University of Zurich[UZH]

Leseaufträge «Mikroökonomik I»

Modul 3: Produktion und Kosten

Unit 5:

- Langfristige Kosten

Quellen:

- **Chapter 11 – Costs**
  Frank, Robert H, & Cartwright, Edward. (2016). *Microeconomics and Behaviour (2nd European ed.)*. London: McGraw-Hill Education.

COSTS IN THE LONG RUN

In the long run all inputs are variable by definition. Kelly's Cleaners, for instance, need no longer take capital fixed at 120 machine-hr/hr. Kelly may well find it desirable to have more or less capital. And note that, if he changes the amount of capital he employs, then he will likely want to also change the amount of labour he employs. Our task in this section is, thus, to find out what the manager of a firm should do if she wishes to produce a given level of output at the lowest possible cost, and is free to choose any input combination she pleases. As we will see, the answer depends critically on the relative prices of capital and labour.

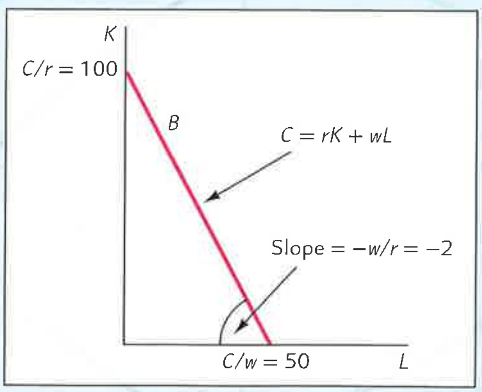
Choosing the Optimal Input Combination

No matter what the structure of industry may be—monopolistic or atomistically competitive, capitalist or socialist, industrialized or less developed—the objective of most producers is to produce any given level and quality of output at the lowest possible cost. Equivalently, the producer wants to produce as much output as possible from any given expenditure on inputs.

Let us begin with the case of a firm that wants to maximize output from a given level of expenditure. Suppose it uses only two inputs, capital (K) and labour (L), whose prices, measured in euros per unit of input per day, are r = €2/day and w = €4/day, respectively. What different combinations of inputs can this firm purchase for a total expenditure of C = €200/day? Notice that this question has the same structure as the one we encountered in the theory of consumer behaviour in Chapter 4 ('With an income of M, and facing prices of P_X and P_Y, what combinations of X and Y can the consumer buy?'). In the consumer's case, recall, the answer was easily summarized by the budget constraint. The parallel information in the case of the firm is summarized by the **isocost line**, shown in Figure 11.10 for the example given. Any of the input combinations on the locus labelled B can be purchased for a total expenditure of €200/day. Analogously to the budget constraint case, the slope of the isocost line is the negative of the ratio of the input prices, $-w/r$.

isocost line a set of input bundles each of which costs the same amount.

FIGURE 11.10
The Isocost Line
For given input prices ($r = 2$ and $w = 4$ in the diagram), the isocost line is the locus of all possible input bundles that can be purchased for a given level of total expenditure C (€200 in the diagram). The slope of the isocost line is the negative of the input price ratio, $-w/r$.



EXERCISE 11.5 **If $w = 3$ and $r = 6$, draw the isocost lines that correspond to total expenditure of €90 and €180 per unit of time.**

The analytic approach for finding the maximum output that can be produced for a given cost turns out to be similar to the one for finding the optimal consumption bundle. Just as a given level of satisfaction can be achieved by any of a multitude of possible consumption bundles (all of

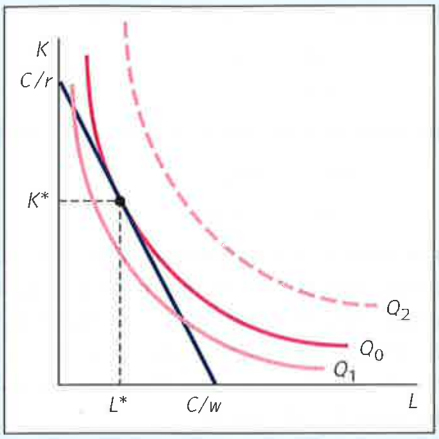


FIGURE 11.11
The Maximum Output for a Given Expenditure
A firm that is trying to produce the largest possible output for an expenditure of *C* will select the input combination at which the isocost line for *C* is tangent to an isoquant.

which lie on the same indifference curve), so too can a given amount of output be produced by any of a host of different input combinations (all of which lie on the same isoquant). In the consumer case, we found the optimum bundle by superimposing the budget constraint onto the indifference map and locating the relevant point of tangency.[9] Here, we superimpose the isocost line onto the isoquant map. In Figure 11.11, the tangency point (L^*, K^*) is the input combination that yields the highest possible output (Q_0) for an expenditure of C.

As noted, the problem of producing the largest output for a given expenditure is solved in essentially the same way as the problem of producing a given level of output for the lowest possible cost. The only difference is that in the latter case we begin with a specific isoquant (the one that corresponds to the level of output we are trying to produce), then superimpose a map of isocost lines, each corresponding to a different cost level. In our first exercise, cost was fixed and output varied; this time, output is fixed and costs vary. As shown in Figure 11.12, the least-cost input bundle (L^*, K^*) corresponds to the point of tangency between an isocost line and the specified isoquant (Q_0).

Recall from Chapter 10 that the slope of the isoquant at any point is equal to $-MP_L/MP_K$, the negative of the ratio of the marginal product of L to the marginal product of K at that point. (Recall also from Chapter 10 that the absolute value of this ratio is called the marginal rate of

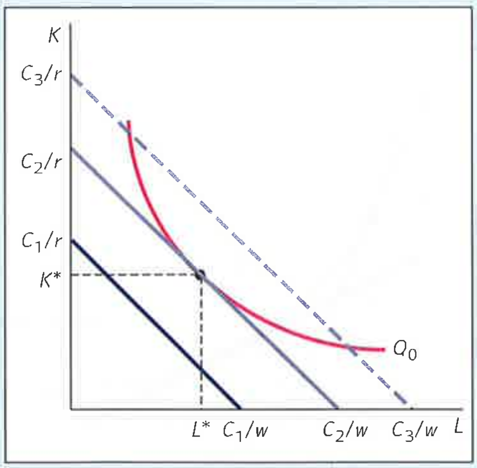


FIGURE 11.12
The Minimum Cost for a Given Level of Output
A firm that is trying to produce a given level of output, Q_0, at the lowest possible cost will select the input combination at which an isocost line is tangent to the Q_0 isoquant.

[9] Except, of course, in the case of corner solutions.

technical substitution.) Combining this with the result that minimum cost occurs at a point of tangency with the isocost line (whose slope is $-w/r$), it follows that

$$\frac{MP_{L^*}}{MP_{K^*}} = \frac{w}{r} \tag{11.20}$$

where K^* and L^* again denote the minimum-cost values of K and L. Cross-multiplying, we have

$$\frac{MP_{L^*}}{w} = \frac{MP_{K^*}}{r} \tag{11.21}$$

Equation 11.21 has a straightforward economic interpretation. Note first that MP_L^* is simply the extra output obtained from an extra unit of L at the cost-minimizing point, and; w is the cost, in euros, of an extra unit of L. The ratio MP_{L^*}/w is thus the extra output we get from the last euro spent on L. Similarly, MP_{K^*}/r is the extra output we get from the last euro spent on K. In words, Equation 11.21 tells us that when costs are at a minimum, the *extra output we get from the last euro spent on an input must be the same for all inputs.*

It is easy to show why, if that were not the case, costs would not be at a minimum. Suppose, for example, that the last units of both labour and capital increased output by 4 units. That is, suppose $MP_L = MP_K = 4$. And again, suppose that $r = €2$/day and $w = €4$/day. We would then have achieved only 1 unit of output for the last euro spent on L, but 2 units for the last euro spent on K. We could reduce spending on L by a euro, increase spending on K by only 50 cents, and get the same output level as before, saving 50 cents in the process. Whenever the ratios of marginal products to input prices differ across inputs, it will always be possible to make a similar cost-saving substitution in favour of the input with the higher MP/P ratio.[10]

More generally, we may consider a production process that employs not two but N inputs, $X_1, X_2, \ldots, X_N$. In this case, the condition for production at minimum cost is a straightforward generalization of Equation 11.21:

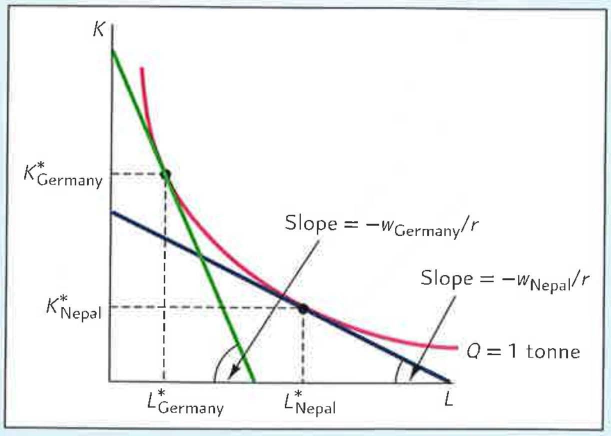
$$\frac{MP_{X_1}}{P_{X_1}} = \frac{MP_{X_2}}{P_{X_2}} = \cdots \frac{MP_{X_N}}{P_{X_N}} \tag{11.22}$$

ECONOMIC NATURALIST 11.1

Why is gravel made by hand in Nepal but by machine in Europe?

For simplicity, suppose that capital (K) and labour (L) are employed to transform rocks into gravel. And suppose that any of the input combinations on the isoquant labelled $Q = 1$ tonne in Figure 11.13 will yield 1 tonne of gravel. Thus, the combination labelled ($L^*_{\text{Germany}}, K^*_{\text{Germany}}$) might correspond to the highly capital-intensive technique used in Germany and ($L^*_{\text{Nepal}}, K^*_{\text{Nepal}}$) to the highly labour-intensive technique used in Nepal.

FIGURE 11.13
Different Ways of Producing One Tonne of Gravel
Countries where labour is cheap relative to capital will select labour-intensive techniques of production. Those where labour is more expensive will employ relatively more capital-intensive techniques.



[10]Again, this statement is true except in the case of corner solutions.

The reason the chosen techniques differ between countries is not that Germany is richer; rather, it is that the relative prices of labour and capital differ so dramatically in the two countries. In Nepal, labour is cheaper than in almost any other nation. Wages in Germany, by contrast, are among the highest in the world. Construction equipment is traded in world markets and, aside from shipping costs, its price does not differ much from one country to another. If the price of capital, r, is roughly the same in the two countries and the price of labour, w, is much higher in Germany, it follows that the isocost line is much flatter in Nepal. And as shown in Figure 11.13, this fact alone is sufficient to account for the dramatic difference in production techniques. ■

EXERCISE 11.6 Suppose capital and labour are perfect complements in a one-to-one ratio. That is, suppose that $Q = \min(L, K)$. Currently, the wage is $w = 5$ and the rental rate is $r = 10$. What is the minimum cost and method of producing $Q = 20$ units of output? Suppose the wage rises to $w' = 20$. If we keep total cost the same, what level of output can now be produced and what method of production (input mix) is used?

EXERCISE 11.7 Repeat the previous exercise but now suppose capital and labour are perfect substitutes in a one-to-one ratio: $Q = K + L$.

ECONOMIC NATURALIST 11.2

Why do unions support minimum wage laws so strongly?

Labour unions have historically been among the most outspoken proponents of minimum wage legislation. They favour not only higher levels of the minimum wage, but also broader coverage. UNISON, for example, represents well over a million public sector workers in the UK. It campaigned strongly for a minimum wage to be introduced in the UK (it was in 1998). And it continues to campaign strongly for a higher minimum, and greater enforcement of the legislation. For instance, it expresses concern that migrant workers are not being paid the minimum. Most union members, however, earn substantially more than the minimum wage, and few are migrant workers. Why, then, do unions like UNISON devote such great effort to lobbying in favour of minimum wages?

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Why do union members, who earn substantially more than the minimum wage, favour increasing the minimum wage?

One reason might be that their members are genuinely concerned about the economic well-being of workers less fortunate than themselves. No doubt many do feel such concern. But there are other disadvantaged groups—many of them even more deserving of help than low-wage workers—on whose behalf the unions might also have lobbied. Why not, for example, try to get extra benefits for homeless children or for the physically disabled?

An understanding of the condition for production at minimum cost helps answer these questions. Note first that, on the average, union workers tend to be more skilled than non-union workers. Unskilled labour and skilled labour are substitutes for one another in many production processes, giving rise to isoquants shaped something like the one shown in Figure 11.14. What mix of the two skill categories the firm chooses to use will depend strongly on relative prices. Figure 11.14 shows the least costly mix for producing $Q = Q_0$ both before and after the enactment of the minimum wage statute. The wage rate for skilled labour is denoted by w. The pre-legislation price of unskilled labour is w_1, which rises to w_2 after enactment of the law. The immediate effect is to increase the absolute value of the slope of the isocost line from w_1/w to w_2/w, causing the firm to increase its employment of skilled labour from S_1 to S_2, simultaneously reducing its employment of unskilled (non-union) labour from U_1 to U_2.

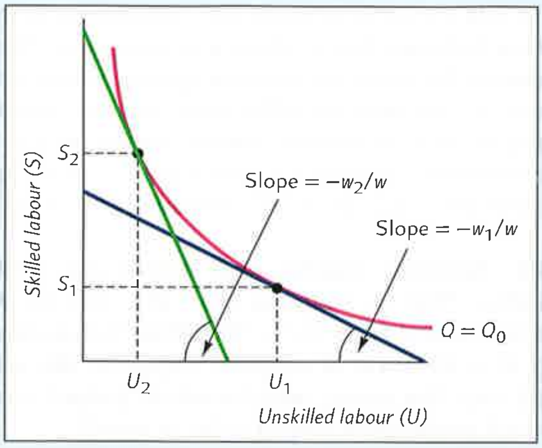


FIGURE 11.14
The Effect of a Minimum Wage Law on Employment of Skilled Labour
Unskilled labour and skilled labour are substitutes for one another in many production processes. When the price of unskilled labour rises, the slope of the isocost line rises, causing many firms to increase their employment of skilled (unionized) labour.

Although most union workers are not affected directly by the minimum wage laws, these laws have the indirect consequence of increasing the demand for union labour.[11] Even if unions lacked their avowed concern for the well-being of unskilled, largely non-union workers, there would thus be little mystery as to why unions devote so much of their resources in support of extensions of minimum wage legislation. ■

ECONOMIC NATURALIST 11.3

Why would a bathroom equipment manufacturer bake the image of a housefly onto the centre of its ceramic urinals?

The substitution of capital for labour is sometimes motivated not by a change in factor prices, but by the introduction of new ideas. Consider, for example, the 'official toilet project' initiated by Jos van Bedaf, then head manager of cleaning for the Schiphol airport in Amsterdam.[12] His problem was that the men's toilets at the airport, which were used by more than 100,000 patrons a year, had a tendency to become messy and smelly despite frequent cleanings. Mr van Bedaf's solution was not to intensify the efforts of maintenance crews but to make a minor change in the restroom equipment. Specifically, he requested that his sanitation equipment manufacturer supply the airport with urinals with the image of a housefly baked onto the centre of each fixture's glazed ceramic surface. His theory was that the presence of this target would cause patrons to be much more accurate in their use of the facilities. The result? Dramatically cleaner facilities and a 20 per cent reduction in cleaning costs. A national newspaper in the Netherlands rated the Schiphol facilities first on a list of clean toilets. ■

The Relationship between Optimal Input Choice and Long-Run Costs

output expansion path the locus of tangencies (minimum-cost input combinations) traced out by an isocost line of given slope as it shifts outward into the isoquant map for a production process.

Given sufficient time to adjust, the firm can always buy the cost-minimizing input bundle that corresponds to any particular output level and relative input prices. To see how the firm's costs vary with output in the long run, we need only compare the costs of the respective optimal input bundles.

The curve labelled *EE* in Figure 11.15 shows the firm's **output expansion path**. It is the set of cost-minimizing input bundles when the input price ratio is fixed at w/r. Thus, when the price of K is r and the price of L is w, the cheapest way to produce Q_1 units of output is to use the input bundle S, which contains K_1^* units of K, L_1^* units of L and costs TC_1. The bundle S is therefore one point on the output expansion path.

[11] Note that this example assumes that the firm will produce the same level of output after the minimum wage hike as before. As we will see in the next chapter, however, the firm will generally produce less output than before. If the output reduction is large enough, it could offset the firm's switch to skilled labour.

[12] This example is based on Stefan Verhagen, 'Fly in the Pot', *Cornell Business*, 21 April 1992.

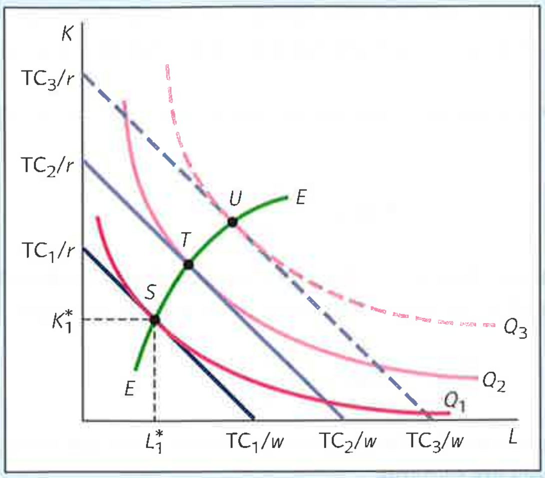


FIGURE 11.15
The Long-Run Expansion Path
With fixed input prices r and w, bundles S, T, U and others along the locus EE represent the least costly ways of producing the corresponding levels of output.

In like fashion, the output level Q_2 is associated with bundle T, which has a total cost of TC_2; Q_3 is associated with U, which costs TC_3; and so on. In the theory of firm behaviour, the long-run expansion path is the analogue to the income-consumption curve in the theory of the consumer.

To go from the long-run expansion path to the long-run total cost curve, we simply plot the relevant quantity–cost pairs from Figure 11.15. Thus, the output level Q_1 corresponds to a long-run total cost of TC_1, Q_2 to TC_2, and so on. The result is the curve labelled LTC in the top panel in Figure 11.16. In the long run there is no need to distinguish between total, fixed and variable costs, since all costs are variable.

The LTC curve will always pass through the origin because in the long run the firm can liquidate all of its inputs. If the firm elects to produce no output, it need not retain, or pay for, the services of any of its inputs. The shape of the LTC curve shown in the top panel looks very much

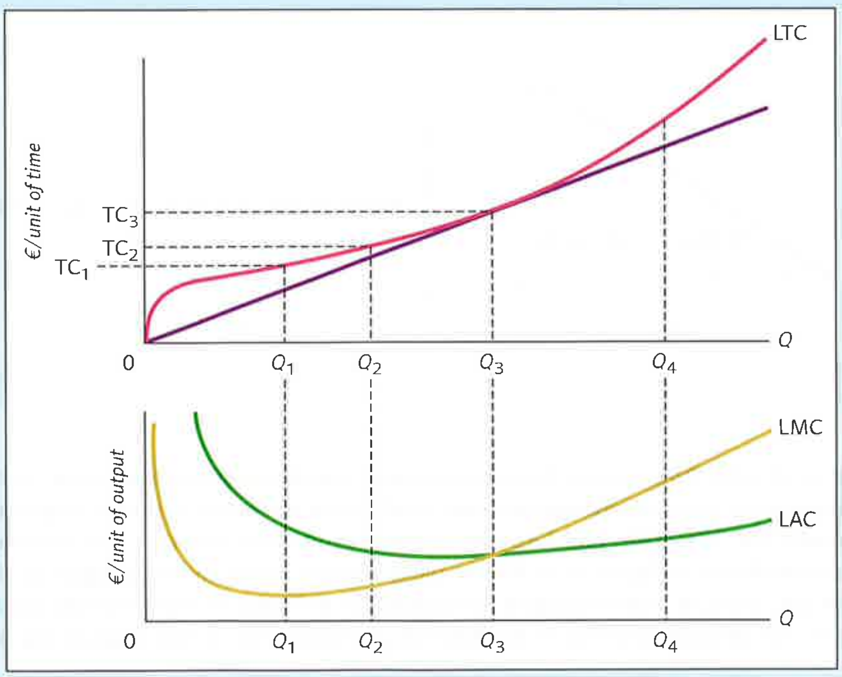


FIGURE 11.16
The Long-Run Total, Average and Marginal Cost Curves
In the long run, the firm always has the option of ceasing operations and ridding itself of all its inputs. This means that the long-run total cost curve (top panel) will always pass through the origin. The long-run average and long-run marginal cost curves (bottom panel) are derived from the long-run total cost curves in a manner completely analogous to the short-run case.

like that of the short-run total cost curve shown in Figure 11.2. But this need not always be the case, as we will presently see. For the moment, though, let us take the shape of the LTC curve in the top panel in Figure 11.16 as given and ask what it implies for the long-run average and marginal cost curves.

Analogously to the short-run case, long-run marginal cost (LMC) is the slope of the long-run total cost curve:

$$\mathrm{LMC}_Q = \frac{\Delta \mathrm{LTC}_Q}{\Delta Q} \tag{11.23}$$

In words, LMC is the cost to the firm, in the long run, of expanding its output by 1 unit.

Long-run average cost (LAC) is the ratio of long-run total cost to output:

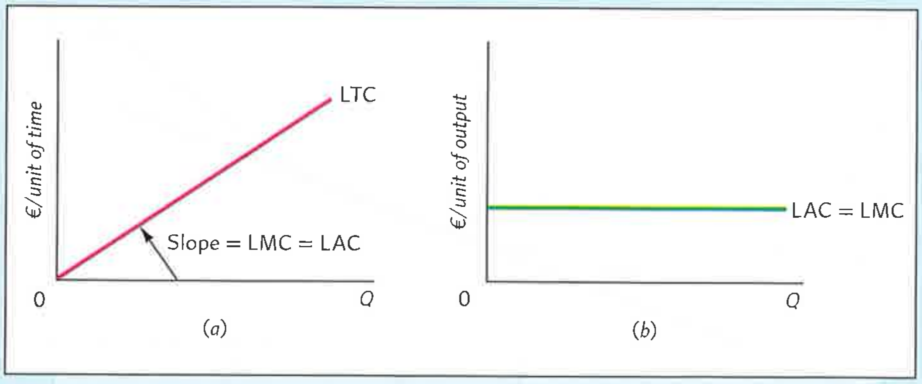
$$\mathrm{LAC}_Q = \frac{\mathrm{LTC}_Q}{Q} \tag{11.24}$$

Again, there is no need to discuss the distinctions between average total, fixed and variable costs, since all long-run costs are variable.

The bottom panel in Figure 11.16 shows the LAC and LMC curves that correspond to the LTC curve shown in the top panel. The slope of the LTC curve is diminishing up to the output level Q_1 and increasing thereafter, which means that the LMC curve takes its minimum value at Q_1. The slope of LTC and the slope of the ray to LTC are the same at Q_3, which means that LAC and LMC intersect at that level of output. And again as before, the traditional average–marginal relationship holds: LAC is declining whenever LMC lies below it, and rising whenever LMC lies above it.

For a constant returns to scale production function, doubling output exactly doubles costs.[13] Tripling all inputs triples output and triples costs, and so on. For the case of constant returns to scale, long-run total costs are thus exactly proportional to output. As shown in Figure 11.17(a), the LTC curve for a production function with constant returns to scale is a straight line through the origin. Because the slope of LTC is constant, the associated LMC curve is a horizontal line, and is exactly the same as the LAC curve, as in Figure 11.17(b).

FIGURE 11.17
The LTC, LMC and LAC Curves with Constant Returns to Scale
(a) With constant returns, long-run total cost is strictly proportional to output. (b) Long-run marginal cost is constant and equal to long-run average cost.



When the production function has decreasing returns to scale, a given proportional increase in output requires a greater proportional increase in all inputs and hence a greater proportional increase in costs. The LTC, LMC and LAC curves for a production function with decreasing returns to scale are shown in Figure 11.18. For the particular LTC curve shown in Figure 11.18(a), the associated LAC and LMC curves happen to be linear, as in Figure 11.18(b), but this need not always happen. The general property of the decreasing returns case is that it gives rise to an

[13]Assuming, of course, that input prices remain the same as output varies.

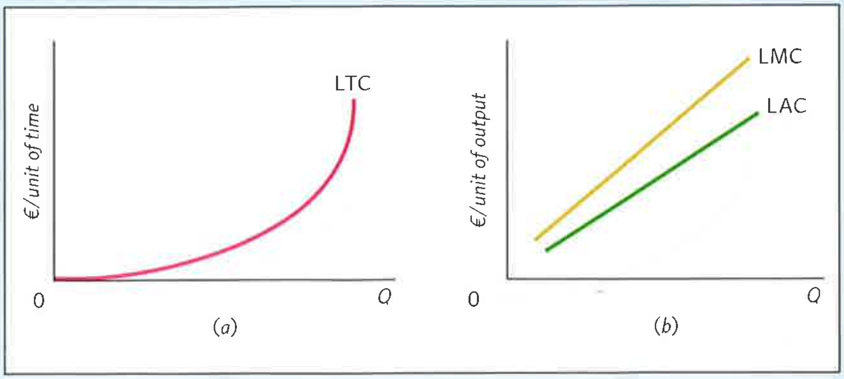


FIGURE 11.18
The LTC, LAC and LMC Curves for a Production Process with Decreasing Returns to Scale
Under decreasing returns, output grows less than in proportion to the growth in inputs, which means that total cost grows more than in proportion to growth in output.

upward-curving LTC curve and upward-sloping LAC and LMC curves. Note yet another application of the average–marginal relationship: the fact that LMC exceeds LAC ensures that LAC must rise with output.

Consider, finally, the case of increasing returns to scale. Here, output grows more than in proportion to the increase in inputs. In consequence, long-run total cost rises less than in proportion to increases in output, as shown in Figure 11.19(*a*). The accompanying LAC and LMC curves are shown in Figure 11.19(*b*). The distinguishing feature of the LAC and LMC curves under increasing returns to scale is not the linear form shown in this particular example, but the fact that they are downward sloping.

The production processes whose long-run cost curves are pictured in Figures 11.17, 11.18 and 11.19 are 'pure cases', exhibiting constant, decreasing and increasing returns to scale, respectively, over their entire ranges of output. As discussed in Chapter 10, however, the degree of returns to scale of a production function need not be the same over the whole range of output.

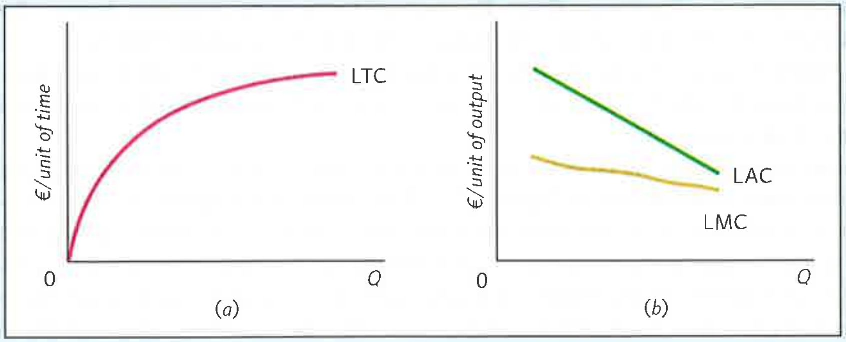


FIGURE 11.19
The LTC, LAC and LMC Curves for a Production Process with Increasing Returns to Scale
With increasing returns, the large-scale firm has lower average and marginal costs than the smaller-scale firm.

APPENDIX 11

MATHEMATICAL EXTENSIONS OF THE THEORY OF COSTS

THE CALCULUS APPROACH TO COST MINIMIZATION

With a basic understanding of calculus it is possible to derive cost functions and optimal factor inputs if we know the production function. We begin with the following example before moving on to the more general case.

EXAMPLE A.11.1 **For the production function $Q = F(K, L) = \sqrt{K}\sqrt{L}$ with $r = 4$ and $w = 2$, find the values of K and L that minimize the cost of producing 2 units of output.**

Our problem here is to minimize $4K + 2L$ subject to $F(K, L) = \sqrt{K}\sqrt{L} = 2$. Here, the production function constraint is $Q = 2 = \sqrt{K}\sqrt{L}$, which yields $K = 4/L$. So our problem is to minimize $4(4/L) + 2L$ with respect to L. The first-order condition for a minimum is given by

$$\frac{d[(16/L) + 2L]}{dl} = 2 - \frac{16}{L^2} = 0 \tag{A.11.1}$$

which yields $L = 2\sqrt{2}$. Substituting back into the production function constraint, we have $K = 4/(2\sqrt{2}) = \sqrt{2}$. ◆

The Cost-Minimization Problem

The objectives of a firm when producing Q_0 units of output are given by the following cost-minimization problem:

$$\min_{K,\,\mathrm{L}} rK + wL \quad \text{subject to } F(K, L) = Q_0 \tag{A.11.2}$$

where w is the price of labour and r the price of capital. To find the values of K and L that minimize costs, we can use the Lagrangian technique discussed in the Appendix to Chapter 4. The Lagrangian expression is (we will use LG to denote the Lagrangian to avoid confusion over Ls):

$$LG = wK + rL + \lambda[F(K, L) - Q_0] \tag{A.11.3}$$

The first-order condition for a minimum is given by

$$\frac{\partial LG}{\partial K} = r + \lambda\frac{\partial F}{\partial K} = 0 \tag{A.11.4}$$

$$\frac{\partial LG}{\partial L} = w + \lambda\frac{\partial F}{\partial L} = 0 \tag{A.11.5}$$

and

$$\frac{\partial LG}{\partial \lambda} = F(K, L) - Q_0 = 0 \tag{A.11.6}$$

Dividing Equation A.11.3 by Equation A.11.4 and rearranging terms, we have

$$\frac{\partial F/\partial K}{r} = \frac{\partial F/\partial L}{w} \tag{A.11.7}$$

which is the result of Equation 11.21.

Deriving Cost Functions

With Equation A.11.7 to hand we can derive the minimum cost of producing any output Q_0. This, in turn, allows us to derive the cost functions. The following examples will illustrate.

EXAMPLE A.11.2 For the production function $Q = F(K, L) = \sqrt{K}\,\sqrt{L}$ with $r = 9$ and $w = 1$, derive short-run and long-run cost functions.

It is convenient to rearrange Equation A.11.7 to get

$$\frac{\partial F/\partial K}{\partial F/\partial L} = \frac{r}{w} \tag{A.11.8}$$

In this case

$$\frac{\partial F}{\partial K} = \frac{\sqrt{L}}{2\sqrt{K}} \quad \text{and} \quad \frac{\partial F}{\partial L} = \frac{\sqrt{K}}{2\sqrt{L}} \tag{A.11.9}$$

So,

$$\frac{\partial F/\partial K}{\partial F/\partial L} = \frac{\sqrt{L}}{2\sqrt{K}} \times \frac{2\sqrt{L}}{\sqrt{K}} = \frac{L}{K} \tag{A.11.10}$$

Substituting this into Equation A.11.8 we get condition $L/K = 9$, or, equivalently, $L = 9K$. This tells us the optimal amount of labour as a function of the amount of capital.

The next step is to substitute the expression $L = 9K$ into the production function to give

$$Q = \sqrt{K}\sqrt{L} = \sqrt{K}\sqrt{9K} = 3K \tag{A.11.11}$$

From this we can infer that the optimal amount of capital to produce Q units of output is $K = Q/3$. Given that $L = 9K$ we also learn that the optimal amount of labour is $L = 3Q$. We are now in a position to write down an expression for long-run total costs:

$$\text{LTC}_Q = 9K + L = 9\frac{Q}{3} + 3Q = 6Q \tag{A.11.12}$$

With this you can easily derive long-run average and marginal costs are 6. So, this production function has constant returns to scale.

In order to derive short-run cost curves we need to fix a specific value of K. For instance, suppose that $K = 36$. Then the short-run production function is $Q = 6\sqrt{L}$. Rearranging gives the required amount of labour:

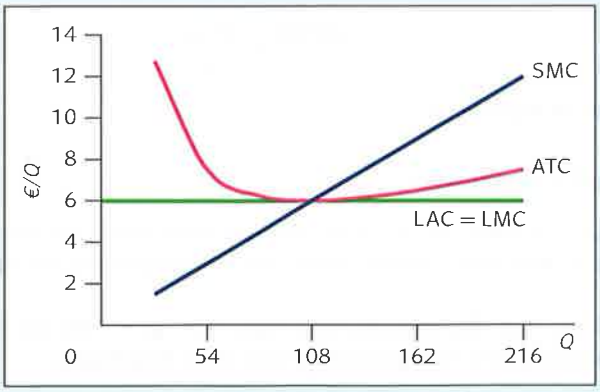
$$L = \frac{Q^2}{36} \tag{A.11.13}$$

Short-run total costs are then given by the sum of fixed costs from capital and variable costs from labour of

$$\text{TC}_Q = 9K + L = 324 + \frac{Q^2}{36} \tag{A.11.14}$$

With this you can derive short-run average costs (which are U-shaped) and short-run marginal costs (which are increasing in output). Figure A.11.6 depicts the cost curves we have derived and can be compared with Figure A.11.3. ◆

FIGURE A.11.6
The LAC, LMC, ATC and SMC when K = 36
There are constant returns to scale. The short-run average cost curve is U-shaped and tangent to the long-run average cost curve at $Q = 108$.



EXERCISE A.11.2 Derive the short-run marginal cost for Example A.11.2 when $K = 4$.

In consumer theory we saw that order-preserving transformations of the utility function do not change the optimal consumption bundle. As the following example illustrates, cost functions are not invariant to such changes in the production function.

EXAMPLE A.11.3 For the production function $Q = F(K, L) = KL$ with $r = 9$ and $w = 1$, derive short-run and long-run cost functions.

In this example

$$\frac{\partial F}{\partial K} = L \quad \text{and} \quad \frac{\partial F}{\partial L} = K \tag{A.11.15}$$

Substituting this into Equation A.11.8 we get condition $L/K = 9$, or, equivalently, $L = 9K$. Note that this is identical to Example A.11.2. So, the optimal ratio between capital and labour is the same in this example as the previous one. Differences emerge, however, when we substitute $L = 9K$ into the production function.

In the current example we get $Q = KL = 9K^2$. From this we can infer that the optimal amount of capital to produce Q units of output is $K = \sqrt{Q}/3$. Given that $L = 9K$ we also learn that the optimal amount of labour is $L = 3\sqrt{Q}$. The expression for long-run total costs is, therefore, given by

$$\mathrm{LTC}_Q = 9K + L = 9\,\frac{\sqrt{Q}}{3} + 3\sqrt{Q} = 6\sqrt{Q} \tag{A.11.16}$$

From this we obtain long-run average and marginal cost curves of $\mathrm{LAC}_Q = 6/\sqrt{Q}$ and $\mathrm{LMC}_Q = 3/\sqrt{Q}$. Note that this production function has increasing returns to scale.

In order to derive some short-run cost curves let us, again, fix $K = 4$. Then the short-run production function is $Q = 4L$. Rearranging gives the required amount of labour, $L = Q/4$. Short-run total costs are then given by

$$\mathrm{TC}_Q = 9K + L = 36 + \frac{Q}{4} \tag{A.11.17}$$

So, short-run marginal costs are given by $\mathrm{SMC}_Q = 1/4$. Figure A.11.7 depicts the relevant cost curves.

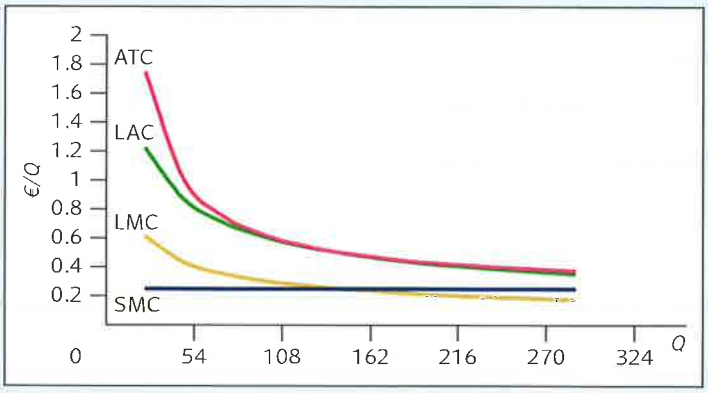


FIGURE A.11.7
The LAC, LMC, ATC and SMC when $Q = KL$
There are increasing returns to scale. The short-run average cost curve is U-shaped and tangent to the long-run average cost curve at $Q = 144$.

We finish by noting that Equation A.11.7 only applies in the case of an interior solution. It is important, therefore, to also consider the possibility for boundary solutions.

EXERCISE A.11.3 For the production function $Q = K + L$ derive an expression for the long-run marginal cost curve if $r = 6$ and $w = 2$.