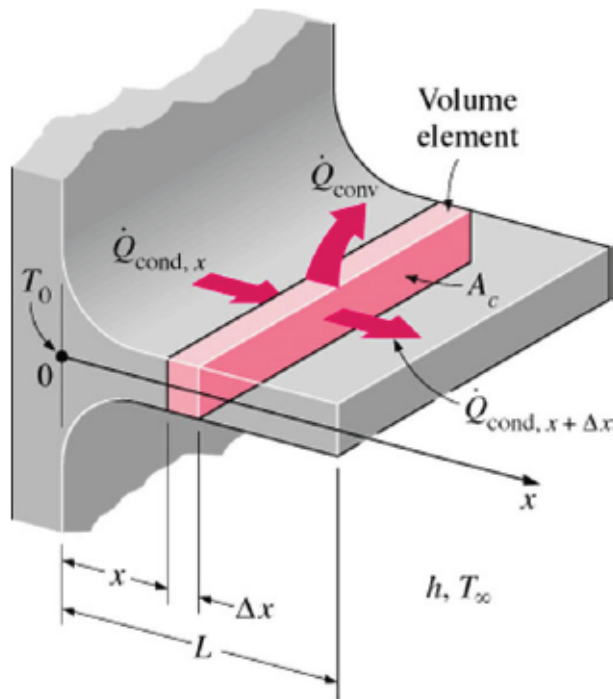
$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0$$

$$\dot{Q}_{\text{cond}} = -kA_c \frac{dT}{dx}$$

$$\frac{d}{dx} \left( kA_c \frac{dT}{dx} \right) - hp(T - T_\infty) = 0$$



# • Vinvergelijking



- (a) Specified temperature
- (b) Negligible heat loss
- (c) Convection
- (d) Convection and radiation

$$\frac{d}{dx} \left( k A_c \frac{dT}{dx} \right) - hp(T - T_\infty) = 0$$

$$m^2 = \frac{hp}{kA_c} \quad A_c \text{ en } k \text{ zijn constant}$$

$$\theta = T - T_\infty$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

# • Vinvergelijking

## • 1. Oneindig lange vin ( $T_{\text{vintip}} = T_{\infty}$ )

$$x = 0: \theta(0) = \theta_b = T_b - T_{\infty}$$

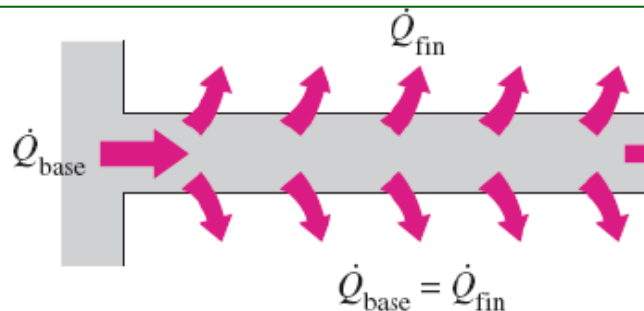
$$x = \infty: \theta(\infty) = T_{\infty} - T_{\infty} = 0$$

$$\begin{matrix} C_1 = 0 \\ C_2 = \theta(b) \end{matrix} \longrightarrow \theta(x) = \theta_b e^{-mx}$$

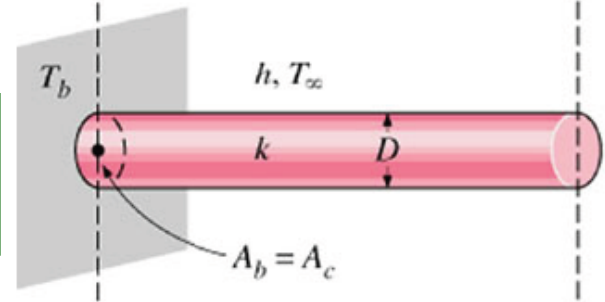
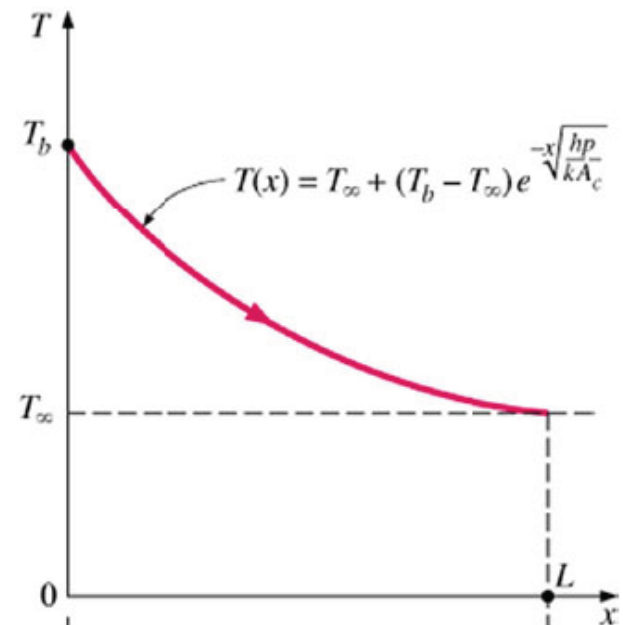
$$\frac{T(x) - T_{\infty}}{T_b - T_{\infty}} = e^{-mx} = e^{-x\sqrt{hp/kA_c}}$$

Stationair warmtedebiet vanaf de ganse vin:

$$\dot{Q}_{\text{long fin}} = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = \sqrt{hp k A_c} (T_b - T_{\infty})$$



Onder stationaire condities is de warmteoverdracht vanaf de vinoppervlakken gelijk aan de warmtegeleiding aan de vinbasis.



( $p = \pi D$ ,  $A_c = \pi D^2/4$  for a cylindrical fin)

- Vinvergelijking:
  - Eindige vin met tipeinde geïsoleerd (adiabatische vintip,  $\dot{Q}_{\text{vintip}} = 0$ )

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\left. \frac{d\theta}{dx} \right|_{x=L} = C_1 m e^{mL} + C_2 (-m) e^{-mL} = 0 \Leftrightarrow C_1 e^{mL} = C_2 e^{-mL}$$

$$x = 0: \theta(0) = \theta_b = C_1 e^{m0} + C_2 e^{-m0} = C_1 + C_2 \Leftrightarrow C_1 = \theta_b - C_2$$

$$\theta_b e^{mL} = C_2 (e^{mL} + e^{-mL}) \Leftrightarrow C_2 = \frac{\theta_b e^{mL}}{(e^{mL} + e^{-mL})}$$

$$C_1 = \theta_b - \frac{\theta_b e^{mL}}{(e^{mL} + e^{-mL})} = \theta_b \left( \frac{e^{-mL}}{e^{mL} + e^{-mL}} \right)$$

$$\Rightarrow \theta = \theta_b \left( \frac{e^{-mL}}{e^{mL} + e^{-mL}} \right) e^{mx} + \frac{\theta_b e^{mL}}{(e^{mL} + e^{-mL})} e^{-mx} = \theta_b \left( \frac{e^{-m(L-x)} + e^{m(L-x)}}{e^{mL} + e^{-mL}} \right) = \theta_b \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)} \Rightarrow T(x) = (T_b - T_\infty) \frac{\cosh[m(L-x)]}{\cosh(mL)} + T_\infty$$

$$\text{Randvoorwaarden: } x = 0: \theta(0) = \theta_b = T_b - T_\infty$$

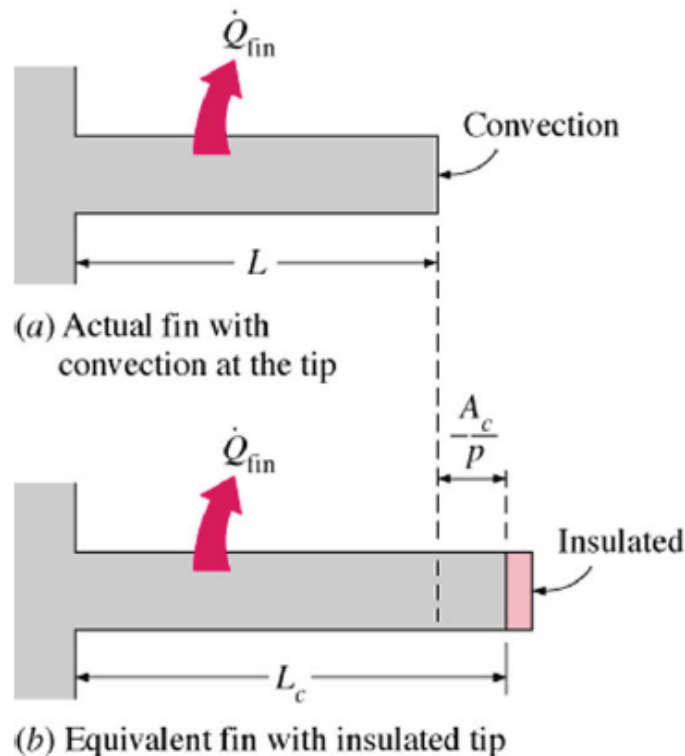
$$x = L: \left. \frac{d\theta}{dx} \right|_{x=L} = 0$$

- Vinvergelijking: Eindige vin met tpeinde geïsoleerd

Stationair warmtedebiet vanaf de ganse vin:

$$\begin{aligned}\dot{Q}_{\text{adiabatic tip}} &= -kA_c \left. \frac{dT}{dx} \right|_{x=0} \\ &= \sqrt{hp k A_c} (T_b - T_\infty) \tanh mL\end{aligned}$$

- Vinvergelijking:
  - 3. Convectie vanaf de tip (of gecombineerde convectie en straling)



Gecorrigeerde vinlengte  $L_c$ :  $L_c = L + \frac{A_c}{p}$

$$L_{c, \text{rectangular fin}} = L + \frac{t}{2}$$

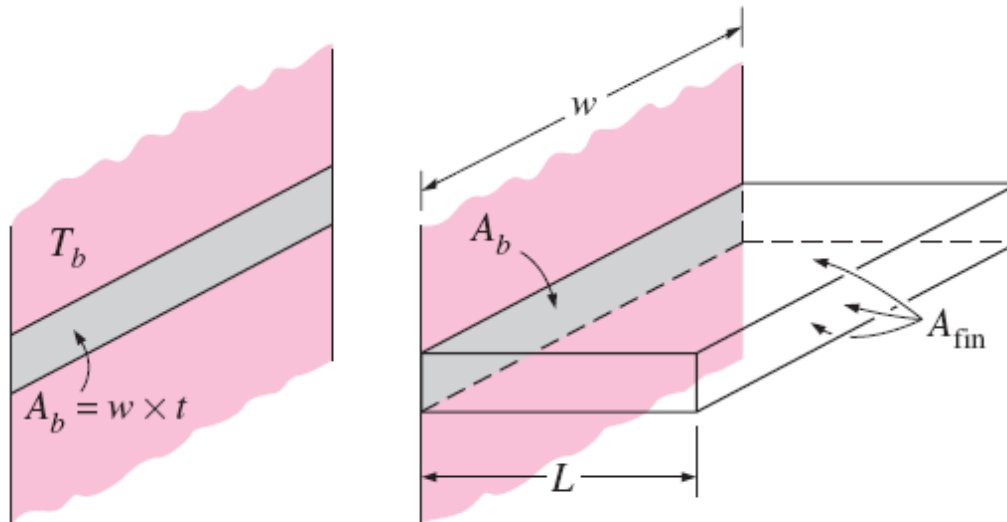
$$L_{c, \text{cylindrical fin}} = L + \frac{D}{4}$$

$t$  = de dikte van rechthoekige vinnen

$D$  = de diameter van cilindervormige vinnen

Vinnen met convectie aan het vinuiteinde kunnen behandeld worden als vinnen met een geïsoleerd vinuiteinde door de gecorrigeerde vinlengte  $L_c$  te gebruiken.

- Vinefficiëntie,  $\eta_{\text{vin}}$

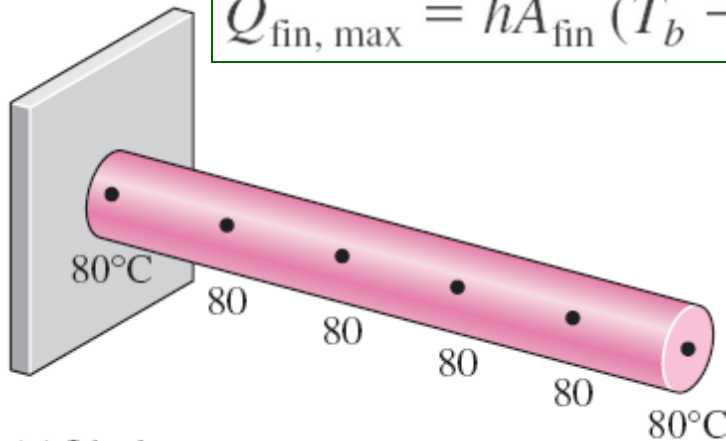


$$\begin{aligned} A_{\text{vin}} &= 2(w.L) + 2(t.L) + (t.w) \\ &= L(2w+2t) + (t.w) \\ &= L.p + (t.w) \\ &= L.p \end{aligned}$$

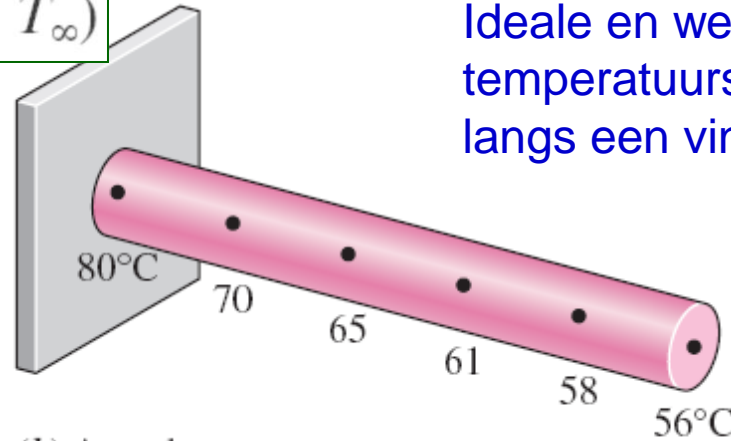
Vinnen verhogen het warmtetransport van een opp. door het opp. te vergroten

(a) Surface without fins (b) Surface with a fin

$$\dot{Q}_{\text{fin, max}} = hA_{\text{fin}} (T_b - T_{\infty})$$



(a) Ideal



(b) Actual


Ideale en werkelijke temperatuursgradient langs een vin.

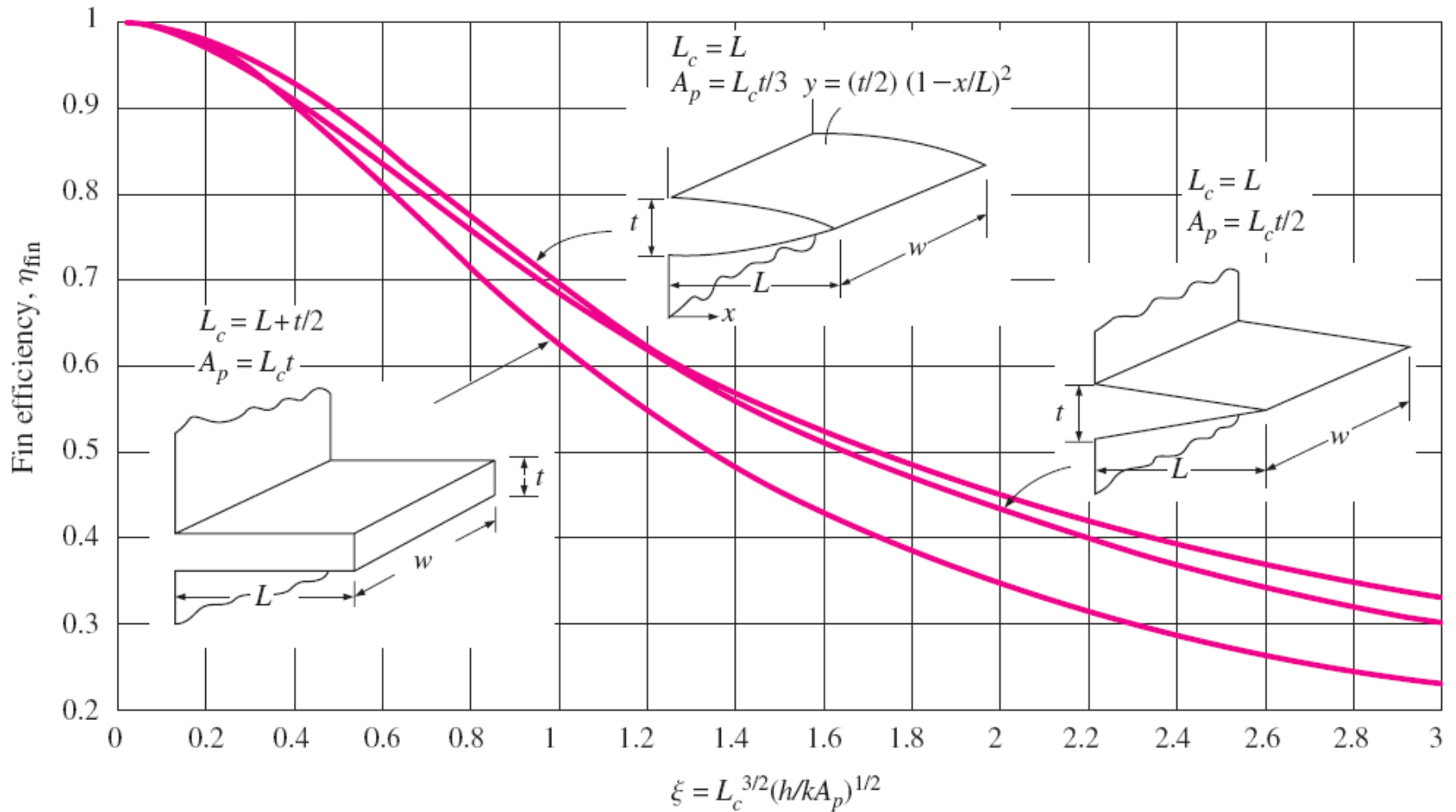
$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} \dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} h A_{\text{fin}} (T_b - T_{\infty})$$

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hp k A_c} (T_b - T_{\infty})}{h A_{\text{fin}} (T_b - T_{\infty})} = \frac{1}{L} \sqrt{\frac{k A_c}{hp}} = \frac{1}{mL}$$

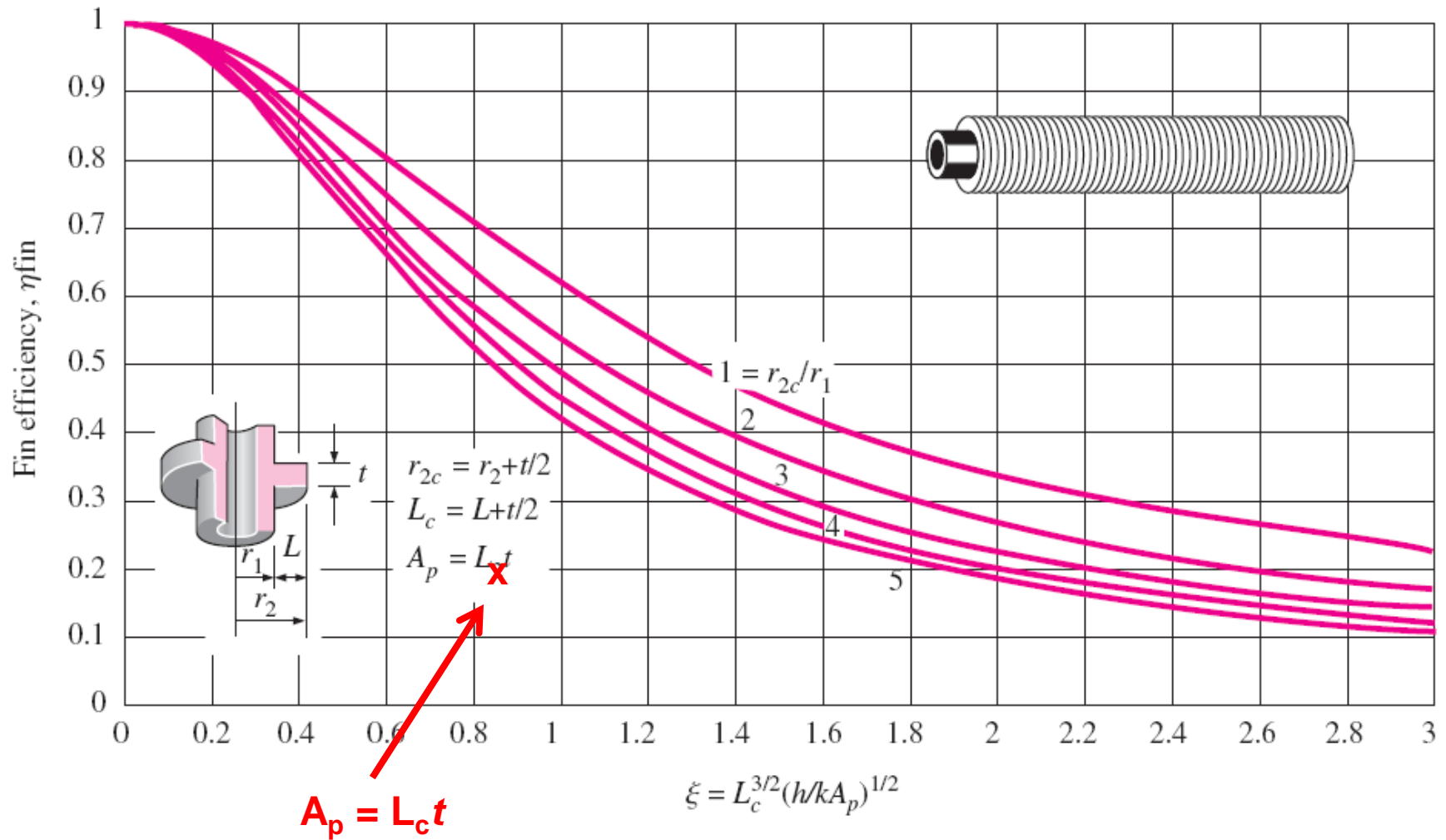
$$\eta_{\text{adiabatic tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hp k A_c} (T_b - T_{\infty}) \tanh mL}{h A_{\text{fin}} (T_b - T_{\infty})} = \frac{\tanh mL}{mL}$$


  
**m**



Efficiëntie van rechte vinnen met rechthoekige, driehoekige en parabolische profielen.





Efficiëntie van annulaire vinnen met constante dikte  $t$

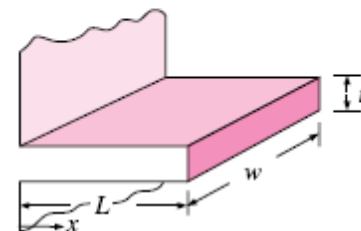
### Straight rectangular fins

$$m = \sqrt{2h/kt}$$

$$L_c = L + t/2$$

$$A_{fin} = 2wL_c$$

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$

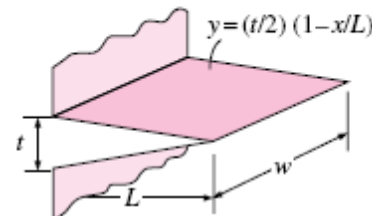


### Straight triangular fins

$$m = \sqrt{2h/kt}$$

$$A_{fin} = 2w\sqrt{L^2 + (t/2)^2}$$

$$\eta_{fin} = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$



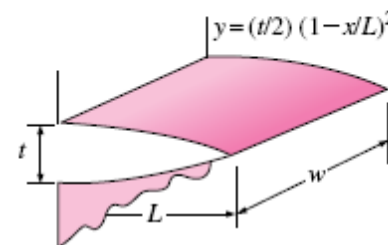
### Straight parabolic fins

$$m = \sqrt{2h/kt}$$

$$A_{fin} = wL[C_1 + (L/t)\ln(t/L + C_1)]$$

$$C_1 = \sqrt{1 + (t/L)^2}$$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$



### Circular fins of rectangular profile

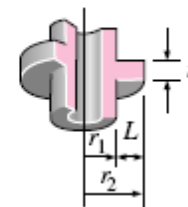
$$m = \sqrt{2h/kt}$$

$$r_{2c} = r_2 + t/2$$

$$A_{fin} = 2\pi(r_{2c}^2 - r_1^2)$$

$$\eta_{fin} = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$



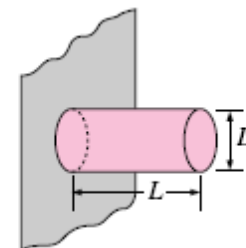
### Pin fins of rectangular profile

$$m = \sqrt{4h/kD}$$

$$L_c = L + D/4$$

$$A_{fin} = \pi DL_c$$

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$

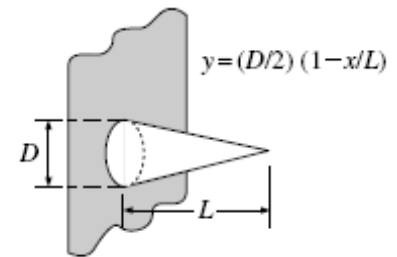


### Pin fins of triangular profile

$$m = \sqrt{4h/kD}$$

$$A_{\text{fin}} = \frac{\pi D}{2} \sqrt{L^2 + (D/2)^2}$$

$$\eta_{\text{fin}} = \frac{2}{mL} \frac{I_2(2mL)}{I_1(2mL)}$$



### Pin fins of parabolic profile

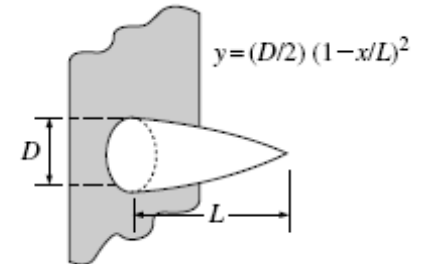
$$m = \sqrt{4h/kD}$$

$$A_{\text{fin}} = \frac{\pi L^3}{8D} [C_3 C_4 - \frac{L}{2D} \ln(2DC_4/L + C_3)]$$

$$C_3 = 1 + 2(D/L)^2$$

$$C_4 = \sqrt{1 + (D/L)^2}$$

$$\eta_{\text{fin}} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$

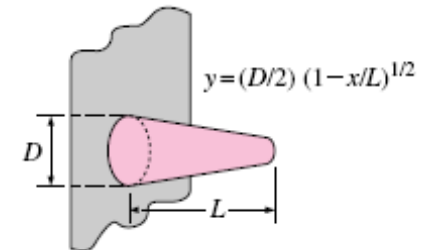


### Pin fins of parabolic profile (blunt tip)

$$m = \sqrt{4h/kD}$$

$$A_{\text{fin}} = \frac{\pi D^4}{96L^2} \left\{ [16(L/D)^2 + 1]^{3/2} - 1 \right\}$$

$$\eta_{\text{fin}} = \frac{3}{2mL} \frac{I_1(4mL/3)}{I_0(4mL/3)}$$



- Vinnen met een **driehoekig en parabolisch profiel** bevatten minder materiaal en zijn efficiënter dan vinnen met een rechthoekig profiel
- De vinefficiëntie neemt af met toenemende vinlengte
- **Hoe moet men de vinlengte bepalen?** Een vin nemen die langer is dan een bepaalde lengte is enkel te verantwoorden als de voordelen groter zijn dan de extra kost
- De vinefficiëntie van de meeste vinnen ligt in de praktijk **boven 90%.**

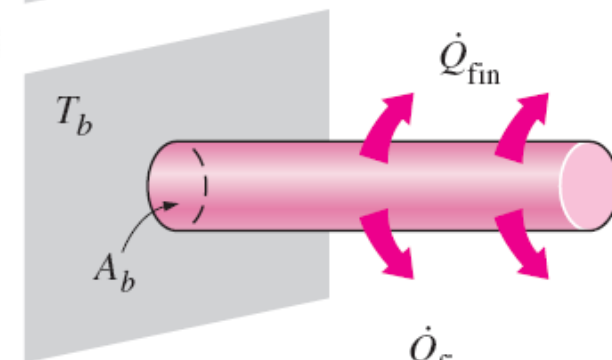
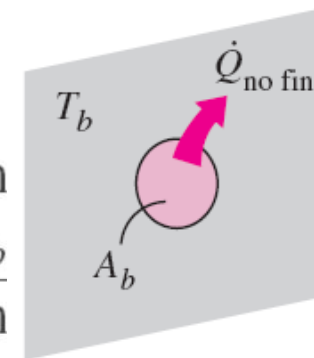
- Vineffectiviteit,  $\varepsilon_{\text{fin}}$

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b (T_b - T_{\infty})} = \frac{\text{Heat transfer rate from the fin of base area } A_b}{\text{Heat transfer rate from the surface of area } A_b}$$

$\varepsilon_{\text{fin}} = 1$     Geen invloed op warmteoverdracht

$\varepsilon_{\text{fin}} < 1$     Isolatie

$\varepsilon_{\text{fin}} > 1$     Warmteoverdracht vergroten



$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}}$$

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b (T_b - T_{\infty})} = \frac{\eta_{\text{fin}} hA_{\text{fin}} (T_b - T_{\infty})}{hA_b (T_b - T_{\infty})} = \frac{A_{\text{fin}}}{A_b} \eta_{\text{fin}}$$

- Vineffectiviteit

Voor een lange vin met uniforme doorsnede ( $A_c=A_b$ ):

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c} (T_b - T_{\infty})}{hA_b (T_b - T_{\infty})} = \sqrt{\frac{kp}{hA_c}}$$

- *Conclusies:*

- $k$  van het vinmateriaal moet zo groot mogelijk zijn => bvb. aluminium, koper, ijzer.
- De verhouding  $p/A_c$  moet zo groot mogelijk zijn => dunne, platte vinnen.
- Meest effectief als  $h$  laag is => medium is gas, natuurlijke convection; Plaats vin aan de gas (lucht)-zijde bij vloeistof-naar-gas-warmtewisselaars.

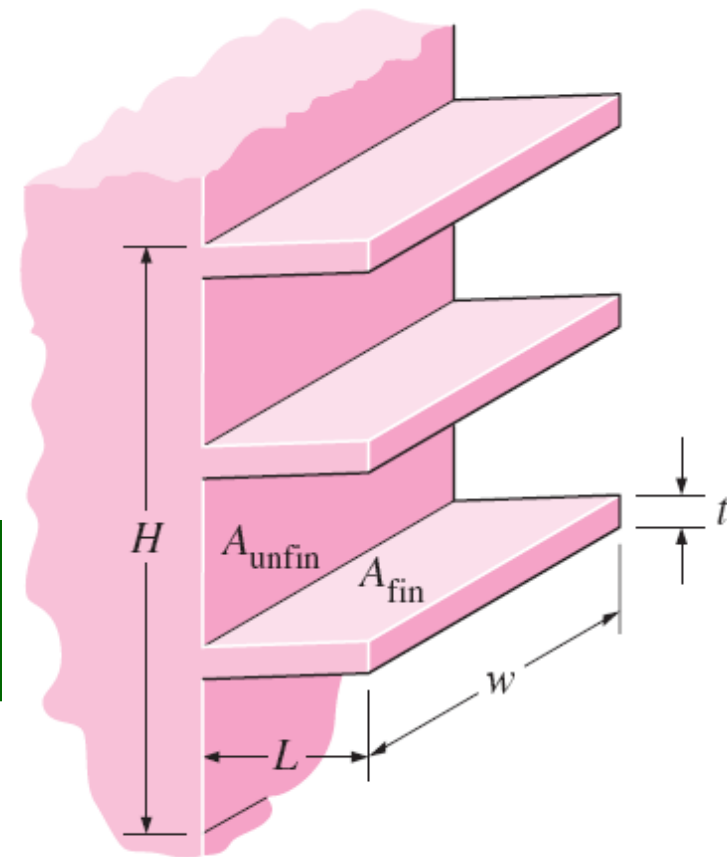
# • Totale vineffectiviteit

$$\begin{aligned}
 \dot{Q}_{\text{total, fin}} &= \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}} \\
 &= hA_{\text{unfin}} (T_b - T_{\infty}) + \eta_{\text{fin}} hA_{\text{fin}} (T_b - T_{\infty}) \\
 &= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_{\infty})
 \end{aligned}$$

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_{\infty})}{hA_{\text{no fin}} (T_b - T_{\infty})}$$

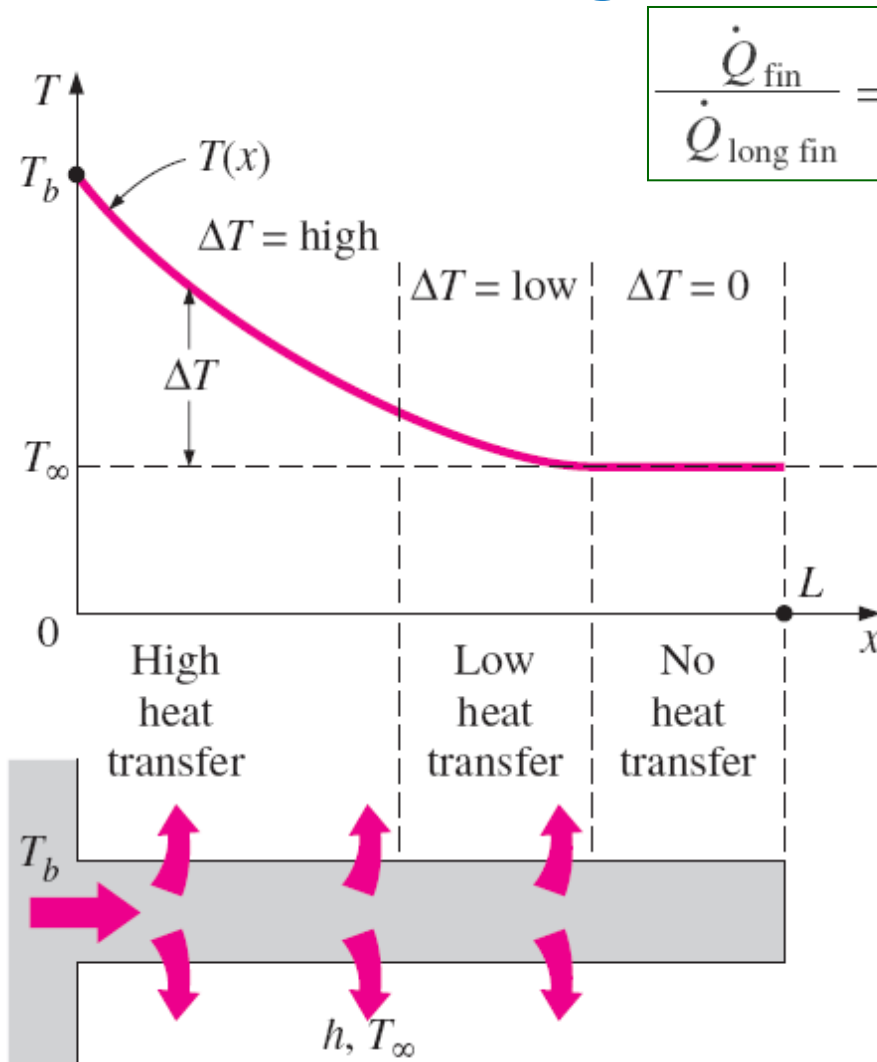
De totale vineffectiviteit hangt af van de vindensiteit (aantal vinnen per eenheidslengte) en de effectiviteit van de individuele vinnen.

De totale vineffectiviteit is een betere maat van de performantie van een gevind oppervlak dan de effectiviteit van de individuele vinnen.



$$\begin{aligned}
 A_{\text{no fin}} &= w \times H \\
 A_{\text{unfin}} &= w \times H - 3 \times (t \times w) \\
 A_{\text{fin}} &= 2 \times L \times w + t \times w \\
 &\cong 2 \times L \times w \text{ (one fin)}
 \end{aligned}$$

- Juiste vinlengte?



$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpkA_c} (T_b - T_\infty) \tanh mL}{\sqrt{hpkA_c} (T_b - T_\infty)} = \tanh mL$$

The variation of heat transfer from a fin relative to that from an infinitely long fin

$mL$	$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \tanh mL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

$mL = 5 \rightarrow$  oneindig lange vin

$mL = 1$  geeft een goed compromis tussen warmteoverdrachtsperformantie en vinlengte

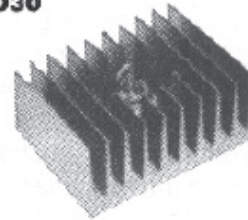
# ● Electronica: “heat sinks”

- “Heat sinks”: speciaal ontworpen vinnen die gebruikt worden om elektronische apparatuur te koelen; hebben een complexe geometrie.
- De warmteoverdrachtspreformantie wordt uitgedrukt in termen van hun *thermische weerstanden*  $R$ .
- Een kleine waarde van de thermische weerstand duidt op een hoge vinefficiëntie.

$$\dot{Q}_{\text{fin}} = \frac{T_b - T_{\infty}}{R} = hA_{\text{fin}} \eta_{\text{fin}} (T_b - T_{\infty})$$

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in) long.

**HS 5030**



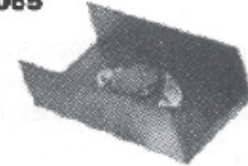
$R = 0.9^{\circ}\text{C}/\text{W}$  (vertical)

$R = 1.2^{\circ}\text{C}/\text{W}$  (horizontal)

Dimensions: 76 mm × 105 mm × 44 mm

Surface area: 677 cm<sup>2</sup>

**HS 6065**

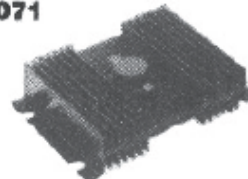


$R = 5^{\circ}\text{C}/\text{W}$

Dimensions: 76 mm × 38 mm × 24 mm

Surface area: 387 cm<sup>2</sup>

**HS 6071**



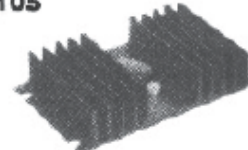
$R = 1.4^{\circ}\text{C}/\text{W}$  (vertical)

$R = 1.8^{\circ}\text{C}/\text{W}$  (horizontal)

Dimensions: 76 mm × 92 mm × 26 mm

Surface area: 968 cm<sup>2</sup>

**HS 6105**



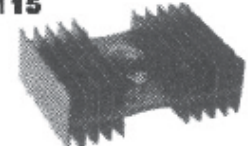
$R = 1.8^{\circ}\text{C}/\text{W}$  (vertical)

$R = 2.1^{\circ}\text{C}/\text{W}$  (horizontal)

Dimensions: 76 mm × 127 mm × 91 mm

Surface area: 677 cm<sup>2</sup>

**HS 6115**



$R = 1.1^{\circ}\text{C}/\text{W}$  (vertical)

$R = 1.3^{\circ}\text{C}/\text{W}$  (horizontal)

Dimensions: 76 mm × 102 mm × 25 mm

Surface area: 929 cm<sup>2</sup>



- **Voorbeeld 1:** selectie van een “heat sink” voor een transistor

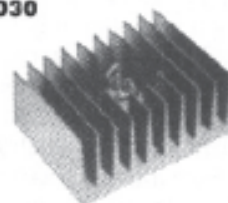
Koelen van een 60 W-transistor door het vast te hechten aan een koelvin (“heat sink”).

Selecteer een koelvin uit Tabel 17-6 zodat de temperatuur van de transistor 90°C niet zal overschrijden bij een omgevingstempertuur van 30°C.

$$\dot{Q} = \frac{\Delta T}{R} \longrightarrow R = \frac{\Delta T}{\dot{Q}} = \frac{(90 - 30)^{\circ}\text{C}}{60 \text{ W}} = 1.0^{\circ}\text{C/W}$$

Tabel 17-6 => HS 5030

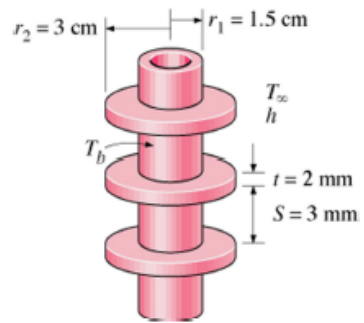
HS 5030



$R = 0.9^{\circ}\text{C/W}$  (vertical)  
 $R = 1.2^{\circ}\text{C/W}$  (horizontal)

Dimensions: 76 mm × 105 mm × 44 mm  
Surface area: 677 cm<sup>2</sup>

• **Voorbeeld 2: Effect van vinnen op de warmteoverdracht van een stoompijp**



Aluminium vin:  $k = 180 \text{ W/m} \cdot ^\circ\text{C}$   
 200 vinnen per meter  
 $T_b = 120^\circ\text{C}$   
 $T_\infty = 25^\circ\text{C}$   
 $h_{\text{gecombineerd}} = 60 \text{ W/m}^2 \cdot ^\circ\text{C}$

Bepaal de toename in warmteoverdracht per meter als gevolg van het toevoegen van 200 vinnen.

$$\begin{aligned} A_{\text{no fin}} &= \pi D_1 L = \pi (0.03 \text{ m}) (1 \text{ m}) = 0.0942 \text{ m}^2 \\ \dot{Q}_{\text{no fin}} &= h A_{\text{no fin}} (T_b - T_\infty) \\ &= (60 \text{ W/m}^2 \cdot ^\circ\text{C}) (0.0942 \text{ m}^2) (120 - 25)^\circ\text{C} \\ &= 537 \text{ W} \end{aligned}$$

Bepalen  $\eta_{\text{vin}}$  via Figuur 17-43:

$$\begin{aligned} L &= (D_2 - D_1)/2 = 0.015 \text{ m} \\ R_{2c} &= r_2 + t/2 = 0.031 \text{ m} \\ L_c &= L + t/2 = 0.016 \text{ m} \\ A_p &= L_c t = 3.2 \cdot 10^{-5} \text{ m}^2 \end{aligned}$$

$$r_{2c}/r_1 = 3.07$$

$$L_c^{3/2} \sqrt{\frac{h}{k A_p}} = 0.207$$

$$\Rightarrow \eta_{\text{vin}} = 0.95$$

$$\begin{aligned} A_{\text{fin}} &= 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t \\ &= 2\pi[(0.03 \text{ m})^2 - (0.015 \text{ m})^2] + 2\pi(0.03 \text{ m})(0.002 \text{ m}) \\ &= 0.00462 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \dot{Q}_{\text{fin}} &= \eta_{\text{fin}} \dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} h A_{\text{fin}} (T_b - T_\infty) \\ &= 0.95 (60 \text{ W/m}^2 \cdot ^\circ\text{C}) (0.00462 \text{ m}^2) (120 - 25)^\circ\text{C} \\ &= 25.0 \text{ W} \end{aligned}$$

$$\begin{aligned} A_{\text{unfin}} &= \pi D_1 S = \pi (0.03 \text{ m}) (0.003 \text{ m}) = 0.000283 \text{ m}^2 \\ \dot{Q}_{\text{unfin}} &= h A_{\text{unfin}} (T_b - T_\infty) \\ &= (60 \text{ W/m}^2 \cdot ^\circ\text{C}) (0.000283 \text{ m}^2) (120 - 25)^\circ\text{C} \\ &= 1.60 \text{ W} \end{aligned}$$

$$\dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.0 + 1.6) \text{ W} = 5320 \text{ W}$$

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 5320 - 537 = \mathbf{4783 \text{ W}} \quad (\text{per m tube length})$$

# 7. Warmteoverdracht in algemene configuraties

- Warmteoverdracht tussen 2 oppervlakken gehouden bij een constante temperatuur  $T_1$  en  $T_2$
- **Geleidingsvormfactor,  $S$**  (m)

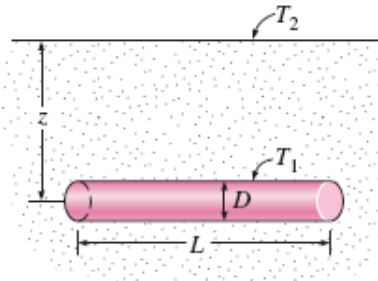
$$\dot{Q} = Sk(T_1 - T_2)$$

$k$  = warmtegeleidingscoëfficiënt van het medium tussen de 2 oppervlakken

Conduction shape factors  $S$  for several configurations for use in  $\dot{Q} = kS(T_1 - T_2)$  to determine the steady rate of heat transfer through a medium of thermal conductivity  $k$  between the surfaces at temperatures  $T_1$  and  $T_2$

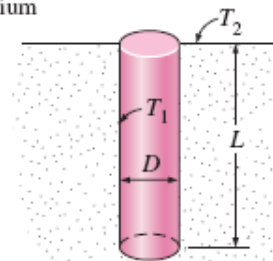
- (1) Isothermal cylinder of length  $L$   
buried in a semi-infinite medium  
( $L \gg D$  and  $z > 1.5D$ )

$$S = \frac{2\pi L}{\ln(4z/D)}$$



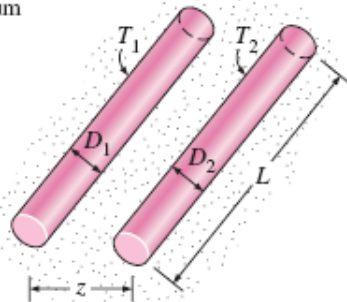
- (2) Vertical isothermal cylinder of length  $L$   
buried in a semi-infinite medium  
( $L \gg D$ )

$$S = \frac{2\pi L}{\ln(4L/D)}$$



- (3) Two parallel isothermal cylinders  
placed in an infinite medium  
( $L \gg D_1, D_2, z$ )

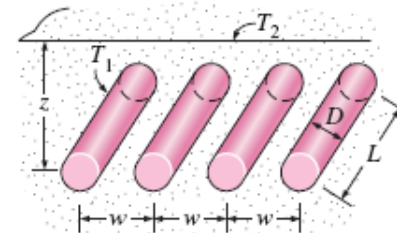
$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{4z^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$$



- (4) A row of equally spaced parallel isothermal  
cylinders buried in a semi-infinite medium  
( $L \gg D, z$  and  $w > 1.5D$ )

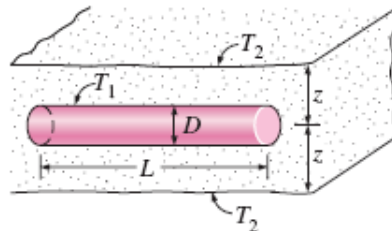
$$S = \frac{2\pi L}{\ln\left(\frac{2w}{\pi D} \sinh \frac{2\pi z}{w}\right)}$$

(per cylinder)



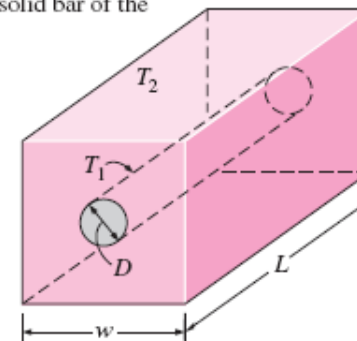
- (5) Circular isothermal cylinder of length  $L$   
in the midplane of an infinite wall  
( $z > 0.5D$ )

$$S = \frac{2\pi L}{\ln(8z/\pi D)}$$



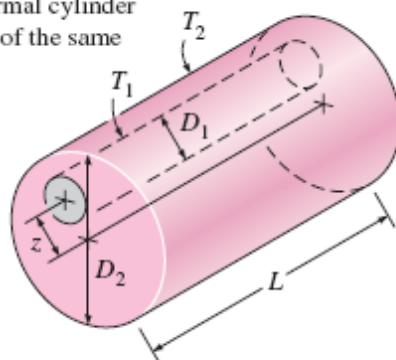
- (6) Circular isothermal cylinder of length  $L$   
at the center of a square solid bar of the  
same length

$$S = \frac{2\pi L}{\ln(1.08w/D)}$$



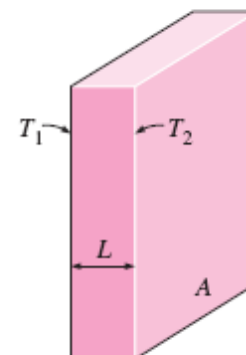
- (7) Eccentric circular isothermal cylinder of length  $L$  in a cylinder of the same length ( $L > D_2$ )

$$S = \frac{2\pi L}{\cosh^{-1} \left( \frac{D_1^2 + D_2^2 - 4z^2}{2D_1 D_2} \right)}$$



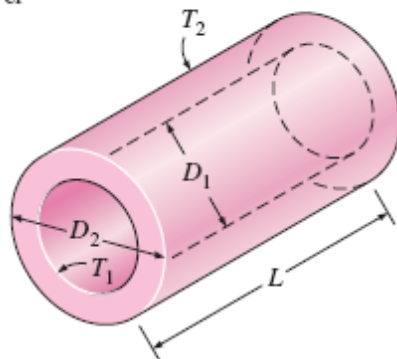
- (8) Large plane wall

$$S = \frac{A}{L}$$



- (9) A long cylindrical layer

$$S = \frac{2\pi L}{\ln(D_2/D_1)}$$



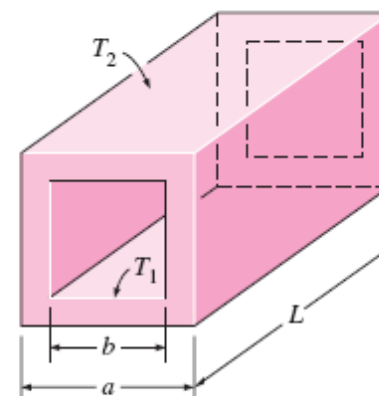
- (10) A square flow passage

- (a) For  $a/b > 1.4$ ,

$$S = \frac{2\pi L}{0.93 \ln(0.948a/b)}$$

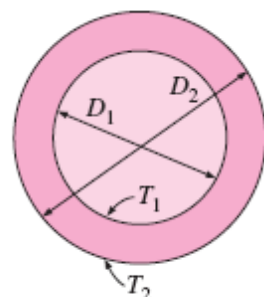
- (b) For  $a/b < 1.41$ ,

$$S = \frac{2\pi L}{0.785 \ln(a/b)}$$



- (11) A spherical layer

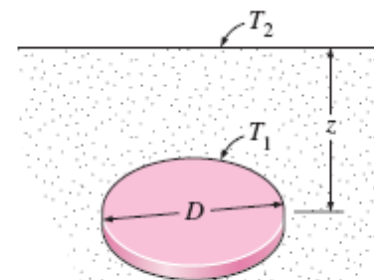
$$S = \frac{2\pi D_1 D_2}{D_2 - D_1}$$



- (12) Disk buried parallel to the surface in a semi-infinite medium ( $z \gg D$ )

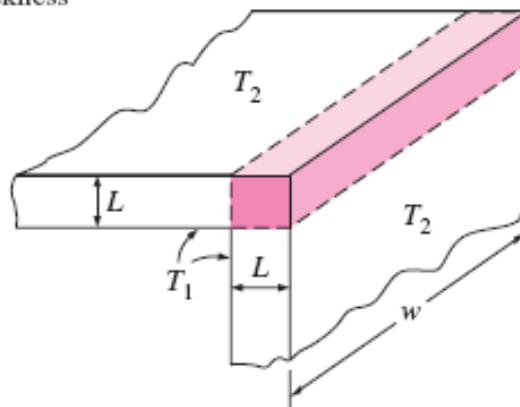
$$S = 4D$$

$$(S = 2D \text{ when } z = 0)$$



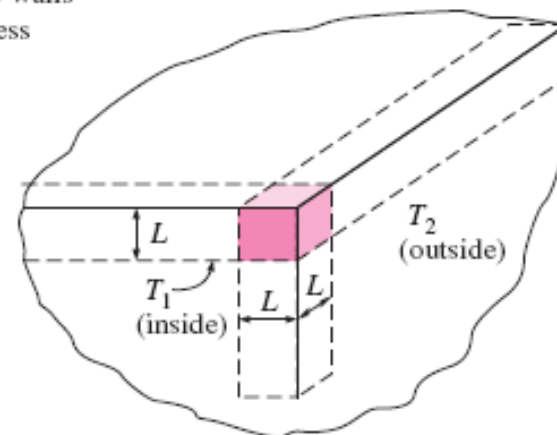
- (13) The edge of two adjoining walls of equal thickness

$$S = 0.54 w$$



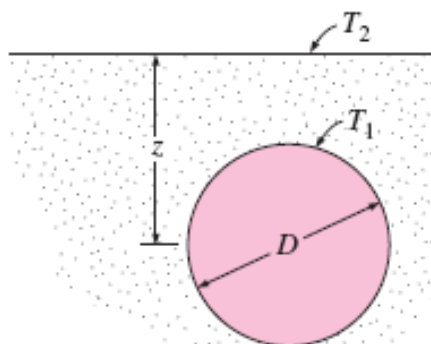
- (14) Corner of three walls of equal thickness

$$S = 0.15 L$$



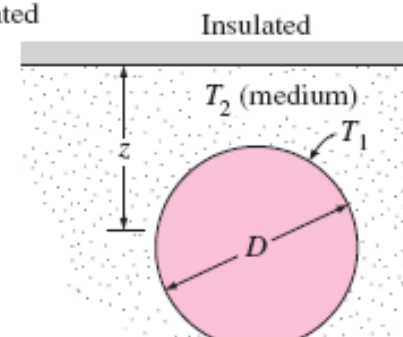
- (15) Isothermal sphere buried in a semi-infinite medium

$$S = \frac{2\pi D}{1 - 0.25D/z}$$

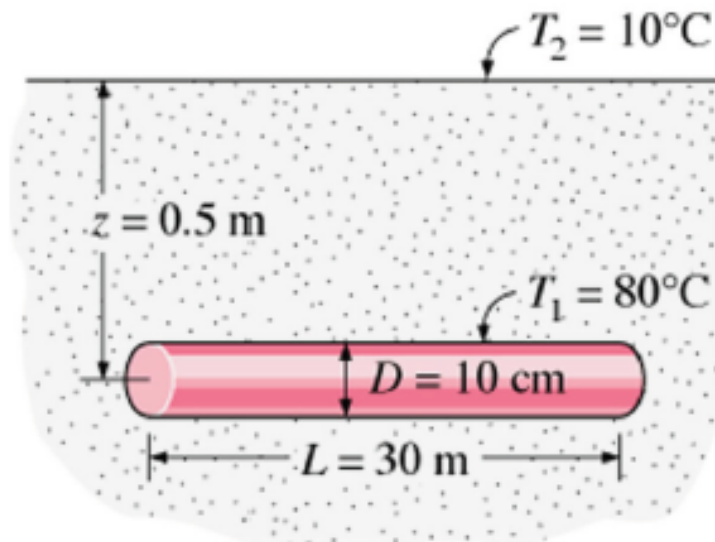


- (16) Isothermal sphere buried in a semi-infinite medium at  $T_2$  whose surface is insulated

$$S = \frac{2\pi D}{1 + 0.25D/z}$$



- Voorbeeld 1:** Warmteverlies vanaf een begraven stoompijp



Grond:  $k=0,9 \text{ W/m } ^\circ\text{C}$

Bepaal het warmtedebiet die door de buis wordt afgestaan.

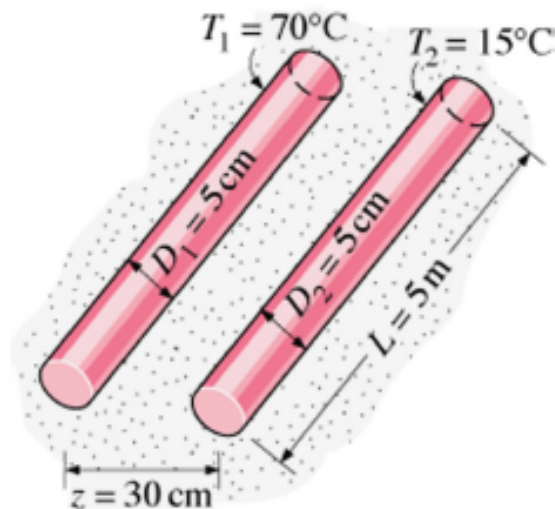
Tabel 17-7:

$$S = \frac{2\pi L}{\ln(4z/D)}$$

$$S = \frac{2\pi \times (30 \text{ m})}{\ln(4 \times 0.5/0.1)} = 62.9 \text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (62.9 \text{ m})(0.9 \text{ W/m} \cdot ^\circ\text{C})(80 - 10)^\circ\text{C} = \mathbf{3963 \text{ W}}$$

- **Voorbeeld 2:** Warmteoverdracht tussen warme en koude buis



Beton:  $k = 0,75\text{ W/m } ^\circ\text{C}$

Bepaal het warmtedebiet tussen de buizen.

Tabel 17-7:

$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{4z^2 - D_1^2 - D_2^2}{2D_1 D_2}\right)}$$

$$S = \frac{2\pi \times (5\text{ m})}{\cosh^{-1}\left(\frac{4 \times 0.3^2 - 0.05^2 - 0.05^2}{2 \times 0.05 \times 0.05}\right)} = 6.34\text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (6.34\text{ m})(0.75\text{ W/m } ^\circ\text{C})(70 - 15)^\circ\text{C} = \mathbf{262\text{ W}}$$