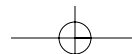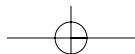# FLUID MECHANICS

PART

2

CHAPTER

10

INTRODUCTION TO FLUID MECHANICS

I n the second part of the text we present the fundamentals of fluid mechanics. In this introductory chapter, we introduce the basic concepts commonly used in the analysis of fluid flow to avoid any misunderstandings. We start with a discussion of the numerous ways of classification of fluid flow, such as *viscous versus inviscid regions of flow, internal versus external flow, compressible versus incompressible flow, laminar versus turbulent flow, natural versus forced flow,* and *steady versus unsteady flow.* We also discuss the no-slip condition, which is responsible for the development of boundary layers adjacent to the solid surfaces, and the no-temperature-jump condition. We continue with a brief history of the development of fluid mechanics. Then we discuss the property *viscosity*, which plays a dominant role in most aspects of fluid flow. Finally, we present the property *surface tension*, and determine the *capillary rise* from static equilibrium conditions.

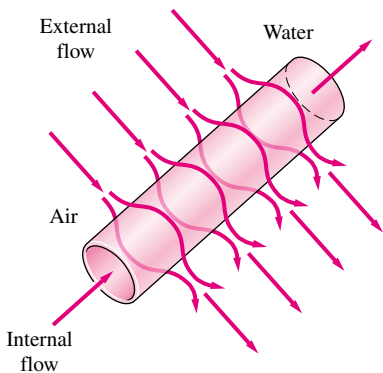## CONTENTS

10–1 ▪ CLASSIFICATION OF FLUID FLOWS

In Chap. 1 we defined *fluid mechanics* as the science that deals with the behavior of fluids at rest or in motion, and the interaction of fluids with solids or other fluids at the boundaries. There is a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups. There are many ways to classify the fluid flow problems, and here we present some general categories.

Viscous versus Inviscid Regions of Flow

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is called the **viscosity,** which is a measure of internal stickiness of the fluid. Viscosity is caused by cohesive forces between the molecules in liquids, and by molecular collisions in gases. There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the effects of viscosity are significant are called **viscous flows.** However, in many flows of practical interest, there are *regions* (typically regions not close to solid surfaces) where viscous forces are negligibly small compared to inertial or pressure forces. Neglecting the viscous terms in such **inviscid flow regions** greatly simplifies the analysis without much loss in accuracy.

Internal versus External Flow

A fluid flow is classified as being internal or external, depending on whether the fluid is forced to flow in a confined channel or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow.** The flow in a pipe or duct is **internal flow** if the fluid is completely bounded by solid surfaces. Water flow in a pipe, for example, is internal flow, and airflow over an exposed pipe during a windy day is external flow (Fig. 10–1). The flow of liquids in a duct is called *open-channel flow* if the duct is only partially filled with the liquid and there is a free surface. The flow of water in rivers and irrigation ditches are examples of such flows.

Compressible versus Incompressible Flow

A fluid flow is classified as being *compressible* or *incompressible,* depending on the density variation of the fluid during flow. The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually classified as *incompressible substances.* A pressure of 210 atm, for example, causes the density of liquid water at 1 atm to change by just 1 percent. Gases, on the other hand, are highly compressible. A pressure change of just 0.01 atm, for example, causes a change of 1 percent in the density of atmospheric air. However, gas flows can be treated as incompressible if the density changes are under about 5 percent, which is usually the case when the flow velocity is less than 30 percent of the speed of sound in

FIGURE 10–1
Internal flow of water in a pipe and the external flow of air over the same pipe.

that gas (i.e., the Mach number of flow is less than 0.3). The speed of sound in air at room temperature is 346 m/s. Therefore, the compressibility effects of air can be neglected at speeds under about 100 m/s. Note that the flow of a gas is not necessarily a compressible flow.

Small density changes of liquids corresponding to large pressure changes can still have important consequences. The irritating "water hammer" in water pipes, for example, is caused by the vibrations of the pipe generated by the reflection of pressure waves following the sudden closing of the valves.

Laminar versus Turbulent Flow

Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth streamlines is called **laminar.** The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called **turbulent.** The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the required power for pumping.

Natural (or Unforced) versus Forced Flow

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In **forced flow,** a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In **natural flows,** any fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid. This thermosiphoning effect is commonly used to replace pumps in solar water heating systems by placing the water tank sufficiently above the solar collectors (Fig. 10–2).

Steady versus Unsteady Flow

The terms *steady* and *uniform* are used frequently in engineering, and thus it is important to have a clear understanding of their meanings. The term **steady** implies *no change at a point with time.* The opposite of steady is **unsteady.** The term *uniform* implies *no change with location* over a specified region. These meanings are consistent with their everyday use (steady girlfriend, uniform distribution, etc.).

The terms *unsteady* and *transient* are often used interchangeably, but these terms are not synonyms. In fluid mechanics, *unsteady* is the most general term that applies to any flow that is not steady, but *transient* is typically used for developing flows. When a rocket engine is fired up, for example, there are transient effects as the pressure builds up inside the rocket engine, the flow accelerates, etc., until the engine settles down and operates steadily. The term *periodic* refers to the kind of unsteady flow in which the flow oscillates about a steady mean.

Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as *steady-flow devices.* (Note that the flow field near the
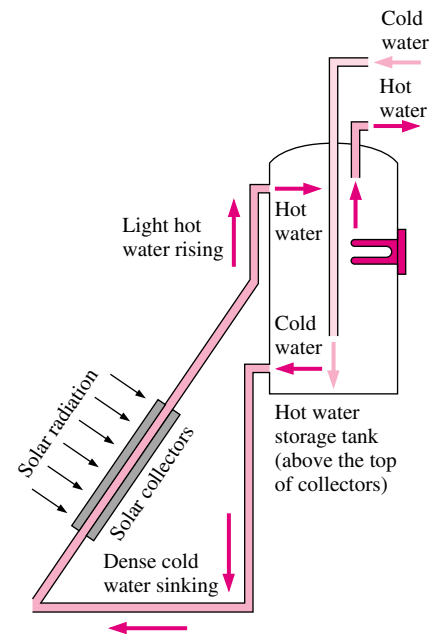
FIGURE 10–2

Natural circulation of water in a solar water heater by thermosiphoning.

rotating blades of a turbomachine is of course unsteady, but we consider the overall flow field rather than the details at some localities when we classify devices.) During steady flow, the fluid properties can change from point to point within a device, but at any fixed point they remain constant. Therefore, the volume, the mass, and the total energy content of a steady-flow device or flow section remain constant in steady operation.

Steady-flow conditions can be closely approximated by devices that are intended for continuous operation such as turbines, pumps, boilers, condensers, and heat exchangers of power plants or refrigeration systems. Some cyclic devices, such as reciprocating engines or compressors, do not satisfy the steady-flow conditions since the flow at the inlets and the exits is pulsating and not steady. However, the fluid properties vary with time in a periodic manner, and the flow through these devices can still be analyzed as a steady-flow process by using time-averaged values for the properties.

Some fascinating visualizations of fluid flow are provided in the book *An Album of Fluid Motion* by Milton Van Dyke (1982). A nice illustration of an unsteady-flow field is shown in Fig. 10–3, taken from Van Dyke's book. Figure 10–3*a* is an instantaneous snapshot from a high-speed motion picture; it reveals large, alternating, swirling, turbulent eddies that are shed into the periodically oscillating wake from the blunt base of the object. The eddies produce shock waves that move upstream alternately over the top and bottom surfaces of the airfoil in an unsteady fashion. Figure 10–3*b* shows the *same* flow field, but the film is exposed for a longer time so that the image is time averaged over 12 cycles. The resulting time-averaged flow field appears "steady" since the details of the unsteady oscillations have been lost in the long exposure.

One of the most important jobs of an engineer is to determine whether it is sufficient to study only the time-averaged "steady" flow features of a problem, or whether a more detailed study of the unsteady features is required. If he or she were interested only in the overall properties of the flow field, such as the time-averaged drag coefficient, the mean velocity and pressure fields, etc., a time-averaged description like that of Fig. 10–3*b*, time-averaged experimental measurements, or an analytical or numerical calculation of the time-averaged flow field would be sufficient. However, if he or she were interested in details about the unsteady-flow field, such as flow induced vibrations, unsteady pressure fluctuations, or the sound waves emitted from the turbulent eddies or the shock waves, a time-averaged description of the flow field would be insufficient.

Most of the analytical and computational examples provided in this textbook deal with steady or time-averaged flows, although we occasionally point out some relevant unsteady-flow features as well when appropriate.

One-, Two-, and Three-Dimensional Flows

A flow field is best characterized by the velocity distribution, and thus a flow is said to be one-, two-, or three-dimensional if the flow velocity varies in one, two, or three primary dimensions, respectively. A typical fluid flow involves a three-dimensional geometry and the velocity may vary in all three dimensions rendering the flow three-dimensional [$\vec{V}$ (x, y, z) in rectangular or $\vec{V}$ (r, θ, z) in cylindrical coordinates]. However, the variation of velocity in certain directions can be small relative to the variation in other directions, and can be

(*a*)

(*b*)
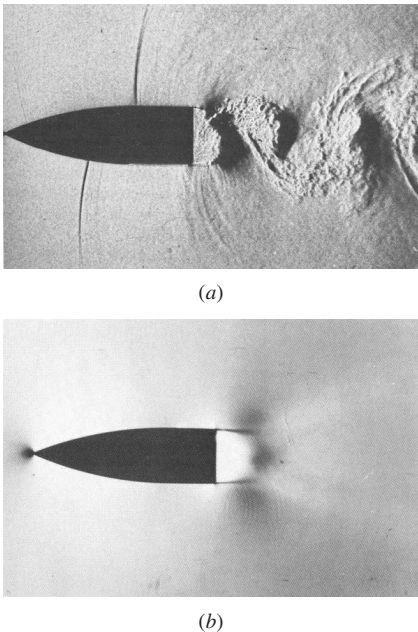
FIGURE 10–3
Oscillating wake of a blunt-based airfoil at Mach number 0.6. Photo (*a*) is an instantaneous image while photo (*b*) is a long-exposure (time-averaged) image. Photos by A. Dyment, J.P. Flodrops, and P. Gryson. Used by permission of Arthur Dyment.

ignored with negligible error. In such cases, the flow can be modeled conveniently as being one- or two-dimensional, which is easier to analyze.

When the entrance effects are far upstream of the region of interest, fully developed fluid flow in a circular pipe is *one-dimensional* since the velocity varies in the radial r-direction but not in the angular θ- or axial z-directions (Fig. 10–4). That is, the velocity profile is the same at any axial z-location, and it is symmetric about the axis of the pipe. Note that even in this simple flow, the velocity cannot be uniform across the cross section of the pipe because of the no-slip condition. However, at a well-rounded entrance to the pipe, the velocity profile may be approximated as nearly **uniform** across the pipe, since the velocity is constant at all radii except very close to the pipe wall (Fig. 10–5)

A flow may be approximated as *two-dimensional* when the aspect ratio is large and the flow does not change appreciably along the longer dimension. For example, the flow of air over a car antenna can be considered two-dimensional except near its ends since the antenna's length is much greater than its diameter, and the airflow hitting the antenna is fairly uniform (Fig. 10–6).
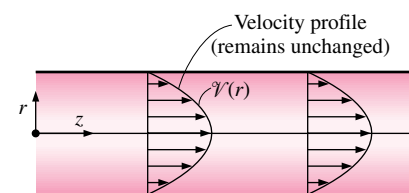

FIGURE 10–4
One-dimensional flow in a circular pipe.

FIGURE 10–5
The velocity profile at the well-rounded entrance to a pipe is nearly uniform (one-dimensional).
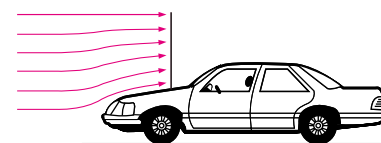
FIGURE 10–6
Flow over a car antenna is approximately two-dimensional except near the top and bottom of the antenna.

EXAMPLE 10–1   Axisymmetric Flow over a Bullet

Consider a bullet piercing through calm air. Determine if airflow over the bullet during its flight is one-, two-, or three-dimensional (Fig. 10–7).
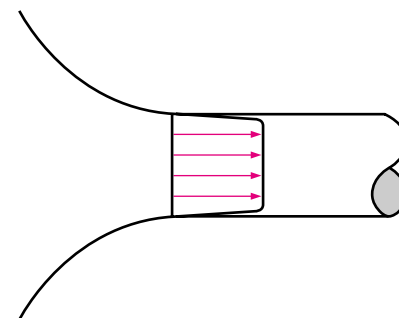
SOLUTION   It is to be determined whether airflow over a bullet is one-, two-, or three- dimensional.

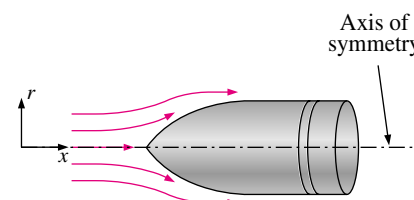*Assumptions*   There are no significant winds and the bullet is not spinning.

Analysis   The bullet possesses an axis of symmetry, and airflow is parallel to this axis. Therefore, airflow is symmetric about the axis, and such flows are said to be axisymmetric. A bullet piercing through air is an example of axisymmetric flow. The velocity in this case varies with the axial distance x and the radial distance r, but not with the angle θ. Therefore, airflow over the bullet is two-dimensional.

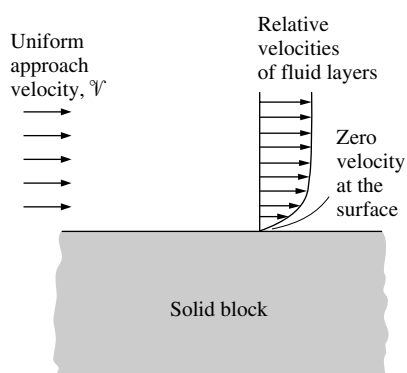
10–2 ▪ THE NO-SLIP CONDITION

Consider the flow of a fluid in a stationary pipe or over a solid surface that is nonporous (i.e., impermeable to the fluid). All experimental observations indicate that a fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, a fluid in direct contact with a solid sticks to the surface and there is no slip. This is known as the **no-slip condition,** and it is due to the fluid viscosity.

The no-slip condition is responsible for the development of the velocity profile. Because of friction between the fluid layers, the layer that sticks to the wall slows the adjacent fluid layer, which slows the next layer, and so on. A fluid layer adjacent to a moving surface has the same velocity as the surface. A consequence of the no-slip condition is that all velocity profiles must have zero values with respect to the surface at the points of contact between a fluid and a solid surface (Fig. 10–8). The only exception to the no-slip condition occurs in extremely rarefied gases.
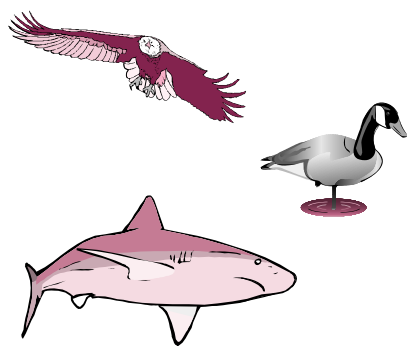

FIGURE 10–7
Axisymmetric flow over a bullet.

Uniform approach velocity, $\mathcal{V}$

Relative velocities of fluid layers

Zero velocity at the surface

Solid block

FIGURE 10–8
A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

FIGURE 10–9
A lot can be learned about fluid mechanics by observing nature.

A similar phenomenon occurs for the temperature. When two bodies at different temperatures are brought into contact, heat transfer occurs until both bodies assume the same temperature at the points of contact. Therefore, a fluid and a solid surface have the same temperature at the points of contact. This is known as **no-temperature-jump condition.**

10–3 ▪ HISTORY OF FLUID MECHANICS

Nature is like an exhibit hall for the applications of fluid mechanics, and we can learn a lot about the fundamentals of fluid mechanics and be inspired by carefully observing nature (Fig. 10–9). The development of fluid mechanics started in ancient times with the need to build irrigation systems and to design better ships with oars. The earliest contribution to the field was made by Archimedes (285–212 B.C.) who formulated the principles of buoyancy of submerged bodies and flotation, and applied those principles to determine the gold content of the crown of King Hiero I. At about the same time, the Roman engineers built an extensive network of fresh-water supply. There was no significant development in fluid mechanics through the Middle Ages until Leonardo da Vinci (1459–1519) conducted several experiments and derived the conservation of mass equation for one-dimensional steady flow.

The development of fluid mechanics continued along two different paths: on one, the mathematicians and physicists developed the theory and applied it to "idealized" problems that did not have much practical value. On the other path, engineers developed empirical equations that could be used in the design of fluid systems in a limited range. The lack of communication between these two groups hindered the development of fluid mechanics for a long time. The development of the laws of motion by Isaac Newton (1649–1727) and the linear law of viscosity for the so-called Newtonian fluids set the stage for advances in fluid mechanics. Applying these laws to a fluid element, Leonhard Euler (1707–1783) obtained the differential equations for fluid motion in 1755. Daniel Bernoulli (1700–1782) developed the energy equation for incompressible flow in 1738. Lord Rayleigh (1849–1919) developed the powerful dimensional analysis technique. Osborn Reynolds (1849–1912) conducted extensive experiments with pipe flow and in 1883 came up with the dimensionless number that bears his name.

The general equations of fluid motion that include the effects of fluid friction, known as the Navier–Stokes equations, were developed by Claude Louis Marie Navier (1785–1836) in 1827 and independently by George Gabriel Stokes (1819–1903) in 1845. These equations were of little use at the time because they were too difficult to solve. Then in a pioneering paper in 1904, Ludwig Prandtl (1875–1953) showed that fluid flows can be divided into a layer near the walls, called the *boundary layer,* where the friction effects are significant and an outer layer where such effects are negligible, thus the Euler and Bernoulli equations are applicable. Theodore von Karman (1889–1963) and Sir Geoffrey I. Taylor (1886–1975) also contributed greatly to the development of fluid mechanics in the twentieth century. The availability of high-speed computers in the last decades and the development of numerical methods have made it possible to solve a variety of real-world fluids problems and to conduct design and optimization studies through numerical simulation.

10–4 ▪ VAPOR PRESSURE AND CAVITATION

Recall that temperature and pressure are dependent properties for pure substances during phase-change processes, and there is one-to-one correspondence between temperatures and pressures. At a given pressure, the temperature at which a pure substance changes phase is called the **saturation temperature,** T_{sat}. Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the **saturation pressure,** P_{sat}. At an absolute pressure of 1 standard atmosphere (1 atm or 101.325 kPa), for example, the saturation temperature of water is 100°C. Conversely, at a temperature of 100°C, the saturation pressure of water is 1 atm.

Atmospheric air can be viewed as a mixture of dry air and water vapor, and the atmospheric pressure is the sum of the pressure of dry air and the pressure of water vapor. The pressure of a vapor, whether it exists alone or in a mixture with other gases, is called the **vapor pressure** P_v. The vapor pressure constitutes a small fraction (usually under 3 percent) of the atmospheric pressure since air is mostly nitrogen and oxygen, but the rate of evaporation from open water bodies such as lakes is controlled by the vapor pressure. For example, the saturation pressure of water at 20°C is 2.34 kPa. Therefore, a glass of water at 20°C left in a room at 1 atm will continue evaporating until the pressure of the water vapor in the room rises to 2.34 kPa at which point phase equilibrium is established. This explains why water at 1 atm "evaporates" at temperatures below 100°C.

For phase-change processes between the liquid and vapor phases of a pure substance, the saturation pressure and the vapor pressure are equivalent since the vapor is pure. Note that the pressure value would be the same whether it is measured in the vapor or liquid phase (provided that it is measured at a location close to the liquid–vapor interface to avoid the hydrostatic effects). Vapor pressure increases with temperature. Thus, a substance at higher temperatures boils at higher pressures. For example, water boils at 134°C in a pressure cooker operating at 3 atm absolute pressure, but it boils at 93°C in an ordinary pan at a 2000-m elevation, where the atmospheric pressure is 0.8 atm. The saturation (or vapor) pressures are given in Appendices 1 and 2 for various substances. A mini table for water is given in Table 10–1 for easy reference.
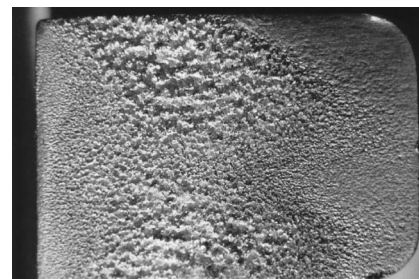
The reason for our interest in vapor pressure is the possibility of the liquid pressure in liquid-flow systems dropping below the vapor pressure at some locations, and the resulting unplanned vaporization. For example, water at 10°C will flash into vapor and form bubbles at locations (such as the tip regions of impellers or suction sides of pumps) where the pressure drops below 1.23 kPa. The vapor bubbles (called **cavitation bubbles** since they form "cavities" in the liquid) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves. This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called **cavitation,** and it is an important consideration in the design of hydraulic turbines and pumps (Fig. 10–10).

Cavitation must be avoided (or at least minimized) in flow systems since it reduces performance, generates annoying vibrations and noise, and causes damage to equipment. The pressure spikes resulting from the large number of bubbles collapsing near a solid surface over a long period of time may cause erosion, surface pitting, fatigue failure, and the eventual destruction of the

TABLE 10–1

Saturation (boiling) pressure of water at various temperatures

Temperature, T, °C	Saturation pressure, P_{sat}, kPa
−10	0.260
−5	0.403
0	0.611
5	0.872
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.38
50	12.35
100	101.3 (1 atm)
150	475.8
200	1554
250	3973
300	8581

FIGURE 10–10
Cavitation damage on a 16-mm by 23-mm aluminum sample tested at 60 m/s for 2.5 h. The sample was located at the cavity collapse region downstream of a cavity generator specifically designed to produce high damage potential (Photo by David Stinebring, Pennsylvania State University).

components or machinery. The presence of cavitation in a flow system can be sensed by its characteristic tumbling sound.

EXAMPLE 10–2   Minimum Pressure to Avoid Cavitation

In a water distribution system, the temperature of water is observed to be as high as 30°C. Determine the minimum pressure allowed in the system to avoid cavitation.

SOLUTION   The minimum pressure in a water distribution system to avoid cavitation is to be determined.
Properties   The vapor pressure of water at 30°C is 4.25 kPa (Table 10–1).
Analysis   To avoid cavitation, the pressure anywhere in flow should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\min} = P_{\text{sat@30°C}} = 4.25 \text{ kPa}$$

Therefore, the pressure should be maintained above 4.25 kPa everywhere in flow.
Discussion   Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.
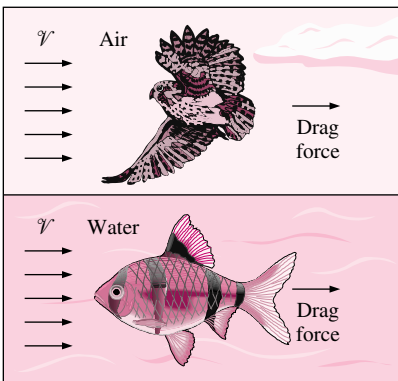

10–5 ▪ VISCOSITY

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force. The magnitude of the force needed to move the table depends on the *friction coefficient* between the table and the floor.

The situation is similar when a fluid moves relative to a solid, or when two fluids move relative to each other. We move with relative ease in air, but not so in water. Moving in oil would be even more difficult, as can be observed by the slower downward motion of a glass ball dropped in a tube filled with oil. It appears that there is a property that represents the internal resistance of a fluid to motion or the "fluidity," and that property is the **viscosity.** The force a flowing fluid exerts on a body in the flow direction is called the **drag force,** and the magnitude of this force depends, in part, on viscosity (Fig. 10–11).

To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a distance ℓ (Fig. 10–12). Now a constant parallel force F is applied to the upper plate while the lower plate is held fixed. After the initial transients, it is observed that the upper plate moves continuously under the influence of this force at a constant velocity $\mathcal{V}$. The fluid in contact with the upper plate sticks to the plate surface and moves with it at the same velocity, and the shear stress τ acting on this fluid layer is

$$\tau = \frac{F}{A} \tag{10–1}$$

FIGURE 10–11
A moving fluid exerts a drag force on a body, partly because of friction caused by viscosity.

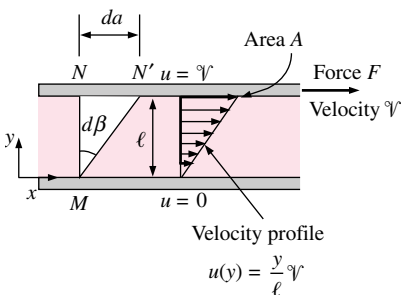**FIGURE 10–12**
The behavior of a fluid between two parallel plates when the upper plate moves with a constant velocity.

where *A* is the contact area between the plate and the fluid. Note that the fluid layer deforms continuously under the influence of shear stress, as expected.

The fluid in contact with the lower plate assumes the velocity of that plate, which is zero (again because of the no-slip condition). In steady operation, the fluid velocity between the plates in most cases varies linearly between 0 and $\mathcal{V}$, and thus the *velocity profile* and the *velocity gradient* are

$$u(y) = \frac{y}{\ell}\mathcal{V} \qquad \text{and} \qquad \frac{du}{dy} = \frac{\mathcal{V}}{\ell} \tag{10–2}$$

where *y* is the vertical distance from the lower plate.

During a differential time interval *dt*, the fluid particles along a vertical line *MN* rotate through a differential angle *dβ* while the upper plate moves a differential distance *da* = $\mathcal{V}dt$. The angular displacement or deformation (or shear strain) can be expressed as

$$d\beta \approx \tan\beta = \frac{da}{\ell} = \frac{\mathcal{V}dt}{\ell} = \frac{du}{dy}\,dt \tag{10–3}$$

Rearranging, the rate of deformation under the influence of shear stress *τ* becomes

$$\frac{d\beta}{dt} = \frac{du}{dy} \tag{10–4}$$

Thus we conclude that the rate of deformation of a fluid element is equivalent to the velocity gradient *du/dy*. Further, it can be verified experimentally that for most fluids the rate of deformation (and thus the velocity gradient) is directly proportional to the shear stress *τ*,

$$\tau \propto \frac{d\beta}{dt} \qquad \text{or} \qquad \tau \propto \frac{du}{dy} \tag{10–5}$$

Fluids for which the rate of deformation is proportional to the shear stress are called **Newtonian fluids** after Sir Isaac Newton, who expressed it first in 1687. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids. Blood and liquid plastics are examples of non-Newtonian fluids.

In one-dimensional flow of Newtonian fluids, shear stress can be expressed by the linear relationship,

Shear stress: $$\tau = \mu\frac{du}{dy} \qquad (N/m^2) \tag{10–6}$$

where the constant of proportionality *μ* is called the **dynamic** (or **absolute**) **viscosity** of the fluid, whose unit is kg/m · s, or equivalently, N · s/m² (or Pa · s where Pa is the pressure unit pascal). A common viscosity unit is **poise**, which is equivalent to 0.1 Pa · s (or *centipoise*, which is one-hundredth of a poise). A plot of shear stress versus the rate of deformation (velocity gradient) for a Newtonian fluid is a straight line whose slope is the viscosity of the fluid, as shown in Fig. 10–13. Note that viscosity is independent of the rate of deformation.

The shear force acting on a Newtonian fluid layer (or, by Newton's third law, the force acting on the plate) is

Shear force: $$F = \tau A = \mu A\frac{du}{dy} \qquad (N) \tag{10–7}$$
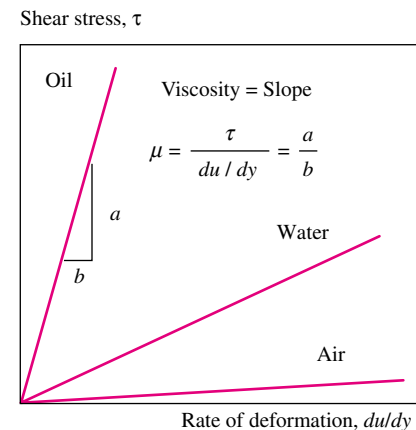
FIGURE 10–13
The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.

where again A is the contact area between the plate and the fluid. Then the force F required to move the upper plate in Fig. 10–12 at a constant velocity of $\mathcal{V}$ while the lower plate remains stationary is

$$F = \mu A \frac{\mathcal{V}}{\ell} \qquad \text{(N)} \qquad \textbf{(10–8)}$$

This relation can alternately be used to calculate μ when the force F is measured. Therefore, the experimental setup just described can be used to measure the viscosity of fluids. Note that under identical conditions, the force F will be very different for different fluids.

For non-Newtonian fluids, the relationship between shear stress and rate of deformation is not linear, as shown in Fig. 10–14. The slope of the curve on the τ versus. du/dy chart is referred to as the *apparent viscosity* of the fluid. Fluids for which the apparent viscosity increases with the rate of deformation (such as solutions with suspended starch or sand) are referred to as *dilatant* or *shear thickening fluids*, and those that exhibit the opposite behavior (the fluid becoming less viscous as it is sheared harder, such as some paints, polymer solutions, and fluids with suspended particles) are referred to as *pseudoplastic* or *shear thinning fluids*. Some materials such as toothpaste can resist a finite shear stress and thus behave as a solid, but deform continuously when the shear stress exceeds the yield stress and thus behave as a fluid. Such materials are referred to as *bingham plastic*.

In fluid mechanics and heat transfer, the ratio of dynamic viscosity to density appears frequently. For convenience, this ratio is given the name **kinematic viscosity** υ and is expressed as $\upsilon = \mu/\rho$. Two common units of kinematic viscosity are m^2/s and **stoke** (1 stoke $= 1$ $cm^2/s = 0.0001$ m^2/s).

In general, the viscosity of a fluid depends on both temperature and pressure, although the dependence on pressure is rather weak. For *liquids*, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usually disregarded, except at extremely high pressures. For *gases*, this is also the case for dynamic viscosity (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure (Fig. 10–15).
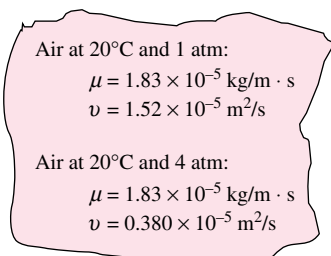
The viscosity of a fluid is a measure of its "stickiness" or "resistance to shear." It is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids, and by the molecular collisions in gases, and it varies greatly with temperature. The viscosity of liquids decreases with temperature, whereas the viscosity of gases increases with temperature (Fig. 10–16). This is because in a liquid the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. As a result, the energized liquid molecules can move more freely.

In a gas, on the other hand, the intermolecular forces are negligible, and the gas molecules at high temperatures move randomly at higher velocities. This results in more molecular collisions per unit volume per unit time, and therefore in greater resistance to flow. The viscosity of a fluid is directly related to power needed to transport a fluid in a pipe or to move a body (such as a car in air or a submarine in the sea) through a fluid.
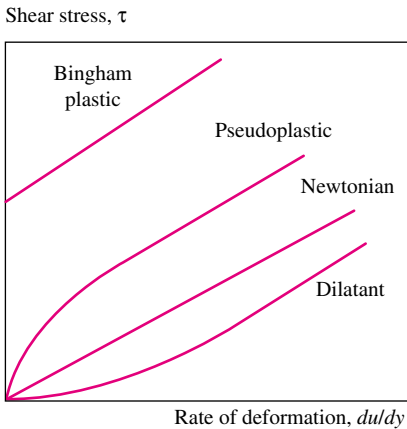
Shear stress, τ

FIGURE 10–14
Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point).

Air at 20°C and 1 atm:
$\mu = 1.83 \times 10^{-5}$ kg/m · s
$\upsilon = 1.52 \times 10^{-5}$ m^2/s

Air at 20°C and 4 atm:
$\mu = 1.83 \times 10^{-5}$ kg/m · s
$\upsilon = 0.380 \times 10^{-5}$ m^2/s

FIGURE 10–15
Dynamic viscosity, in general, does not depend on pressure, but kinematic viscosity does.

The kinetic theory of gases predicts the viscosity of gases to be proportional to the square root of temperature. That is, $\mu_{gas} \propto \sqrt{T}$. This prediction is confirmed by practical observations, but deviations for different gases need to be accounted for by incorporating some correction factors. The viscosity of *gases* is given as a function of temperature by the Sutherland correlation (from The U.S. Standard Atmosphere) as

Gases:
$$\mu = \frac{aT^{1/2}}{1 + b/T} \tag{10–9}$$

where T is absolute temperature, and a and b are experimentally determined constants. Note that measuring viscosities at two different temperatures is sufficient to determine these constants. For air, the values of these constants are $a = 1.458 \times 10^{-6}$ kg/(m · s · K$^{1/2}$) and $b = 110.4$ K at atmospheric conditions. The viscosity of gases is independent of pressure at low to moderate pressures (from a few percent of 1 atm to several atm). But viscosity increases at high pressures due to the increase in density.

For *liquids*, the viscosity is given as

Liquids:
$$\mu = a10^{b/(T-c)} \tag{10–10}$$

where again T is absolute temperature, and a, b, and c are experimentally determined constants. For water, using the values $a = 2.414 \times 10^{-5}$ N · s/m^2, $b = 247.8$ K, and $c = 140$ K results in less than 2.5 percent error in viscosity in the temperature range of 0°C to 370°C (Touloukian et al., 1975).

Consider a fluid layer of thickness ℓ within a small gap between two concentric cylinders, such as the thin layer of oil in a journal bearing. The gap between the cylinders can be modeled as two parallel flat plates separated by a fluid. Noting that torque is $\mathbf{T} = FR$ (force times the moment arm, which is the radius R of the inner cylinder in this case), the tangential velocity is $\mathcal{V} = \omega R$ (angular velocity times the radius), and taking the wetted surface area of the inner cylinder to be $A = 2\pi R L$ by disregarding the shear stress acting on the two ends of the inner cylinder, torque can be expressed as

$$\mathbf{T} = FR = \mu \frac{2\pi R^3 \omega L}{\ell} = \mu \frac{4\pi^2 R^3 \dot{n} L}{\ell} \tag{10–11}$$

where L is the length of the cylinder and $\dot{n}$ is the number of revolutions per unit time, which is usually expressed in rpm (revolutions per minute). Note that the angular distance traveled during one rotation is 2π radians, and thus the relation between the angular velocity in rad/min and the rpm is $\omega = 2\pi\dot{n}$. Equation 10–11 can be used to calculate the viscosity of a fluid by measuring torque at a specified angular velocity. Therefore, two concentric cylinders can be used as a *viscometer*, a device that measures viscosity.

The viscosities of some fluids at room temperature are listed in Table 10–2. They are plotted against the temperature in Fig. 10–17. Note that the viscosities of different fluids differ by several orders of magnitude. Also note that it is more difficult to move an object in a higher-viscosity fluid such as engine oil than it is in a lower-viscosity fluid such as water. Liquids, in general, are much more viscous than gases.
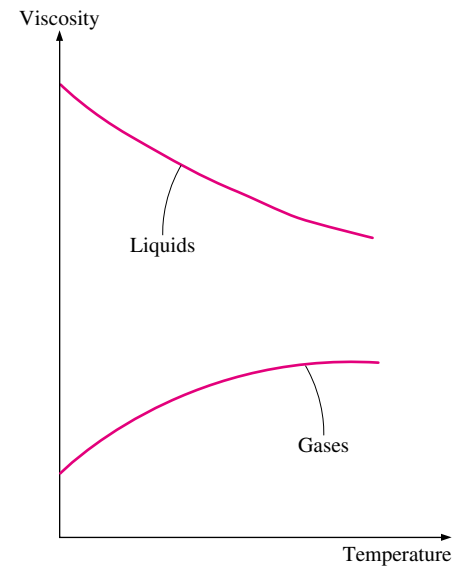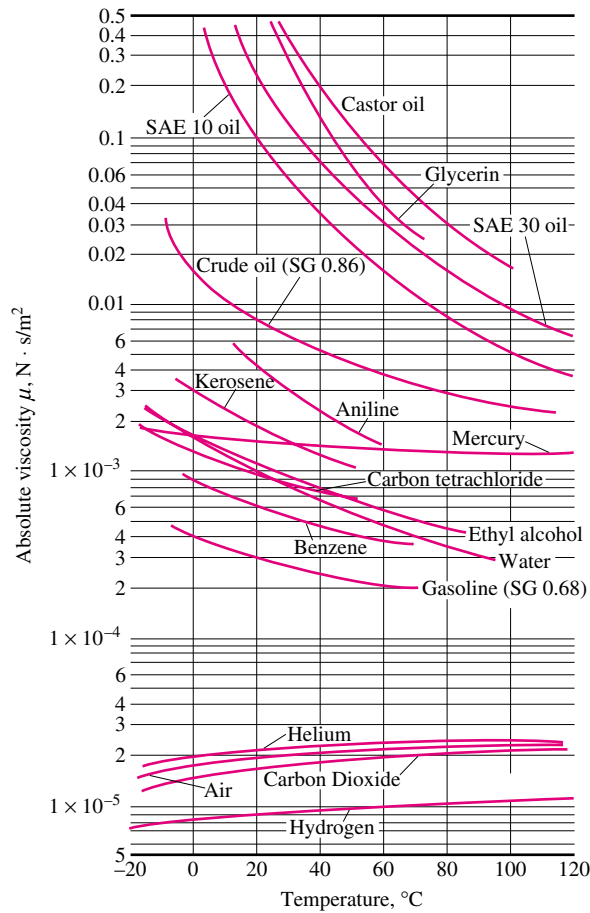

FIGURE 10–16
The viscosity of liquids decreases and the viscosity of gases increases with temperature.

TABLE 10–2

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

FIGURE 10–17
The variation of dynamic (absolute) viscosities of common fluids with temperature at 1 atm
$(1 \text{ N} \cdot \text{s/m}^2 = 1 \text{ kg/m} \cdot \text{s} = 0.020886 \text{ lbf} \cdot \text{s/ft}^2)$
(from White).
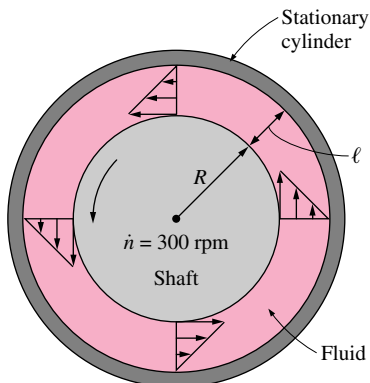
FIGURE 10–18
Schematic for Example 10–3.

EXAMPLE 10–3   Determining the Viscosity of a Fluid

The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders (Fig. 10–18). The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 N · m. Determine the viscosity of the fluid.

SOLUTION   The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions   **1** The inner cylinder is completely submerged in oil. **2** The viscous effects on the two ends of the inner cylinder are negligible.
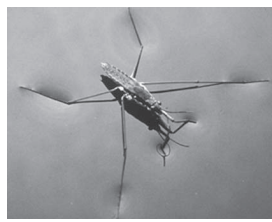
Analysis   Solving Eq. 10–11 for viscosity and substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathbf{T}\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.8 \text{ N} \cdot \text{m})(0.0015 \text{ m})}{4\pi^2 (0.06 \text{ m})^3 (300/60 \text{ 1/s})(0.4 \text{ m})} = \mathbf{0.158 \text{ N/m}^2 \cdot \text{s}}$$

Courtesy Visuals
Unlimited, photo
by Pegasus

Courtesy Visuals
Unlimited, photo
by Dennis Drenner

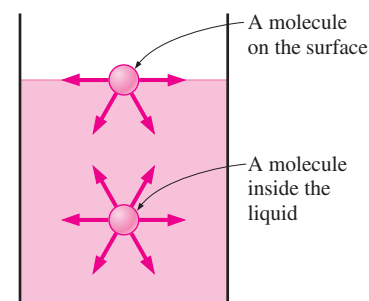**FIGURE 10–19**
Some consequences of
surface tension.

Discussion   Viscosity is a strong function of temperature, and a viscosity value
without a temperature is of little value. Therefore, the temperature of the fluid
should have also been measured during this experiment, and reported with this
calculation.

10–6 ▪ SURFACE TENSION AND CAPILLARY EFFECT

It is often observed that a drop of blood forms a hump on a horizontal glass,
a drop of mercury forms a near-perfect sphere and can be rolled just like a
steel ball over a smooth surface; water droplets from rain or dew hang from
branches or leaves of trees; a liquid fuel injected into an engine forms a mist
of spherical droplets; water dripping from a leaky faucet falls as spherical
droplets; and a soap bubble released into the air forms a spherical shape
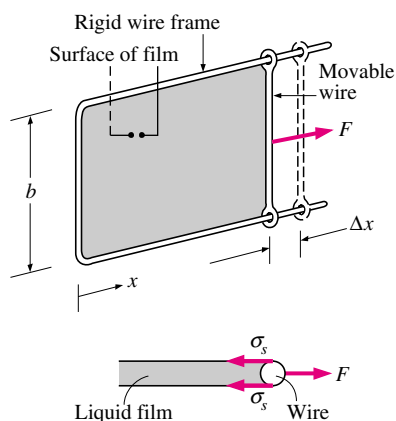(Fig. 10–19).

In these and other observances, liquid droplets behave like small spherical
balloons filled with the liquid, and the surface of the liquid acts like a
stretched elastic membrane under tension. The pulling force that causes this
tension acts parallel to the surface and is due to the attractive forces between
the molecules of the liquid. The magnitude of this force per unit length is
called **surface tension** σ_s and is usually expressed in the unit N/m (or lbf/ft in
English units). This effect is also called *surface energy* and is expressed in the
equivalent unit of N · m/m^2 or J/m^2. In this case, σ_s represents the stretching
work that needs to be done to increase the surface area of the liquid by a unit
amount.

To visualize how surface tension arises, we present a microscopic view in
Fig. 10–20 by considering two liquid molecules, one at the surface and one
deep within the liquid body. The attractive forces applied on the interior mol-
ecule by the surrounding molecules balance each other because of symmetry.
But the attractive forces acting on the surface molecule are not symmetric, and
the attractive forces applied by the gas molecules above are usually very
small. Therefore, there is a net attractive force acting on the molecule at the
surface of the liquid, which tends to pull the molecules on the surface toward
the interior of the liquid. This force is balanced by the repulsive forces from

A molecule
on the surface

A molecule
inside the
liquid

FIGURE 10–20
Attractive forces acting on a liquid
molecule at the surface and
deep inside the liquid.

the molecules below the surface that are being compressed. The resulting compression effect causes the liquid to minimize its surface area. This is the reason for the tendency of the liquid droplets to attain a spherical shape, which has the minimum surface area for a given volume.

You also may have observed, with amusement, that some insects can land on water (even walk on water) and that small steel needles can float on water. These are again made possible by surface tension that balances the weights of these objects.

To understand the surface tension effect better, consider a liquid film (such as the film of a soap bubble) suspended on a U-shaped wire frame with a movable side (Fig. 10–21). Normally, the liquid film tends to pull the movable wire inward in order to minimize its surface area. A force F needs to be applied on the movable wire in the opposite direction to balance this pulling effect. The thin film in the device has two surfaces (the top and bottom surfaces) exposed to air, and thus the length along which the tension acts in this case is $2b$. Then a force balance on the movable wire gives $F = 2b\sigma_s$, and thus the surface tension can be expressed as

$$\sigma_s = \frac{F}{2b} \tag{10–12}$$

Note that for $b = 0.5$ m, the force F measured (in N) is simply the surface tension in N/m. An apparatus of this kind with sufficient precision can be used to measure the surface tension of various fluids.

In the U-shaped wire, the force F remains constant as the movable wire is pulled to stretch the film and increase its surface area. When the movable wire is pulled a distance Δx, the surface area increases by $\Delta A = 2b \Delta x$, and the work done W during this stretching process is

$$W = \text{Force} \times \text{Distance} = F \Delta x = 2b\sigma_s \Delta x = \sigma_s \Delta A$$

since the force remains constant in this case. This result can also be interpreted as *the surface energy of the film is increased by an amount $\sigma_s \Delta A$ during this stretching process,* which is consistent with the alternative interpretation of σ_s as surface energy. This is similar to a rubber band having more potential (elastic) energy after it is stretched further. In the case of liquid film, the work is used to move liquid molecules from the interior parts to the surface against the attraction forces of other molecules. Therefore, surface tension also can be defined as *the work done per unit increase in the surface area of the liquid.*

The surface tension varies greatly from substance to substance, and with temperature for a given substance, as shown in Table 10–3. At 20°C, for example, the surface tension is 0.073 N/m for water and 0.440 N/m for mercury surrounded by atmospheric air. The very high surface tension also explains why mercury droplets form spherical balls that can be rolled like a solid ball on a surface without wetting the surface. The surface tension of a liquid, in general, decreases with temperature and becomes zero at the critical point (and thus there is no distinct liquid–vapor interface at temperatures above the critical point). The effect of pressure on surface tension is usually negligible.

FIGURE 10–21
Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length b.

TABLE 10–3

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface tension σ_s, N/m*
Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028

*Multiply by 0.06852 to convert to lbf/ft.

The surface tension of a substance can be changed considerably by *impurities.* Therefore, certain chemicals, called *surfactants,* can be added to a liquid to decrease its surface tension. For example, soaps and detergents lower the surface tension of water and enable it to penetrate through the small openings between fibers for more effective washing. But this also means that devices whose operation depends on surface tension (such as heat pipes) can be destroyed by the presence of impurities due to poor workmanship.

We speak of surface tension for liquids only at liquid–liquid or liquid–gas interfaces. Therefore, it is important to specify the adjacent liquid or gas when specifying surface tension. Also, surface tension determines the size of the liquid droplets that form. A droplet that keeps growing by the addition of more mass will break down when the surface tension can no longer hold it together. This is like a balloon that will burst while being inflated when the pressure inside rises above the strength of the balloon material.

A curved interface indicates a pressure difference (or "pressure jump") across the interface with pressure being higher on the concave side. The excess pressure ΔP inside a droplet or bubble above the atmospheric pressure, for example, can be determined by considering the free-body diagram of half a droplet or bubble (Fig. 10–22). Noting that surface tension acts along the circumference and the pressure acts on the area, horizontal force balances for the droplet and the bubble give

Droplet: $\quad (2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{droplet}} \quad \rightarrow \quad \Delta P_{\text{droplet}} = P_i - P_o = \dfrac{2\sigma_s}{R}$ **(10–13)**

Bubble: $\quad 2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}} \quad \rightarrow \quad \Delta P_{\text{bubble}} = P_i - P_o = \dfrac{4\sigma_s}{R}$ **(10–14)**

where P_i and P_o are the pressures inside and outside the droplet or bubble, respectively. When the droplet or bubble is in the atmosphere, P_o is simply atmospheric pressure. The factor 2 in the force balance for the bubble is due to the bubble consisting of a film with *two* surfaces (inner and outer surfaces) and thus two circumferences in the cross section.

The excess pressure in a droplet (or bubble) also can be determined by considering a differential increase in the radius of the droplet due to the addition of a differential amount of mass and interpreting the surface tension as the increase in the surface energy per unit area. Then the increase in the surface energy of the droplet during this differential expansion process becomes

$$\delta W_{\text{surface}} = \sigma_s \, dA = \sigma_s \, d(4\pi R^2) = 8\pi R \sigma_s \, dR$$

The expansion work done during this differential process is determined by multiplying the force by distance to obtain

$$\delta W_{\text{expansion}} = \text{Force} \times \text{Distance} = F \, dR = (\Delta P A) \, dR = 4\pi R^2 \, \Delta P \, dR$$

Equating the two expressions above gives $\Delta P_{\text{droplet}} = 2\sigma_s/R,$ which is the same relation obtained before and given in Eq. 10–13. Note that the excess pressure in a droplet or bubble is inversely proportional to the radius.

$(2\pi R)\sigma_s$

$(\pi R^2)\Delta P_{\text{droplet}}$

(*a*) Half a droplet

$2(2\pi R)\sigma_s$

$(\pi R^2)\Delta P_{\text{bubble}}$
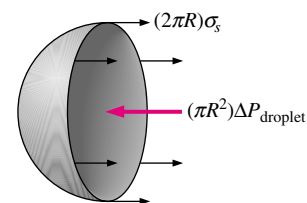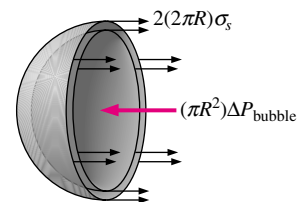
(*b*) Half a bubble

FIGURE 10–22
The free-body diagram of half a droplet and half a bubble.

Capillary Effect

Another interesting consequence of surface tension is the **capillary effect,** which is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. Such narrow tubes or confined flow channels are called **capillaries.** The rise of kerosene through a cotton wick inserted into the reservoir of a kerosene lamp is due to this effect. The capillary effect is also partially responsible for the rise of water to the top of tall trees. The curved free surface of a liquid in a capillary tube is called the **meniscus.**

It is commonly observed that water in a glass container curves up slightly at the edges where it touches the glass surface; but the opposite occurs for mercury: it curves down at the edges (Fig. 10–23). This effect is usually expressed by saying that water *wets* the glass (by sticking to it) while mercury does not. The strength of the capillary effect is quantified by the **contact** (or *wetting*) **angle** ϕ, defined as *the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.* The surface tension force acts along this tangent line toward the solid surface. A liquid is said to wet the surface when $\phi < 90º$ and not to wet the surface when $\phi > 90º$. In atmospheric air, the contact angle of water (and most other organic liquids) with glass is nearly zero, $\phi \approx 0º$ (Fig. 10–24). Therefore, the surface tension force acts upward on water in a glass tube along the circumference, tending to pull the water up. As a result, water rises in the tube until the weight of the liquid in the tube above the liquid level of the reservoir balances the surface tension force. The contact angle is 130º for mercury–glass and 26º for kerosene–glass in air. Note that the contact angle, in general, is different in different environments (such as another gas or liquid in place of air).

The phenomenon of capillary effect can be explained microscopically by considering *cohesive forces* (the forces between like molecules, like water) and *adhesive forces* (the forces between dislike molecules, like water and glass). The liquid molecules at the solid–liquid interface are subjected to both cohesive forces by other liquid molecules and adhesive forces by the molecules of the solid. The relative magnitudes of these forces determine whether a liquid wets a solid surface or not. Obviously, the water molecules are more strongly attracted to the glass molecules than they are to other water molecules, and thus water tends to rise along the glass surface. The opposite occurs for mercury, which causes the liquid surface near the glass wall to be suppressed (Fig. 10–25).

The magnitude of the capillary rise in a circular tube can be determined from a force balance on the cylindrical liquid column of height h in the tube (Fig. 10–26). The bottom of the liquid column is at the same level as the free surface of the reservoir, and thus the pressure there must be atmospheric pressure. This balances the atmospheric pressure acting at the top surface, and thus these two effects cancel each other. The weight of the liquid column is approximately
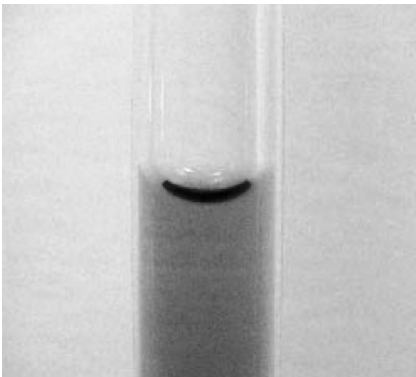
$$W = mg = \rho Vg = \rho g(\pi R^2 h)$$

Equating the vertical component of the surface tension force to the weight gives

$$W = F_{\text{surface}} \quad \rightarrow \quad \rho g(\pi R^2 h) = 2\pi R \sigma_s \cos \phi$$
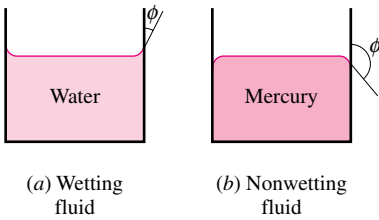
FIGURE 10–23

The contact angle for wetting and nonwetting fluids.

(*a*) Wetting fluid   (*b*) Nonwetting fluid

FIGURE 10–24

The meniscus of colored water in a 4-mm inner diameter glass tube. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle. (Photo by Gabrielle Tremblay, Pennsylvania State University.)

Solving for *h* gives the capillary rise to be

Capillary rise: $h = \dfrac{2\sigma_s}{\rho g R} \cos \phi$ $(R = \text{constant})$ **(10–15)**

This relation is also valid for nonwetting liquids (such as mercury in glass) and gives the capillary drop. In this case $\phi > 90°$ and thus $\cos \phi < 0$, which makes *h* negative. Therefore, a negative value of capillary rise corresponds to a capillary drop (Fig. 10–25).

Note that the capillary rise is inversely proportional to the radius of the tube. Therefore, the thinner the tube is, the greater the rise (or fall) of the liquid in the tube. In practice, the capillary effect is usually negligible in tubes whose diameter is greater than 1 cm. When pressure measurements are made using manometers and barometers, it is important to use sufficiently large tubes to minimize the capillary effect. The capillary rise is also inversely proportional to the density of the liquid, as expected. Therefore, lighter liquids experience greater capillary rises. Finally, it should be kept in mind that Eq. 10–15 is derived for constant diameter tubes and should not be used for tubes of variable cross section.

FIGURE 10–25
The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.

FIGURE 10–26
The forces acting on a liquid column that has risen in a tube due to the capillary effect.

FIGURE 10–27
Schematic for Example 10–4.

EXAMPLE 10–4   **The Capillary Rise of Water in a Tube**

A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube (Fig. 10–27).

SOLUTION   The rise of water in a slender tube as a result of the capillary effect is to be determined.
Assumptions   **1** There are no impurities in the water and no contamination on the surfaces of the glass tube. **2** The experiment is conducted in atmospheric air.
Properties   The surface tension of water at 20°C is 0.073 N/m (Table 10–3). The contact angle of water with glass is 0° (from preceding text). We take the density of liquid water to be 1000 kg/m³.
Analysis   The capillary rise is determined directly from Eq. 10–15 by substituting the given values, yielding

$$h = \frac{2\sigma_s}{\rho g R} \cos \phi = \frac{2(0.073 \text{ N/m})}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \times 10^{-3}\text{m})} \left(\frac{1\text{kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) \cos 0°$$

$$= 0.050 \text{ m} = \textbf{5.0 cm}$$

Therefore, water rises in the tube 5 cm above the liquid level in the cup.

Discussion   Note that if the tube diameter were 1 cm, the capillary rise would be 0.3 mm, which is hardly noticeable to the eye. Actually, the capillary rise in a large-diameter tube occurs only at the rim. The center does not rise at all. Therefore, the capillary effect can be ignored for large-diameter tubes.
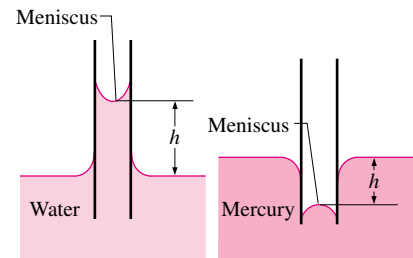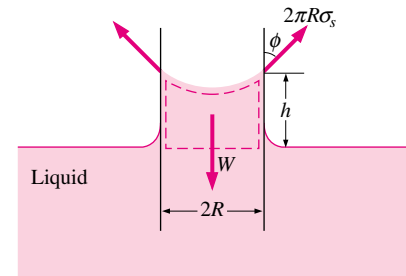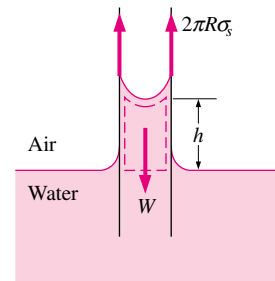

SUMMARY

In this chapter the basic concepts of fluid mechanics are introduced and discussed. *Fluid mechanics* is the science that deals with the behavior of fluids at rest or in motion, and the interaction of fluids with solids or other fluids at the boundaries.

The flow of an unbounded fluid over a surface is *external flow*, and the flow in a pipe or duct is *internal flow* if the fluid is completely bounded by solid surfaces. A fluid flow is classified as being *compressible* or *incompressible*, depending on the density variation of the fluid during flow. The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. The term *steady* implies *no change with time*. The opposite of steady is *unsteady*, or *transient*. The term *uniform* implies *no change with location* over a specified region. A flow is said to be *one-dimensional* when the velocity changes in one dimension only. A fluid in direct contact with a solid surface sticks to the surface and there is no slip. This is known as the *no-slip condition*, and it is due to the viscosity of the fluid.

At a given temperature, the pressure at which a pure substance changes phase is called the *saturation pressure*. For phase-change processes between the liquid and vapor phases of a pure substance, the saturation pressure is commonly called the *vapor pressure* P_v.

The *viscosity* of a fluid is a measure of its "stickiness" or "resistance to deformation." The tangential force per unit area is called *shear stress*, and is expressed for simple shear flow between plates (one-dimensional flow) as

$$\tau = \mu \frac{du}{dy}$$

where μ is the *dynamic* (or *absolute*) *viscosity* of the fluid, u is the velocity component in the flow direction, and y is the direction normal to flow direction. The fluids that obey this linear relationship are called *Newtonian fluids*. The ratio of dynamic viscosity to density is called the *kinematic viscosity, v.*

For liquids, both the dynamic and kinematic viscosities are essentially independent of pressure. For gases, this is also the case for dynamic viscosity, but not for kinematic viscosity since the density of a gas is proportional to its pressure.

The pulling effect on the liquid molecules at an interface caused by the attractive forces of molecules per unit length is called *surface tension σ_s*. The excess pressure ΔP inside a spherical droplet or bubble is given by

$$\Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R}$$

and

$$\Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R}$$

where P_i and P_o are the pressures inside and outside the droplet or bubble. The rise or fall of a liquid in a small-diameter tube inserted into the liquid due to surface tension is called the *capillary effect*. The strength of the capillary effect is quantified by the *contact angle ϕ*, defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact. A liquid is said to wet the surface when $\phi < 90°$, and not to wet the surface when $\phi > 90°$. The capillary rise or drop is given by

$$h = \frac{2\sigma_s}{\rho g R} \cos \phi$$

The capillary rise is inversely proportional to the radius of the tube, and is negligible for tubes whose diameter is larger than about 1 cm.

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PROBLEMS*

Classification of Fluid Flows, Vapor Pressure, and Cavitation

10–1C Define internal, external, and open-channel flows.

10–2C Define incompressible flow and incompressible fluid. Must the flow of a compressible fluid necessarily be treated as compressible?

10–3C What is the no-slip condition? What causes it?

10–4C What is forced flow? How does it differ from natural flow? Is flow caused by wind forced or natural flow?

10–5C What is vapor pressure? How is it related to saturation pressure?

10–6C Does water boil at higher temperatures at higher pressures? Explain.

10–7C If the pressure of a substance is increased during a boiling process, will the temperature also increase or will it remain constant? Why?

10–8C What is cavitation? What causes it?

10–9 In a piping system, the water temperature remains under 40°C. Determine the minimum pressure allowed in the system to avoid cavitation.

10–10 The analysis of a propeller that operates in water at 20°C shows that the pressure at the tips of the propeller drops to 2 kPa at high speeds. Determine if there is a danger of cavitation for this propeller.

10–11E The analysis of a propeller that operates in water at 70°F shows that the pressure at the tips of the propeller drops to 0.1 psia at high speeds. Determine if there is a danger of cavitation for this propeller.

10–12 A pump is used to transport water to a higher reservoir. If the water temperature is 25°C, determine the lowest pressure that can exist in the pump without cavitation.

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*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with a CD-EES icon ⊛ are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon ▦ are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

Viscosity

10–13C What is viscosity? What is the cause of it in liquids and in gases? Do liquids or gases have higher dynamic viscosities?

10–14C What is a Newtonian fluid? Is water a Newtonian fluid?

10–15C Consider two identical small glass balls dropped into two identical containers, one filled with water and the other with oil. Which ball will reach the bottom of the container first? Why?

10–16C How does the dynamic viscosity of (a) liquids and (b) gases vary with temperature?

10–17C How does the kinematic viscosity of (a) liquids and (b) gases vary with temperature?

10–18 A 50-cm $\times$ 30-cm $\times$ 20-cm block weighing 150 N is to be moved at a constant velocity of 0.8 m/s on an inclined surface with a friction coefficient of 0.27. (a) Determine the force F that needs to be applied in the horizontal direction. (b) If a 0.4-mm-thick oil film with a dynamic viscosity of 0.012 Pa · s is applied between the block and inclined surface, determine the percent reduction in the required force.
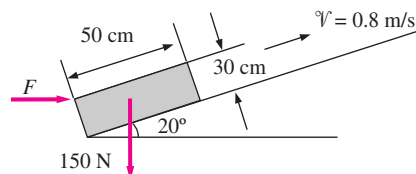

FIGURE P10–18

10–19 Consider the flow of a fluid with viscosity μ though a circular pipe. The velocity profile in the pipe is given as $u(r) = u_{max}(1 - r^n/R^n)$, where u_{max} is maximum flow velocity, which occurs at the centerline; r is the radial distance from the centerline; and $u(r)$ is the flow velocity at any position r. Develop a relation for the drag force exerted on the pipe wall by the fluid in the flow direction per unit length of the pipe.
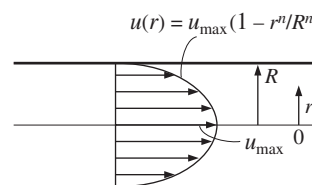

FIGURE P10–19

10–20 A thin 20-cm $\times$ 20-cm flat plate is pulled at 1 m/s horizontally through a 3.6-mm-thick oil layer sandwiched between two plates, one stationary and the other moving at a constant

velocity of 0.3 m/s, as shown in the figure. The dynamic viscosity of oil is 0.027 Pa · s. Assuming the velocity in each oil layer to vary linearly, (a) plot the velocity profile and find the location where the oil velocity is zero and (b) determine the force that needs to be applied on the plate to maintain this motion.
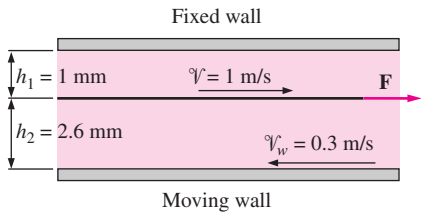

FIGURE P10–20

10–21 A frustum-shaped body is rotating at a constant angular speed of 200 rad/s in a container filled with SAE 10W oil at 20°C ($\mu = 0.1$ Pa · s), as shown in the figure. If the thickness of the oil film on all sides is 1.2 mm, determine the power required to maintain this motion. Also determine the reduction in the required power input when the oil temperature rises to 80°C ($\mu = 0.0078$ Pa · s).
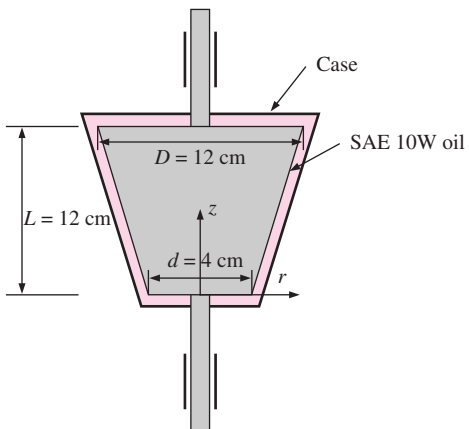
FIGURE P10–21

10–22 The clutch system shown in the figure is used to transmit torque through a 3-mm-thick oil film with $\mu = 0.38$ N · s/m^2 between two identical 30-cm-diameter disks. When the driving shaft rotates at a speed of 1450 rpm, the driven shaft is observed to rotate at 1398 rpm. Assuming a linear velocity profile for the oil film, determine the transmitted torque.
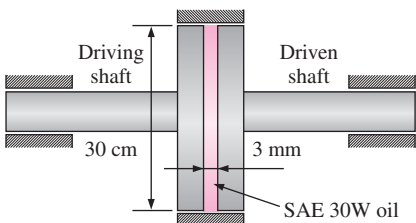

FIGURE P10–22

10–23 Reconsider Prob. 10–22. Using EES (or other) software, investigate the effect of oil film thickness on the torque transmitted. Let the film thickness vary from 0.1 mm to 10 mm. Plot your results, and state your conclusions.

10–24 The viscosity of some fluids changes when a strong electric field is applied on them. This phenomenon is known as the electrorheological (ER) effect, and fluids that exhibit such behavior are known as ER fluids. The Bingham plastic model for shear stress, which is expressed as $\tau = \tau_y + \mu(du/dy)$ is widely used to describe ER fluid behavior because of its simplicity. One of the most promising applications of ER fluids is the ER clutch. A typical multidisk ER clutch consists of several equally spaced steel disks of inner radius R_1 and outer radius R_2, N of them attached to the input shaft. The gap h between the parallel disks is filled with a viscous fluid. (a) Find a relationship for the torque generated by the clutch when the output shaft is stationary and (b) calculate the torque for an ER clutch with $N = 11$ for $R_1 = 50$ mm, $R_2 = 200$ mm, and $\dot{n} = 2400$ rpm if the fluid is SAE 10 with $\mu = 0.1$ Pa · s, $\tau_y = 2.5$ kPa, and $h = 1.2$ mm. *Answer: (b)* 2060 N · m
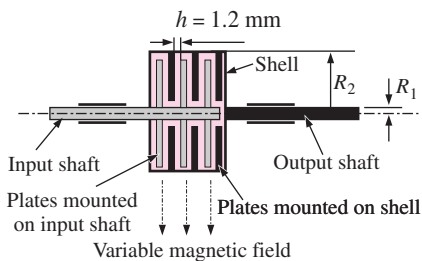

FIGURE P10–24

10–25 The viscosity of some fluids, called magnetorheological (MR) fluids, changes when a magnetic field is applied. Such fluids involve micron-sized magnetizable particles suspended in an appropriate carrier liquid, and are suitable for use in controllable hydraulic clutches. See Fig. P10–24. The MR fluids can have much higher viscosities than the ER fluids, and they often exhibit shear-thinning behavior in which the viscosity of the fluid decreases as the applied shear force increases. This behavior is also known as pseudoplastic behavior, and can be successfully represented by Herschel–Bulkley constitutive model expressed as $\tau = \tau_y + K(du/dy)^m$. Here τ is the shear stress applied, τ_y is the yield stress, K is the consistency index, and m is the power index. For a Herschel–Bulkley fluid with $\tau_y = 900$ Pa, $K = 58$ Pa · s^m, and $m = 0.82$, (a) find a relationship for the torque transmitted by an MR clutch for N plates attached to the input shaft when the input shaft is rotating at an angular speed of ω while the output shaft is stationary and (b) calculate the torque transmitted by such a clutch with $N = 11$ plates for $R_1 = 50$ mm, $R_2 = 200$ mm, $\dot{n} = 2400$ rpm, and $h = 1.2$ mm.

10–26 The viscosity of a fluid is to be measured by a viscometer constructed of two 75-cm-long concentric cylinders. The outer diameter of the inner cylinder is 15 cm, and the gap between the two cylinders is 0.12 cm. The inner cylinder is

rotated at 200 rpm, and the torque is measured to be 0.8 N · m. Determine the viscosity of the fluid.

10–27E   The viscosity of a fluid is to be measured by a viscometer constructed of two 3-ft-long concentric cylinders. The inner diameter of the outer cylinder is 6 in, and the gap between the two cylinders is 0.05 in. The outer cylinder is rotated at 250 rpm, and the torque is measured to be 1.2 lbf · ft. Determine the viscosity of the fluid.
Answer: 0.000648 lbs · s/ft²
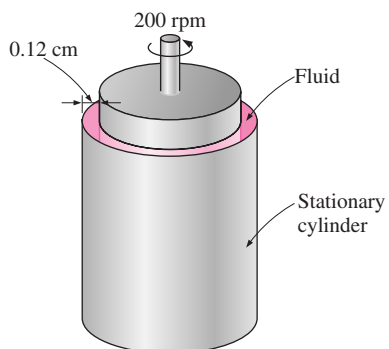
FIGURE P10–26

10–28   In regions far from the entrance, fluid flow through a circular pipe is one-dimensional, and the velocity profile for laminar flow is given by $u(r) = u_{max}(1 - r^2/R^2)$ where R is the radius of the pipe, r is the radial distance from the center of the pipe, and u_{max} is the maximum flow velocity, which occurs at the center. Obtain (*a*) a relation for the drag force applied by the fluid on a section of the pipe of length L and (*b*) the value of the drag force for water flow at 20°C with $R = 0.08$ m, $L = 15$ m, $u_{max} = 3$ m/s, and $\mu = 0.0010$ kg/m · s.

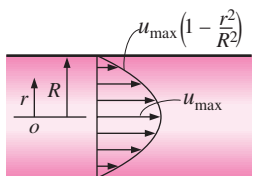**FIGURE P10–28**

10–29   Repeat Prob. 10–28 for $u_{max} = 5$ m/s.
Answer: (*b*) 0.942 N

Surface Tension and Capillary Effect

10–30C   What is surface tension? What is it caused by? Why is the surface tension also called surface energy?

10–31C   Consider a soap bubble. Is the pressure inside the bubble higher or lower than the pressure outside?

10–32C   What is the capillary effect? What is it caused by? How is it affected by the contact angle?

10–33C   A small-diameter tube is inserted into a liquid whose contact angle is 110°. Will the level of liquid in the tube rise or drop? Explain.

10–34C   Is the capillary rise greater in small- or large-diameter tubes?

10–35E   A 0.03-in-diameter glass tube is inserted into kerosene at 68°F. The contact angle of kerosene with a glass surface is 26°. Determine the capillary rise of kerosene in the tube.   *Answer:* 0.65 in
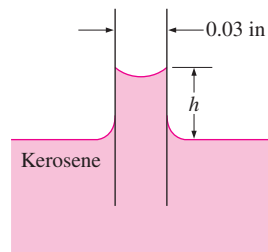

FIGURE P10–35E

10–36   A 1.9-mm-diameter tube is inserted into an unknown liquid whose density is 960 kg/m³, and it is observed that the liquid rises 5 mm in the tube, making a contact angle of 15°. Determine the surface tension of the liquid.

10–37   Determine the gage pressure inside a soap bubble of diameter (*a*) 0.2 cm and (*b*) 5 cm at 20°C.

10–38   Nutrients dissolved in water are carried to upper parts of plants by tiny tubes partly because of the capillary effect. Determine how high the water solution will rise in a tree in a 0.005-mm-diameter tube as a result of the capillary effect. Treat the solution as water at 20°C with a contact angle of 15°.
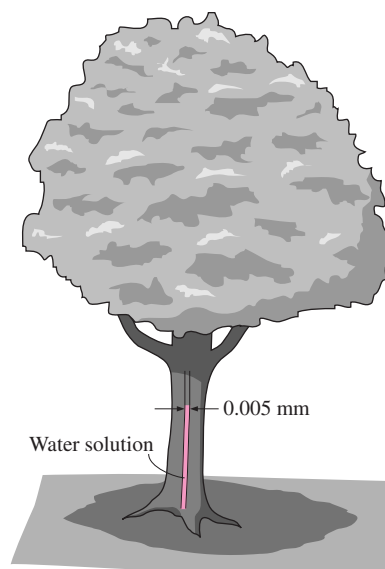*Answer:* 5.75 m

FIGURE P10–38

10–39   The surface tension of a liquid is to be measured using a liquid film suspended on a U-shaped wire frame with an 8-cm-long movable side. If the force needed to move the wire is 0.012 N, determine the surface tension of this liquid in air.

10–40 Contrary to what you might expect, a solid steel ball can float on water due to the surface tension effect. Determine the maximum diameter of a steel ball that would float on water at 20°C. What would your answer be for an aluminum ball? Take the densities of steel and aluminum balls to be 7800 kg/m^3 and 2700 kg/m^3, respectively.

Review Problems

10–41E The pressure on the suction side of pumps is typically low, and the surfaces on that side of the pump are susceptible to cavitation, especially at high fluid temperatures. If the minimum pressure on the suction side of a water pump is 0.95 psia absolute, determine the maximum water temperature to avoid the danger of cavitation.

10–42 A closed tank is partially filled with water at 60°C. If the air above the water is completely evacuated, determine the absolute pressure in the evacuated space. Assume the temperature to remain constant.

10–43 [EES] The variation of the dynamic viscosity of water with absolute temperature is given as

T (K)	μ (Pa · s)
273.15	1.787×10^{-3}
278.15	1.519×10^{-3}
283.15	1.307×10^{-3}
293.15	1.002×10^{-3}
303.15	7.975×10^{-4}
313.15	6.529×10^{-4}
333.15	4.665×10^{-4}
353.15	3.547×10^{-4}
373.15	2.828×10^{-4}

Using tabulated data, develop a relation for viscosity in the form of $\mu = \mu(T) = A + BT + CT^2 + DT^3 + ET^4$. Using the relation developed, predict the dynamic viscosity of water at 50°C at which the reported value is 5.468×10^{-4} Pa · s. Compare your result with the results of Andrade's equation, which is given in the form of $\mu = D \cdot e^{B/T}$, where D and B are constants whose values are to be determined using the viscosity data given.

10–44 Consider laminar flow of a Newtonian fluid of viscosity μ between two parallel plates. The flow is one-dimensional, and the velocity profile is given as $u(y) = 4u_{max}[y/h - (y/h)^2]$, where y is the vertical coordinate from the bottom surface, h is the distance between the two plates, and u_{max} is the maximum flow velocity that occurs at midplane. Develop a relation for the drag force exerted on both plates by the fluid in the flow direction per unit area of the plates.
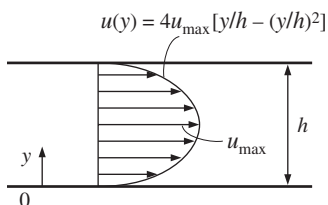

$u(y) = 4u_{max}[y/h - (y/h)^2]$

FIGURE P10–44

10–45 Some non-Newtonian fluids behave as a Bingham plastic for which shear stress can be expressed as $\tau = \tau_y + \mu(du/dr)$. For laminar flow of a Bingham plastic in a horizontal pipe of radius R, the velocity profile is given as $u(r) = (\Delta P/4\mu L)(r^2 - R^2) + (\tau_y/\mu)(r - R)$, where $\Delta P/L$ is the constant pressure drop along the pipe per unit length, μ is the dynamic viscosity, r is the radial distance from the centerline, and τ_y is the yield stress of Bingham plastic. Determine (a) the shear stress at the pipe wall and (b) the drag force acting on a pipe section of length L.

10–46 In some damping systems, a circular disk immersed in oil is used as a damper, as shown in the figure. Show that the damping torque is proportional to angular speed in accordance with the relation $T_{damping} = C\omega$ where $C = 0.5\pi\mu(1/a + 1/b)R^4$. Assume linear velocity profiles on both sides of the disk and neglect the tip effects.

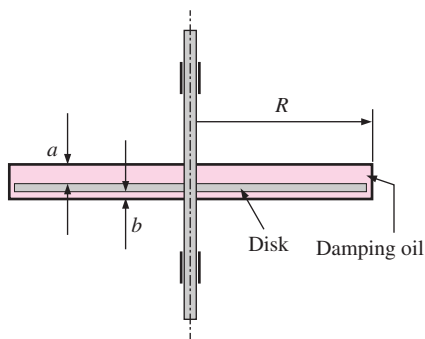**FIGURE P10–46**

10–47E A 0.9-in-diameter glass tube is inserted into mercury, which makes a contact angle of 140° with glass. Determine the capillary drop of mercury in the tube at 68°F.
Answer: 0.0175 in

10–48 Derive a relation for the capillary rise of a liquid between two large parallel plates a distance t apart inserted into the liquid vertically. Take the contact angle to be ϕ.

10–49 Consider a 30-cm-long journal bearing that is lubricated with oil whose viscosity is 0.1 kg/m · s at 20°C at the beginning of operation and 0.008 kg/m · s at the anticipated steady operating temperature of 80°C. The diameter of the shaft is 8 cm, and the average gap between the shaft and the journal is 0.08 cm. Determine the torque needed to overcome the bearing friction initially and during steady operation when the shaft is rotated at 500 rpm.

Design and Essay Problems

10–50 Design an experiment to measure the viscosity of liquids using a vertical funnel with a cylindrical reservoir of height h and a narrow flow section of diameter D and length L. Making appropriate assumptions, obtain a relation for viscosity in terms of easily measurable quantities such as density and volume flow rate.

10–51 Write an essay on the rise of the fluid to the top of the trees by the capillary and other effects.

10–52 Write an essay on the oils used in car engines in different seasons and their viscosities.