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2019

/1 1. De functie f is gegeven (we noteren $2\mathbb{N}$ voor de even natuurlijke getallen):

$$f : \mathbb{N} \to 2\mathbb{N} \times (\mathbb{N} \setminus 2\mathbb{N}) : n \longmapsto (2n, 2n+1)$$

Is f injectief, surjectief, bijectief?

injection

$f(n) = f(n') \Rightarrow (2n, 2n+1) = (2n', 2n'+1)$

$\Rightarrow \begin{cases} 2n = 2n' \\ 2n+1 = 2n'+1 \end{cases}$

$\Rightarrow n = n'$ f is injectief

Surjective

$f(n) = (x, y)$ $x, y \in 2\mathbb{N} \times (\mathbb{N} \setminus 2\mathbb{N})$

$(2n, 2n+1) = (x, y)$

$2n = x$

$2n+1 = y$

even

$\Rightarrow \left(\frac{x}{2}, \frac{y-1}{2}\right)$

$h : \mathbb{R}^2 \longrightarrow \mathbb{R}^2 : (x,y) \longmapsto (x+y,\ x-y)$

Injection

$h(x,y) = h(x',y') \qquad (x,y), (x',y') \in \mathbb{R}^2$

$h(x,y) = h(x',y') \Longrightarrow (x+y,\ x-y) = (x'+y',\ x'-y')$

$$\Longrightarrow \begin{cases} x+y = x'+y' \\ x-y = x'-y' \end{cases}$$

$$\underset{vg_2 - vg_1}{\overset{vg_1+vg_2}{\Longrightarrow}} \begin{cases} 2x = 2x' \\ 2 = 2y' \end{cases}$$

$\Rightarrow (x,y) = (x',y)$

h unjective

Surjection

$(x,y)\ (X,Y) \in \mathbb{R}^2$

$h(x,y) = (X,Y)$

$h(x,y) = (X,y) \Rightarrow (x+y,\ x-y) = (X,\ y)$

$$\Rightarrow \begin{cases} x+y = X \\ x-y = Y \end{cases}$$

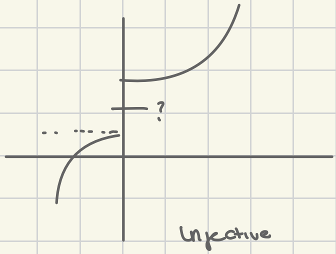
$$\underset{vg_2 - vg_1}{\overset{vg_1+vg_1}{\Rightarrow}} \begin{cases} 2x = X+Y \\ -2y = Y-X \Longleftrightarrow 2y = X-Y \end{cases}$$

Snypunt

$\Rightarrow (x,y) = \left(\frac{X+Y}{2},\ \frac{X-Y}{2}\right) \longrightarrow \in (x,y)$

$h(x,y) = (X,Y)$

Surjectif



Injective

$f : \mathbb{N} \longrightarrow \mathbb{N} : n \longmapsto n+1$

Surjection

$y=0$ n'a pas d'antecedent

f n'est PAS surjective

$g : \mathbb{Z} \to \mathbb{Z} : n \longrightarrow n+1$

est surjective

$m \in \mathbb{Z} \quad n = m-1$

$g(n) = m-1+1 = m$

$]-\infty, 0[$

$f :]4, +\infty[\longrightarrow \mathbb{R} : x \longrightarrow \dfrac{-1}{\sqrt{x}-2}$

$f(5) = \dfrac{-1}{\sqrt{5}-2} = \dfrac{-1}{2.1-2} = \dfrac{-1}{0{,}2} = -5$ $\qquad f(6) = \dfrac{-1}{\sqrt{6}-2} = \dfrac{-1}{0{,}4} = -2{,}5$

Injection

$f(x) = f(x') \qquad x, x' \in]4, +\infty[$

$$f(x) = f(x') \Longrightarrow \frac{-1}{\sqrt{x}-2} = \frac{-1}{\sqrt{x'}-2}$$

$$\Longrightarrow -(\sqrt{x}-2) = -(\sqrt{x'}-2)$$
$$\Rightarrow \sqrt{x}-2 = \sqrt{x'}-2$$
$$\Rightarrow \sqrt{x} = \sqrt{x'}$$
$$\Rightarrow x = x'$$

f is injective

Surjection

$f(x) = y \qquad y \in \mathbb{R} \qquad x \in]4, +\infty[$

$$f(x) = y \Rightarrow \frac{-1}{\sqrt{x}-2} = \frac{y}{1}$$

$$\Rightarrow y(\sqrt{x}-2) = -1$$

$$\Rightarrow y\sqrt{x} - 2y = -1$$

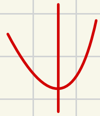
$$\Rightarrow x = \left(\frac{-1+2y}{y}\right)^2$$

kunnen we het inverteren

y	$-\infty$		0		$\frac{1}{2}$		$+\infty$
y		$-$	0	$+$	$+$	$+$	
$-1+2y$		$-$	$-$	$-$	0	$+$	
		$+$	$\Vert$	$-$	0	$+$	

- olduğu yerler var daarom niet surj

$f: \mathbb{R} \longrightarrow [1, +\infty[: x \longmapsto x^4 + 1$



Injection

$g: [0, +\infty[\to$

$f(x) = f(x')$ $\quad x, x' \in \mathbb{R}$

$f(x) = f(x') \Rightarrow x^4 + 1 = x'^4 + 1$ $\quad f(-1) = f(1)$

$\Rightarrow x^4 = x'^4$

$(-2)^4 \quad (2)^4 + 1$ $\quad x = x'$ $\quad$ ~~injective~~

$-$

$a^4 - b^4 = 0$

$(a^2)^2 - (b^2)^2 = 0$

$(a^2+b^2)(a^2-b^2) = 0$

$(a^2+b^2)(a+b)(a-b) = 0$

$a^2 = -b^2 \wedge a = -b \wedge a = b$

2 uitkomsten $\quad$ niet injectief

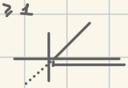
Surjection

$f(x) = y$ $\quad x \in \cancel{\mathbb{R}}$ $\quad]0, +\infty[$, $y \in [1, +\infty[$

$f(x) = y \Rightarrow x^4 + 1 = y$

$\Rightarrow x^4 = y - 1$

$y \geq 1$ $\quad x^2 = \sqrt{y-1}$ $\quad x^2 = \sqrt{\sqrt{x} - 1}$



$g^{-1} :]1, +\infty[\to [0, +\infty[$

$x \longmapsto \sqrt{\sqrt{x-1}}$

$f:]1, +\infty[\longrightarrow \mathbb{R} : x \longrightarrow \sqrt{x^2 + 4x - 5}$

$f(a) = f(b)$ $\qquad$ $a, b \in]1, +\infty[$

$\Rightarrow \sqrt{a^2 + 4a - 5} = \sqrt{b^2 + 4b - 5}$

$\Rightarrow a^2 + 4a - 5 = b^2 + 4b - 5$

$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$

$\Rightarrow (a - b)(a + b) + 4(a - b) = 0$

$(a - b)(a + b + 4) = 0$

$a = b \wedge a + b = -4$ ✗

kan niet positief

Surj

$\sqrt{x^2 + 4x - 5} = y$

$g:$

$f : \mathbb{R} \setminus \{2\} \longrightarrow \mathbb{R} : x \longmapsto \dfrac{1}{x-2}$

Injectivity

$f(a) = f(b)$ $\qquad a, b \in \mathbb{R} \setminus \{2\}$

$$f(a) = f(b) \Longrightarrow \frac{1}{a-2} = \frac{1}{b-2}$$

$$\Longrightarrow a-2 = b-2$$

$$\Rightarrow a = b$$

f is injective

Surjectivity

We compare the Range(f) and Codomain of f

↳ if Range(f) = Codomain of f then f is surjective

Codomain = $\mathbb{R}$

What is the Codomain?

$$f(x) = \left(\frac{1}{x-2} \right)$$

$\Rightarrow f(x) = y$ $\qquad y \in \mathbb{R}$

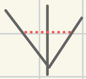
$$\Rightarrow \frac{1}{x-2} = y$$

$\Rightarrow x-2 = \frac{1}{y}$ $\qquad y \neq 0$

$f : \mathbb{Z} \to \mathbb{N} : x \longmapsto |x| + 2$

Injective ?

$|a| = |-a|$



$f(a) = f(b) \implies |a|+2 = |b|+2$

$|a| = |b|$

↳ does this imply $a=b$

$|-1| = |1|$ does not imply $-1 = 1$

aanpassing : $\mathbb{Z}^+ \longrightarrow \mathbb{N}$

Surjectivity

$f(x) = a$ $\quad a \in \mathbb{N}$

$\Rightarrow |x| = a-2$

$>$ $a - 2 \geqslant 0$

$a \geqslant 2$ Contradiction

$a \in \mathbb{N}$ $\quad$ Not Surjective

$\mathbb{Z}^+ \longrightarrow [2, +\infty[$

$$f: \mathbb{R} \setminus \{2\} \longrightarrow \mathbb{R} \setminus \{5\}: \; x \longrightarrow \frac{5x+1}{x-2}$$

Injectivity

$f(a) = f(b)$ $\qquad$ $a, b \in \mathbb{R} \setminus \{2\}$

$$f(a) = f(b) \Rightarrow \frac{5a+1}{a-2} = \frac{5b+1}{b-2}$$

$$\Rightarrow \frac{5a+10-10+1}{a-2} = \frac{5b+10-10+1}{b-2}$$

$$\Rightarrow \frac{5a-10}{a-2} + \frac{11}{a-2} = \frac{5b-10}{b-2} - \frac{11}{b-2}$$

$$\Rightarrow \cancel{5} + \frac{11}{a-2} = \cancel{5} + \frac{11}{b-2}$$

$$\Rightarrow \cancel{11}(b-\cancel{2}) = \cancel{11}(a-\cancel{2})$$

$$\Rightarrow b = a$$ $\qquad$ injective

Surjetivity

$f(x) = a$ $\qquad$ $a \in \mathbb{R} \setminus 5$

$$\Rightarrow \frac{5x+1}{x-2} = a$$

$$\Rightarrow 5x+1 = a(x-2)$$

$$5x+1 = ax - 2a$$

$$x(5-a) = -1-2a$$

$$x = \frac{-1-2a}{5-a}$$

↑ a only can not be 5
cuase $5-5=0$. in the teller geen 0
So $\mathbb{R} \setminus \{5\}$ klopt ✓
Function is bijective

$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2 : (x, y) \longmapsto (2x, x+y)$

injectiviteit

$a, b \in \mathbb{R}^2$

$f(a,b) = f(a',b') \Rightarrow (2a, a+b) = (2a', a'+b')$

$$\Rightarrow \begin{cases} 2a = 2a' \\ a+b = a+b' \end{cases}$$

$$\Rightarrow \begin{cases} a = a' \\ a+b = a+b' \Rightarrow b = b' \end{cases}$$ injective

Surjectivite

$\Rightarrow f(x,y) = (a,b) \quad (a,b) \in \mathbb{R} \quad x, y \in \mathbb{R}$

$\Rightarrow (2x, x+y) = (a, b)$

$$\Rightarrow \begin{cases} 2x = a \\ x+y = b \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{a}{2} \\ y = b - \frac{a}{2} \end{cases}$$

$$f : \mathbb{R} \longrightarrow \mathbb{R} : x \longmapsto x^2$$

Injectivite

$\Rightarrow \quad f(a) = f(b)$

$\Rightarrow \quad a^2 = b^2$

$\Rightarrow \quad a^2 - b^2 = 0$

$\Rightarrow \quad (a+b)(a-b) = 0$

$a = -b \quad \vee \quad a = +b$ — 2 uitkomsten kan niet — AANpassing $\mathbb{N} \longrightarrow \mathbb{R}$

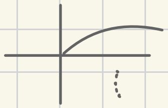
Surjectivite

$f(x) = y \qquad x \in \mathbb{N} \quad y \in \mathbb{R}$

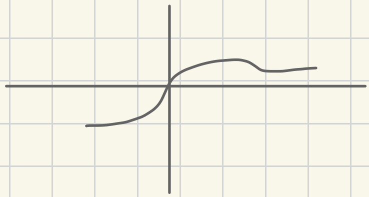
$x^2 = y$

$x = \sqrt{y}$ — AANPASSING $\mathbb{N} \rightarrow \mathbb{N} : x \mapsto x^2$

y kan niet negatief zijn



$\mathbb{R}$: $\longrightarrow \dfrac{x}{x^2+1}$



$$f: M_{2\times 2} \longrightarrow \mathbb{R}^4 : \begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow (d, -c, 3a, b)$$

Injective

$$f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = f\left(\begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix}\right)$$

$$\Rightarrow (d, -c, 3a, b) = (d', -c', 3a', b')$$

$$\Rightarrow \begin{cases} d = d' \\ -c = -c' & \Rightarrow c = c' \\ 3a = 3a' & \Rightarrow a = a' \\ b = b' \end{cases}$$

injection

Surjective

$$f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (x, y, z, w) \qquad x, y, z, w \in \mathbb{R}^4$$

$$(d, -c, 3a, b) = (x, y, z, w)$$

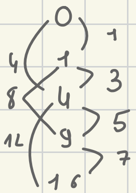
$$\begin{cases} d = x \\ -c = y & \Rightarrow c = -y \\ 3a = z & \Rightarrow a = \frac{z}{3} \\ b = w \end{cases} \qquad \begin{bmatrix} \frac{z}{3} & w \\ -y & x \end{bmatrix}$$

Proof Given Any $(x, y, z, w) \in \mathbb{R}$ we have

$$\begin{bmatrix} z/3 & w \\ -y & x \end{bmatrix} \in M_{2\times 2} \quad \text{Such that } f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (d, -c, 3a, b)$$

$$f\left(\begin{bmatrix} z/3 & w \\ -y & x \end{bmatrix}\right) = \left(x, -(-y), 3\frac{z}{3}, w\right)$$

$$f: \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z} : (m,n) \longrightarrow m^2 - n^2$$



② ? doesnt exist (Not Surjective)

$$f: \mathbb{Z}^+ \longrightarrow \mathbb{Z} : n \longmapsto (-1)^n \cdot n$$

Injection

$$m, n \in \mathbb{Z}^+$$

$$f(m) = f(n) \Rightarrow (-1)^n \cdot n = (-1)^m \cdot m$$

$$\Rightarrow \frac{(-1)^n \cdot n}{(-1)^m} = m$$

$$\Rightarrow \overbrace{(-1)^{n-m} \cdot \underset{>0}{n}}^{\text{has to be } >0} = \underset{>0}{m} \qquad (-1)^{n-1} = 1$$

$$m = 1 \cdot n \qquad \text{injective}$$

$f : \mathbb{N}^2 \longrightarrow \mathbb{N} : (n, m) \longmapsto n.m$

Injective

$f(n,m) = f(n',m')$ $\quad\quad$ $n,m \; n',m' \in \mathbb{N}_2$

$\Rightarrow n m = n' m'$

$20 = 1 \times 20 = f(1, 20)$
$\quad = 2 \times 10 = f(2, 10)$

$f(1, 20) = f(2, 10)$
n'est pas injective

Surjective

$f(n, m) = a$ $\quad\quad$ $a \in \mathbb{N}$
$n.m = a$

$a = 1 \times a$
$f(1, a) = a \quad \in \mathbb{N}$ $\quad\quad$ c'est surjective

$g : \mathbb{N} \longrightarrow \mathbb{N}^2 : n \longmapsto (n, (n+1)^2)$

g is not surjective

$f(a) = f(b)$ $\quad\quad$ $a, b \in \mathbb{N}$

$\Rightarrow (a, (a+1)^2) = (b, (b+1)^2)$

$\Rightarrow \begin{cases} a = b \\ (a+1)^2 = (b+1)^2 \end{cases}$ ook zelfde aangezien $a = b$

g is injective

Surjective

g to be surjective, every element in $\mathbb{N}^2$ must be image of some $n \in \mathbb{N}$

$\hookrightarrow$ 3 wordt nooit bereikt bv

$(n, (n+1)^2)$ 2'nd component always square of n

(2 , 3) krijg je nooit dus het is niet surjectief

$$f : \mathbb{N}^2 \longrightarrow \mathbb{N} : (n, p) \longrightarrow 2^n (2p + 1)$$

$$f(a, b) = f(a', b') \qquad \begin{matrix} a, b \\ a', b' \end{matrix} \in \mathbb{N}$$

$$\Longrightarrow 2^a \underbrace{(2b+1)}_{\text{oneven}} = 2^{a'} \underbrace{(2b'+1)}_{\text{oneven}}$$

$a = a'$

moet waar zijn
zodat de vergelijking zou kloppen

$2^x \underbrace{(2.4+1)}_{9} \qquad \underbrace{2^x}_{?} \underbrace{(2.4+1)}_{9}$

moet
zelfde
zijn want er zitten
geen andere priemfactoren
van 2 in 9

Surjectivite

$\mathbb{N}$