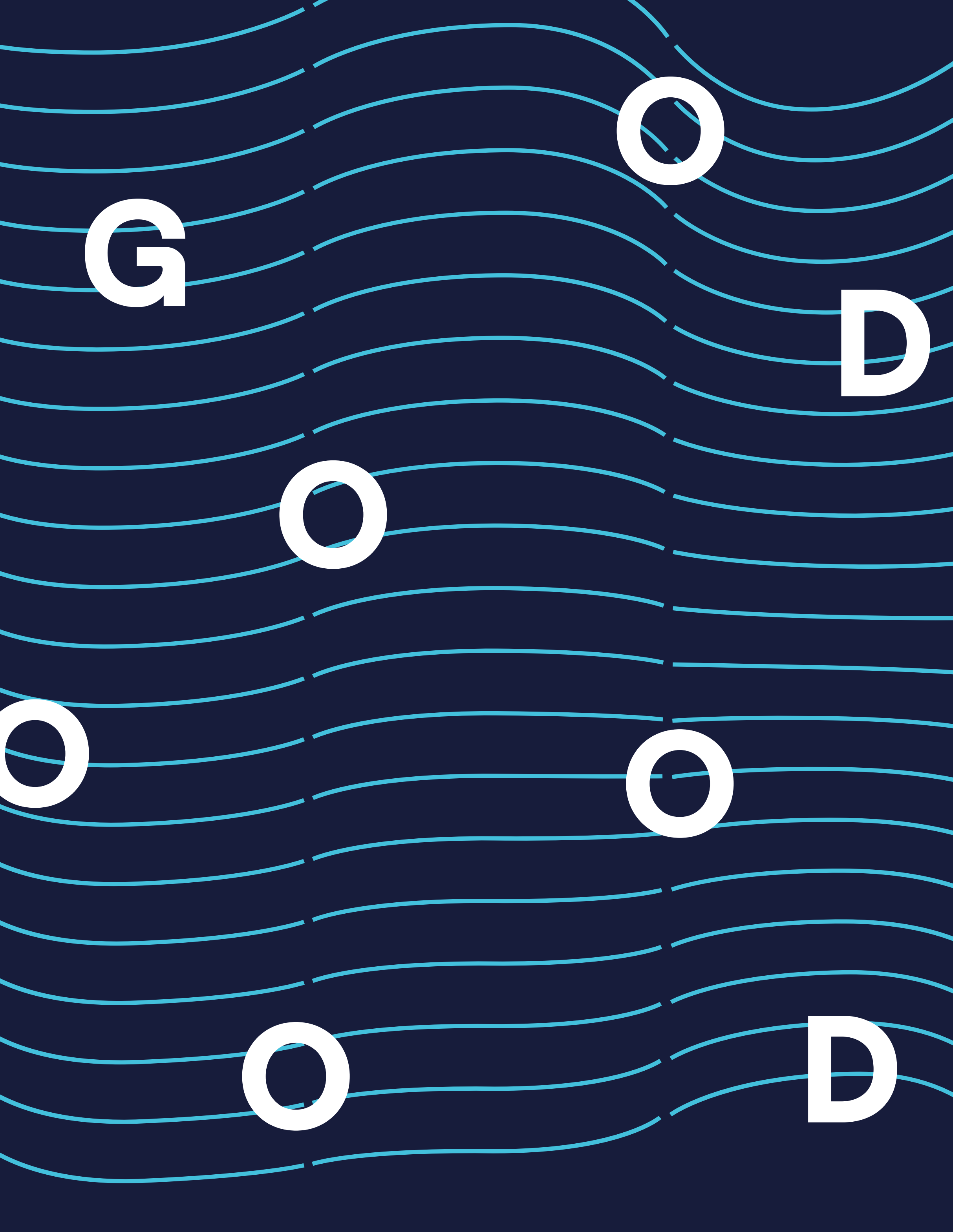
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Belangrijke formules

$\frac{1}{1-x}$ $\qquad \sum_{k=0}^{\infty} x^k$

$\frac{1}{1-ax}$ $\qquad \sum_{k=0}^{\infty} a^k x^k$

$\frac{1}{1-x^m}$ $\qquad \sum_{k=0}^{\infty} x^{mk}$

$\frac{1}{(1-x)^2}$ $\qquad \sum_{k=0}^{\infty} \binom{2+k-1}{k} x^k$

$(k+1)\, x^k$

breuk splitsen $\qquad \frac{A(y)+B(x)}{x \cdot y}$

ontbinden

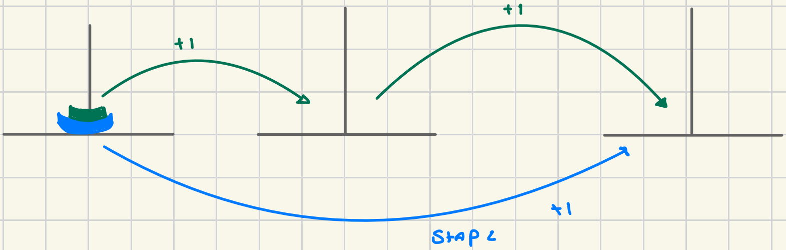
old	new
ax^2+bx+c	$a+bx+cx^2$
$D=b^2-4ac$	$D=b^2-4ac$
$x_1,x_2=\frac{-b\pm\sqrt{D}}{2a}$	$x_1\, x_2=\frac{-b\pm\sqrt{D}}{2a}$
$=a(x-x_1)(x-x_2)$	$a(1-x_1x)(1-x_2x)$

Haakjes: x buiten zetten

$x^2+x^3+x^4+x^5+\dots$ $\qquad$ heeft geen inverse?

opl: $x^2(\underbrace{1+x+x^2+\dots}_{\frac{1}{1-x}}) = x^2 \cdot \frac{1}{1-x}$

Torens van Hanoi



$F_2 = 3$

Genererende Functie

H_n = minimum van het aantal moves om dit te realiseren

$H_0 = 0$, $H_1 = 1$, $H_2 = 3$, $H_3 = 7$

Algemeen

$$H_n = 2H_{n-1} + 1$$

$$(H_n) = 0, 1, 3, 7, 15, 31$$

$$H_n = 2^n - 1$$

recursive

$$\begin{cases} H_0 = 0 \\ H_n = 2H_{n-1} + 1 \end{cases}$$

genererende

$$H(x) = \sum_{k=0}^{\infty} H_k x^k$$

$$H(x) = H_0 + H_1 x + H_2 x^2 + H_3 x^3 + H_4 x^4 + \dots$$

$$\overset{\text{Rec}}{=} H_0 + (2H_0 + 1)\underline{x} + (H_1 + 1)\underline{x^2} + (H_2 + 1)x^3 + \dots$$

2 oneind

$$= H_0 + 2x(H_0 + H_1 x + H_2 x^2 + H_3 x^3) + (x + x^2 + x^3 + x^4 + \dots)$$

$$\Rightarrow H(x) = 0 + 2x\,H(x) + \frac{x}{1-x}$$

$$(1-2x)H(x) = \frac{x}{1-x} \iff$$

Splitsen in parciel breuken

$$H(x) = \frac{x}{(1-x)(1-2x)} = \frac{A}{1-x} + \frac{B}{1-2x}$$

$$= \frac{A(1-2x) + B(1-x)}{(1-x)(1-2x)} = \frac{(A+B) + (-2A - B)x}{(1-x)(1-2x)}$$

Stelsel

cte

teller

① $x = A(1-2x) + B(1-x)$

$$\begin{cases} A + B = 0 \\ -2A - B = 1 \end{cases}$$

$Vgl_2 + Vgl$

$-A = 1 \Rightarrow A = -1$

$B = +1$

$$H(x) = \frac{-1}{1-x} + \frac{1}{1-2x} = -\sum_{k=0}^{\infty} x^k + \sum_{k=0}^{\infty} 2^k x^k$$

$$H(x) = \sum_{k=0}^{\infty} x^k (-1 + 2^k)$$

Vb oefening

$$\begin{cases} a_n = 1 & \text{als } n=0 \text{ of } n=1 \\ a_n = -a_{n-1} + 6a_{n-2} & \text{als } n \geq 2 \end{cases}$$

$1, 1, 5, 1, 29, -23$ $\qquad a_n = $ Formule in n

$$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$$

$$\overset{\text{Rec}}{=} 1 + x + (-a_1 + 6a_0)x^2 + (-a_2 + 6a_1)x^3 + (-a_3 + 6a_2)x^4$$

$$= 1 + x - x\underbrace{(a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \dots)}_{S(x) - a_0} + 6x^2 \underbrace{(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots)}_{S(x)}$$

$$S(x) = 1 + x - x S(x) + x + 6x^2 S(x)$$

$$S(x)(1 + x - 6x^2) = 1 + 2x$$

$$S(x) = \frac{1+2x}{1+x-6x^2}$$

$$= \frac{1+2x}{(1+3x)(1-2x)}$$

Splitsing

$$= \frac{A}{1-2x} + \frac{B}{1+3x} \quad \text{Andere}$$

$$= \frac{A(1+3x) \ B(1-2x)}{(1+3x)(1-2x)}$$

$D = 1 - 4 \cdot 1 \cdot -6 = 1 + 24 = 25$

$x_1 = \frac{-1-5}{2} = \frac{-6}{2} = -3$

$x_2 = \frac{-1+5}{2} = 2$

$1(1+3x)(1-2x)$

ontbinden

$a + bx + cx^2$

$D = b^2 - 4ac$

$x_1, x_2 = \frac{-b - \sqrt{D}}{2a}$

$a(1 - x x_1)(1 - x x_2)$

$$= \frac{(A + B) + (-2B + 3A)x}{(1+3x)(1-2x)}$$

Stelsel

$$\begin{cases} A + B = 1 & \cdot 2 \\ -2B + 3A = 2 \end{cases} \qquad \begin{matrix} c^{te} \\ \text{Macht } x \end{matrix}$$

$+5$

$vgl_2 + 2\, vgl_1$

$+5A = \frac{4}{5} \qquad B = \frac{1}{5}$

$$= \frac{4/5}{1-2x} + \frac{1/5}{1+3x}$$

$$= \frac{1}{5}\sum_{k=0}^{\infty} 2^k x^k - \frac{4}{5}\sum_{k=0}^{\infty} 3^k x^k = \sum_{k=0}^{\infty} x^k \frac{(4 \cdot 2^k + (-3^k))}{5}$$

$$a_n = \frac{2^{n+2} + (-3)^n}{5}$$

Vb oefening 2

Vb 7. 11

$$\begin{cases} a_0 = -6 \quad , \quad a_1 = 10 \\ a_n = a_{n-1} + 6a_{n-2} + 6n - 1 \end{cases}$$

$$a(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

$$= -6 + 10 + (a_1 + 6a_0 + 6 \cdot 2 - 1)x^2 + (a_2 + 6a_1 + 6 \cdot 3 - 1)x^3 + (a_3 + 6a_2 + 6 \cdot 4 - 1)x^4 + \dots$$

$$= -6 + 10 + x\underbrace{(a_1 x + a_2 x^2 + a_3 x^3 + \dots)}_{S(x) - a(0)} + 6x^2\underbrace{(a_0 + a_1 x + a_2 x^2 \dots)}_{S(x) - a_0} + 6\underbrace{(2x^2 + 3x^3 \dots)}_{\frac{x}{(1-x)^2} - x} - 1\underbrace{(x^2 + x^3 + x^4 + x^5 \dots)}_{\frac{-x^2}{1-x}}$$

$$= -6 + 10x + x\,a(x) + 6x^2 S(x) + 36x^2 + \frac{6x}{1-x^2} \cancel{-6x} \; \frac{-x^2}{1-x}$$

$x\,S(x)$

$$S(x) - x\,a(x) - 6x^2 a(x) = (-6 + 4x) + 6x^2(S(x)) + 36x^2 + \frac{6x}{1-x^2} - \frac{x^2}{1-x}$$

$$S(x)(1 - x - 6x^2) = \frac{(-6 + 10x)(1-x^2) + 6x - x^2(1-x)}{4 - x^2)}$$

$$\sum_{k=0}^{\infty} k x^k$$

$$\underbrace{0 + x}_{-} + 2x^2 + 3x^2$$

in ons tabel

$$\sum_{k=0}^{\infty} (k+1) x^k$$

$$1 + 2x + 3x^2 + 4x^3$$

Aanpassing ons vershuift

$$x(2x + 3x^2 + 4x^3 \dots$$

$$\frac{x}{(1-x)^2} - x$$

ontbinding

$$D = 1 - 4 \cdot 1 \cdot -6 = 25$$

$$x_1 x_2 = \frac{-b \pm \sqrt{D}}{2} \quad \begin{matrix} + \; 3 \\ - \; -2 \end{matrix}$$

$$S(x) = \frac{11x^3 - 27x^2 + 18x - 6}{(1-x)^2 (1-3x)(1+2x)}$$

$$S(x) = \frac{A}{(1-x)} + \frac{B}{(1-x)^2} + \frac{C}{1-3x} + \frac{D}{1+2x}$$

2^{de} macht	1	2	3^{macht}
3^{de} macht	1	2	

$$= \frac{A(1-x)(1-3x)(1+2x) + B(1-3x)(1+2x) + C(1+2x)(1-x)^2 + D(1+3x)(1-x)^2}{(1-x)^2(1-3x)(1+2x)}$$

$$= A(1 - 2x + 5x^2 + 6x^3) + B(1 - x - 6x^2) + C(1 - 3x^2 + 2x^3) + D(1 - 5x + 7x^2 - 3x^2)$$

Sklnel

$\Longrightarrow$ $D = -5$

$A = -1$

$C = 1$

$B = -1$

$$\frac{-1}{1-x} + \frac{-1}{(1-x)^2} + \frac{1}{1-3x} - \frac{5}{1+2x}$$

$$-\sum_{k=0}^{\infty} x^k + \sum_{k=0}^{\infty} (k+1)x^k + \sum 3^k x^k - 5\sum 2^k x^k$$

$$\sum \left(-1 - k - 1 + 3^k - 5(-2)^k \right) x^k)$$

$$a_k = 3^k - 5(-2)^k (-2)^k - k - 2$$