ALGORITMIC RIDDLES

Sharpen your problem solving skills

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INTRODUCTION

Welcome to the World of Riddles and Algorithms!

This book is a collection of some of the most intriguing and thought-provoking riddles ever created, designed for whoever is curious and eager to sharpen their problem-solving skills. No specific background knowledge is required—just an open mind and a willingness to learn.
What makes this book special is that it does more than just present riddles; it also guides you through the mathematical and algorithmic concepts necessary to solve them. Each riddle is an opportunity to pause, think deeply, and challenge yourself before checking the provided solution.
How to Approach This Book: You're not meant to simply read through it, though you might enjoy the narrative and the puzzle-solving journey. Instead, I encourage you to engage actively with each challenge. Take your time, work through the riddles, and savor the "aha!" moments when you crack a tough problem.
A Gradual Learning Curve: The book is structured to take you on a journey of increasing complexity. The sections are arranged in a way that introduces concepts progressively, so it's beneficial to read from beginning to end. However, each section is self-contained, allowing you to jump around if a particular type of riddle catches your interest.
A Word of Encouragement: The ultimate aim of this book is to inspire a love for algorithms and to ignite your curiosity about the power of logical thinking. Along the way, you'll find joy in the creative and often surprising ways problems can be solved.

Happy Riddle Solving and Happy Learning!

First draft.

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RIVER CROSSING RIDDLES

A wolf, a goat and a cabbage ★☆☆☆☆

You reach the first of the Three Great rivers and notice that there is no bridge.

Problem:
You need to cross the river with a wolf, a goat and a cabbage. The boat can only carry you and one other item (either the wolf, the goat, or the cabbage) at a time.
You find a little boat where you can only fit with one more thing.
If left alone together at any deck, the wolf would eat the goat and the goat would eat the cabbage.

Goal:
Find a sequence of trips across the river that ensures all three items reach the other side.

The Jealous Husband problem ★☆☆☆☆

Description:
Three husbands and their respective wives need to cross a river using a boat. The boat can only carry two people at a time. No woman can be left with another man unless her husband is present (due to jealousy).

Goal:
To find a sequence of trips across the river that ensures everyone is transported to the other side

The Torch problem ★☆☆☆☆

Description:
You and your comrades need to cross a fragile bridge over the third river at night. You only have one torch, and the bridge is too dangerous to cross without it. The bridge can only hold two people at a time, and everyone takes a different amount of time to cross.

- You take 1 minute to cross.
- Brian takes 2 minutes to cross.
- Carl takes 5 minutes to cross.
- David takes 8 minutes to cross

Goal:
Determine how to make everyone reach the other side if the torch only lasts 15 minutes.

Follow Up question: ★★★★★
Write an algorithm that solves the problem with any set of (natural) crossing times.

BOOLEAN LOGIC RIDDLES

Boxes of Coins ★☆☆☆☆

Problem statement:
You have three boxes. One contains only gold coins, one contains only silver coins, and the third contains both gold and silver coins. Each box is labeled, but all labels are incorrect. You can pick one coin from one box.
Question:
How can you determine which box contains which type of coins?

The fork in the road ★☆☆☆☆

Problem statement:

You're at a fork in the road in which:

- One direction leads to the City of Liars, where every citizen always lies;
- The other leads to the City of Truth-tellers, where every citizen always tells the truth;

There's a person at the fork who lives in one of the cities, but you're not sure which one.

Question:

What question could you ask the person to find out which road leads to the City of Truth?

Lion and unicorn ★☆☆☆☆

Problem statement:
You meet a lion and a unicorn in the forest.

- The lion lies every Monday, Tuesday and Wednesday and the other days he speaks the truth.
- The unicorn lies on Thursdays, Fridays and Saturdays, and the other days he speaks the truth.

The lion tells you: “Yesterday I was lying,”, “So was I,” says the unicorn.

Question:
What day is it?

Island of truth-tellers and liars ★☆☆☆☆

Problem statement:
On an island, there are two types of inhabitants: knights, who always tell the truth, and knaves, who always lie. You meet two inhabitants: Alex and Brian.

- Alex says, 'Brian is a knight.'
- Brian says: 'Alex and I are of opposite types.'

Question:
Can you determine who is a knight and who is a knave?

Two doors two guards ★★☆☆☆

Problem statement:

There are two doors in front of you.

- Behind one door there is a treasure,
- Behind the other a ferocious monster.
- In front of each door there is one guardian.
- One of them always tells the truth, while the other always lies.
- You can ask one question.

Question:

What question do you ask to find out which door has a treasure behind?

The five hats riddle ★★☆☆☆

Problem statement:
You are one of three people each wearing a hat. The hats are chosen randomly and without looking from a set of five hats: three white and two black.

- You are all standing in a line
- The last person in the line can see your hat and the one of the other person.
- The middle person can see the hat of the person in front of them.
- You cannot see any hats.

They try to guess the color of their hat from the last to the first person, so that you can hear their answer and the middle person can hear the answer of the last person.

The last person doesn't know the color of his hat. The middle person doesn't know either.

Question:
What's the color of your hat?

Prisoner’s dilemma ★★★☆☆

Problem statement:
You are a prisoner sentenced to death. The king, in a twist of mercy, offers you a chance to save your life. He tells you:

"You must make a single statement. If the statement you make is true, you will be hanged. If the statement is false, you will be beheaded."

Question:
What do you say to save your life?

The three gods riddle ★★★★★

Problem statement:
You find yourself in front of three gods:

- One always speaks truly,
- One always speaks falsely,
- One may speak truly or falsely.

You can ask them three yes-no questions. You can ask more than one question to the same god. They understand English but will answer in their own language with ja or da. They mean yes and no but you don't know which is which.

Question:
Which questions do you ask to know which god is which?

RECURSION RIDDLES

Matryoshka ★☆☆☆☆

Description:
You have a series of 5 matryoshka dolls stacked one inside the other. Each doll can be opened to reveal an even smaller doll, until you reach the smallest one. If the smallest doll has a diameter of 1 cm, and each doll has a diameter twice that of the one it contains,

Goal:
Find the diameter of the largest doll

Follow Up question: ★☆☆☆☆
Find the general solution for N matryoshka dolls

Guess the number ★☆☆☆☆

Description:
You have to find out the number I'm thinking of. But don't worry, it is between 1 and 100 and I won't cheat. After each guess, I tell you if your guess was correct, too high or too low.

Question:
Which is the optimal strategy to find it?

Follow Up question: ★★☆☆☆
How many time do you have to guess in the worst case

Fake ingots ★☆☆☆☆

Description:
You have 9 gold ingots, but one of them is fake and weighs slightly less than the others. You can compare any amount of ingots by placing them on a two-pan scale. Unfortunately, you don't own the scale, and every time you want to weigh anything, you are charged a fee!

Goal:
Find a way to identify the fake ingot using the scale only 2 times.

Follow Up Question: ★★☆☆☆
What's the minimum number of weighings needed to identify the fake ingot if you have N ingots in total?

Fake ingots - SOLUTION

$\lceil log_3 N \rceil$

1. Divide the ingots in two groups of size $\lceil \frac{N}{3} \rceil$ and put the remaining $\lfloor \frac{N}{3} \rfloor$ in a third group
2. Compare the two groups. If they weigh the same, the fake ingot is in the third group. Otherwise it is in the lightest group.
3. Repeat the scale problem with $N = \lceil \frac{N}{3} \rceil$ until $N > 1$.

We weigh W times where W is the time you can divide N by 3 before it gets to 1, which is exactly $\lceil log_3 N \rceil$.

Tower of Hanoi ★★★☆☆

Problem statement:
You are given three discs of different sizes and three columns labeled A, B, and C. The discs are initially stacked on column A in increasing size from top to bottom. You also have to follow those rules to move the discs:

1. You can move one disc at a time;
2. A disc can only be placed on top of a larger disc or an empty place.

Question: ★★☆☆☆
How can you move all three discs from column A to C?

Follow Up Question: ★★★★☆
Find a recursive way to solve this problem with any quantity of discs and find how many times you need to move a disc to move all N discs from column A to C.

Pirates problem ★★★☆☆

Problem statement:
Five pirates (let's call them A, B, C, D, and E) have to divide 100 gold coins among themselves. They follow a strict hierarchy and rules for dividing the loot:

1. Pirate A is the captain, followed by B, C, D, and E.
2. The highest-ranking pirate proposes how to distribute the coins. All pirates, including the proposer, vote on the proposal.
3. If the proposal gets at least 50% of the votes (including the proposer's vote), it is accepted, and the coins are distributed accordingly. If not, the proposer is thrown overboard, and the next highest-ranking pirate makes a new proposal to be voted on.

All pirates are perfectly rational:
1. They know that every other pirate is perfectly rational;
2. They consider their life as of utmost importance;
3. If they don't die anyway, they prefer to get more money;
4. If they don't die nor get more money anyway, they vote to kill the captain.

Question: ★★★☆☆
How will the 100 coins be distributed?

Followup: ★★★☆☆
What if there are N pirates? What happens when there are more than 100 pirates and only 100 coins?

DYNAMIC PROGRAMMING RIDDLES

Flags problem ★☆☆☆☆

Problem statement:

You are playing a game with a friend of yours

1. There are 21 flags placed on the beach.
2. You can make the first move.
3. In each turn you can take between 1 and 3 flags.
4. The player that takes the last flag (or flags) wins.

Question: ★☆☆☆☆

How many flags do you have to take in the first turn to win?

Followup: ★★☆☆☆

How many flags do you have to take in the first turn to win if there are N flags?

The Coin Change Problem ★★☆☆☆

Description:
You have a lot (∞) of each coin denomination {1, 3, 5, 10, 20, 50, 100},

Question:
What's the fewest number of coins needed to buy an ice cream that costs 76?

Follow Up: ★★★☆☆
Write an algorithm that determines the fewest number of coins needed to buy an ice cream that costs N.

Stairs ★★★☆☆

Problem statement:
A knight has to climb a 10 steps staircase. He can take steps one or two at a time.

Question:
In how many different ways can he reach the top?
*the order of the steps matters (e.g. taking two steps at the beginning or at the top should be counter as two different ways)

Follow Up question: ★★★☆☆
Find a recursive solution for a N steps staircase

Knapsack problem ★☆☆☆☆

Problem statement:

You're a treasure hunter exploring a dungeon filled with valuable items of various values. You found a lot of goodies and you are satisfied with the day. But you can only fit 3 items in your knapsack, because of the weight.

Question: ★☆☆☆☆

What's the optimal strategy to bring home the most value?

Follow Up: ★☆☆☆☆

What's the optimal strategy to bring home the most value if you can fit N items in your knapsack?

Real Knapsack problem ★★★★☆

Problem statement:

I made a little mistake in the statement of the previous problem. Not only every treasure has a value V but also a weight W and you can fit in your knapsack only N kilos of goodies. I wasn't considering that some treasures weigh more than others.

Question: ★☆☆☆☆

If you found:

1. A Dragon's Hoard, that weighs 8 kg and is 500$ worth
2. Elven Silverleaf Sword, that weighs 4 kg and is 200$ worth
3. Amulet of Eternal Night, that weighs 1 kg and is 50$ worth
4. Enchanted Mirror of Truth, that weighs 10 kg and is 100$ worth

What do you bring home if you can fit 10 kg in your knapsack?

Followup: ★★★★☆

Write an algorithm that maximizes the value you can fit in your knapsack.

EXTRA

The 100 prisoners and a lightbulb

★★★☆☆

Problem statement:
You and the other 99 prisoners live in solitary cells. There is a central room with one light bulb, which is initially turned off. Each day, one random prisoner can visit the central room and toggle the bulb as he wishes. Every prisoner can also claim that everyone has visited the central room. If it's true, everybody is free, otherwise they will be executed.
Before the experiment all the prisoners have one day to discuss a strategy.

Question:
Which strategy would lead to freedom?

The 100 prisoners ★★★★★

Disclaimer: this is a really hard riddle that does not really involve the concept learned in the previous ones. Attempt at your own risk.
Problem statement:
You and the other 99 prisoners are in a room when a guard announces the following challenge:

- Every prisoner will be numbered from 1 to 100.
- One prisoner at a time must enter a room containing 100 boxes, also numbered from 1 to 100.
- Inside each box is a slip of paper with a number printed on it, which may differ from the box number.
- Each prisoner has up to 50 attempts to open boxes and find the slip with their assigned number.
- The current prisoner will be escorted to a third room, without the possibility of any kind of communication.
- The box room will be resetted every time. With the numbers in the boxes remaining the same. So there is no difference between the prisoners' attempts.
- If every prisoner succeeds in finding their number within their attempts, all prisoners will be granted freedom. However, if even just one prisoner fails to find their number, all prisoners will be sentenced to death.

Question:
Which is the optimal strategy for survival?
Tip: with a random strategy every prisoner has a 50% chance of success. The probability of surviving would be $\frac{1}{2}^{100} \approx 0{,}00000000000000000000000000000....$

Conclusion

I hope you enjoyed the riddles and that you learned something new.
Special thanks to all my friends that patiently tried to solve my riddles and to the ones that suggested to me some riddles to add. You really helped me to understand how this book could have been improved.
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