OEFENING 3 ✓

$$2y' + 4y = 3e^{-2t}$$
$$y(0) = 1$$

1. $2r + 4 = 0$

$$r = -2$$
$$\rightarrow y_h = Ae^{-2t}$$

2. $y_p = Ke^{-2t}$

$$y_p' = -2Ke^{-2t}$$
$$\rightarrow (-4K + 4K)e^{-2t} = 3e^{-2t}$$

→ part opl zit al in homogene opl. (rechterlid is reeds deel van y_h)

?

$$y_p = Kte^{-2t}$$
$$y_p' = Ke^{-2t} - 2Kte^{-2t}$$
$$\rightarrow (2K - 4Kt + 4Kt)e^{-2t} = 3e^{-2t}$$
$$2K = 3$$
$$K = \frac{3}{2}$$

$$\rightarrow y_p = \frac{3}{2}te^{-2t}$$

3. Totale oplossing

$$y = Ae^{-2t} + \frac{3}{2}te^{-2t}$$

$$\cancel{y' = -2Ae^{-2t} + \frac{3}{2}e^{-2t} - 3te^{-2t}}$$

$$y(0) = 1 : \quad A = 1$$

$$\rightarrow y = e^{-2t} + \frac{3}{2}te^{-2t} \quad ✓$$

OEFENING 4

$$y'' + 3y' + 2y = t + e^{-2t}$$

$$y(0) = -\frac{3}{4} \qquad y'(0) = \frac{1}{2}$$

1. $r^2 + 3r + 2 = 0$

$$D = 9 - 8 = 1 \rightarrow \begin{cases} r_1 = -1 \\ r_2 = -2 \end{cases}$$

$$y_h = Ae^{-t} + Be^{-2t}$$

2. $y_p = at + b + cte^{-2t}$ $\rightarrow ce^{-2t}$ zit al in y_h

$$y_p' = a + ce^{-2t} - 2cte^{-2t}$$

$$y_p'' = -2ce^{-2t} - 2ce^{-2t} + 4cte^{-2t}$$

$$= -4ce^{-2t} + 4cte^{-2t}$$

$$\rightarrow (-4c + \cancel{4ct} + 3c - \cancel{6ct} + \cancel{2ct})e^{-2t} + 3a + 2b + 2at = t + e^{-2t}$$

$$-c = 1 \qquad c = -1$$

$$3a + 2b = 0 \qquad b = -\frac{3}{4}$$

$$2a = 1 \qquad a = \frac{1}{2}$$

$$\rightarrow y_p = \frac{1}{2}t - \frac{3}{4} - te^{-2t}$$

3. $y = Ae^{-t} + Be^{-2t} - te^{-2t} + \frac{1}{2}t - \frac{3}{4}$

$$y' = -Ae^{-t} - 2Be^{-2t} - e^{-2t} + 2te^{-2t} + \frac{1}{2}$$

$$y(0) = \frac{-3}{4} \rightarrow A + B - \frac{3}{4} = \frac{-3}{4} \rightarrow A + B = 0$$

$$y'(0) = \frac{1}{2} \rightarrow -A - 2B - 1 + \frac{1}{2} = 0 \rightarrow -A - 2B = \frac{1}{2}$$

$$+B = \frac{-1}{2}$$

$$A = \frac{1}{2}$$

? $\rightarrow y = -\cancel{\frac{1}{2}}e^{-2t} - te^{-2t} + \cancel{\frac{1}{2}}e^{-t} + \frac{1}{2}t - \frac{3}{4}$

OEFENING 5 ✓

$y' + y = 3 \sin t$

1. $r^2 + 1 = 0$

$r^2 = -1 \rightarrow r_1 = i \qquad a \pm bi$

$r_2 = -i$

$\rightarrow y_h = e^{at}(A_1 \cos bt + A_2 \sin bt) \quad \leftarrow ?$

$= A_1 \cos t + A_2 \sin t$

2. $y_p = at \sin t + bt \cos t \quad \leftarrow ?$

$y_p' = a \sin t + at \cos t + b \cos t - bt \sin t$

$y_p'' = a \cos t + a \cos t - at \sin t - b \sin t - b \sin t - bt \cos t$

$= 2a \cos t - at \sin t - 2b \sin t - bt \cos t$

$\rightarrow \cancel{2a \cos t - at \sin t + at \sin t = 3 \sin t}$

$\cancel{2a \cos t = 3 \sin t}$

$\rightarrow (2a - \cancel{bt} + \cancel{bt}) \cos t + (-\cancel{at} - 2b + \cancel{at}) \sin t = 3 \sin t$

$2a = 0 \rightarrow a = 0$

$-2b = 3 \rightarrow b = -\frac{3}{2}$

$\rightarrow y_p = -\frac{3}{2} t \cos t$

3. $y = A_1 \cos t + A_2 \sin t - \frac{3}{2} t \cos t$ ✓

$\boxed{\text{OEFENING 6}}$ ✓

$$y(0)=0 \qquad y'(0)=1 \qquad y''(0)=0 \qquad y'''(0)=1$$

$$y'''' - 4y''' + 6y'' - 4y' + y = 0$$

1. $r^4 - 4r^3 + 6r^2 - 4r + 1 = 0$

$r_{1234} = 1$

	1	−4	6	−4	1
1		1	−3	3	−1
	1	−3	3	−1	0
1		1	−2	1	
	1	−2	1	0	
1		1	−1		
	1	−1	0		

2. $y = Ae^t + Bte^t + Ct^2e^t + Dt^3e^t$

$y' = Ae^t + Be^t + Bte^t + 2Cte^t + Ct^2e^t + 3Dt^2e^t + Dt^3e^t$

$y'' = Ae^t + \underbrace{Be^t + Be^t} + Bte^t + 2Ce^t + \underbrace{2Cte^t + 2Cte^t} + Ct^2e^t + 6Dte^t + \underbrace{3Dt^2e^t + 3Dt^2e^t} + Dt^3e^t$

$y''' = Ae^t + \underbrace{2Be^t + Be^t} + Bte^t + \underbrace{2Ce^t + 4Ce^t} + \underbrace{4Cte^t + 2Cte^t} + Ct^2e^t + 6De^t + \underbrace{6Dte^t + 12Dte^t} + \underbrace{6Dt^2e^t + 3Dt^2e^t} + Dt^3e^t$

$0\,y'''' = Ae^t + \underbrace{3Be^t + Be^t} + Bte^t + 6Ce^t + 6Ce^t + 6Cte^t + 2Cte^t + Ct^2e^t + 6De^t + 18De^t + 18Dte^t + 18Dte^t + 9Dt^2e^t + 3Dt^2e^t + Dt^3e^t$

$= Ae^t + 4Be^t + Bte^t + 12Ce^t + 8Cte^t + Ct^2e^t + 14De^t + 36Dte^t + 12Dt^2e^t + Dt^3e^t$

$y(0)=0 \rightarrow A = 0$

$y'(0)=1 \rightarrow A + B = 1$

$y''(0)=0 \rightarrow A + 2B + 2C = 0$

$y'''(0)=1 \rightarrow A + 3B + 6C + 6D = 1$

$$\begin{cases} A = 0 \\ B = 1 \\ C = -1 \\ D = \frac{2}{3} \end{cases}$$

$$\rightarrow y = te^t - t^2e^t + \frac{2}{3}t^3e^t$$ ✓

$\boxed{\text{OEFENING 7}}$ ✓

$y'' - 2y' + 2y = \cos t$

$y(0) = 1 \qquad y'(0) = 0$

1. $r^2 - 2r + 2 = 0$

$D = 4 - 8 = -4 < 0 \quad = 4i$

$r_1 = \dfrac{2 + \sqrt{4i}}{2} = 1 + i$

$r_2 = \dfrac{2 - \sqrt{4i}}{2} = 1 - i$

$\rightarrow y_h = e^t (A_1 \cos t + A_2 \sin t)$

2. $y_p = a \cos t + b \sin t$

$y_p' = -a \sin t + b \cos t$

$y_p'' = -a \cos t - b \sin t$

$\rightarrow (-a - 2b + 2a) \cos t + (-b + 2a + 2b) \sin t = \cos t$

$\begin{cases} a - 2b = 1 \\ b + 2a = 0 \end{cases} \qquad \begin{cases} a = \frac{1}{5} \\ b = \frac{-2}{5} \end{cases}$

$\rightarrow y_p = \dfrac{1}{5} \cos t - \dfrac{2}{5} \sin t$

3. $y = e^t (A_1 \cos t + A_2 \sin t) + \frac{1}{5} \cos t - \frac{2}{5} \sin t$

$y' = e^t (A_1 \cos t + A_2 \sin t) + e^t (-A_1 \sin t + A_2 \cos t) - \frac{1}{5} \sin t - \frac{2}{5} \text{co}$

$y(0) = 1 : \quad A_1 + \frac{1}{5} = 1$

$y'(0) = 0 : \quad A_1 + A_2 - \frac{2}{5} = 0 \qquad \begin{cases} A_1 = \frac{4}{5} \\ A_2 = \frac{-2}{5} \end{cases}$

$\rightarrow y = \dfrac{1}{5} \left(\cos t - 2 \sin t + 4 e^t \cos t - 2 e^t \sin t \right)$ ✓

$\boxed{\text{OEFENING 8}}$ ✓

$$y'' - 2y' + y = e^t \sin t$$

1. $r^2 - 2r + 1 = 0$

$D = 4 - 4 = 0 \rightarrow r_{1,2} = 1$

$\rightarrow y_h = A_1 e^t + A_2 t e^t$

2. $y_p = a e^t \sin t + b e^t \cos t$

$y_p' = a e^t \sin t + a e^t \cos t + b e^t \cos t - b e^t \sin t$

$y_p'' = a e^t \cancel{\sin t} + a e^t \cos t + a e^t \cos t - \cancel{a e^t \sin t} + b e^t \cancel{\cos t}$

$- b e^t \sin t - b e^t \sin t - b \cancel{e^t \cos t}$

$\rightarrow (-\cancel{2b} - 2a + \cancel{2b} + a) e^t \sin t + (\cancel{2a} - \cancel{2a} - 2b + b) e^t \cos t = e^t \sin t$

$\begin{cases} -a = 1 \\ -b = 0 \end{cases} \rightarrow \begin{cases} a = -1 \\ b = 0 \end{cases}$

$\rightarrow y_p = -e^t \sin t$

3. $y = A_1 e^t + A_2 t e^t - e^t \sin t)$

$= e^t (A_1 + A_2 t - \sin t)$ ✓

$\boxed{\text{OEFENING 9}}$ ✓

$$y'' - 4y' + 3y = 2\sin t - 4\cos t \qquad y(0) = 0$$
$$y'(0) = 1$$

1. $r^2 - 4r + 3 = 0$

$D = 16 - 12 = 4 \rightarrow r_1 = 3$

$r_2 = 1$

$\rightarrow y_h = A e^{3t} + B e^t$

2. $y_p = a\sin t + b\cos t$

$y_p' = a\cos t - b\sin t$

$y_p'' = -a\sin t - b\cos t$

$\rightarrow (-a + 4b + 3a)\sin t + (-b - 4a + 3b)\cos t = 2\sin t - 4\cos t$

$$\begin{cases} 2a + 4b = 2 \\ 2b - 4a = -4 \end{cases} \qquad \begin{cases} a = 1 \\ b = 0 \end{cases}$$

$\rightarrow y_p = \sin t$

3. $y = Ae^{3t} + Be^{t} + \sin t$

$y' = 3Ae^{3t} + Be^{t} + \cos t$

$$\begin{matrix} y(0) = 0 \rightarrow \\ y'(0) = 1 \rightarrow \end{matrix} \begin{cases} A + B = 0 \\ 3A + B + 1 = 1 \end{cases} \qquad \begin{cases} A = 0 \\ B = 0 \end{cases}$$

$\rightarrow y = \sin t$ ✓

TOPIC 16: Differentiaalvergelijkingen in het Laplacedomein

$\boxed{\text{OEFENING 1}}$ ✓

$$Y(s) = \frac{s^2+3s-4}{s^3+3s^2+4s+2}$$

	1	3	4	2
-1		-1	-2	-2
	1	2	2	0

<0

$$= \frac{A}{s+1} + \frac{Bs+C}{s^2+2s+2}$$

$$= \frac{A(s^2+2s+2)+(Bs+C)(s+1)}{(s+1)(s^2+2s+2)}$$

$$s=-1 \rightarrow A-\cancel{2A}+\cancel{2A} = 1-3-4 \rightarrow A=-6$$

$$\rightarrow -6s^2-12s-12+7s^2+7s+8s+8$$

$$B=7 \qquad C=8$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{-6}{s+1}\right) + \mathcal{L}^{-1}\left(\frac{7s+8}{s^2+2s+2}\right)$$

?

$$= -6e^{-t} + \mathcal{L}^{-1}\left(\frac{7(s+1)}{(s+1)^2+1}\right) + \mathcal{L}^{-1}\left(\frac{1}{(s+1)^2+1}\right)$$

$$= -6e^{-t} + \mathcal{L}^{-1}\left(\left.\frac{7s}{s^2+1}\right|_{s\rightarrow s+1}\right) + \mathcal{L}^{-1}\left(\left.\frac{1}{s^2+1}\right|_{s\rightarrow s+1}\right)$$

$$= -6e^{-t} + 7\cos t|_{s\rightarrow s+1} + \sin t|_{s\rightarrow s+1}$$

$$= -6e^{-t} + 7e^{-t}\cos t + e^{-t}\sin t \quad ✓ \qquad t \geq 0$$

$\boxed{\text{OEFENING 2}}$ ✓

$$Y(s) = \frac{s-1}{s^2+2s+2}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{s+1}{(s+1)^2+1}\right) - \mathcal{L}^{-1}\left(\frac{2}{(s+1)^2+1}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\Big|_{s\to s+1}\right) - 2\,\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\Big|_{s\to s+1}\right)$$

$$= \cos t\,\Big|_{s\to s+1} - 2\sin t\,\Big|_{s\to s+1}$$

$$= e^{-t}\cos t - 2e^{-t}\sin t \quad ; \quad t \geq 0 \quad ✓$$

$\boxed{\text{OEFENING 3}}$ ✓

$$Y(s) = \frac{s^2-4s+3}{s^2+4s+3}$$

$\cancel{D = 16 - 12 = 4 \to s_1 = -1,\ s_2 = -3}$

$$\cancel{= \frac{A}{(s+1)} + \frac{B}{(s+3)}}$$

$\cancel{= A(s+3)}$

Graad N = Graad T → eerst euclidisch delen

$$\begin{array}{r|l} s^2-4s+3 & s^2+4s+3 \\ \cline{2-2} -s^2 \mp 4s \mp 3 & 1 \\ \cline{1-1} -8s & \end{array}$$

$$= 1 - \frac{8s}{s^2+4s+3}$$

$$\frac{1}{\mathcal{L}} \;!$$

$$y(t) = \mathcal{L}^{-1}(1) - \mathcal{L}^{-1}\left(\frac{8s}{s^2+4s+3}\right)$$ $D = 16-12 = 4 \rightarrow \begin{cases} s_1 = -1 \\ s_2 = -3 \end{cases}$

$$= \delta(t) - \mathcal{L}^{-1}\left(\frac{8s}{(s+1)(s+3)}\right)$$

$$\frac{8s}{(s+1)(s+3)} = \frac{A(s+3)+B(s+1)}{(s+1)(s+3)}$$

$s=-3 \rightarrow -2B = -24 \rightarrow B = 12$

$s=-1 \rightarrow 2A = -8 \rightarrow A = -4$

$$= \frac{-4}{s+1} + \frac{12}{s+3}$$

$$y(t) = \delta(t) - \mathcal{L}^{-1}\left(\frac{-4}{s+1}\right) - \mathcal{L}^{-1}\left(\frac{12}{s+3}\right)$$

$$= \delta(t) + 4e^{-t} - 12e^{-3t} \quad ; \; t \geq 0 \quad \checkmark$$

$\boxed{\text{OEFENING 4}}$ ✓

$y'' - y' - 6y = 0$

$y(0) = 1 \qquad y'(0) = -1$

$\mathcal{L}(y'' - y' - 6y) = \mathcal{L}(0)$

$s^2 Y - s\,1 - (-1) - (sY - 1) - 6Y = 0$

$s^2 Y - s + 1 - sY + 1 - 6Y = 0$

$(s^2 - s - 6)Y - s + 2 = 0$

$$Y(s) = \frac{s-2}{s^2-s-6}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{s-2}{s^2-s-6}\right)$$

$\hookrightarrow D = 1 + 24 = 25 \rightarrow s_1 = 3$

$s_2 = -2$

$$y(t) = \mathcal{L}^{-1}\left(\frac{A}{s-3} + \frac{B}{s+2}\right)$$

$$\frac{s-2}{(s-3)(s+2)} = \frac{A(s+2)+B(s-3)}{(s-3)(s+2)}$$

$$s=-2 \rightarrow -5B=-4 \rightarrow B=\frac{4}{5}$$

$$s=3 \rightarrow 5A=1 \rightarrow A=\frac{1}{5}$$

$$y(t) = \frac{1}{5}\mathcal{L}^{-1}\left(\frac{1}{s-3}\right) + \frac{4}{5}\mathcal{L}^{-1}\left(\frac{1}{s+2}\right)$$

$$= \frac{1}{5}e^{3t} + \frac{4}{5}e^{-2t} \quad ; \quad t \geq 0 \quad \checkmark$$

$\boxed{\text{OEFENING 5}}$ ✓

$$y'' - 2y' + 2y = 0$$

$$y(0) = 0 \qquad y'(0) = 1$$

$$s^2Y - 1 - 2(sY) + 2Y = 0$$

$$(s^2 - 2s + 2)Y = 1$$

$$Y = \frac{1}{s^2-2s+2} \qquad D = 4 - 4.2 < 0$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{1}{(s-1)^2+1}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\bigg|_{s\to s-1}\right)$$

$$= \sin t \big|_{s\to s-1}$$

$$= e^t \sin t \quad ; \quad t \geq 0 \quad \checkmark$$

OEFENING 6 ✓

$y'' - 4y' + 4y = 0$
$y(0) = 1 \qquad y'(0) = 1$

$s^2 Y - s - 1 - 4(sY - 1) + 4Y = 0$
$(s^2 - 4s + 4) Y = s - 3$
$Y = \dfrac{s-3}{s^2 - 4s + 4} \qquad D = 16 - 16 = 0 \rightarrow s_{1,2} = 2$

$y(t) = \mathcal{L}^{-1}\left(\dfrac{s-2}{(s-2)^2}\right) - \mathcal{L}^{-1}\left(\dfrac{1}{(s-2)^2}\right)$
$= \mathcal{L}^{-1}\left(\dfrac{1}{s-2}\right) - \mathcal{L}^{-1}\left(\dfrac{1}{s^2}\Big|_{s \to s-2}\right)$
$= e^{2t} - t e^{2t} \quad ; \; t \geq 0$ ✓

OEFENING 7 ✓

$y'' + 3y' + 2y = 0$
$y(0) = 1 \qquad y'(0) = 0$

$s^2 Y - s + 3sY - 3 + 2Y = 0$
$(s^2 + 3s + 2) Y = s + 3$
$Y = \dfrac{s+3}{s^2 + 3s + 2} \qquad D = 9 - 8 = 1 \rightarrow s_1 = -1$
$s_2 = -2$

$y(t) = \mathcal{L}^{-1}\left(\dfrac{A}{s+1} + \dfrac{B}{s+2}\right)$

$$\frac{s+3}{s^2+3s+2} = \frac{A(s+2)+B(s+1)}{(s+1)(s+2)}$$

$s=-1 \rightarrow A=2$

$s=-2 \rightarrow -B=1 \rightarrow B=-1$

$$y(t) = \mathcal{L}^{-1}\left(\frac{2}{s+1}\right) + \mathcal{L}^{-1}\left(\frac{-1}{s+2}\right)$$

$$= 2e^{-t} - e^{-2t} \quad , \quad t \geq 0 \quad \checkmark$$

$\boxed{\text{OEFENING 8}}$ ✓

$$y^{(iv)} - y = 0$$

$y(0)=0 \quad y'(0)=1 \quad y''(0)=0 \quad y'''(0)=0$

$$\mathcal{L}[y'''](s) = s^3Y - s^2y(0) - s\,y'(0) - y''(0)$$
$$\mathcal{L}[y''''](s) = s^4Y - s^3y(0) - s^2y'(0) - s\,y''(0) - y'''(0)$$

→ Hier is opgave: $y'''' - y = 0$ want tot $y'''(0)$ gegeven

$$s^4Y - s^2 - Y = 0$$
$$(s^4-1)\,Y = s^2$$
$$Y = \frac{s^2}{s^4-1}$$

	1	0	0	0	-1
1		1	1	1	1
	1	1	1	1	0
-1		-1	0	-1	
	1	0	1	0	

$$= \frac{s^2}{(s-1)(s+1)(s^2+1)}$$

$$= \frac{A(s+1)(s^2+1) + B(s-1)(s^2+1) + C(s-1)(s+1)}{(s-1)(s+1)(s^2+1)}$$

$s=-1 \rightarrow -4B = 1 \rightarrow B = -1/4$

$s=1 \rightarrow 4A = 1 \rightarrow A = 1/4$

$s^2: \; 1/4 + 1/4 + C = 1 \rightarrow C = 1/2$

$$y(t) = \frac{1}{4}\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) - \frac{1}{4}\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) + \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)$$
$$= \frac{1}{4}e^{t} - \frac{1}{4}e^{-t} + \frac{1}{2}\sin t \quad , \; t \geq 0 \; ✓$$

$\boxed{\text{OEFENING 9}}$ ✓

$$y' - y = e^{t}\cos(2t) + e^{t}$$
$$y(0) = 0$$

$$sY - Y = \mathcal{L}(e^{t}\cos 2t) + \mathcal{L}(e^{t})$$
$$= \mathcal{L}(\cos 2t)(s)\Big|_{s \to s-1} + \mathcal{L}(1)(s)\Big|_{s \to s-1}$$
$$= \frac{s}{s^2+4}\Big|_{s \to s-1} + \frac{1}{s}\Big|_{s \to s-1}$$
$$= \frac{s-1}{(s-1)^2+4} + \frac{1}{s-1}$$
$$Y = \frac{1}{(s-1)^2+4} + \frac{1}{(s-1)^2}$$

$$y(t) = \frac{1}{2}\mathcal{L}^{-1}\left(\frac{2}{(s-1)^2+4}\right) + \mathcal{L}^{-1}\left(\frac{1}{(s-1)^2}\right)$$
$$= \frac{1}{2}\mathcal{L}^{-1}\left(\frac{2}{s^2+4}\Big|_{s \to s-1}\right) + \mathcal{L}^{-1}\left(\frac{1}{s^2}\Big|_{s \to s-1}\right)$$
$$= \frac{1}{2}e^{t}\sin 2t + te^{t} \quad ; \; t \geq 0 \; ✓$$

$\boxed{\text{OEFENING 10}}$ ✓

$$y'' - 3y' + 2y = t^2 - 3$$
$$y(0) = 0 \qquad y'(0) = 0$$

$$s^2 Y - 3sY + 2Y = \mathcal{L}(t^2 - 3)$$
$$= \frac{2!}{s^3} - \frac{3}{s}$$

$$Y = \frac{\frac{2}{s^3} - \frac{3}{s}}{s^2 - 3s + 2}$$
$$= \frac{2 - 3s^2}{s^3(s^2 - 3s + 2)} \qquad D = 9 - 4.2 = 1 \rightarrow \begin{cases} s_1 = 2 \\ s_2 = 1 \end{cases}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-1} + \frac{E}{s-2}\right)$$

$$\frac{2 - 3s^2}{s^3(s^2 - 3s + 2)} = \frac{As^2(s-1)(s-2) + Bs(s-1)(s-2) + C(s-1)(s-2) + Ds^3(s-2) + Es^3(}{s^3(s-1)(s-2)}$$

$$s = 1 \rightarrow -D = -1 \rightarrow D = 1$$
$$s = 2 \rightarrow 8E = -10 \rightarrow E = -5/4$$
$$s = 0 \rightarrow 2C = 2 \rightarrow C = 1$$
$$s^4 : A + D + E = 0 \rightarrow A = -1 + 5/4 = 1/4$$
$$s^3 : -2A - A + B - 2D - E = 0 \rightarrow B = \frac{3}{2} + 2 - \frac{5}{4} = \frac{3}{2}$$

$$y(t) = \frac{1}{4}\mathcal{L}^{-1}\left(\frac{1}{s}\right) + \frac{3}{2}\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) + \mathcal{L}^{-1}\left(\frac{1}{s^3}\right) + \mathcal{L}^{-1}\left(\frac{1}{s-1}\right) - \frac{5}{4}\mathcal{L}^{-1}\left(\frac{1}{s-2}\right)$$
$$= \frac{1}{4} + \frac{3}{2}t + \frac{1}{2}t^2 + e^t - \frac{5}{4}e^{2t} \quad ; t \geq 0 \checkmark$$

OEFENING 11 ✓

$$y'' - 2y' + 2y = e^{-t}$$
$$y(0) = 0 \qquad y'(0) = 1$$

$$s^2 Y - 1 - 2sY + 2Y = \mathcal{L}(e^{-t})$$
$$(s^2 - 2s + 2)Y - 1 = \frac{1}{s+1}$$
$$(s^2 - 2s + 2)Y = \frac{1 + s + 1}{s+1}$$
$$Y = \frac{s+2}{(s+1)(s^2-2s+2)} \qquad D = 4 - 4.2 < 0$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{A}{s+1} + \frac{Bs+C}{s^2-2s+2}\right)$$
$$\frac{s+2}{(s+1)(s^2-2s+2)} = \frac{A(s^2-2s+2) + (Bs+C)(s+1)}{(s+1)(s^2-2s+2)}$$
$$s = -1 \rightarrow 5A = 1 \rightarrow A = 1/5$$
$$s^2 : A + B = 0 \rightarrow B = -1/5$$
$$s : -2A + B + C = 1 \rightarrow C = 8/5$$

$$y(t) = \frac{1}{5}\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) + \mathcal{L}^{-1}\left(\frac{\left(\frac{-1}{5}s + \frac{8}{5}\right)}{(s-1)^2+1}\right)$$
$$= \frac{1}{5}e^{-t} + \mathcal{L}^{-1}\left(\frac{\frac{-1}{5}(s-1)}{(s-1)^2+1}\right) + \mathcal{L}^{-1}\left(\frac{7/5}{(s-1)^2+1}\right)$$
$$= \frac{1}{5}e^{-t} - \frac{1}{5}\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\bigg|_{s\to s-1}\right) + \frac{7}{5}\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\bigg|_{s\to s-1}\right)$$
$$= \frac{1}{5}e^{-t} - \frac{1}{5}e^t\cos t + \frac{7}{5}e^t\sin t$$
$$= \frac{1}{5}e^{-t} - \frac{1}{5}(e^t\cos t - 7e^t\sin t) \quad ; \quad t \geq 0 \quad ✓$$

$\boxed{\text{OEFENING 12}}$ ✓

$$y'' + \omega^2 y = \cos(2t) \qquad \text{met } \omega^2 \neq 4$$
$$y(0) = 1 \qquad y'(0) = 0$$

$$s^2 Y - s + \omega^2 Y = \frac{s}{s^2+4}$$

$$(s^2+\omega^2) Y = \frac{s + s(s^2+4)}{s^2+4}$$

$$Y = \frac{s^3 + 5s}{(s^2+4)(s^2+\omega^2)}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+\omega^2}\right)$$

$$\frac{s^3+5s}{(s^2+4)(s^2+\omega^2)} = \frac{(As+B)(s^2+\omega^2) + (Cs+D)(s^2+4)}{(s^2+4)(s^2+\omega^2)}$$

$$s^3 : A + C = 1$$
$$s^2 : B + D = 0$$
$$s : \omega^2 A + 4C = 5$$
$$\cdot : \omega^2 B + 4D = 0$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ \omega^2 & 0 & 4 & 0 & 5 \\ 0 & \omega^2 & 0 & 4 & 0 \end{array}\right] \xrightarrow[r_4 - \omega_2 r_2]{r_3 - \omega_2 r_1} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 4-\omega^2 & 0 & 5-\omega^2 \\ 0 & 0 & 0 & 4-\omega^2 & 0 \end{array}\right]$$

$$\Rightarrow \quad D = 0$$

$$C = \frac{5-\omega^2}{4-\omega^2}$$

$$B = 0$$

$$A = 1 - \frac{5-\omega^2}{4-\omega^2}$$

$$y(t) = \frac{(4-\omega^2)-(5-\omega^2)}{4-\omega^2}\,\mathcal{L}^{-1}\left(\frac{s}{s^2+4}\right) + \frac{5-\omega^2}{4-\omega^2}\,\mathcal{L}^{-1}\left(\frac{s}{s^2+\omega^2}\right)$$

$$= \frac{-1}{4-\omega^2}\cos 2t + \frac{5-\omega^2}{4-\omega^2}\cos \omega t$$

$$= \frac{1}{\omega^2-4}\left(\cos 2t + (\omega^2-5)\cos \omega t\right), \quad t \geq 0 \checkmark$$

TOPIC 2a: Fysische interpretatie van DV^n

OEFENING 1 ✓ Bepaal de impulsrespons van de volgende DV

Impulsresponsie: $h(t)$ berekenen bij $u(t) = \delta(t)$
(berekenen Laplace met $u(t) = \delta(t) \Rightarrow U(s) = 1$)

$y''(t) + 7y'(t) + 12y(t) = u(t) \qquad y(0) = 0 \;,\; y'(0) = 0$

$s^2 Y(s) - s.0 - 0 + 7sY(s) - 0 + 12Y(s) = \mathcal{L}(\delta(t)) = 1$

$(s^2 + 7s + 12)Y(s) = 1$

$$Y(s) = \frac{1}{s^2 + 7s + 12}$$

$D = 49 - 48 = 1 \rightarrow s_1 = -3$
$s_2 = -4$

$$= \frac{1}{(s+3)(s+4)}$$

$$= \frac{A}{s+3} + \frac{B}{s+4} = \frac{A(s+4) + B(s+3)}{(s+3)(s+4)}$$

$s = -4 \rightarrow -B = 1 \rightarrow B = -1$

$s = -3 \rightarrow A = 1$

$$= \frac{1}{s+3} - \frac{1}{s+4}$$

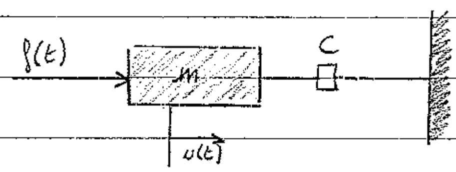
$$y(t) = h(t) = \mathcal{L}^{-1}\left(\frac{1}{s+3}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+4}\right)$$

$$= e^{-3t} - e^{-4t} \;;\; t \geq 0 \quad ✓$$

Zie ook bemerking p9

M

$\boxed{\text{OEFENING 2}}$ ✓



$m = 1\,kg$ $\qquad v(0) = 0\ m/s$

$C = 0{,}5\ Ns/m$ dempingscte

$f(t) = 1N$ $\qquad v(t) = ?$

$t \geq 0$

1. $$m\frac{dv}{dt} = f(t) - Cv$$

$$m\frac{dv}{dt} + Cv = f(t) = u = \delta(t)$$

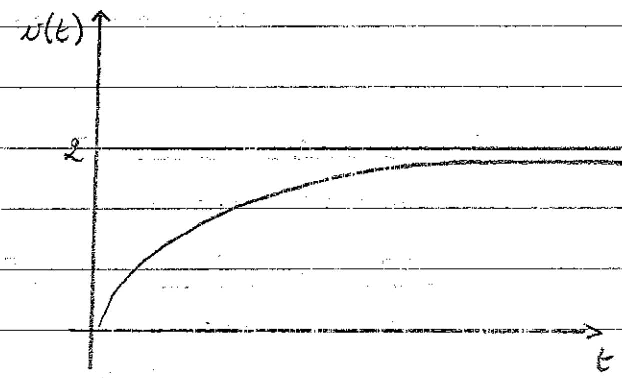
2. $$m s Y(s) + 0 + CY(s) = 1$$

$$(ms + C)Y(s) = 1$$

$$Y(s) = \frac{1}{ms+C} = \frac{1/m}{s + C/m}$$

$$v(t) = \frac{1}{m} e^{-\frac{C}{m}t} \quad ; t \geq 0$$

$$= e^{-\frac{t}{2}}$$



3. Stapresponsie : $u(t) = 1(t)$

$$m sY(s) + CY(s) = \frac{1}{s} \qquad \leftarrow ?$$

$$Y(s) = \frac{1}{s(ms+C)}$$

$$= \frac{A}{s} + \frac{B}{ms+C} = \frac{A(ms+C) + Bs}{s(ms+C)}$$

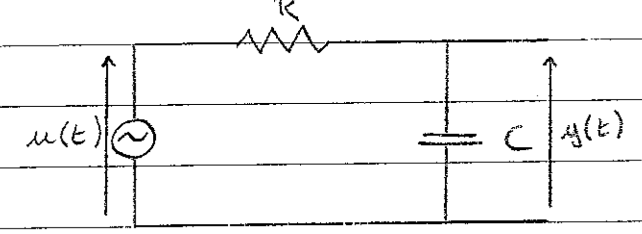
$$s = 0 \rightarrow CA = 1 \rightarrow A = 1/C$$

$$s = -C/m \rightarrow -C/m \cdot B = 1 \rightarrow B = -m/C$$

$$= \frac{1}{C}\frac{1}{s} + \frac{-m}{C}\frac{1}{ms+C} = \frac{1}{C}\frac{1}{s} - \frac{1}{C}\frac{1}{s+\frac{C}{m}}$$

$$v(t) = \frac{1}{C} - \frac{1}{C}e^{-\frac{C}{m}t} = 2 - 2e^{-\frac{t}{2}} = 2\left(1 - e^{-\frac{t}{2}}\right), \ t \geq 0 ✓$$

OEFENING 3 ✓



$u(t) = 3\sin(2t)$

$R \times C = 1$

$y(t) = ?$

1. $v(t) = \dfrac{1}{C}\int_0^t i(t)\,dt + R\,i(t)$

$$\left(V(s) = \left(\frac{1}{sC} + R\right) I(s)\right)$$

$$u(t) = \frac{dy}{dt} + y$$

2. $1 = sY(s) - Y(0) + Y(s)$

$$Y(s) = \frac{1}{s+1}$$

$$y(t) = e^{-t}$$

3. $sY(s) + Y(s) = \mathcal{L}\left(3\sin(2t)\right)$

$$= 3\,\frac{2}{s^2+4}$$

$$Y(s) = \frac{6}{(s^2+4)(s+1)} = \frac{As+B}{(s^2+4)} + \frac{C}{(s+1)}$$

$$= \frac{(As+B)(s+1) + C(s^2+4)}{(s^2+4)(s+1)}$$

$s = -1 \rightarrow 5C = 6 \rightarrow C = 6/5$

$A + C = 0 \rightarrow A = -6/5$

$A + B = 0 \rightarrow B = 6/5$

$$= \frac{-6}{5}\,\frac{s}{s^2+4} + \frac{6}{5}\,\frac{1}{s^2+4} + \frac{6}{5}\,\frac{1}{s+1}$$

$$y(t) = \frac{-6}{5}\cos(2t) + \frac{6}{10}\sin(2t) + \frac{6}{5}e^{-t} \quad ; \ t \geq 0$$

$$= \frac{6}{5}\left[\frac{1}{2}\sin(2t) - \cos(2t) + e^{-t}\right] ✓$$

EXTRA

$u = \; = y + Ri$

$U(s) = Y(s) + RI(s)$

$Y(s) = \dfrac{1}{sC}\, I(s)$

$y(t) = \dfrac{1}{C}\int_0^t i(t)\,dt$

$u = y + \dfrac{dy}{dt}$ $\qquad C\dfrac{dy}{dt} = i$

Spanningsbron: cte spanning, stroom variabel (gaat rro)
→ regimewaarde v y(t) $= u(t)$

OEFENING 4

$$u(t)\begin{cases} 0 > t & : 0 \\ 0 \leq t < 1 & : 1 \\ 1 \leq t < 2 & : t \\ 2 \leq t < 4 & : 4-t \quad \leftarrow ? \\ 4 \leq t < 6 & : 1 \\ t > 6 & : 0 \end{cases}$$

Topic 2b Systeemdynamica 1

OEFENING 1 ✓

Indien beginvoorwaarden 0 zijn
→ $\int h(t)$ = stapantwoord

• $0 \leq t \leq 1$: $(1,0)$ $(0,2)$

$$h - h_1 = \frac{h_2 - h_1}{t_2 - t_1}(t - t_1)$$

$$h = \frac{2}{-1}(t-1) = -2t + 2$$

$$\int h(t) = \frac{-2t^2}{2} + 2t = -t^2 + 2t = y(t)\ ✓$$

$$\left[-t^2 + 2t\right]_0^1 = -1 + 2 = 1 = \text{opp onder curve}$$

• $1 \leq t < 2$: $y(t) = 1$ ✓ = oppervlak van ervoor (komt niks bij)

• $2 \leq t < 3$: $(3,0)$ $(2,1)$

$$h = \frac{1}{-1}(t-3) = -t + 3 \quad ?$$

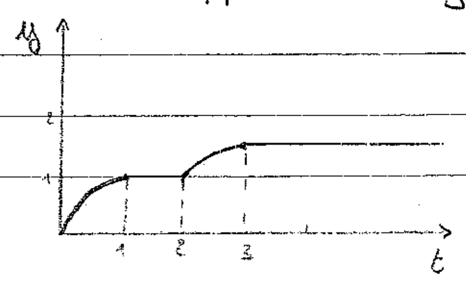
$$\int h(t) = \frac{-t^2}{2} + 3t + C$$ punt $(2,1)$ invullen → $C = -3$

klopt nt met $(3,0)$ → Waarom?

$$\left[-t^2/2 + 3t\right]_2^3 = 0{,}5 = \frac{-9}{2} + 9 + \frac{4}{2} - 6 = \frac{1}{2}$$

$$\rightarrow y(t) = 3t - \frac{t^2}{2} - 3\ ✓$$

• $3 \leq t < \infty$: oppervlak blijft 1,5 → $y(t) = \frac{3}{2}$ ✓



OEFENING 2 ✓

$C = 1\,m^2$ $\quad$ $y(t) = C^{te}$

$R = 0{,}5\ s/m^2$ $\quad$ $u(t) = 1\ m^3/s \leftarrow ?$

$Q_{in} = 3\ m^2/s$

1. DV : $C\frac{dy}{dt} = Q_{in} - Q_{uit}$ ✓ met $Q_{uit} = \frac{y(t)}{R}$? $\quad h(t) - 0 = R\,Q_{uit}$

2. Regimetoestand vòòr verstoring

$y(t) = C^{te} \rightarrow \frac{dy}{dt} = 0 \rightarrow Q_{in} = Q_{uit}$

$u(t) = \frac{y(t)}{R} \rightarrow y(t) = R \cdot u(t)$

$= 1{,}5\ m = \frac{3}{2}\ m$ ✓

3. $\begin{cases} y(t) = y_0 + y^* & \rightarrow \text{met } y_0 = 1{,}5 \\ u(t) = u_0 + u^* & \rightarrow u^* = u(t) - u_0 = 1 - 3 = -2 \end{cases}$

$\rightarrow$ invullen in DV

$C\frac{dy^*}{dt} = u_0 + u^* - \frac{y_0}{R} - \frac{y^*}{R}$ ← waarom ne u^*

$u_0 = 3$ en $y_0 = 1{,}5$ en $R = 0{,}5$

$C\frac{dy^*}{dt} = u^* - \frac{y^*}{R}$

$C\frac{dy^*}{dt} + \frac{y^*}{R} = u^*$ ✓

4. $\cancel{C}\left(sY^*(s) - y^*(0)\right) + \frac{Y^*(s)}{R} = \frac{-2}{s}$?

$(s+2)\,Y^*(s) = \frac{-2}{s}$

$Y^*(s) = \frac{-2}{s(s+2)} = \frac{A(s+2) + Bs}{s(s+2)}$ $\quad$ $s = 0 \rightarrow 2A = -2 \rightarrow A = -1$

$s = -2 \rightarrow -2B = -2 \rightarrow B = 1$

$= \frac{-1}{s} + \frac{1}{s+2}$

1

$y^*(t) = -1 + e^{-2t}$; $t \geq 0$ ✓

5. $\cancel{1 = -1 + e^{-2t}}$

$\cancel{e^{-2t} = 2}$

$\cancel{-2t = \ln 2}$

$\cancel{t = \frac{-\ln 2}{2} = -0{,}347 s}$

?

$y = y_0 + y^*$

$= y_0 - 1 + e^{-2t}$

$1 = 3/2 - 1 + e^{-2t}$

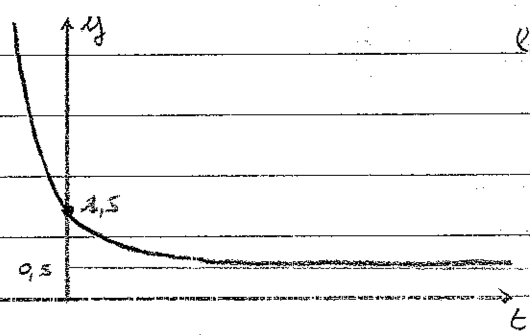
$e^{-2t} = 1/2$

$-2t = -\ln 2$

$t = \frac{\ln 2}{2} = 0{,}347 s$ ✓

6. Nieuwe regimetoestand:

$y(t) = \frac{3}{2} - 1 + e^{-2t}$



$\lim_{t \to \infty} y(t) = \frac{3}{2} - 1 = \frac{1}{2}$

?

Opmerking: moet niet met afwijkingsvariabelen

$C(sY(s) - Y(0)) = \frac{1}{s} - \frac{Y(s)}{R}$

$\rightarrow \left(Cs + \frac{1}{R}\right) Y(s) = \frac{1}{s} + Y(0) = \frac{1}{s} + \frac{3}{2}$

$(s+2)\,Y(s) = \frac{1}{s} + \frac{3}{2}$

$Y(s) = \frac{1}{s+2} \cdot \frac{1 + 3/2\,s}{s} = \frac{A}{s+2} + \frac{B}{s} = \frac{As + B(s+2)}{s(s+2)}$

$s = 0 \rightarrow 2B = 1 \rightarrow B = \frac{1}{2}$

$s = -2 \rightarrow -2A = -2 \rightarrow A = 1$

$= \frac{1}{s+2} + \frac{1}{s}\,\frac{1}{2}$

$y(t) = e^{-2t} + \frac{1}{2}$ ✓

20

Topic 3: Systeemdynamica 2

$\boxed{\text{OEFENING 1}}$ ✓

* $F = m\dfrac{d^2y}{dt^2} + C\dfrac{dy}{dt} + ky$

$$U(s) = ms^2Y(s) - msy(\overset{0}{0}) - y'(\overset{0}{0}) + CsY(s) - Cy(\overset{0}{0}) + kY(s)$$
$$= (ms^2 + Cs + k)\,Y(s)$$
$$Y(s) = \frac{U(s)}{ms^2 + Cs + k}$$

$$H(s) = \frac{1}{ms^2 + Cs + k} = \frac{1/m \cdot k/k}{s^2 + \frac{C}{m}s + \frac{k}{m}}$$

* $\dfrac{Y(s)}{U(s)} = H(s) = \dfrac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\rightarrow K = \dfrac{1}{k}$ → versterker

$\omega_n = \sqrt{\dfrac{k}{m}}$ → natuurlijke pulsatie

$2\zeta\omega_n = \dfrac{C}{m} \rightarrow \zeta = \dfrac{C}{2\sqrt{km}}$ → demping

* zie cursus p 18 → 20

$\zeta > 1 \qquad H(s) \qquad \dfrac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 - \zeta^2\omega_n^2 + \zeta^2\omega_n^2}$

$$= \frac{K\omega_n^2 \dfrac{\sqrt{\zeta^2-1}}{\sqrt{\zeta^2-1}}}{(s+\zeta\omega_n)^2 - \omega_n^2(\zeta^2-1)}$$

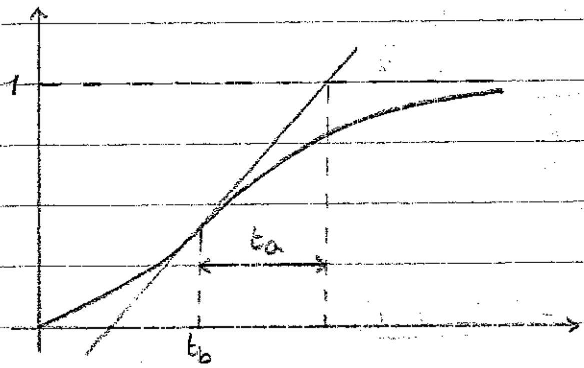
$$y(t) = \frac{K\omega_n}{\sqrt{\zeta^2-1}}\, e^{-\zeta\omega_n t} \cdot \sinh(\omega_n\sqrt{\zeta^2-1}\, t)$$

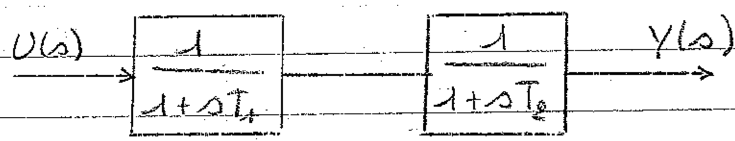
polen : $-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$

$\rightarrow$ overgedempt

is niet gevraagd

OEFENING 2





① $T_1 \neq T_2$

$$* \quad Y(s) = \overbrace{\frac{1}{1+sT_1}\,\frac{1}{1+sT_2}}^{H(s)}\,\overbrace{\frac{1}{s}}^{U(s)} = \frac{A}{1+sT_1} + \frac{B}{1+sT_2} + \frac{C}{s}$$

$$= \frac{As(1+sT_2) + Bs(1+sT_1) + C(1+sT_1)(1+sT_2)}{(1+sT_1)(1+sT_2)s}$$

$$s=0 \rightarrow C=1$$

$$s=\frac{-1}{T_2} \rightarrow \frac{-B}{T_2}\left(1-\frac{T_1}{T_2}\right)=1 \rightarrow B=\frac{T_2^2}{T_1-T_2}$$

$$s=\frac{-1}{T_1} \rightarrow \frac{-A}{T_1}\left(1-\frac{T_2}{T_1}\right)=1 \rightarrow A=\frac{-T_1^2}{T_1-T_2}$$

$$= \frac{-T_1^2}{T_1-T_2}\,\frac{1}{1+sT_1} + \frac{T_2^2}{T_1-T_2}\,\frac{1}{1+sT_2} + \frac{1}{s}$$

$$* \quad y(t) = 1 - \frac{1}{T_1-T_2}\left(T_1 e^{\frac{-t}{T_1}} - T_2 e^{\frac{-t}{T_2}}\right), \quad t \geq 0 \quad ✓$$

$*$ t_b is buigpunt $\rightarrow$ nulpunt van 2^e afgeleide

$$y'(t) = \frac{+\cancel{T_1}}{T_1-T_2}\,\frac{+1}{\cancel{T_1}}\,e^{\frac{-t}{T_1}} + \frac{\cancel{T_2}}{T_1-T_2}\,\frac{-1}{\cancel{T_2}}\,e^{\frac{-t}{T_2}}$$

$$y''(t) = \frac{-1}{T_1(T_1-T_2)}\,e^{\frac{-t}{T_1}} + \frac{1}{T_2(T_1-T_2)}\,e^{\frac{-t}{T_2}}$$

$$y''(t) = 0 = \frac{-1}{T_1(T_1-T_2)} e^{\frac{-t}{T_1}} + \frac{1}{T_2(T_1-T_2)} e^{\frac{-t}{T_2}}$$

$$\rightarrow \frac{1}{T_1\cancel{(T_1-T_2)}} e^{\frac{-t}{T_1}} = \frac{1}{T_2\cancel{(T_1-T_2)}} e^{\frac{-t}{T_2}}$$

$$\frac{T_1}{T_2} = \frac{e^{\frac{-t}{T_1}}}{e^{\frac{-t}{T_2}}} = e^{\left(\frac{-t}{T_1} + \frac{t}{T_2}\right)}$$

$$\ln \frac{T_1}{T_2} = \frac{-t}{T_1} + \frac{t}{T_2} = t_b \frac{-T_2+T_1}{T_1 T_2}$$

$$\rightarrow t_b = \frac{T_1 T_2}{T_1 - T_2} \ln \frac{T_1}{T_2} \checkmark$$

* t_a vinden de vgl van raaklijn op te stellen en snijpunt met $y(t)$

zie Karen p80

OEFENING 3

Stapantwoord berekenen: $u(t) = 1(t)$

$$Y(s) = H(s)\,U(s)$$
$$= H(s)\,\frac{1}{s}$$

$$H(s) = s\,Y(s) = \frac{s\frac{m}{k}}{ms^2 + Cs + k} = \frac{sK\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n^2 = \frac{k}{m}$ deler door m $K = \frac{1}{k}$; $2\zeta\omega_n = \frac{C}{m} \Rightarrow \zeta = \frac{C}{2\sqrt{km}}$

* $\zeta = 0$

$$Y(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\,\frac{1}{s}$$
$$= \frac{K\omega_n^2}{s^2 + \omega_n^2}\,\frac{1}{s}$$
$$= \frac{As + B}{s^2 + \omega_n^2} + \frac{C}{s} = \frac{(As + B)s + C(s^2 + \omega_n^2)}{s(s^2 + \omega_n^2)}$$

$$\begin{cases} s = 0 \rightarrow \omega_n^2 C = K\omega_n^2 \rightarrow C = K \\ A + C = 0 \rightarrow A = -K \\ B = 0 \end{cases}$$

$$= \frac{-Ks}{s^2 + \omega_n^2} + \frac{K}{s}$$

$$y(t) = -K\cos(\omega_n t) + K$$
$$= K[1 - \cos(\omega_n t)] \quad , t \geq 0 \checkmark$$

* $\zeta = 1$

$$Y(s) = \frac{K\omega_n^2}{(s + \omega_n)^2}\,\frac{1}{s}$$
$$= \frac{A}{s + \omega_n} + \frac{B}{(s + \omega_n)^2} + \frac{C}{s} = \frac{As(\omega_n + s) + Bs + C(\omega_n + s)^2}{s(\omega_n + s)^2}$$

$$\begin{cases} s = 0 \rightarrow \omega_n^2 C = K\omega_n^2 \rightarrow C = K \\ s = -\omega_n \rightarrow -\omega_n B = K\omega_n^2 \rightarrow B = -K\omega_n \\ A + C = 0 \rightarrow A = -K \end{cases}$$

$$Y(s) = \frac{-K}{s+\omega_n} - \frac{K\omega_n}{(s+\omega_n)^2} + \frac{K}{s}$$

$$y(t) = -Ke^{-\omega_n t} - Kt\omega_n e^{-\omega_n t} + K$$

$$= K\left[1 - (1 + t\omega_n) e^{-\omega_n t}\right] \quad ; \quad t \geq 0 \quad \checkmark$$

OEFENING 4

Algemeen: $Y(s) = H(s)\,C(s)\,[Y_{ref}(s) - M(s)\,Y(s)]$

a) $Y(s) = (G_2 + G_3) \cdot Z \cdot [Y_{ref}(s) - H_2 Y(s)]$

$\rightarrow$ notatie zonder (s)

$$Y = (G_2 + G_3)\, Z\, [Y_{ref} - H_2 Y]$$

met $Z = \dfrac{G_1 G_4}{1 - G_1 G_4 H_1}$ Komt van: $Z = G_1 G_4 (1 + Z H_1)$

$$Y = (G_2 + G_3) \frac{G_1 G_4}{1 - G_1 G_4 H_1} [Y_{ref} - H_2 Y]$$

$$Y = \frac{\dfrac{(G_2 + G_3)\, G_1 G_4\, Y_{ref}}{1 - G_1 G_4 H_1}}{1 + \dfrac{H_2 (G_2 + G_3)\, G_1 G_4}{1 - G_1 G_4 H_1}} \quad \checkmark$$

b) $Y = (G_1 + G_2)(Y_{ref} + H_1 Y)$

$$Y = \frac{(G_1 + G_2)\, Y_{ref}}{1 - H_1 (G_1 + G_2)} \quad \checkmark$$

c) $Z = G_1 (Y_{ref} + H_1 Z)$

$$\rightarrow Z = \frac{G_1 Y_{ref}}{1 - H_1 G_1}$$

$$Y = Z + G_2 (Y_{ref} + H_1 Z)$$

$$= \frac{G_1 Y_{ref}}{1 - H_1 G_1} + G_2 Y_{ref} + \frac{G_2 H_1 G_1}{1 - H_1 G_1} Y_{ref}$$

$$= \left(\frac{G_1 + G_2 - \cancel{H_1 G_1 G_2} + \cancel{H_1 G_1 G_2}}{1 - H_1 G_1} \right) Y_{ref}$$

$$= \frac{G_1 + G_2}{1 - H_1 G_1} Y_{ref} \quad \checkmark$$

d) $Y = G_2 Y_{ref} + Z$

met $Z = G_1 (Y_{ref} + Z H_1)$

$\rightarrow Z = \dfrac{G_1 Y_{ref}}{1 - H_1 G_1}$

$$Y = G_2 Y_{ref} + \frac{G_1 Y_{ref}}{1 - H_1 G_1}$$

$$= \left(\frac{G_2 - G_2 G_1 H_1 + G_1}{1 - H_1 G_1} \right) Y_{ref} \checkmark$$

e) $Y = G_2 X$

$X = G_1 Z + H_2 Y$

$Z = G_1 Z H_1 + W$

$W = Y_{ref} - Y H_3$

$$\rightarrow Z = \frac{W}{1 - G_1 H_1} = \frac{Y_{ref} - Y H_3}{1 - G_1 H_1}$$

$$\rightarrow X = G_1 \left(\frac{Y_{ref} - Y H_3}{1 - G_1 H_1} \right) + H_2 Y$$

$$\rightarrow Y = G_1 G_2 \left(\frac{Y_{ref} - Y H_3}{1 - G_1 H_1} \right) + G_2 H_2 Y$$

$$\rightarrow \left(1 + \frac{G_1 G_2 H_3}{1 - G_1 H_1} - G_2 H_2 \right) Y = \frac{G_1 G_2}{1 - G_1 H_1} Y_{ref}$$

$$Y = \frac{\dfrac{G_1 G_2}{1 - G_1 H_1}}{1 + \dfrac{G_1 G_2 H_3}{1 - G_1 H_1} - G_2 H_2} Y_{ref}$$ nt helemaal hetzelfde!

oefening 4

c) $Y(s) = G_1(s)\left(U(s) + H_1(s)\,V(s)\right)$

$\qquad + G_2(s)\left(U(s) + H_1(s)\,V(s)\right)$

$V(s) = G_1(s)\left(U(s) + H_1(s)\,V(s)\right)$

$$V(s) = \frac{G_1}{1 - G_1 H_1}\,U(s)$$

$\Rightarrow$ invullen