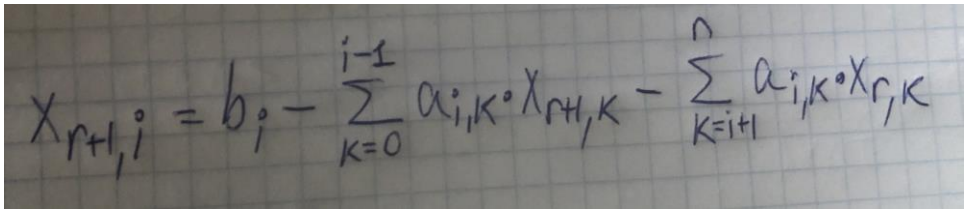
<u>שאלות 20 ו-29:</u>

בשני השיטות, המערכת קודם עוברת בדיקה האם המערכת בעלת אלכסון דומיננטי, ובמידה וצריך, נהפכת לכזאת. אם הדבר לא ניתן לביצוע, השיטה מחזורה NONE בתור פלט, מה שאומר שלא ניתן לפתור את הבעיה בשיטה זו.

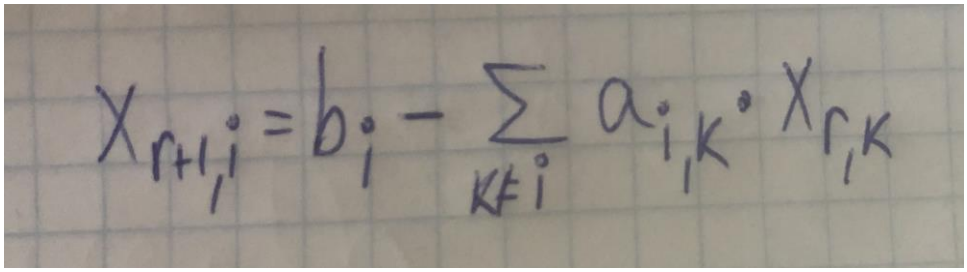
אחרי זה, נקבע ניחוש התחלתי במידה ולא ניתן, ומתחילות האיטרציות עד למציאת ניחוש קרוב מספיק לקודמו.

נוסחאות:

גאוס-זיידל:



יעקובי:



בשני אלו, x(r+1)(i) זה העיבר ה-i של הניחוש החדש x(r+1)

גודל השגיאה נקבע להיות 1E-8 בשני השיטות.

- **שאלה מס׳ 20**

פתרו את המטריצה הבאה בשתי דרכים והשוו בין התוצאות

$$\begin{pmatrix} 10 & 8 & 1 \\ 4 & 10 & -5 \\ 5 & 1 & 10 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -7 \\ 2 \\ 1.5 \end{pmatrix}$$

המערכת לאחר היפוך לצורה בעלת אלכסון דומיננטי:

```
System of equations:
[ 10*x1 +   8*x2 +   1*x3] = [ -7]
[  4*x1 +  10*x2 +  -5*x3] = [  2]
[  5*x1 +   1*x2 +  10*x3] = [1.5]
```

שיטת יעקובי:

```
Iteration 0: guess = [0. 0. 0.]
Iteration 1: guess = [-0.7   0.2   0.15]
Iteration 2: guess = [-0.875  0.555  0.48 ]
Iteration 3: guess = [-1.192  0.79   0.532]
Iteration 4: guess = [-1.3852  0.9428  0.667 ]
Iteration 5: guess = [-1.52094  1.08758  0.74832]
Iteration 6: guess = [-1.644896  1.182536  0.801712]
Iteration 7: guess = [-1.7262     1.2588144  0.8541944]
Iteration 8: guess = [-1.79247096  1.3175772   0.88721856]
Iteration 9: guess = [-1.84278362  1.36059766  0.91447776]
Iteration 10: guess = [-1.87992591  1.39435233  0.93533204]
Iteration 11: guess = [-1.90901507  1.41963638  0.95052772]
Iteration 12: guess = [-1.93076188  1.43886989  0.96254389]
Iteration 13: guess = [-1.9473503   1.4535767   0.97149395]
Iteration 14: guess = [-1.96001075  1.46468709  0.97831748]
Iteration 15: guess = [-1.96958142  1.47316304  0.98353667]
Iteration 16: guess = [-1.9768841   1.4796009   0.98747441]
Iteration 17: guess = [-1.98242816  1.48449084  0.99048196]
Iteration 18: guess = [-1.98664087  1.48821225  0.992765  ]
Iteration 19: guess = [-1.9898463   1.49103885  0.99449921]
Iteration 20: guess = [-1.992281    1.49318812  0.99581926]
```

```
Iteration 21: guess = [-1.99413243  1.49482203  0.99682169]
Iteration 22: guess = [-1.99553979  1.49606381  0.99758401]
Iteration 23: guess = [-1.99660945  1.49700792  0.99816352]
Iteration 24: guess = [-1.99742269  1.49772554  0.99860393]
Iteration 25: guess = [-1.99804082  1.49827104  0.99893879]
Iteration 26: guess = [-1.99851071  1.49868573  0.99919331]
Iteration 27: guess = [-1.99886791  1.49900094  0.99938678]
Iteration 28: guess = [-1.99913943  1.49924056  0.99953386]
Iteration 29: guess = [-1.99934583  1.4994227   0.99964566]
Iteration 30: guess = [-1.99950273  1.49956116  0.99973065]
Iteration 31: guess = [-1.99962199  1.49966641  0.99979525]
Iteration 32: guess = [-1.99971266  1.49974642  0.99984436]
Iteration 33: guess = [-1.99978157  1.49980724  0.99988169]
Iteration 34: guess = [-1.99983396  1.49985347  0.99991006]
Iteration 35: guess = [-1.99987378  1.49988862  0.99993163]
Iteration 36: guess = [-1.99990406  1.49991533  0.99994803]
Iteration 37: guess = [-1.99992707  1.49993564  0.99996049]
Iteration 38: guess = [-1.99994456  1.49995107  0.99996997]
Iteration 39: guess = [-1.99995786  1.49996281  0.99997717]
Iteration 40: guess = [-1.99996796  1.49997173  0.99998265]
```

```
Iteration 41: guess = [-1.99997565  1.49997851  0.99998681]
Iteration 42: guess = [-1.99998149  1.49998366  0.99998997]
Iteration 43: guess = [-1.99998593  1.49998758  0.99999238]
Iteration 44: guess = [-1.9999893   1.49999056  0.99999421]
Iteration 45: guess = [-1.99999187  1.49999282  0.9999956 ]
Iteration 46: guess = [-1.99999382  1.49999455  0.99999665]
Iteration 47: guess = [-1.9999953   1.49999585  0.99999745]
Iteration 48: guess = [-1.99999643  1.49999685  0.99999807]
Iteration 49: guess = [-1.99999728  1.4999976   0.99999853]
Iteration 50: guess = [-1.99999794  1.49999818  0.99999888]
Iteration 51: guess = [-1.99999843  1.49999862  0.99999915]
Iteration 52: guess = [-1.99999881  1.49999895  0.99999935]
Iteration 53: guess = [-1.99999909  1.4999992   0.99999951]
Iteration 54: guess = [-1.99999931  1.49999939  0.99999963]
Iteration 55: guess = [-1.99999948  1.49999954  0.99999972]
Iteration 56: guess = [-1.9999996   1.49999965  0.99999978]
Iteration 57: guess = [-1.9999997   1.49999973  0.99999984]
Iteration 58: guess = [-1.99999977  1.4999998   0.99999988]
Iteration 59: guess = [-1.99999983  1.49999985  0.99999991]
```

```
Iteration 60: guess = [-1.99999987  1.49999988  0.99999993]
Iteration 61: guess = [-1.9999999   1.49999991  0.99999995]
Total iterations: 61
Final result: [-1.99999992  1.49999993  0.99999996]
```

שיטת גאוס-זיידל:

```
Iteration 0: guess = [0. 0. 0.]
Iteration 1: guess = [-0.7    0.48   0.452]
Iteration 2: guess = [-1.1292    0.87768   0.626832]
Iteration 3: guess = [-1.4648272   1.09934688  0.77247891]
Iteration 4: guess = [-1.6567254   1.24892961  0.85346974]
Iteration 5: guess = [-1.78449066  1.34053113  0.90819222]
Iteration 6: guess = [-1.86324413  1.39939376  0.94168269]
Iteration 7: guess = [-1.91368328  1.43631466  0.96321017]
Iteration 8: guess = [-1.94537274  1.45975418  0.97671095]
Iteration 9: guess = [-1.96547444  1.47454525  0.9852827 ]
Iteration 10: guess = [-1.97816447  1.48390714  0.99069152]
Iteration 11: guess = [-1.98619486  1.48982371  0.99411506]
Iteration 12: guess = [-1.99127047  1.49356572  0.99627866]
Iteration 13: guess = [-1.99448044  1.49593151  0.99764707]
Iteration 14: guess = [-1.99650991  1.4974275   0.99851221]
Iteration 15: guess = [-1.99779322  1.49837339  0.99905927]
Iteration 16: guess = [-1.99860464  1.49897149  0.99940517]
Iteration 17: guess = [-1.99911771  1.49934967  0.99962389]
Iteration 18: guess = [-1.99944212  1.49958879  0.99976218]
Iteration 19: guess = [-1.99964725  1.49973999  0.99984963]
Iteration 20: guess = [-1.99977696  1.4998356   0.99990492]
```

```
Iteration 21: guess = [-1.99985897  1.49989605  0.99993988]
Iteration 22: guess = [-1.99991083  1.49993427  0.99996199]
Iteration 23: guess = [-1.99994361  1.49995844  0.99997596]
Iteration 24: guess = [-1.99996435  1.49997372  0.9999848 ]
Iteration 25: guess = [-1.99997746  1.49998338  0.99999039]
Iteration 26: guess = [-1.99998575  1.49998949  0.99999392]
```

```
Iteration 27: guess = [-1.99999099  1.49999336  0.99999616]
Iteration 28: guess = [-1.9999943   1.4999958   0.99999757]
Iteration 29: guess = [-1.9999964   1.49999734  0.99999846]
Iteration 30: guess = [-1.99999772  1.49999832  0.99999903]
Iteration 31: guess = [-1.99999856  1.49999894  0.99999939]
Iteration 32: guess = [-1.99999909  1.49999933  0.99999961]
Iteration 33: guess = [-1.99999942  1.49999958  0.99999975]
Iteration 34: guess = [-1.99999964  1.49999973  0.99999984]
Iteration 35: guess = [-1.99999977  1.49999983  0.9999999 ]
Iteration 36: guess = [-1.99999985  1.49999989  0.99999994]
Iteration 37: guess = [-1.99999991  1.49999993  0.99999996]
Iteration 38: guess = [-1.99999994  1.49999996  0.99999998]
Total iterations: 38
Final result: [-1.99999996  1.49999997  0.99999998]
```

השוואה בין הפתרונות של שני השיטות:

```
gauss seidel solution: [-1.99999996  1.49999997  0.99999998]
jacobi solution: [-1.99999992  1.49999993  0.99999996]
```

- **שאלה מס׳ 29**

פתרו את המטריצה הבאה בשתי דרכים והשוו בין התוצאות

$$\begin{pmatrix} 0 & 1 & 2 \\ -2 & 1 & 0.5 \\ 1 & -2 & -0.5 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ -4 \end{pmatrix}$$

המערכת לאחר היפוך לצורה בעלת אלכסון דומיננטי:

```
System of equations:
[ -2*x1 +   1*x2 + 0.5*x3] = [  4]
[  1*x1 +  -2*x2 + -0.5*x3] = [ -4]
[  0*x1 +   1*x2 +   2*x3] = [  0]
```

שיטת גאוס-זיידל:

```
Iteration 0: guess = [0. 0. 0.]
Iteration 1: guess = [-2.   1.  -0.5]
Iteration 2: guess = [-1.625    1.3125  -0.65625]
Iteration 3: guess = [-1.5078125   1.41015625 -0.70507812]
Iteration 4: guess = [-1.47119141  1.44067383 -0.72033691]
Iteration 5: guess = [-1.45974731  1.45021057 -0.72510529]
Iteration 6: guess = [-1.45617104  1.4531908  -0.7265954 ]
Iteration 7: guess = [-1.45505345  1.45412213 -0.72706106]
Iteration 8: guess = [-1.4547042   1.45441316 -0.72720658]
Iteration 9: guess = [-1.45459506  1.45450411 -0.72725206]
Iteration 10: guess = [-1.45456096  1.45453254 -0.72726627]
Iteration 11: guess = [-1.4545503   1.45454142 -0.72727071]
Iteration 12: guess = [-1.45454697  1.45454419 -0.7272721 ]
Iteration 13: guess = [-1.45454593  1.45454506 -0.72727253]
Iteration 14: guess = [-1.4545456   1.45454533 -0.72727267]
Iteration 15: guess = [-1.4545455   1.45454542 -0.72727271]
Iteration 16: guess = [-1.45454547  1.45454544 -0.72727272]
Total iterations: 16
Final result: [-1.45454546  1.45454545 -0.72727273]
```

שיטת יעקובי:

```
Iteration 0: guess = [0. 0. 0.]
Iteration 1: guess = [-2.  2.  0.]
Iteration 2: guess = [-1.  1. -1.]
Iteration 3: guess = [-1.75  1.75 -0.5 ]
Iteration 4: guess = [-1.25   1.25  -0.875]
Iteration 5: guess = [-1.59375  1.59375 -0.625  ]
Iteration 6: guess = [-1.359375  1.359375 -0.796875]
Iteration 7: guess = [-1.51953125  1.51953125 -0.6796875 ]
Iteration 8: guess = [-1.41015625  1.41015625 -0.75976562]
Iteration 9: guess = [-1.48486328  1.48486328 -0.70507812]
Iteration 10: guess = [-1.43383789  1.43383789 -0.74243164]
Iteration 11: guess = [-1.46868896  1.46868896 -0.71691895]
Iteration 12: guess = [-1.44488525  1.44488525 -0.73434448]
Iteration 13: guess = [-1.46114349  1.46114349 -0.72244263]
Iteration 14: guess = [-1.45003891  1.45003891 -0.73057175]
Iteration 15: guess = [-1.45762348  1.45762348 -0.72501945]
Iteration 16: guess = [-1.45244312  1.45244312 -0.72881174]
Iteration 17: guess = [-1.45598137  1.45598137 -0.72622156]
Iteration 18: guess = [-1.4535647   1.4535647  -0.72799069]
Iteration 19: guess = [-1.45521532  1.45521532 -0.72678235]
Iteration 20: guess = [-1.45408793  1.45408793 -0.72760766]
```

```
Iteration 21: guess = [-1.45485795  1.45485795 -0.72704396]
Iteration 22: guess = [-1.45433202  1.45433202 -0.72742898]
Iteration 23: guess = [-1.45469124  1.45469124 -0.72716601]
Iteration 24: guess = [-1.45444588  1.45444588 -0.72734562]
Iteration 25: guess = [-1.45461346  1.45461346 -0.72722294]
Iteration 26: guess = [-1.454499    1.454499   -0.72730673]
Iteration 27: guess = [-1.45457718  1.45457718 -0.7272495 ]
Iteration 28: guess = [-1.45452379  1.45452379 -0.72728859]
Iteration 29: guess = [-1.45456026  1.45456026 -0.72726189]
Iteration 30: guess = [-1.45453535  1.45453535 -0.72728013]
Iteration 31: guess = [-1.45455236  1.45455236 -0.72726767]
Iteration 32: guess = [-1.45454074  1.45454074 -0.72727618]
```

```
Iteration 33: guess = [-1.45454868  1.45454868 -0.72727037]
Iteration 34: guess = [-1.45454325  1.45454325 -0.72727434]
Iteration 35: guess = [-1.45454696  1.45454696 -0.72727163]
Iteration 36: guess = [-1.45454443  1.45454443 -0.72727348]
Iteration 37: guess = [-1.45454616  1.45454616 -0.72727221]
Iteration 38: guess = [-1.45454498  1.45454498 -0.72727308]
Iteration 39: guess = [-1.45454578  1.45454578 -0.72727249]
Iteration 40: guess = [-1.45454523  1.45454523 -0.72727289]
Iteration 41: guess = [-1.45454561  1.45454561 -0.72727262]
Iteration 42: guess = [-1.45454535  1.45454535 -0.7272728 ]
Iteration 43: guess = [-1.45454553  1.45454553 -0.72727268]
Iteration 44: guess = [-1.45454541  1.45454541 -0.72727276]
Iteration 45: guess = [-1.45454549  1.45454549 -0.7272727 ]
Iteration 46: guess = [-1.45454543  1.45454543 -0.72727274]
Iteration 47: guess = [-1.45454547  1.45454547 -0.72727272]
Iteration 48: guess = [-1.45454544  1.45454544 -0.72727274]
Total iterations: 48
Final result: [-1.45454546  1.45454546 -0.72727272]
```

השוואה בין הפתרונות של שני השיטות:

```
gauss seidel solution: [-1.45454546  1.45454545 -0.72727273]
jacobi solution: [-1.45454546  1.45454546 -0.72727272]
```