Physics Club - Mathematical Prerequisites for higher Physics and Engineering

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July 2025

1 Introduction

This is a short workbook that will introduce you to some of the mathematical tools that will be vital for the pursuit of degrees such as Physics and Engineering. This will be divided into three large sections: methods in complex numbers (expanding on the modulus-argument form), methods in linear algebra (on eigenvectors and eigenvalues), and methods in calculus (common ODEs and methods in integration).

2 Complex Numbers - Euler's formula

In the year 1748, Leonhard Euler, who many consider to be the greatest mathematician of all time, published his monumental work in a two volume text called " Introductio in analysin infinitorum". It was this work that contained what many consider to be the most beautiful mathematical identity, and it's one you may have seen before:

$$e^{i\pi} + 1 = 0$$

But where does this come from? Let's take a step back to the modulus-argument form of complex numbers

$$a + bi = r(\cos(\theta) + i\sin(\theta))$$

. Euler proved (see proof here) that this exact formula could be compacted down into

$$a + bi = e^{i\theta}$$

where θ is the argument of the complex number and r is the modulus. Since when $\theta = \pi$, on the Argand diagram, (if the modulus is assumed as 1), this corresponds to the number -1. Thus, it is generalised to say that:

$$a + bi = r(\cos(\theta) + i\sin(\theta) = re^{i\theta}$$

Now, try solving some problems where you can get used to this new form:

1.) Write the complex number $5 + 6i$ in exponential form.
2.) Write the complex number $-6 + 3i$ in exponential form.
3.) Write the number $-3 + 0i$ in exponential form.

3 Linear Algebra - Eigenvectors and Eigenvalues

As a prerequisite, we must be familiar with how matrices and vectors interact, how they multiply, and some key properties. So far, we have heard of the concept of **invariant points and lines** on a matrix transformation, those points and lines on a coordinate grid that are uneffected by a matrix transformation. But what about points and vectors that get **scaled** during a matrix transformations? These are called **eigenvectors**, where "eigen" is German for "same". The formal notation for an eigenvector is as follows:

$$\mathbf{M}\vec{\mathbf{v}} = \lambda\vec{\mathbf{v}}$$

Here, $\vec{\mathbf{v}}$ is the eigenvector, and λ, which is the scalar by which the eigenvector was scaled, is called the corresponding **eigenvalue**. One matrix can have more than one unique eigenvector or none at all, and the eigenvector multiplied by some scalar will always yield another eigenvector. So how do you find the eigenvector aside from just trial and error? Let us follow the example of the transformation represented by the matrix:

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 7 & -5 \\ 0 & 2 & 9 \end{bmatrix}$$

For the eigenvalue, λ and $\vec{\mathbf{v}}$, the property $\det(\mathbf{M} - \lambda\mathbf{I})\vec{\mathbf{v}} = 0$. Thus, since $\mathbf{M} - \lambda\mathbf{I}$ is represented by:

$$\begin{bmatrix} 1-\lambda & 2 & 4 \\ 3 & 7-\lambda & -5 \\ 0 & 2 & 9-\lambda \end{bmatrix}$$

The determinant of this matrix must be equal to zero. The algebra of calculating the determinant and setting it equal to zero is left to the reader. This gives us the eigenvalue(s). Then, rearrange the eigenvalues in the original equation $\mathbf{M}\vec{\mathbf{v}} = \lambda\vec{\mathbf{v}}$ to find the eigenvector.

4 Methods in calculus - ODEs and integration

ODEs or ordinary differential equations are equations that relate a function to one or more of its derivatives, e.g.:

$$f''(x) - 3f'(x) - 2f(x) = 0$$

These kinds of equations come up all the time in physics and engineering. Some common examples that you should look out for include:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

The general solution to this equation is:

$$x(t) = A\sin(\omega t + \phi) + B\cos(\omega t + \phi)$$

Where A, B and ϕ are all constants to be adjusted based on the initial conditions. This is called the **simple harmonic oscillator**. This ODE comes up in the motion of springs, waves, but also in quantum mechanics, electronics and many more. Another common differential equation is:

$$\frac{df}{dt} = kt$$

or

$$\frac{df}{dt} = -kt$$

the general solutions to which are

$$f = Ae^{kt}$$

and

$$f = Ae^{-kt}$$

respectively. This is called an exponential/exponential decay equation. It or its variants comes up very often in quantum physics (as solutions to the Schrödinger equation), in nuclear physics (modelling half-lives) and thermodynamics (modelling heat transfer), but also in biology, modelling population dynamics (anywhere where the rate of change of something may be proportional the amount of something you already have, in this case, the rate of growth of population being proportional to the existing population).

In order to solve more complicated differential equations, it is useful to be aware of some more advanced methods in integration. For example, the rule of integration by parts:

$$\int uv'dx = uv - \int u'vdx$$

It is also useful to know of methods such as integration by substitution or trig substitution. For demonstration, take the following example:

$$\int \frac{1}{\sqrt{R^2 - x^2}}dx$$

Let $x = R\sin\theta \Rightarrow dx = R\cos\theta d\theta$. Then:

$$\int \frac{1}{\sqrt{R^2 - R^2\sin^2\theta}}dx$$

$$\Rightarrow \int \frac{1}{R\sqrt{1 - \sin^2\theta}}dx$$

$$\Rightarrow \int \frac{1}{R\cos\theta}dx$$

$$\Rightarrow \int \frac{1}{R\cos\theta}R\cos\theta d\theta$$

$$\Rightarrow \int 1d\theta$$

$$\Rightarrow \theta + C$$

Let us remember that $x = R\sin\theta$ therefore $\theta = \arcsin(\frac{x}{R})$, and therefore our final answer is:

$$\int \frac{1}{\sqrt{R^2 - x^2}}dx = \arcsin(\frac{x}{R}) + C$$

To practise, try to find the indefinite integral of the following using substitution or by parts:

1.) $\int \frac{x}{\sqrt{x+1}}dx$
2.) $\int \frac{1}{1-x^2}dx$
3.) $\int x\ln xdx$