Physics Society - The Infinite Square Well

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1 Introduction

We have thus far thoroughly examined the Schrödinger equation. Now it is finally time to take a look at how to actually solve it. Let us re-examine it for the sake of refreshing our memories and try to label each term in the equation, and what it means. The following is the time-independent version:

$$E\psi = \frac{-\hbar^2}{2m}\frac{d^2\psi}{dt^2} + V\psi$$

Question 1 Label each term in the Schrödinger equation and qualitatively describe its meaning.

Question 2 What happens to the Schrödinger equation if our potential term (V) is 0? What happens if $V = \infty$?

We will be looking at solutions of the Schrödinger equation for one of the simplest cases: the infinite square well potential. This is a situation in which the graph of potential energy against displacement looks like so:

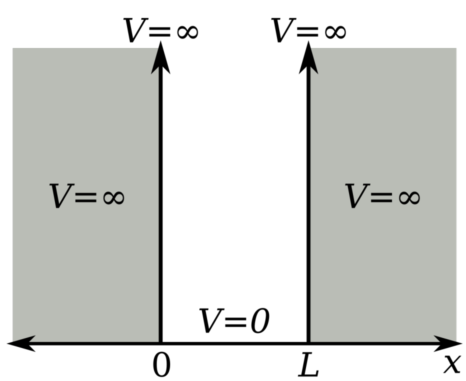

Figure 1: Infinite square well potential

Therefore, the Schrödinger equation for $0 < x < L$ collapses down to $\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$. Can we therefore think of an equation for ψ such that its second derivative is negatively proportional to the function itself? That's right! It is $A\sin(kx) + B\cos(kx)$. We can remember from SHM that $k = \sqrt{\frac{2mE}{\hbar^2}}$.

Question 3 If we say that our potential well is $V = 0$ for $-a < x < a$, fit the function for ψ to this such that $\psi(-a) = \psi(a) = 0$. Find the conditions for k in which this is true. Hint: $\sin(n\pi) = 0$ where $n \in \mathbb{Z}$

Question 4 Rearrange the equation you gathered above to find the allowed energies, E_n, that the particle can exist in.

2 Extension

We know that the probability of the particle existing at any x must sum to 1. We know that $P(x) = \int \psi^2 dx$ from $x = \infty$ to $x = -\infty$. This needs what's called a **normalisation factor**. Find the normalisation factor such that:

$$A \int_{-\infty}^{\infty} \psi^2 dx = 1$$

Take a look at the Isaac Science Quantum Mechanics Primer Chapter 2 if you are interested!

Thank you all for attending!