Overview

- focus on passive circuits vs active circuits (later course)
  - resistors, capacitors, inductors

- topics
  1) Basics of circuit analysis (DC - only)
  2) Transient Analysis (DC + time varying)
  3) AC Analysis (complex frequency domain)

Lecture 1: Charge, Current, Voltage, Work

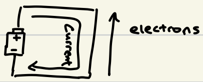
General

- time (s) & current (A) are SI units
-

10^{-15}	10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{0}	10^{3}	10^{6}	10^{9}
f	p	n	μ	m	/	k	M	G

Current and Charge

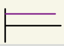
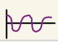
- 1 Ampere (A) = $\frac{1 \text{ coulomb}}{1 \text{ second}}$
- 1 electron's charge = -1.602×10^{-19} coulomb (C)
- current flows from positive terminal to negative
  - ↳ electrons flow from negative to positive



- averge current = $\frac{\text{averge charge moved through cross sectional area}}{\text{time}}$ $\boxed{\left(I_{averge} = \frac{\Delta Q}{\Delta t} \right)}$
- instantaneous current = $\boxed{i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}}$
- current has magnitude & direction

$$\boxed{\begin{array}{l} I = \text{averge current} \\ i = \text{instantaneous current} \end{array}}$$

Types of Current

- direct current (DC) 
- alternating current (AC) 
- both (AC + DC) 

Lecture 2: Voltage, Power, Current

Voltage

- voltage is the work required to move an electron from point A to B
  - ↳ must be between two points
- work = Voltage · charge → $V = \frac{W}{C}$ (Joules / current)
- voltage has value & polarity
- voltage is measured in parallel / current measured in series
- voltmeter positive (red) should point to higher voltage

Power

- power = $\frac{\text{work}}{\text{time}}$ ($P = \frac{W}{t}$) (Watt = $\frac{\text{Joule}}{\text{Second}}$)
- power = voltage · current ($P = VI = vi$)

Efficency

- unit (η)
- $\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{E_{in}}{E_{out}} \times 100\%$
- in a cascaded system: $\eta_{total} = \eta_1 \cdot \eta_2 \cdot \eta_3$

Lecture 3: Ohm's Law, KVL, KCL

Resistance

- certain materials impede the flow of current
- electrons collide with other electrons, ions, impurities
- $R = \frac{V}{I}$: need voltage V to push current I through material
  - ↳ Ohm's Law
- resistors absorb energy
- open circuit $\infty\Omega$, short circuit 0Ω

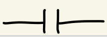
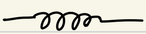
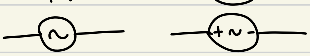
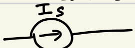
Conductance

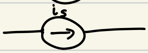
- conductance $= G = \frac{1}{R} = \frac{I}{V}$
- unit : siemens = mho = Ω^{-1}

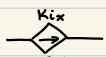
Specific Resistance

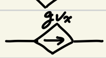
- $R = \rho \frac{l}{A}$ = resistivity (Ωm) $\frac{\text{length } (m)}{\text{cross sectional area } (m^2)}$
- $\sigma = \frac{1}{\rho}$ conductivity $\Omega^{-1} m^{-1}$

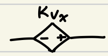
Electric Symbol

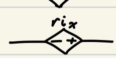
- resistor 
- capacity 
- inductor 
- DC Voltage Source 
- AC Voltage Source 
  - ↳ ideal voltage source can supply infinite current
- DC Current Source 

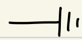
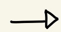
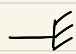
- AC Current Source 

- Dependent Sources : current controlled current 

  voltage controlled current 

  voltage controlled voltage 

  current controlled voltage 

- ground 
- common ground 
- chassis / frame 

Kirchoff's Laws

▷ Voltage Law

- $\sum_{\text{around loop}} V_n = 0$
- choose direction

▷ Current Law

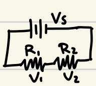
- $\sum_{\text{into node}} i_n = \sum_{\text{out}} i_o$
- current in = current out

General Rules

- ▷ voltage sources in series can be combined additively
- ▷ current sources in parallel can be combined additively
- ▷ parallel components can be arranged in any order

Circuit

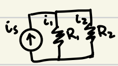
- Ohm's Law $V = IR$
- KCL $\sum i_{enter} = \sum i_{out}$
- KVL $\sum_{\text{loop}} V_i = 0$
- Voltage Divider



$$V_1 = V_s \frac{R_1}{R_1 + R_2}$$

$$V_2 = V_s \frac{R_2}{R_1 + R_2}$$

- Current Divider



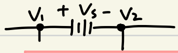
$$i_1 = i_s \frac{R_2}{R_1 + R_2} \qquad i_2 = i_s \frac{R_1}{R_1 + R_2}$$

Lecture 4 Nodal & Mesh Analysis

Nodal Analysis

- steps:
  1. identify all nodes (area with same voltage)
  2. select a reference node
  3. label node voltages with variables $V_1, V_2, \ldots, V_{N-1}$
  4. write KCL at $(N-1)$ number of nodes
     - ↳ if voltage sources / dependent sources exist, may need more
  5. solve for $V_1, V_2, \ldots, V_{N-1}$

- supernode: if voltage source exists

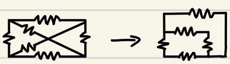


$$\boxed{V_1 - V_2 = V_s}$$

Mesh Analysis

- steps:
  1. check if circuit is planar (can be drawn on flat surface / no wires cross)
  2. label mesh currents $I_1, I_2, \ldots, I_M$
  3. write KVL for M meshes
     - ↳ may need to supermesh if current source exists
  4. Solve for M currents

- redraw:



- supermesh: combine meshes that enclose the current source
  - ↳ 1 less mesh, but new equation $\boxed{i_s = i_1 + i_2}$

Lecture 5 Resistor Construction

Ideal Resistors

- value fixed (±0% tolerance)
- temperature 25°C

Carbon Composition

- general purpose
- 5 ~ 20% tolerance
- 1/8 ~ 2 W
- 2.2 Ω ~ 20 MΩ
- reading
  - color → number
  - digit 1, digit 2, multiplier ($\times 10^n$), tolerance

Carbon Film

- general purpose
- 1 ~ 10% tolerance
- 1/8 ~ 3 W
- 1 Ω ~ 22 MΩ

Precision Metalwire

- precision & high power
- 0.01 ~ 5% tolerance
- 1/8 ~ 2500 W
- 0.01 Ω ~ 200 kΩ

Metal Film

- precision
- 0.01 ~ 5% tolerance
- 0.05 ~ 3 W
- 0.01 Ω ~ 2 GΩ

Thick Film

- general purpose & precision
- 0.1 ~ 20% tolerance
- 1/16 W ~ 250 W
- 0.01 Ω ~ 500 GW

Thin Film

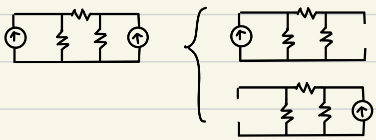
- general purpose & precision
- 0.01 ~ 20% tolerance
- 1/32 ~ 40 W
- 0.001 Ω ~ 22 MΩ

Generally, select resistor twice the maximum anticipated value

Lecture 6 Superposition

- general approach: split problem into circuits and solve each individually

  ↳ eg.

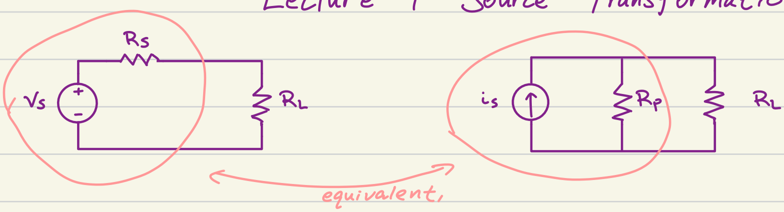


Superposition Theorem: in a linear network, the voltage across or the current through any element may be calculated by adding algebraicly the individual voltages or currents caused by independent sources

voltage source → short circuit
current source → open circuit

- never turn off dependent source

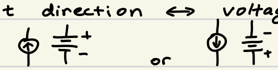
Lecture 7 Source Transformation



$R_s = R_p$ $V_s = i_s \cdot R_s$

Ideas

- current direction ↔ voltage source positive

  ↳ 

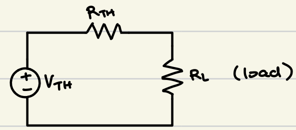
- voltage source in series, current source in parallel
- transformed circuits are not the same internally

  ↳ current & voltage

Lecture 8 Thevenin and Equivalents

General

- a linear network can be replaced by its Thevenin equivalent circuit
- repeatedly apply source transformation to find Thevenin equivalent
  - ↳ not all circuits can be transformed



Approach

① Disconnect load

② Find voltage across disconnected (V_{oc})

③ Find equivalent resistance across disconnect (R_{eq})

④ $V_{oc} = V_{TH}$ $\quad R_{eq} = R_{TH}$

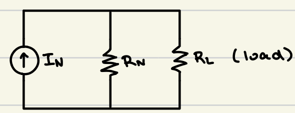
Norton Equivalent Circuits

- a linear network can be replaced by its Norton equivalent circuit
- ① replace load with short circuit

  ② find current in short circuit (I_{sc})

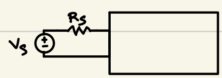
  ③ find equivalent resistance of network (R_{eq})

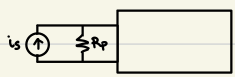
  ④ $I_N = I_{sc}$ $\quad R_N = R_{eq}$



Lecture 9 Maximum Power Transfer

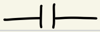
- real battery has small resistance
- maximum power in load when $R_L = R_s$





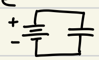
Lecture 10 Capacitance

Symbols

- resistor 
- capacitor 
- inductor 

Capacitor

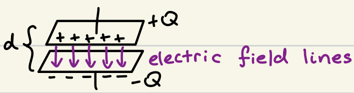
- most common: parallel plate capacitor
- with current in resistor, one plate becomes positively charged, the other negative

↳ 

- capacitor can "store" charge / energy
- capacitor can then discharge stored energy
- discharge: $V = V_s \cdot e^{-\frac{t}{RC}}$

$$I = \frac{V}{R} = \frac{V_s}{R} e^{-\frac{t}{RC}}$$

- electric field:



$$E = \frac{V}{d} \rightarrow Q = \varepsilon A E$$

$$Q = CV$$

$$C = \frac{\varepsilon A}{d}$$

- capacitance unit: $\frac{Q}{V} = \frac{\text{coulomb}}{\text{volt}} = \text{Farad (F)}$
- permittivity: $\varepsilon = \varepsilon_r \cdot \varepsilon_0$

ε_0 = permittivity of free space = $8.85 \times 10^{-12}\ C^2/Nm^2$

ε_r = material property → vacuum = 1.0 air ≈ 1.00059

- multiple-plate capacitor: $C = \frac{\varepsilon A}{d}(n-1)$
- current ↔ voltage $i = C \cdot \frac{dv}{dt}$

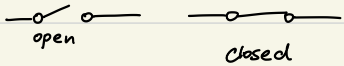
$$v = 1/C \cdot \int i \, dt$$

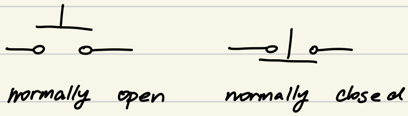
- energy stored in capacitor: $W = \frac{1}{2} C V^2$ (J)

- Series: $C_{eq} = 1/(1/C_1 + 1/C_2 + 1/C_3 + \dots + 1/C_n)$
- parallel: $C_{eq} = C_1 + C_2 + \dots + C_n$
- misc. characteristics:
  - capacitors are open circuits to DC Voltages
  - voltages on real capacitors cannot change instantaneously
  - capacitors do not dissipate energy
    - ↳ only store or deliver energy

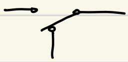
Lecture 10.5 Switches

Single pole single throw (SPST)

- schematic throw switch 

- schematic momentary switch 

Single Pole Double Throw

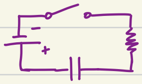
- schematic throw 

Lecture 11: Inductors

Magnetic Field Induced by Current

right hand rule

RC Circuit Independent

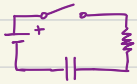


at time $t=0$: ▷ charge of capacitor is 0
(just closed)

▷ no potential difference across capacitor

▷ exists a current in circuit

▷ exists voltage over resistor

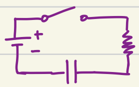


at time $t=\infty$: ▷ no current flowing through circuit

▷ can find charge on capacitor using $Q=CV$

capacitors in series: $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$

$Q_1 = Q_2 = Q_T = \ldots$



$V_T = V_R + V_C$

$= IR + \frac{Q}{C}$ $\Rightarrow$ Since voltage is constant, as Q increase, capacitor has more voltage and resistor has less

$t\uparrow$ $\vec{V_T}$ $I\downarrow$ $Q\uparrow$

through discharging $t \to \infty$ $Q \to 0$, $V_c \to 0$, $I \to 0$

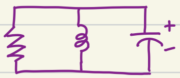
capacitor charging

$V(t) = V_B(1 - e^{-t/RC})$

$I(t) = I_0(e^{-t/RC})$

Lecture 13: RLC Circuits

Parallel RLC Derivation



use KCL:

$$\frac{V}{R} + C\frac{dv}{dt} = i$$

$$\frac{V}{R} + C\frac{dv}{dt} = -\frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$$

$$\frac{V}{R} + C\frac{dv}{dt} + \frac{1}{L}\int_{t_0}^{t} v\,dt - i(t_0) = 0$$

take derivative w/ respect to t

$$\frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} + \frac{1}{L}\cdot v = 0$$

Solving RC Network

- analyze $t = 0^-$: DC analysis to find V_c ($I_c = 0$)
- analyze $t = 0^+$: DC analysis to find V_c ($I_c = 0$)
- analyze $t = \infty$: DC analysis to find V_c (C becomes voltage source)
- find R_{TH} across C, $\tau = RC$
- $v(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau}$
- $i(t) = I(\infty) + [I(0^+) - I(\infty)]e^{-t/\tau}$

Solving RL Network