Physics Club Problem Sheet - More Gravitation and Orbits

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1 Introduction

The week before last, we had a look at radial fields, in which we looked more deeply at gravitation. Now, we are going to inspect how to analyse bodies in orbit, and in other scenarios involving large masses. The escape velocity (the minimum velocity with which a body must 'take off' from another body to be able to leave its gravitational field) is given by:

$$v_e = \sqrt{\frac{2GM}{r}}$$

Where M is the mass of the planetary body and r is the radius of the planet from which the object is launched. Similarly, the velocity of a body in orbit a distance R from the centre of mass of a planet is given by:

$$v_o = \sqrt{\frac{GM}{R}}$$

Question 1 (Physics Club) Calculate the escape velocity for Earth.

Question 2 (Physics Club) The ISS orbits roughly 410 km above the Earth's surface. Calculate its orbital speed and its orbital period

Kepler's laws generally govern how orbits behave. Kepler's first law states that planetary orbits are elliptical. The second law states that the imaginary line joining a planet and the Sun sweeps equal areas of space during equal time intervals as the planet orbits. The third law states that the square of the orbital period is proportional to the orbital radius cubed.

Question 3 (Physics Club) Prove Kepler's third law for a circular orbit.

How do gravitational fields behave inside of planets? The shell theorem tells us that A spherically symmetric body affects external objects gravitationally as though all of its mass were concentrated at a point at its centre **and** If the body is a spherically symmetric shell (i.e., a hollow ball), no net gravitational force is exerted by the shell on any object inside, regardless of the object's location within the shell.

Question 4 (Physics Club) Using the average density of the Earth ρ, the radius as R, and taking the Earth as a sphere, calculate an expression $g(h)$ for the gravitational acceleration experienced by an object a distance h below the surface of the Earth.

2 Extension problems and further reading

In reality, orbits are not circular, they are elliptical - learn about the geometry of an ellipse. What is the significance of the major and semimajor axes?

What is the difference between aphelion and perihelion? How does this relate to ellipse geometry?

Gravitational orbits must always be describable by a **conic section**. What are conic sections? Discuss and think about their geometry.

As for further reading, take a look at the BAAO Astro Round 1 Astrophysics Self-Study Guide! The link is:

`https://www.bpho.org.uk/baao/round-1/`

From this website, scroll down to where it says "Round One Astrophysics Self Study Guide". This will also be helpful if you want to take part in the Astro Olympiad later in the year!

Thank you all for coming!