Angular Mechanics

Aditya C

April 2025

1 Introduction

Angular mechanics is all about describing objects in rotational motion. The theory will be covered in the club meeting on Friday. The fundamental equations that govern angular mechanics that you may find useful are:

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\tau = I\alpha = \vec{F} \times \vec{r}$$

$$I = \int r^2 dm$$

$$\vec{L} = I\vec{\omega} = m\vec{v}r$$

$$I = I_{com} + Mh^2$$

2 Problems

Question 1 (MIT): A thin uniform disc of mass M and radius R is mounted on an axis passing through the centre of the disc, perpendicular to the plane of the disc. Determine the moment of inertia (i) about the axis passing through the centre of the disc and (ii) an axis passing through a point $\frac{R}{2}$ from the centre.

Question 2 (created by Physics Club): A shape has rotational inertia $I = 2.5\mathrm{kg\,m}^2$. A force $\vec{F_1} = (3, -5, 2)\mathrm{N}$ acts at $\vec{r_1} = (4, 2, -3)\mathrm{m}$ and another force $\vec{F_2} = (-6, 4, 0)\mathrm{N}$ acts at $\vec{r_2} = (2, 9, 1)\mathrm{m}$. Calculate the angular acceleration α.

Question 3 (Isaac Physics): In the 'Hadley Cell' in the atmosphere of the Earth, the air that rises at the equator travels towards the poles, then sinks and travels back to the equator. Consider a 'parcel' of air of mass $m = 1$kg which moves north from the equator. At the equator, the speed in the West-East direction relative to the Earth surface is zero. Using the conservation of angular momentum, find an equation for the West-East (zonal) velocity component u as the air parcel moves north, as a function of latitude ϕ, the radius of the Earth r and the magnitude of the angular velocity of the Earth about its own axis Ω.

3 Extra extension problems

Question 4 (Principles of Physics): Two discs are mounted like merry-go rounds on low-friction bearings on the same axle and can be brought together so that they rotate as one unit. The first disk, with rotational inertia $3.3\mathrm{kg\,m}^2$ about its central axis is set spinning counter-clockwise at $450\mathrm{min}^-1$. The second disc, with rotational inertia $6.6\mathrm{kgm}^2$ about its central axis is set spinning counter-clockwise at $900\mathrm{min}^-1$. They then couple together. What is the final angular velocity and direction of rotation?

Question 5 (created by Physics club): Prove the parallel-axis theorem

4 Further reading for those interested

MIT Open Course Ware Take a look at lectures 26-33 if you're interested:
`https://ocw.mit.edu/courses/8-01l-physics-i-classical-mechanics-fall-2005/pages/lecture-notes/`
Isaac Physics pages for Moment of Inertia and Angular Momentum:
`https://isaacphysics.org/concepts/cp_moment_inertia` and
`https://isaacphysics.org/concepts/cp_angular_momentum`

Thank you very much for attending! See you next week!