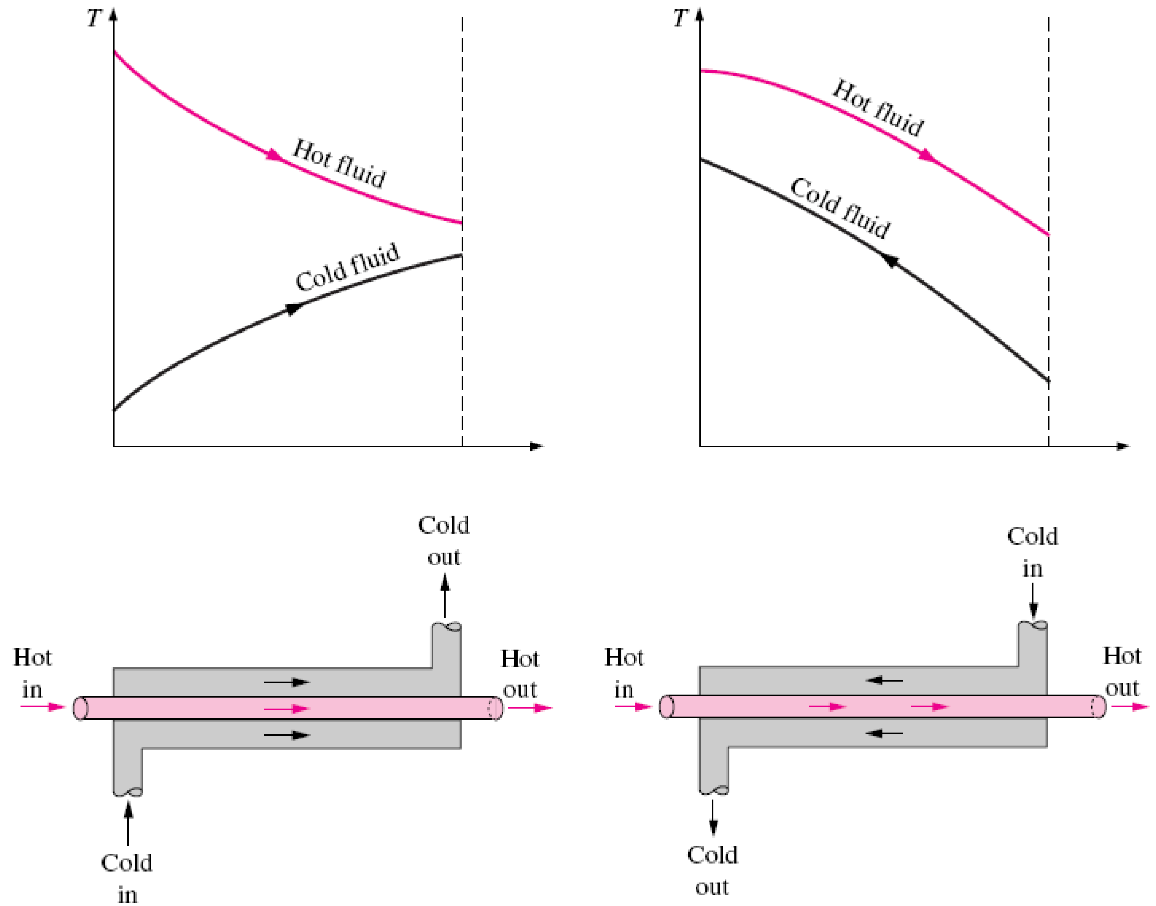
Hfdst 13: Warmtewisselaars

1. Warmtewisselaarstypes
2. De totale warmteoverdrachtscoëfficiënt
3. De analyse van warmtewisselaars
4. De logaritmisch-gemiddelde-temperatuursverschil-methode
5. De effectiviteit-NTU-methode
6. Selectie van warmtewisselaars

1. Warmtewisselaarstypes

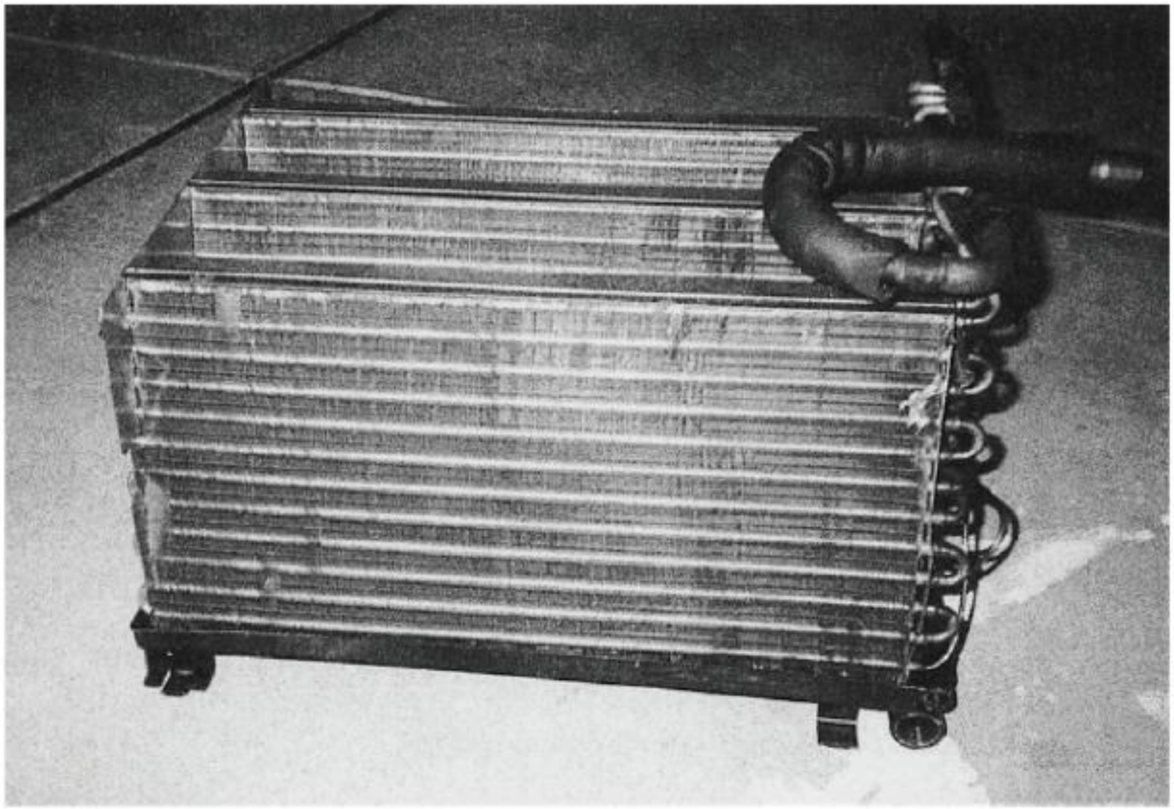
- **Dubbele-buis-warmtewisselaar**

Gelijk- en tegenstroom warmtewisselaars



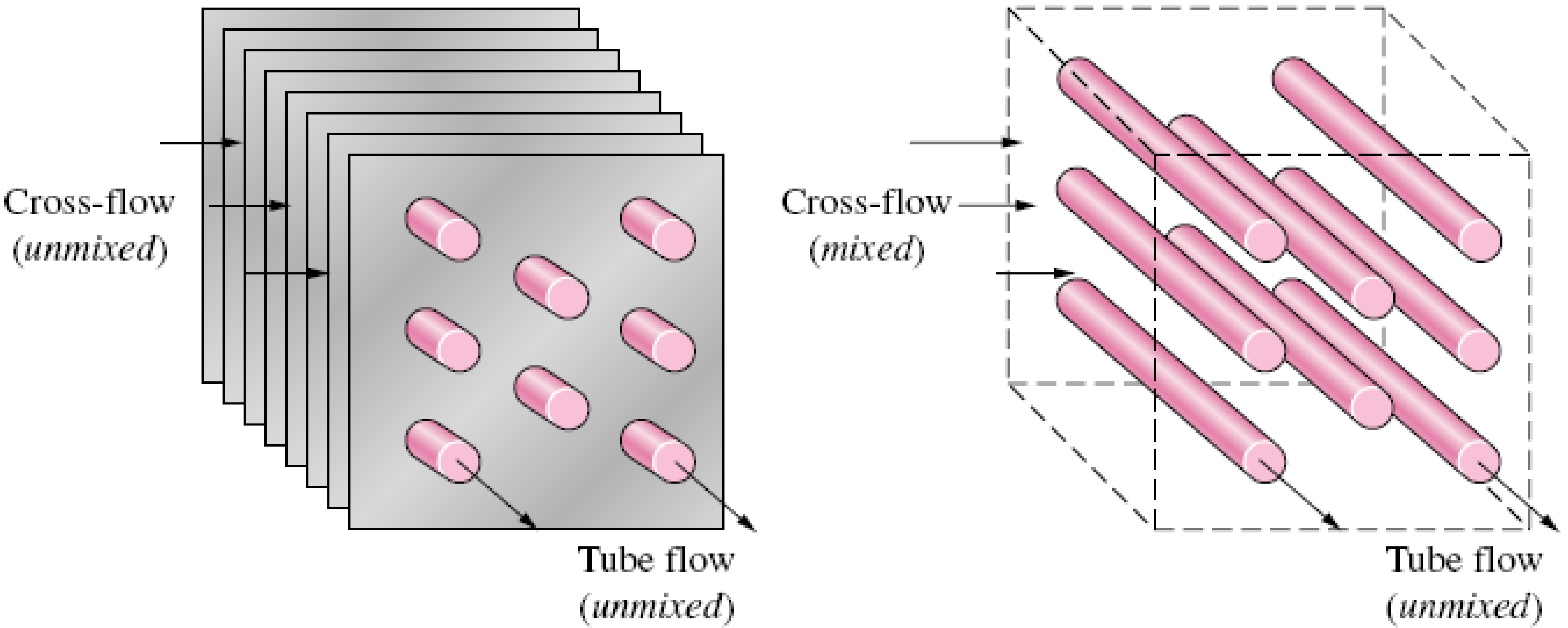
(a) Parallel flow (b) Counter flow

- Gevinde buizen/compacte warmtewisselaar
  - oppervlaktedensiteit β = opp./volume $> 700\ m^2/m^3$
  - Bvb. autoradiator: $\beta \approx 1000\ m^2/m^3$



Een gas-vloeistof compacte warmtewisselaar voor een air-conditioning-systeem.

- Gevinde buizen (compacte warmtewisselaar)

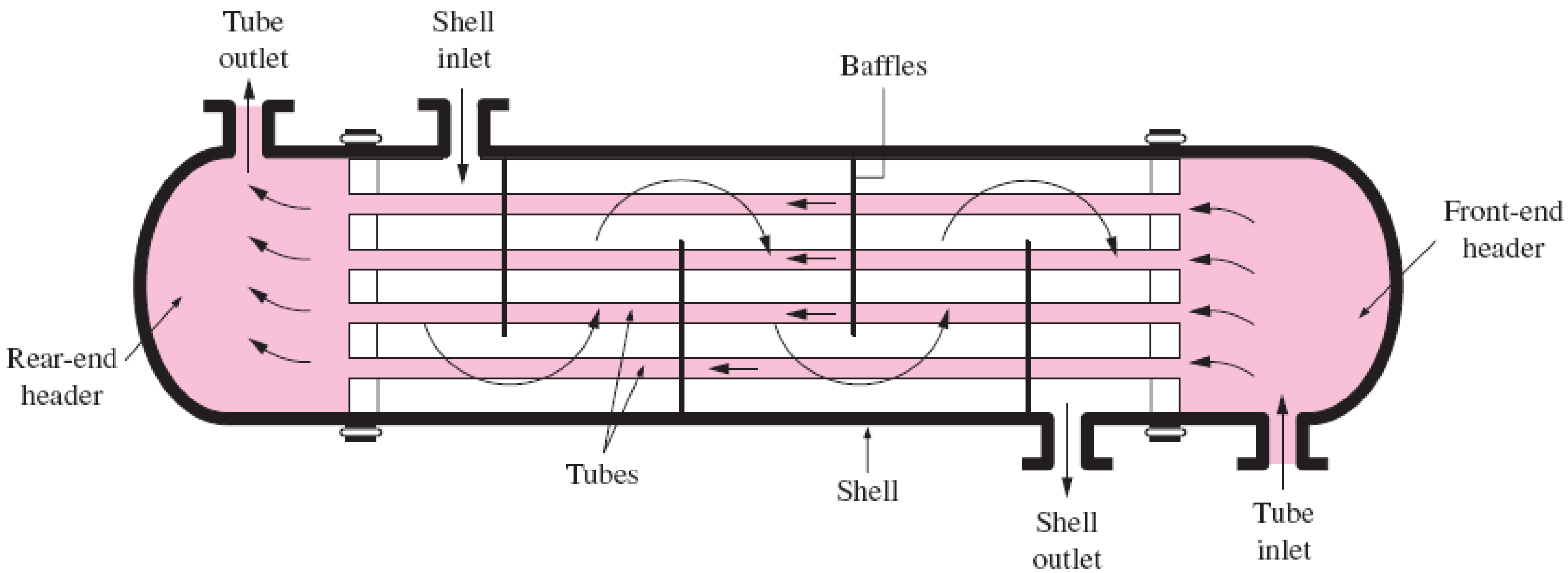


(a) Both fluids unmixed

(b) One fluid mixed, one fluid unmixed

Dwarsstroming (cross-flow): de 2 fluïda bewegen meestal *loodrecht* t.o.v. elkaar in een compacte warmtewisselaar. De dwarsstroming wordt verder geclassificeerd als *ongemengde* en *gemengde stroming*.

• Shell-and tube warmtewisselaar

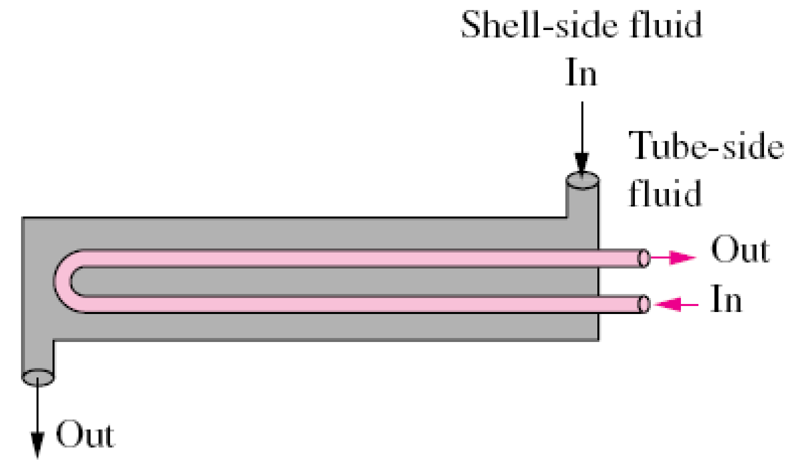


Buizenwarmtewisselaar: het meest gebruikte type voor industriële toepassingen.

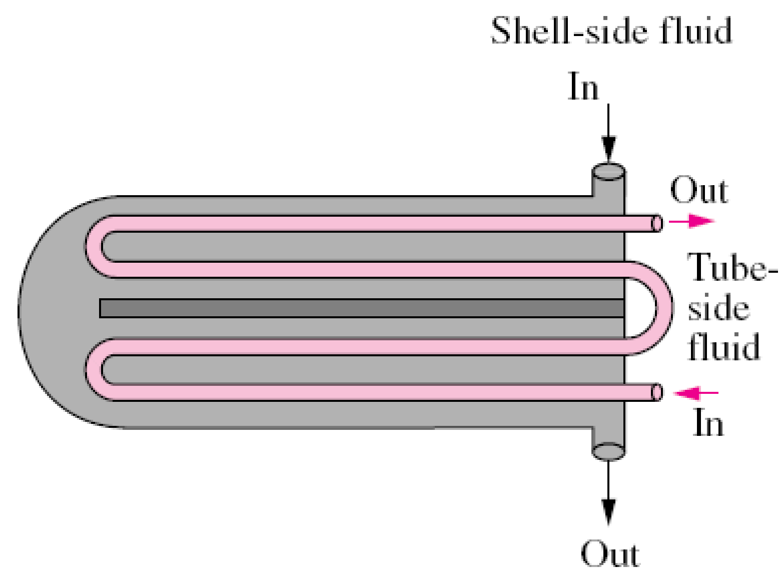
Een groot aantal buizen zijn gepakt (dikwijls >100). Warmteoverdracht grijpt plaats doordat het ene fluïdum door de buizen stroomt en het andere door de shell.

- # Buizenwarmtewisselaar

Buizenwarmtewisselaars worden verder onderverdeeld naargelang het aantal buis- en shell-passages

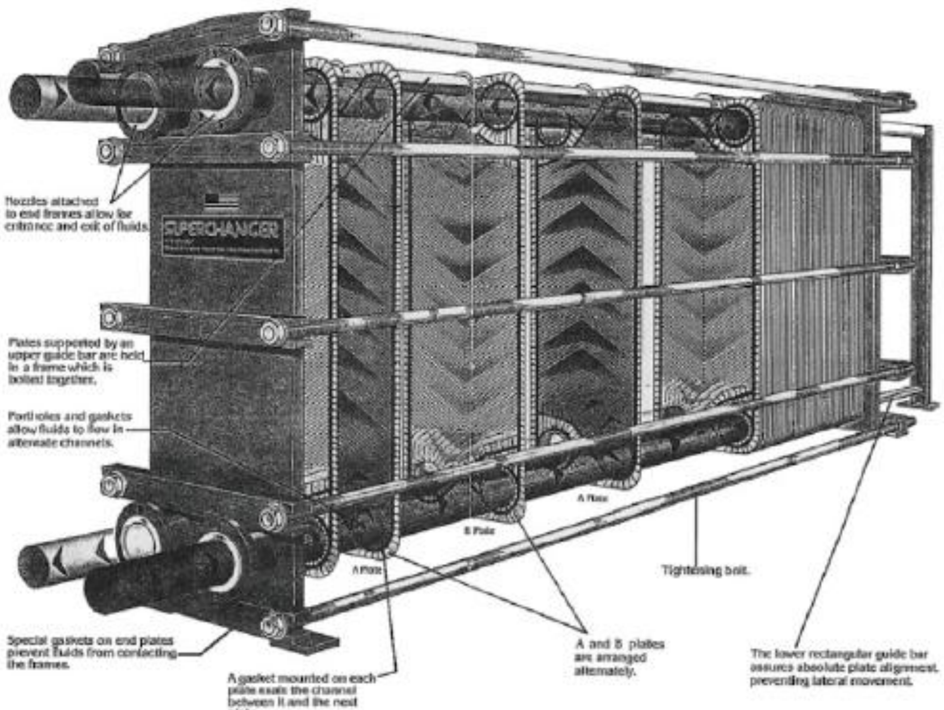


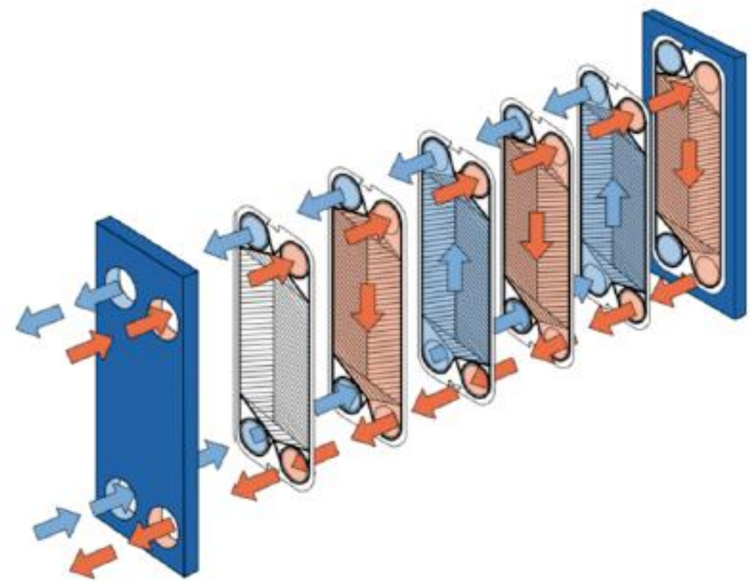
(*a*) One-shell pass and two-tube passes



(*b*) Two-shell passes and four-tube passes

- Platenwarmtewisselaar
  - Vooral geschikt voor vloeistof-vloeistof-warmteoverdracht
  - Vb. afkoelen van wort (bierproductie); opwarmen (pasteuriseren) van dranken (melk, bier)





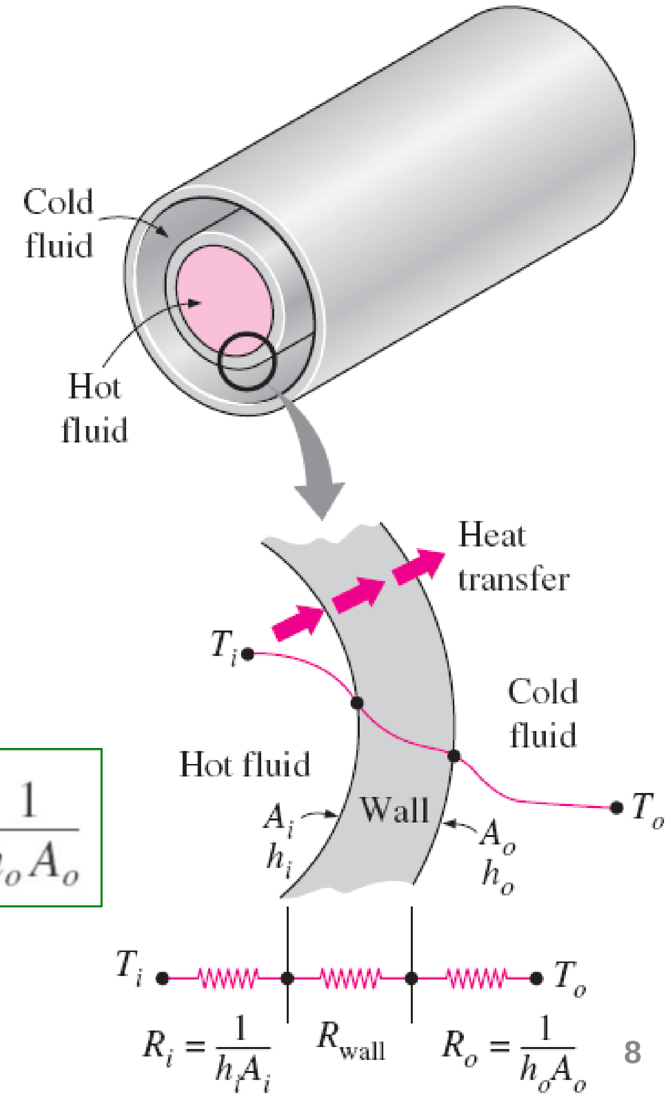
2. De totale warmteoverdrachtscoëfficiënt

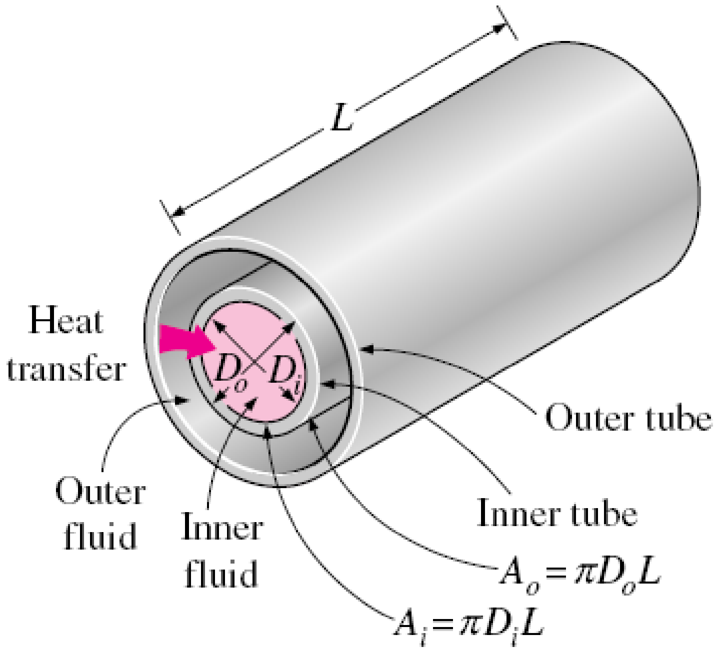
- Thermisch weerstandsnetwerk geassocieerd met warmteoverdracht in een dubbele-buis-warmtewisselaar

$$R_{\text{wall}} = \frac{\ln (D_o/D_i)}{2\pi kL}$$

$$\boxed{R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln (D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o}}$$

$$A_i = \pi D_i L \text{ and } A_o = \pi D_o L$$





$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

U de totale warmteoverdrachtscoëfficiënt, W/m$^2 \cdot$ °C

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o}$$

$$U_i A_i = U_o A_o, \text{ but } U_i \neq U_o \text{ unless } A_i = A_o$$

Als $R_{\text{wall}} \approx 0$ **en** $A_i \approx A_o \approx A_s$

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \qquad U \approx U_i \approx U_o$$

Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	U, W/m^2 · °C*
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60†
	400–850†
Steam-to-air in finned tubes (steam in tubes)	30–300†
	400–4000‡

De totale warmteoverdrachtcoëfficiënt varieert van ongeveer 10 W/m^2 · °C voor gas-gas-warmtewisselaars tot ongeveer 10 000 W/m^2 · °C voor warmtewisselaars met een faseverandering (condensors, boilers).

Bevuilingsfactor R_f

- Bevuiling = bijkomende weerstand voor warmteoverdacht
- Precipitatie: Ca-neerslag
- Corrosie
- Chemische bevuiling: producten van chemische reacties
- Biologische bevuiling: algen
- R_f = 0 voor nieuwe warmtewisselaars, de waarde stijgt bij gebruik

Neerslagbevuiling van asdeeltjes op hete buizen.

- R_f hangt af van: de temperatuur, vloeistofsnelheid, gebruiksperiode
- R_f neemt toe met toenemende temperatuur en dalende snelheid

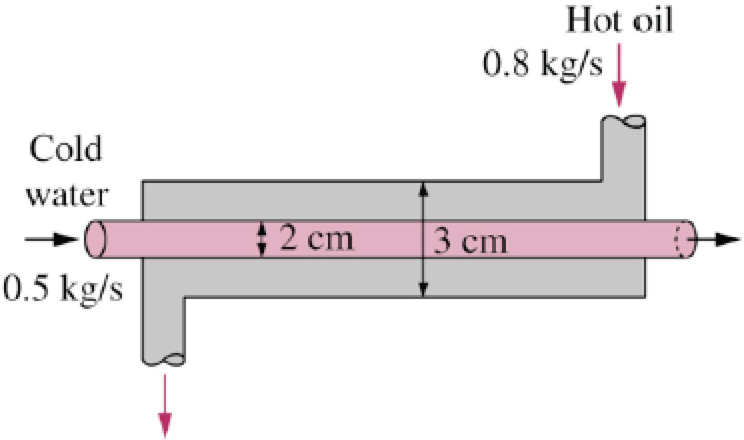
- Bevuilingsfactor R_f

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

Representative fouling factors (thermal resistance due to fouling for a unit surface area)

Fluid	R_f, $m^2 \cdot °C/W$
Distilled water, seawater, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

• Voorbeeld 1: De totale warmteoverdrachtscoëffiënt



Eigenschappen bij van water bij 45°C:
k = 0,637 W/(m °C); Pr = 3,91
ν = 0,602 10^{-6} m²/s; ρ = 990 kg/m³

Eigenschappen bij van olie bij 80°C:
k = 0,138 W/(m °C); Pr = 490
ν = 37,5 10^{-6} m²/s; ρ = 852 kg/m³

Bepaal de totale warmteoverdrachtscoëfficiënt.

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

De dikte van de binnenste buis is te verwaarlozen.

$$V_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{1}{4}\pi D^2)} = \frac{0.5 \text{ kg/s}}{(990 \text{ kg/m}^3)[\frac{1}{4}\pi(0.02 \text{ m})^2]} = 1.61 \text{ m/s}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(1.61 \text{ m/s})(0.02 \text{ m})}{0.602 \times 10^{-6} \text{ m}^2\text{/s}} = 53{,}490$$

=> turbulente stroming

$$\text{Nu} = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8}\text{Pr}^{0.4} = 0.023(53{,}490)^{0.8}(3.91)^{0.4} = 240.6$$

$$h = \frac{k}{D_h}\mathrm{Nu} = \frac{0.637\ \mathrm{W/m \cdot {}^\circ C}}{0.02\ \mathrm{m}}(240.6) = 7663\ \mathrm{W/m^2 \cdot {}^\circ C}$$

Olie:

$$D_h = D_o - D_i = 0.03 - 0.02 = 0.01\ \mathrm{m}$$

$$V_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho[\frac{1}{4}\pi(D_o^2 - D_i^2)]} = \frac{0.8\ \mathrm{kg/s}}{(852\ \mathrm{kg/m^3})[\frac{1}{4}\pi(0.03^2 - 0.02^2)]\ \mathrm{m^2}} = 2.39\ \mathrm{m/s}$$

$$\mathrm{Re} = \frac{V_m D_h}{\upsilon} = \frac{(2.39\ \mathrm{m/s})(0.01\ \mathrm{m})}{37.5 \times 10^{-6}\ \mathrm{m^2/s}} = 637$$

$\mathrm{Nu}_o = 4.57$

$D_i/D_o = 0.02/0.03 = 0.667$ $\quad \mathrm{Nu}_i \cancel{=} 5.45$ $\quad$ (Tabel 22-3)

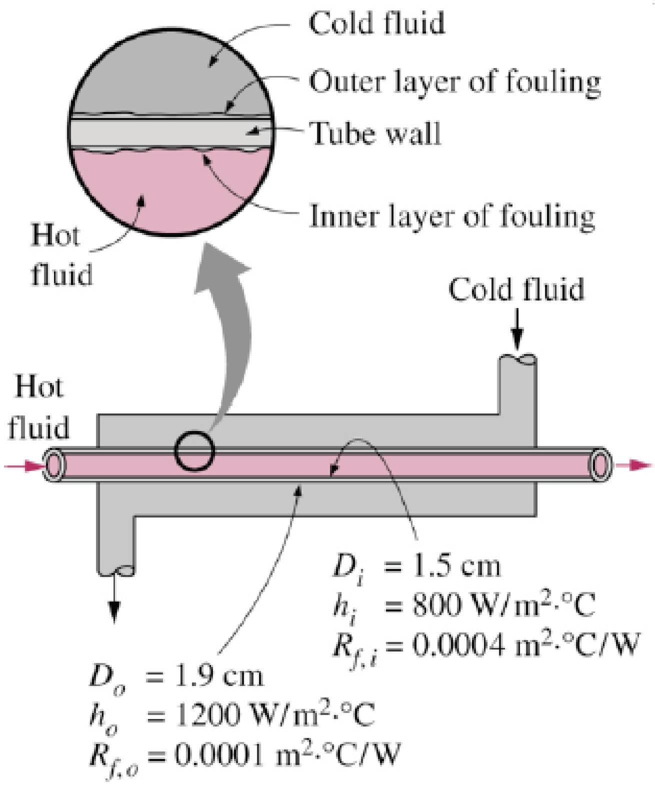
63.1

$$h_o = \frac{k}{D_h}\mathrm{Nu} = \frac{0.138\ \mathrm{W/m \cdot {}^\circ C}}{0.01\ \mathrm{m}}(5.\cancel{4}5) = 7\cancel{5}.2\ \mathrm{W/m^2 \cdot {}^\circ C}$$

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7663\ \mathrm{W/m^2 \cdot {}^\circ C}} + \frac{1}{7\cancel{5}.2\ \mathrm{W/m^2 \cdot {}^\circ C}}} = 7\cancel{4}.5\ \mathrm{W/m^2 \cdot {}^\circ C}$$

62.6

- Voorbeeld 2: Effect van bevuiling



Bepaal: (a) de thermische weerstand van de warmtewisselaar per eenheidslengte, en (b) de totale warmteoverdrachtscoëfficiënten U_i en U_o.

(a)

$$R = \frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

$$A_i = \pi D_i L = \pi(0.015 \text{ m})(1 \text{ m}) = 0.0471 \text{ m}^2$$

$$A_o = \pi D_o L = \pi(0.019 \text{ m})(1 \text{ m}) = 0.0597 \text{ m}^2$$

$$
\begin{aligned}
R &= \frac{1}{(800\ \mathrm{W/m^2\cdot{}^\circ C})(0.0471\ \mathrm{m^2})} + \frac{0.0004\ \mathrm{m^2\cdot{}^\circ C/W}}{0.0471\ \mathrm{m^2}} \\
&\quad + \frac{\ln(0.019/0.015)}{2\pi(15.1\ \mathrm{W/m\cdot{}^\circ C})(1\ \mathrm{m})} \\
&\quad + \frac{0.0001\ \mathrm{m^2\cdot{}^\circ C/W}}{0.0597\ \mathrm{m^2}} + \frac{1}{(1200\ \mathrm{W/m^2\cdot{}^\circ C})(0.0597\ \mathrm{m^2})} \\
&= (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396)\mathrm{^\circ C/W} \\
&= \mathbf{0.0532^\circ C/W}
\end{aligned}
$$

(b)

$$
U_i = \frac{1}{RA_i} = \frac{1}{(0.0532\ \mathrm{^\circ C/W})(0.0471\ \mathrm{m^2})} = \mathbf{399\ W/m^2\cdot{}^\circ C}
$$

$$
U_o = \frac{1}{RA_o} = \frac{1}{(0.0532\ \mathrm{^\circ C/W})(0.0597\ \mathrm{m^2})} = \mathbf{315\ W/m^2\cdot{}^\circ C}
$$

3. De analyse van warmtewisselaars

- Warmtebalansen

$$\boxed{\dot{Q} = \dot{m}_c c_{pc}(T_{c,\,\mathrm{out}} - T_{c,\,\mathrm{in}})}$$

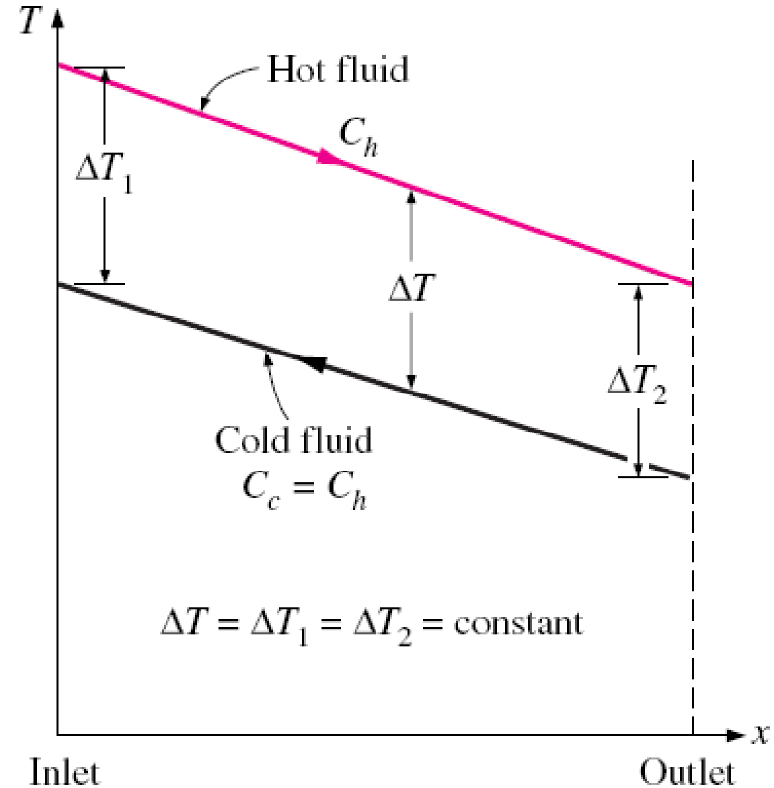
$$\boxed{\dot{Q} = \dot{m}_h c_{ph}(T_{h,\,\mathrm{in}} - T_{h,\,\mathrm{out}})}$$

$\dot{m}_c, \dot{m}_h$ = mass flow rates

c_{pc}, c_{ph} = specific heats

$T_{c,\,\mathrm{out}}, T_{h,\,\mathrm{out}}$ = outlet temperatures

$T_{c,\,\mathrm{in}}, T_{h,\,\mathrm{in}}$ = inlet temperatures



Warmtecapaciteitsdebiet:

$$C_h = \dot{m}_h c_{ph} \qquad \text{and} \qquad C_c = \dot{m}_c c_{pc}$$

$$\dot{Q} = C_c(T_{c,\,\mathrm{out}} - T_{c,\,\mathrm{in}}) \qquad \dot{Q} = C_h(T_{h,\,\mathrm{in}} - T_{h,\,\mathrm{out}})$$

- Condensors en boilers

$$\boxed{\dot{Q} = \dot{m} h_{fg}}$$

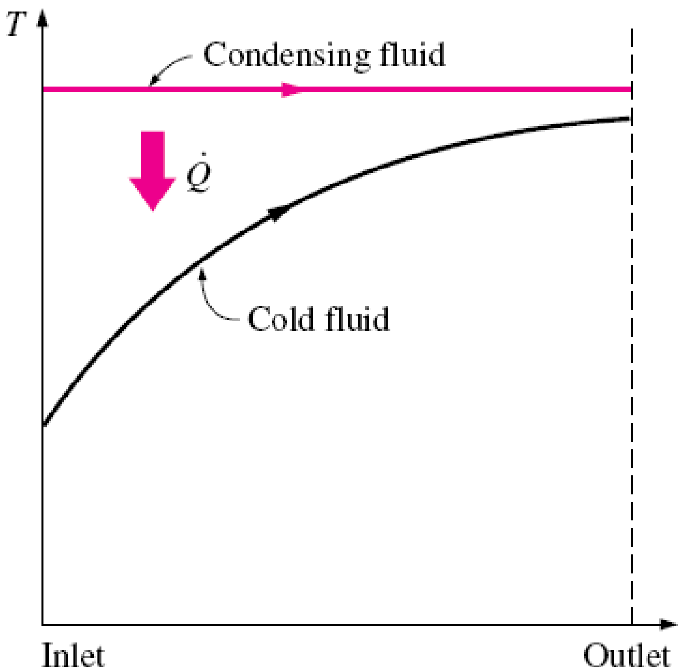
Condensatie- of verdampingswarmte (enthalpie)

$$\dot{Q} = \dot{m} C_p \Delta T$$

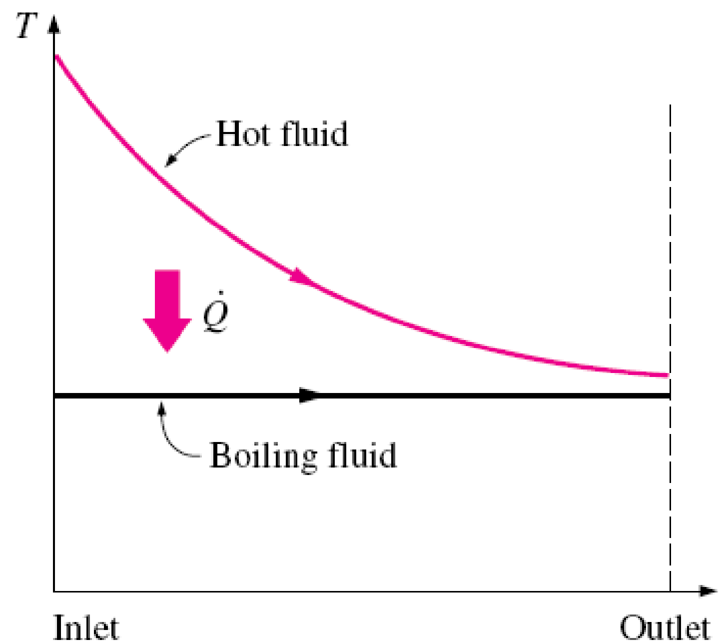
$C_h \rightarrow \infty$ of $C_c \rightarrow \infty$

dan $C = \dot{m} C_p \rightarrow \infty$ vermits $\Delta T \rightarrow 0$

De warmtecapaciteitsnelheid van een fluïdum gedurende een faseveranderingsproces gaat naar oneindig vermits de temperatuursverandering praktisch 0 is.



(a) Condenser $\boxed{(C_h \rightarrow \infty)}$



(b) Boiler $\boxed{(C_c \rightarrow \infty)}$

4. De logaritmisch-gemiddelde-temperatuursverschil-methode

- Het temperatuursverschil tussen de warme en koude vloeistof varieert langs de warmtewisselaar => werken met een gemiddeld temperatuursverschil

$$\boxed{\dot{Q} = UA_s\,\Delta T_m}$$

- Gelijkstroom-dubbelbuiswarmtewisselaar:

$$\delta\dot{Q} = -\dot{m}_h c_{ph}\, dT_h \qquad \delta\dot{Q} = \dot{m}_c c_{pc}\, dT_c$$

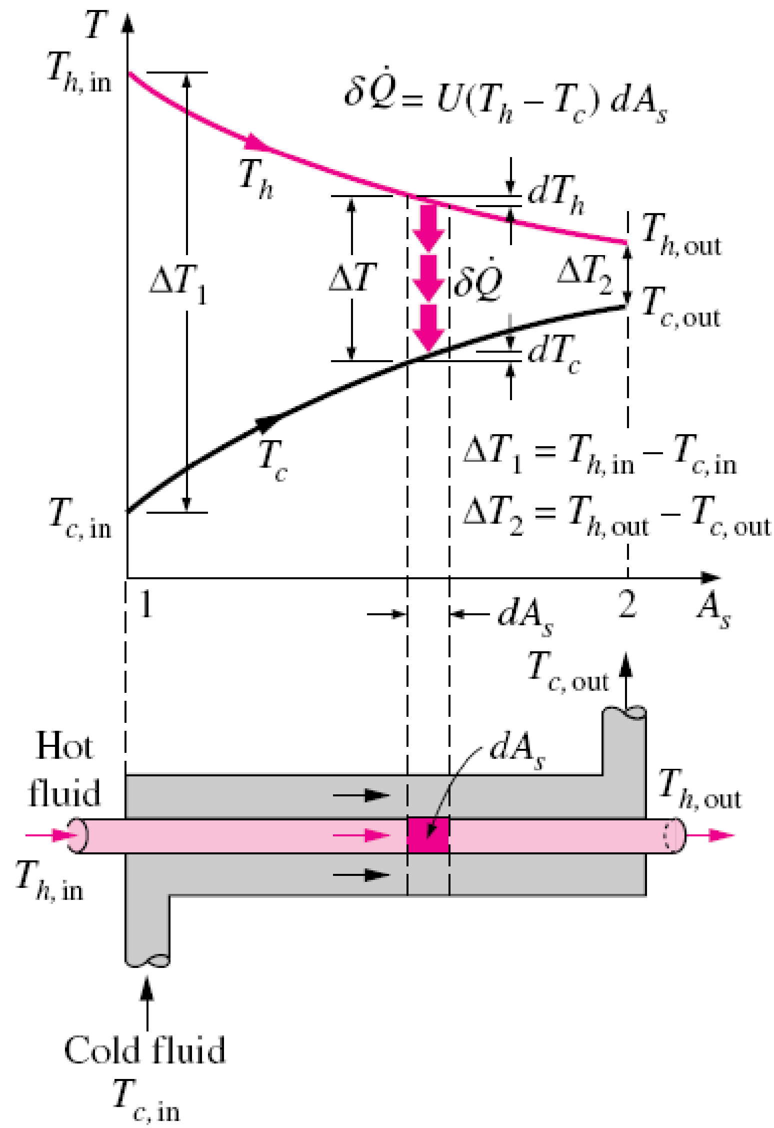
$$dT_h = -\frac{\delta\dot{Q}}{\dot{m}_h c_{ph}} \qquad dT_c = \frac{\delta\dot{Q}}{\dot{m}_c c_{pc}}$$

$$dT_h - dT_c = d(T_h - T_c) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$

$$\delta\dot{Q} = U(T_h - T_c)\, dA_s$$

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\, dA_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$

$$\ln\frac{T_{h,\text{out}} - T_{c,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = -UA_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$



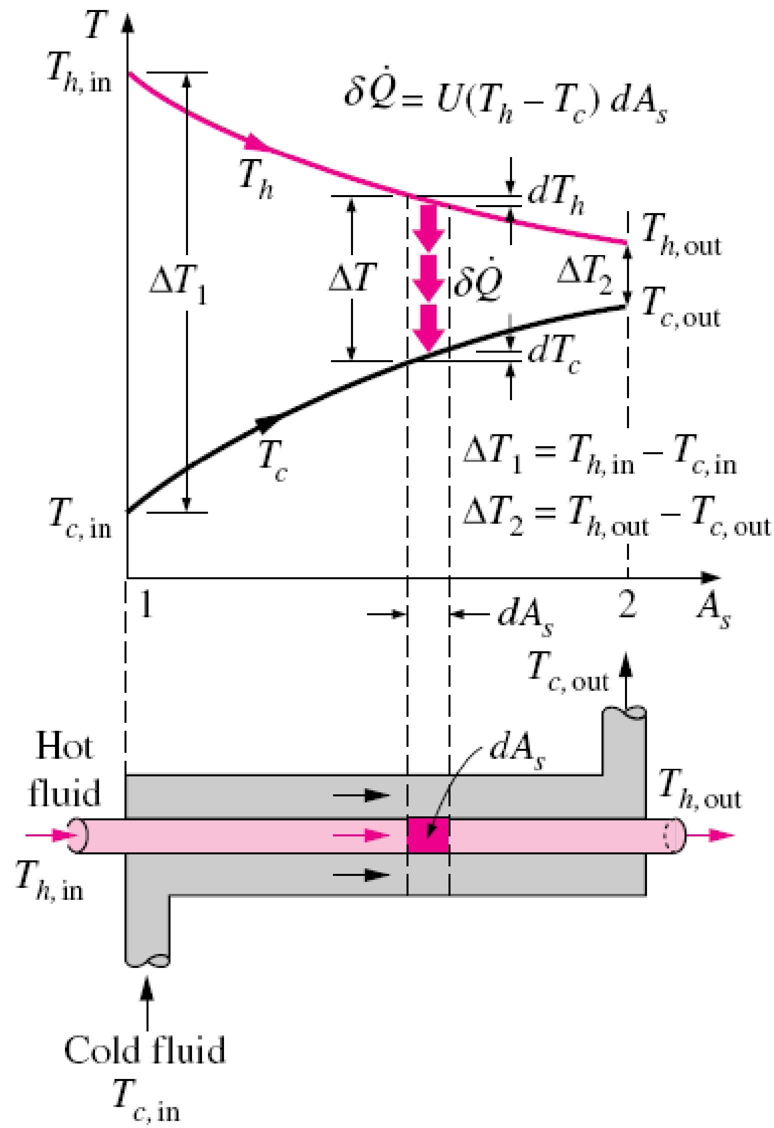
$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,\text{ out}} - T_{c,\text{ in}})$$

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,\text{ in}} - T_{h,\text{ out}})$$

$$\ln\frac{T_{h,\text{ out}} - T_{c,\text{ out}}}{T_{h,\text{ in}} - T_{c,\text{ in}}} = -UA_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}}$$

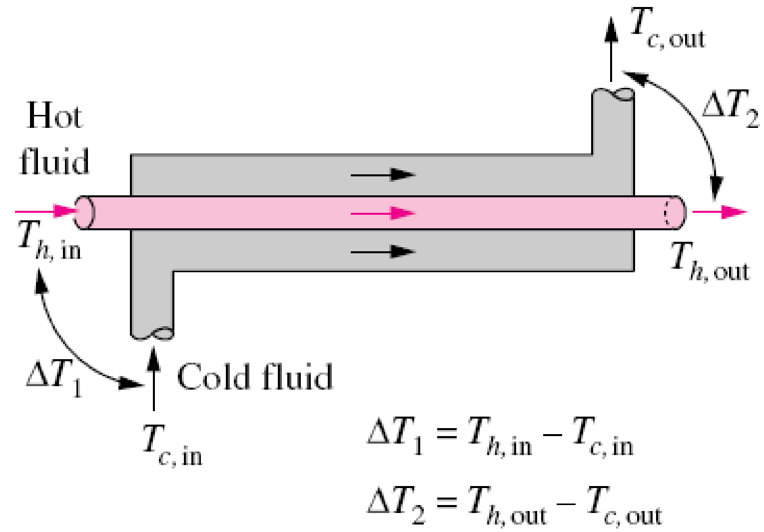
$$\boxed{\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}}$$



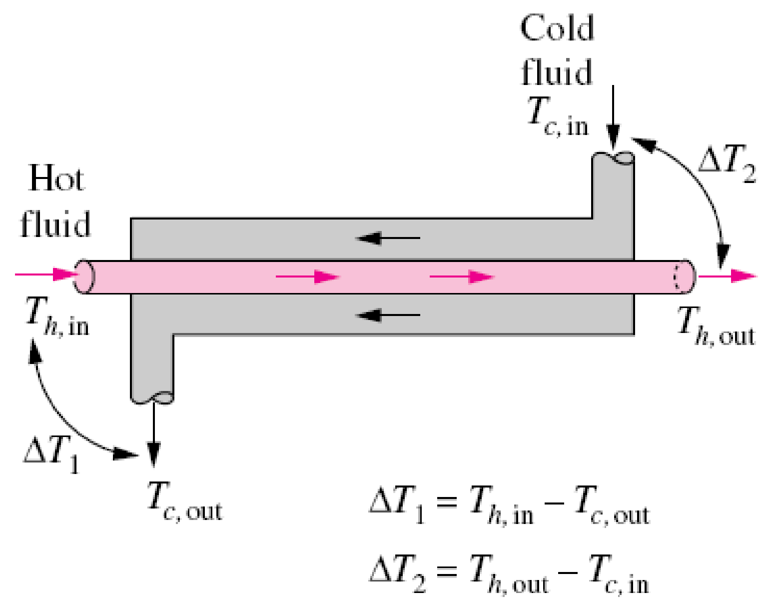
- Gelijkstroomwarmtewisselaar

- Tegenstroomwarmtewisselaar

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)}$$



(*a*) Parallel-flow heat exchangers



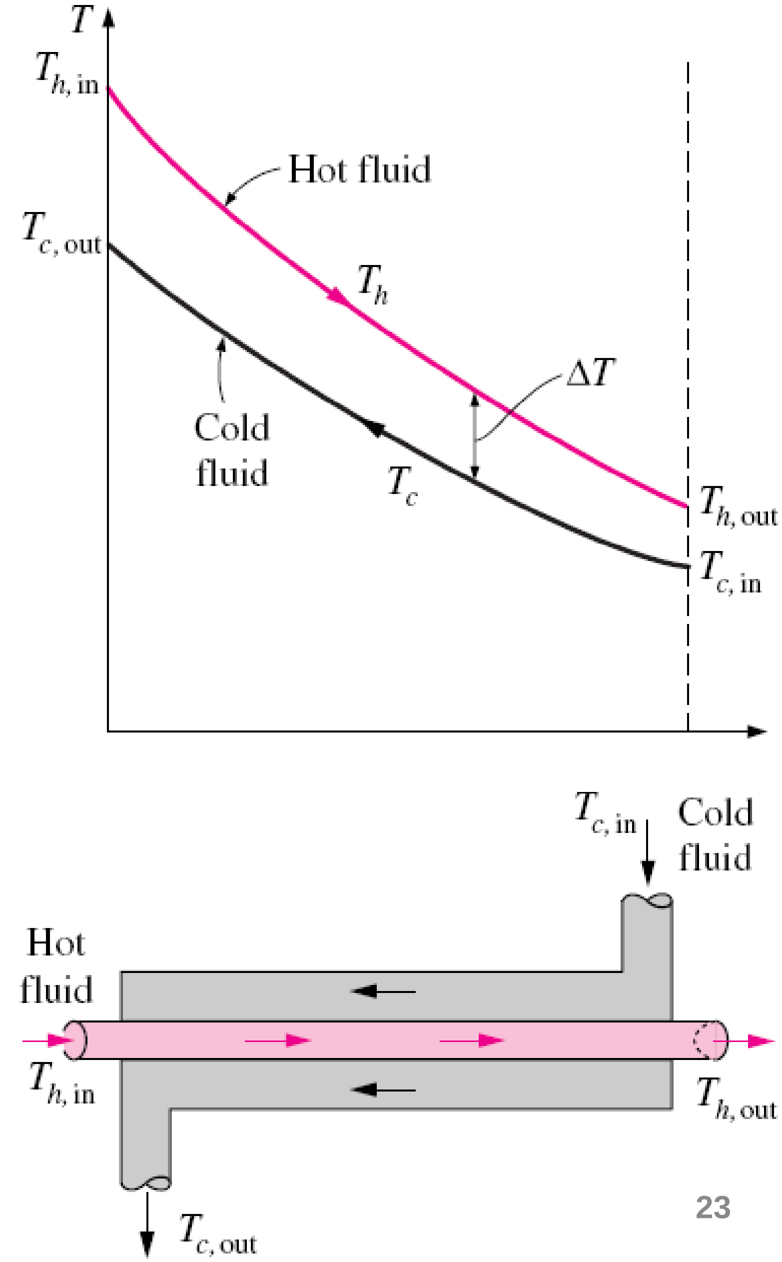
(*b*) Counter-flow heat exchangers

- Tegenstroomwarmtewisselaar

$$\boxed{\Delta T_{\text{lm, CF}} > \Delta T_{\text{lm, PF}}}$$

⇨ Kleiner oppervlak

(= kleinere warmtewisselaar) is nodig bij tegenstroom



- **“Multipass”- en “cross flow”-warmtewisselaars**: het gebruik van een correctiefactor *F*

$$\boxed{\Delta T_{\text{lm}} = F\,\Delta T_{\text{lm, CF}}}$$

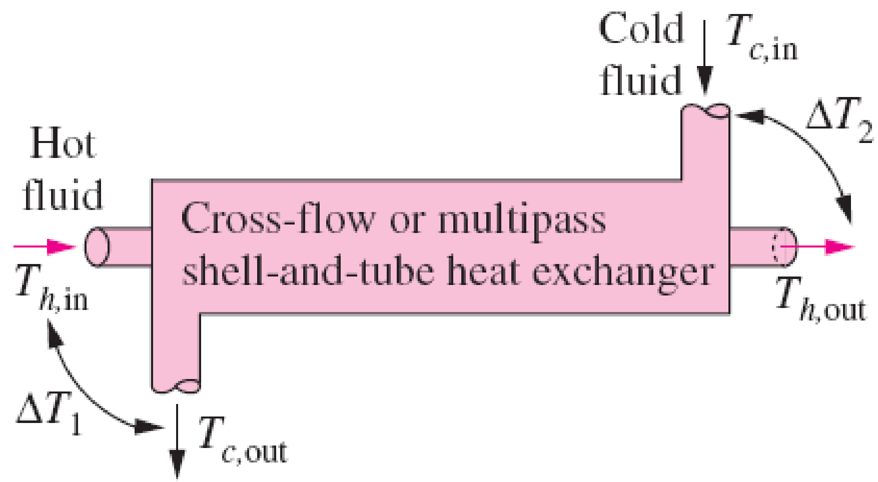
F – correctiefactor hangt van de *geometrie* van de warmtewisselaar af en van de in- en uitlaattemperaturen van de warme en koude stromen.

F wordt bepaald m.b.v. figuren:

$$P = \frac{t_2 - t_1}{T_1 - t_1} \qquad R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}c_p)_{\text{tube side}}}{(\dot{m}c_p)_{\text{shell side}}}$$

1 en 2: *inlaat* en *uitlaat*
T en *t*: *shell*- en *tube-side*-temperaturen

F = 1 voor een condensor of boiler



Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{\text{lm,CF}}$$

where $$\Delta T_{\text{lm,CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$$

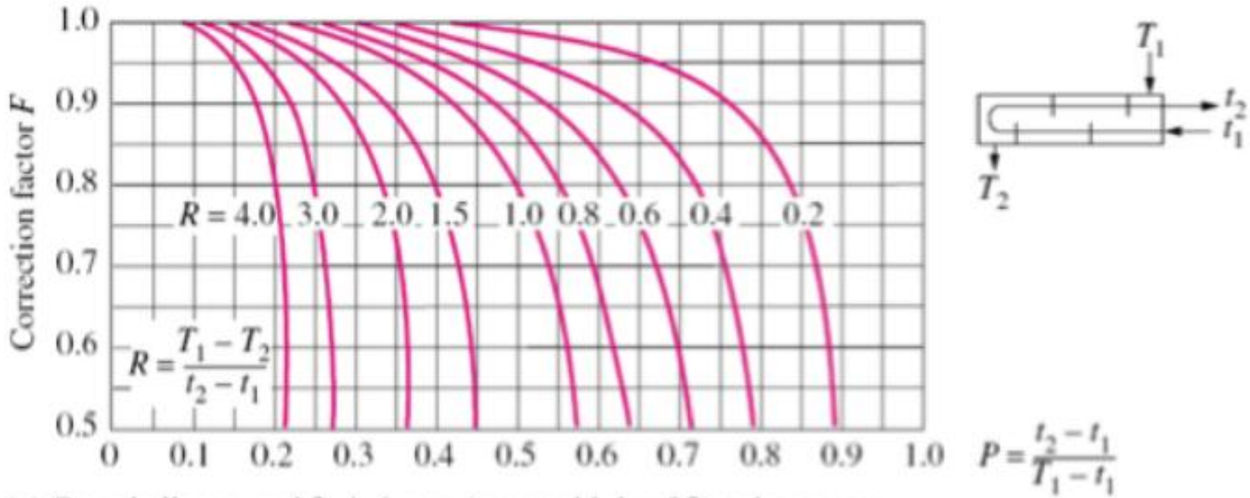
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$$

and $F = \ldots$ (Fig. 22–18)

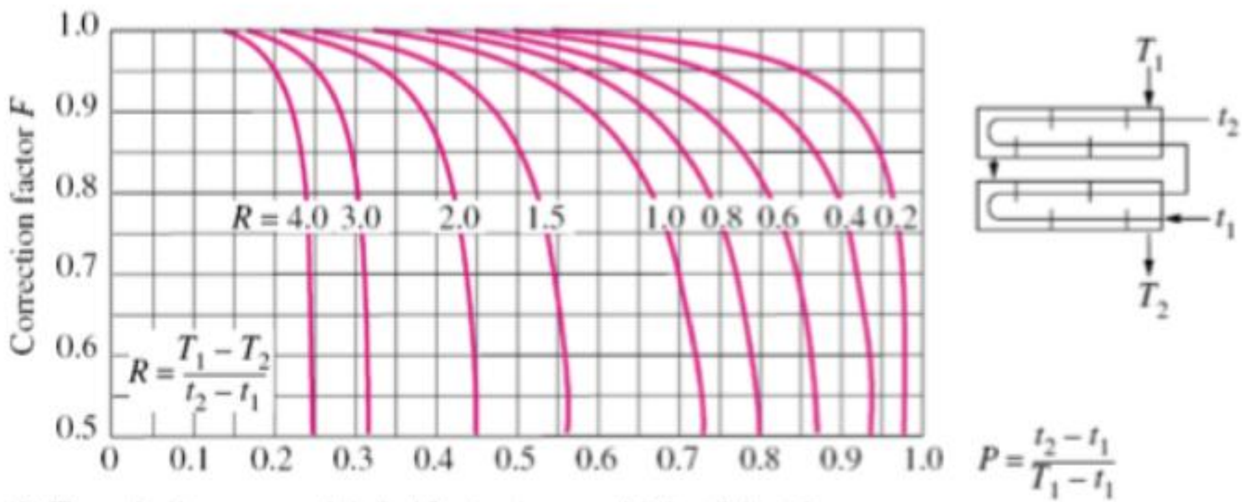
- "Multipass"- buizenwarmtewisselaars: het gebruik van een correctiefactor *F*

$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}c_p)_{\text{tube side}}}{(\dot{m}c_p)_{\text{shell side}}}$$

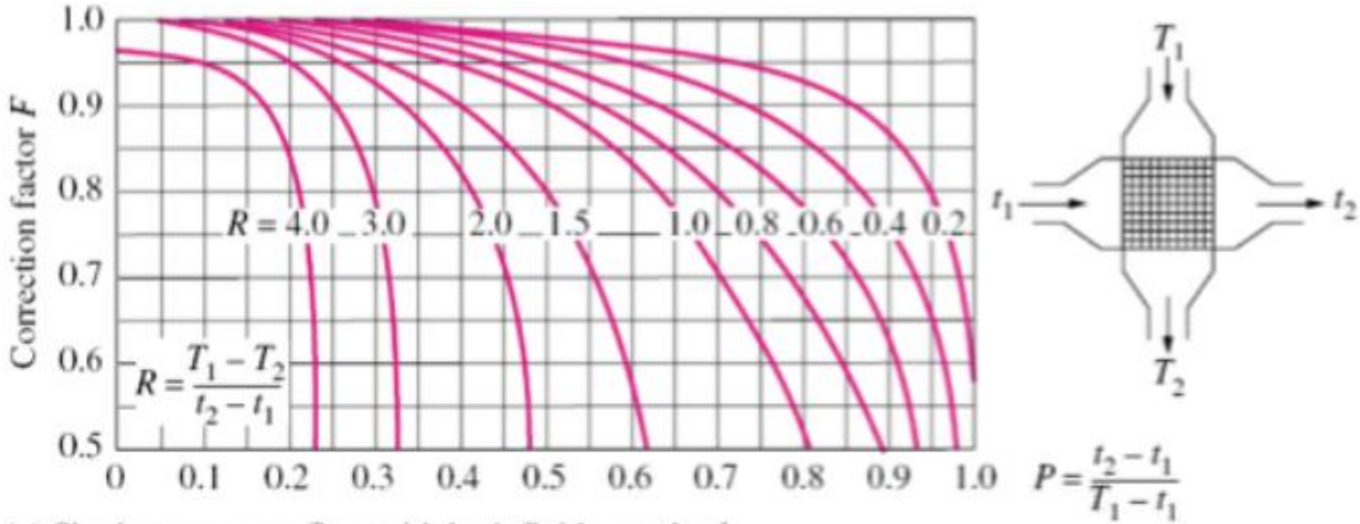


(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

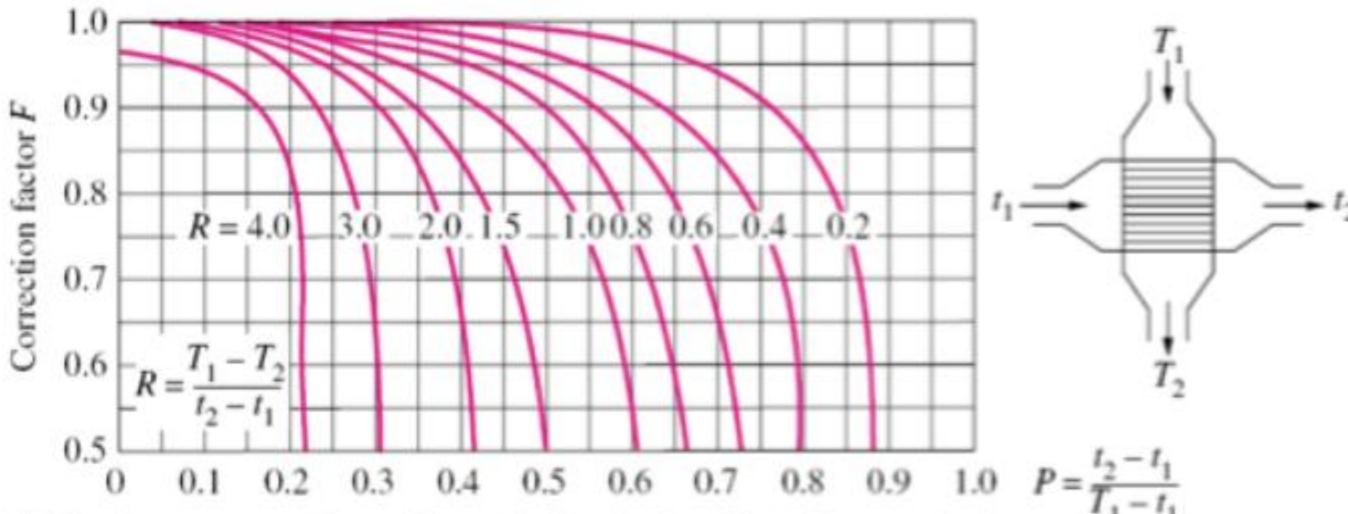


(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

- “Cross flow”-warmtewisselaars: het gebruik van een correctiefactor *F*

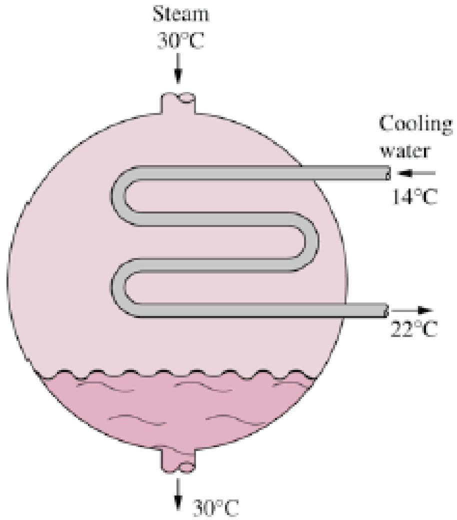


(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

• Voorbeeld 1: **Condensatie van stoom in een condenser**



Oppervlakte van de buizen = 45 m^2
U = 2100 W/(m^2 °C)

Bepaal het massadebiet van het koelwater en het massadebiet van het condensaat.

$$\Delta T_1 = T_{h,\,\text{in}} - T_{c,\,\text{out}} = (30 - 22)°\text{C} = 8°\text{C}$$
$$\Delta T_2 = T_{h,\,\text{out}} - T_{c,\,\text{in}} = (30 - 14)°\text{C} = 16°\text{C}$$

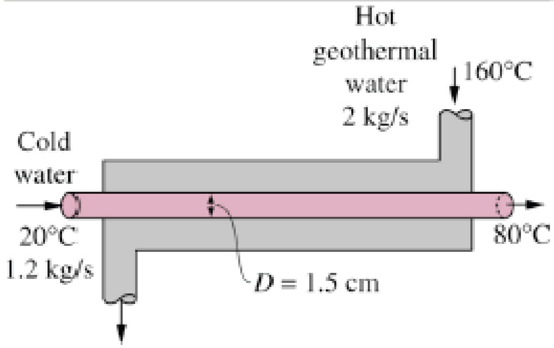
$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8-16}{\ln(8/16)} = 11.5°\text{C}$$

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}} = (2100\ \text{W/m}^2\cdot°\text{C})(45\ \text{m}^2)(11.5°\text{C}) = \mathbf{1.087 \times 10^6\ W = 1087\ kW}$$

$$\dot{m}_{\text{cooling water}} = \frac{\dot{Q}}{C_p(T_{\text{out}} - T_{\text{in}})} = \frac{1087\ \text{kJ/s}}{(4.184\ \text{kJ/kg}\cdot°\text{C})(22-14)°\text{C}} = \mathbf{32.5\ kg/s}$$

$$\dot{m}_{\text{steam}} = \frac{\dot{Q}}{h_{fg}} = \frac{1087\ \text{kJ/s}}{2431\ \text{kJ/kg}} = \mathbf{0.45\ kg/s}$$

• Voorbeeld 2: Tegenstroomwarmtewisselaar



U = 640 W/(m² °C)

Bepaal de lengte van de warmtewisselaar.

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C})(80-20)^\circ\text{C} = 301\ \text{kW}$$

$$\dot{Q} = [\dot{m}C_p(T_{\text{in}} - T_{\text{out}})]_{\text{geothermal}} \longrightarrow T_{\text{out}} = T_{\text{in}} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 160^\circ\text{C} - \frac{301\ \text{kW}}{(2\ \text{kg/s})(4.31\ \text{kJ/kg}\cdot{}^\circ\text{C})}$$

$$= 125^\circ\text{C}$$

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = (160-80)^\circ\text{C} = 80^\circ\text{C}$$

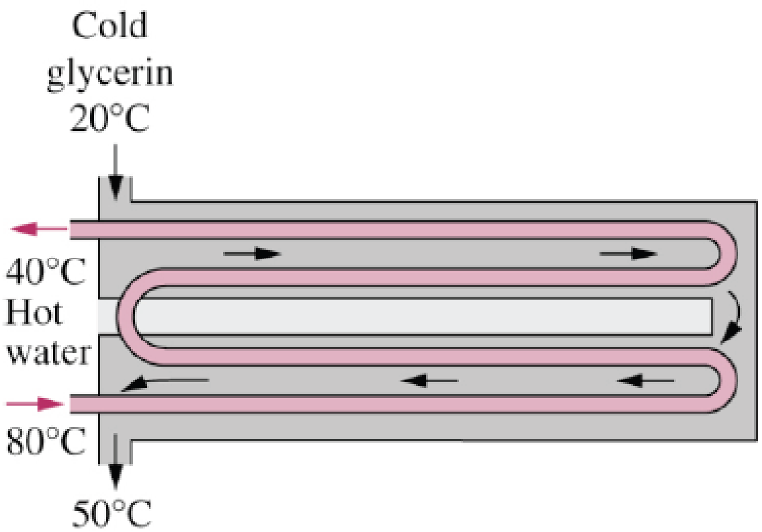
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = (125-20)^\circ\text{C} = 105^\circ\text{C}$$

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{80-105}{\ln(80/105)} = 92.0^\circ\text{C}$$

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}} \longrightarrow A_s = \frac{\dot{Q}}{U\,\Delta T_{\text{lm}}} = \frac{301{,}000\ \text{W}}{(640\ \text{W/m}^2\cdot{}^\circ\text{C})(92.0^\circ\text{C})} = 5.11\ \text{m}^2$$

$$A_s = \pi D L \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11\ \text{m}^2}{\pi(0.015\ \text{m})} = 108\ \text{m}$$

- Voorbeeld 3: "Multipass"-warmtewisselaar



Shell side: h = 25 W/(m² °C)
Tube side: h = 160 W/(m² °C)
R_f = 0,0006 m² °C/W

Bepaal het warmtedebiet (a) voordat bevuiling optrad, en (b) na bevuiling.

(a)

$$A_s = \pi D L = \pi(0.02\text{ m})(60\text{ m}) = 3.77\text{ m}^2$$

$$\dot{Q} = UA_s F\,\Delta T_{\text{lm, CF}}$$

$$\Delta T_1 = T_{h,\text{ in}} - T_{c,\text{ out}} = (80 - 50)°\text{C} = 30°\text{C}$$

$$\Delta T_2 = T_{h,\text{ out}} - T_{c,\text{ in}} = (40 - 20)°\text{C} = 20°\text{C}$$

$$\Delta T_{\text{lm, CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{30 - 20}{\ln(30/20)} = 24.7°\text{C}$$

$$
\left.\begin{aligned}
P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67 \\
R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75
\end{aligned}\right\} F = 0.91 \qquad \text{(Fig. 2}\cancel{3}\text{2–18}b\text{)}
$$

$$
U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2\cdot{}^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2\cdot{}^\circ\text{C}}} = 21.6\ \text{W/m}^2\cdot{}^\circ\text{C}
$$

$$
\dot{Q} = UA_s F\,\Delta T_{\text{lm, CF}} = (21.6\ \text{W/m}^2\cdot{}^\circ\text{C})(3.77\text{m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1830\ W}
$$

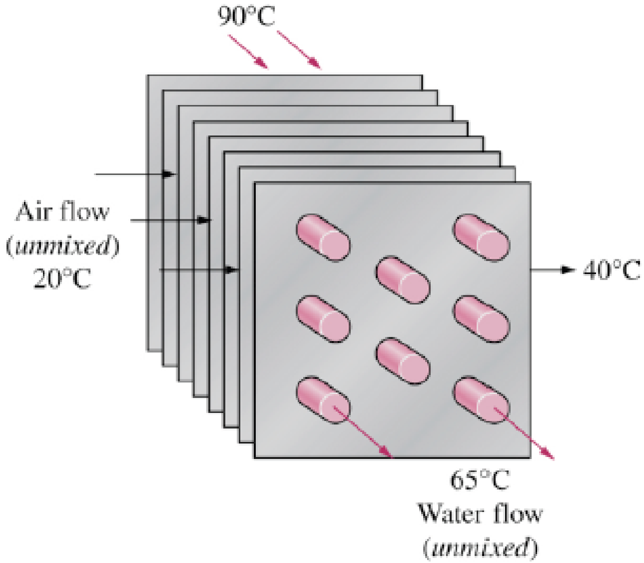
(b)

$$
U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o} + R_f} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2\cdot{}^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2\cdot{}^\circ\text{C}} + 0.0006\ \text{m}^2\cdot{}^\circ\text{C/W}}
$$

$$
= 21.3\ \text{W/m}^2\cdot{}^\circ\text{C}
$$

$$
\dot{Q} = UA_s F\,\Delta T_{\text{lm, CF}} = (21.3\ \text{W/m}^2\cdot{}^\circ\text{C})(3.77\ \text{m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1805\ W}
$$

Voorbeeld 4: Koeling van een radiator



40 buizen
Interne diameter = 0.5 cm, lengte = 65 cm
Massadebiet warm water = 0.6 kg/s
c_p = 4,195 kJ/(kg °C) bij de gemiddelde temperatuur
= (90 + 65)/2 = 77,5°C

Bepaal de totale warmteoverdrachtscoëffiënt U_i.

$$\dot{Q} = [\dot{m}C_p(T_{\text{in}} - T_{\text{out}})]_{\text{water}} = (0.6 \text{ kg/s})(4.195 \text{ kJ/kg} \cdot {}^\circ\text{C})(90 - 65)^\circ\text{C} = 62.93 \text{ kW}$$

$$A_i = n\pi D_i L = (40)\pi(0.005 \text{ m})(0.65 \text{ m}) = 0.408 \text{ m}^2$$

$$\dot{Q} = U_i A_i F \Delta T_{\text{lm, CF}} \longrightarrow U_i = \frac{\dot{Q}}{A_i F \Delta T_{\text{lm, CF}}}$$

$$\Delta T_1 = T_{h,\text{ in}} - T_{c,\text{ out}} = (90 - 40)°\text{C} = 50°\text{C}$$

$$\Delta T_2 = T_{h,\text{ out}} - T_{c,\text{ in}} = (65 - 20)°\text{C} = 45°\text{C}$$

$$\Delta T_{\text{lm, CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{50 - 45}{\ln(50/45)} = 47.6°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{65 - 90}{20 - 90} = 0.36 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 40}{65 - 90} = 0.80 \end{aligned}\right\} F = 0.97 \qquad \text{(Fig. 22–18}c\text{)}$$

$$U_i = \frac{\dot{Q}}{A_i F \Delta T_{\text{lm, CF}}} = \frac{62{,}930 \text{ W}}{(0.408 \text{ m}^2)(0.97)(47.6°\text{C})} = \mathbf{3341 \text{ W/m}^2 \cdot °\text{C}}$$

5. De effectiviteit-NTU-methode

- **LMTD-methode**: inlaat- en uitlaattemperaturen zijn gekend of kunnen bepaald worden via de energiebalans => bepalen van nodige oppervlakte A_s met $$\boxed{\dot{Q} = UA_s\,\Delta T_{\mathrm{lm}}}$$
- **Procedure**
  - Selecteer een bepaald type warmtewisselaar
  - Bepaal onbekende inlaat- of uitlaattemperatuur en warmtedebiet m.b.v. energiebalans
  - Bereken ΔT_{lm} en correctiefactor F (indien nodig)
  - Selecteer/bereken de waarde van U
  - Bereken de warmteoverdrachtsoppervlakte A_s

- **2e type probleem:**

Bepaal het warmtedebiet en uitlaattemperatuur van de warme en koude vloeistoffen voor een gekend massadebiet en inlaattemperaturen als type en grootte van de warmtewisselaar gespecificeerd zijn (uitlaattemperaturen onbekend)

→ Gebruik de effectiviteit-NTU-methode

- Warmteoverdrachtseffectiviteit ε

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) = C_h(T_{h,\text{ in}} - T_{h,\text{ out}})$$

$$C_c = \dot{m}_c c_{pc} \text{ and } C_h = \dot{m}_c c_{ph}$$

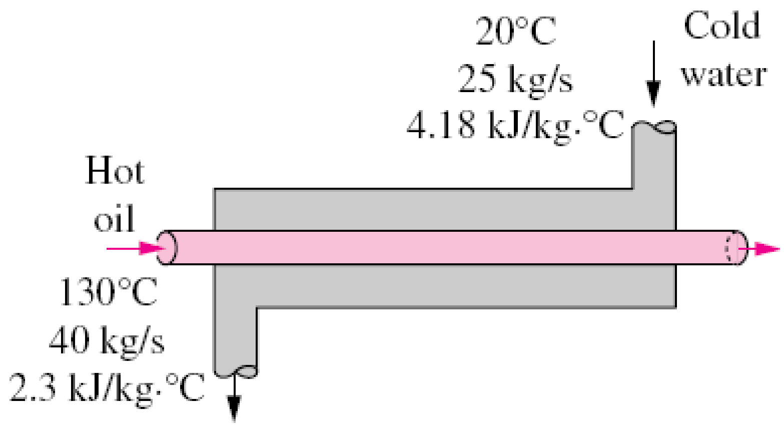
$$\Delta T_{\max} = T_{h,\text{ in}} - T_{c,\text{ in}}$$

$$\boxed{\dot{Q}_{\max} = C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}})}$$

Met C_{min} de kleinste waarde voor

$$C_c = \dot{m}_c c_{pc} \text{ of } C_h = \dot{m}_c c_{ph}$$

$$\boxed{\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}})}$$



$$C_c = \dot{m}_c c_{pc} = 104.5 \text{ kW/°C}$$

$$C_h = \dot{m}_c c_{ph} = 92 \text{ kW/°C}$$

$$C_{\min} = 92 \text{ kW/°C}$$

$$\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} = 110\text{°C}$$

$$\dot{Q}_{\max} = C_{\min}\,\Delta T_{\max} = 10{,}120 \text{ kW}$$

- Effectiviteitsrelatie voor een dubbele-buis-warmtewisselaar met gelijkstroming

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,\,\mathrm{in}} - T_{c,\,\mathrm{in}})$$

$$\ln \frac{T_{h,\,\mathrm{out}} - T_{c,\,\mathrm{out}}}{T_{h,\,\mathrm{in}} - T_{c,\,\mathrm{in}}} = -UA_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right) \longrightarrow \ln \frac{T_{h,\,\mathrm{out}} - T_{c,\,\mathrm{out}}}{T_{h,\,\mathrm{in}} - T_{c,\,\mathrm{in}}} = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

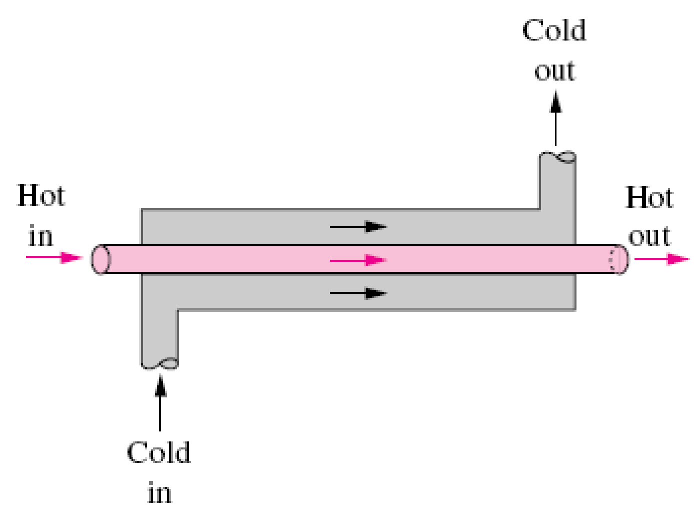
$$\dot{Q} = C_c(T_{c,\,\mathrm{out}} - T_{c,\,\mathrm{in}}) = C_h(T_{h,\,\mathrm{in}} - T_{h,\,\mathrm{out}}) \longrightarrow T_{h,\,\mathrm{out}} = T_{h,\,\mathrm{in}} - \frac{C_c}{C_h}(T_{c,\,\mathrm{out}} - T_{c,\,\mathrm{in}})$$

$$\ln \frac{T_{h,\,\mathrm{in}} - T_{c,\,\mathrm{in}} + T_{c,\,\mathrm{in}} - T_{c,\,\mathrm{out}} - \dfrac{C_c}{C_h}(T_{c,\,\mathrm{out}} - T_{c,\,\mathrm{in}})}{T_{h,\,\mathrm{in}} - T_{c,\,\mathrm{in}}} = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

$$\ln\left[1 - \left(1 + \frac{C_c}{C_h}\right)\frac{T_{c,\,\mathrm{out}} - T_{c,\,\mathrm{in}}}{T_{h,\,\mathrm{in}} - T_{c,\,\mathrm{in}}}\right] = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,\,\mathrm{out}} - T_{c,\,\mathrm{in}})}{C_{\min}(T_{h,\,\mathrm{in}} - T_{c,\,\mathrm{in}})} \longrightarrow \frac{T_{c,\,\mathrm{out}} - T_{c,\,\mathrm{in}}}{T_{h,\,\mathrm{in}} - T_{c,\,\mathrm{in}}} = \varepsilon\frac{C_{\min}}{C_c}$$

$$\boxed{\varepsilon_{\text{parallel flow}} = \frac{1 - \exp\left[-\dfrac{UA_s}{C_c}\left(1 + \dfrac{C_c}{C_h}\right)\right]}{\left(1 + \dfrac{C_c}{C_h}\right)\dfrac{C_{\min}}{C_c}}}$$



(*a*) Parallel flow

$$\varepsilon_{\text{parallel flow}} = \frac{1 - \exp\left[-\dfrac{UA_s}{C_c}\left(1 + \dfrac{C_c}{C_h}\right)\right]}{\left(1 + \dfrac{C_c}{C_h}\right)\dfrac{C_{\min}}{C_c}}$$

Stel C_{min} de kleinste waarde voor C_c of C_h

$$\boxed{\varepsilon_{\text{parallel flow}} = \frac{1 - \exp\left[-\dfrac{UA_s}{C_{\min}}\left(1 + \dfrac{C_{\min}}{C_{\max}}\right)\right]}{1 + \dfrac{C_{\min}}{C_{\max}}}}$$

Aantal transfereenheden (NTU):

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{UA_s}{(\dot{m}c_p)_{\min}}$$

Capaciteitsverhouding c:

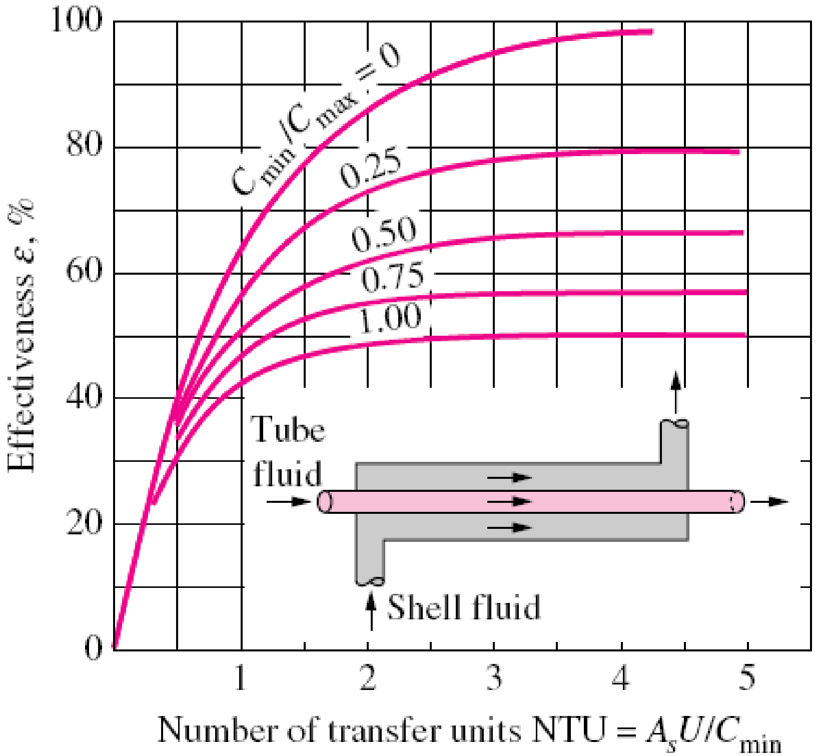
$$c = \frac{C_{\min}}{C_{\max}}$$

De effectiviteit van een warmtewisselaar is een functie van het aantal transfereenheden (NTU) en de capaciteitsverhouding *c*:

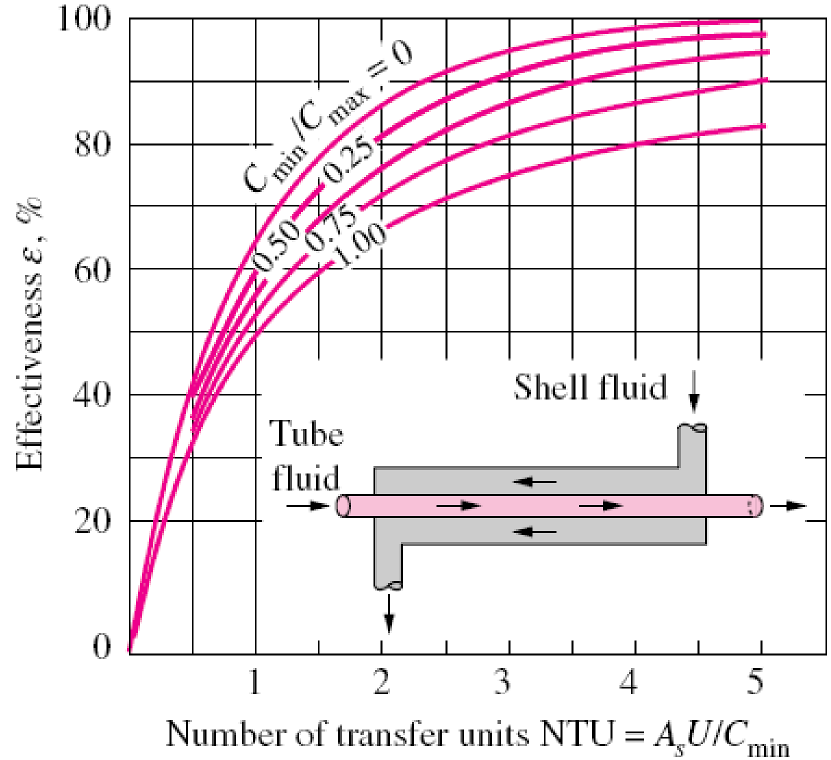
$$\varepsilon = \text{function}\ (UA_s/C_{\min},\ C_{\min}/C_{\max}) = \text{function}\ (\text{NTU},\ c)$$

Effectiveness relations for heat exchangers: NTU $= UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}c_p)_{\min}/(\dot{m}c_p)_{\max}$

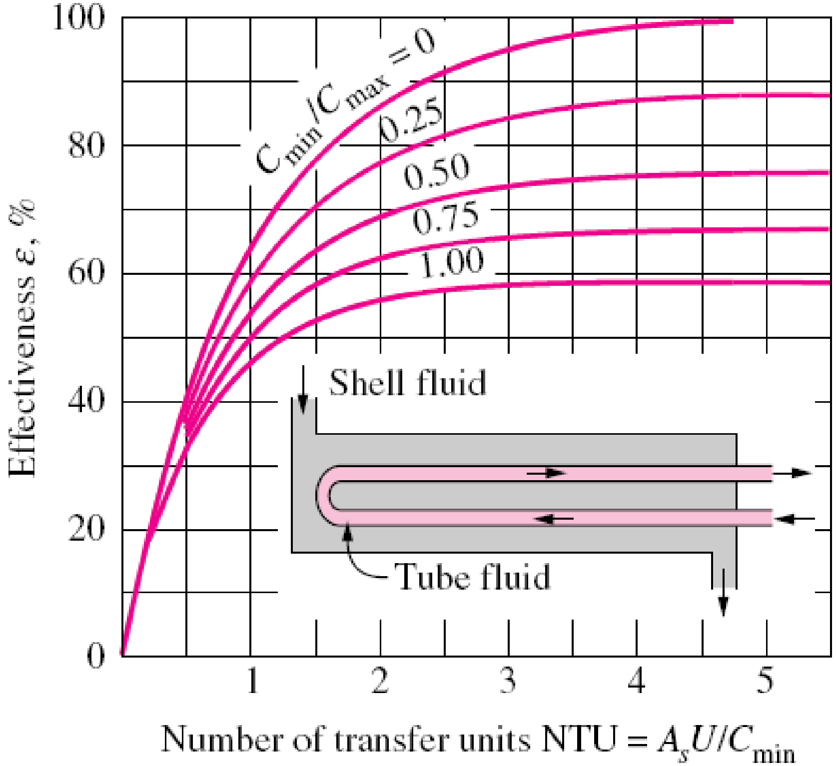
Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1+c)]}{1+c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1-c)]}{1 - c\exp[-\text{NTU}(1-c)]}$
2 *Shell-and-tube:*	
One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1+c^2}\,\dfrac{1 + \exp[-\text{NTU}\sqrt{1+c^2}]}{1 - \exp[-\text{NTU}\sqrt{1+c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}[\exp(-c\ \text{NTU}^{0.78}) - 1]\right\}$
$C_{\max}$ mixed, $C_{\min}$ unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{-c[1 - \exp(-\text{NTU})]\})$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\,\text{NTU})]\right\}$
4 *All heat exchangers with $c = 0$*	$\varepsilon = 1 - \exp(-\text{NTU})$



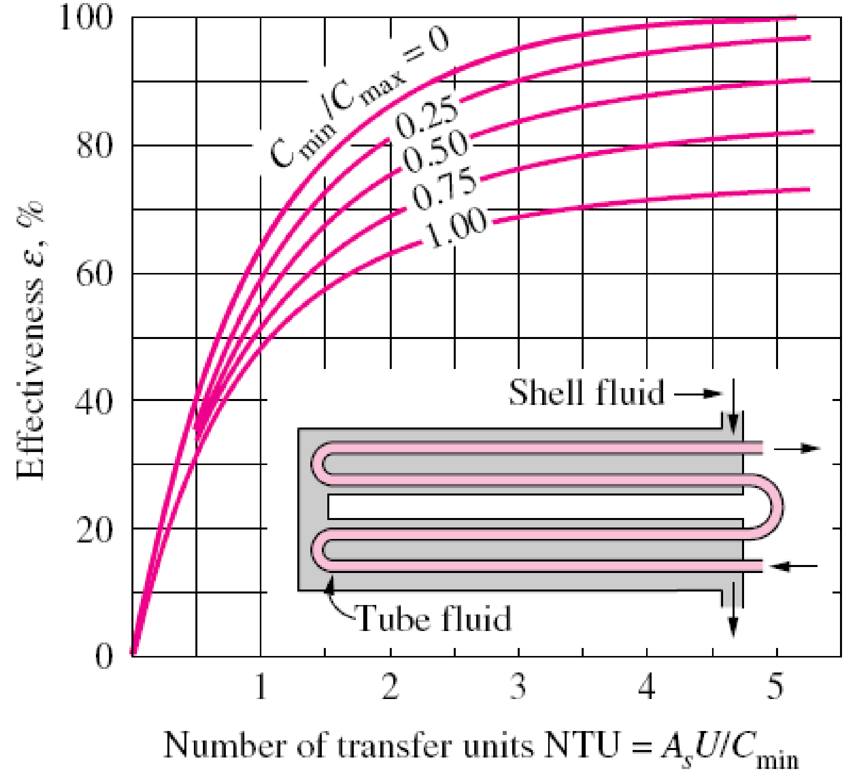
(*a*) Parallel-flow



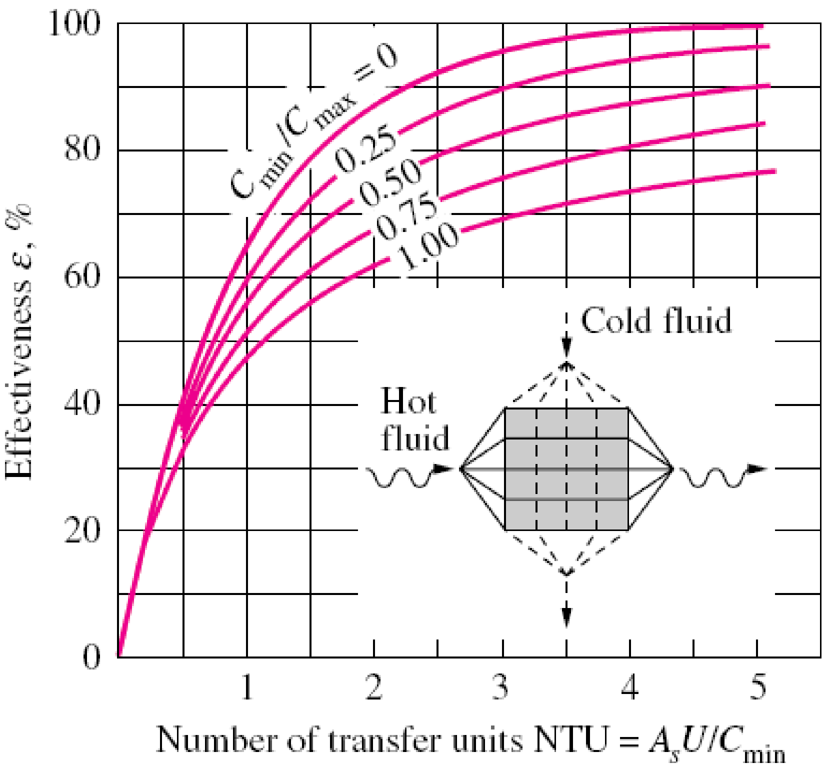
(*b*) Counter-flow



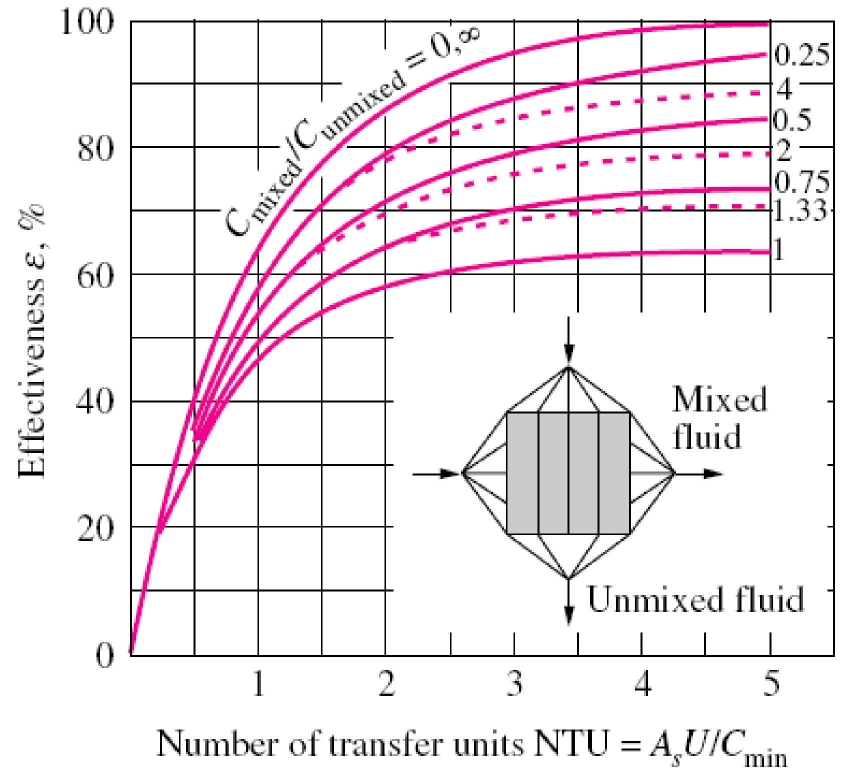
(c) One-shell pass and 2, 4, 6, … tube passes



(d) Two-shell passes and 4, 8, 12, … tube passes

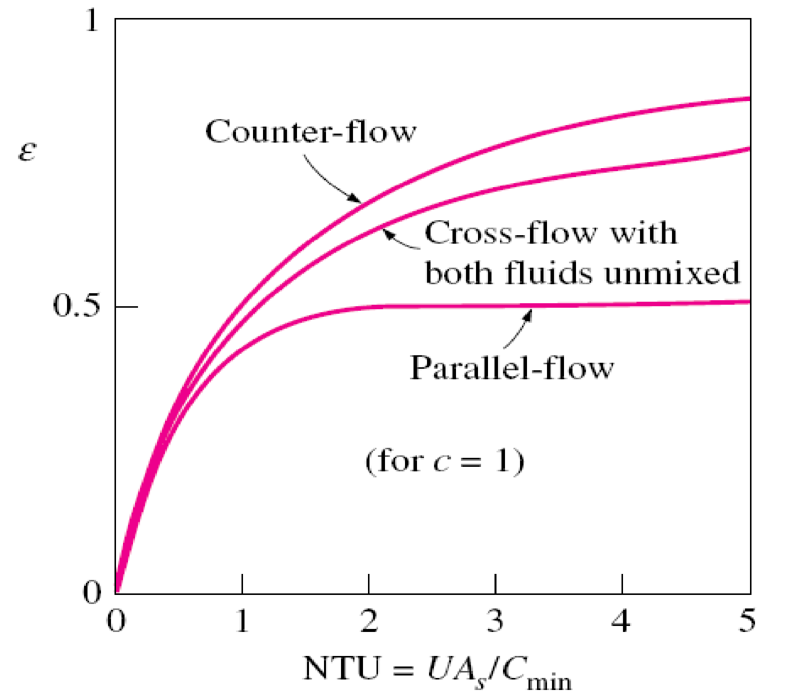


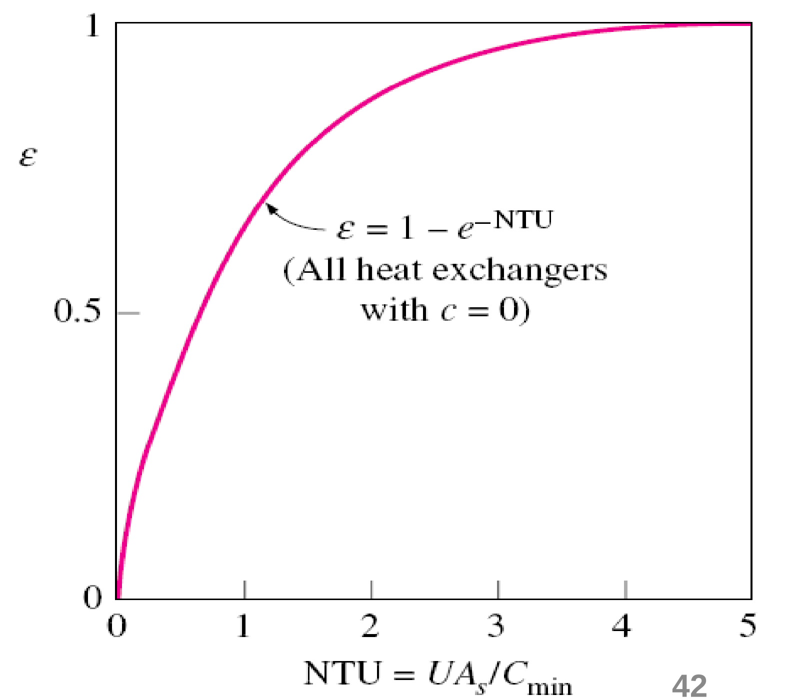
(e) Cross-flow with both fluids unmixed



(f) Cross-flow with one fluid mixed and the other unmixed

- Effectiviteit verandert snel tussen 0 en 1,5. Warmtewisselaar met NTU > 3 is te groot om economisch goed te zijn
- Tegenstroom geeft grootste effectiviteit; gelijkstroom laagste
- Effectiviteit is onafhankelijk van c voor NTU < 0,3
- Bij bepaalde NTU is ε maximaal als *c* = 0 en minimaal voor *c* = 1.



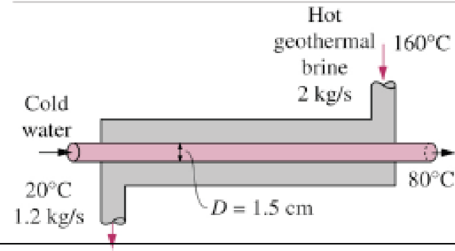


NTU relations for heat exchangers: $NTU = UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}c_p)_{min}/(\dot{m}c_p)_{max}$

Heat exchanger type	NTU relation
1 *Double-pipe:* Parallel-flow	$NTU = -\dfrac{\ln[1-\varepsilon(1+c)]}{1+c}$
Counter-flow	$NTU = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon c-1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$NTU = -\dfrac{1}{\sqrt{1+c^2}}\ln\left(\dfrac{2/\varepsilon-1-c-\sqrt{1+c^2}}{2/\varepsilon-1-c+\sqrt{1+c^2}}\right)$
3 *Cross-flow* (*single-pass*): C_{max} mixed, C_{min} unmixed	$NTU = -\ln\left[1+\dfrac{\ln(1-\varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$NTU = -\dfrac{\ln[c\ln(1-\varepsilon)+1]}{c}$
4 *All heat exchangers with $c = 0$*	$NTU = -\ln(1-\varepsilon)$

- Als alle in- en uitlaattemperaturen gekend zijn, dan kan de *grootte* van de warmtewisselaar gemakkelijk bepaald worden m.b.v. de LMTD-methode.
- De grootte kan ook bepaald worden met de effectiviteit-NTU-methode door eerst de effectiviteit te berekenen via de definitie en dan NTU m.b.v. de geschikte NTU-vergelijking

- Voorbeeld 1: Gebruik van de effectiviteit-NTU-methode



$U = 640$ W/(m² °C)

Bepaal de lengte van de warmtewisselaar (zie ook vorige oefening met LMTD-methode).

$$C_h = \dot{m}_h C_{ph} = (2 \text{ kg/s})(4.31 \text{ kJ/kg}\cdot{}^\circ\text{C}) = 8.62 \text{ kW/}^\circ\text{C}$$

$$C_c = \dot{m}_c C_{pc} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg}\cdot{}^\circ\text{C}) = 5.02 \text{ kW/}^\circ\text{C}$$

$$C_{\min} = C_c = 5.02 \text{ kW/}^\circ\text{C}$$

$$c = C_{\min}/C_{\max} = 5.02/8.62 = 0.583$$

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \\ &= (5.02 \text{ kW/}^\circ\text{C})(160 - 20)^\circ\text{C} \\ &= 702.8 \text{ kW}\end{aligned}$$

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg}\cdot{}^\circ\text{C})(80 - 20)^\circ\text{C} = 301.0 \text{ kW}$$

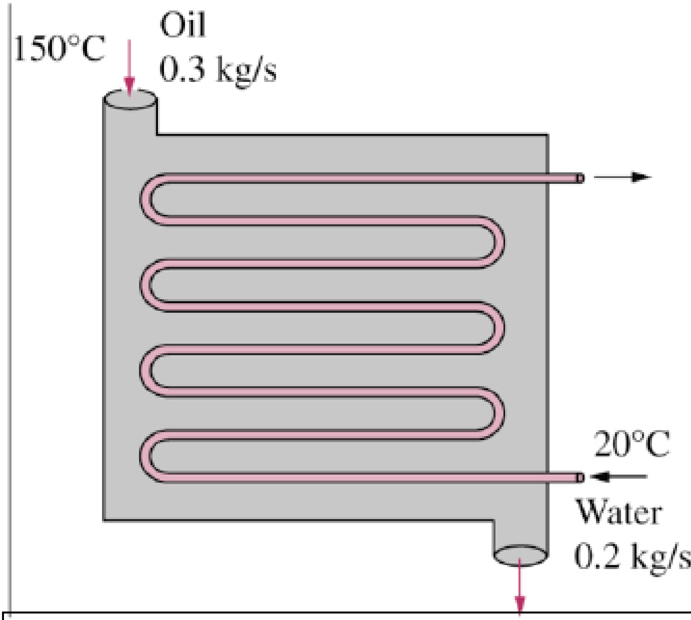
$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{301.0 \text{ kW}}{702.8 \text{ kW}} = 0.428$$

$$\text{NTU} = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.583 - 1}\ln\left(\frac{0.428 - 1}{0.428 \times 0.583 - 1}\right) = 0.651$$

$$\text{NTU} = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{\text{NTU } C_{\min}}{U} = \frac{(0.651)(5020 \text{ W/}^\circ\text{C})}{640 \text{ W/m}^2\cdot{}^\circ\text{C}} = 5.11 \text{ m}^2$$

$$A_s = \pi D L \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11 \text{ m}^2}{\pi(0.015 \text{ m})} = \mathbf{108 \text{ m}}$$

• Voorbeeld 2: "Multipass"-warmtewisselaar



U = 310 W/(m² °C)
Buizen: dunne wand, interne diameter=1.4 cm,
lengte = 5 m/buispassage

Bepaal het warmtedebiet en de uitlaattemperaturen van de olie en het water.

$$C_h = \dot{m}_h C_{ph} = (0.3 \text{ kg/s})(2.13 \text{ kJ/kg} \cdot {}^\circ\text{C}) = 0.639 \text{ kW/}^\circ\text{C}$$
$$C_c = \dot{m}_c C_{pc} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot {}^\circ\text{C}) = 0.836 \text{ kW/}^\circ\text{C}$$

$$C_{\min} = C_h = 0.639 \text{ kW/}^\circ\text{C}$$

$$c = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \\ &= (0.639 \text{ kW/}^\circ\text{C})(150 - 20)^\circ\text{C} = 83.1 \text{ kW}\end{aligned}$$

$$A_s = n(\pi DL) = 8\pi(0.014 \text{ m})(5 \text{ m}) = 1.76 \text{ m}^2$$

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(310 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.76 \text{ m}^2)}{639 \text{ W/}^\circ\text{C}} = 0.853$$

Fig. 22-26c: $\varepsilon = 0.47$

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.47)(83.1 \text{ kW}) = \mathbf{39.1\ kW}$$

$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) \longrightarrow T_{c,\text{ out}} = T_{c,\text{ in}} + \frac{\dot{Q}}{C_c}$$

$$= 20^\circ\text{C} + \frac{39.1 \text{ kW}}{0.836 \text{ kW/}^\circ\text{C}} = \mathbf{66.8^\circ C}$$

$$\dot{Q} = C_h(T_{h,\text{ in}} - T_{h,\text{ out}}) \longrightarrow T_{h,\text{ out}} = T_{h,\text{ in}} - \frac{\dot{Q}}{C_h}$$

$$= 150^\circ\text{C} - \frac{39.1 \text{ kW}}{0.639 \text{ kW/}^\circ\text{C}} = \mathbf{88.8^\circ C}$$

6. Selectie van warmtewisselaars

- Onzekerheid in de voorspelde *U*-waarde kan groter zijn dan 30% -> overdimensioneren

- Verbeterd warmteoverdracht gepaard met grotere drukverlies -> groter pompvermogen

- Meest viskeuze fluïdum aan de shell-zijde (grotere oppervlakte, minder drukverlies) en fluïdum met de grootste druk aan de buis-zijde

- Praktijk:
  - Doel: opwarmen of afkoelen van een fluïdum aan gekend massadebiet en temperatuur tot gewenste temperatuur

$$\dot{Q} = \dot{m} c_p (T_{\text{in}} - T_{\text{out}})$$

  - Selectie:
    - Warmtedebiet
    - Kost: Kant-en-klaar of op maat gedimensioneerd
    - Pompvermogen

$$\text{Operating cost} = (\text{Pumping power, kW}) \times (\text{Hours of operation, h}) \times (\text{Unit cost of electricity, \$/kWh})$$

    - Grootte en gewicht
    - Type
    - Materialen