Chapter 13
MOMENTUM ANALYSIS OF FLOW SYSTEMS

Newton's Laws and Conservation of Momentum

13-1C Mass, energy, momentum, and electric charge are conserved, and volume and entropy are not conserved during a process.

13-2C Newton's first law states that "*a body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero*." Therefore, a body tends to preserve its state or inertia. Newton's second law states that "*the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.*" Newton's third law states "*when a body exerts a force on a second body, the second body exerts an equal and opposite force on the first*."

13-3C Since momentum ($m\vec{V}$) is the product of a vector (velocity) and a scalar (mass), momentum must be a vector that points in the same direction as the velocity vector.

13-4C The *conservation of momentum principle* is expressed as "the momentum of a system remains constant when the net force acting on it is zero, and thus the momentum of such systems is conserved". The momentum of a body remains constant if the net force acting on it is zero.

13-5C Newton's second law of motion, also called the angular momentum equation, is expressed as "*the rate of change of the angular momentum of a body is equal to the net torque acting it.*" For a non-rigid body with zero net torque, the angular momentum remains constant, but the angular velocity changes in accordance with $I\omega = constant$ where I is the moment of inertia of the body.

13-6C No. Two rigid bodies having the same mass and angular speed will have different angular momentums unless they also have the same moment of inertia I.

Linear Momentum Equation

13-7C The relationship between the time rates of change of an extensive property for a system and for a control volume is expressed by the Reynolds transport theorem, which provides the link between the system and control volume concepts. The linear momentum equation is obtained by setting $b = \vec{V}$ and thus $B = m\vec{V}$ in the Reynolds transport theorem.

13-8C The forces acting on the control volume consist of **body forces** that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and **surface forces** that act on the control surface (such as the pressure forces and reaction forces at points of contact). The net force acting on a control volume is the sum of all body and surface forces. Fluid weight is a body force, and pressure is a surface force (acting per unit area).

13-9C All of these surface forces arise as the control volume is isolated from its surroundings for analysis, and the effect of any detached object is accounted for by a force at that location. We can minimize the number of surface forces exposed by choosing the control volume such that the forces that we are not interested in remain internal, and thus they do not complicate the analysis. A well-chosen control volume exposes only the forces that are to be determined (such as reaction forces) and a minimum number of other forces.

13-10C The momentum-flux correction factor β enables us to express the momentum flux in terms of the mass flow rate and mean flow velocity as $\int_{A_c} \rho \vec{V}(\vec{V} \cdot \vec{n})dA_c = \beta \dot{m}\vec{V}_m$. The value of β is unity for uniform flow, such as a jet flow, nearly unity for turbulent flow (between 1.01 and 1.04), but about 1.3 for laminar flow. So it should be considered in laminar flow.

13-11C The momentum equation for steady one-dimensional flow for the case of no external forces is

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

where the left hand side is the net force acting on the control volume, and first term on the right hand side is the incoming momentum flux and the second term is the outgoing momentum flux by mass.

13-12C In the application of the momentum equation, we can disregard the atmospheric pressure and work with gage pressures only since the atmospheric pressure acts in all directions, and its effect cancels out in every direction.

13-13C The fireman who holds the hose backwards so that the water makes a U-turn before being discharged will experience a greater reaction force since the numerical values of momentum fluxes across the nozzle are added in this case instead of being subtracted.

13-14C No, **V** is not the upper limit to the rocket's ultimate velocity. Without friction the rocket velocity will continue to increase as more gas exits the nozzle.

13-15C A helicopter hovers because the strong downdraft of air, caused by the overhead propeller blades, manifests a momentum in the air stream. This momentum must be countered by the helicopter lift force.

13-16C As the air density decreases, it requires more energy for a helicopter to hover, because more air must be forced into the downdraft by the helicopter blades to provide the same lift force. Therefore, it takes more power for a helicopter to hover on the top of a high mountain than it does at sea level.

13-17C In winter the air is generally colder, and thus denser. Therefore, less air must be driven by the blades to provide the same helicopter lift, requiring less power.

13-18C The force required to hold the plate against the horizontal water stream will increase by a factor of 4 when the velocity is doubled since

$$F = \dot{m}\mathbf{V} = (\rho A\mathbf{V})\mathbf{V} = \rho A\mathbf{V}^2$$

and thus the force is proportional to the square of the velocity.

13-19C The acceleration will not be constant since the force is not constant. The impulse force exerted by water on the plate is $F = \dot{m}\mathbf{V} = (\rho A\mathbf{V})\mathbf{V} = \rho A\mathbf{V}^2$, where **V** is the relative velocity between the water and the plate, which is moving. The plate acceleration will be **a = F/m.** But as the plate begins to move, **V** decreases, so the acceleration must also decrease.

13-20C The maximum velocity possible for the plate is the velocity of the water jet. As long as the plate is moving slower than the jet, the water will exert a force on the plate, which will cause it to accelerate, until terminal jet velocity is reached.

13-21 It is to be shown that the force exerted by a liquid jet of velocity **V** on a stationary nozzle is proportional to $\mathbf{V}^2$, or alternatively, to $\dot{m}^2$. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The nozzle is given to be stationary. **3** The nozzle involves a 90° turn and thus the incoming and outgoing flow streams are normal to each other. **4** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

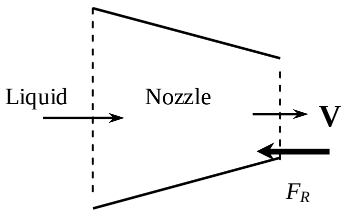
Analysis We take the nozzle as the control volume, and the flow direction at the exit as the x axis. Note that the nozzle makes a 90° turn, and thus the flow direction at the inlet is in the normal direction to exit flow, and thus it does not contribute to any pressure force or momentum flux term at the inlet in the x direction. Noting that $\dot{m} = \rho A\mathbf{V}$ where A is the nozzle exit area and **V** is the mean nozzle exit velocity, the momentum equation for steady one-dimensional flow in the x direction reduces to

$$\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i \quad\rightarrow\quad F_{Rx} = \dot{m}_e\mathbf{V}_e = \dot{m}\mathbf{V}$$

where F_{Rx} is the reaction force on the nozzle due to liquid jet at the nozzle exit. Then,

$$\dot{m} = \rho A\mathbf{V} \quad\rightarrow\quad F_{Rx} = \dot{m}\mathbf{V} = \rho A\mathbf{V}\mathbf{V} = \rho A\mathbf{V}^2 \quad \text{or} \quad F_{Rx} = \dot{m}\mathbf{V} = \dot{m}\frac{\dot{m}}{\rho A} = \frac{\dot{m}^2}{\rho A}$$

Therefore, the force exerted by a liquid jet of velocity **V** on this stationary nozzle is proportional to $\mathbf{V}^2$, or alternatively, to $\dot{m}^2$.



13-22 A water jet of velocity **V** impinges on a plate moving toward the water jet with velocity ½**V**. The force required to move the plate towards the jet is to be determined in terms of *F* acting on the stationary plate.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The plate is vertical and the jet is stationary and normal to plate. **3** The pressure on both sides of the plate is atmospheric pressure (and thus its effect can be disregarded). **4** Fiction during motion is negligible. **5** There is no acceleration of the plate. **6** The water splashes off the sides of the plate in a plane normal to the jet.

Analysis We take the plate as the control volume. The relative velocity between the plate and the jet is **V** when the plate is stationary, and 1.5**V** when the plate is moving with a velocity ½**V** towards the plate. Then the momentum equation for steady one-dimensional flow in the horizontal direction reduces to

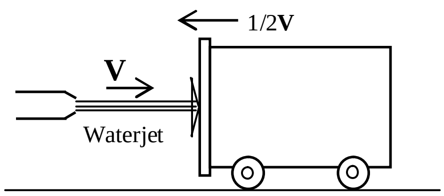
$$\sum \vec{F} = \sum_{\text{out}} \dot{m}\vec{\mathbf{V}} - \sum_{\text{in}} \dot{m}\vec{\mathbf{V}} \quad \rightarrow \quad -F_R = -\dot{m}_i \mathbf{V}_i \quad \rightarrow \quad F_R = \dot{m}_i \mathbf{V}_i$$

Stationary plate: $(\mathbf{V}_i = \mathbf{V} \text{ and } \dot{m}_i = \rho A \mathbf{V}_i = \rho A \mathbf{V}) \rightarrow F_R = \rho A \mathbf{V}^2 = F$

Moving plate: $(\mathbf{V}_i = 1.5\mathbf{V} \text{ and } \dot{m}_i = \rho A \mathbf{V}_i = \rho A(1.5\mathbf{V})) \rightarrow F_R = \rho A(1.5\mathbf{V})^2 = 2.25\rho A \mathbf{V}^2 = 2.25F$

Therefore, the force required to hold the plate stationary against the oncoming water jet becomes **2.25 times** when the jet velocity becomes 1.5 times.

Discussion Note that when the plate is stationary, **V** is also the jet velocity. But if the plate moves toward the stream with velocity ½**V**, then the relative velocity is 1.5**V**, and the amount of mass striking the plate (and falling off its sides) per unit time also increases by 50%.



13-23 A 90° elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The gage pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place are to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *y*. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho A \mathbf{V}$, the mean inlet and exit velocities of water are

$$\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V} = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\text{ kg/s}}{(1000\text{ kg/m}^3)[\pi(0.1\text{ m})^2/4]} = 3.18\text{ m/s}$$

Noting that $\mathbf{V}_1 = \mathbf{V}_2$ and $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \;\rightarrow\; P_1 - P_2 = \rho g(y_2 - y_1) \;\rightarrow\; P_{1,\text{ gage}} = \rho g(y_2 - y_1)$$

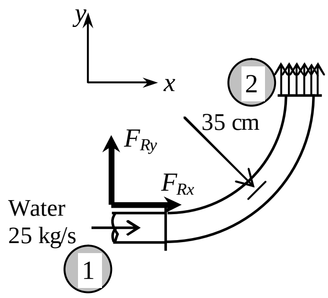
Substituting,

$$P_{1,\text{ gage}} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.35\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 3.434\text{ kN/m}^2 = \mathbf{3.434\text{ kPa}}$$

(*b*) The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. We let the *x*- and *y*- components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *y* axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = 0 - \dot{m}(+\mathbf{V}_1) = -\dot{m}\mathbf{V}$$

$$F_{Ry} = \dot{m}(+\mathbf{V}_2) = \dot{m}\mathbf{V}$$



Solving for F_{Rx} and F_{Ry}, and substituting the given values,

$$\begin{aligned} F_{Rx} &= -\dot{m}\mathbf{V} - P_{1,\text{ gage}} A_1 \\ &= -(25\text{ kg/s})(3.18\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) - (3434\text{ N/m}^2)[\pi(0.1\text{ m})^2/4] \\ &= -107\text{ N} \end{aligned}$$

$$F_{Ry} = \dot{m}\mathbf{V} = (25\text{ kg/s})(3.18\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 79.5\text{ N}$$

and $\quad F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-107)^2 + 79.5^2} = \mathbf{133\text{ N}}, \quad \theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{79.5}{-107} = -36.6° = \mathbf{143°}$

Discussion Note that the magnitude of the anchoring force is 133 N, and its line of action makes 143.4° from the positive *x* direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

13-24 An 180° elbow forces the flow to make a U-turn and discharges it to the atmosphere at a specified rate. The gage pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place are to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *y*. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30\ \text{kg/s}$. Noting that $\dot{m} = \rho A \mathbf{V}$, the mean inlet and exit velocities of water are

$$\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V} = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.1\ \text{m})^2/4]} = 3.18\ \text{m/s}$$

Noting that $\mathbf{V}_1 = \mathbf{V}_2$ and $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \ \rightarrow\ P_1 - P_2 = \rho g(y_2 - y_1) \ \rightarrow\ P_{1,\,\text{gage}} = \rho g(y_2 - y_1)$$

Substituting,

$$P_{1,\,\text{gage}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.70\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 6.867\ \text{kN/m}^2 = \mathbf{6.867\ kPa}$$

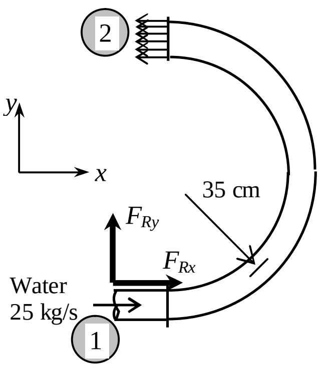
(*b*) The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. We let the *x*- and *y*- components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *y* axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = \dot{m}(-\mathbf{V}_2) - \dot{m}(+\mathbf{V}_1) = -2\dot{m}\mathbf{V}$$

$$F_{Ry} = 0$$

Solving for F_{Rx} and substituting the given values,

$$\begin{aligned} F_{Rx} &= -2\dot{m}\mathbf{V} - P_{1,\,\text{gage}} A_1 \\ &= -2(25\ \text{kg/s})(3.18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) - (6867\ \text{N/m}^2)[\pi(0.1\ \text{m})^2/4] \\ &= -213\ \text{N} \end{aligned}$$



and $F_R = F_{Rx}$ **= - 213 N** since the *y*-component of the anchoring force is zero. Therefore, the anchoring force has a magnitude of 213 N and it acts in the negative *x* direction.

Discussion Note that a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

13-25E A horizontal water jet strikes a vertical stationary plate normally at a specified velocity. For a given anchoring force needed to hold the plate in place, the flow rate of water is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water splatters off the sides of the plate in a plane normal to the jet. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on the entire control surface. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the plate as the control volume such that it contains the entire plate and cuts through the water jet and the support bar normally, and the direction of flow as the positive direction of x axis. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad -F_{Rx} = -\dot{m}\mathbf{V}_1 \quad \rightarrow \quad F_R = \dot{m}\mathbf{V}_1$$

We note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Solving for $\dot{m}$ and substituting the given values,

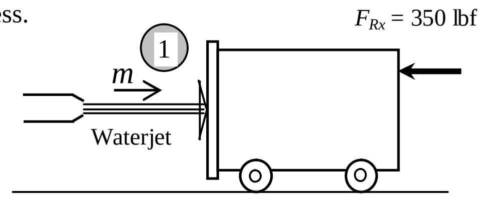
$$\dot{m} = \frac{F_{Rx}}{\mathbf{V}_1} = \frac{350 \text{ lbf}}{30 \text{ ft/s}}\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right) = 376 \text{ lbm/s}$$

Then the volume flow rate becomes

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{376 \text{ lbm/s}}{62.4 \text{ lbm/ft}^3} = \mathbf{6.02 \text{ ft}^3/\text{s}}$$

Therefore, the volume flow rate of water under stated assumptions must be 3.45 ft^3/s.

Discussion In reality, some water will be scattered back, and this will add to the reaction force of water. The flow rate in that case will be less.



13-26 A reducing elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The anchoring force needed to hold the elbow in place is to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N} = 0.4905\text{ kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by y. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30\text{ kg/s}$. Noting that $\dot{m} = \rho A\mathbf{V}$, the inlet and exit velocities of water are

$$\mathbf{V}_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0150\text{ m}^2)} = 2.0\text{ m/s}$$

$$\mathbf{V}_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0025\text{ m}^2)} = 12\text{ m/s}$$

Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

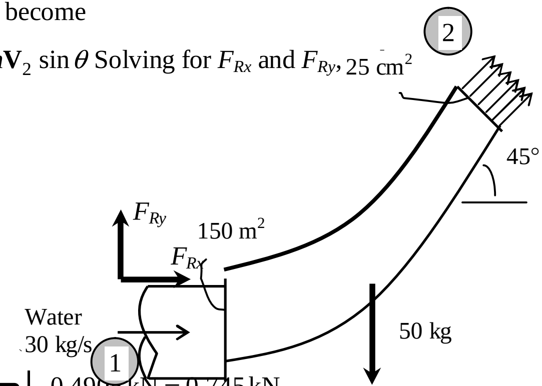
$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \;\rightarrow\; P_1 - P_2 = \rho g\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g} + y_2 - y_1\right) \;\rightarrow\; P_{1,\text{ gage}} = \rho g\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g} + y_2\right)$$

Substituting,

$$P_{1,\text{ gage}} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(12\text{ m/s})^2 - (2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0.4\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 73.9\text{ kN/m}^2 = 73.9\text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\sum\vec{F} = \sum\dot{m}_e\vec{\mathbf{V}}_e - \sum\dot{m}_i\vec{\mathbf{V}}_i$. We let the x- and y-components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and y axes become

$F_{Rx} + P_{1,\text{gage}}A_1 = \dot{m}\mathbf{V}_2\cos\theta - \dot{m}\mathbf{V}_1$ and $F_{Ry} - W = \dot{m}\mathbf{V}_2\sin\theta$ Solving for F_{Rx} and F_{Ry}, substituting the given values,



$$\begin{aligned} F_{Rx} &= \dot{m}(\mathbf{V}_2\cos\theta - \mathbf{V}_1) - P_{1,\text{ gage}}A_1 \\ &= (30\text{ kg/s})[(12\cos45° - 2)\text{ m/s}]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) \\ &\quad - (73.9\text{ kN/m}^2)(0.0150\text{ m}^2) \\ &= -0.914\text{ kN} \end{aligned}$$

$$F_{Ry} = \dot{m}\mathbf{V}_2\sin\theta - W = (30\text{ kg/s})(12\sin30°\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) - 0.4905\text{ kN} = 0.745\text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-0.914)^2 + 0.745^2} = \mathbf{1.18\text{ kN}}, \qquad \theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{0.745}{-0.914} = -39.2° = \mathbf{140.8°}$$

Discussion Note that the magnitude of the anchoring force is 1.18 kN, and its line of action makes 140.8° from the positive x direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

13-27 A reducing elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The anchoring force needed to hold the elbow in place is to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\,\text{kg})(9.81\,\text{m/s}^2) = 490.5\,\text{N} = 0.4905\ \text{kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *y*. The continuity equation for this one-inlet one-exit steady flow system is $\dot m_1 = \dot m_2 = \dot m = 30$ kg/s. Noting that $\dot m = \rho A\mathbf{V}$, the inlet and exit velocities of water are

$$\mathbf{V}_1 = \frac{\dot m}{\rho A_1} = \frac{30\,\text{kg/s}}{(1000\ \text{kg/m}^3)(0.0150\,\text{m}^2)} = 2.0\,\text{m/s}$$

$$\mathbf{V}_2 = \frac{\dot m}{\rho A_2} = \frac{30\,\text{kg/s}}{(1000\ \text{kg/m}^3)(0.0025\,\text{m}^2)} = 12\,\text{m/s}$$

Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \ \rightarrow\ P_1 - P_2 = \rho g\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g} + y_2 - y_1\right) \rightarrow\ P_{1,\text{gage}} = \rho g\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g} + y_2\right)$$

or, $P_{1,\text{gage}} = (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)\left(\dfrac{(12\,\text{m/s})^2 - (2\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} + 0.4\right)\left(\dfrac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) = 73.9\,\text{kN/m}^2 = 73.9\,\text{kPa}$

The momentum equation for steady one-dimensional flow is $\sum \vec F = \sum \dot m_e \vec{\mathbf{V}}_e - \sum \dot m_i \vec{\mathbf{V}}_i$. We let the *x*- and *y*-components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *y* axes become

$$F_{Rx} + P_{1,\text{gage}}A_1 = \dot m\mathbf{V}_2\cos\theta - \dot m\mathbf{V}_1 \quad \text{and} \quad F_{Ry} - W = \dot m\mathbf{V}_2\sin\theta$$

Solving for F_{Rx} and F_{Ry}, and substituting the given values,

$$F_{Rx} = \dot m(\mathbf{V}_2\cos\theta - \mathbf{V}_1) - P_{1,\text{gage}}A_1$$

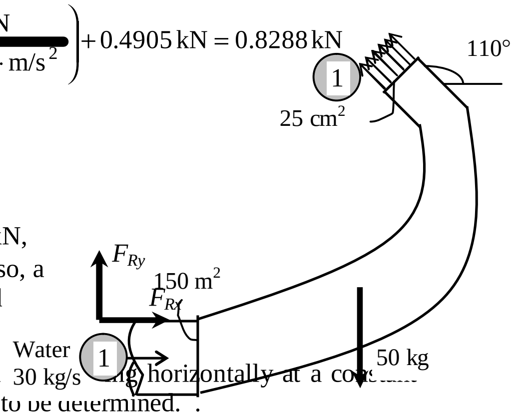
$$= (30\,\text{kg/s})[(12\cos 110° - 2)\,\text{m/s}]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) - (73.9\,\text{kN/m}^2)(0.0150\,\text{m}^2) = -1.292\,\text{kN}$$

$$F_{Ry} = \dot m\mathbf{V}_2\sin\theta + W = (30\,\text{kg/s})(12\sin 110°\,\text{m/s})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) + 0.4905\,\text{kN} = 0.8288\,\text{kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-1.292)^2 + 0.8288^2} = \mathbf{1.53\ kN}$$

and

$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{0.8288}{-1.292} = \mathbf{-32.7°}$$

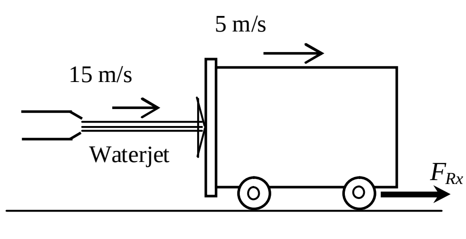


Discussion Note that the magnitude of the anchoring force is 1.53 kN, and its line of action makes –32.7° from the positive *x* direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

13-28 Water accelerated by a nozzle strikes the back surface of a cart moving horizontally at a constant velocity. The braking force and the power wasted by the brakes are to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water splatters off the sides of the plate in all directions in the plane of the back surface. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force. **5** Fiction during motion is negligible. **6** There is no acceleration of the cart. **7** The motions of the water jet and the cart are horizontal.

Analysis We take the cart as the control volume, and the direction of flow as the positive direction of *x* axis. The relative velocity



between the cart and the jet is

$$\mathbf{V}_r = \mathbf{V}_{\text{jet}} - \mathbf{V}_{\text{cart}} = 15 - 10 = 10 \text{ m/s}$$

Therefore, we can assume the cart to be stationary and the jet to move with a velocity of 10 m/s. The momentum equation for steady one-dimensional flow in the *x* (flow) direction reduces in this case to

$$\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad F_{Rx} = -\dot{m}_i \mathbf{V}_i \quad \rightarrow \quad F_{\text{brake}} = -\dot{m}\mathbf{V}_r$$

We note that the brake force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative *x*-direction. Substituting the given values,

$$F_{\text{brake}} = -\dot{m}\mathbf{V}_r = -(25 \text{ kg/s})(+10 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = \mathbf{-250 \text{ N}}$$

The negative sign indicates that the braking force acts in the opposite direction to motion, as expected. Noting that work is force times distance and the distance traveled by the cart per unit time is the cart velocity, the power wasted by the brakes is

$$\dot{W} = F_{\text{brake}} \mathbf{V}_{\text{cart}} = (250 \text{ N})(5 \text{ m/s})\left(\frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}}\right) = \mathbf{1.25 \text{ kW}}$$

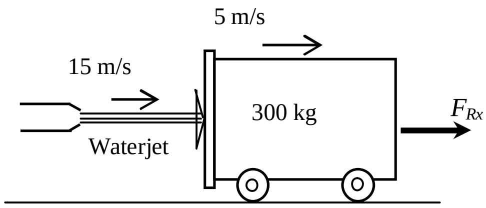
Discussion Note that the power wasted is equivalent to the maximum power that can be generated as the cart velocity is maintained constant.

13-29 Water accelerated by a nozzle strikes the back surface of a cart moving horizontally. The acceleration of the cart if the brakes fail is to be determined.

Analysis The braking force was determined in previous problem to be 250 N. When the brakes fail, this force will propel the cart forward, and the accelerating will be

$$a = \frac{F}{m_{\text{cart}}} = \frac{250\ \text{N}}{300\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = \mathbf{0.833\ m/s^2}$$

Discussion This is the acceleration at the moment the brakes fail. The acceleration will decrease as the relative velocity between the water jet and the cart (and thus the force) decreases.



13-30E A water jet hits a stationary splitter, such that half of the flow is diverted upward at 45°, and the other half is directed down. The force required to hold the splitter in place is to be determined. √EES

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet before and after the split is the atmospheric pressure which is disregarded since it acts on all surfaces. **3** The gravitational effects are disregarded.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis The mass flow rate of water jet is

$$\dot{m} = \rho \dot{V} = (62.4\ \text{lbm/ft}^3)(100\ \text{ft}^3\text{/s}) = 6240\ \text{lbm/s}$$

We take the splitting section of water jet, including the splitter as the control volume, and designate the entrance by 1 and the exit of either arm by 2 (both arms have the same velocity and mass flow rate). We also designate the horizontal coordinate by x with the direction of flow as being the positive direction and the vertical coordinate by y.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. We let the x- and y- components of the anchoring force of the splitter be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Noting that $\mathbf{V_2} = \mathbf{V_1} = \mathbf{V}$ and $\dot{m}_2 = \frac{1}{2}\dot{m}$, the momentum equations along the x and y axes become

$$F_{Rx} = 2(\tfrac{1}{2}\dot{m})\mathbf{V}_2 \cos\theta - \dot{m}\mathbf{V}_1 = \dot{m}\mathbf{V}(\cos\theta - 1)$$

$$F_{Ry} = \tfrac{1}{2}\dot{m}(+\mathbf{V}_2 \sin\theta) + \tfrac{1}{2}\dot{m}(-\mathbf{V}_2 \sin\theta) - 0 = 0$$

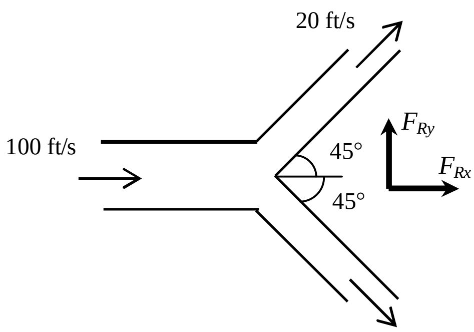
Substituting the given values,

$$F_{Rx} = (6240\ \text{lbm/s})(20\ \text{ft/s})(\cos 45° - 1)\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{-1135\ lbf}$$

$$F_{Ry} = \mathbf{0}$$

The negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed. Therefore, a force of 1135 lbf must be applied to the splitter in the opposite direction to flow to hold it in place. No holding force is necessary in the vertical direction. This can also be concluded from the symmetry.

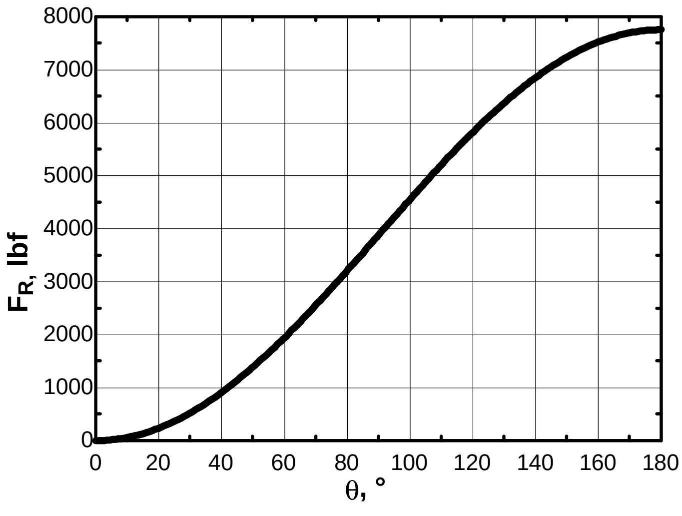
Discussion In reality, the gravitational effects will cause the upper stream to slow down and the lower stream to speed up after the split. But for short distances, these effects are indeed negligible.



13-31E Problem 13-30E is reconsidered. The effect of splitter angle on the force exerted on the splitter as the half splitter angle varies from 0 to 180° in increments of 10° is to be investigated.

```
g=32.2 "ft/s2"
rho=62.4 "lbm/ft3"
V_dot=100 "ft3/s"
V=20 "ft/s"
m_dot=rho*V_dot
F_R=-m_dot*V*(cos(theta)-1)/g "lbf"
```

θ, °	$\dot{m}$, lbm/s	F_R, lbf
0	6240	0
10	6240	59
20	6240	234
30	6240	519
40	6240	907
50	6240	1384
60	6240	1938
70	6240	2550
80	6240	3203
90	6240	3876
100	6240	4549
110	6240	5201
120	6240	5814
130	6240	6367
140	6240	6845
150	6240	7232
160	6240	7518
170	6240	7693
180	6240	7752



13-32 A horizontal water jet impinges normally upon a vertical plate which is held on a frictionless track and is initially stationary. The initial acceleration of the plate, the time it takes to reach a certain velocity, and the velocity at a given time are to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water always splatters in the plane of the retreating plate. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal motion. **5** The tract is frictionless, and thus fiction during motion is negligible. **6** The motions of the water jet and the cart are horizontal. **7** The velocity of the jet relative to the plate remains constant, $\mathbf{V}_r = \mathbf{V}_{jet} = \mathbf{V}$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the vertical plate on the frictionless track as the control volume, and the direction of flow as the positive direction of *x* axis. The mass flow rate of water in the jet is

$$\dot{m} = \rho \mathbf{V} A = (1000\ \text{kg/m}^3)(18\ \text{m/s})[\pi(0.05\ \text{m})^2/4] = 35.34\ \text{kg/s}$$

The momentum equation for steady one-dimensional flow in the *x* (flow) direction reduces in this case to

$$\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad F_{Rx} = -\dot{m}_i \mathbf{V}_i \quad \rightarrow \quad F_{Rx} = -\dot{m}\mathbf{V}$$

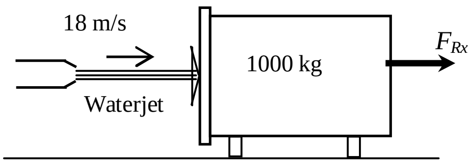
where F_{Rx} is the reaction force required to hold the plate in place. When the plate is released, an equal and opposite impulse force acts on the plate, which is determined to

$$F_{\text{plate}} = -F_{Rx} = \dot{m}\mathbf{V} = (35.34\ \text{kg/s})(18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 636\ \text{N}$$

Then the initial acceleration of the plate becomes

$$a = \frac{F_{\text{plate}}}{m_{\text{plate}}} = \frac{636\ \text{N}}{1000\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = \mathbf{0.636\ m/s^2}$$

This acceleration will remain constant during motion since the force acting on the plate remains constant.



(*b*) Noting that $a = d\mathbf{V}/dt = \Delta\mathbf{V}/\Delta t$ since the acceleration *a* is constant, the time it takes for the plate to reach a velocity of 9 m/s is

$$\Delta t = \frac{\Delta \mathbf{V}_{\text{plate}}}{a} = \frac{(9-0)\ \text{m/s}}{0.636\ \text{m/s}^2} = \mathbf{14.2\ s}$$

(*c*) Noting that $a = d\mathbf{V}/dt$ and thus $d\mathbf{V} = adt$ and that the acceleration *a* is constant, the plate velocity in 20 s becomes

$$\mathbf{V}_{\text{plate}} = \mathbf{V}_{0,\text{ plate}} + a\Delta t = 0 + (0.636\ \text{m/s}^2)(20\ \text{s}) = \mathbf{12.7\ m/s}$$

Discussion The assumption that the relative velocity between the water jet and the plate remains constant is valid only for the initial moments of motion when the plate velocity is low unless the water jet is moving with the plate at the same velocity as the plate.

13-33 A 90° reducer elbow deflects water downwards into a smaller diameter pipe. The resultant force exerted on the reducer by water is to be determined.

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is disregarded since the gravitational effects are negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *y*. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 353.4$ kg/s. Noting that $\dot{m} = \rho A \mathbf{V}$, the mass flow rate of water and its exit velocity are

$$\dot{m} = \rho \mathbf{V}_1 A_1 = \rho \mathbf{V}_1 (\pi D_1^2/4) = (1000\ \text{kg/m}^3)(5\ \text{m/s})[\pi(0.3\ \text{m})^2/4] = 353.4\ \text{kg/s}$$

$$\mathbf{V}_2 = \frac{\dot{m}}{\rho A_2} = \frac{\dot{m}}{\rho \pi D_2^2/4} = \frac{353.4\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.15\ \text{m})^2/4]} = 20\ \text{m/s}$$

The Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \quad \rightarrow \quad P_2 = P_1 + \rho g\left(\frac{\mathbf{V}_1^2 - \mathbf{V}_2^2}{2g} + y_1 - y_2\right)$$

Substituting, the gage pressure at the exit becomes

$$P_2 = (300\ \text{kPa}) + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(\frac{(5\ \text{m/s})^2 - (20\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.5\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = 117.4\ \text{kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. We let the *x*- and *y*-components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Then the momentum equations along the *x* and *y* axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = 0 - \dot{m}\mathbf{V}_1$$
$$F_{Ry} - P_{2,\text{gage}} A_2 = \dot{m}(-\mathbf{V}_2) - 0$$

Note that we should not forget the negative sign for forces and velocities in the negative *x* or *y* direction. Solving for F_{Rx} and F_{Ry}, and substituting the given values,

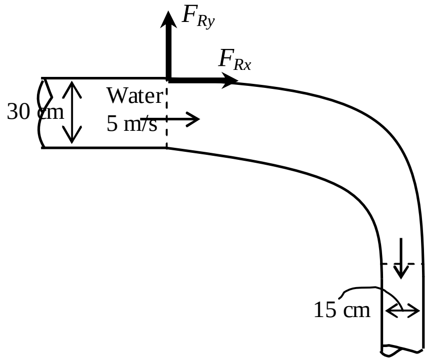
$$F_{Rx} = -\dot{m}\mathbf{V}_1 - P_{1,\text{gage}} A_1 = -(353.4\ \text{kg/s})(5\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) - (300\ \text{kN/m}^2)\frac{\pi(0.3\ \text{m})^2}{4} = -23.0\ \text{kN}$$

$$F_{Ry} = -\dot{m}\mathbf{V}_2 + P_{2,\text{gage}} A_1 = -(353.4\ \text{kg/s})(20\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) + (117.4\ \text{kN/m}^2)\frac{\pi(0.15\ \text{m})^2}{4} = -5.0\ \text{kN}$$

and

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-23.0)^2 + (-5.0)^2} = \mathbf{23.5\ kN},$$

$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{-5.0}{-23.0} = \mathbf{12.3°}$$

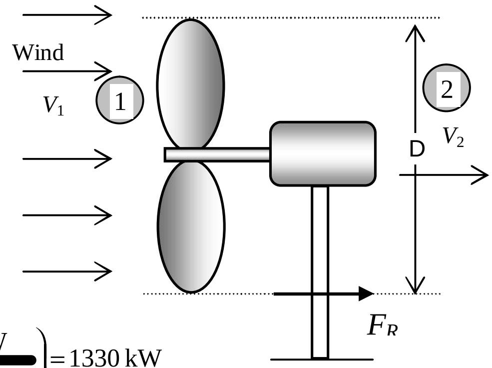


Discussion Note that the magnitude of the anchoring force is 23.5 kN, and its line of action makes 12.3° from the positive *x* direction. Also, negative values for F_{Rx} and F_{Ry} indicate that the assumed directions are wrong, and should be reversed.

13-34 A wind turbine with a given span diameter and efficiency is subjected to steady winds. The power generated and the horizontal force on the supporting mast of the turbine are to be determined. √EES

Assumptions **1** The wind flow is steady, one-dimensional, and incompressible. **2** The efficiency of the turbine-generator is independent of wind speed. **3** The frictional effects are negligible, and thus none of the incoming kinetic energy is converted to thermal energy.

Properties The density of air is given to be 1.25 kg/m^3.



Analysis (*a*) The power potential of the wind is its kinetic energy, which is $\mathbf{V}^2/2$ per unit mass, and $\dot{m}\mathbf{V}^2/2$ for a given mass flow rate:

$$\mathbf{V}_1 = (25\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 6.94\ \text{m/s}$$

$$\dot{m} = \rho_1 \mathbf{V}_1 A_1 = \rho_1 \mathbf{V}_1 \frac{\pi D^2}{4} = (1.25\ \text{kg/m}^3)(6.94\ \text{m/s})\frac{\pi (90\ \text{m})^2}{4} = 55{,}200\ \text{kg/s}$$

$$\dot{W}_{\max} = \dot{m}ke_1 = \dot{m}\frac{\mathbf{V}_1^2}{2} = (55{,}200\ \text{kg/s})\frac{(6.94\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right) = 1330\ \text{kW}$$

Then the actual power produced becomes

$$\dot{W}_{\text{act}} = \eta_{\text{wind turbine}}\dot{W}_{\max} = (0.32)(1330\ \text{kW}) = \mathbf{426\ kW}$$

(*b*) The frictional effects are assumed to be negligible, and thus the portion of incoming kinetic energy not converted to electric power leaves the wind turbine as outgoing kinetic energy. Therefore,

$$\dot{m}ke_2 = \dot{m}ke_1(1-\eta_{\text{wind turbine}}) \quad \rightarrow \quad \dot{m}\frac{\mathbf{V}_2^2}{2} = \dot{m}\frac{\mathbf{V}_1^2}{2}(1-\eta_{\text{wind turbine}})$$

or

$$\mathbf{V}_2 = \mathbf{V}_1\sqrt{1-\eta_{\text{wind turbine}}} = (6.94\ \text{m/s})\sqrt{1-0.32} = 5.72\ \text{m/s}$$

We choose the control volume around the wind turbine such that the wind is normal to the control surface at the inlet and the exit, and the entire control surface is at the atmospheric pressure. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. Writing it along the *x*-direction (without forgetting the negative sign for forces and velocities in the negative *x*-direction) and assuming the flow velocity through the turbine to be equal to the wind velocity give

$$F_R = \dot{m}\mathbf{V}_2 - \dot{m}\mathbf{V}_1 = \dot{m}(\mathbf{V}_2 - \mathbf{V}_1) = (55{,}200\ \text{kg/s})(5.72 - 6.94\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{-67.3\ kN}$$

The negative sign indicates that the reaction force acts in the negative *x* direction, as expected.

Discussion This force acts on top of the tower where the wind turbine is installed, and the bending moment it generates at the bottom of the tower is obtained by multiplying this force by the tower height.

13-35E A horizontal water jet strikes a curved plate, which deflects the water back to its original direction. The force required to hold the plate against the water stream is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Friction between the plate and the surface it is on is negligible (or the friction force can be included in the required force to hold the plate). **4** There is no splattering of water or the deformation of the jet, and the reversed jet leaves horizontally at the same velocity and flow rate.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the plate together with the curved water jet as the control volume, and designate the jet inlet by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of incoming flow as being the positive direction). The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$ where

$$\dot{m} = \rho \mathbf{V} A = \rho \mathbf{V}[\pi D^2/4] = (62.4\ \text{lbm/ft}^3)(140\ \text{ft/s})[\pi(3/12\ \text{ft})^2/4] = 428.8\ \text{lbm/s}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. Letting the reaction force to hold the plate be F_{Rx} and assuming it to be in the positive direction, the momentum equation along the x axis becomes

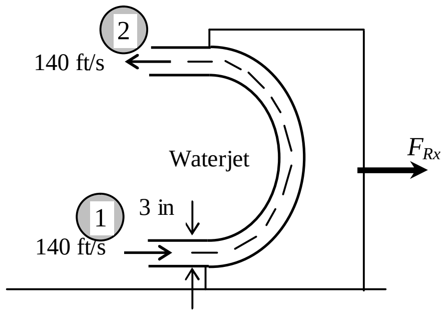
$$F_{Rx} = \dot{m}(-\mathbf{V}_2) - \dot{m}(+\mathbf{V}_1) = -2\dot{m}\mathbf{V}$$

Substituting,

$$F_{Rx} = -2(428.8\ \text{lbm/s})(140\ \text{ft/s})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = -3729\ \textbf{lbf}$$

Therefore, a force of 3729 lbm must be applied on the plate in the negative x direction to hold it in place.

Discussion Note that a negative value for F_{Rx} indicates the assumed direction is wrong (as expected), and should be reversed. Also, there is no need for an analysis in the vertical direction since the fluid streams are horizontal.



13-36E A horizontal water jet strikes a bent plate, which deflects the water by 135° from its original direction. The force required to hold the plate against the water stream is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Frictional and gravitational effects are negligible. **4** There is no splattering of water or the deformation of the jet, and the reversed jet leaves horizontally at the same velocity and flow rate.

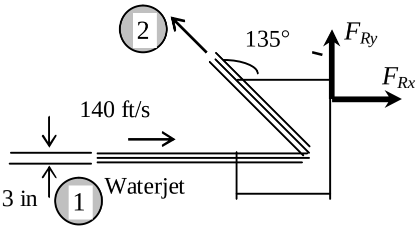
Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the plate together with the curved water jet as the control volume, and designate the jet inlet by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of incoming flow as being the positive direction), and the vertical coordinate by y. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$ where

$$\dot{m} = \rho \mathbf{V} A = \rho \mathbf{V}[\pi D^2/4] = (62.4\ \text{lbm/ft}^3)(140\ \text{ft/s})[\pi(3/12\ \text{ft})^2/4] = 428.8\ \text{lbm/s}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. We let the x- and y-components of the anchoring force of the plate be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Then the momentum equations along the x and y axes become

$$F_{Rx} = \dot{m}(-\mathbf{V}_2)\cos 45° - \dot{m}(+\mathbf{V}_1) = -\dot{m}\mathbf{V}(1+\cos 45°)$$
$$F_{Ry} = \dot{m}(+\mathbf{V}_2)\sin 45° = \dot{m}\mathbf{V}\sin 45°$$



Substituting the given values,

$$F_{Rx} = -2(428.8\ \text{lbm/s})(140\ \text{ft/s})(1+\cos 45°)\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)$$
$$= -6365\ \text{lbf}$$
$$F_{Ry} = (428.8\ \text{lbm/s})(140\ \text{ft/s})\sin 45°\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = 1318\ \text{lbf}$$

and

$$F_{Ry} = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-6365)^2 + 1318^2} = 6500\ \textbf{lbf}, \qquad \theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{1318}{-6365} = -11.7° = \mathbf{168.3°}$$

Discussion Note that the magnitude of the anchoring force is 6500 lbf, and its line of action makes 168.3° from the positive x direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

13-37 Firemen are holding a nozzle at the end of a hose while trying to extinguish a fire. The average water exit velocity and the resistance force required of the firemen to hold the nozzle are to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Gravitational effects and vertical forces are disregarded since the horizontal resistance force is to be determined.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the nozzle and the horizontal portion of the hose as the system such that water enters the control volume vertically and exits horizontally (this way the pressure force and the momentum flux at the inlet are in the vertical direction, with no contribution to the force balance in the horizontal direction), and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction). The average exit velocity and the mass flow rate of water are determined from

$$\mathbf{V} = \frac{\dot{V}}{A} = \frac{\dot{V}}{\pi D^2/4} = \frac{5\ \text{m}^3/\text{min}}{\pi(0.06\ \text{m})^2/4} = 1768\ \text{m/min} = \mathbf{29.5\ m/s}$$

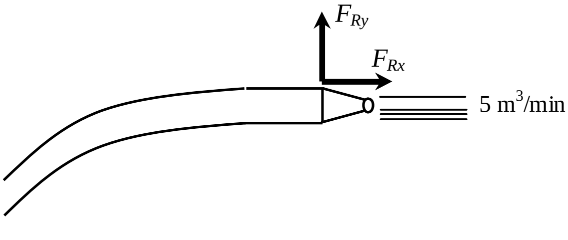
$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(5\ \text{m}^3/\text{min}) = 5000\ \text{kg/min} = 83.3\ \text{kg/s}$$

(*b*) The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. We let horizontal force applied by the firemen to the nozzle to hold it be F_{Rx}, and assume it to be in the positive *x* direction. Then the momentum equation along the *x* direction gives

$$F_{Rx} = \dot{m}\mathbf{V}_e - 0 = \dot{m}\mathbf{V} = (83.3\ \text{kg/s})(29.5\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{2457\ N}$$

Therefore, the firemen must be able to resist a force of 2457 N to hold the nozzle in place.

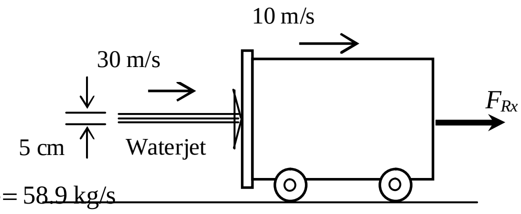
Discussion The force of 2457 N is equivalent to the weight of about 250 kg. That is, holding the nozzle requires the strength of holding a weight of 250 kg, which cannot be done by a single person. This demonstrates why several firemen are used to hold a hose with a high flow rate.



13-38 A horizontal jet of water with a given velocity strikes a flat plate that is moving in the same direction at a specified velocity. The force that the water stream exerts against the plate is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water splatters in all directions in the plane of the plate. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal force exerted on the plate. **5** The velocity of the plate, and the velocity of the water jet relative to the plate, are constant.

Properties We take the density of water to be 1000 kg/m^3.



Analysis We take the plate as the control volume, and the flow direction as the positive direction of *x* axis. The mass flow rate of water in the jet is

$$\dot{m} = \rho \mathbf{V}_{jet} A = \rho \mathbf{V}_{jet} \frac{\pi D^2}{4} = (1000\ \text{kg/m}^3)(30\ \text{m/s})\frac{\pi (0.05\ \text{m})^2}{4} = 58.9\ \text{kg/s}$$

The relative velocity between the plate and the jet is

$$\mathbf{V}_r = \mathbf{V}_{\text{jet}} - \mathbf{V}_{\text{plate}} = 30 - 10 = 20\ \text{m/s}$$

Therefore, we can assume the plate to be stationary and the jet to move with a velocity of 20 m/s. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. We let the horizontal reaction force applied to the plate in the negative *x* direction to counteract the impulse of the water jet be F_{Rx}. Then the momentum equation along the *x* direction gives

$$-F_{Rx} = 0 - \dot{m}\mathbf{V}_i \quad \rightarrow \quad F_{Rx} = \dot{m}\mathbf{V}_r = (58.9\ \text{kg/s})(20\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{1178\ N}$$

Therefore, the water jet applies a force of 1178 N on the plate in the direction of motion, and an equal and opposite force must be applied on the plate if its velocity is to remain constant.

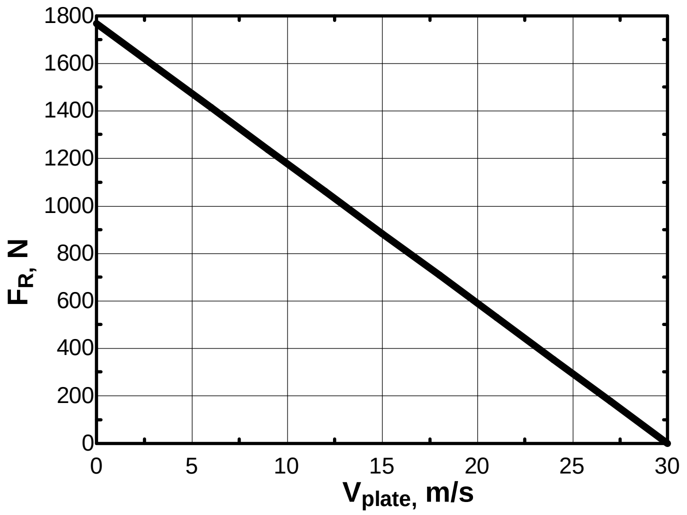
Discussion Note that we used the relative velocity in the determination of the mass flow rate of water in the momentum analysis since water will enter the control volume at this rate. (In the limiting case of the plate and the water jet moving at the same velocity, the mass flow rate of water relative to the plate will be zero since no water will be able to strike the plate).

13-39 Problem 13-38 is reconsidered. The effect of the plate velocity on the force exerted on the plate as the plate velocity varies from 0 to 30 m/s in increments of 3 m/s is to be investigated.

```
rho=1000 "kg/m3"
D=0.05 "m"
V_jet=30 "m/s"

Ac=pi*D^2/4
V_r=V_jet-V_plate
m_dot=rho*Ac*V_jet
F_R=m_dot*V_r "N"
```

V_{plate}, m/s	V_r, m/s	F_R, N
0	30	1767
3	27	1590
6	24	1414
9	21	1237
12	18	1060
15	15	883.6
18	12	706.9
21	9	530.1
24	6	353.4
27	3	176.7
30	0	0



13-40E A fan moves air at sea level at a specified rate. The force required to hold the fan and the minimum power input required for the fan are to be determined. √

Assumptions **1** The flow of air is steady, one-dimensional, and incompressible. **2** Standard atmospheric conditions exist so that the pressure at sea level is 1 atm. **3** Air leaves the fan at a uniform velocity at atmospheric pressure. **4** Air approaches the fan through a large area at atmospheric pressure with negligible velocity. **5** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects).

Properties The gas constant of air is R = 0.3704 psi·ft^3/lbm·R. The standard atmospheric pressure at sea level is 1 atm = 14.7 psi.

Analysis (a) We take the control volume to be a horizontal hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) and the fan located at the narrow cross-section at the end (section 2), and let its centerline be the x axis. The density, mass flow rate, and discharge velocity of air are

$$\rho = \frac{P}{RT} = \frac{14.7\text{ psi}}{(0.3704\text{ psi}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(530\text{ R})} = 0.0749\text{ lbm/ft}^3$$

$$\dot{m} = \rho\dot{V} = (0.0749\text{ lbm/ft}^3)(2000\text{ ft}^3/\text{min}) = 149.8\text{ lbm/min} = 2.50\text{ lbm/s}$$

$$\mathbf{V}_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{2000\text{ ft}^3/\text{min}}{\pi(2\text{ ft})^2/4} = 636.6\text{ ft/min} = 10.6\text{ ft/s}$$

The momentum equation for steady one-dimensional flow is $\sum\vec{F} = \sum\dot{m}_e\vec{\mathbf{V}}_e - \sum\dot{m}_i\vec{\mathbf{V}}_i$. Letting the reaction force to hold the fan be F_{Rx} and assuming it to be in the positive x (i.e., the flow) direction, the momentum equation along the x axis becomes

$$F_{Rx} = \dot{m}(\mathbf{V}_2) - 0 = \dot{m}\mathbf{V} = (2.50\text{ lbm/s})(10.6\text{ ft/s})\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{0.82\text{ lbf}}$$

Therefore, a force of 0.82 lbf must be applied (through friction at the base, for example) to prevent the fan from moving in the horizontal direction under the influence of this force.

(b) Noting that $P_1 = P_2 = P_{atm}$ and $\mathbf{V}_1 \cong 0$, the energy equation for the selected control volume reduces to

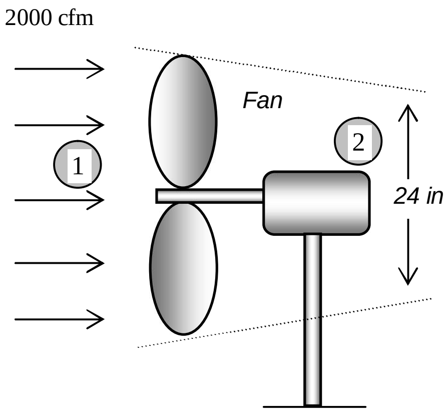
$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad\rightarrow\quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{fan,u}} = \dot{m}\frac{\mathbf{V}_2^2}{2} = (2.50\text{ lbm/s})\frac{(10.6\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ W}}{0.73756\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{5.91\text{ W}}$$

Therefore, a useful mechanical power of 5.91 W must be supplied to air. This is the *minimum* required power input required for the fan.

Discussion The actual power input to the fan will be larger than 5.91 W because of the fan inefficiency in converting mechanical power to kinetic energy.



13-41 A helicopter hovers at sea level while being loaded. The volumetric air flow rate and the required power input during unloaded hover, and the rpm and the required power input during loaded hover are to be determined. √

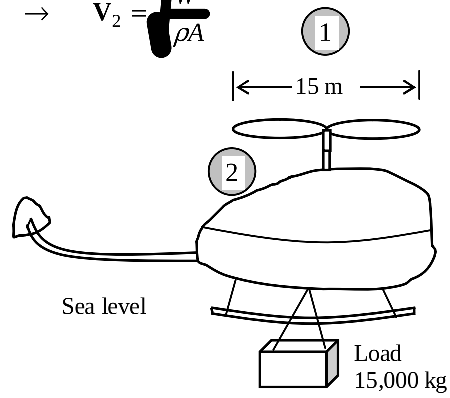
Assumptions **1** The flow of air is steady, one-dimensional, and incompressible. **2** Air leaves the blades at a uniform velocity at atmospheric pressure. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects). **5** The change in air pressure with elevation is negligible because of the low density of air. **6** There is no acceleration of the helicopter, and thus the lift generated is equal to the total weight.

Properties The density of air is given to be 1.18 kg/m³.

Analysis (*a*) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the *z* axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{V}_e - \sum \dot{m}_i \vec{V}_i$. Noting that the only force acting on the control volume is the total weight W and it acts in the negative z direction, the momentum equation along the z axis gives

$$-W = \dot{m}(-V_2) - 0 \quad \rightarrow \quad W = \dot{m}V_2 = (\rho A V_2)V_2 = \rho A V_2^2 \quad \rightarrow \quad V_2 = \sqrt{\frac{W}{\rho A}}$$



where A is the blade span area,

$$A = \pi D^2/4 = \pi(15\ \text{m})^2/4 = 176.7\ \text{m}^2$$

Then the discharge velocity, volume flow rate, and the mass flow rate of air in the unloaded mode become

$$V_{2,\text{unloaded}} = \sqrt{\frac{m_{\text{unloaded}}\, g}{\rho A}} = \sqrt{\frac{(10{,}000\ \text{kg})(9.81\ \text{m/s}^2)}{(1.18\ \text{kg/m}^3)(176.7\ \text{m}^2)}} = 21.7\ \text{m/s}$$

$$\dot{V}_{\text{unloaded}} = A V_{2,\text{unloaded}} = (176.7\ \text{m}^2)(21.7\ \text{m/s}) = \mathbf{3834\ m^3/s}$$

$$\dot{m}_{\text{unloaded}} = \rho \dot{V}_{\text{unloaded}} = (1.18\ \text{kg/m}^3)(3834\ \text{m}^3/\text{s}) = 4524\ \text{kg/s}$$

Noting that $P_1 = P_2 = P_{\text{atm}}$, $V_1 \cong 0$, the elevation effects are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{V_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{unloaded fan,u}} = \left(\dot{m}\frac{V_2^2}{2}\right)_{\text{unloaded}} = (4524\ \text{kg/s})\frac{(21.7\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right) = \mathbf{1065\ kW}$$

(*b*) We now repeat the calculations for the *loaded* helicopter, whose mass is 10,000+15,000 = 25,000 kg:

$$V_{2,\text{loaded}} = \sqrt{\frac{m_{\text{loaded}}\, g}{\rho A}} = \sqrt{\frac{(25{,}000\ \text{kg})(9.81\ \text{m/s}^2)}{(1.18\ \text{kg/m}^3)(176.7\ \text{m}^2)}} = 34.3\ \text{m/s}$$

$$\dot{m}_{\text{loaded}} = \rho \dot{V}_{\text{loaded}} = \rho A V_{2,\text{ loaded}} = (1.18\ \text{kg/m}^3)(176.7\ \text{m}^2)(34.3\ \text{m/s}) = 7152\ \text{kg/s}$$

$$\dot{W}_{\text{loaded fan,u}} = \left(\dot{m}\frac{V_2^2}{2}\right)_{\text{loaded}} = (7152\ \text{kg/s})\frac{(34.3\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right) = \mathbf{4207\ kW}$$

Noting that the average flow velocity is proportional to the overhead blade rotational velocity, the rpm of the loaded helicopter blades becomes

$$\mathbf{V}_2 = k\dot{n} \quad \rightarrow \quad \frac{\mathbf{V}_{2,\text{loaded}}}{\mathbf{V}_{2,\text{unloaded}}} = \frac{\dot{n}_{\text{loaded}}}{\dot{n}_{\text{unloaded}}} \quad \rightarrow \quad \dot{n}_{\text{loaded}} = \frac{\mathbf{V}_{2,\text{loaded}}}{\mathbf{V}_{2,\text{unloaded}}}\dot{n}_{\text{unloaded}} = \frac{34.3}{21.7}(400\text{ rpm}) = \mathbf{632\ rpm}$$

Discussion The actual power input to the helicopter blades will be considerably larger than the calculated power input because of the fan inefficiency in converting mechanical power to kinetic energy.

13-42 A helicopter hovers on top of a high mountain where the air density considerably lower than that at sea level. The blade rotational velocity to hover at the higher altitude and the percent increase in the required power input to hover at high altitude relative to that at sea level are to be determined. √

Assumptions **1** The flow of air is steady, one-dimensional, and incompressible. **2** The air leaves the blades at a uniform velocity at atmospheric pressure. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects). **5** The change in air pressure with elevation while hovering at a given location is negligible because of the low density of air. **6** There is no acceleration of the helicopter, and thus the lift generated is equal to the total weight.

Properties The density of air is given to be 1.18 kg/m^3 at sea level, and 0.79 kg/m^3 on top of the mountain.

Analysis (a) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the z axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. Noting that the only force acting on the control volume is the total weight W and it acts in the negative z direction, the momentum equation along the z axis gives

$$-W = \dot{m}(-\mathbf{V}_2) - 0 \quad \rightarrow \quad W = \dot{m}\mathbf{V}_2 = (\rho A \mathbf{V}_2)\mathbf{V}_2 = \rho A \mathbf{V}_2^2 \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{\frac{W}{\rho A}}$$

where A is the blade span area. Then for a given weight W, the ratio of discharge velocities becomes

$$\frac{\mathbf{V}_{2,\text{mountain}}}{\mathbf{V}_{2,\text{sea}}} = \frac{\sqrt{W/\rho_{\text{mountain}}A}}{\sqrt{W/\rho_{\text{sea}}A}} = \sqrt{\frac{\rho_{\text{sea}}}{\rho_{\text{mountain}}}} = \sqrt{\frac{1.18\ \text{kg/m}^3}{0.79\ \text{kg/m}^3}} = 1.222$$

Noting that the average flow velocity is proportional to the overhead blade rotational velocity, the rpm of the helicopter blades on top of the mountain becomes

$$\dot{n} = k\mathbf{V}_2 \quad \rightarrow \quad \frac{\dot{n}_{\text{mountain}}}{\dot{n}_{\text{sea}}} = \frac{\mathbf{V}_{2,\text{mountain}}}{\mathbf{V}_{2,\text{sea}}} \quad \rightarrow \quad \dot{n}_{\text{mountain}} = \frac{\mathbf{V}_{2,\text{mountain}}}{\mathbf{V}_{2,\text{sea}}}\dot{n}_{\text{sea}} = 1.222(400\ \text{rpm}) = \mathbf{489\ rpm}$$

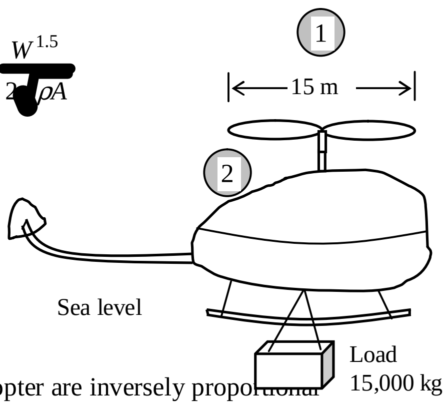
Noting that $P_1 = P_2 = P_{\text{atm}}$, $\mathbf{V}_1 \cong 0$, the elevation effect are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2}$$

or $$\dot{W}_{\text{fan,u}} = \dot{m}\frac{\mathbf{V}_2^2}{2} = \rho A \mathbf{V}_2 \frac{\mathbf{V}_2^2}{2} = \rho A \frac{\mathbf{V}_2^3}{2} = \frac{1}{2}\rho A\left(\sqrt{\frac{W}{\rho A}}\right)^3 = \frac{1}{2}\rho A\left(\frac{W}{\rho A}\right)^{1.5} = \frac{W^{1.5}}{2\sqrt{\rho A}}$$

Then the ratio of the required power input on top of the mountain to that at sea level becomes

$$\frac{\dot{W}_{\text{mountain fan,u}}}{\dot{W}_{\text{sea fan,u}}} = \frac{0.5W^{1.5}/\sqrt{\rho_{\text{mountain}}A}}{0.5W^{1.5}/\sqrt{\rho_{\text{sea}}A}} = \sqrt{\frac{\rho_{\text{sea}}}{\rho_{\text{mountain}}}} = \sqrt{\frac{1.18\ \text{kg/m}^3}{0.79\ \text{kg/m}^3}} = 1.222$$



Therefore, the required power input will increase by **22.2%** on top of the mountain relative to the sea level.

Discussion Note that both the rpm and the required power input to the helicopter are inversely proportional to the square root of air density. Therefore, more power is required at higher elevations for the helicopter to operate because air is less dense, and more air must be forced by the blades into the downdraft.

13-43 The flow rate in a channel is controlled by a sluice gate by raising or lowering a vertical plate. A relation for the force acting on a sluice gate of width w for steady and uniform flow is to be developed.

Assumptions **1** The flow is steady, incompressible, frictionless, and uniform (and thus the Bernoulli equation is applicable.) **2** Wall shear forces at surfaces are negligible. **3** The channel is exposed to the atmosphere, and thus the pressure at free surfaces is the atmospheric pressure. **4** The flow is horizontal.

Analysis We take point 1 at the free surface of the upstream flow before the gate and point 2 at the free surface of the downstream flow after the gate. We also take the bottom surface of the channel as the reference level so that the elevations of points 1 and 2 are y_1 and y_2, respectively. The application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+y_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+y_2 \quad\rightarrow\quad \mathbf{V}_2^2-\mathbf{V}_1^2=2\,\mathrm{g}(y_1-y_2) \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad\rightarrow\quad A_1\mathbf{V}_1=A_2\mathbf{V}_2=\dot{V} \quad\rightarrow\quad \mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{wy_1} \quad\text{and}\quad \mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{wy_2} \qquad (2)$$

Substituting into Eq. (1),

$$\left(\frac{\dot{V}}{wy_2}\right)^2-\left(\frac{\dot{V}}{wy_1}\right)^2=2g(y_1-y_2) \;\rightarrow\; \dot{V}=w\sqrt{\frac{2g(y_1-y_2)}{1/y_2^2-1/y_1^2}} \;\rightarrow\; \dot{V}=wy_2\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \qquad (3)$$

Substituting Eq. (3) into Eqs. (2) gives the following relations for velocities,

$$\mathbf{V}_1=\frac{y_2}{y_1}\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \quad\text{and}\quad \mathbf{V}_2=\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \qquad (4)$$

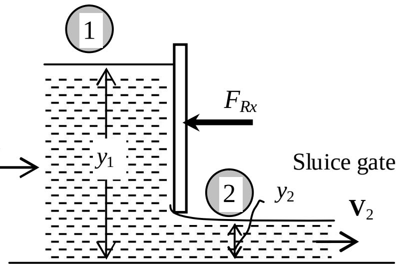
We choose the control volume as the water body surrounded by the vertical cross-sections of the upstream and downstream flows, free surfaces of water, the inner surface of the sluice gate, and the bottom surface of the channel. The momentum equation for steady one-dimensional flow is $\sum \vec{F}=\sum \dot{m}_e\vec{\mathbf{V}}_e-\sum \dot{m}_i\vec{\mathbf{V}}_i$. The force acting on the sluice gate F_{Rx} is horizontal since the wall shear at the surfaces is negligible, and it is equal and opposite to the force applied on water by the sluice gate. Noting that the pressure force acting on a vertical surface is equal to the product of the pressure at the centroid of the surface and the surface area, the momentum equation along the x direction gives

$$-F_{Rx}+P_1A_1-P_2A_2=\dot{m}\mathbf{V}_2-\dot{m}\mathbf{V}_1 \quad\rightarrow\quad -F_{Rx}+\left(\rho g\frac{y_1}{2}\right)(wy_1)-\left(\rho g\frac{y_2}{2}\right)(wy_2)=\dot{m}(\mathbf{V}_2-\mathbf{V}_1)$$

Rearranging, the force acting on the sluice gate is determined to be

$$F_{Rx}=\dot{m}(\mathbf{V}_1-\mathbf{V}_2)+\frac{w}{2}\rho g(y_1^2-y_2^2) \qquad (5)$$

where $\mathbf{V}_1$ and $\mathbf{V}_2$ are given in Eq. (4).



Discussion Note that for $y_1 >> y_2$, Eq. (3) simplifies to $\dot{V}=y_2w\sqrt{2gy_1}$ or $\mathbf{V}_2=\sqrt{2gy_1}$ which is the Toricelli equation for frictionless flow from a tank through a hole a distance y_1 below the free surface.

13-44 Water enters a centrifugal pump axially at a specified rate and velocity, and leaves in the normal direction along the pump casing. The force acting on the shaft in the axial direction is to be determined.

Properties We take the density of water to be 1000 kg/m^3.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The forces acting on the piping system in the horizontal direction are negligible. **3** The atmospheric pressure is disregarded since it acts on all surfaces.

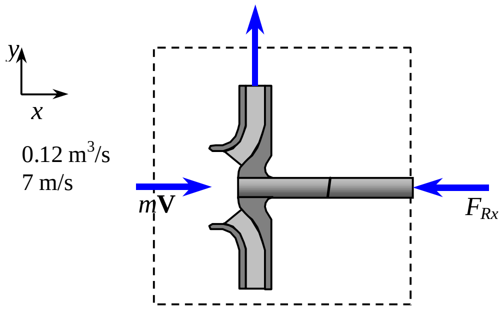
Analysis We take the pump as the control volume, and the inlet direction of flow as the positive direction of x axis. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\sum \vec{F} = \sum \dot{m}_e \vec{V}_e - \sum \dot{m}_i \vec{V}_i \quad \rightarrow \quad -F_{Rx} = -\dot{m}\mathbf{V}_i \quad \rightarrow \quad F_{Rx} = \dot{m}\mathbf{V}_i = \rho \dot{\mathcal{V}} \mathbf{V}_i$$

Note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative *x*-direction. Substituting the given values,

$$F_{\text{brake}} = (1000\ \text{kg/m}^3)(0.12\ \text{m}^3\text{/s})(70\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{8.40\ kN}$$

Discussion To find the total force acting on the shaft, we also need to do a force balance for the vertical direction, and find the vertical component of the reaction force.



Angular Momentum Equation

13-45C The angular momentum equation is obtained by replacing B in the Reynolds transport theorem by the total angular momentum $\vec{H}_{sys}$, and b by the angular momentum per unit mass $\vec{r} \times \vec{V}$.

13-46C The angular momentum equation in this case is expressed as $I\vec{\alpha} = -\vec{r} \times \dot{m}\vec{V}$ where $\vec{\alpha}$ is the angular acceleration of the control volume, and $\vec{r}$ is the position vector from the axis of rotation to any point on the line of action of $\vec{F}$.

13-47C The angular momentum equation in this case is expressed as $I\vec{\alpha} = -\vec{r} \times \dot{m}\vec{V}$ where $\vec{\alpha}$ is the angular acceleration of the control volume, and $\vec{r}$ is the position vector from the axis of rotation to any point on the line of action of $\vec{F}$.

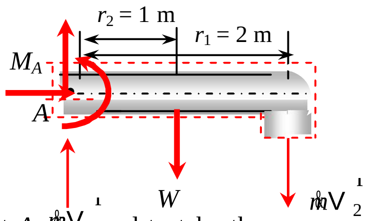
13-48 Water is pumped through a piping section. The moment acting on the elbow for the cases of downward and upward discharge is to be determined.

Assumptions **1** The flow is steady and uniform. **2** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **3** Effects of water falling down during upward discharge is disregarded.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the entire pipe as the control volume, and designate the inlet by 1 and the outlet by 2. We also take the x and y coordinates as shown. The control volume and the reference frame are fixed.

The conservation of mass equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and $V_1 = V_2 = V$ since A_c = constant. The mass flow rate and the weight of the horizontal section of the pipe are



$$\dot{m} = \rho A_c V = (1000\ \text{kg/m}^3)[\pi(0.12\ \text{m})^2/4](4\ \text{m/s}) = 45.24\ \text{kg/s}$$

$$W = mg = (15\ \text{kg/m})(2\ \text{m})(9.81\ \text{m/s}^2)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 294.3\ \text{N/m}$$

(*a*) **Downward discharge**: To determine the moment acting on the pipe at point *A*, we need to take the moment of all forces and momentum flows about that point. This is a steady and uniform flow problem, and all forces and momentum flows are in the same plane. Therefore, the angular momentum equation in this case can be expressed as $\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$ where r is the moment arm, all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative.

The free body diagram of the pipe section is given in the figure. Noting that the moments of all forces and momentum flows passing through point *A* are zero, the only force that will yield a moment about point *A* is the weight *W* of the horizontal pipe section, and the only momentum flow that will yield a moment is the exit stream (both are negative since both moments are in the clockwise direction). Then the angular momentum equation about point *A* becomes

$$M_A - r_1 W = -r_2 \dot{m} V_2$$

Solving for M_A and substituting,

$$M_A = r_1 W - r_2 \dot{m} V_2 = (1\ \text{m})(294.3\ \text{N}) - (2\ \text{m})(45.54\ \text{kg/s})(4\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{-70.0\ N\cdot m}$$

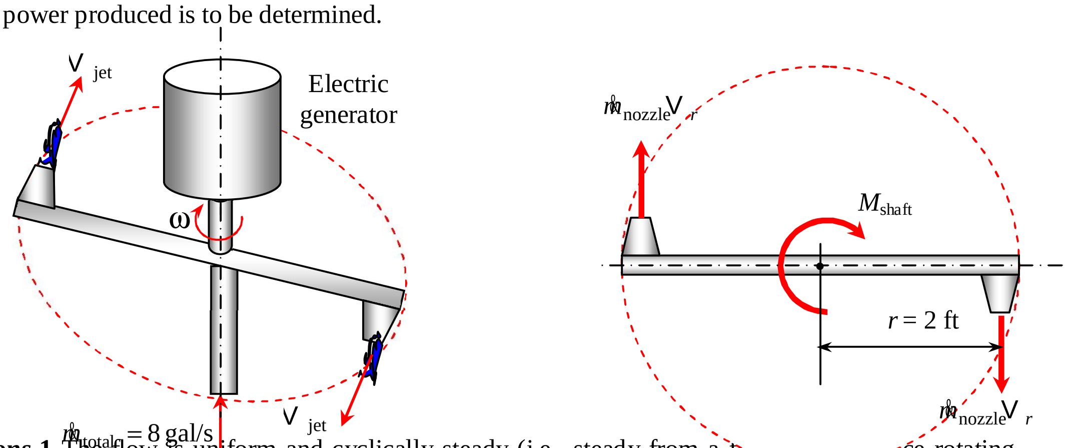
The negative sign indicates that the assumed direction for M_A is wrong, and should be reversed. Therefore, a moment of 70 N·m acts at the stem of the pipe in the clockwise direction.

(*b*) **Upward discharge**: The moment due to discharge stream is positive in this case, and the moment acting on the pipe at point *A* is

$$M_A = r_1 W + r_2 \dot{m} V_2 = (1\ \text{m})(294.3\ \text{N}) + (2\ \text{m})(45.54\ \text{kg/s})(4\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{657\ N\cdot m}$$

Discussion Note direction of discharge can make a big difference in the moments applied on a piping system. This problem also shows the importance of accounting for the moments of momentums of flow streams when performing evaluating the stresses in pipe materials at critical cross-sections.

13-49E A two-armed sprinkler is used to generate electric power. For a specified flow rate and rotational speed, the power produced is to be determined.



Assumptions **1** The flow is uniform and cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle exit is zero. **3** Generator losses and air drag of rotating components are neglected.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot m_1 = \dot m_2 = \dot m$. Noting that the two nozzles are identical, we have $\dot m_{nozzle} = \dot m/2$ or $\dot V_{nozzle} = \dot V/2$ since the density of water is constant. The average jet exit velocity relative to the nozzle is

$$V_{jet} = \frac{\dot V_{nozzle}}{A_{jet}} = \frac{4\text{ gal/s}}{[\pi(0.5/12\text{ ft})^2/4]}\left(\frac{1\text{ ft}^3}{7.480\text{ gal}}\right) = 392.2\text{ ft/s}$$

The angular and tangential velocities of the nozzles are

$$\omega = 2\pi\dot n = 2\pi(250\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 26.18\text{ rad/s}$$

$$V_{nozzle} = r\omega = (2\text{ ft})(26.18\text{ rad/s}) = 52.36\text{ ft/s}$$

The velocity of water jet relative to the control volume (or relative to a fixed location on earth) is

$$V_r = V_{jet} - V_{nozzle} = 392.2 - 52.36 = 339.8\text{ ft/s}$$

The angular momentum equation can be expressed as $\sum M = \sum_{out} r\dot m V - \sum_{in} r\dot m V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-M_{shaft} = -2r\dot m_{nozzle}V_r \qquad \text{or} \qquad M_{shaft} = r\dot m_{total}V_r$$

Substituting, the torque transmitted through the shaft is determined to be

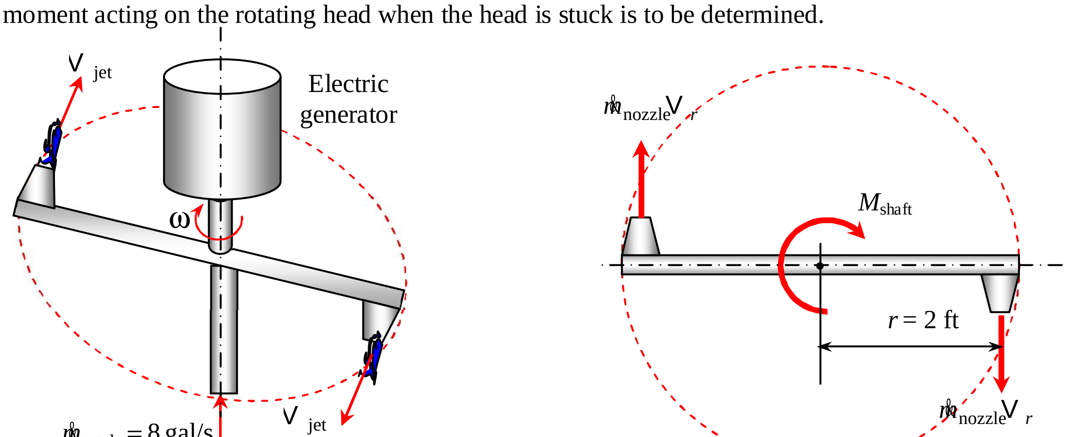
$$M_{shaft} = r\dot m_{total}V_r = (2\text{ ft})(66.74\text{ lbm/s})(339.8\text{ ft/s})\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 1409\text{ lbf}\cdot\text{ft}$$

since $\dot m_{total} = \rho\dot V_{total} = (62.4\text{ lbm/ft}^3)(8/7.480\text{ ft}^3\text{/s}) = 66.74\text{ lbm/s}$. Then the power generated becomes

$$\dot W = 2\pi\dot n M_{shaft} = \omega M_{shaft} = (26.18\text{ rad/s})(1409\text{ lbf}\cdot\text{ft})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{50.0\ kW}$$

Therefore, this sprinkler-type turbine has the potential to produce 50 kW of power.

13-50E A two-armed sprinkler is used to generate electric power. For a specified flow rate and rotational speed, the moment acting on the rotating head when the head is stuck is to be determined.



Assumptions **1** The flow is uniform and cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle exit is zero.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot m_1 = \dot m_2 = \dot m$. Noting that the two nozzles are identical, we have $\dot m_{\text{nozzle}} = \dot m / 2$ or $\dot V_{\text{nozzle}} = \dot V / 2$ since the density of water is constant. The average jet exit velocity relative to the nozzle is

$$V_{\text{jet}} = \frac{\dot V_{\text{nozzle}}}{A_{\text{jet}}} = \frac{4 \text{ gal/s}}{[\pi (0.5/12 \text{ ft})^2/4]}\left(\frac{1 \text{ ft}^3}{7.480 \text{ gal}}\right) = 392.2 \text{ ft/s}$$

The angular momentum equation can be expressed as $\sum M = \sum_{\text{out}} r\dot m V - \sum_{\text{in}} r\dot m V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-M_{\text{shaft}} = -2r\dot m_{\text{nozzle}} V_{jet} \qquad \text{or} \qquad M_{\text{shaft}} = r\dot m_{\text{total}} V_{jet}$$

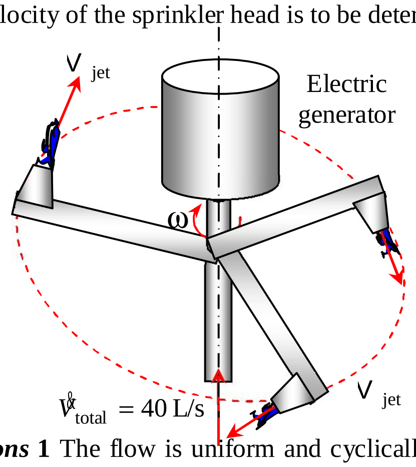
Substituting, the torque transmitted through the shaft is determined to be

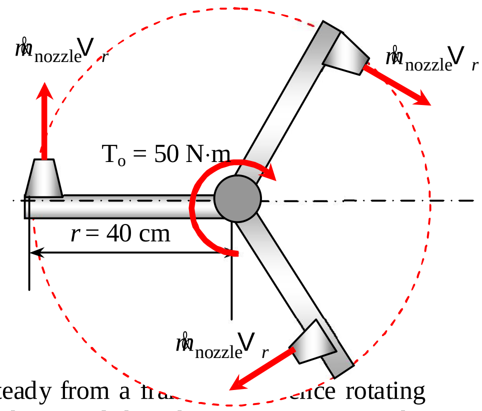
$$M_{\text{shaft}} = r\dot m_{\text{total}} V_{jet} = (2 \text{ ft})(66.74 \text{ lbm/s})(392.2 \text{ ft/s})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{1626\ lbf\cdot ft}$$

since $\dot m_{\text{total}} = \rho \dot V_{\text{total}} = (62.4 \text{ lbm/ft}^3)(8/7.480 \text{ ft}^3\text{/s}) = 66.74 \text{ lbm/s}$.

Discussion When the sprinkler is stuck and thus the angular velocity is zero, the torque developed is maximum since $V_{\text{nozzle}} = 0$ and thus $V_r = V_{\text{jet}} = 392.2$ ft/s, giving $M_{\text{shaft, max}} = 1626 \text{ lbf}\cdot\text{ft}$. But the power generated is zero in this case since the shaft does not rotate.

13-51 A three-armed sprinkler is used to water a garden. For a specified flow rate and resistance torque, the angular velocity of the sprinkler head is to be determined.





Assumptions **1** The flow is uniform and cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle exit is zero. **3** Air drag of rotating components are neglected.

Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Noting that the three nozzles are identical, we have $\dot{m}_{nozzle} = \dot{m}/3$ or $\dot{V}_{nozzle} = \dot{V}/3$ since the density of water is constant. The average jet exit velocity relative to the nozzle and the mass flow rate are

$$V_{jet} = \frac{\dot{V}_{nozzle}}{A_{jet}} = \frac{40\text{ L/s}}{3[\pi(0.012\text{ m})^2/4]}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right) = 117.9\text{ m/s}$$

$$\dot{m}_{total} = \rho\dot{V}_{total} = (1\text{ kg/L})(40\text{ L/s}) = 40\text{ kg/s}$$

The angular momentum equation can be expressed as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-T_0 = -3r\dot{m}_{nozzle}V_r \qquad \text{or} \qquad T_0 = r\dot{m}_{total}V_r$$

Solving for the relative velocity V_r and substituting,

$$V_r = \frac{T_0}{r\dot{m}_{total}} = \frac{50\text{ N}\cdot\text{m}}{(0.40\text{ m})(40\text{ kg/s})}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 3.1\text{ m/s}$$

Then the tangential and angular velocity of the nozzles become

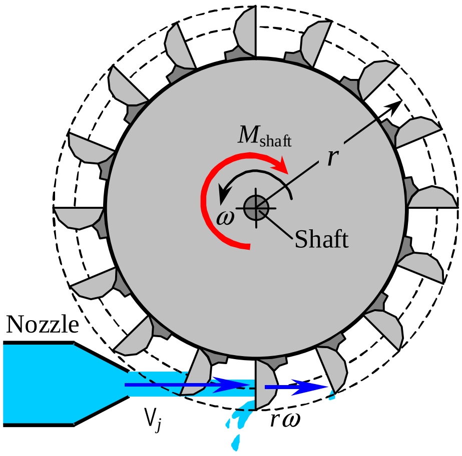
$$V_{nozzle} = V_{jet} - V_r = 117.9 - 3.1 = 114.8\text{ m/s}$$

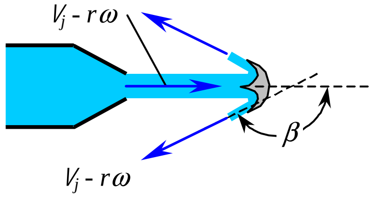
$$\omega = \frac{V_{nozzle}}{r} = \frac{114.8\text{ m/s}}{0.4\text{ m}} = \mathbf{287\ rad/s}$$

$$\dot{n} = \frac{\omega}{2\pi} = \frac{287\text{ rad/s}}{2\pi}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \mathbf{2741\ rpm}$$

Therefore, this sprinkler will rotate at 2741 revolutions per minute.

13-52 A Pelton wheel is considered for power generation in a hydroelectric power plant. A relation is to be obtained for power generation, and its numerical value is to be obtained.





Assumptions **1** The flow is uniform and cyclically steady. **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle exit is zero. **3** Generator losses and air drag of rotating components are neglected.

Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis The tangential velocity of buckets corresponding to an angular velocity of $\omega = 2\pi\dot{n}$ is $V_{bucket} = r\omega$. Then the relative velocity of the jet (relative to the bucket) becomes

$$V_r = V_j - V_{bucket} = V_j - r\omega$$

We take the imaginary disk that contains the Pelton wheel as the control volume. The inlet velocity of the fluid into this control volume is V_r, and the component of outlet velocity normal to the moment arm is $V_r\cos\beta$. The angular momentum equation can be expressed as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-M_{shaft} = r\dot{m}V_r\cos\beta - r\dot{m}V_r \quad \text{or} \quad M_{shaft} = r\dot{m}V_r(1-\cos\beta) = r\dot{m}(V_j - r\omega)(1-\cos\beta)$$

Noting that $\dot{W}_{shaft} = 2\pi\dot{n}M_{shaft} = \omega M_{shaft}$ and $\dot{m} = \rho\dot{V}$, the power output of a Pelton turbine becomes

$$\dot{W}_{shaft} = \rho\dot{V}r\omega(V_j - r\omega)(1-\cos\beta)$$

which is the desired relation. For given values, the power output is determined to be

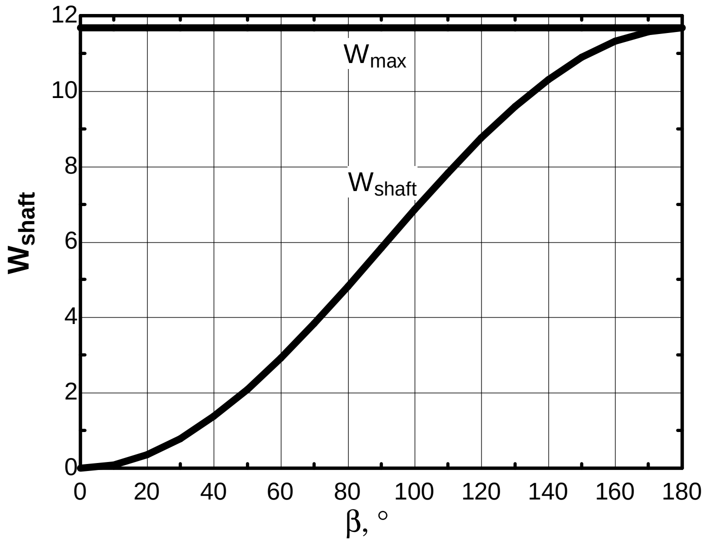
$$\dot{W}_{shaft} = (1000\text{ kg/m}^3)(10\text{ m}^3\text{/s})(2\text{ m})(15.71\text{ rad/s})(50\text{-}2\times 15.71\text{ m/s})(1\text{-}\cos 160°)\left(\frac{1\text{ MW}}{10^6\text{ N}\cdot\text{m/s}}\right) = \mathbf{11.3\ MW}$$

where $\qquad \omega = 2\pi\dot{n} = 2\pi(150\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 15.71\text{ rad/s}$

13-53 Problem 13-52 is reconsidered. The effect of β on the power generation as β varies from 0° to 180° is to be determined, and the fraction of power loss at 160° is to be assessed.

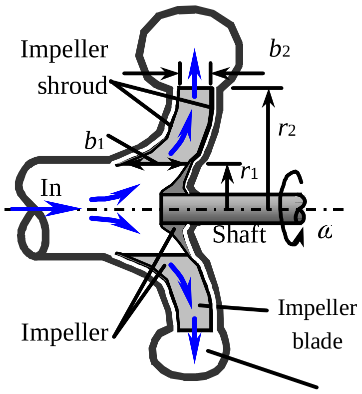
```
rho=1000 "kg/m3"
r=2 "m"
V_dot=10 "m3/s"
V_jet=50 "m/s"
n_dot=150 "rpm"
omega=2*pi*n_dot/60
V_r=V_jet-r*omega
m_dot=rho*V_dot
W_dot_shaft=m_dot*omega*r*V_r*(1-cos(Beta))/1E6 "MW"
W_dot_max=m_dot*omega*r*V_r*2/1E6 "MW"
Effectiveness=W_dot_shaft/W_dot_max
```

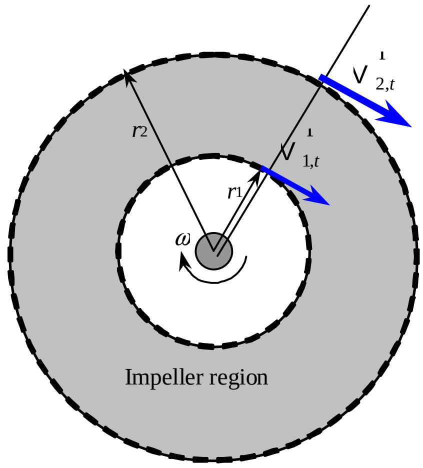
Angle, $\beta°$	Max power, $\dot{W}_{max}$, MW	Actual power, $\dot{W}_{shaft}$, MW	Effectiveness, η
0	11.7	0.00	0.000
10	11.7	0.09	0.008
20	11.7	0.35	0.030
30	11.7	0.78	0.067
40	11.7	1.37	0.117
50	11.7	2.09	0.179
60	11.7	2.92	0.250
70	11.7	3.84	0.329
80	11.7	4.82	0.413
90	11.7	5.84	0.500
100	11.7	6.85	0.587
110	11.7	7.84	0.671
120	11.7	8.76	0.750
130	11.7	9.59	0.821
140	11.7	10.31	0.883
150	11.7	10.89	0.933
160	11.7	11.32	0.970
170	11.7	11.59	0.992
180	11.7	11.68	1.000



The effectiveness of Pelton wheel for β =160° is 0.97. Therefore, at this angle, only 3% of power is lost.

13-54 A centrifugal blower is used to deliver atmospheric air. For a given angular speed and power input, the volume flow rate of air is to be determined.





Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible. **3** The tangential components of air velocity at the inlet and the outlet are said to be equal to the impeller velocity at respective locations.

Properties The gas constant of air is 0.287 kPa·m^3/kg·K. The density of air at 20°C and 95 kPa is

$$\rho = \frac{P}{RT} = \frac{95 \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293 \text{ K})} = 1.130 \text{ kg/m}^3$$

Analysis In the idealized case of the tangential fluid velocity being equal to the blade angular velocity both at the inlet and the exit, we have $V_{1,t} = \omega r_1$ and $V_{2,t} = \omega r_2$, and the torque is expressed as

$$\text{T}_{\text{shaft}} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = \dot{m}\omega(r_2^2 - r_1^2) = \rho\dot{V}\omega(r_2^2 - r_1^2)$$

where the angular velocity is

$$\omega = 2\pi\dot{n} = 2\pi(800 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 83.78 \text{ rad/s}$$

Then the shaft power becomes

$$\dot{W}_{\text{shaft}} = \omega \text{T}_{\text{shaft}} = \rho\dot{V}\omega^2(r_2^2 - r_1^2)$$

Solving for $\dot{V}$ and substituting, the volumetric flow rate of air is determined to

$$\dot{V} = \frac{\dot{W}_{\text{shaft}}}{\rho\omega^2(r_2^2 - r_1^2)} = \frac{120 \text{ N}\cdot\text{m/s}}{(1.130 \text{ kg/m}^3)(83.78 \text{ rad/s})^2[(0.30 \text{ m})^2 - (0.15 \text{ m})^2]}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right) = \mathbf{0.224\ m^3/s}$$

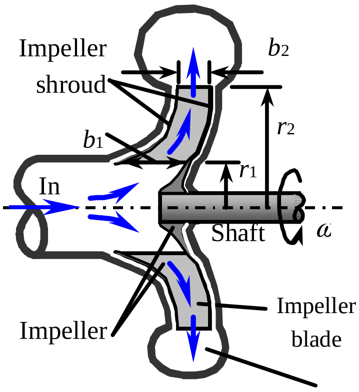
The normal velocity components at the inlet and the outlet are

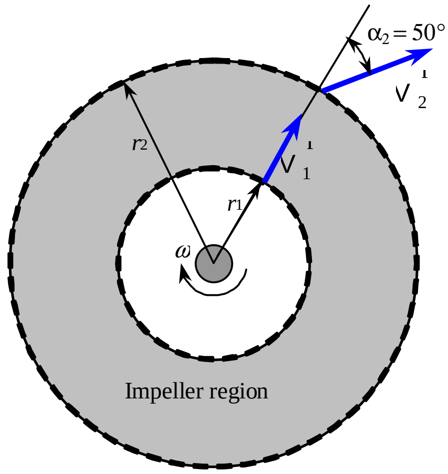
$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.224 \text{ m}^3/\text{s}}{2\pi(0.15 \text{ m})(0.061 \text{ m})} = \mathbf{3.90\ m/s}$$

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{0.224 \text{ m}^3/\text{s}}{2\pi(0.30 \text{ m})(0.034 \text{ m})} = \mathbf{3.50\ m/s}$$

Discussion Note that the irreversible losses are not considered in analysis. In reality, the flow rate and the normal components of velocities will be smaller.

13-55 A centrifugal blower is used to deliver atmospheric air at a specified rate and angular speed. The minimum power consumption of the blower is to be determined.





Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.

Properties The density of air is given to be 1.25 kg/m^3.

Analysis We take the impeller region as the control volume. The normal velocity components at the inlet and the outlet are

$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.70\ \text{m}^3/\text{s}}{2\pi(0.20\ \text{m})(0.082\ \text{m})} = 6.793\ \text{m/s}$$

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{0.70\ \text{m}^3/\text{s}}{2\pi(0.45\ \text{m})(0.056\ \text{m})} = 4.421\ \text{m/s}$$

The tangential components of absolute velocity are:

$\alpha_1 = 0°$: $\quad V_{1,t} = V_{1,n} \tan\alpha_1 = 0$

$\alpha_2 = 60°$: $\quad V_{2,t} = V_{2,n} \tan\alpha_1 = (4.421\ \text{m/s})\tan 50° = 5.269\ \text{m/s}$

The angular velocity of the propeller is

$$\omega = 2\pi\dot{n} = 2\pi(700\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 73.30\ \text{rad/s}$$

$$\dot{m} = \rho\dot{V} = (1.25\ \text{kg/m}^3)(0.7\ \text{m}^3/\text{s}) = 0.875\ \text{kg/s}$$

Normal velocity components $V_{1,n}$ and $V_{2,n}$ as well pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque. Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

$$T_{\text{shaft}} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = (0.875\ \text{kg/s})[(0.45\ \text{m})(5.269\ \text{m/s}) - 0]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 2.075\ \text{N}\cdot\text{m}$$

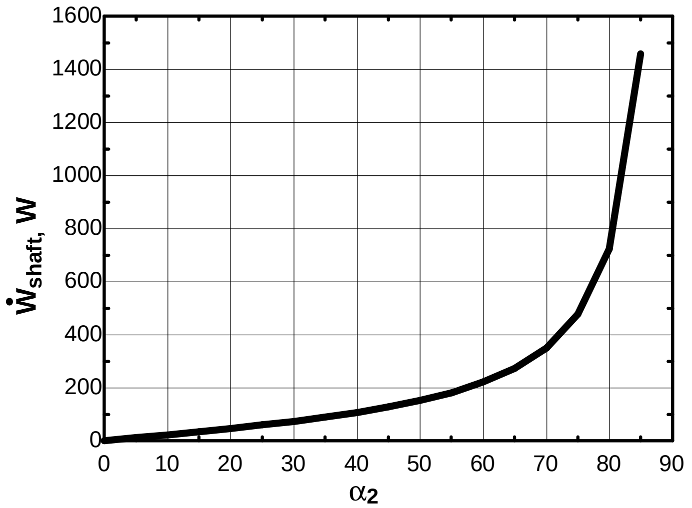
Then the shaft power becomes

$$\dot{W} = \omega T_{\text{shaft}} = (73.30\ \text{rad/s})(2.075\ \text{N}\cdot\text{m})\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{152\ W}$$

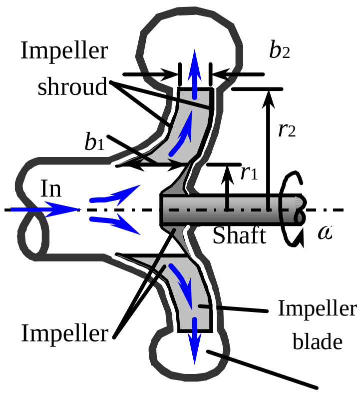
13-56 Problem 13-55 is reconsidered. The effect of discharge angle α_2 on the minimum power input requirements as α_2 varies from 0° to 85° in increments of 5° is to be investigated.

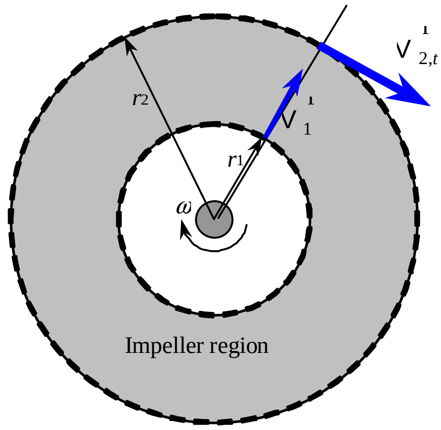
```
rho=1.25 "kg/m3"
r1=0.20 "m"
b1=0.082 "m"
r2=0.45 "m"
b2=0.056 "m"
V_dot=0.70 "m3/s"
V1n=V_dot/(2*pi*r1*b1) "m/s"
V2n=V_dot/(2*pi*r2*b2) "m/s"
Alpha1=0
V1t=V1n*tan(Alpha1) "m/s"
V2t=V2n*tan(Alpha2) "m/s"
n_dot=700 "rpm"
omega=2*pi*n_dot/60 "rad/s"
m_dot=rho*V_dot "kg/s"
T_shaft=m_dot*(r2*V2t-r1*V1t) "Nm"
W_dot_shaft=omega*T_shaft "W"
```

Angle, α_2°	$V_{2,t}$, m/s	Torque, T_{shaft}, Nm	Shaft power, $\dot{W}_{shaft}$, W
0	0.00	0.00	0
5	0.39	0.15	11
10	0.78	0.31	23
15	1.18	0.47	34
20	1.61	0.63	46
25	2.06	0.81	60
30	2.55	1.01	74
35	3.10	1.22	89
40	3.71	1.46	107
45	4.42	1.74	128
50	5.27	2.07	152
55	6.31	2.49	182
60	7.66	3.02	221
65	9.48	3.73	274
70	12.15	4.78	351
75	16.50	6.50	476
80	25.07	9.87	724
85	50.53	19.90	1459



13-57E Water enters the impeller of a centrifugal pump radially at a specified flow rate and angular speed. The torque applied to the impeller is to be determined.





Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis Water enters the impeller normally, and thus $V_{1,t} = 0$. The tangential component of fluid velocity at the outlet is given to be $V_{2,t} = 180\text{ ft/s}$. The inlet radius r_1 is unknown, but the outlet radius is given to be r_2 = 1 ft. The angular velocity of the propeller is

$$\omega = 2\pi\dot{n} = 2\pi(500\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 52.36\text{ rad/s}$$

The mass flow rate is

$$\dot{m} = \rho\dot{V} = (62.4\text{ lbm/ft}^3)(80/60\text{ ft}^3/\text{s}) = 83.2\text{ lbm/s}$$

Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

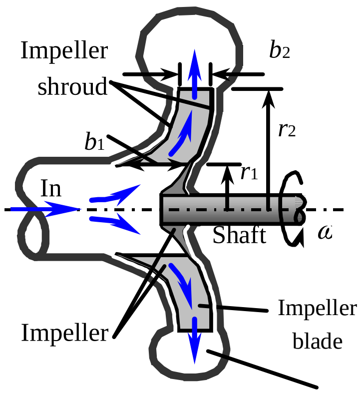
$$T_{\text{shaft}} = \dot{m}(r_2V_{2,t} - r_1V_{1,t}) = (83.2\text{ lbm/s})[(1\text{ ft})(180\text{ ft/s}) - 0]\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{465\text{ lbf}\cdot\text{ft}}$$

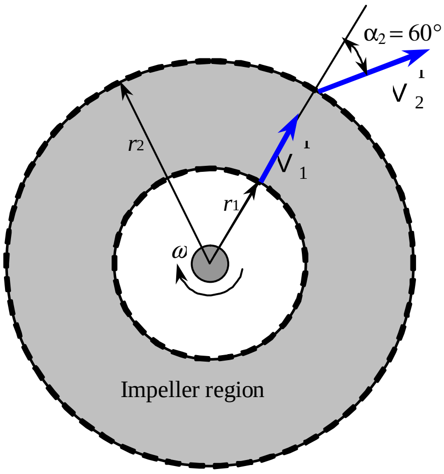
Discussion This shaft power input corresponding to this torque is

$$\dot{W} = 2\pi\dot{n}T_{\text{shaft}} = \omega T_{\text{shaft}} = (52.36\text{ rad/s})(465\text{ lbf}\cdot\text{ft})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{33.0\text{ kW}}$$

Therefore, the minimum power input to this pump should be 33 kW.

13-58 A centrifugal pump is used to supply water at a specified rate and angular speed. The minimum power consumption of the pump is to be determined.





Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the impeller region as the control volume. The normal velocity components at the inlet and the outlet are

$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.15\ \text{m}^3/\text{s}}{2\pi(0.13\ \text{m})(0.080\ \text{m})} = 2.296\ \text{m/s}$$

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{0.15\ \text{m}^3/\text{s}}{2\pi(0.30\ \text{m})(0.035\ \text{m})} = 2.274\ \text{m/s}$$

The tangential components of absolute velocity are:

$$\alpha_1 = 0°: \qquad V_{1,t} = V_{1,n} \tan\alpha_1 = 0$$

$$\alpha_2 = 60°: \qquad V_{2,t} = V_{2,n} \tan\alpha_1 = (2.274\ \text{m/s}) \tan 60° = 3.938\ \text{m/s}$$

The angular velocity of the propeller is

$$\omega = 2\pi\dot{n} = 2\pi(1200\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 125.7\ \text{rad/s}$$

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(0.15\ \text{m}^3/\text{s}) = 150\ \text{kg/s}$$

Normal velocity components $V_{1,n}$ and $V_{2,n}$ as well pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque. Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

$$T_{\text{shaft}} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = (150\ \text{kg/s})[(0.30\ \text{m})(3.938\ \text{m/s}) - 0]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 177.2\ \text{kN}\cdot\text{m}$$

Then the shaft power becomes

$$\dot{W} = \omega T_{\text{shaft}} = (125.7\ \text{rad/s})(177.2\ \text{kN}\cdot\text{m})\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right) = \mathbf{22.3\ kW}$$

Discussion Note that the irreversible losses are not considered in analysis. In reality, the required power input will be larger.

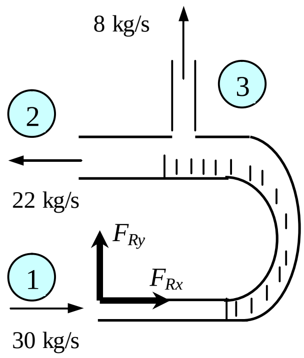
Review Problems

13-59 Water is flowing into and discharging from a pipe U-section with a secondary discharge section normal to return flow. Net x- and y- forces at the two flanges that connect the pipes are to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The weight of the U-turn and the water in it is negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The flow velocities of the 3 streams are



$$\mathbf{V}_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{\dot{m}_1}{\rho(\pi D_1^2/4)} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)[\pi(0.05\text{ m})^2/4]} = 0.51\text{ m/s}$$

$$\mathbf{V}_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{\dot{m}_2}{\rho(\pi D_2^2/4)} = \frac{22\text{ kg/s}}{(1000\text{ kg/m}^3)[\pi(0.10\text{ m})^2/4]} = 2.80\text{ m/s}$$

$$\mathbf{V}_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{\dot{m}_3}{\rho(\pi D_3^2/4)} = \frac{8\text{ kg/s}}{(1000\text{ kg/m}^3)[\pi(0.03\text{ m})^2/4]} = 11.3\text{ m/s}$$

We take the entire U-section as the control volume. We designate the horizontal coordinate by x with the direction of incoming flow as being the positive direction and the vertical coordinate by y. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. We let the x- and y- components of the anchoring force of the cone be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Then the momentum equations along the x and y axes become

$$F_{Rx} + P_1A_1 + P_2A_2 = \dot{m}_2(-\mathbf{V}_2) - \dot{m}_1\mathbf{V}_1 \quad \rightarrow \quad F_{Rx} = -P_1A_1 - P_2A_2 - \dot{m}_2\mathbf{V}_2 - \dot{m}_1\mathbf{V}_1$$

$$F_{Ry} + 0 = \dot{m}_3\mathbf{V}_3 - 0 \quad \rightarrow \quad F_{Ry} = \dot{m}_3\mathbf{V}_3$$

Substituting the given values,

$$F_{Rx} = -[(200-100)\text{ kN/m}^2]\frac{\pi(0.05\text{ m})^2}{4} - [(150-100)\text{ kN/m}^2]\frac{\pi(0.10\text{ m})^2}{4}$$
$$-(22\text{ kg/s})(2.80\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) - (30\text{ kg/s})(0.51\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$
$$= -0.666\ k\mathbf{N} = \mathbf{-666\ N}$$

$$F_{Ry} = (8\text{ kg/s})(11.3\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{90.4\ N}$$

The negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed. Therefore, a force of 666 N acts on the flanges in the opposite direction. A vertical force of 90.4 N acts on the flange in the vertical direction.

Discussion To assess the significance of gravity forces, we estimate the weight of the weight of water in the U-turn and compare it to the vertical force. Assuming the length of the U-turn to be 0.5 m and the average diameter to be 7.5 cm, the mass of the water becomes

$$m = \rho V = \rho A L = \rho\frac{\pi D^2}{4}L = (1000\text{ kg/m}^3)\frac{\pi(0.075\text{ m})^2}{4}(0.5\text{ m}) = 2.2\text{ kg}$$

whose weight is 2.2×9.81 = 22 N, which is much less than 90.4 N, but still significant. Therefore, disregarding the gravitational effects is a reasonable assumption if great accuracy is not required.

13-60 A fireman was hit by a nozzle held by a tripod with a rated holding force. The accident is to be investigated by calculating the water velocity, the flow rate, and the nozzle velocity.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Gravitational effects and vertical forces are disregarded since the horizontal resistance force is to be determined.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the nozzle and the horizontal portion of the hose as the system such that water enters the control volume vertically and exits horizontally (this way the pressure force and the momentum flux at the inlet are in the vertical direction, with no contribution to the force balance in the horizontal direction, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction).

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. We let the horizontal force applied by the tripod to the nozzle to hold it be F_{Rx}, and assume it to be in the positive *x* direction. Then the momentum equation along the *x* direction becomes

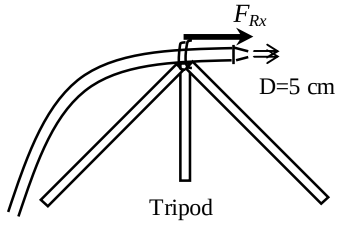
$$F_{Rx} = \dot{m}\mathbf{V}_e - 0 = \dot{m}\mathbf{V} = \rho A\mathbf{V}\mathbf{V} = \rho\frac{\pi D^2}{4}\mathbf{V}^2 \quad \to \quad (1800\text{ N})\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right) = (1000\text{ kg/m}^3)\frac{\pi(0.05\text{ m})^2}{4}\mathbf{V}^2$$

Solving for the water exit velocity gives **V** = **30.2 m/s.** Then the water flow rate becomes

$$\dot{V} = A\mathbf{V} = \frac{\pi D^2}{4}\mathbf{V} = \frac{\pi(0.05\text{ m})^2}{4}(30.2\text{ m/s}) = \mathbf{0.0593\ m^3/s}$$

When the nozzle was released, its acceleration must have been

$$a_{\text{nozzle}} = \frac{F}{m_{\text{nozzle}}} = \frac{1800\text{ N}}{10\text{ kg}}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 180\text{ m/s}^2$$



Assuming the reaction force acting on the nozzle and thus its acceleration to remain constant, the time it takes for the nozzle to travel 60 cm and the nozzle velocity at that moment were (note that both the distance *x* and the velocity **V** are zero at time *t* = 0)

$$x = \frac{1}{2}at^2 \quad \to \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(0.6\text{ m})}{180\text{ m/s}^2}} = 0.0816\ s$$

$$\mathbf{V} = at = (180\text{ m/s}^2)(0.0816\text{ s}) = \mathbf{14.7\ m/s}$$

Thus we conclude that the nozzle hit the fireman with a velocity of 14.7 m/s.

Discussion Engineering analyses such as this one are frequently used in accident reconstruction cases, and they often form the basis for judgment in courts.

13-61 During landing of an airplane, the thrust reverser is lowered in the path of the exhaust jet, which deflects the exhaust and provides breaking. The thrust of the engine and the braking force produced after the thrust reverser is deployed are to be determined. √EES

Assumptions **1** The flow of exhaust gases is steady and one-dimensional. **2** The exhaust gas stream is exposed to the atmosphere, and thus its pressure is the atmospheric pressure. **3** The velocity of exhaust gases remains constant during reversing.

Analysis (*a*) The thrust exerted on an airplane is simply the momentum flux of the combustion gases in the reverse direction,

$$\text{Thrust} = \dot{m}_{ex}\mathbf{V}_{ex} = (18\text{ kg/s})(250\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{4500\ N}$$

(*b*) We take the thrust reverser as the control volume such that it cuts through both exhaust streams normally and the connecting bars to the airplane, and the direction of airplane as the positive direction of *x* axis. The momentum equation for steady one-dimensional flow in the *x* direction reduces to

$$\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i \quad \rightarrow \quad F_{Rx} = \dot{m}(\mathbf{V})\cos 20° - \dot{m}(-\mathbf{V}) \quad \rightarrow \quad F_{Rx} = (1+\cos 20°)\dot{m}\mathbf{V}_i$$

Substituting, the reaction force is determined to be

$$F_{Rx} = (1+\cos 20°)(18\text{ kg/s})(250\text{ m/s}) = 8729\text{ N}$$

The breaking force acting on the plane is equal and opposite to this force,

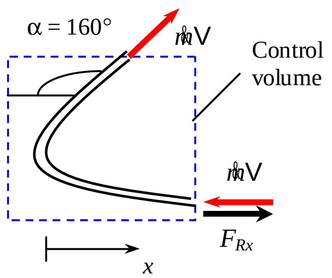
$$F_{\text{breaking}} = \mathbf{8729\ N}$$

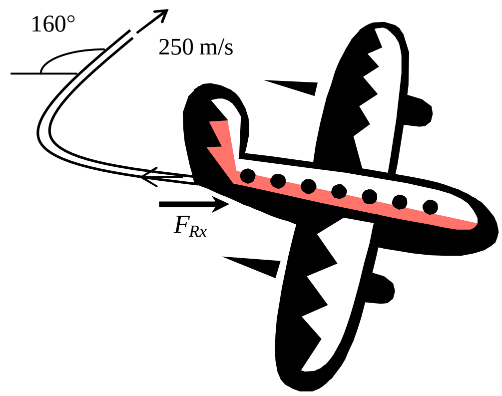
Therefore, a braking force of 8729 N develops in the opposite direction tot flight.

Discussion This problem can be solved more generally by measuring the reversing angle from the direction of exhaust gases ($\alpha = 0$ when there is no reversing). When $\alpha < 90°$, the reversed gases are discharged in the negative *x* direction, and the momentum equation reduces to

$$F_{Rx} = \dot{m}(-\mathbf{V})\cos\alpha - \dot{m}(-\mathbf{V}) \quad \rightarrow \quad F_{Rx} = (1-\cos\alpha)\dot{m}\mathbf{V}_i$$

This equation is also valid for $\alpha > 90°$ since $\cos(180°-\alpha) = -\cos\alpha$. Using $\alpha = 160°$, for example, gives $F_{Rx} = (1-\cos 160)\dot{m}\mathbf{V}_i = (1+\cos 20)\dot{m}\mathbf{V}_i$, which is identical to the solution above.



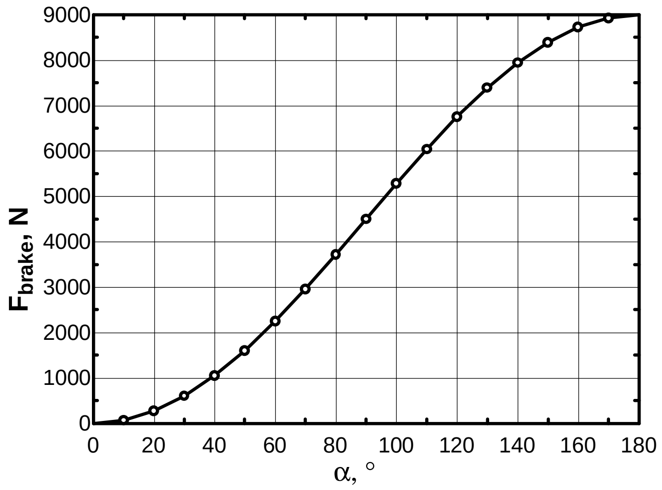


13-62 Problem 13-61 reconsidered. The effect of thrust reverser angle on the braking force exerted on the airplane as the reverser angle varies from 0 (no reversing) to 180° (full reversing) in increments of 10° is to be investigated.

```
V_jet=250 "m/s"

m_dot=18 "kg/s"
F_Rx=(1-cos(alpha))*m_dot*V_jet "N"
```

Reversing angle, α°	Braking force F_{brake}, N
0	0
10	68
20	271
30	603
40	1053
50	1607
60	2250
70	2961
80	3719
90	4500
100	5281
110	6039
120	6750
130	7393
140	7947
150	8397
160	8729
170	8932
180	9000



13-63E The rocket of a spacecraft is fired in the opposite direction to motion. The acceleration, the velocity change, and the thrust are to be determined.

Assumptions **1** The flow of combustion gases is steady and one-dimensional during firing period, but the flight of spacecraft is unsteady. **2** There are no external forces acting on the spacecraft, and the effect of pressure force at the nozzle exit is negligible. **3** The mass of discharged fuel is negligible relative to the mass of the spacecraft, and thus the spacecraft may be treated as a solid body with a constant mass.

Analysis (*a*) A body moving at constant velocity can be considered to be stationary for convenience. Then the velocities of fluid steams become simply their relative velocities relative to the moving body. We take the direction of motion of the spacecraft as the positive direction along the *x* axis. There are no external forces acting on the spacecraft, and its mass is nearly constant. Therefore, the spacecraft can be treated as a solid body with constant mass, and the momentum equation in this case is

$$\frac{d(m\vec{V})_{CV}}{dt} = \sum_{\text{in}} \dot{m}_i \vec{V}_i - \sum_{\text{out}} \dot{m}_e \vec{V}_e \quad \rightarrow \quad m_{\text{space}} \frac{d\vec{V}_{\text{space}}}{dt} = -\dot{m}_f \vec{V}_f$$

Noting that the motion is on a straight line and the discharged gases move in the negative *x* direction, we can write the momentum equation using magnitudes as

$$m_{\text{space}} \frac{d\mathbf{V}_{\text{space}}}{dt} = \dot{m}_f \mathbf{V}_f \quad \rightarrow \quad \frac{d\mathbf{V}_{\text{space}}}{dt} = \frac{\dot{m}_f}{m_{\text{space}}} \mathbf{V}_f$$

Substituting, the acceleration of the spacecraft during the first 5 seconds is determined to be

$$a_{\text{space}} = \frac{d\mathbf{V}_{\text{space}}}{dt} = \frac{\dot{m}_f}{m_{\text{space}}} \mathbf{V}_f = \frac{150\ \text{lbm/s}}{18{,}000\ \text{lbm}} (5000\ \text{ft/s}) = \mathbf{41.7\ ft/s^2}$$

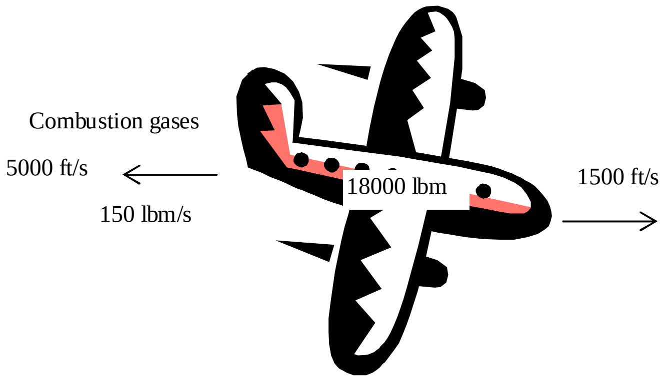
(*b*) Knowing acceleration, which is constant, the velocity change of the spacecraft during the first 5 seconds is determined from the definition of acceleration $a_{\text{space}} = d\mathbf{V}_{\text{space}} / dt$ to be

$$d\mathbf{V}_{\text{space}} = a_{\text{space}} dt \quad \rightarrow \quad \Delta\mathbf{V}_{\text{space}} = a_{\text{space}} \Delta t = (41.7\ \text{ft/s}^2)(5\ \text{s}) = \mathbf{209\ ft/s}$$

(*c*) The thrust exerted on the system is simply the momentum flux of the combustion gases in the reverse direction,

$$\text{Thrust} = F_R = -\dot{m}_f \mathbf{V}_f = -(150\ \text{lbm/s})(-5000\ \text{ft/s}) \left(\frac{1\ \text{lbf}}{32.2\ \text{lbm} \cdot \text{ft/s}^2} \right) = \mathbf{23{,}290\ lbf}$$

Therefore, if this spacecraft were attached somewhere, it would exert a force of 23,290 lbf (equivalent to the weight of 23,290 lbm of mass) to its support.



13-64 A horizontal water jet strikes a vertical stationary flat plate normally at a specified velocity. For a given flow velocity, the anchoring force needed to hold the plate in place is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water splatters off the sides of the plate in a plane normal to the jet. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on the entire control surface. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the plate as the control volume such that it contains the entire plate and cuts through the water jet and the support bar normally, and the direction of flow as the positive direction of *x* axis. We take the reaction force to be in the negative *x* direction. The momentum equation for steady one-dimensional flow in the *x* (flow) direction reduces in this case to

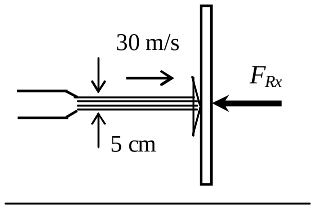
$$\Sigma \vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad -F_{Rx} = -\dot{m}_i \mathbf{V}_i \quad \rightarrow \quad F_{Rx} = \dot{m}\mathbf{V}$$

We note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative *x*-direction. The mass flow rate of water is

$$\dot{m} = \rho \dot{V} = \rho A \mathbf{V} = \rho \frac{\pi D^2}{4}\mathbf{V} = (1000\,\text{kg/m}^3)\frac{\pi (0.05\,\text{m})^2}{4}(30\,\text{m/s}) = 58.90\,\text{kg/s}$$

Substituting, the reaction force is determined to be

$$F_{Rx} = (58.90\,\text{kg/s})(30\,\text{m/s}) = \mathbf{1767\,N}$$



Therefore, a force of 1767 N must be applied to the plate in the opposite direction to flow to hold it in place.

Discussion In reality, some water will be scattered back, and this will add to the reaction force of water.

13-65 A water jet hits a stationary cone, such that the flow is diverted equally in all directions at 45°. The force required to hold the cone in place against the water stream is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet before and after the split is the atmospheric pressure which is disregarded since it acts on all surfaces. **3** The gravitational effects are disregarded.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The mass flow rate of water jet is

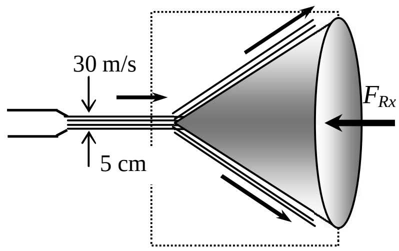
$$\dot{m} = \rho\dot{\mathcal{V}} = \rho A\mathbf{V} = \rho\frac{\pi D^2}{4}\mathbf{V} = (1000\,\text{kg/m}^3)\frac{\pi(0.05\,\text{m})^2}{4}(30\,\text{m/s}) = 58.90\,\text{kg/s}$$

We take the diverting section of water jet, including the cone as the control volume, and designate the entrance by 1 and the exit after divergence by 2. We also designate the horizontal coordinate by x with the direction of flow as being the positive direction and the vertical coordinate by y.

The momentum equation for steady one-dimensional flow is $\sum\vec{F} = \sum\dot{m}_e\vec{\mathbf{V}}_e - \sum\dot{m}_i\vec{\mathbf{V}}_i$. We let the x- and y- components of the anchoring force of the cone be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Noting that $\mathbf{V_2} = \mathbf{V_1} = \mathbf{V}$ and $\dot{m}_2 = \dot{m}_1 = \dot{m}$, the momentum equations along the x and y axes become

$$F_{Rx} = \dot{m}\mathbf{V}_2\cos\theta - \dot{m}\mathbf{V}_1 = \dot{m}\mathbf{V}(\cos\theta - 1)$$

$$F_{Ry} = 0 \quad \text{(because of symmetry about x axis)}$$



Substituting the given values,

$$F_{Rx} = (58.90\text{ kg/s})(30\text{ m/s})(\cos45° - 1)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{-518\ N}$$

$$F_{Ry} = \mathbf{0}$$

The negative value for F_{Rx} indicates that the assumed direction is wrong, and should be reversed. Therefore, a force of 518 N must be applied to the cone in the opposite direction to flow to hold it in place. No holding force is necessary in the vertical direction due to symmetry and neglecting gravitational effects.

Discussion In reality, the gravitational effects will cause the upper part of flow to slow down and the lower part to speed up after the split. But for short distances, these effects are negligible.

13-66 An ice skater is holding a flexible hose (essentially weightless) which directs a stream of water horizontally at a specified velocity. The velocity and the distance traveled in 5 seconds, and the time it takes to move 5 m and the velocity at that moment are to be determined.

Assumptions **1** Friction between the skates and ice is negligible. **2** The flow of water is steady and one-dimensional (but the motion of skater is unsteady). **3** The ice skating arena is level, and the water jet is discharged horizontally. **4** The mass of the hose and the water in it is negligible. **5** The skater is standing still initially at $t = 0$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) The mass flow rate of water through the hose is

$$\dot{m} = \rho A \mathbf{V} = \rho \frac{\pi D^2}{4}\mathbf{V} = (1000\ \text{kg/m}^3)\frac{\pi(0.02\ \text{m})^2}{4}(10\ \text{m/s}) = 3.14\ \text{kg/s}$$

The thrust exerted on the skater by the water stream is simply the momentum flux of the water stream, and it acts in the reverse direction,

$$F = \text{Thrust} = \dot{m}\mathbf{V} = (3.14\ \text{kg/s})(10\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 31.4\ \text{N}\ \ \text{(constant)}$$

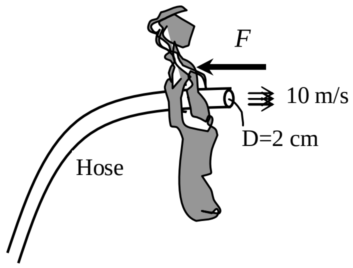
The acceleration of the skater is determined from Newton's 2nd law of motion $F = ma$ where m is the mass of the skater,

$$a = \frac{F}{m} = \frac{31.4\ \text{N}}{60\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 0.523\ \text{m/s}^2$$

Note that thrust and thus the acceleration of the skater is constant. The velocity of the skater and the distance traveled in 5 s are

$$\mathbf{V}_{\text{skater}} = at = (0.523\ \text{m/s}^2)(5\ \text{s}) = \mathbf{2.62\ m/s}$$

$$x = \tfrac{1}{2}at^2 = \tfrac{1}{2}(0.523\ \text{m/s}^2)(5\ \text{s})^2 = \mathbf{6.54\ m}$$



(*b*) The time it will take to move 5 m and the velocity at that moment are

$$x = \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(5\ \text{m})}{0.523\ \text{m/s}^2}} = \mathbf{4.4\ s}$$

$$\mathbf{V}_{\text{skater}} = at = (0.523\ \text{m/s}^2)(4.4\ \text{s}) = \mathbf{2.3\ m/s}$$

Discussion In reality, the velocity of the skater will be lower because of friction on ice and the resistance of the hose to follow the skater.

13-67 Indiana Jones is to ascend a building by building a platform, and mounting four water nozzles pointing down at each corner. The minimum water jet velocity needed to raise the system, the time it will take to rise to the top of the building and the velocity of the system at that moment, the additional rise when the water is shut off, and the time he has to jump from the platform to the roof are to be determined.

Assumptions **1** The air resistance is negligible. **2** The flow of water is steady and one-dimensional (but the motion of platform is unsteady). **3** The platform is still initially at $t = 0$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (a) The total mass flow rate of water through the 4 hoses and the total weight of the platform are

$$\dot{m} = \rho A\mathbf{V} = 4\rho\frac{\pi D^2}{4}\mathbf{V} = 4(1000\ \text{kg/m}^3)\frac{\pi(0.05\ \text{m})^2}{4}(15\ \text{m/s}) = 118\ \text{kg/s}$$

$$W = mg = (150\ \text{kg})(9.81\ \text{m/s}^2)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 1472\ \text{N}$$

We take the platform as the system. The momentum equation for steady one-dimensional flow is $\sum\vec{F} = \sum\dot{m}_e\vec{\mathbf{V}}_e - \sum\dot{m}_i\vec{\mathbf{V}}_i$. The minimum water jet velocity needed to raise the platform is determined by setting the net force acting on the platform equal to zero,

$$-W = \dot{m}(-\mathbf{V}_{\min}) - 0 \quad\rightarrow\quad W = \dot{m}\mathbf{V}_{\min} = \rho A\mathbf{V}_{\min}\mathbf{V}_{\min} = 4\rho\frac{\pi D^2}{4}\mathbf{V}_{\min}^2$$

Solving for $\mathbf{V}_{\min}$ and substituting,

$$\mathbf{V}_{\min} = \sqrt{\frac{W}{\rho\pi D^2}} = \sqrt{\frac{1472\ \text{N}}{(1000\ \text{kg/m}^3)\pi(0.05\ \text{m})^2}\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right)} = \mathbf{13.7\ m/s}$$

(b) We let the vertical reaction force (assumed upwards) acting on the platform be F_{Ry}. Then the momentum equation in the vertical direction becomes

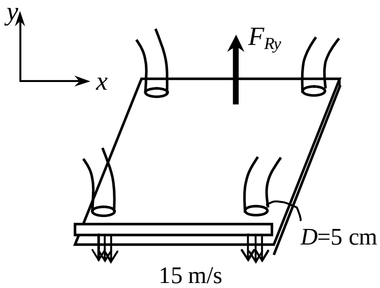
$$F_{Ry} - W = \dot{m}(-\mathbf{V}) - 0 = \dot{m}\mathbf{V} \quad\rightarrow\quad F_{Ry} = W - \dot{m}\mathbf{V} = (1472\ \text{N}) - (118\ \text{kg/s})(15\ \text{m/s})\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right) = -298\ \text{N}$$

The upward thrust acting on the platform is equal and opposite to this reaction force, and thus F = 298 N. Then the acceleration and the ascending time to rise 10 m and the velocity at that moment become

$$a = \frac{F}{m} = \frac{298\ \text{N}}{150\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 2.0\ \text{m/s}^2$$

$$x = \frac{1}{2}at^2 \quad\rightarrow\quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(10\ \text{m})}{2\ \text{m/s}^2}} = \mathbf{3.2\ s}$$

$$\mathbf{V} = at = (2\ \text{m/s}^2)(3.2\ \text{s}) = \mathbf{6.4\ m/s}$$



(c) When water is shut off at 10 m height (where the velocity is 6.4 m/s), the platform will decelerate under the influence of gravity, and the time it takes to come to a stop and the additional rise above 10 m become

$$\mathbf{V} = \mathbf{V}_0 - gt = 0 \quad\rightarrow\quad t = \frac{\mathbf{V}_0}{g} = \frac{6.4\ \text{m/s}}{9.81\ \text{m/s}^2} = \mathbf{0.65\ s}$$

$$x = \mathbf{V}_0 t - \frac{1}{2}gt^2 = (6.4\ \text{m/s})(0.65\ \text{s}) - \frac{1}{2}(9.81\ \text{m/s}^2)(0.65\ \text{s})^2 = \mathbf{2.1\ m}$$

Therefore, Jones has 2×0.65 = 1.3 s to jump off from the platform to the roof since it takes another 0.65 s for the platform to descend to the 10 m level.

13-68E A box-enclosed fan is faced down so the air blast is directed downwards, and it is to be hovered by increasing the blade rpm. The required blade rpm, air exit velocity, the volumetric flow rate, and the minimum mechanical power are to be determined.

Assumptions **1** The flow of air is steady, one-dimensional, and incompressible. **2** The air leaves the blades at a uniform velocity at atmospheric pressure. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects). **5** The change in air pressure with elevation is negligible because of the low density of air. **6** There is no acceleration of the fan, and thus the lift generated is equal to the total weight.

Properties The density of air is given to be 0.078 lbm/ft^3.

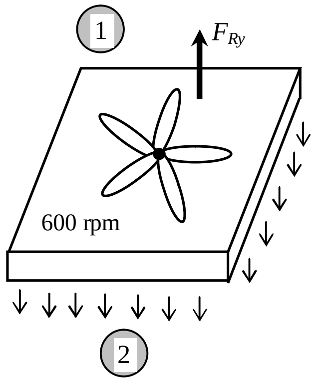
Analysis (*a*) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the *z* axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{V}_e - \sum \dot{m}_i \vec{V}_i$. Noting that the only force acting on the control volume is the total weight *W* and it acts in the negative *z* direction, the momentum equation along the *z* axis gives

$$-W = \dot{m}(-\mathbf{V}_2) - 0 \quad \rightarrow \quad W = \dot{m}\mathbf{V}_2 = (\rho A \mathbf{V}_2)\mathbf{V}_2 = \rho A \mathbf{V}_2^2 \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{\frac{W}{\rho A}}$$

where *A* is the blade span area,

$$A = \pi D^2/4 = \pi(3\text{ ft})^2/4 = 7.069\text{ ft}^2$$



Then the discharge velocity to produce 5 lbf of upward force becomes

$$\mathbf{V}_2 = \sqrt{\frac{5\text{ lbf}}{(0.078\text{ lbm/ft}^3)(7.069\text{ ft}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)} = \mathbf{17.1\ ft/s}$$

(*b*) The volume flow rate and the mass flow rate of air are determined from their definitions,

$$\dot{V} = A\mathbf{V}_2 = (7.069\text{ ft}^2)(17.1\text{ ft/s}) = \mathbf{121\ ft^3/s}$$

$$\dot{m} = \rho\dot{V} = (0.078\text{ lbm/ft}^3)(121\text{ ft}^3/\text{s}) = 9.43\text{ lbm/s}$$

(*c*) Noting that $P_1 = P_2 = P_{atm}$, $\mathbf{V}_1 \cong 0$, the elevation effects are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{fan,u}} = \dot{m}\frac{\mathbf{V}_2^2}{2} = (9.43\text{ lbm/s})\frac{(18.0\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ W}}{0.73756\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{64.3\ W}$$

Therefore, the minimum mechanical power that must be supplied to the air stream is 64.3 W.

Discussion The actual power input to the fan will be considerably larger than the calculated power input because of the fan inefficiency in converting mechanical work to kinetic energy.

13-69 A parachute slows a soldier from his terminal velocity $\mathbf{V}_T$ to his landing velocity of $\mathbf{V}_F$. A relation is to be developed for the soldier's velocity after he opens the parachute at time $t = 0$.

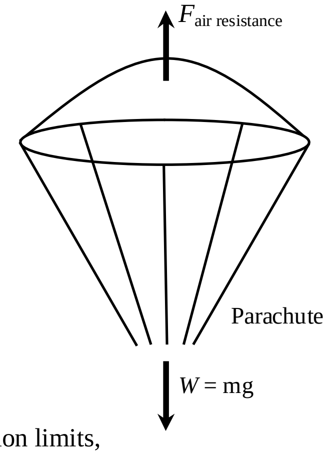
Assumptions **1** The air resistance is proportional to the velocity squared (i.e. $F = -k\mathbf{V}^2$). **2** The variation of the air properties with altitude is negligible. **3** The buoyancy force applied by air to the person (and the parachute) is negligible because of the small volume occupied and the low density of air. **4** The final velocity of the soldier is equal to its terminal velocity with his parachute open.

Analysis The terminal velocity of a free falling object is reached when the air resistance (or air drag) equals the weight of the object, less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_{\text{air resistance}} = W \quad \to \quad k\mathbf{V}_F^2 = mg \quad \to \quad k = \frac{mg}{\mathbf{V}_F^2}$$

This is the desired relation for the constant of proportionality k. When the parachute is deployed and the soldier starts to decelerate, the net downward force acting on him is his weight less the air resistance,

$$F_{\text{net}} = W - F_{\text{air resistance}} = mg - k\mathbf{V}^2 = mg - \frac{mg}{\mathbf{V}_F^2}\mathbf{V}^2 = mg\left(1 - \frac{\mathbf{V}^2}{\mathbf{V}_F^2}\right)$$



Substituting it into Newton's 2nd law relation $F_{\text{net}} = ma = m\frac{d\mathbf{V}}{dt}$ gives

$$mg\left(1 - \frac{\mathbf{V}^2}{\mathbf{V}_F^2}\right) = m\frac{d\mathbf{V}}{dt}$$

Canceling m and separating variables, and integrating from $t = 0$ when $\mathbf{V} = \mathbf{V}_T$ to $t = t$ when $\mathbf{V} = \mathbf{V}$ gives

$$\frac{d\mathbf{V}}{1 - \mathbf{V}^2/\mathbf{V}_F^2} = g\,dt \quad \to \quad \int_{\mathbf{V}_T}^{\mathbf{V}} \frac{d\mathbf{V}}{\mathbf{V}_F^2 - \mathbf{V}^2} = \frac{g}{\mathbf{V}_F^2}\int_0^t dt$$

Using $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\frac{a+x}{a-x}$ from integral tables and applying the integration limits,

$$\frac{1}{2\mathbf{V}_F}\left(\ln\frac{\mathbf{V}_F + \mathbf{V}}{\mathbf{V}_F - \mathbf{V}} - \ln\frac{\mathbf{V}_F + \mathbf{V}_T}{\mathbf{V}_F - \mathbf{V}_T}\right) = \frac{gt}{\mathbf{V}_F^2}$$

Rearranging, the velocity can be expressed explicitly as a function of time as

$$\mathbf{V} = \mathbf{V}_F \frac{\mathbf{V}_T + \mathbf{V}_F + (\mathbf{V}_T - \mathbf{V}_F)e^{-2gt/\mathbf{V}_F}}{\mathbf{V}_T + \mathbf{V}_F - (\mathbf{V}_T - \mathbf{V}_F)e^{-2gt/\mathbf{V}_F}}$$

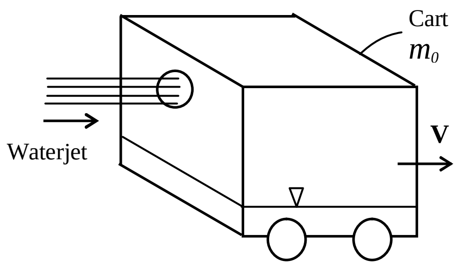
Discussion Note that as $t \to \infty$, the velocity approaches the landing velocity of $\mathbf{V}_F$, as expected.

13-70 An empty cart is to be driven by a horizontal water jet that enters from a hole at the rear of the cart. A relation is to be developed for cart velocity versus time.

Assumptions **1** The flow of water is steady, one-dimensional, incompressible, and horizontal. **2** All the water which enters the cart is retained. **3** The path of the cart is level and frictionless. **4** The cart is initially empty and stationary, and thus **V** = 0 at time t = 0. **5** Friction between water jet and air is negligible, and the entire momentum of water jet is used to drive the cart with no losses.

Analysis We note that the water jet velocity $\mathbf{V}_J$ is constant, but the car velocity **V** is variable. Noting that $\dot{m} = \rho A(\mathbf{V}_J - \mathbf{V})$ where A is the cross-sectional area of the water jet and $\mathbf{V}_J$ - **V** is the velocity of the water jet relative to the cart, the mass of water in the cart at any time t is

$$m_w = \int_0^t \dot{m}dt = \int_0^t \rho A(\mathbf{V}_J - \mathbf{V})dt = \rho A\mathbf{V}_J t - \rho A\int_0^t \mathbf{V}dt \quad (1)$$



Also,

$$\frac{dm_w}{dt} = \dot{m} = \rho A(\mathbf{V}_J - \mathbf{V})$$

We take the cart as the system. The net force acting on the cart in this case is equal to the momentum flux of the water jet. Newton's 2nd law $F = ma = d(m\mathbf{V})/dt$ in this case can be expressed as

$$F = \frac{d(m_{\text{total}}\mathbf{V})}{dt} \quad \text{where} \quad F = \dot{m}(\mathbf{V}_J - \mathbf{V}) = \rho A(\mathbf{V}_J - \mathbf{V})^2$$

and

$$\frac{d(m_{\text{total}}\mathbf{V})}{dt} = \frac{d[(m_c + m_w)\mathbf{V}]}{dt} = m_c\frac{d\mathbf{V}}{dt} + \frac{d(m_w\mathbf{V})}{dt} = m_c\frac{d\mathbf{V}}{dt} + m_w\frac{d\mathbf{V}}{dt} + \mathbf{V}\frac{dm_w}{dt}$$
$$= (m_c + m_w)\frac{d\mathbf{V}}{dt} + \rho A(\mathbf{V}_J - \mathbf{V})\mathbf{V}$$

Substituting,

$$\rho A(\mathbf{V}_J - \mathbf{V})^2 = (m_c + m_w)\frac{d\mathbf{V}}{dt} + \rho A(\mathbf{V}_J - \mathbf{V})\mathbf{V} \quad \rightarrow \quad \rho A(\mathbf{V}_J - \mathbf{V})(\mathbf{V}_J - 2\mathbf{V}) = (m_c + m_w)\frac{d\mathbf{V}}{dt}$$

Noting that m_w is a function of t (as given by Eq. 1) and separating variables,

$$\frac{d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})(\mathbf{V}_J - 2\mathbf{V})} = \frac{dt}{m_c + m_w} \quad \rightarrow \quad \frac{d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})(\mathbf{V}_J - 2\mathbf{V})} = \frac{dt}{m_c + \rho A\mathbf{V}_J t - \rho A\int_0^t \mathbf{V}dt}$$

Integrating from t = 0 when **V** = 0 to t = t when **V** = **V** gives the desired integral,

$$\int_0^{\mathbf{V}} \frac{d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})(\mathbf{V}_J - 2\mathbf{V})} = \int_o^t \frac{dt}{m_c + \rho A\mathbf{V}_J t - \rho A\int_0^t \mathbf{V}dt}$$

Discussion Note that the time integral involves the integral of velocity, which complicates the solution.

13-71 A plate is maintained in a horizontal position by frictionless vertical guide rails. The underside of the plate is subjected to a water jet. The minimum mass flow rate $\dot{m}_{\min}$ to just levitate the plate is to be determined, and a relation is to be obtained for the steady state upward velocity. Also, the integral that relates velocity to time when the water is first turned on is to be obtained.

Assumptions **1** The flow of water is steady and one-dimensional. **2** The water jet splatters in the plane of he plate. **3** The vertical guide rails are frictionless. **4** Times are short, so the velocity of the rising jet can be considered to remain constant with height. **5** At time t = 0, the plate is at rest.

Analysis (*a*) We take the plate as the system. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. Noting that $\dot{m} = \rho A \mathbf{V}_J$ where A is the cross-sectional area of the water jet and $W = m_p g$, the minimum mass flow rate of water needed to raise the plate is determined by setting the net force acting on the plate equal to zero,

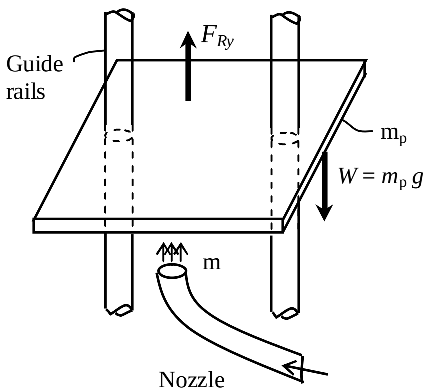
$$-W = 0 - \dot{m}_{\min}\mathbf{V}_{\mathrm{J}} \quad \rightarrow \quad W = \dot{m}_{\min}\mathbf{V}_{\mathrm{J}} \ \rightarrow \quad m_p g = \dot{m}_{\min}(\dot{m}_{\min} / A\mathbf{V}_{\mathrm{J}}) \rightarrow \quad \dot{m}_{\min} = \sqrt{\rho A m_p g}$$

For $\dot{m} > \dot{m}_{\min}$, a relation for the steady state upward velocity **V** is obtained setting the upward impulse applied by water jet to the weight of the plate (during steady motion, the plate velocity **V** is constant, and the velocity of water jet relative to plate is $\mathbf{V}_J - \mathbf{V}$),

$$W = \dot{m}(\mathbf{V}_J - \mathbf{V}) \quad \rightarrow \quad m_p g = \rho A(\mathbf{V}_J - \mathbf{V})^2 \quad \rightarrow \quad \mathbf{V}_J - \mathbf{V} = \sqrt{\frac{m_p g}{\rho A}} \quad \rightarrow \quad \mathbf{V} = \frac{\dot{m}}{\rho A} - \sqrt{\frac{m_p g}{\rho A}}$$

(*b*) At time $t = 0$ the plate is at rest (**V** = 0), and it is subjected to water jet with $\dot{m} > \dot{m}_{\min}$ and thus the net force acting on it is greater than the weight of the plate, and the difference between the jet impulse and the weight will accelerate the plate upwards. Therefore, Newton's 2nd law $F = ma = md\mathbf{V}/dt$ in this case can be expressed as

$$\dot{m}(\mathbf{V}_J - \mathbf{V}) - W = m_p a \quad \rightarrow \quad \rho A(\mathbf{V}_J - \mathbf{V})^2 - m_p g = m_p \frac{d\mathbf{V}}{dt}$$

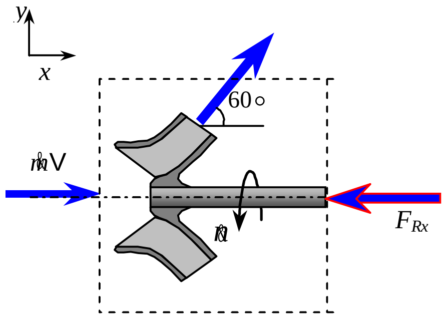


Separating the variables and integrating from $t = 0$ when **V** = 0 to $t = t$ when **V** = **V** gives the desired integral,

$$\int_0^{\mathbf{V}} \frac{m_p d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})^2 - m_p g} = \int_{t=0}^{t} dt \quad \rightarrow \quad t = \int_0^{\mathbf{V}} \frac{m_p d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})^2 - m_p g}$$

Discussion This integral can be performed with the help of integral tables. But the relation obtained will be implicit in **V**.

13-72 Water enters a centrifugal pump axially at a specified rate and velocity, and leaves at an angle from the axial direction. The force acting on the shaft in the axial direction is to be determined.



Properties We take the density of water to be 1000 kg/m^3.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The forces acting on the piping system in the horizontal direction are negligible. **3** The atmospheric pressure is disregarded since it acts on all surfaces.

Analysis From conservation of mass we have $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and thus $\dot{V}_1 = \dot{V}_2$ and $A_{c1}\mathbf{V}_1 = A_{c2}\mathbf{V}_2$. Noting that the discharge area is half the inlet area, the discharge velocity is twice the inlet velocity. That is,

$$A_{c1}\mathbf{V}_2 = \frac{A_{c1}}{A_{c2}}\mathbf{V}_1 = 2\mathbf{V}_1 = 2(5\text{ m/s}) = 10\text{ m/s}$$

We take the pump as the control volume, and the inlet direction of flow as the positive direction of x axis. The momentum equation in this case in the x direction reduces to

$$\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad -F_{Rx} = \dot{m}\mathbf{V}_2\cos\theta - \dot{m}\mathbf{V}_1 \quad \rightarrow \quad F_{Rx} = \dot{m}(\mathbf{V}_1 - \mathbf{V}_2\cos\theta)$$

where the mass flow rate it

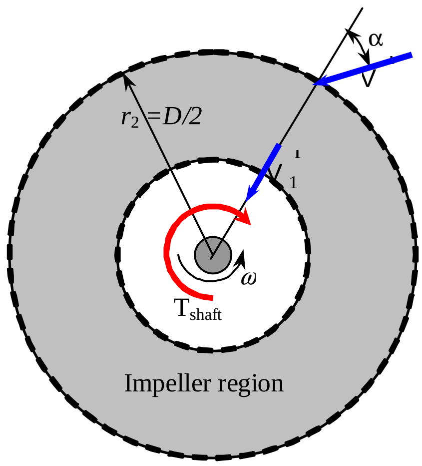
$$\dot{m} = \rho\dot{V} = (1000\text{ kg/m}^3)(0.20\text{ m}^3\text{/s}) = 200\text{ kg/s}$$

Substituting the known quantities, the reaction force is determined to be (note that cos60° = 0.5)

$$F_{\text{Rx}} = (200\text{ kg/s})[(5\text{ m/s}) - (10\text{ m/s})\cos 60]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{0}$$

Discussion Note that at this angle of discharge, the bearing is not subjected to any horizontal loading. Therefore, the loading in the system can be controlled by adjusting the discharge angle.

13-73 Water enters the impeller of a turbine through its outer edge of diameter D with velocity V making an angle α with the radial direction at a mass flow rate of $\dot{m}$, and leaves the impeller in the radial direction. The maximum power that can be generated is to be shown to be $\dot{W}_{\text{shaft}} = \pi \dot{n} \dot{m} D V \sin\alpha$.



Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.

Analysis We take the impeller region as the control volume. The tangential velocity components at the inlet and the outlet are $V_{1,t} = 0$ and $V_{2,t} = V \sin\alpha$.

Normal velocity components as well pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque. Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

$$T_{\text{shaft}} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = \dot{m} r_2 V_{2,t} - 0 = \dot{m} D (V \sin\alpha)/2$$

The angular velocity of the propeller is $\omega = 2\pi\dot{n}$. Then the shaft power becomes

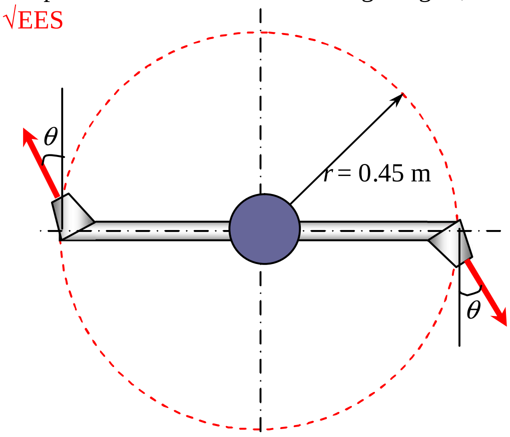
$$\dot{W}_{\text{shaft}} = \omega T_{\text{shaft}} = 2\pi\dot{n}\dot{m} D (V \sin\alpha)/2$$

Simplifying, the maximum power generated becomes

$$\dot{W}_{\text{shaft}} = \pi\dot{n}\dot{m} D V \sin\alpha$$

which is the desired relation.

13-74 A two-armed sprinkler is used to water a garden. For specified flow rate and discharge angles, the rates of rotation of the sprinkler head are to be determined. √EES



Assumptions **1** The flow is uniform and cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle exit is zero. **3** Frictional effects and air drag of rotating components are neglected.

Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Noting that the two nozzles are identical, we have $\dot{m}_{\text{nozzle}} = \dot{m}/2$ or $\dot{V}_{\text{nozzle}} = \dot{V}/2$ since the density of water is constant. The average jet exit velocity relative to the nozzle is

$$V_{\text{jet}} = \frac{\dot{V}_{\text{nozzle}}}{A_{\text{jet}}} = \frac{60\text{ L/s}}{2[\pi(0.02\text{ m})^2/4]}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right) = 95.49\text{ m/s}$$

The angular momentum equation can be expressed as $\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$. Noting that there are no external moments acting, the angular momentum equation about the axis of rotation becomes

$$0 = -2r\dot{m}_{\text{nozzle}}V_r \cos\theta \quad \rightarrow \quad V_r = 0 \quad \rightarrow \quad V_{\text{jet,t}} - V_{\text{nozzle}} = 0$$

Noting that the tangential component of jet velocity is $V_{\text{jet,t}} = V_{\text{jet}}\cos\theta$, we have

$$V_{\text{nozzle}} = V_{\text{jet}}\cos\theta = (95.49\text{ m/s})\cos\theta$$

Also noting that $V_{\text{nozzle}} = \omega r = 2\pi\dot{n}r$, and angular speed and the rate of rotation of sprinkler head become

1) θ = 0°: $\omega = \frac{V_{\text{nozzle}}}{r} = \frac{(95.49\text{ m/s})\cos 0}{0.45\text{ m}} = \mathbf{212\ rad/s}$ and $\dot{n} = \frac{\omega}{2\pi} = \frac{212\text{ rad/s}}{2\pi}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \mathbf{2026\ rpm}$

2) θ = 30°: $\omega = \frac{V_{\text{nozzle}}}{r} = \frac{(95.49\text{ m/s})\cos 30°}{0.45\text{ m}} = \mathbf{184\ rad/s}$ and $\dot{n} = \frac{\omega}{2\pi} = \frac{184\text{ rad/s}}{2\pi}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \mathbf{1755\ rpm}$

3) θ = 60°: $\omega = \frac{V_{\text{nozzle}}}{r} = \frac{(95.49\text{ m/s})\cos 60°}{0.45\text{ m}} = \mathbf{106\ rad/s}$ and $\dot{n} = \frac{\omega}{2\pi} = \frac{106\text{ rad/s}}{2\pi}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \mathbf{1013\ rpm}$

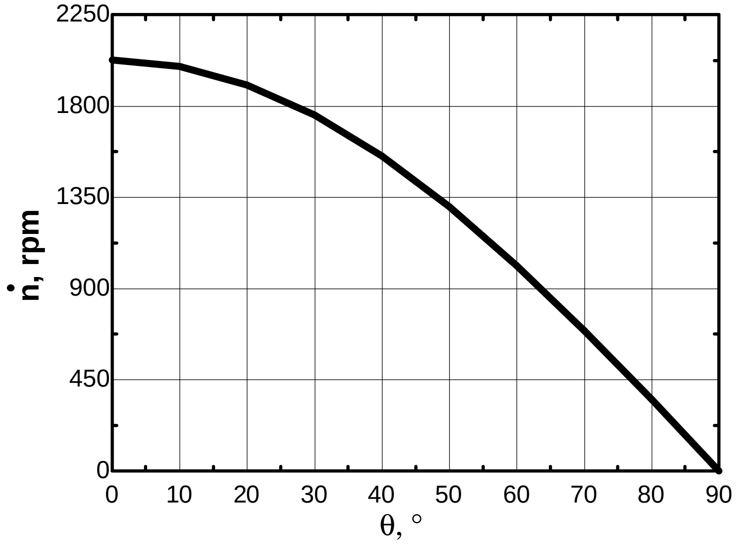
Discussion The rate of rotation in reality will be lower because of frictional effects and air drag.

13-75 Problem 13-74 is reconsidered. The effect of discharge angle θ on the rate of rotation $\dot{n}$ as θ varies from 0 to 90° in increments of 10° is to be investigated.

```
D=0.02 "m"
r=0.45 "m"
n_nozzle=2 "number of nozzles"
Ac=pi*D^2/4
V_jet=V_dot/Ac/n_nozzle

V_nozzle=V_jet*cos(theta)
V_dot=0.060 "m3/s"
omega=V_nozzle/r
n_dot=omega*60/(2*pi)
```

Angle, θ°	V_{nozzle}, m/s	ω rad/s	$\dot{n}$ rpm
0	95.5	212	2026
10	94.0	209	1996
20	89.7	199	1904
30	82.7	184	1755
40	73.2	163	1552
50	61.4	136	1303
60	47.7	106	1013
70	32.7	73	693
80	16.6	37	352
90	0.0	0	0



13-76 A stationary water tank placed on wheels on a frictionless surface is propelled by a water jet that leaves the tank through a smooth hole. Relations are to be developed for the acceleration, the velocity, and the distance traveled by the tank as a function of time as water discharges.

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). **3** The surface under the wheeled tank is level and frictionless. **4** The water jet is discharged horizontally and rearward. **5** The mass of the tank and wheel assembly is negligible compared to the mass of water in the tank.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the hole, which is also taken to be the reference level ($z_2 = 0$) so that the water height above the hole at any time is z. Noting that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), it is open to the atmosphere ($P_1 = P_{atm}$), and water discharges into the atmosphere (and thus $P_2 = P_{atm}$), the Bernoulli equation simplifies to

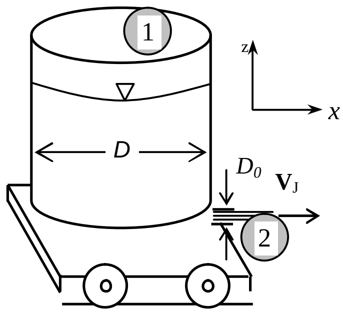
$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 \quad \rightarrow \quad z = \frac{\mathbf{V}_J^2}{2g} + 0 \quad \rightarrow \quad \mathbf{V}_J = \sqrt{2gz}$$

The discharge rate of water from the tank through the hole is

$$\dot{m} = \rho A \mathbf{V}_J = \rho \frac{\pi D_0^2}{4} \mathbf{V}_J = \rho \frac{\pi D_0^2}{4} \sqrt{2gz}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum \dot{m}_e \vec{\mathbf{V}}_e - \sum \dot{m}_i \vec{\mathbf{V}}_i$. Applying it to the water tank, the horizontal force that acts on the tank is determined to be

$$F = \dot{m}\mathbf{V}_e - 0 = \dot{m}\mathbf{V}_J = \rho \frac{\pi D_0^2}{4} 2gz = \rho g z \frac{\pi D_0^2}{2}$$



The acceleration of the water tank is determined from Newton's 2nd law of motion $F = ma$ where m is the mass of water in the tank, $m = \rho V_{tank} = \rho(\pi D^2/4)z$,

$$a = \frac{F}{m} = \frac{\rho g z(\pi D_0^2/2)}{\rho z(\pi D^2/4)} \quad \rightarrow \quad a = 2g\frac{D_0^2}{D^2}$$

Note that the acceleration of the tank is constant.

(*b*) Noting that $a = d\mathbf{V}/dt$ and thus $d\mathbf{V} = adt$ and acceleration a is constant, the velocity is expressed as

$$\mathbf{V} = at \quad \rightarrow \quad \mathbf{V} = 2g\frac{D_0^2}{D^2}t$$

(*c*) Noting that $\mathbf{V} = dx/dt$ and thus $dx = \mathbf{V}dt$, the distance traveled by the water tank is determined by integration to be

$$dx = \mathbf{V}dt \quad \rightarrow \quad dx = 2g\frac{D_0^2}{D^2}tdt \quad \rightarrow \quad x = g\frac{D_0^2}{D^2}t^2$$

since $x = 0$ at $t = 0$.

Discussion In reality, the flow rate discharge velocity and thus the force acting on the water tank will be less because of the frictional losses at the hole. But these losses can be accounted for by incorporating a discharge coefficient.

13-77 … Design and Essay Problems

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