University of Zurich[UZH]

# Leseaufträge «Mikroökonomik I»

# Modul 3: Produktion und Kosten

## Unit 4:

– Kurzfristige Kosten

**Quellen:**

- **Chapter 11 – Costs**
  Frank, Robert H, & Cartwright, Edward. (2016). *Microeconomics and Behaviour (2nd European ed.)*. London: McGraw-Hill Education.
- **The great distortion**
  The Economists, May $16^{th}$ 2015

# CHAPTER 11

# COSTS

Just after finishing college, one of us (Robert) was a high school maths and science teacher in Sanischare, a small village in eastern Nepal. During the two years spent there, one of the country's few roads was in the process of being built through Sanischare. Once the right-of-way was cleared and the culverts and bridges laid in, the next step was to spread gravel over the roadbed. As at almost every other stage of the process, the methods employed at this step were a page torn from another century. The Nepalese workmen squatted by the side of the road in the blazing sun, tapping away at large rocks with their hammers. In a 12-hour day, each worker would produce a small mound of gravel, not enough to cover even one running foot of roadbed. But there were a lot of people working, and eventually the job was done.

In Europe, of course, we do not hire people to hammer rocks into gravel by hand. Instead, we have huge machines that pulverize several tonnes of rock each minute. The reason for this difference may seem obvious: Nepal, being a very poor country, simply could not afford the expensive equipment used in industrialized nations. But this explanation is wrong. As we will see, it still would have made sense for Nepal to make gravel with manual labour even if it had had vast surplus revenues in its national treasury, because labour is very cheap relative to capital equipment there.

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## CHAPTER PREVIEW

In this chapter our goal is to translate the theory of production developed in Chapter 10 into a coherent theory of costs. In Chapter 10 we established the relationship between the quantities of inputs employed and the corresponding level of output. Here, we will forge the link between the quantity of output produced and the cost of producing it.

Our first step will be to tackle the question of how costs vary with output in the short run. This question turns out to be more involved than it sounds, for there are seven different types of cost to keep track of—total cost, variable cost, fixed cost, marginal cost, average total cost, average variable cost and average fixed cost. This array sounds bewildering at first, but the links between the different cost concepts are actually clear and simple. And each turns out to be important for the study of firm behaviour, our principal concern in the chapters to follow.

Of even greater importance for the structure and conduct of industry is the question of how costs vary with output in the long run. Here, we will begin with the question of how to produce a given level of output—say, a mile of road—at the lowest possible cost. A given quantity can be produced in many ways: we need to find the cheapest way, the most appropriate method for existing factor prices. The answer to this question enables us to explore how costs are related to returns to scale in production.

## COSTS IN THE SHORT RUN

To see how costs vary with output in the short run, it is convenient to begin with a simple production example of the sort we discussed in Chapter 10. Suppose Kelly's Cleaners washes bags of laundry using labour ($L$) and capital ($K$). Labour is purchased in the open market at a wage rate $w$ = €10/person-hr.[1] Capital is fixed in the short run. The relationship between the variable input and the total number of bags washed per hour is summarized in Table 11.1. Note that output initially grows at an increasing rate with additional units of the variable input (as $L$ grows from 0 to 4 units), then grows at a diminishing rate (as $L$ grows from 4 to 8 units).

**TABLE 11.1**
**The Short-Run Production Function for Kelly's Cleaners**

| Quantity of labour (person-hr/hr) | Quantity of output (bags/hr) |
|---|---|
| 0 | 0 |
| 1 | 4 |
| 2 | 14 |
| 3 | 27 |
| 4 | 43 |
| 5 | 58 |
| 6 | 72 |
| 7 | 81 |
| 8 | 86 |

The entries in each row of the right column tell the quantity of output produced by the quantity of variable input in the corresponding row of the left column. This production function initially exhibits increasing, then diminishing, returns to the variable input.

The total cost of producing the various levels of output is simply the cost of all the factors of production employed. If Kelly owns his own capital, its implicit rental value is an opportunity cost, the money Kelly could have earned if he had sold his capital and invested the proceeds in, say, a

[1] A person-hour is one person working for 1 hour. In Chapter 14 we will consider how input prices are determined. For the present, we simply take them as given.

government bond (see Chapter 1). Suppose Kelly's capital is fixed at 120 machine-hr/hr, the rental value of each of which is $r$ = €0.25/machine-hr,[2] for a total capital rental of €30/hr. This cost is **fixed cost (FC)**, which means that it does not vary in the short run as the level of output varies. More generally, if $K_0$ denotes the amount of capital and $r$ is its rental price per unit, we have

***fixed cost (FC)*** cost that does not vary with the level of output in the short run (the cost of all fixed factors of production).

$$\text{FC} = rK_0 \tag{11.1}$$

Other examples of fixed cost might include property taxes, insurance payments, interest on loans and other payments to which the firm is committed in the short run and which do not vary as the level of output varies. Business managers often refer to fixed costs as *overhead costs*.

**Variable cost (VC)** is defined as the total cost of the variable factor of production at each level of output.[3] To calculate VC for any given level of output in this example, we simply multiply the amount of labour needed to produce that level of output by the hourly wage rate. Thus, the variable cost of 27 bags/hr is (€10/person-hr) (3 person-hr/hr) = €30/hr. More generally, if $L_1$ is the quantity of labour required to produce an output level of $Q_1$ and $w$ is the hourly wage rate, we have

***variable cost (VC)*** cost that varies with the level of output in the short run (the cost of all variable factors of production).

$$\text{VC}_{Q_1} = wL_1 \tag{11.2}$$

Note the explicit dependence of VC on output in the notation on the left-hand side of Equation 11.2, which is lacking in Equation 11.1. This is to emphasize that variable cost depends on the output level produced, whereas fixed cost does not.

**Total cost (TC)** is the sum of FC and VC. If Kelly wishes to wash 43 bags/hr, the total cost of doing so will be €30/hr + (€10/person-hr) (4 person-hr/hr) = €70/hr. More generally, the expression for total cost of producing an output level of $Q_1$ is written

***total cost (TC)*** all costs of production: the sum of variable cost and fixed cost.

$$\text{TC}_{Q_1} = \text{FC} + \text{VC}_{Q_1} = rK_0 + wL_1 \tag{11.3}$$

Table 11.2 shows fixed, variable and total cost for corresponding output levels for the production function given in Table 11.1. The relationships among the various cost categories are

**TABLE 11.2**
**Outputs and Costs**

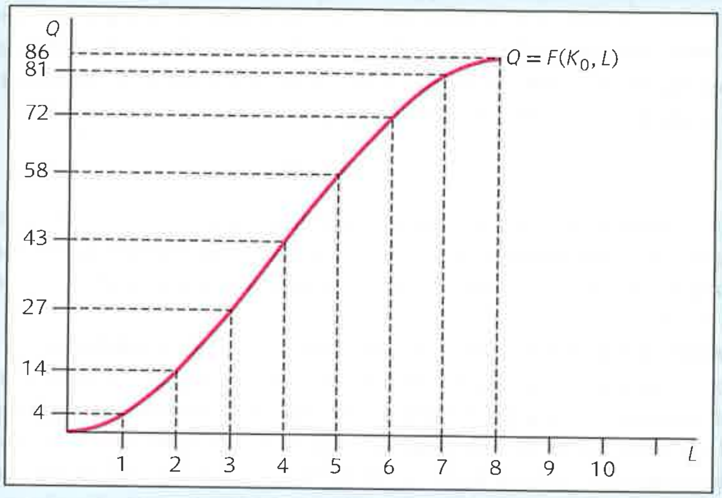
| Q | FC | VC | TC |
|---|---|---|---|
| 0 | 30 | 0 | 30 |
| 4 | 30 | 10 | 40 |
| 14 | 30 | 20 | 50 |
| 27 | 30 | 30 | 60 |
| 43 | 30 | 40 | 70 |
| 58 | 30 | 50 | 80 |
| 72 | 30 | 60 | 90 |
| 81 | 30 | 70 | 100 |
| 86 | 30 | 80 | 110 |

The fixed cost of capital is €30/hr, and the cost per unit of the variable factor ($L$) is €10/hr. Total cost is calculated as the sum of fixed cost and variable cost.

[2] A machine-hour is one machine working for 1 hour. To say that Kelly's capital is fixed at 120 machine-hr/hr means that he has 120 machines that can operate simultaneously.

[3] In production processes with more than one variable input, variable cost refers to the cost of *all* such inputs.

**FIGURE 11.1**
**Short-Run Production Function**
The short-run production function of Kelly's cleaners when capital is fixed at 120 machine-hr/hr. There are increasing returns to labour up to $L = 4$, and diminishing returns thereafter.




most clearly seen by displaying the information graphically, not in tabular form. The short-run production function from Table 11.1 is plotted in Figure 11.1. Recall from Chapter 10 that the initial region of upward curvature ($0 \leq L \leq 4$) of the production function corresponds to increasing returns to the variable input. Beyond the point $L = 4$, the production function exhibits diminishing returns to the variable input.

## Graphing the Total, Variable and Fixed Cost Curves

Not surprisingly, the shape of the variable cost curve is systematically related to the shape of the short-run production function. The connection arises because the production function tells us how much labour we need to produce a given level of output, and this quantity of labour, when multiplied by the wage rate, gives us variable cost. Suppose, for example, we want to plot the variable cost of producing 58 units of output. We first note from the production function shown in Figure 11.1 that 58 units of output require 5 units of labour, which, at a wage rate of €10/person-hr, gives rise to a variable cost of (5)(10) = €50/hr. So in Figure 11.2, the output level of 58 is plotted against a variable cost of €50/hr. Similarly, note from the production function that 43 units of output require 4 units of labour, which, at the €10 wage rate, gives rise in Figure 11.2 to a variable cost of €40/hr. In like fashion, we can generate as many additional points on the variable cost curve as we choose.

Of particular interest is the relationship between the curvature of the production function and that of the variable cost curve. Note in Figure 11.1 that $L = 4$ is the point at which diminishing returns to the variable factor of production set in. For values of $L$ less than 4, there are increasing returns to $L$, which means that increments in $L$ produce successively larger increments in $Q$ in that region. Put another way, in this region a given increase in output, $Q$, requires successively smaller increments in the variable input, $L$. As a result, variable cost grows at a diminishing rate for output levels less than 43. This is reflected in Figure 11.2 by the concave shape of the variable cost curve for output levels between 0 and 43.

Once $L$ exceeds 4 in Figure 11.1, we enter the region of diminishing returns. Here, successively larger increments in $L$ are required to produce a given increment in $Q$. In consequence, variable cost grows at an increasing rate in this region. This is reflected in the convex shape of the variable cost curve in Figure 11.2 for output levels in excess of 43.

Because fixed costs do not vary with the level of output, their graph is simply a horizontal line. Figure 11.2 shows the fixed, variable and total cost curves (FC, VC and TC) for the production function shown in Figure 11.1. Note in the figure that the variable cost curve passes through the origin, which means simply that variable cost is zero when we produce no

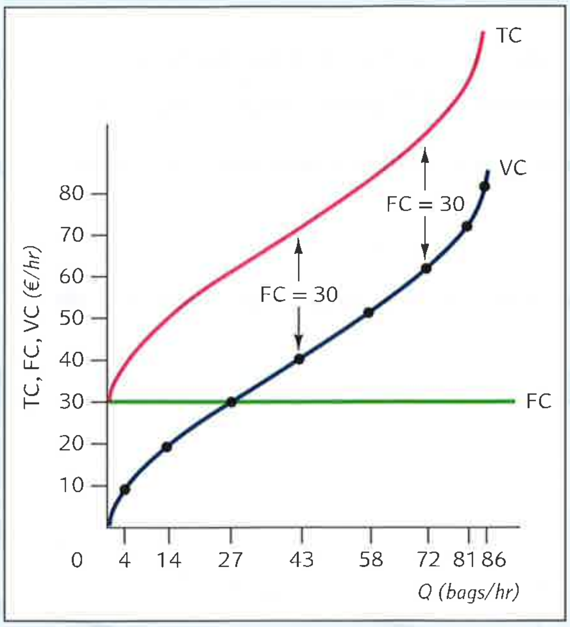



**FIGURE 11.2**
**The Total, Variable and Fixed Cost Curves of Kelly's Cleaners**
Fixed costs are €30 and do not depend on output. So, the FC curve is a horizontal line. Variable costs mirror the production function, shown in Figure 11.1, with variable costs growing at a diminishing rate for Q less than 43. The TC curve, which is the sum of the FC and VC curves, is parallel to the VC curve and lies FC = 30 units above it.

output. The total cost of producing zero output is equal to fixed costs, FC. Note also in the figure that the vertical distance between the VC and TC curves is everywhere equal to FC. This means that the total cost curve is parallel to the variable cost curve and lies FC units above it.

**EXAMPLE 11.1 Suppose the production function is given by $Q = 3KL$, where $K$ denotes capital and $L$ denotes labour. The price of capital is €2/machine-hr, the price of labour is €24/person-hr, and capital is fixed at 4 machine-hr/hr in the short run. Graph the TC, VC and FC curves for this production process.**

Unlike the production process shown in Figure 11.1, the process in this example is one in which there are everywhere constant returns to the variable factor of production. As shown in Figure 11.3, output here is strictly proportional to the variable input.

To derive the total cost function from this production function, we must first discover how much capital and labour are required to produce a given level of output in the short run. Since $K$ is fixed at 4 machine-hr/hr, the required amount of labour input is found by solving

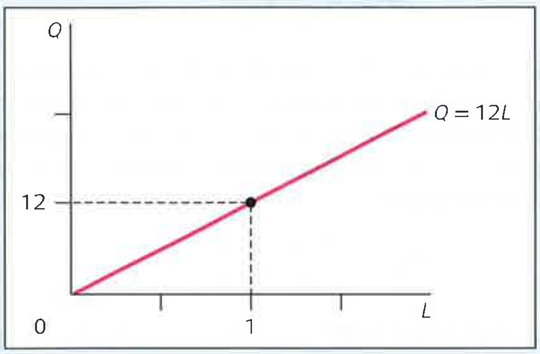



**FIGURE 11.3**
**The Short-Run Production Function**
**$Q = 3KL$, with $K = 4$**
This short-run production function exhibits constant returns to $L$ over the entire range of $L$.

$Q = 3KL = 3(4)L$ for $L = Q/12$. The total cost of producing $Q$ units of output per hour is therefore given by

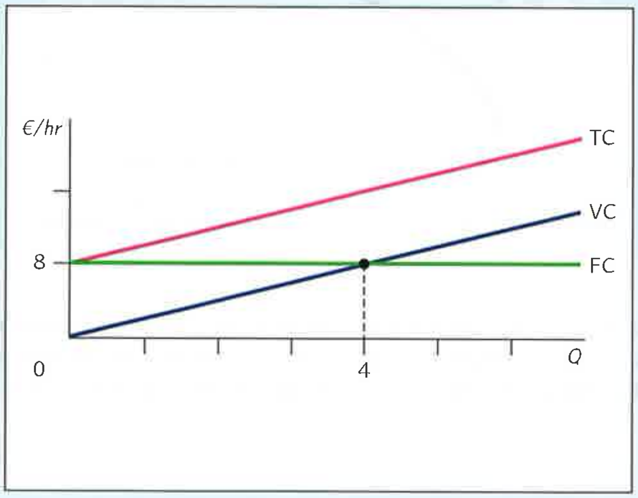
$$\mathrm{TC}_Q = (€2/\text{machine-hr})(4 \text{ machine-hr/hr}) + (€24/\text{person-hr})\left(\frac{Q}{12}\text{ person-hr/hr}\right) = €8/\text{hr} + €2Q/\text{hr} \quad (11.4)$$

The €8/hr expenditure on capital constitutes fixed cost. Variable cost is total cost less fixed cost, or

$$\mathrm{VC}_Q = 2Q \quad (11.5)$$

The total, variable and fixed cost curves are plotted in Figure 11.4. ◆

**FIGURE 11.4**
**The Total, Variable and Fixed Cost Curves for the Production Function $Q = 3KL$**
With $K$ fixed at 4 machine-hr/hr in the short run and a price of $K$ of $r$ = €2/machine-hr, fixed costs are €8/hr. To produce $Q$ units of output per hour requires $Q/12$ person-hr/hr of labour. With a price of labour of €24/person-hr, variable cost is €2$Q$/hr. Total cost is €8/hr + €2$Q$/hr.




**EXERCISE 11.1** **Same as Example 11.1 except the price of capital is $r$ = €4/machine-hr.**

## Other Short-Run Costs

***average fixed cost (AFC)*** fixed cost divided by the quantity of output.

**Average fixed cost (AFC)** is fixed cost divided by the quantity of output. For the production function shown in Table 11.1, for example, the average fixed cost of washing 58 bags/hr is (€30/hr) ÷ (58 bags/hr) = €0.517/bag. More generally, the average fixed cost of producing an output level of $Q_1$ is written

$$\mathrm{AFC}_{Q_1} = \frac{\mathrm{FC}}{Q_1} = \frac{rk_0}{Q_1} \quad (11.6)$$

Note in Equation 11.6 that, unlike FC, AFC depends on the level of output produced.

***average variable cost (AVC)*** variable cost divided by the quantity of output.

**Average variable cost (AVC)** is variable cost divided by the quantity of output. If Kelly washes 72 bags/hr, his AVC will be (€10/person-hr) (6 person-hr/hr) ÷ 72 bags/hr = €0.833/bag. The average variable cost of producing an output level $Q_1$ may be written as

$$\mathrm{AVC}_{Q_1} = \frac{\mathrm{VC}_{Q_1}}{Q_1} = \frac{wL_1}{Q_1} \quad (11.7)$$

***average total cost (ATC)*** total cost divided by the quantity of output.

**Average total cost (ATC)** is total cost divided by the quantity of output. And since total cost is the sum of total fixed cost and total variable cost, it follows that ATC is the sum of AFC and AVC. For example, the ATC of washing 58 bags/hr is (€30/hr) ÷ (58 bags/hr) + (€10/person-hr) (5 person-hr/hr) ÷ (58 bags/hr) = €0.517/bag + €0.862/bag = €1.379/bag. The average total cost of producing $Q_1$ units of output is given by

$$\mathrm{ATC}_{Q_1} = \mathrm{AFC}_{Q_1} + \mathrm{AVC}_{Q_1} = \frac{rK_0 + wL_1}{Q_1} \tag{11.8}$$

**Marginal cost (MC)**, finally, is the change in total cost that results from producing an additional unit of output.[4] In going from 58 to 72 bags/hr, for example, total costs go up by €10/hr, which is the cost of hiring the extra worker needed to achieve that increase in output. Since the extra worker washes an extra 14 bags/hr, the marginal cost of the additional output in per-bag terms is (€10/hr) ÷ (14 bags/hr) = €0.714/bag. More generally, if $\Delta Q$ denotes the change in output from an initial level of $Q_1$, and $\Delta \mathrm{TC}_{Q_1}$ denotes the corresponding change in total cost, marginal cost at $Q_1$ is given by

> ***marginal cost (MC)*** change in total cost that results from a one-unit change in output.

$$\mathrm{MC}_{Q_1} = \frac{\Delta \mathrm{TC}_{Q_1}}{\Delta Q} \tag{11.9}$$

Because fixed cost does not vary with the level of output, the change in total cost when we produce $\Delta Q$ additional units of output is the same as the change in variable cost. Thus an equivalent expression for marginal cost is

$$\mathrm{MC}_{Q_1} = \frac{\Delta \mathrm{VC}_{Q_1}}{\Delta Q} \tag{11.10}$$

where $\Delta \mathrm{VC}_{Q_1}$ represents the change in variable cost when we produce $\Delta Q$ units of additional output.

# The great distortion

**Subsidies that make borrowing irresistible need to be phased out**


THE way that black holes bend light's path through space cannot be smoothed out by human ingenuity. By contrast, a vast distortion in the world economy is wholly man-made. It is the subsidy that governments give to debt. Half the rich world's governments allow their citizens to deduct the interest payments on mortgages from their taxable income; almost all countries allow firms to write off payments on their borrowing against taxable earnings. It sounds prosaic, but the cost—and the harm—is immense.

In 2007, before the financial crisis led to the slashing of interest rates, the annual value of the forgone tax revenues in Europe was around 3% of GDP—or $510 billion—and in America almost 5% of GDP—or $725 billion (see pages 19-22). That means governments on both sides of the Atlantic were spending more on cheapening the cost of debt than on defence. Even today, with interest rates close to zero, America's debt subsidies cost the federal government over 2% of GDP—as much as it spends on all its policies to help the poor.

This hardly begins to capture the full damage, which is aggravated by the behaviour the tax breaks encourage. People borrow more to buy property than they otherwise would, raising house prices and encouraging over-investment in real estate instead of in assets that create wealth. The tax benefits are largely reaped by the rich, worsening inequality. Corporate financial decisions are motivated by maximising the tax relief on debt instead of the needs of the underlying business.

Debt has many wonderful qualities—allowing firms to invest and individuals to benefit today from tomorrow's income. But the tax subsidies have tilted the economy in a woeful direction. They have created a financial system that is prone to crises and biased against productive investment; they have reduced economic growth and worsened inequality. They are a man-made distortion and they need to be fixed.

### Debt and taxes, life's certainties

Start with the fragility. Economies biased towards debt are more prone to crises, because debt imposes a rigid obligation to repay on vulnerable borrowers, whereas equity is expressly designed to spread losses onto investors. Firms without significant equity buffers are more likely to go broke, banks more likely to topple (see page 66). The dotcom crash in 2000-02 caused losses to shareholders worth $4 trillion and a mild recession. Leveraged global banks notched up losses of $2 trillion in 2007-10 and the world economy imploded. Financial regulators have already gone some way to redressing the balance from debt by forcing the banks to fund themselves with more equity. But the bias remains—in large part because of the subsidy for debt. Under a more neutral tax system, firms would sell more equity and carry less debt. Investors would have to get used to greater volatility; but as equity buffers got thicker, shareholders would be taking less risk.

A neutral tax system would also lead to more efficient choices by savers and lenders. Today 60% of bank lending in rich countries is for mortgages. Without a tax break, people would borrow less to buy houses and banks would lend less against property. Investment in new ideas and businesses that enhance productivity would become relatively more attractive, in turn boosting economic growth.

Removing the advantages that debt enjoys would also lead to a fairer system. Relief on mortgage payments is a subsidy that flows to people who need it least: studies show that the richest 20% of American households by income gain the most. Mortgages would become costlier. But new instruments would emerge to allow individuals to bridge the gap between current savings and future income that debt alone now closes—for example, shared-equity mortgages that divide the gains and losses from house-price movements between banks and homeowners.

### Lenders and borrowers

If the arguments for getting rid of the debt distortion are overwhelming, the path to its elimination could hardly be more rocky. Politicians do not much like changes that will lower house prices. There is a big co-ordination problem: tax is a matter for national governments, and few countries will be prepared unilaterally to withdraw subsidies that might make them less appealing to footloose companies. In addition, vested interests will bleat loudly. Businesses that depend heavily on debt—banks, private-equity firms and the like—will be ready to spend some of the billions they gain from the tax subsidy on lobbying to defend it.

This argues for a staged approach. The place to start is the subsidies on residential mortgages. Not only do these subsidies increase financial fragility, they fail to achieve their purported goal of promoting home-ownership. The shares of people owning their own homes in America and Switzerland, two countries with vast subsidies, are 65% and 44% respectively—no more than in other advanced economies like Britain and Canada that offer no tax break. The wisest step would be to phase out tax relief gradually, as Britain did in the 1990s.

Getting rid of the tax breaks for corporate debt will be harder. The few countries that have tried to level the playing field have done so by giving an equivalent handout to equity. Belgium and Italy, for instance, give dividend payments and profits flowing to equity holders some of the same perks enjoyed by interest payments. But such systems are fiddly and lower a country's tax base at a time when governments need money.

The best approach is gradually to phase out tax breaks for debt at the same time as lowering the corporate-tax rate. That would make the policy revenue-neutral, and would also defuse the risk to governments who want to push ahead but fear losing a war waged on tax competition.

Acting in concert or alone, countries should act soon. When interest rates are low, as now, the sweeteners for debt are smaller and thus easier to remove. When rates rise—as, inevitably, they will—the subsidy will become more valuable. This is the moment to tackle the great debt distortion. There may never be a better chance. ■