



University of
Zurich^{UZH}

Leseaufträge «Mikroökonomik I»

Modul 2: Konsument und Nachfrage

Unit 2:

- Präferenzen und Nutzenfunktion

Quellen:

- **Chapter 4 – Rational Consumer Choice**
Frank, Robert H, & Cartwright, Edward. (2016). *Microeconomics and Behaviour (2nd European ed.)*. London: McGraw-Hill Education.

nothing. If something is free then it is a relatively simple decision to take it. Less clear is whether the spinach will be eaten. Recent studies suggest that over a third of bagged salad is thrown away by customers. Making the budget constraint more linear by, say, reducing the size of bags and the number of special offers would help alleviate waste. ■

To recapitulate briefly, the budget constraint or budget line summarizes the combinations of bundles that the consumer is able to buy. Its position is determined jointly by income and prices. From the set of feasible bundles, the consumer's task is to pick the particular one she likes best. To identify this bundle, we need some means of summarizing the consumer's preferences over all possible bundles she might consume. We now turn to this task.

CONSUMER PREFERENCES

For simplicity, let us again begin by considering a world with only two goods: shelter and food. A **preference ordering** enables the consumer to rank any two bundles of goods in terms of their desirability, or order of preference. Consider two bundles, *A* and *B*. For concreteness, suppose that *A* contains 12 sq. m/wk of shelter and 8 kg/wk of food, while *B* has 10 sq. m/wk of shelter and 10 kg/wk of food. Knowing nothing about a consumer's preferences, we can say nothing about which of these bundles he will prefer. *A* has more shelter but less food than *B*. Someone who spends a lot of time at home would probably choose *A*, while someone with a rapid metabolism might be more likely to choose *B*.

preference ordering a ranking of all possible consumption bundles in order of preference.

The preference ordering enables the consumer to rank pairs of bundles but not to make more precise quantitative statements about their relative desirability. Thus, the consumer might be able to say that he prefers bundle *A* to *B* but not that *A* provides twice as much satisfaction as *B*.

Preference orderings often differ widely among consumers. One person will like Rachmaninoff, another the Red Hot Chili Peppers. Despite these differences, however, most preference orderings share several important features. Economists generally assume five simple properties of preference orderings. These properties allow us to construct the concise analytical representation of preferences we need for the budget allocation problem. We shall introduce these five properties over the next couple of pages. Here are the first two.

1. Completeness A preference ordering is *complete* if it enables the consumer to rank all possible combinations of goods and services. For any two bundles *A* and *B*, the consumer is able to make one of three possible statements: (1) *A* is preferred to *B*, (2) *B* is preferred to *A*, or (3) *A* and *B* are equally attractive. Taken literally, the completeness assumption is never satisfied, for there are many goods we know too little about to be able to evaluate. It is nonetheless a useful simplifying assumption for the analysis of choices among bundles of goods with which consumers are familiar. Its real intent is to rule out instances like the one portrayed in the fable of Buridan's ass. The hungry animal was unable to choose between two bales of hay in front of him and starved to death as a result.

2. Transitivity If, at current prices, you like steak better than hamburger and hamburger better than hot dogs, you are probably someone who likes steak better than hot dogs. To say that a consumer's preference ordering is *transitive* means that, for any three bundles *A*, *B* and *C*, if he prefers *A* to *B* and prefers *B* to *C*, then he always prefers *A* to *C*.

The preference relationship is thus assumed to be like the relationship used to compare heights of people. If O'Neal is taller than Nowitzki and Nowitzki is taller than Bryant, we know that O'Neal must be taller than Bryant. Not all comparative relationships are transitive. This is shown by the relationship 'defeats in football'. Some seasons, Manchester United beat Arsenal, and Arsenal beat Chelsea, but that does not tell us that Manchester United will necessarily beat Chelsea.

Transitivity is a simple consistency property and applies as well to the relation 'equally attractive as' and to any combination of it and the 'preferred to' relation. For example, if A is equally attractive as B and B is equally attractive as C , it follows that A is equally attractive as C . Similarly, if A is preferred to B and B is equally attractive as C , it follows that A is preferred to C .

The transitivity assumption can be justified as eliminating the potential for a 'money pump' problem. To illustrate, suppose you prefer A to B and B to C , but you also prefer C over A , so that your preferences are intransitive. If you start with C , you would trade C for B , trade B for A , and then trade A for C . This cycle could continue forever. If in each stage you were charged a tiny fee for the trade, you would eventually transfer all your money to the other trader. Clearly, such preferences are problematic.

ECONOMIC NATURALIST 4.3

Why should you not forget to ask the price?

What would you prefer: (A) a holiday in a 5* hotel with every convenience you could ask for, (B) a holiday in a 3* hotel with few facilities, or (C) a holiday in a cockroach-infested dorm? If you think the answer is easy, then suppose we also add that the 5* holiday would cost €1 million, the 3* holiday would cost €200, and the cockroach holiday is part of a TV game show where you can win €1 million.

The main point to take away from this example is that preferences are over *bundles* of goods and not goods. Clearly, the 5* holiday sounds better than the 3* holiday. But, would you be willing to pay the extra €999,800?

This is important to remember when judging transitivity. If all three holidays were to cost the same then we would clearly expect A is preferred to B and B to C and A to C . Things are less obvious when the different prices of the holidays are factored in. This is why we can observe preferences that violate transitivity. For instance, it does not seem so weird that a consumer would prefer B to A , and A to C yet prefer C over B . ■

As reasonable as the transitivity property sounds, we will see examples in later chapters of behaviour that seems inconsistent with it. But it is an accurate description of preferences in most instances. Unless otherwise stated, we will adopt it.

Taken together, the completeness and transitivity properties are very important and useful. Their main implication is that we can put different bundles in order from least preferred to most preferred. This allows us to use a utility function to represent the consumer's preferences (as discussed in the Appendix). More fundamentally, it means that the budget allocation problem is going to have a solution. Without either of these properties our task would look impossible. For instance, if the consumer's preference ordering is not complete, and he cannot tell us whether he prefers bundle A to B , then how are we to know what bundle is best for him to choose?

Indifference Curves

The two properties of preference orderings we have discussed so far enable us to generate a graphical description of the consumer's preferences. To see how, consider first the bundle A in Figure 4.9, which has 12 sq. m/wk of shelter and 10 kg/wk of food. What we want to do is to find bundles that are equally attractive as A . To make our task easier we shall introduce the third property that we shall assume of preference orderings, namely, more-is-better.

3. More-Is-Better The more-is-better property means simply that, other things equal, more of a good is preferred to less. We can, of course, think of examples of more of something making us worse rather than better off (as with someone who has overeaten). But these examples usually contemplate some sort of practical difficulty, such as having a self-control problem or being unable to store a good for future use. As long as people can freely store or dispose of goods they don't want, having more of something can't make them worse off.

As an example of the application of the more-is-better assumption, consider two more bundles: W , which has 6 sq. m/wk of shelter and 4 kg/wk of food, and Z , which has 28 sq. m/wk of shelter and 12 kg/wk of food. The assumption tells us that Z is preferred to W because it has more

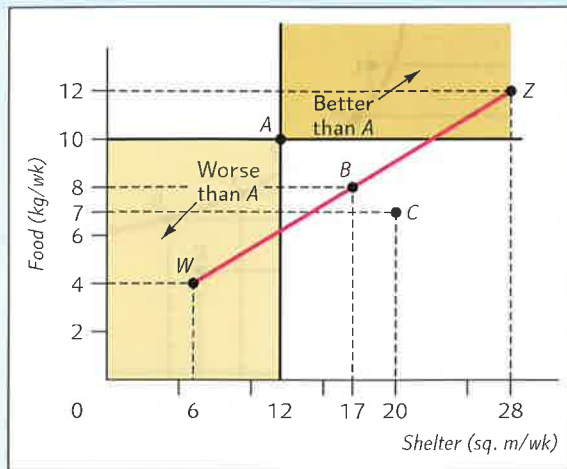


FIGURE 4.9

Generating Equally Preferred Bundles

Z is preferred to A because it has more of each good than A has. For the same reason, A is preferred to W. It follows that on the line joining W and Z there must be a bundle B that is equally attractive as A. In similar fashion, we can find a bundle C that is equally attractive as B.

food and no less shelter. We can also tell that A is preferred to W, and, consistent with transitivity, Z is preferred to A.

Now consider the set of bundles that lie along the line joining W and Z. To say something about these bundles we need the fourth property of preference orderings.

4. Continuity Small changes in the quantity of a good should not lead to sudden ‘jumps’ in preferences. For instance, we would not expect the consumer to have a strong preference over 12 sq. m/wk shelter compared to 11.999 sq. m/wk. A consumer’s preference ordering is *continuous* if, for any two bundles Z and W, where the consumer prefers Z to W then any bundle sufficiently close to Z is also preferred to W.

Because Z is preferred to A and A is preferred to W, the continuity assumption means that as we move from Z to W we must encounter a bundle that is equally attractive as A. (The intuition behind this claim is the same as the intuition that tells us that if we climb on any continuous path on a mountainside from one point at 1,000 metres above sea level to another at 2,000 metres, we must pass through every intermediate altitude along the way.) Let B denote the bundle that is equally attractive as A, and suppose it contains 17 sq. m/wk of shelter and 8 kg/wk of food. (The exact amounts of each good in B will of course depend on the specific consumer whose preferences we are talking about.) The more-is-better assumption also tells us that there will be only one such bundle on the straight line between W and Z. Points on that line to the northeast of B are all better than B; those to the southwest of B are all worse.

In precisely the same fashion, we can find another point—call it C—that is equally attractive as B. C is shown as the bundle (20, 7), where the specific quantities in C again depend on the preferences of the consumer under consideration. By the transitivity assumption, we know that C is also equally attractive as A (since C is equally attractive as B, which is equally attractive as A).

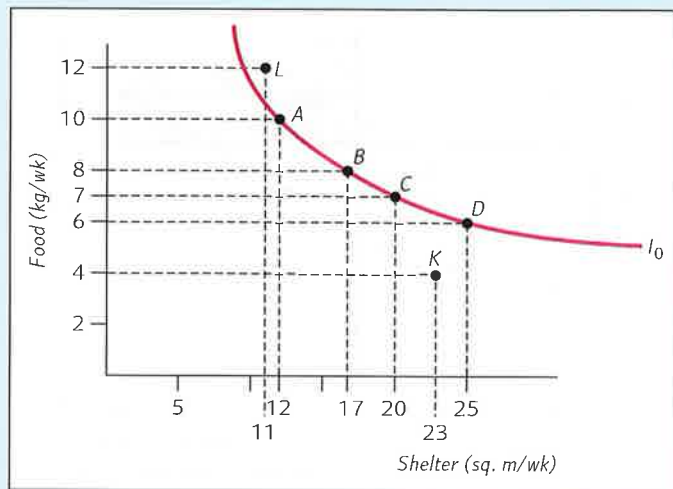
We can repeat this process as often as we like, and the end result will be an **indifference curve**, a set of bundles all of which are equally attractive as the original bundle A, and hence also equally attractive as one another. This set is shown as the curve labelled I in Figure 4.10. It is called an indifference curve because the consumer is indifferent about all the bundles that lie along it.

indifference curve a set of bundles among which the consumer is indifferent.

An indifference curve also permits us to compare the satisfaction implicit in bundles that lie along it with those that lie either above or below it. It permits us, for example, to compare bundle C (20, 7) to bundle K (23, 4), which has less food and more shelter than C has. We know that C is equally attractive as D (25, 6) because both bundles lie along the same indifference curve. D, in turn, is preferred to K because of the more-is-better assumption: it has 2 sq. m/wk more shelter and 2 kg/wk more food than K has. Transitivity, finally, tells us that since C is equally attractive as D and D is preferred to K, C must be preferred to K.

FIGURE 4.10**An Indifference Curve**

An indifference curve is a set of bundles that the consumer considers equally attractive. Any bundle, such as *L*, that lies above an indifference curve is preferred to any bundle on the indifference curve. Any bundle on the indifference curve, in turn, is preferred to any bundle, such as *K*, that lies below the indifference curve.



By analogous reasoning, we can say that bundle *L* is preferred to *A*. In general, bundles that lie above an indifference curve are all preferred to the bundles that lie on it. Similarly, bundles that lie on an indifference curve are all preferred to those that lie below it.

indifference map a representative sample of the set of a consumer's indifference curves, used as a graphical summary of her preference ordering.

The completeness property of preferences implies that there is an indifference curve that passes through every possible bundle. That being so, we can represent a consumer's preferences with an **indifference map**, an example of which is shown in Figure 4.11. This indifference map shows just four of the infinitely many indifference curves that, taken together, yield a complete description of the consumer's preferences.

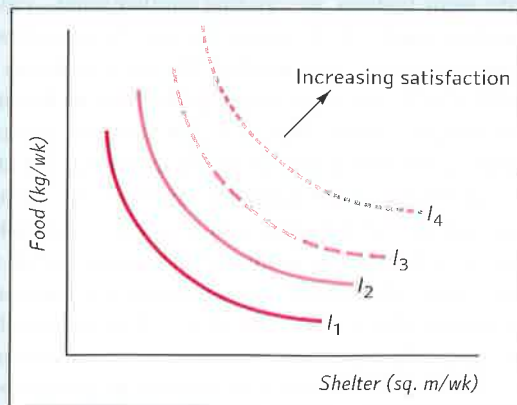
The numbers I_1, \dots, I_4 in Figure 4.11 are index values used to denote the order of preference that corresponds to the respective indifference curves. Any index numbers would do equally well provided they satisfied the property $I_1 < I_2 < I_3 < I_4$. In representing the consumer's preferences, what really counts is the *ranking* of the indifference curves, not the particular numerical values we assign to them.⁴

The four properties of preference orderings we have looked at so far imply three important properties of indifference curves and indifference maps:

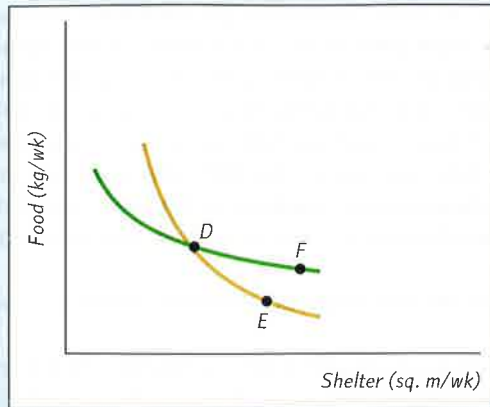
1. Indifference curves are ubiquitous. Any bundle has an indifference curve passing through it. This property is assured by the completeness property of preferences.

FIGURE 4.11**Part of an Indifference Map**

The entire set of a consumer's indifference curves is called the consumer's indifference map. Bundles on any indifference curve are less preferred than bundles on a higher indifference curve, and more preferred than bundles on a lower indifference curve.



⁴For a more complete discussion of this issue, see the Appendix to this chapter.

**FIGURE 4.12**

Why Two Indifference Curves Do Not Cross

If indifference curves were to cross, they would have to violate at least one of the assumed properties of preference orderings.

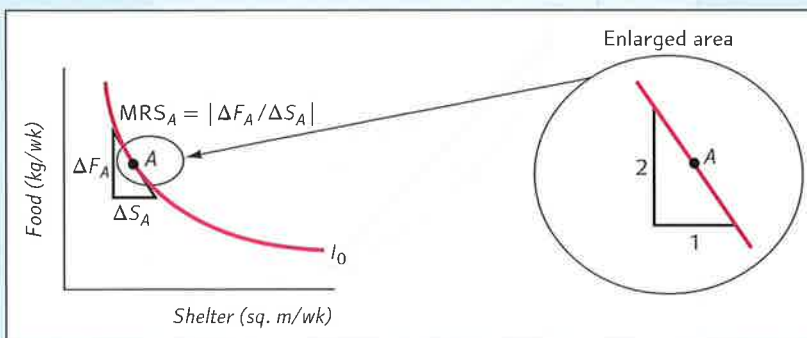
- Indifference curves are downward sloping. An upward-sloping indifference curve would violate the more-is-better property by saying a bundle with more of both goods is equivalent to a bundle with less of both.
- Indifference curves (from the same indifference map) cannot cross. To see why, suppose that two indifference curves did, in fact, cross as in Figure 4.12. The following statements would then have to be true: *E* is equally attractive as *D* (because they each lie on the same indifference curve). *D* is equally attractive as *F* (because they each lie on the same indifference curve). *E* is equally attractive as *F* (by the transitivity assumption). But we also know that *F* is preferred to *E* (because more is better). Because it is not possible for the statements *E* is equally attractive as *F* and *F* is preferred to *E* to be true simultaneously, the assumption that two indifference curves cross thus implies a contradiction. The conclusion is that the original proposition must be true, namely, two indifference curves cannot cross.

Trade-offs between Goods

An important property of a consumer's preferences is the rate at which he is willing to exchange, or 'trade off', one good for another. This rate is represented at any point on an indifference curve by the **marginal rate of substitution (MRS)**, which is given by the absolute value of the slope of the indifference curve at that point. In the left panel of Figure 4.13, for example, the marginal rate of substitution at point *A* is given by the absolute value of the slope of the tangent to the indifference curve at *A*, which is the ratio $\Delta F_A / \Delta S_A$.⁵ (The notation ΔF_A

marginal rate of substitution

(MRS) at any point on an indifference curve is the rate at which the consumer is willing to exchange the good measured on the vertical axis for the good measured along the horizontal axis; equal to the absolute value of the slope of the indifference curve.

**FIGURE 4.13**

The Marginal Rate of Substitution

MRS at any point along an indifference curve is defined as the absolute value of the slope of the indifference curve at that point. It is the amount of food the consumer must be given to compensate for the loss of 1 unit of shelter.

⁵More formally, the indifference curve may be expressed as a function $Y = Y(X)$ and the MRS at point *A* is defined as the absolute value of the derivative of the indifference curve at that point: $MRS = |dY(X)/dX|$.

means 'small change in food from the amount at point A'.) If we take ΔF_A units of food away from the consumer at point A, we have to give him ΔS_A additional units of shelter to make him just as well off as before. The right panel of the figure shows an enlargement of the region surrounding bundle A. If the marginal rate of substitution at A is 2, this means that the consumer must be given 2 kg/wk of food to make up for the loss of 1 sq. m/wk of shelter.

Whereas the slope of the budget constraint tells us the rate at which we can substitute food for shelter without changing total expenditure, the MRS tells us the rate at which we can substitute food for shelter without changing total satisfaction. Put another way, the slope of the budget constraint is the marginal cost of shelter in terms of food, and the MRS is the marginal benefit of shelter in terms of food.

It is now time to introduce the fifth property commonly assumed of preference orderings.

5. Convexity Mixtures of goods are preferable to extremes. If you are indifferent between two bundles A and D, your preferences are convex if you prefer a bundle that contains half of A and half of D (or any other mixture) to either of the original bundles. For example, suppose you are indifferent between $A = (3, 17)$ and $D = (16, 3)$. If your preferences are convex, you will prefer the bundle $E = (9.5, 10)$ to each of the more extreme bundles. This property conveys the sense that we like balance in our mix of consumption goods.

The convexity property of preferences tells us that, along any indifference curve, the more a consumer has of one good, the more she must be given of that good before she will be willing to give up a unit of the other good. Stated differently, MRS declines as we move downward to the right along an indifference curve. Indifference curves with diminishing rates of marginal substitution are thus convex—or bowed outward—when viewed from the origin. The indifference curves shown in Figures 4.10, 4.11 and 4.13 have this property, as does the curve shown in Figure 4.14.

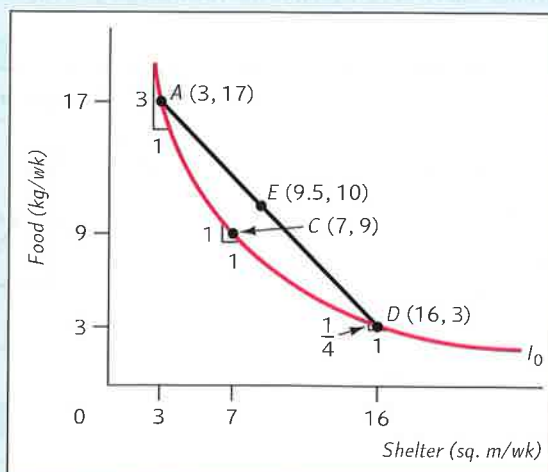
Figure 4.14 shows why they are bowed out. If we pick any two bundles on the same indifference curve, like A and D, and draw a line between them, then we obtain all possible mixtures of A and D. For example, we get bundle E half way along the line. Convexity tells us that all the bundles along this line are preferred to bundles A and D. The indifference curve, therefore, passing through A and D needs to bow below the straight line that connects A and D.

In Figure 4.14, note that at bundle A food is relatively plentiful and the consumer would be willing to sacrifice 3 kg/wk of it in order to obtain an additional square metre of shelter. Her MRS at A is 3. At C, the quantities of food and shelter are more balanced, and there she would be willing to give up only 1 kg/wk to obtain an additional square metre of shelter. Her MRS at C is 1. Finally, note that food is relatively scarce at D, and there she would be willing to give up only $\frac{1}{4}$ kg/wk of food to obtain an additional unit of shelter. Her MRS at D is $\frac{1}{4}$.

FIGURE 4.14

Diminishing Marginal Rate of Substitution

The more food the consumer has, the more she is willing to give up to obtain an additional unit of shelter. The marginal rates of substitution at bundles A, C and D are 3, 1 and $\frac{1}{4}$, respectively. Convexity implies that bundle E is preferred to bundles A and D.



Intuitively, diminishing MRS means that consumers like variety. We are usually willing to give up goods we already have a lot of to obtain more of those goods we now have only a little of. This is particularly the case, as the following Economic Naturalist shows, if we take a long enough time horizon.

Why do restaurants not sell ice cream topped pizza?

We highlighted earlier in the chapter that consumption should always be measured as a flow. The units of time we use are, however, for us to choose. And this can critically alter what a consumer's indifference curves look like.

To make the point, suppose we wanted to describe the preferences of Alice over ice cream and pizza. Alice likes both ice cream and pizza. But, not at the same time! So, if we look at her preferences per course of a meal they will not be convex. She would prefer all ice cream or all pizza over a mixture of the two. If we look at her preferences per meal they will be convex. She would prefer a meal with pizza followed by ice cream to one with only a big pizza or only a lot of ice cream.

Given that people like variety, the longer the time horizon we use the more likely it is that preferences will be convex. Even a student would struggle to survive on pizza, and nothing but pizza, for a year. ■

ECONOMIC NATURALIST 4.4

Using Indifference Curves to Describe Preferences

To get a feel for how indifference maps describe a consumer's preferences, let us see how indifference maps can be used to portray differences in preferences between two consumers. Suppose, for example, that both Tex and Mohan like potatoes but that Mohan likes rice much more than Tex does.

This difference in their tastes is captured by the differing slopes of their indifference curves in Figure 4.15. Note in Figure 4.15(a), which shows Tex's indifference map, that Tex would be willing to exchange 1 kg of potatoes for 1 kg of rice at bundle A. But at the corresponding bundle in Figure 4.15(b), which shows Mohan's indifference map, we see that Mohan would trade 2 kg of potatoes for 1 kg of rice.

The fact that Mohan likes rice much more than Tex means that his indifference curves are steeper. He needs more potato than Tex to compensate for a reduction in the amount of rice consumed. Equivalently his marginal rate of substitution of potatoes for rice is higher.

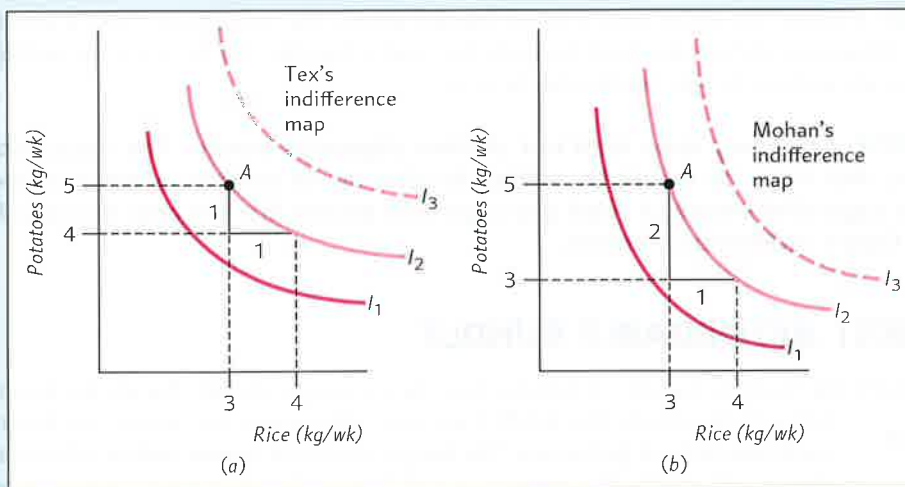


FIGURE 4.15
People with Different Tastes
Relatively speaking, Tex is a potato lover; Mohan, a rice lover. This difference shows up in the fact that at any given bundle Tex's marginal rate of substitution of potatoes for rice is smaller than Mohan's.

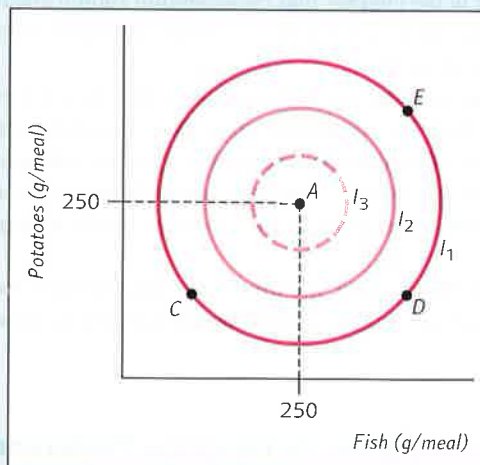
More-Is-Worse and Satiation Points

Preferences do not have to satisfy the five properties we have assumed so far. As long, however, as the preference ordering satisfies completeness, transitivity and continuity we can still represent preferences using an indifference map. To illustrate, consider Mohan at his favourite restaurant. He has ordered a dish containing fish and potatoes. Figure 4.16 shows his indifference map.

FIGURE 4.16

Preferences with a Satiation Point

The best outcome for Mohan is bundle *A*. At bundle *D* he has too much fish and so the indifference curve slopes up. At bundle *E* he has too much fish and potato and so points below the indifference curve are preferred to points on it.



Mohan's perfect meal contains 250 g of fish and 250 g of potatoes. This is bundle *A* and is called his satiation point. To the bottom left of *A*, near bundle *C*, the indifference curves look 'standard' because Mohan would like more fish and potato. The indifference curve slopes down because Mohan is willing to trade off more fish for less potato.

Things are not so standard to the bottom right of *A*. With bundle *D* Mohan's meal has too much fish. The more-is-better property assumes that he can either store or dispose of the fish he does not want, but in an expensive restaurant neither option may be all that viable. Thus, to the right of *A*, more-is-worse in terms of fish. This is captured by an upward-sloping indifference curve at *D*. Mohan is no longer willing to trade off more fish for less potato. He is only willing to trade off more fish (the good he has too much of) for more potato (the good he has too little of). The marginal rate of substitution is measuring the rate at which Mohan is willing to exchange potato for more fish.

Bundle *E* means that Mohan has too much fish and potato. The indifference curve is sloping down but things are still not standard. Less-is-better, and so bundles that lie below the indifference curve are preferred to the bundles that lie on it.

EXERCISE 4.6 Gary likes food but dislikes cigarette smoke. The more food he has, the more he would be willing to give up to achieve a given reduction in cigarette smoke. If food and cigarette smoke are the only two goods, draw Gary's indifference curves.

THE BEST AFFORDABLE BUNDLE

We now have the tools we need to determine how the consumer should allocate his income between two goods. The indifference map tells us how the various bundles are ranked in order of preference. The budget constraint, in turn, tells us which bundles are affordable. The consumer's task is to put the two together and to choose the **best affordable bundle**. (Recall from Chapter 1 that we need not suppose

best affordable bundle the most preferred bundle of those that are affordable.