



# Leseaufträge «Mikroökonomik I»

## Modul 2: Konsument und Nachfrage

### Unit 3:

- Nutzenmaximierung

### Quellen:

- **Chapter 4 – Rational Consumer Choice**  
Frank, Robert H, & Cartwright, Edward. (2016). *Microeconomics and Behaviour (2nd European ed.)*. London: McGraw-Hill Education.

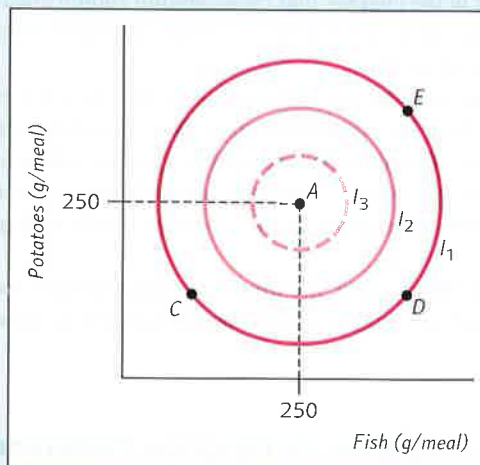
## More-Is-Worse and Satiation Points

Preferences do not have to satisfy the five properties we have assumed so far. As long, however, as the preference ordering satisfies completeness, transitivity and continuity we can still represent preferences using an indifference map. To illustrate, consider Mohan at his favourite restaurant. He has ordered a dish containing fish and potatoes. Figure 4.16 shows his indifference map.

**FIGURE 4.16**

### Preferences with a Satiation Point

The best outcome for Mohan is bundle *A*. At bundle *D* he has too much fish and so the indifference curve slopes up. At bundle *E* he has too much fish and potato and so points below the indifference curve are preferred to points on it.



Mohan's perfect meal contains 250 g of fish and 250 g of potatoes. This is bundle *A* and is called his satiation point. To the bottom left of *A*, near bundle *C*, the indifference curves look 'standard' because Mohan would like more fish and potato. The indifference curve slopes down because Mohan is willing to trade off more fish for less potato.

Things are not so standard to the bottom right of *A*. With bundle *D* Mohan's meal has too much fish. The more-is-better property assumes that he can either store or dispose of the fish he does not want, but in an expensive restaurant neither option may be all that viable. Thus, to the right of *A*, more-is-worse in terms of fish. This is captured by an upward-sloping indifference curve at *D*. Mohan is no longer willing to trade off more fish for less potato. He is only willing to trade off more fish (the good he has too much of) for more potato (the good he has too little of). The marginal rate of substitution is measuring the rate at which Mohan is willing to exchange potato for more fish.

Bundle *E* means that Mohan has too much fish and potato. The indifference curve is sloping down but things are still not standard. Less-is-better, and so bundles that lie below the indifference curve are preferred to the bundles that lie on it.

**EXERCISE 4.6** Gary likes food but dislikes cigarette smoke. The more food he has, the more he would be willing to give up to achieve a given reduction in cigarette smoke. If food and cigarette smoke are the only two goods, draw Gary's indifference curves.

## THE BEST AFFORDABLE BUNDLE

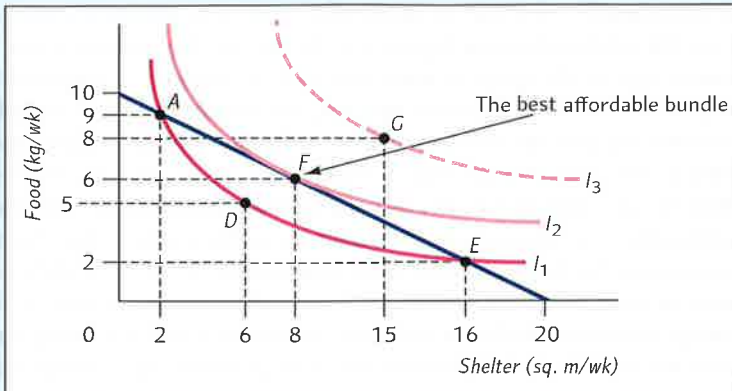
We now have the tools we need to determine how the consumer should allocate his income between two goods. The indifference map tells us how the various bundles are ranked in order of preference. The budget constraint, in turn, tells us which bundles are affordable. The consumer's task is to put the two together and to choose the **best affordable bundle**. (Recall from Chapter 1 that we need not suppose

**best affordable bundle** the most preferred bundle of those that are affordable.

FIGURE 4.17

**The Best Affordable Bundle**

The best the consumer can do is to choose the bundle on the budget constraint that lies on the highest attainable indifference curve. Here, that is bundle *F*, which lies at a tangency between the indifference curve and the budget constraint.



that consumers think explicitly about budget constraints and indifference maps when deciding what to buy. It is sufficient to assume that people make decisions *as if* they were thinking in these terms, just as expert pool players choose between shots as if they knew all the relevant laws of Newtonian physics.)

Let us again consider the choice between food and shelter that confronts a consumer with an income of  $M = €100/\text{wk}$  facing prices of  $P_F = €10/\text{kg}$  and  $P_S = €5/\text{sq. m}$ . Figure 4.17 shows this consumer's budget constraint and part of his indifference map. Of the five labelled bundles—*A*, *D*, *E*, *F* and *G*—in the diagram, *G* is the most preferred because it lies on the highest indifference curve. *G*, however, is not affordable, nor is any other bundle that lies beyond the budget constraint. The more-is-better assumption implies that the best affordable bundle must lie *on* the budget constraint, not inside it. (Any bundle inside the budget constraint would be less preferred than one just slightly to the northeast, which would also be affordable.)

Where exactly is the best affordable bundle located along the budget constraint? We know that it cannot be on an indifference curve that lies partly inside the budget constraint. On the indifference curve  $I_1$ , for example, the only points that are even candidates for the best affordable bundle are the two that lie on the budget constraint, namely, *A* and *E*. But *A* cannot be the best affordable bundle because it is equally attractive as *D*, which in turn is less desirable than *F* by the more-is-better assumption. So by transitivity, *A* is less desirable than *F*. For the same reason, *E* cannot be the best affordable bundle.

Since the best affordable bundle cannot lie on an indifference curve that lies partly inside the budget constraint, and since it must lie on the budget constraint itself, we know it has to lie on an indifference curve that intersects the budget constraint only once. In Figure 4.17, that indifference curve is the one labelled  $I_2$ , and the best affordable bundle is *F*, which lies at the point of tangency between  $I_2$  and the budget constraint. With an income of  $€100/\text{wk}$  and facing prices of  $€5/\text{sq. m}$  for shelter and  $€10/\text{kg}$  for food, the best this consumer can do is to buy 6 kg/wk of food and 8 sq. m/wk of shelter.

The choice of bundle *F* makes perfect sense on intuitive grounds. The consumer's goal, after all, is to reach the highest indifference curve he can, given his budget constraint. His strategy is to keep moving to higher and higher indifference curves until he reaches the highest one that is still affordable. For indifference maps for which a tangency point exists, as in Figure 4.17, the best affordable bundle will always lie at the point of tangency.

In Figure 4.17, note that the marginal rate of substitution at *F* is exactly the same as the absolute value of the slope of the budget constraint. This will always be so when the best affordable bundle occurs at a point of tangency. The condition that must be satisfied in such cases is therefore

$$\text{MRS} = \frac{P_S}{P_F} \quad (4.3)$$

The right-hand side of Equation 4.3 represents the opportunity cost of shelter in terms of food. Thus, with  $P_S = €5/\text{sq. m}$  and  $P_F = €10/\text{kg}$ , the opportunity cost of an additional square metre of shelter is  $\frac{1}{2}$  kg of food. The left-hand side of Equation 4.3 is  $|\Delta F/\Delta S|$ , the absolute value of the slope of the indifference curve at the point of tangency. It is the amount of additional food the consumer must be given in order to compensate him fully for the loss of 1 sq. m of shelter. In the language of cost-benefit analysis discussed in Chapter 1, the slope of the budget constraint represents the opportunity cost of shelter in terms of food, while the slope of the indifference curve represents the benefits of consuming shelter as compared with consuming food. Since the slope of the budget constraint is  $-\frac{1}{2}$  in this example, the tangency condition tells us that  $\frac{1}{2}$  kg of food would be required to compensate for the benefits given up with the loss of 1 sq. m of shelter.

If the consumer were at some bundle on the budget line for which the two slopes are not the same, then it would always be possible for him to purchase a better bundle. To see why, suppose he were at a point where the slope of the indifference curve (in absolute value) is less than the slope of the budget constraint (also in absolute value), as at point  $E$  in Figure 4.17. Suppose, for instance, that the MRS at  $E$  is only  $\frac{1}{4}$ . This tells us that the consumer can be compensated for the loss of 1 sq. m of shelter by being given an additional  $\frac{1}{4}$  kg of food. But the slope of the budget constraint tells us that, by giving up 1 sq. m of shelter, he can purchase an additional  $\frac{1}{2}$  kg of food. Since this is  $\frac{1}{4}$  kg more than he needs to remain equally satisfied, he will clearly be better off if he purchases more food and less shelter than at point  $E$ . The opportunity cost of an additional kg of food is less than the benefit it confers.

**EXERCISE 4.7** Suppose that the marginal rate of substitution at point  $A$  in Figure 4.17 is 1.0. Show that this means the consumer will be better off if he purchases less food and more shelter than at  $A$ .

### Corner Solutions

The best affordable bundle need not always occur at a point of tangency. In some cases, there may simply be no point of tangency—the MRS may be everywhere greater, or less, than the slope of the budget constraint. In this case we get a **corner solution**, like the one

**corner solution** in a choice between two goods, a case in which the consumer does not consume one of the goods.

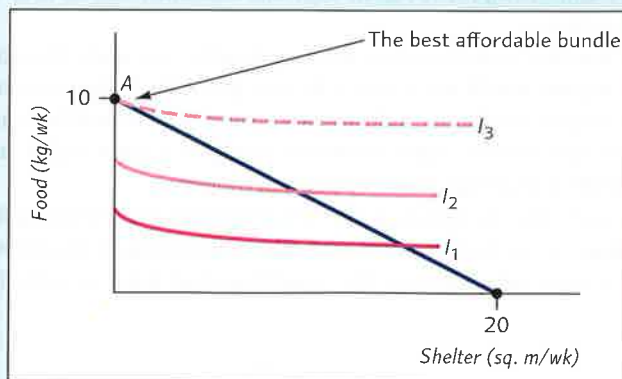
shown in Figure 4.18, where  $M$ ,  $P_F$  and  $P_S$  are again given by €100/wk, €10/kg and €5/sq. m, respectively. The best affordable bundle is the one labelled  $A$ , and it lies at the upper end of the budget constraint. At  $A$  the MRS is less than the absolute value of the slope of the budget constraint. For the sake of illustration, suppose

the MRS at  $A = \frac{1}{4}$ , which means that this consumer would be willing to give up  $\frac{1}{4}$  kg of food to get an additional square metre of shelter. But at market prices the opportunity cost of an additional square metre of shelter is  $\frac{1}{2}$  kg of food. He increases his satisfaction by continuing to give up shelter for more food until it is no longer possible to do so. Even though this consumer regards shelter as a desirable commodity, the best he can do is to spend all his income on food. Market

**FIGURE 4.18**

#### A Corner Solution

When the MRS of food for shelter is always less than the slope of the budget constraint, the best the consumer can do is to spend all his income on food.



prices are such that he would have to give up too much food to make the purchase of even a single unit of shelter worthwhile.

The indifference map in Figure 4.18 satisfies the property of diminishing marginal rate of substitution—moving to the right along any indifference curve, the slope becomes smaller in absolute terms. But because the slopes of the indifference curves start out smaller than the slope of the budget constraint the two never reach equality. The condition that must be satisfied in such cases is therefore

$$\text{MRS} < \frac{P_S}{P_F} \quad (4.4)$$

when the amount of shelter is zero.

## Why do people not consume most goods?

How many bottles of English sparkling wine have you drunk in the past week? How many times have you been to watch football at Old Trafford this year, or to experience ballet at the Mariinsky Theatre? How many Boeing 747s, speedboats, Porsches, four-bedroom houses and tins of baked beans do you own? Hopefully, you have answered none to at least one of these questions. More generally, there are millions of products and services available and the typical consumer will consume only a very, very small fraction of these. So, do not think that Figure 4.17 captures the ‘normal’ situation and Figure 4.18 more ‘extreme’ situations. If anything, it is the other way around.

This tells us something useful about the likely slopes of the budget constraint and indifference curves. To illustrate, consider the demand for English sparkling wine. Some English wine is nice, but it is also relatively expensive (even for someone living in England). For most people the opportunity cost of buying English wine is everywhere greater than the willingness to exchange, say, champagne for English wine. So they do not buy any English wine. Those who do consume English wine must have a greater willingness to exchange champagne for English wine. ■

Indifference curves that are not strongly convex are characteristic of goods that are easily substituted for one another. Corner solutions are more likely to occur for such goods, and indeed are almost certain to occur when goods are perfect substitutes. (See Example 4.2.) For such goods, the MRS does not diminish at all; rather, it is everywhere the same. With perfect substitutes, indifference curves are straight lines. If they happen to be steeper than the budget constraint, we get a corner solution on the horizontal axis; if less steep, we get a corner solution on the vertical axis.

**EXAMPLE 4.2** Mattingly is a caffeinated-cola drinker who spends his entire soft drink budget on Coke and Jolt cola and cares only about total caffeine content. If Jolt has twice the caffeine of Coke, and if Jolt costs €1/litre and Coke costs €0.75/litre, how will Mattingly spend his soft drink budget of €15/wk?

For Mattingly, Jolt and Coke are *perfect substitutes*, which means that his indifference curves will be linear. The top line in Figure 4.19 is the set of all possible Coke–Jolt combinations that provide the same satisfaction as the bundle consisting of 0 litres of Jolt per day and 30 litres of Coke per day. Since each litre of Jolt has twice the caffeine of a litre of Coke, all bundles along this line contain precisely the same amount of caffeine.  $I_2$  is the indifference curve for bundles equivalent to bundle (0, 20); and  $I_1$  is the indifference curve corresponding to (0, 10). Along each of these indifference curves, the marginal rate of substitution of Coke for Jolt is always  $\frac{2}{1}$ , that is, 2 litres of Coke for every litre of Jolt.

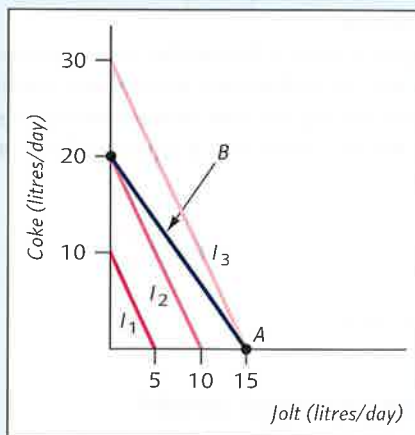
In the same diagram, Mattingly’s budget constraint is shown as  $B$ . The slope of his indifference curves is  $-2$ ; of his budget constraint,  $-\frac{4}{3}$ . The best affordable bundle is the one labelled  $A$ , a corner solution in which he spends his entire budget on Jolt. This makes intuitive sense in light of Mattingly’s peculiar preferences: he cares only about total caffeine content, and Jolt provides more

## ECONOMIC NATURALIST 4.5

**FIGURE 4.19****Equilibrium with Perfect Substitutes**

Here, the MRS of Coke for Jolt is 2 at every point.

Whenever the price ratio  $P_J/P_C$  is less than 2, a corner solution results in which the consumer buys only Jolt. On the budget constraint  $B$ , the consumer does best to buy bundle  $A$ .



caffeine per euro than Coke does. If the Jolt–Coke price ratio,  $P_J/P_C$  had been 3 (or any other amount greater than 2) Mattingly would have spent all his income on Coke. That is, we would again have had a corner solution, only this time on the vertical axis. Only if the price ratio had been exactly  $\frac{2}{1}$  might we have seen Mattingly spend part of his income on each good. In that case, any combination of Coke and Jolt on his budget constraint would have served equally well. ♦

**interior solution** in a choice between two goods, a case in which the consumer consumes a positive amount of both goods.

Most of the time we will deal with problems that have an **interior solution**. The focus on interior solutions is not because boundary solutions are uncommon (see Economic Naturalist 4.5). It is because problems with interior solutions are more interesting and informative to analyse.

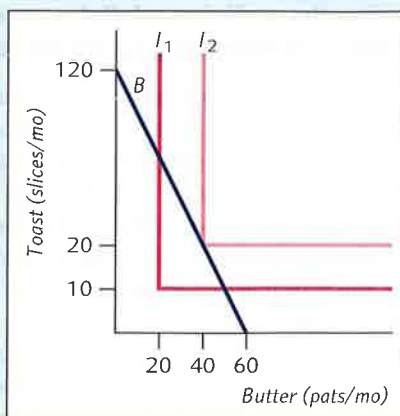
Typically an interior solution will lie at a point of tangency. That is, where the MRS is exactly the same as the slope of the budget constraint. As the following example illustrates, however, it is possible to have an interior solution but no point of tangency.

**EXAMPLE 4.3** Suppose Albert always uses exactly two pats of butter on each piece of toast. If toast costs €0.10/slice and butter costs €0.20/pat, find Albert's best affordable bundle if he has €12/mo to spend on toast and butter.

For Albert, butter and toast are perfect complements, which means that his indifference curves will be L-shaped.  $I_1$  in Figure 4.20 is the set of all butter–toast combinations that provide the same

**FIGURE 4.20****Equilibrium with Perfect Complements**

Here, the MRS of butter for toast is either infinite or zero. Albert wants to consume exactly twice as much butter as toast. We obtain an interior, non-tangency, solution.



satisfaction as 20 pats of butter and 10 slices of toast. Because Albert wants exactly two pats of butter for every slice of toast he is indifferent between the bundle (20, 10) and the bundles (20, 20) or (40, 10). The extra toast or butter is unwanted on its own. Note that this means the more-is-better property is not satisfied.

In the same diagram, Albert's budget constraint is shown as  $B$ . The slope of his budget constraint is  $-2$ . The slope of his indifference curve is either infinite or zero. In interpretation, if he has an excess of toast (e.g. bundle (20, 20)) he would need an infinite amount of toast to compensate for a little less butter. If he has a shortage of toast (e.g. bundle (40, 10)) he would need no toast to compensate for a little less butter.

There can be no point of tangency between the budget constraint and an indifference curve. We still, though, obtain an interior solution of 40 pats/mo of butter and 20 slices/mo of toast. ♦

**EXERCISE 4.8** Suppose Albert starts to watch his cholesterol and therefore alters his preference to using exactly one pat of butter on each piece of toast. How much toast and butter would Albert then consume each month?

## Indifference Curves When There Are More Than Two Goods

In the examples thus far, the consumer cared about only two goods. Where there are more than two, we can construct indifference curves by using the same device we used earlier to represent multi-good budget constraints. We simply view the consumer's choice as being one between a particular good  $X$  and an amalgam of other goods  $Y$ , which is again called the composite good. As before, the composite good is the amount of income the consumer has left over after buying the good  $X$ .

In the multigood case, we may thus continue to represent the consumer's preferences with an indifference map in the  $XY$  plane. Here, the indifference curve tells the rate at which the consumer will exchange the composite good for  $X$ . As in the two-good case, equilibrium occurs when the consumer reaches the highest indifference curve attainable on his budget constraint.

## AN APPLICATION OF THE RATIONAL CHOICE MODEL

As the following example makes clear, the composite good construct enables us to deal with more general questions than we could in the two-good case.

### EXAMPLE 4.4 Is it better to give poor people cash or rent support?

Most governments offer some form of support to families with low income to help them pay for accommodation. For example, housing benefit in the UK entitles low-income consumers to claim up to around £290 a week to cover rent for a two-bedroom property. This money can only be used to pay for rent; indeed, the government will often pay the landlord directly. Any rent in excess of £290/wk must be paid by the family. Would the consumer have been better off had he instead been given £290/wk directly in cash?

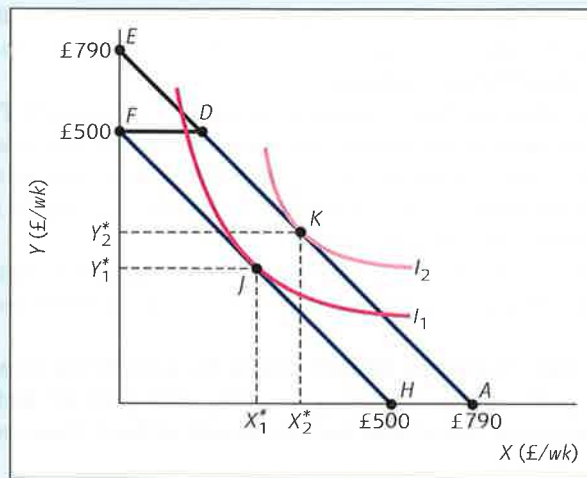
We can try to answer this question by investigating which alternative would get him to a higher indifference curve. Suppose  $Y$  denotes the composite good and  $X$  denotes rent. If the consumer's income is £500/wk, his initial equilibrium is the bundle  $J$  in Figure 4.21. The effect of housing benefit is to increase the total amount of rent he can afford each week from £500 to £790. In terms of the maximum amount of rent he can afford, housing benefit is thus exactly the same as a cash grant of £290.

Where the two alternatives differ is in terms of the maximum amounts of other goods he can buy. With a cash grant of £290, he has a total weekly income of £790, and this is, of course, the maximum amount of other goods (the composite good) he can buy. His budget constraint in this case is thus the line labelled  $AE$  in Figure 4.21.

With housing benefit, by contrast, the consumer is not able to buy £790/wk of other goods because his £290 in housing benefit can be used only for rent. The maximum amount of other goods he can purchase is £500. In Figure 4.21, his budget constraint with housing benefit is

**FIGURE 4.21****Housing Benefit versus Cash Grant**

By comparison with the budget constraint under a cash grant ( $AE$ ), the budget constraint under housing benefits ( $ADF$ ) limits the amount that can be spent on non-housing goods. But for the consumer whose indifference map is shown, the equilibrium bundles are the same under both alternatives.



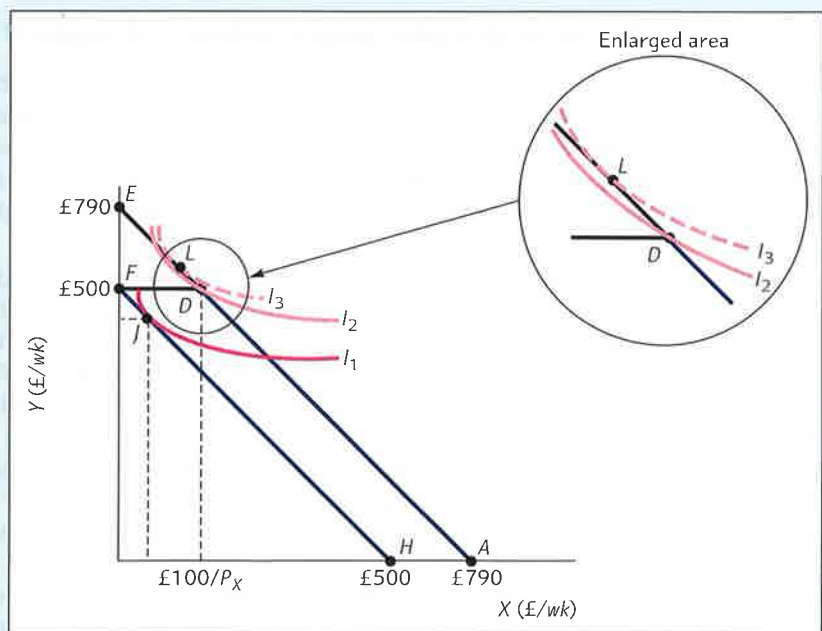
labelled  $ADF$ . For values of  $Y$  less than £500, it is thus exactly the same as his budget constraint with a cash grant. For values of  $Y$  larger than £500, however, his budget constraint with housing benefit is completely flat.

Note that the consumer whose indifference curves are shown in Figure 4.21 buys exactly the same bundle—namely, bundle  $K$ —under both options. The effect of housing benefit here is precisely the same as the effect of the cash grant. In general, this will be true whenever the consumer with a cash grant would have spent more on rent anyway than the amount of housing benefit he would have received.

Figure 4.22 depicts a consumer for whom this is *not* the case. With a cash grant, he would choose bundle  $L$ , which would put him on a higher indifference curve than he could attain with housing benefit, which would lead him to buy bundle  $D$ . Note that bundle  $D$  contains exactly £290 worth of rent, the amount of housing benefit he received. Bundle  $L$ , by contrast, contains

**FIGURE 4.22****Where Housing Benefit and Cash Grants Yield Different Outcomes**

For the consumer whose indifference map is shown, a cash grant would be preferred to housing benefit, which forces him to devote more to rent than he would choose to spend on his own.



less than £290 worth of rent. Here, the effect of housing benefit is to cause the recipient to spend more on rent than he would have if he had instead been given cash. ◆

The analysis in Example 4.4 raises the question of why governments do not just give poor people cash grants in the first place. The ostensible reason is that they want to help poor people rent suitable accommodation, not buy luxury items or even cigarettes and alcohol. And yet if most participants would have spent at least as much on rent as they received in housing benefit, not being able to use housing benefit to buy other things is a meaningless restriction. For instance, if someone would have spent £300 on rent anyway, getting £290 in housing benefit simply lets him take some of the money he would have spent on rent and spend it instead on whatever else he chooses.

On purely economic grounds, there is thus a strong case for replacing housing benefit—and all other benefits—with a much simpler programme of cash grants to the poor. At the very least, this would eliminate the cumbersome step of applying for housing benefit after having agreed a rental contract with a landlord.

As a political matter, however, it is easy to see why governments might have set things up the way they did. Many taxpayers would be distressed to see their tax ‘wasted’. If housing benefit prevents even a tiny minority of participants from spending more on luxury goods, it spares many political difficulties.

Economic Naturalist 4.6 calls our attention to a problem that applies not just to housing benefit but to all other forms of in-kind transfers as well: although the two forms of transfer are sometimes equivalent, gifts in cash seem clearly superior on those occasions when they differ.

### Why do people often give gifts in kind instead of cash?

Occasionally someone receives a gift that is exactly what he would have purchased for himself had he been given an equivalent amount of money. But we are all far too familiar with gifts that miss the mark. Who has never been given an article of clothing that he was embarrassed to wear? The logic of the rational choice model seems to state unequivocally that we could avoid the problem of useless gifts if we followed the simple expedient of giving cash. And yet virtually every society continues to engage in ritualized gift giving.

The fact that this custom has persisted should not be taken as evidence that people are stupid. Rather, it suggests that the rational choice model may fail to capture something important about gift giving. One purpose of a gift is to express affection for the recipient. A thoughtfully chosen gift accomplishes this in a way that cash cannot. Or it may be that some people have difficulty indulging themselves with even small luxuries and would feel compelled to spend cash gifts on purely practical items. For these people, a gift provides a way of enjoying a small luxury without having to feel guilty about it.<sup>6</sup> This interpretation is supported by the observation that we rarely give purely practical gifts like plain cotton underwear or laundry detergent.

Whatever the real reasons people may have for giving in kind rather than in cash, it seems safe to assume that we do not do it because it never occurred to us to give cash. On the contrary, occasionally we do give cash gifts, especially to young relatives with low incomes. But even though there are advantages to gifts in cash, people seem clearly reluctant to abandon the practice of giving in kind. ■

### ECONOMIC NATURALIST 4.6

## THEORY OF CHOICE AND HOUSEHOLD PRODUCTION

In the 1960s and 1970s the traditional model of rational consumer choice (that we have looked at so far in this chapter) was independently criticized by Kelvin Lancaster and Gary Becker.<sup>7</sup> A look at the alternative approaches they proposed allows us to better understand the traditional model and appreciate how it can be extended.

<sup>6</sup>For a discussion of this interpretation, see R. Thaler, ‘Mental Accounting and Consumer Choice’, *Marketing Science*, 4, Summer 1985.

<sup>7</sup>See, for instance, K. L. Lancaster, ‘A New Approach to Consumer Theory’, *Journal of Political Economy*, 74, April 1966, and R. T. Michael and G. S. Becker, ‘On the New Theory of Consumer Behavior’, *Swedish Journal of Economics*, 75, December 1973.

## APPENDIX

# 4

# THE UTILITY FUNCTION APPROACH TO THE CONSUMER BUDGETING PROBLEM



## THE UTILITY FUNCTION APPROACH TO CONSUMER CHOICE

**F**inding the highest attainable indifference curve on a budget constraint is just one way that economists have analysed the consumer choice problem. For many applications, a second approach is also useful. In this approach we represent the consumer's preferences not with an indifference map but with a *utility function*.

For each possible bundle of goods, a utility function yields a number that represents the amount of satisfaction provided by that bundle. Suppose, for example, that Tom consumes only food and shelter and that his utility function is given by  $U(F, S) = FS$ , where  $F$  denotes the number of kilograms of food,  $S$  the number of square metres of shelter he consumes per week, and  $U$  his satisfaction, measured in 'utils' per week. If  $F = 4$  kg/wk and  $S = 3$  sq. m/wk, Tom will receive 12 utils/wk of utility, just as he would if he consumed 3 kg/wk of food and 4 sq. m/wk of shelter. By contrast, if he consumed 8 kg/wk of food and 6 sq. m/wk of shelter, he would receive 48 utils/wk.

In our discussion about how to represent consumer preferences, we assumed that people are able to rank each possible bundle in order of preferences. This is called the *ordinal utility* approach to the consumer budgeting problem. It does not require that people be able to make quantitative statements about how much they like various bundles. Thus it assumes that a consumer will always be able to say whether he prefers A to B, but that he may not be able to make such statements as 'A is 6.43 times as good as B'.

Consequently, what is important for consumer choice is not the actual number of utils various bundles provide, but the rankings of bundles based on their associated utilities. If bundle A is preferred to bundle B then we require that bundle A gives more utils than bundle B. Similarly, if the consumer is indifferent between bundles A and B they should give the same utils. The term 'utils' is, therefore, an arbitrary unit. Any utility function that ranks bundles the same way is equivalent for our purposes.

The utility function is analogous to an indifference map in that both provide a complete description of the consumer's preferences. In the indifference curve framework, we can rank any two bundles by seeing which one lies on a higher indifference curve. In the utility-function framework, we can compare any two bundles by seeing which one yields a greater number of utils. Indeed, as the following example illustrates, it is straightforward to use the utility function to construct an indifference map.

**EXAMPLE A.4.1** If Tom's utility function is given by  $U(F, S) = FS$ , graph the indifference curves that correspond to 1, 2, 3 and 4 utils, respectively.

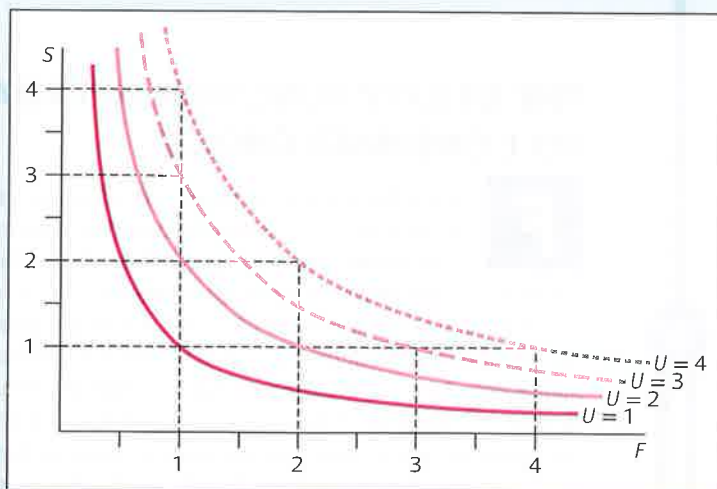
In the language of utility functions, an indifference curve is all combinations of  $F$  and  $S$  that yield the same level of utility—the same number of utils. Suppose we look at the indifference curve that corresponds to 1 unit of utility—that is, the combinations of bundles for which  $FS = 1$ . Solving this equation for  $S$ , we have

$$S = \frac{1}{F} \quad (\text{A.4.1})$$

which is the indifference curve labelled  $U = 1$  in Figure A.4.1. The indifference curve that corresponds to 2 units of utility is generated by solving  $FS = 2$  to get  $S = 2/F$ , and it is shown by the curve labelled  $U = 2$  in Figure A.4.1. In similar fashion, we generate the indifference curves to  $U = 3$  and  $U = 4$ , which are correspondingly labelled in the diagram. More generally, we get the indifference curve corresponding to a utility level of  $U_0$  by solving  $FS = U_0$  to get  $S = U_0/F$ . ♦

**FIGURE A.4.1**  
Indifference Curves  
for the Utility  
Function  $U = FS$

To get the indifference curve that corresponds to all bundles that yield a utility level of  $U_0$ , set  $FS = U_0$  and solve for  $S$  to get  $S = U_0/F$ .



In the indifference curve framework, the best affordable bundle is the bundle on the budget constraint that lies on the highest indifference curve. Analogously, the best affordable bundle in the utility-function framework is the bundle on the budget constraint that provides the highest level of utility. In the indifference curve framework, the best affordable bundle occurs at a point of tangency between an indifference curve and the budget constraint. At the optimal bundle, the slope of the indifference curve, or MRS, equals the slope of the budget constraint. Suppose food and shelter are again our two goods, and  $P_F$  and  $P_S$  are their respective prices. If  $\Delta S/\Delta F$  denotes the slope of the highest attainable indifference curve at the optimal bundle, the tangency condition says that  $\Delta S/\Delta F = P_F/P_S$ . What is the analogous condition in the utility-function framework?

To answer this question, we must introduce the concept of *marginal utility* (the marginal utility of a good is the rate at which total utility changes with consumption of the good), which is the rate at which total utility changes as the quantities of food and shelter change. More specifically, let  $MU_F$  denote the number of additional utils we get for each additional unit of food and  $MU_S$  denote the number of additional utils we get for each additional unit of shelter. If we change the quantities of both food and shelter, the change in total utility is given by the total derivative of  $U$ ,

$$\Delta U = MU_F \Delta F + MU_S \Delta S \quad (\text{A.4.2})$$

This simply says that the change in total utility is equal to the change in utility from consuming a different amount of food plus the change in utility from consuming a different amount of shelter.

In Figure A.4.2, note that bundle  $K$  has  $\Delta F$  fewer units of food and  $\Delta S$  more units of shelter than bundle  $L$ . Thus, if we move from bundle  $K$  to bundle  $L$ , we gain  $MU_F \Delta F$  utils from having more food, but we lose  $MU_S \Delta S$  utils from having less shelter. Because  $K$  and  $L$  both lie on the same indifference curve, we know that both bundles provide the same level of utility. Thus  $\Delta U = 0$  and the utility we lose from having less shelter must be exactly offset by the utility we gain from having more food. This tells us that

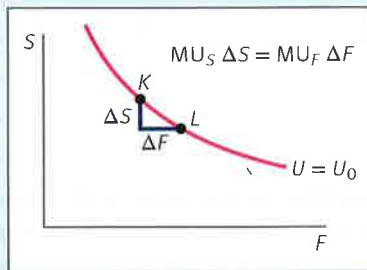
$$|MU_F \Delta F| = |MU_S \Delta S| \quad (\text{A.4.3})$$

Cross-multiplying terms in Equation A.4.3 gives

$$\frac{MU_F}{MU_S} = \left| \frac{\Delta S}{\Delta F} \right| \quad (\text{A.4.4})$$

Suppose that  $K$  and  $L$  are very close together, so that  $\Delta F$  and  $\Delta S$  are both very small. The smaller they become, the closer the ratio  $\Delta S/\Delta F$  becomes to the slope of the indifference curve. We know that the slope of the indifference curve is equal to the marginal rate of substitution. This gives us an important formula

$$\frac{MU_F}{MU_S} = \text{MRS} \quad (\text{A.4.5})$$



**FIGURE A.4.2**

**Utility Along an Indifference Curve Remains Constant**

In moving from  $K$  to  $L$ , the loss in utility from having less shelter,  $MU_S \Delta S$ , is exactly offset by the gain in utility from having more food,  $MU_F \Delta F$ .

The (absolute value of the) marginal rate of substitution is equal to the ratio of the marginal utilities of the two goods. To illustrate why, suppose that  $MU_F = 2$  and  $MU_S = 1$ . If the consumer gets one more unit of food his utility will increase by 2. If he gives up 2 units of shelter his utility will decrease by 2. The consumer would be willing to exchange 2 units of shelter for 1 unit of food. Thus, his MRS is 2.

Excluding the possibility of corner solutions, we know that the slope of the indifference curve at the optimal bundle is the same as that of the budget constraint; the following condition must hold for the optimal bundle:

$$\frac{MU_F}{MU_S} = \frac{P_F}{P_S} \quad (\text{A.4.6})$$

Equation A.4.6 is the condition in the utility-function framework that is analogous to the  $MRS = P_F/P_S$  condition in the indifference curve framework. If we cross-multiply terms in Equation A.4.6, we get an equivalent condition, called *Gossen's Second Law*, that has a very straightforward intuitive interpretation:

$$\frac{MU_F}{P_F} = \frac{MU_S}{P_S} \quad (\text{A.4.7})$$

In words, Equation A.4.7 tells us that the ratio of marginal utility to price must be the same for all goods at the optimal bundle. The following examples illustrate why this condition must be satisfied if the consumer has allocated his budget optimally.

**EXAMPLE A.4.2** Suppose that the marginal utility of the last euro John spends on food is greater than the marginal utility of the last euro he spends on shelter. For example, suppose the prices of food and shelter are €1/kg and €2/sq. m, respectively, and that the corresponding marginal utilities are 6 and 4. Show that John cannot possibly be maximizing his utility.

If John bought 1 sq. m/wk less shelter, he would save €2/wk and would lose 4 utils. But this would enable him to buy 2 kg/wk more food, which would add 12 utils, for a net gain of 8 utils. ♦

Abstracting from the special case of corner solutions, a necessary condition for optimal budget allocation is that the last euro spent on each commodity yields the same increment in utility.

**EXAMPLE A.4.3** Mary has a weekly allowance of €10, all of which she spends on newspapers ( $N$ ) and magazines ( $M$ ), whose respective prices are €1 and €2. Her utility from these purchases is given by  $U(N) + V(M)$ . If the values of  $U(N)$  and  $V(M)$  are as shown in the table, is Mary a utility maximizer if she buys 4 magazines and 2 newspapers each week? If not, how should she reallocate her allowance?

$N$	$U(N)$	$M$	$V(M)$
0	0	0	0
1	12	1	20
2	20	2	32
3	26	3	40
4	30	4	44
5	32	5	46

For Mary to be a utility maximizer, extra utility per euro must be the same for both the last newspaper and the last magazine she purchased. But since the second newspaper provided 8 additional utils per euro spent, which is four times the 2 utils per euro she got from the fourth magazine (4 extra utils at a cost of €2), Mary is not a utility maximizer.

$N$	$U(N)$	$MU(N)$	$MU(N)/P_N$	$M$	$U(M)$	$MU(M)$	$MU(M)/P_M$
0	0			0	0		
		12	12			20	10
1	12			1	20		
		8	8			12	6
2	20			2	32		
		6	6			8	4
3	26			3	40		
		4	4			4	2
4	30			4	44		
		2	2			2	1
5	32			5	46		

To see clearly how she should reallocate her purchases, let us rewrite the table to include the relevant information on marginal utilities. From this table, we see that there are several bundles for which  $MU(N)/P_N = MU(M)/P_M$ —namely, 3 newspapers and 2 magazines; or 4 newspapers and 3 magazines; or 5 newspapers and 4 magazines. The last of these bundles yields the highest total utility but costs €13, and is hence beyond Mary's budget constraint. The first, which costs only €7, is affordable, but so is the second, which costs exactly €10 and yields higher total utility than the first. With 4 newspapers and 3 magazines, Mary gets 4 utils per euro from her last purchase in each category. Her total utility is 70 utils, which is 6 more than she got from the original bundle. ♦

In Example A.4.3, note that if all Mary's utility values were doubled, or cut by half, she would still do best to buy 4 newspapers and 3 magazines each week. This illustrates the claim that consumer choice depends not on the absolute number of utils associated with different bundles, but instead on the ordinal ranking of the utility levels associated with different bundles. If we double all the utils associated with various bundles, or cut them by half, the ordinal ranking of the bundles will be preserved, and thus the optimal bundle will remain the same. This will also be true if we take the logarithm of the utility function, the square root of it, or add 5 to it, or transform it in any other way that preserves the ordinal ranking of different bundles.

## FINDING THE BEST AFFORDABLE BUNDLE ALGEBRAICALLY

The advantage of using a utility function is that this procedure provides a compact algebraic way of summarizing all the information that is implicit in the graphical representation of preferences, as we saw in Example A.4.1. Students who know some calculus are able to solve the consumer's budget allocation problem without direct recourse to the geometry of indifference maps. Let  $U(X, Y)$  be the consumer's utility function; and suppose  $M$ ,  $P_X$  and  $P_Y$  denote income, the price of  $X$ , and the price of  $Y$ , respectively. Formally, the consumer's allocation problem can be stated as follows:

$$\begin{aligned} &\text{Maximize } U(X, Y) \text{ subject to } P_X X + P_Y Y = M \\ &X, Y \end{aligned} \quad (\text{A.4.8})$$

The appearance of the terms  $X$  and  $Y$  below the 'maximize' expression indicates that these are the variables whose values the consumer must choose. The price and income values in the budget constraint are given in advance.

As noted earlier, the function  $U(X, Y)$  itself has no maximum; it simply keeps on increasing with increases in  $X$  or  $Y$ . The maximization problem defined in Equation A.4.8 is called a *constrained maximization problem*, which means we want to find the values of  $X$  and  $Y$  that produce the highest value of  $U$  subject to the constraint that the consumer spends only as much as his income.

There are lots of approaches to solving this problem. We will examine three. All give the same answer and so you should follow the method easiest for you.

## The Method of Lagrangian Multipliers

One way of making sure that the budget constraint is satisfied is to use the so-called method of *Lagrangian multipliers*. In this method, we begin by transforming the constrained maximization problem in Equation A.4.8 into the following unconstrained maximization problem:

$$\begin{aligned} \text{Maximize } L &= U(X, Y) - \lambda(P_X X + P_Y Y - M) \\ X, Y, \lambda \end{aligned} \quad (\text{A.4.9})$$

The term  $\lambda$  is called a Lagrangian multiplier, and its role is to assure that the budget constraint is satisfied. (How it does this will become clear in a moment.) The first-order conditions for a maximum of  $L$  are obtained by taking the first partial derivatives of  $L$  with respect to  $X$ ,  $Y$  and  $\lambda$  and setting them equal to zero:

$$\frac{\partial L}{\partial X} = \frac{\partial U}{\partial X} - \lambda P_X = 0 \quad (\text{A.4.10})$$

$$\frac{\partial L}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda P_Y = 0 \quad (\text{A.4.11})$$

$$\text{and} \quad \frac{\partial L}{\partial \lambda} = M - P_X X - P_Y Y = 0 \quad (\text{A.4.12})$$

The next step is to solve Equations A.4.10–A.4.12 for  $X$ ,  $Y$  and  $\lambda$ . The solutions for  $X$  and  $Y$  are the only ones we really care about here. The role of the equilibrium value of  $\lambda$  is to guarantee that the budget constraint is satisfied. Note in Equation A.4.12 that setting the first partial derivative of  $L$  with respect to  $\lambda$  equal to zero guarantees this result.

**EXAMPLE A.4.4** David's utility function is given by  $U(X, Y) = XY$ . If  $M = 40$ ,  $P_X = 4$  and  $P_Y = 2$  find the best affordable bundle.

Our unconstrained maximization problem can be written as

$$\begin{aligned} \text{Maximize } L &= XY - \lambda(4X + 2Y - 40) \\ X, Y, \lambda \end{aligned} \quad (\text{A.4.13})$$

The first-order conditions for a maximum of  $L$  are given by

$$\frac{\partial L}{\partial X} = \frac{\partial(XY)}{\partial X} - 4\lambda = Y - 4\lambda = 0 \quad (\text{A.4.14})$$

$$\frac{\partial L}{\partial Y} = \frac{\partial(XY)}{\partial Y} - 2\lambda = X - 2\lambda = 0 \quad (\text{A.4.15})$$

$$\text{and} \quad \frac{\partial L}{\partial \lambda} = 40 - 4X - 2Y = 0 \quad (\text{A.4.16})$$

Dividing Equation A.4.14 by Equation A.4.15 and solving for  $Y$ , we get  $Y = 2X$ ; substituting this result into Equation A.4.16 and solving for  $X$ , we get  $X = 5$ , which in turn yields  $Y = 2X = 10$ . Thus  $(5, 10)$  is the utility-maximizing bundle.<sup>1</sup> ◆

## Gossen's Second Law

If we divide Equation A.4.10 by Equation A.4.11 we get

$$\frac{\partial U / \partial X}{\partial U / \partial Y} = \frac{\lambda P_X}{\lambda P_Y} = \frac{P_X}{P_Y} \quad (\text{A.4.17})$$

Equation A.4.17 is the utility function analogue to Equation 4.3 from the text, which says that the optimal values of  $X$  and  $Y$  must satisfy  $MRS = P_X/P_Y$ . The terms  $\partial U / \partial X$  and  $\partial U / \partial Y$  from Equation A.4.17 are called the *marginal utility of  $X$*  and the *marginal utility of  $Y$* , respectively. In

<sup>1</sup>Assuming that the second-order conditions for a local maximum are also met.

words, the marginal utility of a good is the extra utility obtained per additional unit of the good consumed. Equation A.4.17 tells us that the ratio of these marginal utilities is simply the marginal rate of substitution of  $Y$  for  $X$ .

If we rearrange Equation A.4.17 in the form

$$\frac{\partial U/\partial X}{P_X} = \frac{\partial U/\partial Y}{P_Y} \quad (\text{A.4.18})$$

we obtain Gossen's Second Law. In words, the left-hand side of Equation A.4.18 may be interpreted as the extra utility gained from the last euro spent on  $X$ . Equation A.4.18 is thus the calculus derivation of the result shown earlier in Equation A.4.7.

We can directly apply equation A.4.18 to find the consumer's utility-maximizing bundle. To illustrate, again suppose that  $U(X, Y) = XY$  and that  $M = 40$ ,  $P_X = 4$  and  $P_Y = 2$ . Then

$$\frac{\partial U}{\partial X} = Y \quad (\text{A.4.19})$$

and 
$$\frac{\partial U}{\partial Y} = X \quad (\text{A.4.20})$$

giving condition 
$$\frac{Y}{P_X} = \frac{Y}{4} = \frac{X}{P_Y} = \frac{X}{2}. \quad (\text{A.4.21})$$

Equivalently  $2Y = 4X$ . Putting this into the budget constraint  $4X + 2Y = 40$  gives  $Y = 10$  and  $X = 5$  as before.

**EXAMPLE A.4.5** Mary's utility function is given by  $U(X, Y) = \ln(X) + Y$ . If  $M = 40$ ,  $P_X = 2$  and  $P_Y = 20$  find the best affordable bundle.

The marginal utilities are

$$\frac{\partial U}{\partial X} = \frac{1}{X} \quad (\text{A.4.22})$$

and 
$$\frac{\partial U}{\partial Y} = 1 \quad (\text{A.4.23})$$

giving condition 
$$\frac{1}{2X} = \frac{1}{20}. \quad (\text{A.4.24})$$

Thus  $X = 10$ . Putting this into the budget constraint  $2X + 20Y = 40$  gives  $Y = 1$ . ♦

**EXAMPLE A.4.6** Brian's utility function is given by

$$U(X, Y) = \ln(X) + \sqrt{Y}$$

If  $M = 16$ ,  $P_X = 4$  and  $P_Y = 2$  find the best affordable bundle.

The marginal utilities are

$$\frac{\partial U}{\partial X} = \frac{1}{X} \quad (\text{A.4.25})$$

and 
$$\frac{\partial U}{\partial Y} = \frac{1}{2\sqrt{Y}} \quad (\text{A.4.26})$$

giving condition 
$$\frac{1}{4X} = \frac{1}{4\sqrt{Y}}. \quad (\text{A.4.27})$$

Putting this into the budget constraint  $4X + 2Y = 16$  gives

$$4\sqrt{Y} + 2Y = 16. \quad (\text{A.4.28})$$

This has solution  $Y = 4$ . From that we get  $X = 2$ . ♦

## An Alternative Method

There is an alternative way of making sure that the budget constraint is satisfied, one that involves less cumbersome notation than the Lagrangian approach. In this alternative method, we simply solve the budget constraint for  $Y$  in terms of  $X$  and substitute the result wherever  $Y$  appears in the utility function. Utility then becomes a function of  $X$  alone, and we can *maximize* it by taking its first derivative with respect to  $X$  and equating that to zero.<sup>2</sup> The value of  $X$  that solves that equation is the optimal value of  $X$ , which can then be substituted back into the budget constraint to find the optimal value of  $Y$ .

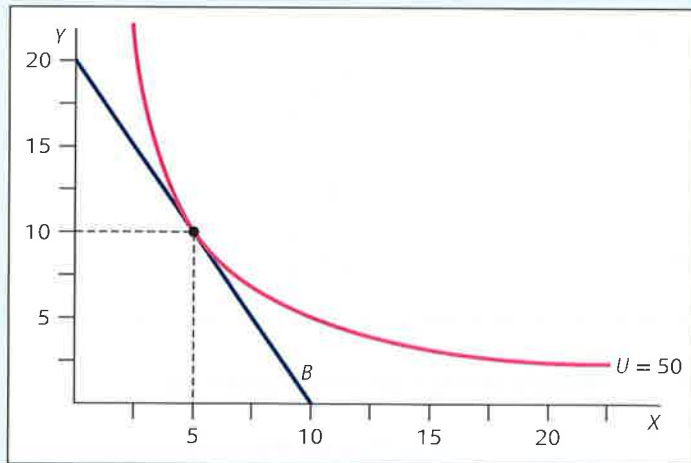
To illustrate, again suppose that  $U(X, Y) = XY$ , with  $M = 40$ ,  $P_X = 4$  and  $P_Y = 2$ . The budget constraint is then  $4X + 2Y = 40$ , which solves for  $Y = 20 - 2X$ . Substituting this expression back into the utility function, we have  $U(XY) = X(20 - 2X) = 20X - 2X^2$ . Taking the first derivative of  $U$  with respect to  $X$  and equating the result to zero, we have

$$\frac{dU}{dX} = 20 - 4X = 0 \quad (\text{A.4.29})$$

which solves for  $X = 5$ . Plugging this value of  $X$  back into the budget constraint, we discover that the optimal value of  $Y$  is 10. So the optimal bundle is again  $(5, 10)$ , just as we found using the Lagrangian approach. For these optimal values of  $X$  and  $Y$ , the consumer will obtain  $5 \times 10 = 50$  units of utility.

Both algebraic approaches to the budget allocation problem yield precisely the same result as the graphical approach described in the text. Note in Figure A.4.3 that the  $U = 50$  indifference curve is tangent to the budget constraint at the bundle  $(5, 10)$ .

**FIGURE A.4.3**  
The Optimal Bundle  
when  $U = XY$ ,  $P_X = 4$ ,  
 $P_Y = 2$  and  $M = 40$



## A Simplifying Technique

Suppose our constrained maximization problem is of the general form

$$\begin{aligned} &\text{Maximize } U(X, Y) \text{ subject to } P_X X + P_Y Y = M \\ &X, Y \end{aligned} \quad (\text{A.4.30})$$

If  $(X^*, Y^*)$  is the optimum bundle for this maximization problem, then we know it will also be the optimum bundle for the utility function  $V[U(X, Y)]$ , where  $V$  is any increasing function.<sup>3</sup> This property often enables us to transform a computationally difficult maximization problem into a simple one, as illustrated by the following example.

<sup>2</sup>Here, the second-order condition for a local maximum is that  $d^2U/dX^2 < 0$ .

<sup>3</sup>Again, an increasing function is one for which  $V(X_1) > V(X_2)$  whenever  $X_1 > X_2$ .

**EXAMPLE A.4.7** Bill has utility function  $X^{1/3}Y^{2/3}$ . The price of good  $X$  is €4, that of good  $Y$  is €2 and he has €24 to spend. What is his best affordable bundle?

Suppose we transform the utility function by taking its logarithm:

$$V = \ln[U(X, Y)] = \left(\frac{1}{3}\right)\ln X + \left(\frac{2}{3}\right)\ln Y \quad (\text{A.4.31})$$

Since the logarithm is an increasing function, when we maximize  $V$  subject to the budget constraint, we will get the same answer we would get using  $U$ . The advantage of the logarithmic transformation here is that the derivative of  $V$  is much easier to calculate than the derivative of  $U$ . Again, solving the budget constraint for  $Y = 12 - 2X$  and substituting the result into  $V$ , we have  $V = \left(\frac{1}{3}\right)\ln X + \left(\frac{2}{3}\right)\ln(12 - 2X)$ . The first-order condition follows almost without effort:

$$\frac{dV}{dX} = \frac{1}{3X} - \frac{2\left(\frac{2}{3}\right)}{12 - 2X} = 0 \quad (\text{A.4.32})$$

which solves easily for  $X = 2$ . Plugging  $X = 2$  back into the budget constraint, we again get  $Y = 8$ . ♦

The best transformation to make will naturally depend on the particular utility function you start with. The logarithmic transformation greatly simplified matters in the example above, but will not necessarily be helpful for other forms of  $U$ .

## Corner Solutions

The two methods for finding the optimal bundle given above are not guaranteed to find the optimal solution. Technically that is because we omitted two other important constraints of the consumer's budget allocation problem, namely,  $X \geq 0$  and  $Y \geq 0$ .

**EXAMPLE A.4.8** Amanda's utility function is given by  $U(X, Y) = \left(\frac{2}{3}\right)X + 2Y$ . Suppose that  $M = 40$  and  $P_X = 4$ . We leave  $P_Y$  unspecified. Find the best affordable bundle.

Our unconstrained maximization problem would then be written as

$$\begin{aligned} \text{Maximize } L &= \left(\frac{2}{3}\right)X + 2Y - \lambda(4X + P_Y Y - 40) \\ X, Y, \lambda \end{aligned} \quad (\text{A.4.33})$$

The first-order conditions for a maximum of  $L$  are given by

$$\frac{\partial L}{\partial X} = \frac{\partial\left(\left(\frac{2}{3}\right)X + 2Y\right)}{\partial X} - 4\lambda = \frac{2}{3} - 4\lambda = 0 \quad (\text{A.4.34})$$

$$\frac{\partial L}{\partial Y} = \frac{\partial\left(\left(\frac{2}{3}\right)X + 2Y\right)}{\partial Y} - P_Y\lambda = 2 - P_Y\lambda = 0 \quad (\text{A.4.35})$$

$$\text{and} \quad \frac{\partial L}{\partial \lambda} = 40 - 4X - P_Y Y = 0 \quad (\text{A.4.36})$$

Dividing Equation A.4.34 by Equation A.4.35 gives

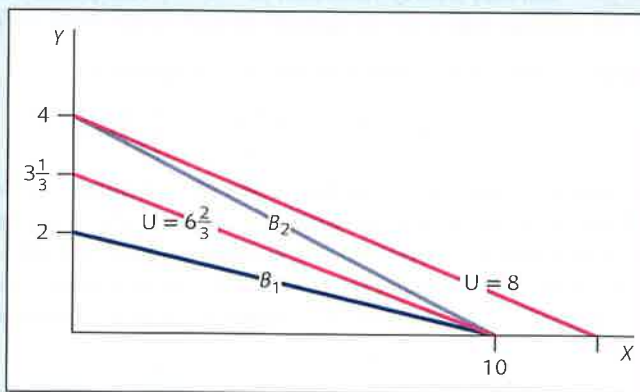
$$\frac{1}{3} = \frac{4}{P_Y} \quad (\text{A.4.37})$$

This can only be satisfied if  $P_Y = 12$ . So, what if  $P_Y$  does not equal 12? In that case, we must have a corner solution. To solve the problem we can go back and look at the indifference curve map as given in Figure A4.4. If  $P_Y$  is more than 12, say 20, the budget constraint is  $B_1$  and the consumer will consume only good  $X$ . Thus  $(10, 0)$  is the utility-maximizing bundle. If  $P_Y$  is less than 12, say 10, the budget constraint is  $B_2$  and the consumer will consume only good  $Y$ . Thus  $(0, 4)$  is the utility-maximizing bundle. ♦

**FIGURE A.4.4**

**The Optimal Bundle when**  
 $U = (\frac{2}{3})X + 2Y$ ,  $P_X = 4$  and  
 $M = 40$

The optimal bundle depends on whether the price of good Y is more or less than 12. If it is more than 12 the consumer should consume only good X. If it is less than 12 the consumer should consume only good Y.



## ■ PROBLEMS ■

1. Tom spends all his €100 weekly income on two goods,  $X$  and  $Y$ . His utility function is given by  $U(X, Y) = XY$ . If  $P_X = 4$  and  $P_Y = 10$ , how much of each good should he buy?
2. Same as Problem 1, except now Tom's utility function is given by  $U(X, Y) = X^{1/2}Y^{1/2}$ . Note the relationship between your answer and that in Problems 1 and 2. What accounts for this relationship?
3. Piers spends all his €170 weekly income on two goods,  $X$  and  $Y$ . His utility function is given by  $U(X, Y) = X^{1/2} + Y^{1/2}$ . If  $P_X = 8$  and  $P_Y = 2$ , how much of each good should he buy?
4. Sue consumes only two goods, food and clothing. The marginal utility of the last euro she spends on food is 12, and the marginal utility of the last euro she spends on clothing is 9. The price of food is €1.20/unit, and the price of clothing is €0.90/unit. Is Sue maximizing her utility?
5. Barbara spends all her €35 weekly income on two goods,  $F$  and  $C$ . Her utility function is given by  $U(F, C) = F^{1/2} + C$ . If  $P_F = 1$  and  $P_C = 10$ , how much of each good should she buy? What if her income were only €15?
6. Hannah spends all her €500 weekly income on two goods,  $F$  and  $C$ . Her utility function is given by  $U(F, C) = F^2 + C^2$ . If  $P_F = 5$  and  $P_C = 10$ , how much of each good should she buy?
7. David spends all his €118 weekly income on two goods,  $X$  and  $Y$ . His utility function is given by  $U(X, Y) = XY + X$ . If  $P_X = 10$  and  $P_Y = 2$ , how much of each good should he buy?
8. Fred consumes two goods,  $X$  and  $Y$ . His utility function is given by  $U(X, Y) = \min(2X, 4Y)$ . He is currently consuming 10 units of good  $X$  and 20 units of good  $Y$ . Is he maximizing his utility?
9. Brian spends all his €300 weekly income on two goods,  $X$  and  $Y$ . His utility function is given by  $U(X, Y) = \min(X, Y) + 2X$ . If  $P_X = 5$  and  $P_Y = 4$ , how much of each good should he buy? What if  $P_Y = 1$ ?
10. Albert has a weekly allowance of €17, all of which he spends on used CDs ( $C$ ) and movie rentals ( $M$ ), whose respective prices are €4 and €3. His utility from these purchases is given by  $U(C) + V(M)$ . If the values of  $U(C)$  and  $V(M)$  are as shown in the table, is Albert a utility maximizer if he buys 2 CDs and rents 3 movies each week? If not, how should he reallocate his allowance?

$C$	$U(C)$	$M$	$V(M)$
0	0	0	0
1	12	1	21
2	20	2	33
3	24	3	39
4	28	4	42