

## CHAPTER

## 20

## NATURAL CONVECTION

In Chap. 19, we considered heat transfer by *forced convection*, where a fluid was *forced* to move over a surface or in a tube by external means such as a pump or a fan. In this chapter, we consider *natural convection*, where any fluid motion occurs by natural means such as buoyancy. The fluid motion in forced convection is quite *noticeable*, since a fan or a pump can transfer enough momentum to the fluid to move it in a certain direction. The fluid motion in natural convection, however, is often not noticeable because of the low velocities involved.

The convection heat transfer coefficient is a strong function of *velocity*: the higher the velocity, the higher the convection heat transfer coefficient. The fluid velocities associated with natural convection are low, typically less than 1 m/s. Therefore, the heat transfer coefficients encountered in natural convection are usually much lower than those encountered in forced convection. Yet several types of heat transfer equipment are designed to operate under natural convection conditions instead of forced convection, because natural convection does not require the use of a fluid mover.

We start this chapter with a discussion of the physical mechanism of *natural convection* and the *Grashof number*. We then present the correlations to evaluate heat transfer by natural convection for various geometries, including finned surfaces and enclosures. Finally, we discuss simultaneous forced and natural convection.

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## 20-1 ■ PHYSICAL MECHANISM OF NATURAL CONVECTION

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipment such as power transistors, TVs, and VCRs; heat transfer from electric baseboard heaters or steam radiators; heat transfer from the refrigeration coils and power transmission lines; and heat transfer from the bodies of animals and human beings. Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

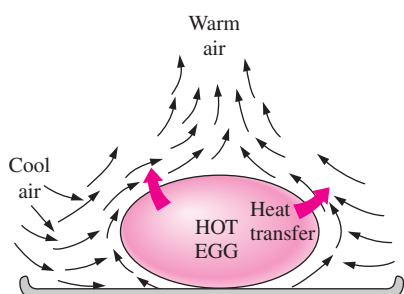
We know that a hot boiled egg (or a hot baked potato) on a plate eventually cools to the surrounding air temperature (Fig. 20-1). The egg is cooled by transferring heat by convection to the air and by radiation to the surrounding surfaces. Disregarding heat transfer by radiation, the physical mechanism of cooling a hot egg (or any hot object) in a cooler environment can be explained as follows:

As soon as the hot egg is exposed to cooler air, the temperature of the outer surface of the egg shell will drop somewhat, and the temperature of the air adjacent to the shell will rise as a result of heat conduction from the shell to the air. Consequently, the egg will soon be surrounded by a thin layer of warmer air, and heat will then be transferred from this warmer layer to the outer layers of air. The cooling process in this case would be rather slow since the egg would always be blanketed by warm air, and it would have no direct contact with the cooler air farther away. We may not notice any air motion in the vicinity of the egg, but careful measurements indicate otherwise.

The temperature of the air adjacent to the egg is higher, and thus its density is lower, since at constant pressure the density of a gas is inversely proportional to its temperature. Thus, we have a situation in which some low-density or “light” gas is surrounded by a high-density or “heavy” gas, and the natural laws dictate that *the light gas rise*. This is no different than the oil in a vinegar-and-oil salad dressing rising to the top (since  $\rho_{\text{oil}} < \rho_{\text{vinegar}}$ ). This phenomenon is characterized incorrectly by the phrase “heat rises,” which is understood to mean *heated air rises*. The space vacated by the warmer air in the vicinity of the egg is replaced by the cooler air nearby, and the presence of cooler air in the vicinity of the egg speeds up the cooling process. The rise of warmer air and the flow of cooler air into its place continues until the egg is cooled to the temperature of the surrounding air. The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a **natural convection current**, and the heat transfer that is enhanced as a result of this natural convection current is called **natural convection heat transfer**. Note that in the absence of natural convection currents, heat transfer from the egg to the air surrounding it would be by conduction only, and the rate of heat transfer from the egg would be much lower.

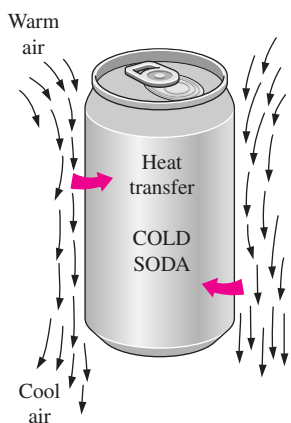
Natural convection is just as effective in the heating of cold surfaces in a warmer environment as it is in the cooling of hot surfaces in a cooler environment, as shown in Fig. 20-2. Note that the direction of fluid motion is reversed in this case.

In a gravitational field, there is a net force that pushes upward a light fluid placed in a heavier fluid. The upward force exerted by a fluid on a body



**FIGURE 20-1**

The cooling of a boiled egg in a cooler environment by natural convection.



**FIGURE 20-2**

The warming up of a cold drink in a warmer environment by natural convection.

completely or partially immersed in it is called the **buoyancy force**. The magnitude of the buoyancy force is equal to the weight of the *fluid displaced* by the body. That is,

$$F_{\text{buoyancy}} = \rho_{\text{fluid}} g V_{\text{body}} \quad (20-1)$$

where  $\rho_{\text{fluid}}$  is the average density of the *fluid* (not the body),  $g$  is the gravitational acceleration, and  $V_{\text{body}}$  is the volume of the portion of the body immersed in the fluid (for bodies completely immersed in the fluid, it is the total volume of the body). In the absence of other forces, the net vertical force acting on a body is the difference between the weight of the body and the buoyancy force. That is,

$$\begin{aligned} F_{\text{net}} &= W - F_{\text{buoyancy}} \\ &= \rho_{\text{body}} g V_{\text{body}} - \rho_{\text{fluid}} g V_{\text{body}} \\ &= (\rho_{\text{body}} - \rho_{\text{fluid}}) g V_{\text{body}} \end{aligned} \quad (20-2)$$

Note that this force is proportional to the difference in the densities of the fluid and the body immersed in it. Thus, a body immersed in a fluid will experience a “weight loss” in an amount equal to the weight of the fluid it displaces. This is known as *Archimedes’ principle*.

To have a better understanding of the buoyancy effect, consider an egg dropped into water. If the average density of the egg is greater than the density of water (a sign of freshness), the egg will settle at the bottom of the container. Otherwise, it will rise to the top. When the density of the egg equals the density of water, the egg will settle somewhere in the water while remaining completely immersed, acting like a “weightless object” in space. This occurs when the upward buoyancy force acting on the egg equals the weight of the egg, which acts downward.

The *buoyancy effect* has far-reaching implications in life. For one thing, without buoyancy, heat transfer between a hot (or cold) surface and the fluid surrounding it would be by *conduction* instead of by *natural convection*. The natural convection currents encountered in the oceans, lakes, and the atmosphere owe their existence to buoyancy. Also, light boats as well as heavy warships made of steel float on water because of buoyancy (Fig. 20–3). Ships are designed on the basis of the principle that the entire weight of a ship and its contents is equal to the weight of the water that the submerged volume of the ship can contain. The “chimney effect” that induces the upward flow of hot combustion gases through a chimney is also due to the buoyancy effect, and the upward force acting on the gases in the chimney is proportional to the difference between the densities of the hot gases in the chimney and the cooler air outside. Note that there is *no gravity* in space, and thus there can be no natural convection heat transfer in a spacecraft, even if the spacecraft is filled with atmospheric air.

In heat transfer studies, the primary variable is *temperature*, and it is desirable to express the net buoyancy force (Eq. 20–2) in terms of temperature differences. But this requires expressing the density difference in terms of a temperature difference, which requires a knowledge of a property that represents the *variation of the density of a fluid with temperature at constant pressure*. The property that provides that information is the **volume expansion coefficient**  $\beta$ , defined as (Fig. 20–4)

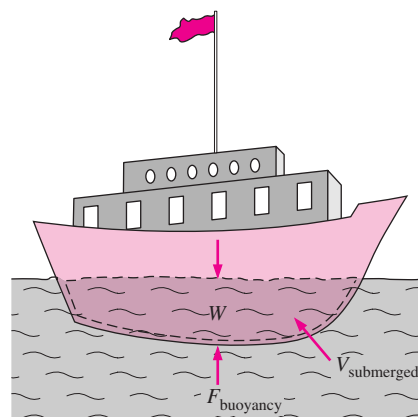
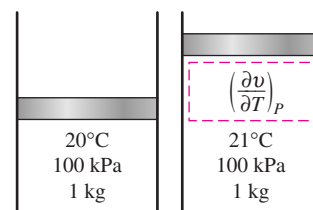
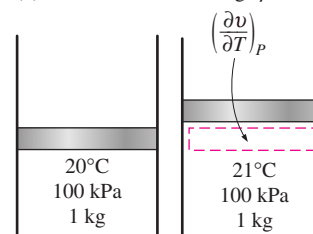


FIGURE 20–3

It is the buoyancy force that keeps the ships afloat in water ( $W = F_{\text{buoyancy}}$  for floating objects).



(a) A substance with a large  $\beta$



(b) A substance with a small  $\beta$

FIGURE 20–4

The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

$$\beta = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_P = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_P \quad (1/\text{K}) \quad (20-3)$$

In natural convection studies, the condition of the fluid sufficiently far from the hot or cold surface is indicated by the subscript “infinity” to serve as a reminder that this is the value at a distance where the presence of the surface is not felt. In such cases, the volume expansion coefficient can be expressed approximately by replacing differential quantities by differences as

$$\beta \approx -\frac{1}{\rho} \frac{\Delta \rho}{\Delta T} = -\frac{1}{\rho} \frac{\rho_\infty - \rho}{T_\infty - T} \quad (\text{at constant } P) \quad (20-4)$$

or

$$\rho_\infty - \rho = \rho \beta (T - T_\infty) \quad (\text{at constant } P) \quad (20-5)$$

where  $\rho_\infty$  is the density and  $T_\infty$  is the temperature of the quiescent fluid away from the surface.

We can show easily that the volume expansion coefficient  $\beta$  of an *ideal gas* ( $P = \rho RT$ ) at a temperature  $T$  is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \quad (1/\text{K}) \quad (20-6)$$

where  $T$  is the *absolute* temperature. Note that a large value of  $\beta$  for a fluid means a large change in density with temperature, and that the product  $\beta \Delta T$  represents the fraction of volume change of a fluid that corresponds to a temperature change  $\Delta T$  at constant pressure. Also note that the buoyancy force is proportional to the *density difference*, which is proportional to the *temperature difference* at constant pressure. Therefore, the larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger* the buoyancy force and the *stronger* the natural convection currents, and thus the *higher* the heat transfer rate.

The magnitude of the natural convection heat transfer between a surface and a fluid is directly related to the *flow rate* of the fluid. The higher the flow rate, the higher the heat transfer rate. In fact, it is the very high flow rates that increase the heat transfer coefficient by orders of magnitude when forced convection is used. In natural convection, no blowers are used, and therefore the flow rate cannot be controlled externally. The flow rate in this case is established by the dynamic balance of *buoyancy* and *friction*.

As we have discussed earlier, the buoyancy force is caused by the density difference between the heated (or cooled) fluid adjacent to the surface and the fluid surrounding it, and is proportional to this density difference and the volume occupied by the warmer fluid. It is also well known that whenever two bodies in contact (solid–solid, solid–fluid, or fluid–fluid) move relative to each other, a *friction force* develops at the contact surface in the direction opposite to that of the motion. This opposing force slows down the fluid and thus reduces the flow rate of the fluid. Under steady conditions, the airflow rate driven by buoyancy is established at the point where these two effects *balance* each other. The friction force increases as more and more solid surfaces are introduced, seriously disrupting the fluid flow and heat transfer. For that reason, heat sinks with closely spaced fins are not suitable for natural convection cooling.

Most heat transfer correlations in natural convection are based on experimental measurements. The instrument often used in natural convection

experiments is the *Mach–Zehnder interferometer*, which gives a plot of isotherms in the fluid in the vicinity of a surface. The operation principle of interferometers is based on the fact that at low pressure, the lines of constant temperature for a gas correspond to the lines of constant density, and that the index of refraction of a gas is a function of its density. Therefore, the degree of refraction of light at some point in a gas is a measure of the temperature gradient at that point. An interferometer produces a map of interference fringes, which can be interpreted as lines of *constant temperature* as shown in Fig. 20–5. The smooth and parallel lines in (a) indicate that the flow is *laminar*, whereas the eddies and irregularities in (b) indicate that the flow is *turbulent*. Note that the lines are closest near the surface, indicating a *higher temperature gradient*.

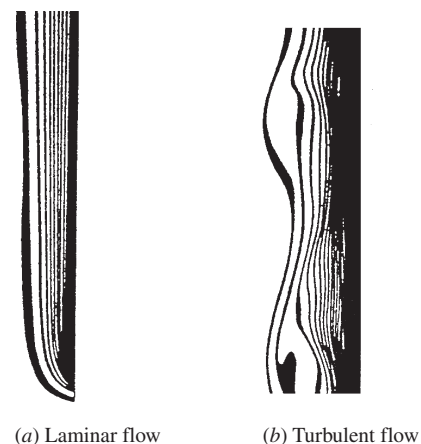
## 20–2 ■ EQUATION OF MOTION AND THE GRASHOF NUMBER

In this section we derive the equation of motion that governs the natural convection flow in laminar boundary layer. The conservation of mass and energy equations derived in Chap. 6 for forced convection are also applicable for natural convection, but the momentum equation needs to be modified to incorporate buoyancy.

Consider a vertical hot flat plate immersed in a quiescent fluid body. We assume the natural convection flow to be steady, laminar, and two-dimensional, and the fluid to be Newtonian with constant properties, including density, with one exception: the density difference  $\rho - \rho_\infty$  is to be considered since it is this density difference between the inside and the outside of the boundary layer that gives rise to buoyancy force and sustains flow. (This is known as the *Boussinesq approximation*.) We take the upward direction along the plate to be  $x$ , and the direction normal to surface to be  $y$ , as shown in Fig. 20–6. Therefore, gravity acts in the  $-x$ -direction. Noting that the flow is steady and two-dimensional, the  $x$ - and  $y$ -components of velocity within boundary layer are  $u = u(x, y)$  and  $v = v(x, y)$ , respectively.

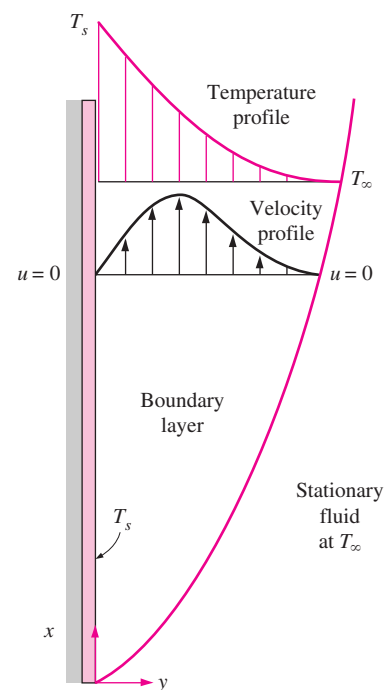
The velocity and temperature profiles for natural convection over a vertical hot plate are also shown in Fig. 20–6. Note that as in forced convection, the thickness of the boundary layer increases in the flow direction. Unlike forced convection, however, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate. This is expected since the fluid beyond the boundary layer is motionless. Thus, the fluid velocity increases with distance from the surface, reaches a maximum, and gradually decreases to zero at a distance sufficiently far from the surface. At the surface, the fluid temperature is equal to the plate temperature, and gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface, as shown in the figure. In the case of *cold surfaces*, the shape of the velocity and temperature profiles remains the same but their direction is reversed.

Consider a differential volume element of height  $dx$ , length  $dy$ , and unit depth in the  $z$ -direction (normal to the paper) for analysis. The forces acting on this volume element are shown in Fig. 20–7. Newton's second law of motion for this control volume can be expressed as



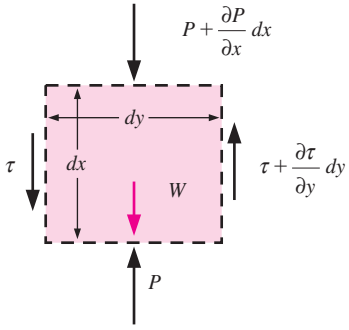
**FIGURE 20–5**

Isotherms in natural convection over a hot plate in air.



**FIGURE 20–6**

Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature  $T_s$  inserted in a fluid at temperature  $T_\infty$ .

**FIGURE 20-7**

Forces acting on a differential control volume in the natural convection boundary layer over a vertical flat plate.

$$\delta m \cdot a_x = F_x \quad (20-7)$$

where  $\delta m = \rho(dx \cdot dy \cdot 1)$  is the mass of the fluid element within the control volume. The acceleration in the  $x$ -direction is obtained by taking the total differential of  $u(x, y)$ , which is  $du = (\partial u/\partial x)dx + (\partial u/\partial y)dy$ , and dividing it by  $dt$ . We get

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \quad (20-8)$$

The forces acting on the differential volume element in the vertical direction are the pressure forces acting on the top and bottom surfaces, the shear stresses acting on the side surfaces (the normal stresses acting on the top and bottom surfaces are small and are disregarded), and the force of gravity acting on the entire volume element. Then the net surface force acting in the  $x$ -direction becomes

$$\begin{aligned} F_x &= \left( \frac{\partial \tau}{\partial y} dy \right) (dx \cdot 1) - \left( \frac{\partial P}{\partial x} dx \right) (dy \cdot 1) - \rho g (dx \cdot dy \cdot 1) \\ &= \left( \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g \right) (dx \cdot dy \cdot 1) \end{aligned} \quad (20-9)$$

since  $\tau = \mu(\partial u/\partial y)$ . Substituting Eqs. 20-8 and 20-9 into Eq. 20-7 and dividing by  $\rho \cdot dx \cdot dy \cdot 1$  gives the *conservation of momentum* in the  $x$ -direction as

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g \quad (20-10)$$

The  $x$ -momentum equation in the quiescent fluid outside the boundary layer can be obtained from the relation above as a special case by setting  $u = 0$ . It gives

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g \quad (20-11)$$

which is simply the relation for the variation of hydrostatic pressure in a quiescent fluid with height, as expected. Also, noting that  $v \ll u$  in the boundary layer and thus  $\partial v/\partial x \approx \partial v/\partial y \approx 0$ , and that there are no body forces (including gravity) in the  $y$ -direction, the force balance in that direction gives  $\partial P/\partial y = 0$ . That is, the variation of pressure in the direction normal to the surface is negligible, and for a given  $x$  the pressure in the boundary layer is equal to the pressure in the quiescent fluid. Therefore,  $P = P(x) = P_\infty(x)$  and  $\partial P/\partial x = \partial P_\infty/\partial x = -\rho_\infty g$ . Substituting into Eq. 20-10,

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g \quad (20-12)$$

The last term represents the net upward force per unit volume of the fluid (the difference between the buoyant force and the fluid weight). This is the force that initiates and sustains convection currents.

From Eq. 20-5, we have  $\rho_\infty - \rho = \rho\beta(T - T_\infty)$ . Substituting it into the last equation and dividing both sides by  $\rho$  gives the desired form of the  $x$ -momentum equation,



$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \quad (20-13)$$

This is the equation that governs the fluid motion in the boundary layer due to the effect of buoyancy. Note that the momentum equation involves the temperature, and thus the momentum and energy equations must be solved simultaneously.

The set of three partial differential equations (the continuity, momentum, and the energy equations) that govern natural convection flow over vertical isothermal plates can be reduced to a set of two ordinary nonlinear differential equations by the introduction of a similarity variable. But the resulting equations must still be solved numerically [Ostrach (1953)]. Interested readers are referred to advanced books on the topic for detailed discussions [e.g., Kays and Crawford (1993)].

## The Grashof Number

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities: all lengths by a characteristic length  $L_c$ , all velocities by an arbitrary reference velocity  $\mathcal{V}$  (which, from the definition of Reynolds number, is taken to be  $\mathcal{V} = \text{Re}_L v/L_c$ ), and temperature by a suitable temperature difference (which is taken to be  $T_s - T_\infty$ ) as

$$x^* = \frac{x}{L_c} \quad y^* = \frac{y}{L_c} \quad u^* = \frac{u}{\mathcal{V}} \quad v^* = \frac{v}{\mathcal{V}} \quad \text{and} \quad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

where asterisks are used to denote nondimensional variables. Substituting them into the momentum equation and simplifying give

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[ \frac{g\beta(T_s - T_\infty)L_c^3}{v^2} \right] \frac{T^*}{\text{Re}_L^2} + \frac{1}{\text{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (20-14)$$

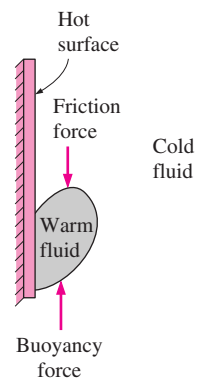
The dimensionless parameter in the brackets represents the natural convection effects, and is called the **Grashof number**  $\text{Gr}_L$ ,

$$\text{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{v^2} \quad (20-15)$$

where

- $g$  = gravitational acceleration, m/s<sup>2</sup>
- $\beta$  = coefficient of volume expansion, 1/K ( $\beta = 1/T$  for ideal gases)
- $T_s$  = temperature of the surface, °C
- $T_\infty$  = temperature of the fluid sufficiently far from the surface, °C
- $L_c$  = characteristic length of the geometry, m
- $v$  = kinematic viscosity of the fluid, m<sup>2</sup>/s

We mentioned in the preceding chapters that the flow regime in forced convection is governed by the dimensionless *Reynolds number*, which represents the ratio of inertial forces to viscous forces acting on the fluid. The flow regime in natural convection is governed by the dimensionless *Grashof number*, which represents the ratio of the *buoyancy force* to the *viscous force* acting on the fluid (Fig. 20–8).



**FIGURE 20–8**

The Grashof number  $\text{Gr}$  is a measure of the relative magnitudes of the *buoyancy force* and the opposing *viscous force* acting on the fluid.

The role played by the Reynolds number in forced convection is played by the Grashof number in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection. For vertical plates, for example, the critical Grashof number is observed to be about  $10^9$ . Therefore, the flow regime on a vertical plate becomes *turbulent* at Grashof numbers greater than  $10^9$ .

When a surface is subjected to external flow, the problem involves both natural and forced convection. The relative importance of each mode of heat transfer is determined by the value of the coefficient  $Gr_L/Re_L^2$ : Natural convection effects are negligible if  $Gr_L/Re_L^2 \ll 1$ , free convection dominates and the forced convection effects are negligible if  $Gr_L/Re_L^2 \gg 1$ , and both effects are significant and must be considered if  $Gr_L/Re_L^2 \approx 1$ .

## 20-3 ■ NATURAL CONVECTION OVER SURFACES

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid involved.

Although we understand the mechanism of natural convection well, the complexities of fluid motion make it very difficult to obtain simple analytical relations for heat transfer by solving the governing equations of motion and energy. Some analytical solutions exist for natural convection, but such solutions lack generality since they are obtained for simple geometries under some simplifying assumptions. Therefore, with the exception of some simple cases, heat transfer relations in natural convection are based on experimental studies. Of the numerous such correlations of varying complexity and claimed accuracy available in the literature for any given geometry, we present here the ones that are best known and widely used.

The simple empirical correlations for the average *Nusselt number*  $Nu$  in natural convection are of the form (Fig. 20-9)

$$Nu = \frac{hL_c}{k} = C(Gr_L Pr)^n = C Ra_L^n \quad (20-16)$$

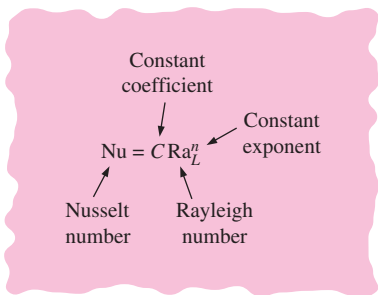
where  $Ra_L$  is the **Rayleigh number**, which is the product of the Grashof and Prandtl numbers:

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr \quad (20-17)$$

The values of the constants  $C$  and  $n$  depend on the *geometry* of the surface and the *flow regime*, which is characterized by the range of the Rayleigh number. The value of  $n$  is usually  $\frac{1}{4}$  for laminar flow and  $\frac{1}{3}$  for turbulent flow. The value of the constant  $C$  is normally less than 1.

Simple relations for the average Nusselt number for various geometries are given in Table 20-1, together with sketches of the geometries. Also given in this table are the characteristic lengths of the geometries and the ranges of Rayleigh number in which the relation is applicable. All fluid properties are to be evaluated at the film temperature  $T_f = \frac{1}{2}(T_s + T_\infty)$ .

When the average Nusselt number and thus the average convection coefficient is known, the rate of heat transfer by natural convection from a solid



**FIGURE 20-9**

Natural convection heat transfer correlations are usually expressed in terms of the Rayleigh number raised to a constant  $n$  multiplied by another constant  $C$ , both of which are determined experimentally.



surface at a uniform temperature  $T_s$  to the surrounding fluid is expressed by Newton's law of cooling as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \quad (\text{W}) \quad (20-18)$$

where  $A_s$  is the heat transfer surface area and  $h$  is the average heat transfer coefficient on the surface.

### Vertical Plates ( $T_s = \text{constant}$ )

For a vertical flat plate, the characteristic length is the plate height  $L$ . In Table 20-1 we give three relations for the average Nusselt number for an isothermal vertical plate. The first two relations are very simple. Despite its complexity, we suggest using the third one (Eq. 20-21) recommended by Churchill and Chu (1975) since it is applicable over the entire range of Rayleigh number. This relation is most accurate in the range of  $10^{-1} < \text{Ra}_L < 10^9$ .

### Vertical Plates ( $\dot{q}_s = \text{constant}$ )

In the case of constant surface heat flux, the rate of heat transfer is known (it is simply  $\dot{Q} = \dot{q}_s A_s$ ), but the surface temperature  $T_s$  is not. In fact,  $T_s$  increases with height along the plate. It turns out that the Nusselt number relations for the constant surface temperature and constant surface heat flux cases are nearly identical [Churchill and Chu (1975)]. Therefore, the relations for isothermal plates can also be used for plates subjected to uniform heat flux, provided that the plate midpoint temperature  $T_{L/2}$  is used for  $T_s$  in the evaluation of the film temperature, Rayleigh number, and the Nusselt number. Noting that  $h = \dot{q}_s / (T_{L/2} - T_\infty)$ , the average Nusselt number in this case can be expressed as

$$\text{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \quad (20-19)$$

The midpoint temperature  $T_{L/2}$  is determined by iteration so that the Nusselt numbers determined from Eqs. 20-21 and 20-19 match.

### Vertical Cylinders

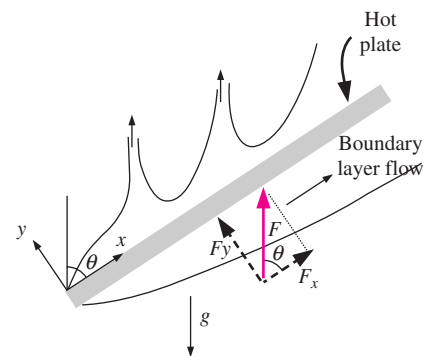
An outer surface of a vertical cylinder can be treated as a vertical plate when the diameter of the cylinder is sufficiently large so that the curvature effects are negligible. This condition is satisfied if

$$D \geq \frac{35L}{\text{Gr}_L^{1/4}} \quad (20-20)$$

When this criteria is met, the relations for vertical plates can also be used for vertical cylinders. Nusselt number relations for slender cylinders that do not meet this criteria are available in the literature [e.g., Cebeci (1974)].

### Inclined Plates

Consider an inclined hot plate that makes an angle  $\theta$  from the vertical, as shown in Fig. 20-10, in a cooler environment. The net force  $F = g(\rho_\infty - \rho)$  (the difference between the buoyancy and gravity) acting on a unit volume of the fluid in the boundary layer is always in the vertical direction. In the case



**FIGURE 20-10**  
Natural convection flows on the upper and lower surfaces of an inclined hot plate.



of inclined plate, this force can be resolved into two components:  $F_y = F \cos \theta$  parallel to the plate that drives the flow along the plate, and  $F_y = F \sin \theta$  normal to the plate. Noting that the force that drives the motion is reduced, we expect the convection currents to be weaker, and the rate of heat transfer to be lower relative to the vertical plate case.

The experiments confirm what we suspect for the lower surface of a hot plate, but the opposite is observed on the upper surface. The reason for this curious behavior for the upper surface is that the force component  $F_y$  initiates upward motion in addition to the parallel motion along the plate, and thus the boundary layer breaks up and forms plumes, as shown in the figure. As a result, the thickness of the boundary layer and thus the resistance to heat transfer decreases, and the rate of heat transfer increases relative to the vertical orientation.

In the case of a cold plate in a warmer environment, the opposite occurs as expected: The boundary layer on the upper surface remains intact with weaker boundary layer flow and thus lower rate of heat transfer, and the boundary layer on the lower surface breaks apart (the colder fluid falls down) and thus enhances heat transfer.

When the boundary layer remains intact (the lower surface of a hot plate or the upper surface of a cold plate), the Nusselt number can be determined from the vertical plate relations provided that  $g$  in the Rayleigh number relation is replaced by  $g \cos \theta$  for  $\theta < 60^\circ$ . Nusselt number relations for the other two surfaces (the upper surface of a hot plate or the lower surface of a cold plate) are available in the literature [e.g., Fujii and Imura (1972)].

## Horizontal Plates

The rate of heat transfer to or from a horizontal surface depends on whether the surface is facing upward or downward. For a hot surface in a cooler environment, the net force acts upward, forcing the heated fluid to rise. If the hot surface is facing upward, the heated fluid rises freely, inducing strong natural convection currents and thus effective heat transfer, as shown in Fig. 20–11. But if the hot surface is facing downward, the plate will block the heated fluid that tends to rise (except near the edges), impeding heat transfer. The opposite is true for a cold plate in a warmer environment since the net force (weight minus buoyancy force) in this case acts downward, and the cooled fluid near the plate tends to descend.

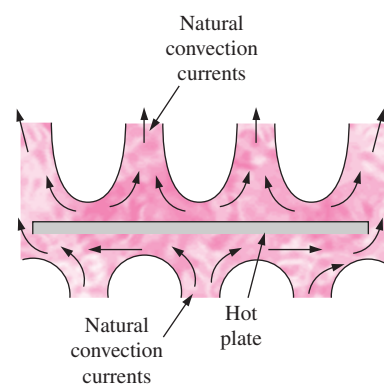
The average Nusselt number for horizontal surfaces can be determined from the simple power-law relations given in Table 20–1. The characteristic length for horizontal surfaces is calculated from

$$L_c = \frac{A_s}{p} \quad (20-21)$$

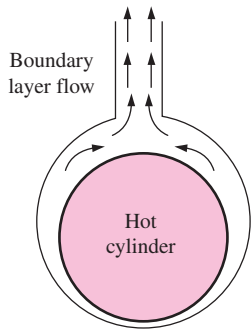
where  $A_s$  is the surface area and  $p$  is the perimeter. Note that  $L_c = a/4$  for a horizontal square surface of length  $a$ , and  $D/4$  for a horizontal circular surface of diameter  $D$ .

## Horizontal Cylinders and Spheres

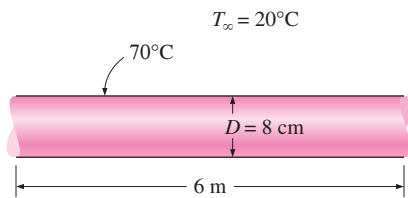
The boundary layer over a hot horizontal cylinder starts to develop at the bottom, increasing in thickness along the circumference, and forming a rising



**FIGURE 20–11**  
Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

**FIGURE 20-12**

Natural convection flow over a horizontal hot cylinder.

**FIGURE 20-13**

Schematic for Example 20-1.

plume at the top, as shown in Fig. 20-12. Therefore, the local Nusselt number is highest at the bottom, and lowest at the top of the cylinder when the boundary layer flow remains laminar. The opposite is true in the case of a cold horizontal cylinder in a warmer medium, and the boundary layer in this case starts to develop at the top of the cylinder and ending with a descending plume at the bottom.

The average Nusselt number over the entire surface can be determined from Eq. 20-26 [Churchill and Chu (1975)] for an isothermal horizontal cylinder, and from Eq. 20-19 for an isothermal sphere [Churchill (1983)] both given in Table 20-1.

### EXAMPLE 20-1 Heat Loss from Hot-Water Pipes

A 6-m-long section of an 8-cm-diameter horizontal hot-water pipe shown in Fig. 20-13 passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.

**SOLUTION** A horizontal hot-water pipe passes through a large room. The rate of heat loss from the pipe by natural convection is to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The local atmospheric pressure is 1 atm.

**Properties** The properties of air at the film temperature of  $T_f = (T_s + T_\infty)/2 = (70 + 20)/2 = 45^\circ\text{C}$  and 1 atm are (Table A-22)

$$k = 0.02699 \text{ W/m} \cdot ^\circ\text{C}$$

$$\text{Pr} = 0.7241$$

$$\nu = 1.749 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\beta = \frac{1}{T_f} = \frac{1}{318 \text{ K}}$$

**Analysis** The characteristic length in this case is the outer diameter of the pipe,  $L_c = D = 0.08 \text{ m}$ . Then the Rayleigh number becomes

$$\begin{aligned} \text{Ra}_D &= \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} \text{Pr} \\ &= \frac{(9.81 \text{ m/s}^2)[1/(318 \text{ K})](70 - 20 \text{ K})(0.08 \text{ m})^3}{(1.749 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7241) = 1.869 \times 10^6 \end{aligned}$$

The natural convection Nusselt number in this case can be determined from Eq. 20-25 to be

$$\begin{aligned} \text{Nu} &= \left\{ 0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}} \right\}^2 = \left\{ 0.6 + \frac{0.387(1.869 \times 10^6)^{1/6}}{[1 + (0.559/0.7241)^{9/16}]^{8/27}} \right\}^2 \\ &= 17.40 \end{aligned}$$

Then,

$$h = \frac{k}{D} \text{Nu} = \frac{0.02699 \text{ W/m} \cdot ^\circ\text{C}}{0.08 \text{ m}} (17.40) = 5.869 \text{ W/m} \cdot ^\circ\text{C}$$

$$A_s = \pi DL = \pi(0.08 \text{ m})(6 \text{ m}) = 1.508 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.869 \text{ W/m}^2 \cdot ^\circ\text{C})(1.508 \text{ m}^2)(70 - 20)^\circ\text{C} = \mathbf{443 \text{ W}}$$

Therefore, the pipe will lose heat to the air in the room at a rate of 443 W by natural convection.

**Discussion** The pipe will lose heat to the surroundings by radiation as well as by natural convection. Assuming the outer surface of the pipe to be black (emissivity  $\varepsilon = 1$ ) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer is determined to be (Fig. 20–14)

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) \\ &= (1)(1.508 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(70 + 273 \text{ K})^4 - (20 + 273 \text{ K})^4] \\ &= 553 \text{ W}\end{aligned}$$

which is larger than natural convection. The emissivity of a real surface is less than 1, and thus the radiation heat transfer for a real surface will be less. But radiation will still be significant for most systems cooled by natural convection. Therefore, a radiation analysis should normally accompany a natural convection analysis unless the emissivity of the surface is low.

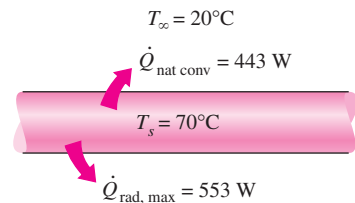


FIGURE 20–14

Radiation heat transfer is usually comparable to natural convection in magnitude and should be considered in heat transfer analysis.

### EXAMPLE 20–2 Cooling of a Plate in Different Orientations

Consider a 0.6-m  $\times$  0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Fig. 20–15. Determine the rate of heat transfer from the plate by natural convection if the plate is (a) vertical, (b) horizontal with hot surface facing up, and (c) horizontal with hot surface facing down.

**SOLUTION** A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

**Assumptions** 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The local atmospheric pressure is 1 atm.

**Properties** The properties of air at the film temperature of  $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60^\circ\text{C}$  and 1 atm are (Table A–22)

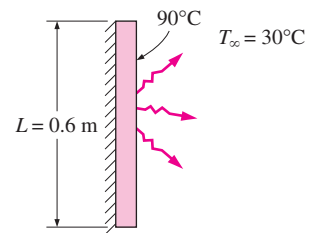
$$\begin{aligned}k &= 0.02808 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 0.7202 \\ \nu &= 1.896 \times 10^{-5} \text{ m}^2/\text{s} & \beta &= \frac{1}{T_f} = \frac{1}{333 \text{ K}}\end{aligned}$$

**Analysis** (a) *Vertical.* The characteristic length in this case is the height of the plate, which is  $L = 0.6 \text{ m}$ . The Rayleigh number is

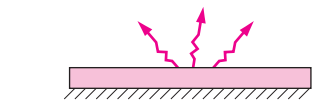
$$\begin{aligned}\text{Ra}_L &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \text{Pr} \\ &= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.6 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.722) = 7.656 \times 10^8\end{aligned}$$

Then the natural convection Nusselt number can be determined from Eq. 20–21 to be

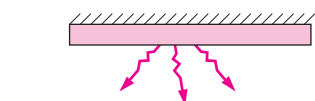
$$\begin{aligned}\text{Nu} &= \left\{ 0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}} \right\}^2 \\ &= \left\{ 0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}} \right\}^2 = 113.4\end{aligned}$$



(a) Vertical



(b) Hot surface facing up



(c) Hot surface facing down

FIGURE 20–15

Schematic for Example 20–2.

Note that the simpler relation Eq. 20–19 would give  $Nu = 0.59 Ra_L^{1/4} = 98.14$ , which is 13 percent lower. Then,

$$h = \frac{k}{L} Nu = \frac{0.02808 \text{ W/m} \cdot ^\circ\text{C}}{0.6 \text{ m}} (113.4) = 5.306 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306 \text{ W/m}^2 \cdot ^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{115 \text{ W}}$$

(b) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{P} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6 \text{ m}}{4} = 0.15 \text{ m}$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.15 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7202) = 1.196 \times 10^7$$

The natural convection Nusselt number can be determined from Eq. 20–22 to be

$$Nu = 0.54 Ra_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

Then,

$$h = \frac{k}{L_c} Nu = \frac{0.0280 \text{ W/m} \cdot ^\circ\text{C}}{0.15 \text{ m}} (31.76) = 5.946 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946 \text{ W/m}^2 \cdot ^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{128 \text{ W}}$$

(c) *Horizontal with hot surface facing down.* The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (b). But the natural convection Nusselt number is to be determined from Eq. 20–24,

$$Nu = 0.27 Ra_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

Then,

$$h = \frac{k}{L_c} Nu = \frac{0.02808 \text{ W/m} \cdot ^\circ\text{C}}{0.15 \text{ m}} (15.86) = 2.973 \text{ W/m}^2 \cdot ^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973 \text{ W/m}^2 \cdot ^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{64.2 \text{ W}}$$

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air is “trapped” under the plate in this case and cannot get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty reaching the plate, which results in a reduced rate of heat transfer.

**Discussion** The plate will lose heat to the surroundings by radiation as well as by natural convection. Assuming the surface of the plate to be black (emissivity



$\varepsilon = 1$ ) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer in this case is determined to be

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) \\ &= (1)(0.36 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(90 + 273 \text{ K})^4 - (30 + 273 \text{ K})^4] \\ &= 182 \text{ W}\end{aligned}$$

which is larger than that for natural convection heat transfer for each case. Therefore, radiation can be significant and needs to be considered in surfaces cooled by natural convection.

## 20-4 ■ NATURAL CONVECTION FROM FINNED SURFACES AND PCBs

Natural convection flow through a channel formed by two parallel plates as shown in Fig. 20-16 is commonly encountered in practice. When the plates are hot ( $T_s > T_\infty$ ), the ambient fluid at  $T_\infty$  enters the channel from the lower end, rises as it is heated under the effect of buoyancy, and the heated fluid leaves the channel from the upper end. The plates could be the fins of a finned heat sink, or the PCBs (printed circuit boards) of an electronic device. The plates can be approximated as being isothermal ( $T_s = \text{constant}$ ) in the first case, and isoflux ( $\dot{q}_s = \text{constant}$ ) in the second case.

Boundary layers start to develop at the lower ends of opposing surfaces, and eventually merge at the midplane if the plates are vertical and sufficiently long. In this case, we will have fully developed channel flow after the merger of the boundary layers, and the natural convection flow is analyzed as channel flow. But when the plates are short or the spacing is large, the boundary layers of opposing surfaces never reach each other, and the natural convection flow on a surface is not affected by the presence of the opposing surface. In that case, the problem should be analyzed as natural convection from two independent plates in a quiescent medium, using the relations given for surfaces, rather than natural convection flow through a channel.

### Natural Convection Cooling of Finned Surfaces ( $T_s = \text{constant}$ )

Finned surfaces of various shapes, called *heat sinks*, are frequently used in the cooling of electronic devices. Energy dissipated by these devices is transferred to the heat sinks by conduction and from the heat sinks to the ambient air by natural or forced convection, depending on the power dissipation requirements. Natural convection is the preferred mode of heat transfer since it involves no moving parts, like the electronic components themselves. However, in the natural convection mode, the components are more likely to run at a higher temperature and thus undermine reliability. A properly selected heat sink may considerably lower the operation temperature of the components and thus reduce the risk of failure.

Natural convection from vertical finned surfaces of rectangular shape has been the subject of numerous studies, mostly experimental. Bar-Cohen and

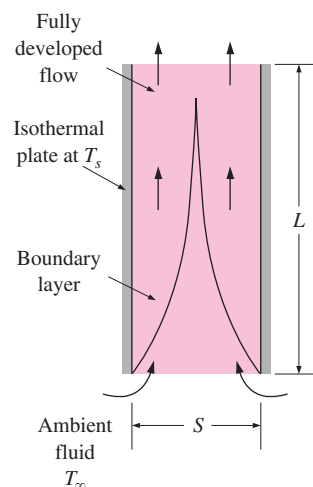
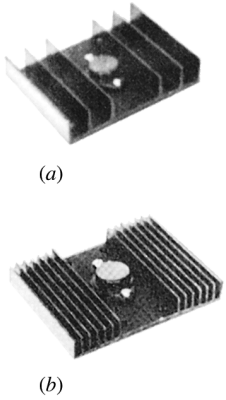
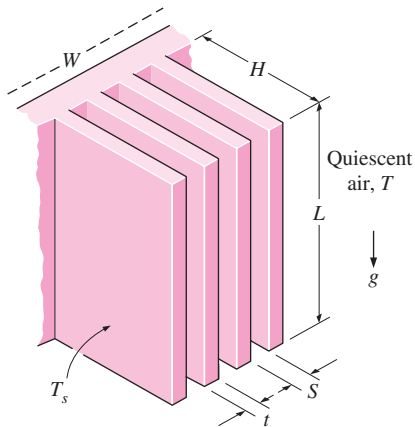


FIGURE 20-16

Natural convection flow through a channel between two isothermal vertical plates.

**FIGURE 20-17**

Heat sinks with (a) widely spaced and (b) closely packed fins.

**FIGURE 20-18**

Various dimensions of a finned surface oriented vertically.

Rohsenow (1984) have compiled the available data under various boundary conditions, and developed correlations for the Nusselt number and optimum spacing. The characteristic length for vertical parallel plates used as fins is usually taken to be the spacing between adjacent fins  $S$ , although the fin height  $L$  could also be used. The Rayleigh number is expressed as

$$Ra_S = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2} \text{Pr} \quad \text{and} \quad Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \text{Pr} = Ra_S \frac{L^3}{S^3} \quad (20-22)$$

The recommended relation for the average Nusselt number for vertical isothermal parallel plates is

$$T_s = \text{constant:} \quad Nu = \frac{hS}{k} = \left[ \frac{576}{(Ra_S S/L)^2} + \frac{2.873}{(Ra_S S/L)^{0.5}} \right]^{-0.5} \quad (20-23)$$

A question that often arises in the selection of a heat sink is whether to select one with *closely packed* fins or *widely spaced* fins for a given base area (Fig. 20-17). A heat sink with closely packed fins will have greater surface area for heat transfer but a smaller heat transfer coefficient because of the extra resistance the additional fins introduce to fluid flow through the interfin passages. A heat sink with widely spaced fins, on the other hand, will have a higher heat transfer coefficient but a smaller surface area. Therefore, there must be an *optimum spacing* that maximizes the natural convection heat transfer from the heat sink for a given base area  $WL$ , where  $W$  and  $L$  are the width and height of the base of the heat sink, respectively, as shown in Fig. 20-18. When the fins are essentially isothermal and the fin thickness  $t$  is small relative to the fin spacing  $S$ , the optimum fin spacing for a vertical heat sink is determined by Bar-Cohen and Rohsenow to be

$$T_s = \text{constant:} \quad S_{\text{opt}} = 2.714 \left( \frac{S^3 L}{Ra_S} \right)^{0.25} = 2.714 \frac{L}{Ra_L^{0.25}} \quad (20-24)$$

It can be shown by combining the three equations above that when  $S = S_{\text{opt}}$ , the Nusselt number is a constant and its value is 1.307,

$$S = S_{\text{opt}}: \quad Nu = \frac{hS_{\text{opt}}}{k} = 1.307 \quad (20-25)$$

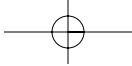
The rate of heat transfer by natural convection from the fins can be determined from

$$\dot{Q} = h(2nLH)(T_s - T_\infty) \quad (20-26)$$

where  $n = W/(S + t) \approx W/S$  is the number of fins on the heat sink and  $T_s$  is the surface temperature of the fins. All fluid properties are to be evaluated at the average temperature  $T_{\text{ave}} = (T_s + T_\infty)/2$ .

## Natural Convection Cooling of Vertical PCBs ( $\dot{q}_s = \text{constant}$ )

Arrays of printed circuit boards used in electronic systems can often be modeled as parallel plates subjected to uniform heat flux  $\dot{q}_s$  (Fig. 20-19). The plate temperature in this case increases with height, reaching a maximum at the



upper edge of the board. The modified Rayleigh number for uniform heat flux on both plates is expressed as

$$\text{Ra}_s^* = \frac{g\beta\dot{q}_s S^4}{k\nu^2} \text{Pr} \quad (20-27)$$

The Nusselt number at the upper edge of the plate where maximum temperature occurs is determined from [Bar-Cohen and Rohsenow (1984)]

$$\text{Nu}_L = \frac{h_L S}{k} = \left[ \frac{48}{\text{Ra}_s^* S/L} + \frac{2.51}{(\text{Ra}_s^* S/L)^{0.4}} \right]^{-0.5} \quad (20-28)$$

The optimum fin spacing for the case of uniform heat flux on both plates is given as

$$\dot{q}_s = \text{constant:} \quad S_{\text{opt}} = 2.12 \left( \frac{S^4 L}{\text{Ra}_s^*} \right)^{0.2} \quad (20-29)$$

The total rate of heat transfer from the plates is

$$\dot{Q} = \dot{q}_s A_s = \dot{q}_s (2nLH) \quad (20-30)$$

where  $n = W/(S + t) \approx W/S$  is the number of plates. The critical surface temperature  $T_L$  occurs at the upper edge of the plates, and it can be determined from

$$\dot{q}_s = h_L (T_L - T_\infty) \quad (20-31)$$

All fluid properties are to be evaluated at the average temperature  $T_{\text{ave}} = (T_L + T_\infty)/2$ .

## Mass Flow Rate through the Space between Plates

As we mentioned earlier, the magnitude of the natural convection heat transfer is directly related to the mass flow rate of the fluid, which is established by the dynamic balance of two opposing effects: *buoyancy* and *friction*.

The fins of a heat sink introduce both effects: *inducing extra buoyancy* as a result of the elevated temperature of the fin surfaces and *slowing down the fluid* by acting as an added obstacle on the flow path. As a result, increasing the number of fins on a heat sink can either enhance or reduce natural convection, depending on which effect is dominant. The buoyancy-driven fluid flow rate is established at the point where these two effects balance each other. The friction force increases as more and more solid surfaces are introduced, seriously disrupting fluid flow and heat transfer. Under some conditions, the increase in friction may more than offset the increase in buoyancy. This in turn will tend to reduce the flow rate and thus the heat transfer. For that reason, heat sinks with closely spaced fins are not suitable for natural convection cooling.

When the heat sink involves closely spaced fins, the narrow channels formed tend to block or “suffocate” the fluid, especially when the heat sink is long. As a result, the blocking action produced overwhelms the extra buoyancy and downgrades the heat transfer characteristics of the heat sink. Then, at a fixed power setting, the heat sink runs at a higher temperature relative to the no-shroud case. When the heat sink involves widely spaced fins, the

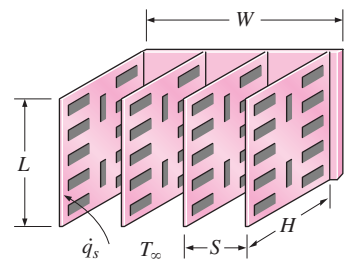
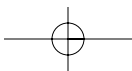


FIGURE 20-19

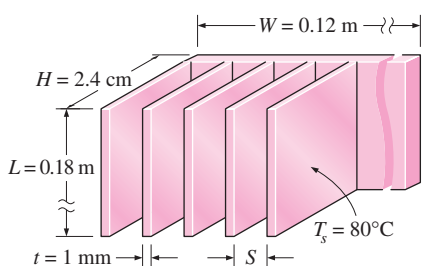
Arrays of vertical printed circuit boards (PCBs) cooled by natural convection.



shroud does not introduce a significant increase in resistance to flow, and the buoyancy effects dominate. As a result, heat transfer by natural convection may improve, and at a fixed power level the heat sink may run at a lower temperature.

When extended surfaces such as fins are used to enhance natural convection heat transfer between a solid and a fluid, the flow rate of the fluid in the vicinity of the solid adjusts itself to incorporate the changes in buoyancy and friction. It is obvious that this enhancement technique will work to advantage only when the increase in buoyancy is greater than the additional friction introduced. One does not need to be concerned with pressure drop or pumping power when studying natural convection since no pumps or blowers are used in this case. Therefore, an enhancement technique in natural convection is evaluated on heat transfer performance alone.

The failure rate of an electronic component increases almost exponentially with operating temperature. The cooler the electronic device operates, the more reliable it is. A rule of thumb is that the semiconductor failure rate is halved for each  $10^\circ\text{C}$  reduction in junction operating temperature. The desire to lower the operating temperature without having to resort to forced convection has motivated researchers to investigate enhancement techniques for natural convection. Sparrow and Prakash have demonstrated that, under certain conditions, the use of discrete plates in lieu of continuous plates of the same surface area increases heat transfer considerably. In other experimental work, using transistors as the heat source, Çengel and Zing have demonstrated that temperature recorded on the transistor case dropped by as much as  $30^\circ\text{C}$  when a shroud was used, as opposed to the corresponding no-shroud case.



**FIGURE 20-20**

Schematic for Example 20-3.

### EXAMPLE 20-3 Optimum Fin Spacing of a Heat Sink

A 12-cm-wide and 18-cm-high vertical hot surface in  $30^\circ\text{C}$  air is to be cooled by a heat sink with equally spaced fins of rectangular profile (Fig. 20-20). The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is  $80^\circ\text{C}$ .

**SOLUTION** A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The atmospheric pressure at that location is 1 atm. 4 The thickness  $t$  of the fins is very small relative to the fin spacing  $S$  so that Eq. 20-24 for optimum fin spacing is applicable. 5 All fin surfaces are isothermal at base temperature.

**Properties** The properties of air at the film temperature of  $T_f = (T_s + T_\infty)/2 = (80 + 30)/2 = 55^\circ\text{C}$  and 1 atm pressure are (Table A-22)

$$\begin{aligned} k &= 0.02772 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 0.7215 \\ \nu &= 1.846 \times 10^{-5} \text{ m}^2/\text{s} & \beta &= 1/T_f = 1/328 \text{ K} \end{aligned}$$

**Analysis** We take the characteristic length to be the length of the fins in the vertical direction (since we do not know the fin spacing). Then the Rayleigh number becomes

$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \text{Pr} \\ &= \frac{(981 \text{ m/s}^2)[1/(328 \text{ K})](80 - 30 \text{ K})(0.18 \text{ m})^3}{(1.846 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7215) = 1.846 \times 10^7 \end{aligned}$$

The optimum fin spacing is determined from Eq. 20–24 to be

$$S_{\text{opt}} = 2.714 \frac{L}{\text{Ra}_L^{0.25}} = 2.714 \frac{0.8 \text{ m}}{(1.846 \times 10^7)^{0.25}} = 7.45 \times 10^{-3} \text{ m} = \mathbf{7.45 \text{ mm}}$$

which is about seven times the thickness of the fins. Therefore, the assumption of negligible fin thickness in this case is acceptable. The number of fins and the heat transfer coefficient for this optimum fin spacing case are

$$n = \frac{W}{S + t} = \frac{0.12 \text{ m}}{(0.00745 + 0.0001) \text{ m}} \approx 15 \text{ fins}$$

The convection coefficient for this optimum in spacing case is, from Eq. 20–25,

$$h = \text{Nu}_{\text{opt}} \frac{k}{S_{\text{opt}}} = 1.307 \frac{0.02772 \text{ W/m} \cdot ^\circ\text{C}}{0.00745 \text{ m}} = 0.2012 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Then the rate of natural convection heat transfer becomes

$$\begin{aligned} \dot{Q} &= hA_s(T_s - T_\infty) = h(2nLH)(T_s - T_\infty) \\ &= (0.2012 \text{ W/m}^2 \cdot ^\circ\text{C})[2 \times 15(0.18 \text{ m})(0.024 \text{ m})](80 - 30)^\circ\text{C} = \mathbf{1.30 \text{ W}} \end{aligned}$$

Therefore, this heat sink can dissipate heat by natural convection at a rate of 1.30 W.

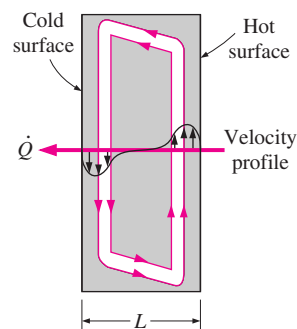


FIGURE 20–21

Convective currents in a vertical rectangular enclosure.

## 20–5 ■ NATURAL CONVECTION INSIDE ENCLOSURES

A considerable portion of heat loss from a typical residence occurs through the windows. We certainly would insulate the windows, if we could, in order to conserve energy. The problem is finding an insulating material that is transparent. An examination of the thermal conductivities of the insulating materials reveals that *air* is a *better insulator* than most common insulating materials. Besides, it is transparent. Therefore, it makes sense to insulate the windows with a layer of air. Of course, we need to use another sheet of glass to trap the air. The result is an *enclosure*, which is known as a *double-pane window* in this case. Other examples of enclosures include wall cavities, solar collectors, and cryogenic chambers involving concentric cylinders or spheres.

Enclosures are frequently encountered in practice, and heat transfer through them is of practical interest. Heat transfer in enclosed spaces is complicated by the fact that the fluid in the enclosure, in general, does not remain stationary. In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotary motion within the enclosure that enhances heat transfer through the enclosure. Typical flow patterns in vertical and horizontal rectangular enclosures are shown in Figs. 20–21 and 20–22.

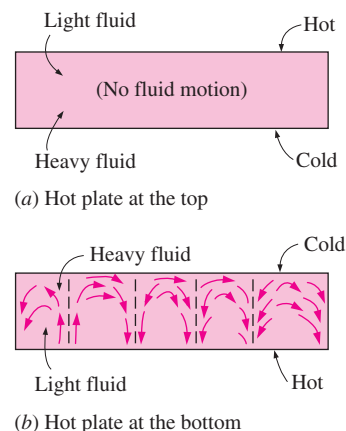


FIGURE 20–22

Convective currents in a horizontal enclosure with (a) hot plate at the top and (b) hot plate at the bottom.

The characteristics of heat transfer through a horizontal enclosure depend on whether the hotter plate is at the top or at the bottom, as shown in Fig. 20–22. When the hotter plate is at the *top*, no convection currents will develop in the enclosure, since the lighter fluid will always be on top of the heavier fluid. Heat transfer in this case will be by *pure conduction*, and we will have  $Nu = 1$ . When the hotter plate is at the *bottom*, the heavier fluid will be on top of the lighter fluid, and there will be a tendency for the lighter fluid to topple the heavier fluid and rise to the top, where it will come in contact with the cooler plate and cool down. Until that happens, however, the heat transfer is still by *pure conduction* and  $Nu = 1$ . When  $Ra > 1708$ , the buoyant force overcomes the fluid resistance and initiates natural convection currents, which are observed to be in the form of hexagonal cells called *Bénard cells*. For  $Ra > 3 \times 10^5$ , the cells break down and the fluid motion becomes turbulent.

The Rayleigh number for an enclosure is determined from

$$Ra_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2} Pr \quad (20-32)$$

where the characteristic length  $L_c$  is the distance between the hot and cold surfaces, and  $T_1$  and  $T_2$  are the temperatures of the hot and cold surfaces, respectively. All fluid properties are to be evaluated at the average fluid temperature  $T_{ave} = (T_1 + T_2)/2$ .

## Effective Thermal Conductivity

When the Nusselt number is known, the rate of heat transfer through the enclosure can be determined from

$$\dot{Q} = hA_s(T_1 - T_2) = kNuA_s \frac{T_1 - T_2}{L_c} \quad (20-33)$$

since  $h = kNu/L$ . The rate of steady heat conduction across a layer of thickness  $L_c$ , area  $A_s$ , and thermal conductivity  $k$  is expressed as

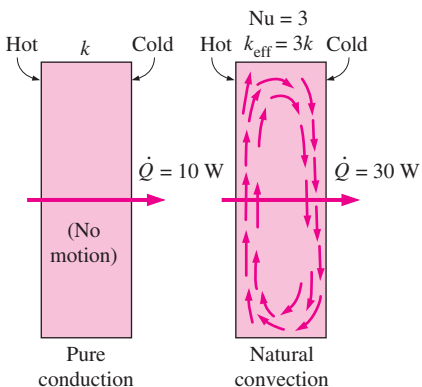
$$\dot{Q}_{cond} = kA_s \frac{T_1 - T_2}{L_c} \quad (20-34)$$

where  $T_1$  and  $T_2$  are the temperatures on the two sides of the layer. A comparison of this relation with Eq. 20–33 reveals that the convection heat transfer in an enclosure is analogous to heat conduction across the fluid layer in the enclosure provided that the thermal conductivity  $k$  is replaced by  $kNu$ . That is, *the fluid in an enclosure behaves like a fluid whose thermal conductivity is  $kNu$  as a result of convection currents*. Therefore, the quantity  $kNu$  is called the **effective thermal conductivity** of the enclosure. That is,

$$k_{eff} = kNu \quad (20-35)$$

Note that for the special case of  $Nu = 1$ , the effective thermal conductivity of the enclosure becomes equal to the conductivity of the fluid. This is expected since this case corresponds to pure conduction (Fig. 20–23).

Natural convection heat transfer in enclosed spaces has been the subject of many experimental and numerical studies, and numerous correlations for the Nusselt number exist. Simple power-law type relations in the form of



**FIGURE 20–23**

A Nusselt number of 3 for an enclosure indicates that heat transfer through the enclosure by *natural convection* is three times that by *pure conduction*.



$Nu = C Ra_L^n$ , where  $C$  and  $n$  are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios. The relations that are more comprehensive are naturally more complex. Next we present some widely used relations for various types of enclosures.

## Horizontal Rectangular Enclosures

We need no Nusselt number relations for the case of the hotter plate being at the top, since there will be no convection currents in this case and heat transfer will be downward by conduction ( $Nu = 1$ ). When the hotter plate is at the bottom, however, significant convection currents set in for  $Ra_L > 1708$ , and the rate of heat transfer increases (Fig. 20–24).

For horizontal enclosures that contain air, Jakob (1949) recommends the following simple correlations

$$Nu = 0.195 Ra_L^{1/4} \quad 10^4 < Ra_L < 4 \times 10^5 \quad (20-36)$$

$$Nu = 0.068 Ra_L^{1/3} \quad 4 \times 10^5 < Ra_L < 10^7 \quad (20-37)$$

These relations can also be used for other gases with  $0.5 < Pr < 2$ . Using water, silicone oil, and mercury in their experiments, Globe and Dropkin (1959) obtained this correlation for horizontal enclosures heated from below,

$$Nu = 0.069 Ra_L^{1/3} Pr^{0.074} \quad 3 \times 10^5 < Ra_L < 7 \times 10^9 \quad (20-38)$$

Based on experiments with air, Hollands et al. (1976) recommend this correlation for horizontal enclosures,

$$Nu = 1 + 1.44 \left[ 1 - \frac{1708}{Ra_L} \right]^+ + \left[ \frac{Ra_L^{1/3}}{18} - 1 \right]^+ \quad Ra_L < 10^8 \quad (20-39)$$

The notation  $[ ]^+$  indicates that if the quantity in the bracket is negative, it should be set equal to zero. This relation also correlates data well for liquids with moderate Prandtl numbers for  $Ra_L < 10^5$ , and thus it can also be used for water.

## Inclined Rectangular Enclosures

Air spaces between two inclined parallel plates are commonly encountered in flat-plate solar collectors (between the glass cover and the absorber plate) and the double-pane skylights on inclined roofs. Heat transfer through an inclined enclosure depends on the **aspect ratio**  $H/L$  as well as the tilt angle  $\theta$  from the horizontal (Fig. 20–25).

For large aspect ratios ( $H/L \geq 12$ ), this equation [Hollands et al., 1976] correlates experimental data extremely well for tilt angles up to  $70^\circ$ ,

$$Nu = 1 + 1.44 \left[ 1 - \frac{1708}{Ra_L \cos \theta} \right]^+ \left( 1 - \frac{1708 (\sin 1.8\theta)^{1.6}}{Ra_L \cos \theta} \right) + \left[ \frac{(Ra_L \cos \theta)^{1/3}}{18} - 1 \right]^+ \quad (20-40)$$

for  $Ra_L < 10^5$ ,  $0 < \theta < 70^\circ$ , and  $H/L \geq 12$ . Again any quantity in  $[ ]^+$  should be set equal to zero if it is negative. This is to ensure that  $Nu = 1$  for  $Ra_L \cos \theta < 1708$ . Note that this relation reduces to Eq. 20–39 for horizontal enclosures for  $\theta = 0^\circ$ , as expected.

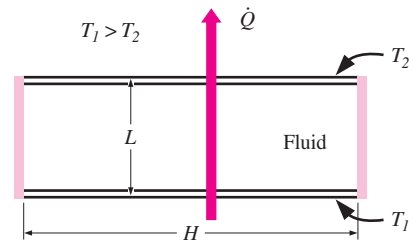


FIGURE 20–24

A horizontal rectangular enclosure with isothermal surfaces.

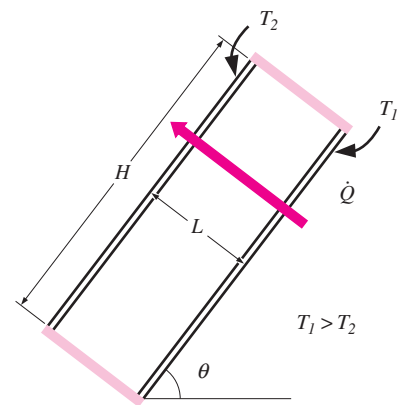


FIGURE 20–25

An inclined rectangular enclosure with isothermal surfaces.

**TABLE 20-2**

Critical angles for inclined rectangular enclosures

Aspect ratio, $H/L$	Critical angle, $\theta_{cr}$
1	25°
3	53°
6	60°
12	67°
> 12	70°

For enclosures with smaller aspect ratios ( $H/L < 12$ ), the next correlation can be used provided that the tilt angle is less than the critical value  $\theta_{cr}$  listed in Table 20-2 [Catton (1978)]

$$Nu = Nu_{\theta=0^\circ} \left( \frac{Nu_{\theta=90^\circ}}{Nu_{\theta=0^\circ}} \right)^{\theta/\theta_{cr}} (\sin \theta_{cr})^{\theta/(4\theta_{cr})} \quad 0^\circ < \theta < \theta_{cr} \quad (20-41)$$

For tilt angles greater than the critical value ( $\theta_{cr} < \theta < 90^\circ$ ), the Nusselt number can be obtained by multiplying the Nusselt number for a vertical enclosure by  $(\sin \theta)^{1/4}$  [Ayyaswamy and Catton (1973)],

$$Nu = Nu_{\theta=90^\circ} (\sin \theta)^{1/4} \quad \theta_{cr} < \theta < 90^\circ, \text{ any } H/L \quad (20-42)$$

For enclosures tilted more than  $90^\circ$ , the recommended relation is [Arnold et al. (1974)]

$$Nu = 1 + (Nu_{\theta=90^\circ} - 1) \sin \theta \quad 90^\circ < \theta < 180^\circ, \text{ any } H/L \quad (20-43)$$

More recent but more complex correlations are also available in the literature [e.g., and ElSherbiny et al. (1982)].

## Vertical Rectangular Enclosures

For vertical enclosures (Fig. 20-26), Catton (1978) recommends these two correlations due to Berkovsky and Polevikov (1977),

$$Nu = 0.18 \left( \frac{Pr}{0.2 + Pr} Ra_L \right)^{0.29} \quad \begin{array}{l} 1 < H/L < 2 \\ \text{any Prandtl number} \\ Ra_L Pr / (0.2 + Pr) > 10^3 \end{array} \quad (20-44)$$

$$Nu = 0.22 \left( \frac{Pr}{0.2 + Pr} Ra_L \right)^{0.28} \left( \frac{H}{L} \right)^{-1/4} \quad \begin{array}{l} 2 < H/L < 10 \\ \text{any Prandtl number} \\ Ra_L < 10^{10} \end{array} \quad (20-45)$$

For vertical enclosures with larger aspect ratios, the following correlations can be used [MacGregor and Emery (1969)]

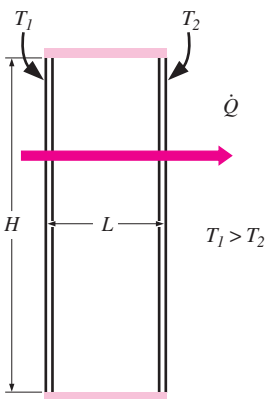
$$Nu = 0.42 Ra_L^{1/4} Pr^{0.012} \left( \frac{H}{L} \right)^{-0.3} \quad \begin{array}{l} 10 < H/L < 40 \\ 1 < Pr < 2 \times 10^4 \\ 10^4 < Ra_L < 10^7 \end{array} \quad (20-46)$$

$$Nu = 0.46 Ra_L^{1/3} \quad \begin{array}{l} 1 < H/L < 40 \\ 1 < Pr < 20 \\ 10^6 < Ra_L < 10^9 \end{array} \quad (20-47)$$

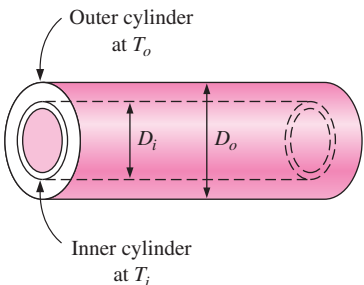
Again all fluid properties are to be evaluated at the average temperature  $(T_1 + T_2)/2$ .

## Concentric Cylinders

Consider two long concentric horizontal cylinders maintained at uniform but different temperatures of  $T_i$  and  $T_o$ , as shown in Figure 20-27. The diameters of the inner and outer cylinders are  $D_i$  and  $D_o$ , respectively, and the characteristic length is the spacing between the cylinders,  $L_c = (D_o - D_i)/2$ . The rate of heat transfer through the annular space between the natural convection unit is expressed as

**FIGURE 20-26**

A vertical rectangular enclosure with isothermal surfaces.

**FIGURE 20-27**

Two concentric horizontal isothermal cylinders.

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)} (T_i - T_o) \quad (\text{W/m}) \quad (20-48)$$

The recommended relation for effective thermal conductivity is [Raithby and Hollands (1975)]

$$\frac{k_{\text{eff}}}{k} = 0.386 \left( \frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{cyl}} \text{Ra}_L)^{1/4} \quad (20-49)$$

where the geometric factor for concentric cylinders  $F_{\text{cyl}}$  is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} \quad (20-50)$$

The  $k_{\text{eff}}$  relation in Eq. 20-49 is applicable for  $0.70 \leq \text{Pr} \leq 6000$  and  $10^2 \leq F_{\text{cyl}} \text{Ra}_L \leq 10^7$ . For  $F_{\text{cyl}} \text{Ra}_L < 100$ , natural convection currents are negligible and thus  $k_{\text{eff}} = k$ . Note that  $k_{\text{eff}}$  cannot be less than  $k$ , and thus we should set  $k_{\text{eff}} = k$  if  $k_{\text{eff}}/k < 1$ . The fluid properties are evaluated at the average temperature of  $(T_i + T_o)/2$ .

## Concentric Spheres

For concentric isothermal spheres, the rate of heat transfer through the gap between the spheres by natural convection is expressed as (Fig. 20-28)

$$\dot{Q} = k_{\text{eff}} \pi \left( \frac{D_i D_o}{L_c} \right) (T_i - T_o) \quad (\text{W}) \quad (20-51)$$

where  $L_c = (D_o - D_i)/2$  is the characteristic length. The recommended relation for effective thermal conductivity is [Raithby and Hollands (1975)]

$$\frac{k_{\text{eff}}}{k} = 0.74 \left( \frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{sph}} \text{Ra}_L)^{1/4} \quad (20-52)$$

where the geometric factor for concentric spheres  $F_{\text{sph}}$  is

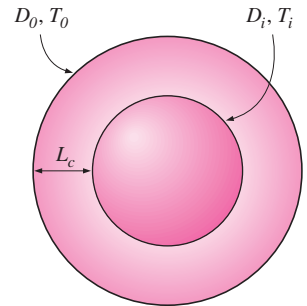
$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} \quad (20-53)$$

The  $k_{\text{eff}}$  relation in Eq. 20-52 is applicable for  $0.70 \leq \text{Pr} \leq 4200$  and  $10^2 \leq F_{\text{sph}} \text{Ra}_L \leq 10^4$ . If  $k_{\text{eff}}/k < 1$ , we should set  $k_{\text{eff}} = k$ .

## Combined Natural Convection and Radiation

Gases are nearly transparent to radiation, and thus heat transfer through a gas layer is by simultaneous convection (or conduction, if the gas is quiescent) and radiation. Natural convection heat transfer coefficients are typically very low compared to those for forced convection. Therefore, radiation is usually disregarded in forced convection problems, but it must be considered in natural convection problems that involve a gas. This is especially the case for surfaces with high emissivities. For example, about half of the heat transfer through the air space of a double-pane window is by radiation. The total rate of heat transfer is determined by adding the convection and radiation components,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \quad (20-54)$$



**FIGURE 20-28**  
Two concentric isothermal spheres.

Radiation heat transfer from a surface at temperature  $T_s$  surrounded by surfaces at a temperature  $T_{\text{surr}}$  (both in absolute temperature unit K) is determined from

$$\dot{Q}_{\text{rad}} = \varepsilon \sigma A_s (T_s^4 - T_{\text{surr}}^4) \quad (\text{W}) \quad (20-55)$$

where  $\varepsilon$  is the emissivity of the surface,  $A_s$  is the surface area, and  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$  is the Stefan–Boltzmann constant.

When the end effects are negligible, radiation heat transfer between two large parallel plates at absolute temperatures  $T_1$  and  $T_2$  is expressed as (see Chap. 22 for details)

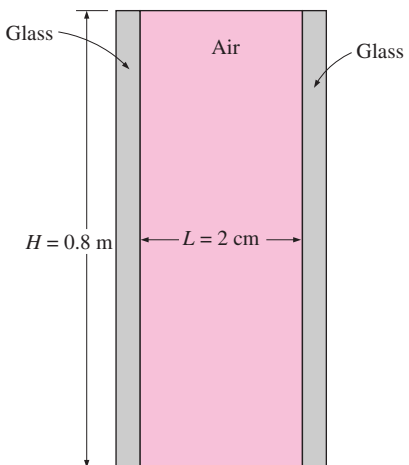
$$\dot{Q}_{\text{rad}} = \frac{\pi A_s (T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \varepsilon_{\text{effective}} \sigma A_s (T_1^4 - T_2^4) \quad (\text{W}) \quad (20-56)$$

where  $\varepsilon_1$  and  $\varepsilon_2$  are the emissivities of the plates, and  $\varepsilon_{\text{effective}}$  is the *effective emissivity* defined as

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \quad (20-57)$$

The emissivity of an ordinary glass surface, for example, is 0.84. Therefore, the effective emissivity of two parallel glass surfaces facing each other is 0.72. Radiation heat transfer between concentric cylinders and spheres is discussed in Chap. 22.

Note that in some cases the temperature of the surrounding medium may be below the surface temperature ( $T_\infty < T_s$ ), while the temperature of the surrounding surfaces is above the surface temperature ( $T_{\text{surr}} > T_s$ ). In such cases, convection and radiation heat transfers are subtracted from each other instead of being added since they are in opposite directions. Also, for a metal surface, the radiation effect can be reduced to negligible levels by polishing the surface and thus lowering the surface emissivity to a value near zero.



**FIGURE 20-29**  
Schematic for Example 20-4.

#### EXAMPLE 20-4 Heat Loss through a Double-Pane Window

The vertical 0.8-m-high, 2-m-wide double-pane window shown in Fig. 20-29 consists of two sheets of glass separated by a 2-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 12°C and 2°C, determine the rate of heat transfer through the window.

**SOLUTION** Two glasses of a double-pane window are maintained at specified temperatures. The rate of heat transfer through the window is to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 Radiation heat transfer is not considered.

**Properties** The properties of air at the average temperature of  $T_{\text{ave}} = (T_1 + T_2)/2 = (12 + 2)/2 = 7^\circ\text{C}$  and 1 atm pressure are (Table A-22)

$$\begin{aligned} k &= 0.02416 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 0.7344 \\ \nu &= 1.399 \times 10^{-5} \text{ m}^2/\text{s} & \beta &= \frac{1}{T_{\text{ave}}} = \frac{1}{280 \text{ K}} \end{aligned}$$

**Analysis** We have a rectangular enclosure filled with air. The characteristic length in this case is the distance between the two glasses,  $L = 0.02 \text{ m}$ . Then the Rayleigh number becomes

$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_1 - T_2)L^3}{\nu^2} \\ &= \frac{(9.81 \text{ m/s}^2)[1/(280 \text{ K})](12 - 2 \text{ K})(0.02 \text{ m})^3}{(1.399 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7344) = 1.051 \times 10^4 \end{aligned}$$

The aspect ratio of the geometry is  $H/L = 0.8/0.02 = 40$ . Then the Nusselt number in this case can be determined from Eq. 20-46 to be

$$\begin{aligned} \text{Nu} &= 0.42\text{Ra}_L^{1/4} \text{Pr}^{0.012} \left(\frac{H}{L}\right)^{-0.3} \\ &= 0.42(1.051 \times 10^4)^{1/4} (0.7344)^{0.012} \left(\frac{0.8}{0.02}\right)^{-0.3} = 1.401 \end{aligned}$$

Then,

$$A_s = H \times W = (0.8 \text{ m})(2 \text{ m}) = 1.6 \text{ m}^2$$

and

$$\begin{aligned} \dot{Q} &= hA_s(T_1 - T_2) = k\text{Nu}A_s \frac{T_1 - T_2}{L} \\ &= (0.02416 \text{ W/m} \cdot ^\circ\text{C})(1.401)(1.6 \text{ m}^2) \frac{(12 - 2)^\circ\text{C}}{0.02 \text{ m}} = \mathbf{27.1 \text{ W}} \end{aligned}$$

Therefore, heat will be lost through the window at a rate of 27.1 W.

**Discussion** Recall that a Nusselt number of  $\text{Nu} = 1$  for an enclosure corresponds to pure conduction heat transfer through the enclosure. The air in the enclosure in this case remains still, and no natural convection currents occur in the enclosure. The Nusselt number in our case is 1.32, which indicates that heat transfer through the enclosure is 1.32 times that by pure conduction. The increase in heat transfer is due to the natural convection currents that develop in the enclosure.

### EXAMPLE 20-5 Heat Transfer through a Spherical Enclosure

The two concentric spheres of diameters  $D_i = 20 \text{ cm}$  and  $D_o = 30 \text{ cm}$  shown in Fig. 20-30 are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are  $T_i = 320 \text{ K}$  and  $T_o = 280 \text{ K}$ , respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

**SOLUTION** Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 Radiation heat transfer is not considered.

**Properties** The properties of air at the average temperature of  $T_{\text{ave}} = (T_i + T_o)/2 = (320 + 280)/2 = 300 \text{ K} = 27^\circ\text{C}$  and 1 atm pressure are (Table A-22)

$$\begin{aligned} k &= 0.02566 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 0.7290 \\ \nu &= 1.580 \times 10^{-5} \text{ m}^2/\text{s} & \beta &= \frac{1}{T_{\text{ave}}} = \frac{1}{300 \text{ K}} \end{aligned}$$

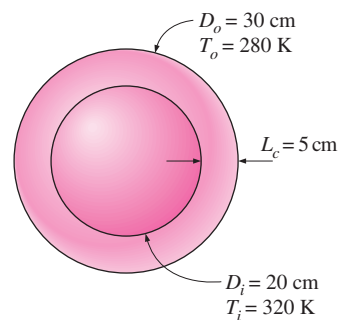


FIGURE 20-30

Schematic for Example 20-5.

**Analysis** We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05 \text{ m}$$

The Rayleigh number is

$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_i - T_o)L^3}{\nu^2} \text{Pr} \\ &= \frac{(9.81 \text{ m/s}^2)[1/(300 \text{ K})](320 - 280 \text{ K})(0.05 \text{ m})^3}{(1.58 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.729) = 4.776 \times 10^5 \end{aligned}$$

The effective thermal conductivity is

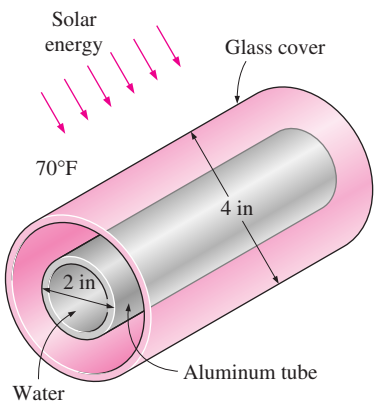
$$\begin{aligned} F_{\text{sph}} &= \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} \\ &= \frac{0.05 \text{ m}}{[(0.2 \text{ m})(0.3 \text{ m})]^4 [(0.2 \text{ m})^{-7/5} + (0.3 \text{ m})^{-7/5}]^5} = 0.005229 \\ k_{\text{eff}} &= 0.74k \left( \frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{sph}} \text{Ra}_L)^{1/4} \\ &= 0.74(0.02566 \text{ W/m} \cdot ^\circ\text{C}) \left( \frac{0.729}{0.861 + 0.729} \right) (0.005229 \times 4.776 \times 10^5)^{1/4} \\ &= 0.1104 \text{ W/m} \cdot ^\circ\text{C} \end{aligned}$$

Then the rate of heat transfer between the spheres becomes

$$\begin{aligned} \dot{Q} &= k_{\text{eff}} \pi \left( \frac{D_i D_o}{L_c} \right) (T_i - T_o) \\ &= (0.1104 \text{ W/m} \cdot ^\circ\text{C}) \pi \left( \frac{(0.2 \text{ m})(0.3 \text{ m})}{0.05 \text{ m}} \right) (320 - 280) \text{ K} = \mathbf{16.7 \text{ W}} \end{aligned}$$

Therefore, heat will be lost from the inner sphere to the outer one at a rate of 16.7 W.

**Discussion** Note that the air in the spherical enclosure will act like a stationary fluid whose thermal conductivity is  $k_{\text{eff}}/k = 0.1104/0.02566 = 4.3$  times that of air as a result of natural convection currents. Also, radiation heat transfer between spheres is usually very significant, and should be considered in a complete analysis.



**FIGURE 20-31**

Schematic for Example 20-6.

### EXAMPLE 20-6 Heating Water in a Tube by Solar Energy

A solar collector consists of a horizontal aluminum tube having an outer diameter of 2 in enclosed in a concentric thin glass tube of 4-in-diameter (Fig. 20-31). Water is heated as it flows through the tube, and the annular space between the aluminum and the glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 30 Btu/h per foot length, and the temperature of the ambient air outside is 70°F. Disregarding any heat loss by radiation, determine the temperature of the aluminum tube when steady operation is established (i.e., when the rate of heat loss from the tube equals the amount of solar energy gained by the tube).



**SOLUTION** The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

**Assumptions** **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** Heat loss by radiation is negligible.

**Properties** The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 110°F, and use properties at an anticipated average temperature of  $(70 + 110)/2 = 90^\circ\text{F}$  (Table A-22E),

$$k = 0.01505 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F} \quad \text{Pr} = 0.7275$$

$$\nu = 0.6310 \text{ ft}^2/\text{h} = 1.753 \times 10^{-4} \text{ ft}^2/\text{s} \quad \beta = \frac{1}{T_{\text{ave}}} = \frac{1}{550 \text{ K}}$$

**Analysis** We have a horizontal cylindrical enclosure filled with air at 1 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30 \text{ Btu/h} \quad (\text{per foot of tube})$$

The heat transfer surface area of the glass cover is

$$A_o = A_{\text{glass}} = (\pi D_o L) = \pi(4/12 \text{ ft})(1 \text{ ft}) = 1.047 \text{ ft}^2 \quad (\text{per foot of tube})$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, it is clear that the solution will require a trial-and-error approach. Assuming the glass cover temperature to be 100°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\begin{aligned} \text{Ra}_{D_o} &= \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2} \text{Pr} \\ &= \frac{(32.2 \text{ ft/s}^2)[1/(550 \text{ R})](110 - 70 \text{ R})(4/12 \text{ ft})^3}{(1.753 \times 10^{-4} \text{ ft}^2/\text{s})^2} (0.7275) = 2.054 \times 10^6 \\ \text{Nu} &= \left\{ 0.6 + \frac{0.387 \text{ Ra}_{D_o}^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}} \right\}^2 = \left\{ 0.6 + \frac{0.387(2.054 \times 10^6)^{1/6}}{[1 + (0.559/0.7275)^{9/16}]^{8/27}} \right\}^2 \\ &= 17.89 \\ h_o &= \frac{k}{D_o} \text{Nu} = \frac{0.0150 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}}{4/12 \text{ ft}} (17.89) = 0.8075 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F} \\ \dot{Q}_o &= h_o A_o (T_o - T_\infty) = (0.8075 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(1.047 \text{ ft}^2)(110 - 70)^\circ\text{F} \\ &= 33.8 \text{ Btu/h} \end{aligned}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 110°F for the glass cover is high. Repeating the calculations with lower temperatures, the glass cover temperature corresponding to 30 Btu/h is determined to be 106°F.

The temperature of the aluminum tube is determined in a similar manner using the natural convection relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (4 - 2)/2 = 1 \text{ in} = 1/12 \text{ ft}$$

We start the calculations by assuming the tube temperature to be 200°F, and thus an average temperature of  $(106 + 200)/2 = 154^\circ\text{F} = 614 \text{ R}$ . This gives

$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_i - T_o)L_c^3}{\nu^2} \text{Pr} \\ &= \frac{(32.2 \text{ ft/s}^2)[1/614 \text{ R}](200 - 106 \text{ R})(1/12 \text{ ft})^3}{(2.117 \times 10^{-4} \text{ ft}^2/\text{s})^2} (0.7184) = 4.579 \times 10^4 \end{aligned}$$

The effective thermal conductivity is

$$\begin{aligned} F_{\text{cyl}} &= \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} \\ &= \frac{[\ln(4/2)]^4}{(1/12 \text{ ft})^3[(2/12 \text{ ft})^{-3/5} + (4/12 \text{ ft})^{-3/5}]^5} = 0.1466 \end{aligned}$$

$$\begin{aligned} k_{\text{eff}} &= 0.386k \left( \frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{cyl}} \text{Ra}_L)^{1/4} \\ &= 0.386(0.01653 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}) \left( \frac{0.7184}{0.861 + 0.7184} \right) (0.1466 \times 4.579 \times 10^4)^{1/4} \\ &= 0.04743 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F} \end{aligned}$$

Then the rate of heat transfer between the cylinders becomes

$$\begin{aligned} \dot{Q} &= \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)} (T_i - T_o) \\ &= \frac{2\pi(0.04743 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F})}{\ln(4/2)} (200 - 106)^\circ\text{F} = 40.4 \text{ Btu/h} \end{aligned}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 200°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **180°F**. Therefore, the tube will reach an equilibrium temperature of 180°F when the pump fails.

**Discussion** Note that we have not considered heat loss by radiation in the calculations, and thus the tube temperature determined is probably too high. This problem is considered in Chap. 22 by accounting for the effect of radiation heat transfer.

## 20-6 ■ COMBINED NATURAL AND FORCED CONVECTION

The presence of a temperature gradient in a fluid in a gravity field always gives rise to natural convection currents, and thus heat transfer by natural convection. Therefore, forced convection is always accompanied by natural convection.

We mentioned earlier that the convection heat transfer coefficient, natural or forced, is a strong function of the fluid velocity. Heat transfer coefficients encountered in forced convection are typically much higher than those encountered in natural convection because of the higher fluid velocities associated with forced convection. As a result, we tend to ignore natural convection in heat transfer analyses that involve forced convection, although we recognize that natural convection always accompanies forced convection. The error involved in ignoring natural convection is negligible at high velocities but may be considerable at low velocities associated with forced convection. Therefore, it is desirable to have a criterion to assess the relative magnitude of natural convection in the presence of forced convection.

For a given fluid, it is observed that the parameter  $Gr/Re^2$  represents the importance of natural convection relative to forced convection. This is not surprising since the convection heat transfer coefficient is a strong function of the Reynolds number  $Re$  in forced convection and the Grashof number  $Gr$  in natural convection.

A plot of the nondimensionalized heat transfer coefficient for combined natural and forced convection on a vertical plate is given in Fig. 20–32 for different fluids. We note from this figure that natural convection is negligible when  $Gr/Re^2 < 0.1$ , forced convection is negligible when  $Gr/Re^2 > 10$ , and neither is negligible when  $0.1 < Gr/Re^2 < 10$ . Therefore, both natural and forced convection must be considered in heat transfer calculations when the  $Gr$  and  $Re^2$  are of the same order of magnitude (one is within a factor of 10 times the other). Note that forced convection is small relative to natural convection only in the rare case of extremely low forced flow velocities.

Natural convection may *help* or *hurt* forced convection heat transfer, depending on the relative directions of *buoyancy-induced* and the *forced convection* motions (Fig. 20–33):

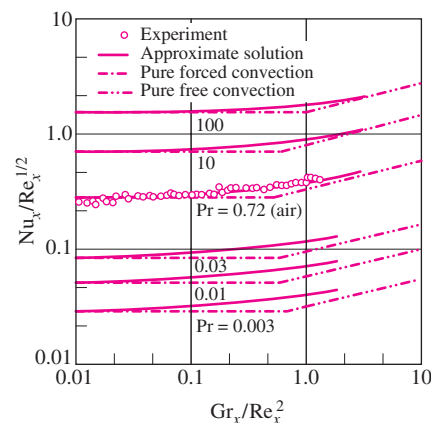


FIGURE 20–32

Variation of the local Nusselt number  $Nu_x$  for combined natural and forced convection from a hot isothermal vertical plate (from Lloyd and Sparrows).

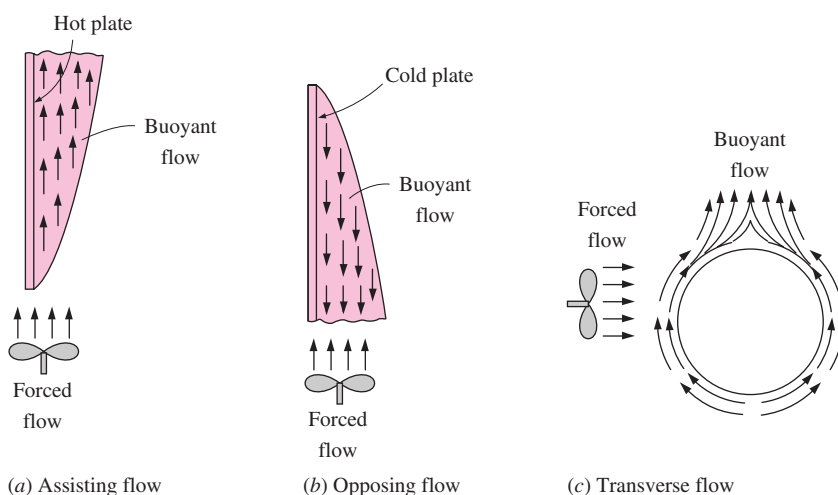


FIGURE 20–33

Natural convection can *enhance* or *inhibit* heat transfer, depending on the relative directions of *buoyancy-induced motion* and the *forced convection motion*.

1. In *assisting flow*, the buoyant motion is in the *same* direction as the forced motion. Therefore, natural convection assists forced convection and *enhances* heat transfer. An example is upward forced flow over a hot surface.
2. In *opposing flow*, the buoyant motion is in the *opposite* direction to the forced motion. Therefore, natural convection resists forced convection and *decreases* heat transfer. An example is upward forced flow over a cold surface.
3. In *transverse flow*, the buoyant motion is *perpendicular* to the forced motion. Transverse flow enhances fluid mixing and thus *enhances* heat transfer. An example is horizontal forced flow over a hot or cold cylinder or sphere.

When determining heat transfer under combined natural and forced convection conditions, it is tempting to add the contributions of natural and forced convection in assisting flows and to subtract them in opposing flows. However, the evidence indicates differently. A review of experimental data suggests a correlation of the form

$$Nu_{\text{combined}} = (Nu_{\text{forced}}^n \pm Nu_{\text{natural}}^n)^{1/n} \quad (20-58)$$

where  $Nu_{\text{forced}}$  and  $Nu_{\text{natural}}$  are determined from the correlations for *pure forced* and *pure natural convection*, respectively. The plus sign is for *assisting* and *transverse* flows and the minus sign is for *opposing* flows. The value of the exponent  $n$  varies between 3 and 4, depending on the geometry involved. It is observed that  $n = 3$  correlates experimental data for vertical surfaces well. Larger values of  $n$  are better suited for horizontal surfaces.

A question that frequently arises in the cooling of heat-generating equipment such as electronic components is whether to use a fan (or a pump if the cooling medium is a liquid)—that is, whether to utilize *natural* or *forced* convection in the cooling of the equipment. The answer depends on the maximum allowable operating temperature. Recall that the convection heat transfer rate from a surface at temperature  $T_s$  in a medium at  $T_\infty$  is given by

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where  $h$  is the convection heat transfer coefficient and  $A_s$  is the surface area. Note that for a fixed value of power dissipation and surface area,  $h$  and  $T_s$  are *inversely proportional*. Therefore, the device will operate at a *higher* temperature when  $h$  is low (typical of natural convection) and at a *lower* temperature when  $h$  is high (typical of forced convection).

Natural convection is the preferred mode of heat transfer since no blowers or pumps are needed and thus all the problems associated with these, such as noise, vibration, power consumption, and malfunctioning, are avoided. Natural convection is adequate for cooling *low-power-output* devices, especially when they are attached to extended surfaces such as heat sinks. For *high-power-output* devices, however, we have no choice but to use a blower or a pump to keep the operating temperature below the maximum allowable level. For *very-high-power-output* devices, even forced convection may not be sufficient to keep the surface temperature at the desirable levels. In such cases, we may have to use *boiling* and *condensation* to take advantage of the very high heat transfer coefficients associated with phase-change processes.

## SUMMARY

In this chapter, we have considered *natural convection* heat transfer where any fluid motion occurs by natural means such as buoyancy. The *volume expansion coefficient* of a substance represents the variation of the density of that substance with temperature at constant pressure, and for an ideal gas, it is expressed as  $\beta = 1/T$ , where  $T$  is the absolute temperature in K or R.

The flow regime in natural convection is governed by a dimensionless number called the *Grashof number*, which represents the ratio of the buoyancy force to the viscous force acting on the fluid and is expressed as

$$\text{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where  $L_c$  is the *characteristic length*, which is the height  $L$  for a vertical plate and the diameter  $D$  for a horizontal cylinder. The correlations for the Nusselt number  $\text{Nu} = hL_c/k$  in natural convection are expressed in terms of the *Rayleigh number* defined as

$$\text{Ra}_L = \text{Gr}_L \text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \text{Pr}$$

Nusselt number relations for various surfaces are given in Table 20–1. All fluid properties are evaluated at the film temperature of  $T_f = \frac{1}{2}(T_s + T_\infty)$ . The outer surface of a vertical cylinder can be treated as a vertical plate when the curvature effects are negligible. The characteristic length for a horizontal surface is  $L_c = A_s/p$ , where  $A_s$  is the surface area and  $p$  is the perimeter.

The average Nusselt number for vertical isothermal *parallel plates* of spacing  $S$  and height  $L$  is given as

$$\text{Nu} = \frac{hS}{k} = \left[ \frac{576}{(\text{Ra}_S S/L)^2} + \frac{2.873}{(\text{Ra}_S S/L)^{0.5}} \right]^{-0.5}$$

The optimum fin spacing for a vertical heat sink and the Nusselt number for optimally spaced fins is

$$S_{\text{opt}} = 2.714 \left( \frac{S^3 L}{\text{Ra}_S} \right)^{0.25} = 2.714 \frac{L}{\text{Ra}_L^{0.25}} \text{ and } \text{Nu} = \frac{hS_{\text{opt}}}{k} = 1.307$$

In a *horizontal rectangular enclosure* with the hotter plate at the top, heat transfer is by pure conduction and  $\text{Nu} = 1$ . When the hotter plate is at the bottom, the Nusselt is

$$\text{Nu} = 1 + 1.44 \left[ 1 - \frac{1708}{\text{Ra}_L} \right]^+ + \left[ \frac{\text{Ra}_L^{1/3}}{18} - 1 \right]^+ \quad \text{Ra}_L < 10^8$$

The notation  $[ ]^+$  indicates that if the quantity in the bracket is negative, it should be set equal to zero. For *vertical horizontal enclosures*, the Nusselt number can be determined from

$$\text{Nu} = 0.18 \left( \frac{\text{Pr}}{0.2 + \text{Pr}} \text{Ra}_L \right)^{0.29} \quad \begin{array}{l} 1 < H/L < 2 \\ \text{any Prandtl number} \\ \text{Ra}_L \text{Pr}/(0.2 + \text{Pr}) > 10^3 \end{array}$$

$$\text{Nu} = 0.22 \left( \frac{\text{Pr}}{0.2 + \text{Pr}} \text{Ra}_L \right)^{0.28} \left( \frac{H}{L} \right)^{-1/4} \quad \begin{array}{l} 2 < H/L < 10 \\ \text{any Prandtl number} \\ \text{Ra}_L < 10^{10} \end{array}$$

For aspect ratios greater than 10, Eqs. 20–46 and 20–47 should be used. For inclined enclosures, Eqs. 20–40 through 20–43 should be used.

For *concentric horizontal cylinders*, the rate of heat transfer through the annular space between the cylinders by natural convection per unit length is

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)} (T_i - T_o)$$

where

$$\frac{k_{\text{eff}}}{k} = 0.386 \left( \frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{cyl}} \text{Ra}_L)^{1/4}$$

and

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3 (D_i^{-3/5} + D_o^{-3/5})^5}$$

For a *spherical enclosure*, the rate of heat transfer through the space between the spheres by natural convection is expressed as

$$\dot{Q} = k_{\text{eff}} \pi \left( \frac{D_i D_o}{L_c} \right) (T_i - T_o)$$

where

$$\frac{k_{\text{eff}}}{k} = 0.74 \left( \frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{sph}} \text{Ra}_L)^{1/4}$$

$$L_c = (D_o - D_i)/2$$

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5}$$

The quantity  $k\text{Nu}$  is called the *effective thermal conductivity* of the enclosure, since a fluid in an enclosure behaves like a quiescent fluid whose thermal conductivity is  $k\text{Nu}$  as a result of convection currents. The fluid properties are evaluated at the average temperature of  $(T_i + T_o)/2$ .

For a given fluid, the parameter  $\text{Gr}/\text{Re}^2$  represents the importance of natural convection relative to forced convection. Natural convection is negligible when  $\text{Gr}/\text{Re}^2 < 0.1$ , forced convection is negligible when  $\text{Gr}/\text{Re}^2 > 10$ , and neither is negligible when  $0.1 < \text{Gr}/\text{Re}^2 < 10$ .

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## PROBLEMS\*

### Physical Mechanism of Natural Convection

**20-1C** What is natural convection? How does it differ from forced convection? What force causes natural convection currents?

**20-2C** In which mode of heat transfer is the convection heat transfer coefficient usually higher, natural convection or forced convection? Why?

**20-3C** Consider a hot boiled egg in a spacecraft that is filled with air at atmospheric pressure and temperature at all times. Will the egg cool faster or slower when the spacecraft is in space instead of on the ground? Explain.

**20-4C** What is buoyancy force? Compare the relative magnitudes of the buoyancy force acting on a body immersed in these mediums: (a) air, (b) water, (c) mercury, and (d) an evacuated chamber.

**20-5C** When will the hull of a ship sink in water deeper: when the ship is sailing in fresh water or in seawater? Why?

**20-6C** A person weighs himself on a waterproof spring scale placed at the bottom of a 1-m-deep swimming pool. Will the person weigh more or less in water? Why?

**20-7C** Consider two fluids, one with a large coefficient of volume expansion and the other with a small one. In what fluid will a hot surface initiate stronger natural convection currents? Why? Assume the viscosity of the fluids to be the same.

**20-8C** Consider a fluid whose volume does not change with temperature at constant pressure. What can you say about natural convection heat transfer in this medium?

**20-9C** What do the lines on an interferometer photograph represent? What do closely packed lines on the same photograph represent?

**20-10C** Physically, what does the Grashof number represent? How does the Grashof number differ from the Reynolds number?

**20-11** Show that the volume expansion coefficient of an ideal gas is  $\beta = 1/T$ , where  $T$  is the absolute temperature.

### Natural Convection over Surfaces

**20-12C** How does the Rayleigh number differ from the Grashof number?



**20-13C** Under what conditions can the outer surface of a vertical cylinder be treated as a vertical plate in natural convection calculations?

**20-14C** Will a hot horizontal plate whose back side is insulated cool faster or slower when its hot surface is facing down instead of up?

**20-15C** Consider laminar natural convection from a vertical hot-plate. Will the heat flux be higher at the top or at the bottom of the plate? Why?

**20-16** A 10-m-long section of a 6-cm-diameter horizontal hot-water pipe passes through a large room whose temperature is 22°C. If the temperature and the emissivity of the outer surface of the pipe are 65°C and 0.8, respectively, determine the rate of heat loss from the pipe by (a) natural convection and (b) radiation.

**20-17** Consider a wall-mounted power transistor that dissipates 0.18 W of power in an environment at 35°C. The transistor is 0.45 cm long and has a diameter of 0.4 cm. The emissivity of the outer surface of the transistor is 0.1, and the average temperature of the surrounding surfaces is 25°C. Disregarding any heat transfer from the base surface, determine the surface temperature of the transistor. Use air properties at 100°C. *Answer: 183°C*

\*Problems designated by a “C” are concept questions, and students are encouraged to answer them all. Problems designated by an “E” are in English units, and the SI users can ignore them. Problems with a CD-EES icon  are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon  are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

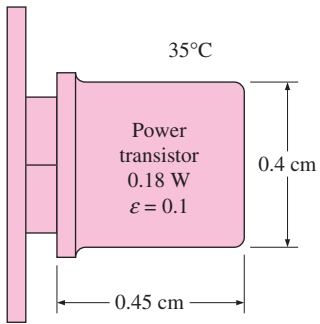




FIGURE P20-17

**20-18**  Reconsider Prob. 20-17. Using EES (or other) software, investigate the effect of ambient temperature on the surface temperature of the transistor. Let the environment temperature vary from 10°C to 40°C and assume that the surrounding surfaces are 10°C colder than the environment temperature. Plot the surface temperature of the transistor versus the environment temperature, and discuss the results.

**20-19E** Consider a 2-ft  $\times$  2-ft thin square plate in a room at 75°F. One side of the plate is maintained at a temperature of 130°F, while the other side is insulated. Determine the rate of heat transfer from the plate by natural convection if the plate is (a) vertical; (b) horizontal with hot surface facing up; and (c) horizontal with hot surface facing down.

**20-20E**  Reconsider Prob. 20-19E. Using EES (or other) software, plot the rate of natural convection heat transfer for different orientations of the plate as a function of the plate temperature as the temperature varies from 80°F to 180°F, and discuss the results.

**20-21** A 400-W cylindrical resistance heater is 1 m long and 0.5 cm in diameter. The resistance wire is placed horizontally in a fluid at 20°C. Determine the outer surface temperature of the resistance wire in steady operation if the fluid is (a) air and (b) water. Ignore any heat transfer by radiation. Use properties at 500°C for air and 40°C for water.

**20-22** Water is boiling in a 12-cm-deep pan with an outer diameter of 25 cm that is placed on top of a stove. The ambient

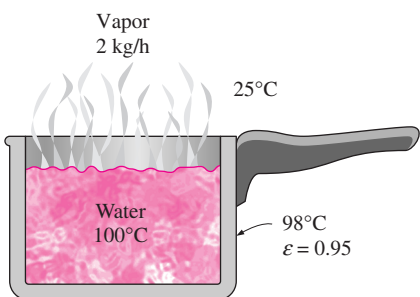


FIGURE P20-22

air and the surrounding surfaces are at a temperature of 25°C, and the emissivity of the outer surface of the pan is 0.95. Assuming the entire pan to be at an average temperature of 98°C, determine the rate of heat loss from the cylindrical side surface of the pan to the surroundings by (a) natural convection and (b) radiation. (c) If water is boiling at a rate of 2 kg/h at 100°C, determine the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water. The heat of vaporization of water at 100°C is 2257 kJ/kg.

**Answers:** 46.2 W, 56.1 W, 0.082

**20-23** Repeat Prob. 20-22 for a pan whose outer surface is polished and has an emissivity of 0.1.

**20-24** In a plant that manufactures canned aerosol paints, the cans are temperature-tested in water baths at 55°C before they are shipped to ensure that they will withstand temperatures up to 55°C during transportation and shelving. The cans, moving on a conveyor, enter the open hot water bath, which is 0.5 m deep, 1 m wide, and 3.5 m long, and move slowly in the hot water toward the other end. Some of the cans fail the test and explode in the water bath. The water container is made of sheet metal, and the entire container is at about the same temperature as the hot water. The emissivity of the outer surface of the container is 0.7. If the temperature of the surrounding air and surfaces is 20°C, determine the rate of heat loss from the four side surfaces of the container (disregard the top surface, which is open).

The water is heated electrically by resistance heaters, and the cost of electricity is \$0.085/kWh. If the plant operates 24 h a day 365 days a year and thus 8760 h a year, determine the annual cost of the heat losses from the container for this facility.

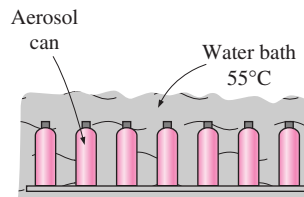


FIGURE P20-24

**20-25** Reconsider Prob. 20-24. In order to reduce the heating cost of the hot water, it is proposed to insulate the side and bottom surfaces of the container with 5-cm-thick fiberglass insulation ( $k = 0.035$  W/m  $\cdot$  °C) and to wrap the insulation with aluminum foil ( $\epsilon = 0.1$ ) in order to minimize the heat loss by radiation. An estimate is obtained from a local insulation contractor, who proposes to do the insulation job for \$350, including materials and labor. Would you support this proposal? How long will it take for the insulation to pay for itself from the energy it saves?

**20-26** Consider a 15-cm  $\times$  20-cm printed circuit board (PCB) that has electronic components on one side. The board is placed in a room at 20°C. The heat loss from the back

surface of the board is negligible. If the circuit board is dissipating 8 W of power in steady operation, determine the average temperature of the hot surface of the board, assuming the board is (a) vertical; (b) horizontal with hot surface facing up; and (c) horizontal with hot surface facing down. Take the emissivity of the surface of the board to be 0.8 and assume the surrounding surfaces to be at the same temperature as the air in the room. **Answers: (a) 46.6°C, (b) 42.6°C, (c) 50.7°C**

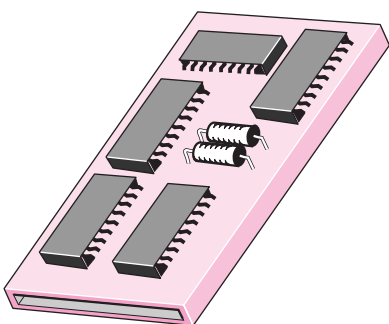




FIGURE P20-26

**20-27**  Reconsider Prob. 20-26. Using EES (or other) software, investigate the effects of the room temperature and the emissivity of the board on the temperature of the hot surface of the board for different orientations of the board. Let the room temperature vary from 5°C to 35°C and the emissivity from 0.1 to 1.0. Plot the hot surface temperature for different orientations of the board as the functions of the room temperature and the emissivity, and discuss the results.

**20-28**  A manufacturer makes absorber plates that are 1.2 m × 0.8 m in size for use in solar collectors. The back side of the plate is heavily insulated, while its front surface is coated with black chrome, which has an absorptivity of 0.87 for solar radiation and an emissivity of 0.09. Consider such a plate placed horizontally outdoors in calm air at 25°C. Solar radiation is incident on the plate at a rate of 700 W/m<sup>2</sup>. Taking the effective sky temperature to be

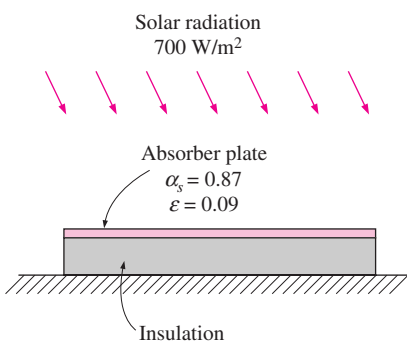


FIGURE P20-28

10°C, determine the equilibrium temperature of the absorber plate. What would your answer be if the absorber plate is made of ordinary aluminum plate that has a solar absorptivity of 0.28 and an emissivity of 0.07?

**20-29** Repeat Prob. 20-28 for an aluminum plate painted flat black (solar absorptivity 0.98 and emissivity 0.98) and also for a plate painted white (solar absorptivity 0.26 and emissivity 0.90).

**20-30** The following experiment is conducted to determine the natural convection heat transfer coefficient for a horizontal cylinder that is 80 cm long and 2 cm in diameter. A 80-cm-long resistance heater is placed along the centerline of the cylinder, and the surfaces of the cylinder are polished to minimize the radiation effect. The two circular side surfaces of the cylinder are well insulated. The resistance heater is turned on, and the power dissipation is maintained constant at 40 W. If the average surface temperature of the cylinder is measured to be 120°C in the 20°C room air when steady operation is reached, determine the natural convection heat transfer coefficient. If the emissivity of the outer surface of the cylinder is 0.1 and a 5 percent error is acceptable, do you think we need to do any correction for the radiation effect? Assume the surrounding surfaces to be at 20°C also.

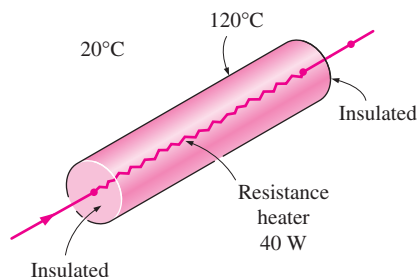


FIGURE P20-30

**20-31** Thick fluids such as asphalt and waxes and the pipes in which they flow are often heated in order to reduce the viscosity of the fluids and thus to reduce the pumping costs. Consider the flow of such a fluid through a 100-m-long pipe of outer diameter 30 cm in calm ambient air at 0°C. The pipe is heated electrically, and a thermostat keeps the outer surface

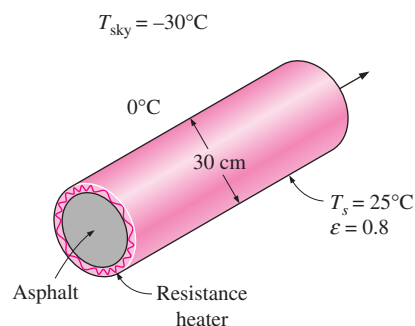


FIGURE P20-31

temperature of the pipe constant at  $25^{\circ}\text{C}$ . The emissivity of the outer surface of the pipe is 0.8, and the effective sky temperature is  $-30^{\circ}\text{C}$ . Determine the power rating of the electric resistance heater, in kW, that needs to be used. Also, determine the cost of electricity associated with heating the pipe during a 10-h period under the above conditions if the price of electricity is  $\$0.09/\text{kWh}$ . **Answers:** 29.1 kW,  $\$26.2$

**20–32** Reconsider Prob. 20–31. To reduce the heating cost of the pipe, it is proposed to insulate it with sufficiently thick fiberglass insulation ( $k = 0.035 \text{ W/m} \cdot ^{\circ}\text{C}$ ) wrapped with aluminum foil ( $\varepsilon = 0.1$ ) to cut down the heat losses by 85 percent. Assuming the pipe temperature to remain constant at  $25^{\circ}\text{C}$ , determine the thickness of the insulation that needs to be used. How much money will the insulation save during this 10-h period? **Answers:** 1.3 cm,  $\$22.3$

**20–33E** Consider an industrial furnace that resembles a 13-ft-long horizontal cylindrical enclosure 8 ft in diameter whose end surfaces are well insulated. The furnace burns natural gas at a rate of 48 therms/h (1 therm = 100,000 Btu). The combustion efficiency of the furnace is 82 percent (i.e., 18 percent of the chemical energy of the fuel is lost through the flue gases as a result of incomplete combustion and the flue gases leaving the furnace at high temperature). If the heat loss from the outer surfaces of the furnace by natural convection and radiation is not to exceed 1 percent of the heat generated inside, determine the highest allowable surface temperature of the furnace. Assume the air and wall surface temperature of the room to be  $75^{\circ}\text{F}$ , and take the emissivity of the outer surface of the furnace to be 0.85. If the cost of natural gas is  $\$0.65/\text{therm}$  and the furnace operates 2800 h per year, determine the annual cost of this heat loss to the plant.

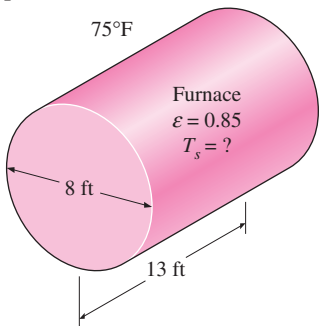


FIGURE P20–33

**20–34** Consider a 1.2-m-high and 2-m-wide glass window with a thickness of 6 mm, thermal conductivity  $k = 0.78 \text{ W/m} \cdot ^{\circ}\text{C}$ , and emissivity  $\varepsilon = 0.9$ . The room and the walls that face the window are maintained at  $25^{\circ}\text{C}$ , and the average temperature of the inner surface of the window is measured to be  $5^{\circ}\text{C}$ . If the temperature of the outdoors is  $-5^{\circ}\text{C}$ , determine (a) the convection heat transfer coefficient on the inner surface of the window; (b) the rate of total heat transfer through the window; and (c) the combined natural convection and radiation heat transfer coefficient on the outer

surface of the window. Is it reasonable to neglect the thermal resistance of the glass in this case?

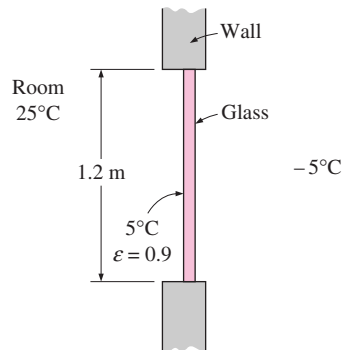


FIGURE P20–34

**20–35** A 3-mm-diameter and 12-m-long electric wire is tightly wrapped with a 1.5-mm-thick plastic cover whose thermal conductivity and emissivity are  $k = 0.15 \text{ W/m} \cdot ^{\circ}\text{C}$  and  $\varepsilon = 0.9$ . Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to calm atmospheric air at  $T_{\infty} = 30^{\circ}\text{C}$ , determine the temperature at the interface of the wire and the plastic cover in steady operation. Take the surrounding surfaces to be at about the same temperature as the air.

**20–36** During a visit to a plastic sheeting plant, it was observed that a 60-m-long section of a 2-in nominal (6.03-cm-outer-diameter) steam pipe extended from one end of the plant to the other with no insulation on it. The temperature measurements at several locations revealed that the average temperature of the exposed surfaces of the steam pipe was  $170^{\circ}\text{C}$ , while the temperature of the surrounding air was  $20^{\circ}\text{C}$ . The outer surface of the pipe appeared to be oxidized, and its emissivity can be taken to be 0.7. Taking the temperature of the surrounding surfaces to be  $20^{\circ}\text{C}$  also, determine the rate of heat loss from the steam pipe.

Steam is generated in a gas furnace that has an efficiency of 78 percent, and the plant pays  $\$0.538$  per therm (1 therm = 105,500 kJ) of natural gas. The plant operates 24 h a day 365 days a year, and thus 8760 h a year. Determine the annual cost of the heat losses from the steam pipe for this facility.

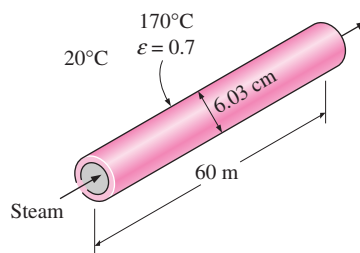




FIGURE P20–36

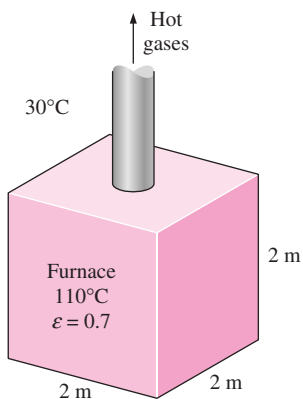
**20–37**  Reconsider Prob. 20–36. Using EES (or other) software, investigate the effect of the surface temperature of the steam pipe on the rate of heat loss from the pipe and the annual cost of this heat loss. Let the surface temperature vary from 100°C to 200°C. Plot the rate of heat loss and the annual cost as a function of the surface temperature, and discuss the results.

**20–38** Reconsider Prob. 20–36. In order to reduce heat losses, it is proposed to insulate the steam pipe with 5-cm-thick fiberglass insulation ( $k = 0.038 \text{ W/m} \cdot ^\circ\text{C}$ ) and to wrap it with aluminum foil ( $\varepsilon = 0.1$ ) in order to minimize the radiation losses. Also, an estimate is obtained from a local insulation contractor, who proposed to do the insulation job for \$750, including materials and labor. Would you support this proposal? How long will it take for the insulation to pay for itself from the energy it saves? Assume the temperature of the steam pipe to remain constant at 170°C.

**20–39** A 30-cm  $\times$  30-cm circuit board that contains 121 square chips on one side is to be cooled by combined natural convection and radiation by mounting it on a vertical surface in a room at 25°C. Each chip dissipates 0.05 W of power, and the emissivity of the chip surfaces is 0.7. Assuming the heat transfer from the back side of the circuit board to be negligible, and the temperature of the surrounding surfaces to be the same as the air temperature of the room, determine the surface temperature of the chips. **Answer:** 33.4°C

**20–40** Repeat Prob. 20–39 assuming the circuit board to be positioned horizontally with (a) chips facing up and (b) chips facing down.

**20–41**  The side surfaces of a 2-m-high cubic industrial furnace burning natural gas are not insulated, and the temperature at the outer surface of this section is measured to be 110°C. The temperature of the furnace room, including its surfaces, is 30°C, and the emissivity of the outer surface of the furnace is 0.7. It is proposed that this section of the furnace wall be insulated with glass wool insulation ( $k = 0.038 \text{ W/m} \cdot ^\circ\text{C}$ ) wrapped by a reflective sheet ( $\varepsilon = 0.2$ ) in order to reduce the

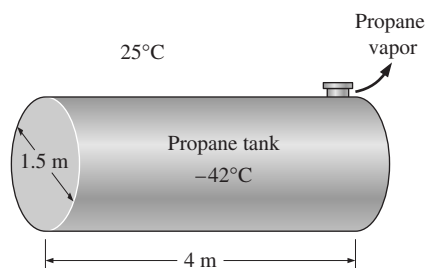


**FIGURE P20–41**

heat loss by 90 percent. Assuming the outer surface temperature of the metal section still remains at about 110°C, determine the thickness of the insulation that needs to be used.

The furnace operates continuously throughout the year and has an efficiency of 78 percent. The price of the natural gas is \$0.55/therm (1 therm = 105,500 kJ of energy content). If the installation of the insulation will cost \$550 for materials and labor, determine how long it will take for the insulation to pay for itself from the energy it saves.

**20–42** A 1.5-m-diameter, 4-m-long cylindrical propane tank is initially filled with liquid propane, whose density is 581 kg/m<sup>3</sup>. The tank is exposed to the ambient air at 25°C in calm weather. The outer surface of the tank is polished so that the radiation heat transfer is negligible. Now a crack develops at the top of the tank, and the pressure inside drops to 1 atm while the temperature drops to –42°C, which is the boiling temperature of propane at 1 atm. The heat of vaporization of propane at 1 atm is 425 kJ/kg. The propane is slowly vaporized as a result of the heat transfer from the ambient air into the tank, and the propane vapor escapes the tank at –42°C through the crack. Assuming the propane tank to be at about the same temperature as the propane inside at all times, determine how long it will take for the tank to empty if it is not insulated.



**FIGURE P20–42**

**20–43E** An average person generates heat at a rate of 287 Btu/h while resting in a room at 77°F. Assuming one-quarter of this heat is lost from the head and taking the emissivity of the skin to be 0.9, determine the average surface temperature of the head when it is not covered. The head can be approximated as a 12-in-diameter sphere, and the interior surfaces of the room can be assumed to be at the room temperature.

**20–44** An incandescent lightbulb is an inexpensive but highly inefficient device that converts electrical energy into light. It converts about 10 percent of the electrical energy it consumes into light while converting the remaining 90 percent into heat. The glass bulb of the lamp heats up very quickly as a result of absorbing all that heat and dissipating it to the surroundings by convection and radiation. Consider an 8-cm-diameter 60-W lightbulb in a room at 25°C. The emissivity of the glass is 0.9. Assuming that 10 percent of the energy passes through the glass bulb as light with negligible absorption and the rest of the energy is absorbed and dissipated by the bulb itself by natural convection and radiation, determine the equilibrium



temperature of the glass bulb. Assume the interior surfaces of the room to be at room temperature. **Answer:** 169°C

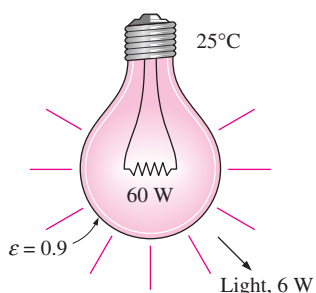



FIGURE P20-44

**20-45** A 40-cm-diameter, 110-cm-high cylindrical hot-water tank is located in the bathroom of a house maintained at 20°C. The surface temperature of the tank is measured to be 44°C and its emissivity is 0.4. Taking the surrounding surface temperature to be also 20°C, determine the rate of heat loss from all surfaces of the tank by natural convection and radiation.

**20-46** A 28-cm-high, 18-cm-long, and 18-cm-wide rectangular container suspended in a room at 24°C is initially filled with cold water at 2°C. The surface temperature of the container is observed to be nearly the same as the water temperature inside. The emissivity of the container surface is 0.6, and the temperature of the surrounding surfaces is about the same as the air temperature. Determine the water temperature in the container after 3 h, and the average rate of heat transfer to the water. Assume the heat transfer coefficient on the top and bottom surfaces to be the same as that on the side surfaces.

**20-47**  Reconsider Prob. 20-46. Using EES (or other) software, plot the water temperature in the container as a function of the heating time as the time varies from 30 min to 10 h, and discuss the results.

**20-48** A room is to be heated by a coal-burning stove, which is a cylindrical cavity with an outer diameter of 32 cm and a height of 70 cm. The rate of heat loss from the room is estimated to be 1.2 kW when the air temperature in the room is maintained constant at 24°C. The emissivity of the stove surface is 0.85 and the average temperature of the surrounding wall surfaces is 17°C. Determine the surface temperature of the stove. Neglect the transfer from the bottom surface and take the heat transfer coefficient at the top surface to be the same as that on the side surface.

The heating value of the coal is 30,000 kJ/kg, and the combustion efficiency is 65 percent. Determine the amount of coal burned a day if the stove operates 14 h a day.

**20-49** The water in a 40-L tank is to be heated from 15°C to 45°C by a 6-cm-diameter spherical heater whose surface temperature is maintained at 85°C. Determine how long the heater should be kept on.

## Natural Convection from Finned Surfaces and PCBs

**20-50C** Why are finned surfaces frequently used in practice? Why are the finned surfaces referred to as heat sinks in the electronics industry?

**20-51C** Why are heat sinks with closely packed fins not suitable for natural convection heat transfer, although they increase the heat transfer surface area more?

**20-52C** Consider a heat sink with optimum fin spacing. Explain how heat transfer from this heat sink will be affected by (a) removing some of the fins on the heat sink and (b) doubling the number of fins on the heat sink by reducing the fin spacing. The base area of the heat sink remains unchanged at all times.

**20-53** Aluminum heat sinks of rectangular profile are commonly used to cool electronic components. Consider a 7.62-cm-long and 9.68-cm-wide commercially available heat sink whose cross section and dimensions are as shown in Fig. P20-53. The heat sink is oriented vertically and is used to cool a power transistor that can dissipate up to 125 W of power. The back surface of the heat sink is insulated. The surfaces of the heat sink are untreated, and thus they have a low emissivity (under 0.1). Therefore, radiation heat transfer from the heat sink can be neglected. During an experiment conducted in room air at 22°C, the base temperature of the heat sink was measured to be 120°C when the power dissipation of the transistor was 15 W. Assuming the entire heat sink to be at the base temperature, determine the average natural convection heat transfer coefficient for this case. **Answer:** 7.1 W/m<sup>2</sup> · °C

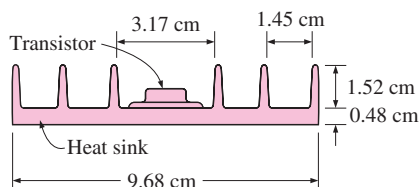


FIGURE P20-53

**20-54** Reconsider the heat sink in Prob. 20-53. In order to enhance heat transfer, a shroud (a thin rectangular metal plate) whose surface area is equal to the base area of the heat sink is

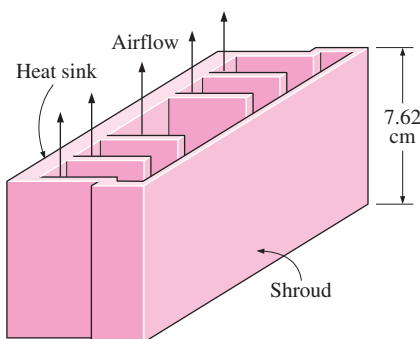



FIGURE P20-54

placed very close to the tips of the fins such that the interfin spaces are converted into rectangular channels. The base temperature of the heat sink in this case was measured to be 108°C. Noting that the shroud loses heat to the ambient air from both sides, determine the average natural convection heat transfer coefficient in this shrouded case. (For complete details, see Çengel and Zing.)

**20-55E** A 6-in-wide and 8-in-high vertical hot surface in 78°F air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.08 in thick and 8 in long in the vertical direction and have a height of 1.2 in from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 180°F.

**20-56E**  Reconsider Prob. 20-55E. Using EES (or other) software, investigate the effect of the length of the fins in the vertical direction on the optimum fin spacing and the rate of heat transfer by natural convection. Let the fin length vary from 2 in to 10 in. Plot the optimum fin spacing and the rate of convection heat transfer as a function of the fin length, and discuss the results.

**20-57** A 12.1-cm-wide and 18-cm-high vertical hot surface in 25°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction. Determine the optimum fin height and the rate of heat transfer by natural convection from the heat sink if the base temperature is 65°C.

### Natural Convection inside Enclosures

**20-58C** The upper and lower compartments of a well-insulated container are separated by two parallel sheets of glass with an air space between them. One of the compartments is to be filled with a hot fluid and the other with a cold fluid. If it is desired that heat transfer between the two compartments be minimal, would you recommend putting the hot fluid into the upper or the lower compartment of the container? Why?

**20-59C** Someone claims that the air space in a double-pane window enhances the heat transfer from a house because of the natural convection currents that occur in the air space and recommends that the double-pane window be replaced by a single sheet of glass whose thickness is equal to the sum of the thicknesses of the two glasses of the double-pane window to save energy. Do you agree with this claim?

**20-60C** Consider a double-pane window consisting of two glass sheets separated by a 1-cm-wide air space. Someone suggests inserting a thin vinyl sheet in the middle of the two glasses to form two 0.5-cm-wide compartments in the window in order to reduce natural convection heat transfer through the window. From a heat transfer point of view, would you be in favor of this idea to reduce heat losses through the window?

**20-61C** What does the effective conductivity of an enclosure represent? How is the ratio of the effective conductivity to thermal conductivity related to the Nusselt number?

**20-62** Show that the thermal resistance of a rectangular enclosure can be expressed as  $R = \delta/(Ak Nu)$ , where  $k$  is the thermal conductivity of the fluid in the enclosure.

**20-63E** A vertical 4-ft-high and 6-ft-wide double-pane window consists of two sheets of glass separated by a 1-in air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 65°F and 40°F, determine the rate of heat transfer through the window by (a) natural convection and (b) radiation. Also, determine the  $R$ -value of insulation of this window such that multiplying the inverse of the  $R$ -value by the surface area and the temperature difference gives the total rate of heat transfer through the window. The effective emissivity for use in radiation calculations between two large parallel glass plates can be taken to be 0.82.

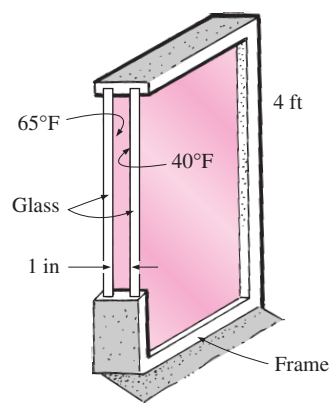




FIGURE P20-63E

**20-64E**  Reconsider Prob. 20-63E. Using EES (or other) software, investigate the effect of the air gap thickness on the rates of heat transfer by natural convection and radiation, and the  $R$ -value of insulation. Let the air gap thickness vary from 0.2 in to 2.0 in. Plot the rates of heat transfer by natural convection and radiation, and the  $R$ -value of insulation as a function of the air gap thickness, and discuss the results.

**20-65** Two concentric spheres of diameters 15 cm and 25 cm are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are  $T_1 = 350$  K and  $T_2 = 275$  K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

**20-66**  Reconsider Prob. 20-65. Using EES (or other) software, plot the rate of natural convection heat transfer as a function of the hot surface temperature of the sphere as the temperature varies from 300 K to 500 K, and discuss the results.

**20-67** Flat-plate solar collectors are often tilted up toward the sun in order to intercept a greater amount of direct solar radiation. The tilt angle from the horizontal also affects the rate of heat loss from the collector. Consider a 2-m-high and



3-m-wide solar collector that is tilted at an angle  $\theta$  from the horizontal. The back side of the absorber is heavily insulated. The absorber plate and the glass cover, which are spaced 2.5 cm from each other, are maintained at temperatures of  $80^\circ\text{C}$  and  $40^\circ\text{C}$ , respectively. Determine the rate of heat loss from the absorber plate by natural convection for  $\theta = 0^\circ$ ,  $20^\circ$ , and  $90^\circ$ .

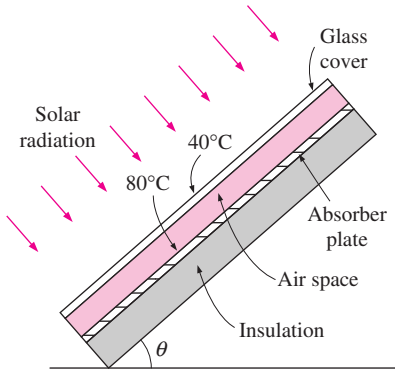


FIGURE P20-67

**20-68** A simple solar collector is built by placing a 5-cm-diameter clear plastic tube around a garden hose whose outer diameter is 1.6 cm. The hose is painted black to maximize solar absorption, and some plastic rings are used to keep the spacing between the hose and the clear plastic cover constant. During a clear day, the temperature of the hose is measured to be  $65^\circ\text{C}$ , while the ambient air temperature is  $26^\circ\text{C}$ . Determine the rate of heat loss from the water in the hose per meter of its length by natural convection. Also, discuss how the performance of this solar collector can be improved. *Answer: 8.2 W*

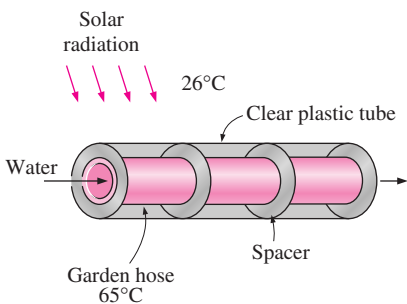



FIGURE P20-68

**20-69**  Reconsider Prob. 20-68. Using EES (or other) software, plot the rate of heat loss from the water by natural convection as a function of the ambient air temperature as the temperature varies from  $4^\circ\text{C}$  to  $40^\circ\text{C}$ , and discuss the results.

**20-70** A vertical 1.3-m-high, 2.8-m-wide double-pane window consists of two layers of glass separated by a 2.2-cm air

gap at atmospheric pressure. The room temperature is  $26^\circ\text{C}$  while the inner glass temperature is  $18^\circ\text{C}$ . Disregarding radiation heat transfer, determine the temperature of the outer glass layer and the rate of heat loss through the window by natural convection.

**20-71** Consider two concentric horizontal cylinders of diameters 55 cm and 65 cm, and length 125 cm. The surfaces of the inner and outer cylinders are maintained at  $46^\circ\text{C}$  and  $74^\circ\text{C}$ , respectively. Determine the rate of heat transfer between the cylinders by natural convection if the annular space is filled with (a) water and (b) air.

### Combined Natural and Forced Convection

**20-72C** When is natural convection negligible and when is it not negligible in forced convection heat transfer?

**20-73C** Under what conditions does natural convection enhance forced convection, and under what conditions does it hurt forced convection?

**20-74C** When neither natural nor forced convection is negligible, is it correct to calculate each independently and add them to determine the total convection heat transfer?

**20-75** Consider a 5-m-long vertical plate at  $85^\circ\text{C}$  in air at  $30^\circ\text{C}$ . Determine the forced motion velocity above which natural convection heat transfer from this plate is negligible.

*Answer: 9.04 m/s*

**20-76** Reconsider Prob. 20-75. Using EES (or other) software, plot the forced motion velocity above which natural convection heat transfer is negligible as a function of the plate temperature as the temperature varies from  $50^\circ\text{C}$  to  $150^\circ\text{C}$ , and discuss the results.

**20-77** Consider a 5-m-long vertical plate at  $60^\circ\text{C}$  in water at  $25^\circ\text{C}$ . Determine the forced motion velocity above which natural convection heat transfer from this plate is negligible. Take  $\beta = 0.0004 \text{ K}^{-1}$  for water.

**20-78** In a production facility, thin square plates  $2 \text{ m} \times 2 \text{ m}$  in size coming out of the oven at  $270^\circ\text{C}$  are cooled by blowing ambient air at  $30^\circ\text{C}$  horizontally parallel to their surfaces. Determine the air velocity above which the natural convection effects on heat transfer are less than 10 percent and thus are negligible.

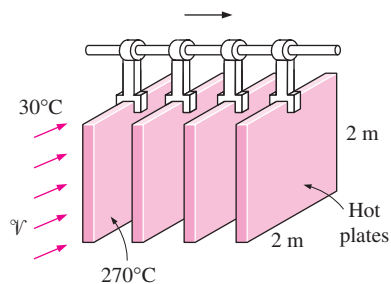


FIGURE P20-78

**20–79** A 12-cm-high and 20-cm-wide circuit board houses 100 closely spaced logic chips on its surface, each dissipating 0.05 W. The board is cooled by a fan that blows air over the hot surface of the board at 35°C at a velocity of 0.5 m/s. The heat transfer from the back surface of the board is negligible. Determine the average temperature on the surface of the circuit board assuming the air flows vertically upward along the 12-cm-long side by (a) ignoring natural convection and (b) considering the contribution of natural convection. Disregard any heat transfer by radiation.

### Review Problems

**20–80E** A 0.1-W small cylindrical resistor mounted on a lower part of a vertical circuit board is 0.3 in long and has a diameter of 0.2 in. The view of the resistor is largely blocked by another circuit board facing it, and the heat transfer through the connecting wires is negligible. The air is free to flow through the large parallel flow passages between the boards as a result of natural convection currents. If the air temperature at the vicinity of the resistor is 120°F, determine the approximate surface temperature of the resistor.

*Answer:* 212°F

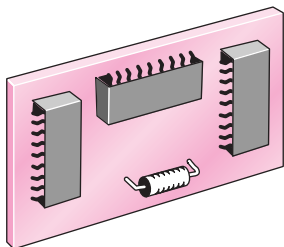


FIGURE P20–80E

**20–81** An ice chest whose outer dimensions are 30 cm × 40 cm × 40 cm is made of 3-cm-thick Styrofoam ( $k = 0.033 \text{ W/m} \cdot ^\circ\text{C}$ ). Initially, the chest is filled with 30 kg of ice at 0°C, and the inner surface temperature of the ice chest can be taken to be 0°C at all times. The heat of fusion of water at 0°C is 333.7 kJ/kg, and the surrounding ambient air is at 20°C. Disregarding any heat transfer from the 40 cm × 40 cm base of the ice chest, determine how long it will take for the ice in the chest to melt completely if the ice chest is subjected to (a) calm air and (b) winds at 50 km/h. Assume the heat transfer coefficient on the front, back, and top surfaces to be the same as that on the side surfaces.

**20–82** An electronic box that consumes 180 W of power is cooled by a fan blowing air into the box enclosure. The dimensions of the electronic box are 15 cm × 50 cm × 50 cm, and all surfaces of the box are exposed to the ambient except the base surface. Temperature measurements indicate that the box is at an average temperature of 32°C when the ambient temperature and the temperature of the surrounding walls are 25°C. If the

emissivity of the outer surface of the box is 0.85, determine the fraction of the heat lost from the outer surfaces of the electronic box.

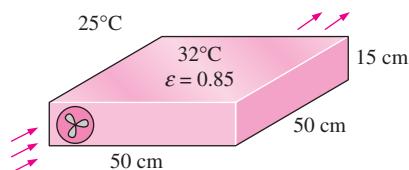


FIGURE P20–82

**20–83** A 6-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel ( $k = 15 \text{ W/m} \cdot ^\circ\text{C}$ ) is used to store iced water at 0°C in a room at 20°C. The walls of the room are also at 20°C. The outer surface of the tank is black (emissivity  $\varepsilon = 1$ ), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. Assuming the entire steel tank to be at 0°C and thus the thermal resistance of the tank to be negligible, determine (a) the rate of heat transfer to the iced water in the tank and (b) the amount of ice at 0°C that melts during a 24-h period.

*Answers:* (a) 15.4 kW, (b) 3988 kg

**20–84** Consider a 1.2-m-high and 2-m-wide double-pane window consisting of two 3-mm-thick layers of glass ( $k = 0.78 \text{ W/m} \cdot ^\circ\text{C}$ ) separated by a 3-cm-wide air space. Determine the steady rate of heat transfer through this window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is 0°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be  $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$  and  $h_2 = 25 \text{ W/m}^2 \cdot ^\circ\text{C}$  and disregard any heat transfer by radiation.

**20–85** An electric resistance space heater is designed such that it resembles a rectangular box 50 cm high, 80 cm long, and 15 cm wide filled with 45 kg of oil. The heater is to be placed against a wall, and thus heat transfer from its back surface is negligible for safety considerations. The surface temperature of the heater is not to exceed 45°C in a room at 25°C. Disregarding heat transfer from the bottom and top surfaces of the heater in anticipation that the top surface will be used as a shelf, determine the power rating of the heater in W. Take the emissivity of the outer surface of the heater to be 0.8 and the average temperature of the ceiling and wall surfaces to be the same as the room air temperature.

Also, determine how long it will take for the heater to reach steady operation when it is first turned on (i.e., for the oil temperature to rise from 25°C to 45°C). State your assumptions in the calculations.

**20–86** Skylights or “roof windows” are commonly used in homes and manufacturing facilities since they let natural light in during day time and thus reduce the lighting costs.

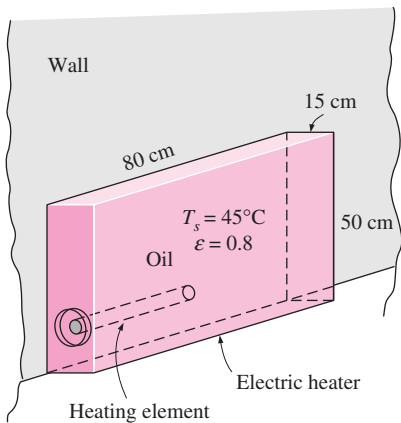


FIGURE P20-85

However, they offer little resistance to heat transfer, and large amounts of energy are lost through them in winter unless they are equipped with a motorized insulating cover that can be used in cold weather and at nights to reduce heat losses. Consider a 1-m-wide and 2.5-m-long horizontal skylight on the roof of a house that is kept at  $20^{\circ}\text{C}$ . The glazing of the skylight is made of a single layer of 0.5-cm-thick glass ( $k = 0.78 \text{ W/m} \cdot ^{\circ}\text{C}$  and  $\varepsilon = 0.9$ ). Determine the rate of heat loss through the skylight when the air temperature outside is  $-10^{\circ}\text{C}$  and the effective sky temperature is  $-30^{\circ}\text{C}$ . Compare your result with the rate of heat loss through an equivalent surface area of the roof that has a common R-5.34 construction in SI units (i.e., a thickness-to-effective-thermal-conductivity ratio of  $5.34 \text{ m}^2 \cdot ^{\circ}\text{C/W}$ ).

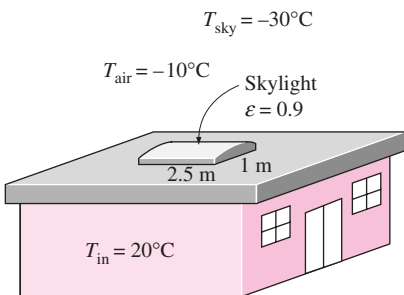


FIGURE P20-86

**20-87** A solar collector consists of a horizontal copper tube of outer diameter 5 cm enclosed in a concentric thin glass tube of 9 cm diameter. Water is heated as it flows through the tube, and the annular space between the copper and glass tube is filled with air at 1 atm pressure. During a clear day, the temperatures of the tube surface and the glass cover are measured to be  $60^{\circ}\text{C}$  and  $32^{\circ}\text{C}$ , respectively. Determine the rate of heat loss from the collector by natural convection per meter length of the tube. **Answer:** 17.4 W

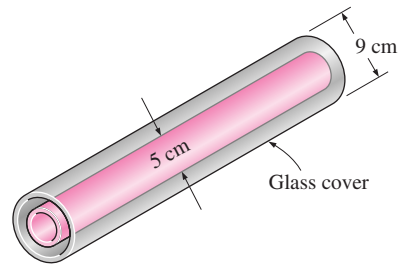


FIGURE P20-87

**20-88** A solar collector consists of a horizontal aluminum tube of outer diameter 4 cm enclosed in a concentric thin glass tube of 7 cm diameter. Water is heated as it flows through the aluminum tube, and the annular space between the aluminum and glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 20 W per meter length, and the temperature of the ambient air outside is  $30^{\circ}\text{C}$ . Approximating the surfaces of the tube and the glass cover as being black (emissivity  $\varepsilon = 1$ ) in radiation calculations and taking the effective sky temperature to be  $20^{\circ}\text{C}$ , determine the temperature of the aluminum tube when equilibrium is established (i.e., when the net heat loss from the tube by convection and radiation equals the amount of solar energy absorbed by the tube).

**20-89E** The components of an electronic system dissipating 180 W are located in a 4-ft-long horizontal duct whose cross section is 6 in  $\times$  6 in. The components in the duct are cooled by forced air, which enters at  $85^{\circ}\text{F}$  at a rate of 22 cfm and leaves at  $100^{\circ}\text{F}$ . The surfaces of the sheet metal duct are not painted, and thus radiation heat transfer from the outer surfaces is negligible. If the ambient air temperature is  $80^{\circ}\text{F}$ , determine (a) the heat transfer from the outer surfaces of the duct to the ambient air by natural convection and (b) the average temperature of the duct.

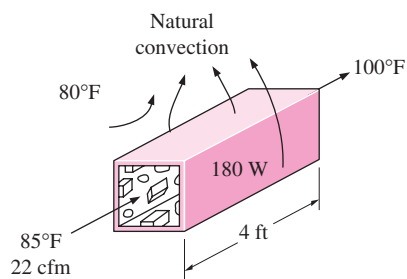


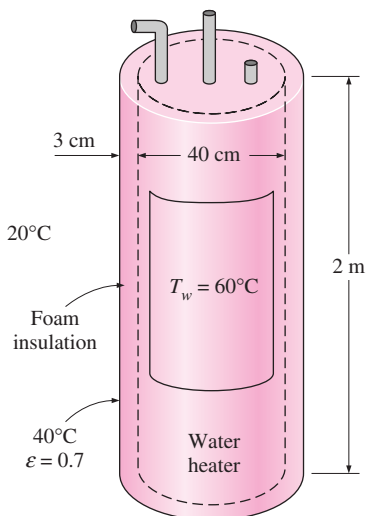
FIGURE P20-89E

**20-90E** Repeat Prob. 20-90E for a circular horizontal duct of diameter 4 in.

**20-91E** Repeat Prob. 20-90E assuming the fan fails and thus the entire heat generated inside the duct must be rejected to the ambient air by natural convection through the outer surfaces of the duct.

**20-92** Consider a cold aluminum canned drink that is initially at a uniform temperature of  $5^{\circ}\text{C}$ . The can is 12.5 cm high and has a diameter of 6 cm. The emissivity of the outer surface of the can is 0.6. Disregarding any heat transfer from the bottom surface of the can, determine how long it will take for the average temperature of the drink to rise to  $7^{\circ}\text{C}$  if the surrounding air and surfaces are at  $25^{\circ}\text{C}$ . **Answer: 12.1 min**

**20-93** Consider a 2-m-high electric hot-water heater that has a diameter of 40 cm and maintains the hot water at  $60^{\circ}\text{C}$ . The tank is located in a small room at  $20^{\circ}\text{C}$  whose walls and the ceiling are at about the same temperature. The tank is placed in a 46-cm-diameter sheet metal shell of negligible thickness, and the space between the tank and the shell is filled with foam insulation. The average temperature and emissivity of the outer surface of the shell are  $40^{\circ}\text{C}$  and 0.7, respectively. The price of electricity is  $\$0.08/\text{kWh}$ . Hot-water tank insulation kits large enough to wrap the entire tank are available on the market for about  $\$30$ . If such an insulation is installed on this water tank by the home owner himself, how long will it take for this additional insulation to pay for itself? Disregard any heat loss from the top and bottom surfaces, and assume the insulation to reduce the heat losses by 80 percent.

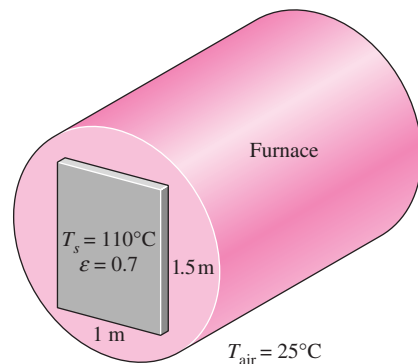


**FIGURE P20-93**

**20-94** During a plant visit, it was observed that a 1.5-m-high and 1-m-wide section of the vertical front section of a natural gas furnace wall was too hot to touch. The temperature measurements on the surface revealed that the average temperature of the exposed hot surface was  $110^{\circ}\text{C}$ , while the temperature of the surrounding air was  $25^{\circ}\text{C}$ . The surface appeared to be

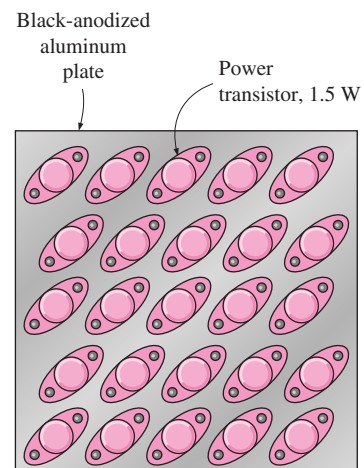
oxidized, and its emissivity can be taken to be 0.7. Taking the temperature of the surrounding surfaces to be  $25^{\circ}\text{C}$  also, determine the rate of heat loss from this furnace.

The furnace has an efficiency of 79 percent, and the plant pays  $\$0.75$  per therm of natural gas. If the plant operates 10 h a day, 310 days a year, and thus 3100 h a year, determine the annual cost of the heat loss from this vertical hot surface on the front section of the furnace wall.



**FIGURE P20-94**

**20-95** A group of 25 power transistors, dissipating 1.5 W each, are to be cooled by attaching them to a black-anodized square aluminum plate and mounting the plate on the wall of a room at  $30^{\circ}\text{C}$ . The emissivity of the transistor and the plate surfaces is 0.9. Assuming the heat transfer from the back side of the plate to be negligible and the temperature of the surrounding surfaces to be the same as the air temperature of the room, determine the size of the plate if the average surface temperature of the plate is not to exceed  $50^{\circ}\text{C}$ . **Answer: 43 cm 3 43 cm**



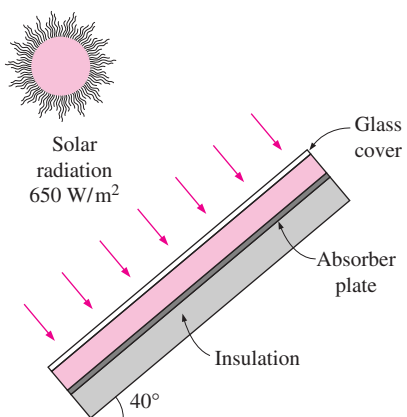
**FIGURE P20-95**

**20-96** Repeat Prob. 20-109 assuming the plate to be positioned horizontally with (a) transistors facing up and (b) transistors facing down.

**20–97E** Hot water is flowing at an average velocity of 4 ft/s through a cast iron pipe ( $k = 30 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}$ ) whose inner and outer diameters are 1.0 in and 1.2 in, respectively. The pipe passes through a 50-ft-long section of a basement whose temperature is  $60^\circ\text{F}$ . The emissivity of the outer surface of the pipe is 0.5, and the walls of the basement are also at about  $60^\circ\text{F}$ . If the inlet temperature of the water is  $150^\circ\text{F}$  and the heat transfer coefficient on the inner surface of the pipe is  $30 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$ , determine the temperature drop of water as it passes through the basement.

**20–98** Consider a flat-plate solar collector placed horizontally on the flat roof of a house. The collector is 1.5 m wide and 6 m long, and the average temperature of the exposed surface of the collector is  $42^\circ\text{C}$ . Determine the rate of heat loss from the collector by natural convection during a calm day when the ambient air temperature is  $15^\circ\text{C}$ . Also, determine the heat loss by radiation by taking the emissivity of the collector surface to be 0.9 and the effective sky temperature to be  $-30^\circ\text{C}$ . **Answers:** 1295 W, 2921 W

**20–99** Solar radiation is incident on the glass cover of a solar collector at a rate of  $650 \text{ W/m}^2$ . The glass transmits 88 percent of the incident radiation and has an emissivity of 0.90. The hot-water needs of a family in summer can be met completely by a collector 1.5 m high and 2 m wide, and tilted  $40^\circ$  from the horizontal. The temperature of the glass cover is measured to be  $40^\circ\text{C}$  on a calm day when the surrounding air temperature is  $20^\circ\text{C}$ . The effective sky temperature for radiation exchange between the glass cover and the open sky is  $-40^\circ\text{C}$ . Water enters the tubes attached to the absorber plate at a rate of 1 kg/min. Assuming the back surface of the absorber plate to be heavily insulated and the only heat loss occurs through the glass cover, determine (a) the total rate of heat loss from the collector; (b) the collector efficiency, which is the ratio of the amount of heat transferred to the water to the solar energy incident on the



**FIGURE P20–99**

collector; and (c) the temperature rise of water as it flows through the collector.

### Design and Essay Problems

**20–100** Write a computer program to evaluate the variation of temperature with time of thin square metal plates that are removed from an oven at a specified temperature and placed vertically in a large room. The thickness, the size, the initial temperature, the emissivity, and the thermophysical properties of the plate as well as the room temperature are to be specified by the user. The program should evaluate the temperature of the plate at specified intervals and tabulate the results against time. The computer should list the assumptions made during calculations before printing the results.

For each step or time interval, assume the surface temperature to be constant and evaluate the heat loss during that time interval and the temperature drop of the plate as a result of this heat loss. This gives the temperature of the plate at the end of a time interval, which is to serve as the initial temperature of the plate for the beginning of the next time interval.

Try your program for 0.2-cm-thick vertical copper plates of  $40 \text{ cm} \times 40 \text{ cm}$  in size initially at  $300^\circ\text{C}$  cooled in a room at  $25^\circ\text{C}$ . Take the surface emissivity to be 0.9. Use a time interval of 1 s in calculations, but print the results at 10-s intervals for a total cooling period of 15 min.

**20–101** Write a computer program to optimize the spacing between the two glasses of a double-pane window. Assume the spacing is filled with dry air at atmospheric pressure. The program should evaluate the recommended practical value of the spacing to minimize the heat losses and list it when the size of the window (the height and the width) and the temperatures of the two glasses are specified.

**20–102** Contact a manufacturer of aluminum heat sinks and obtain their product catalog for cooling electronic components by natural convection and radiation. Write an essay on how to select a suitable heat sink for an electronic component when its maximum power dissipation and maximum allowable surface temperature are specified.

**20–103** The top surfaces of practically all flat-plate solar collectors are covered with glass in order to reduce the heat losses from the absorber plate underneath. Although the glass cover reflects or absorbs about 15 percent of the incident solar radiation, it saves much more from the potential heat losses from the absorber plate, and thus it is considered to be an essential part of a well-designed solar collector. Inspired by the energy efficiency of double-pane windows, someone proposes to use double glazing on solar collectors instead of a single glass. Investigate if this is a good idea for the town in which you live. Use local weather data and base your conclusion on heat transfer analysis and economic considerations.