yyge | yyglumnodt@gmail.com |
is the course in afernoon.

Theoretical Analysis of Complex Systems :

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Evaluation : . ½ Project . 11/12 .
. ½ : → ¼ weekly exercises .
→ ¼ : exam + partial
(16/12) (23/10)

Programming . Python ...

Lecture by Werner KRAUTH .
CFP master
takes place at Orsay Friday Afternoon .
Statistical Physics & Algorithms .
(LPS

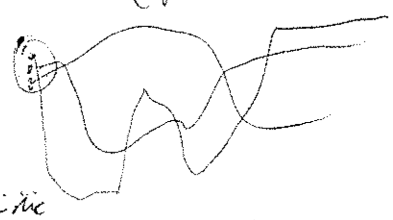
Lecture 1 | 28/08 :

INTRODUCTION TO STOCHASTIC PROCESSES .

→ Understand the emerging properties, the emergence process .

1 - From microscopic to macroscopic description   N constituents $N \gg 1$.
elementary systems of positions $\{\vec{x}_i^{mic}\}$, Newton's law . $m_i \ddot{\vec{x}}_i = \vec{F}(\{\vec{x}_j^{mic}\}, t)$.
in 1 mole : 6.10^{23} constituent → impossible .
⚠ Sensitivity to initial ~~conditions~~ (if we could solve) .

→ Solution in phase space .



Some systems present arbitrary sensitivity .

CHAOS .
↳ evolution of deterministic eq.
we will describe effective facts , from stochastic point of view .
(complementary : mesoscopic)
→ random gradients .

Complex Quantum systems are also possible .
Comparison between 2 points of view : → individual trajectories .
→ distributions . (stochastic) .

. 1st solution (historically) : Thermodynamics . (Gibbs, Boltzmann, Fermi) .
Hypothesis on the distribution after a long time .
× microcanonical point of view : every configuration with the same energy E has the same probability . (equi -) .

× canonical p. of view : (not isolated systems) $\mathrm{Prob}(\mathrm{conf}) \propto e^{-\beta \,\mathrm{Energy}(\mathrm{conf})}$ $\beta = \frac{1}{\mathrm{Temp}}$

↳ Applies only to systems
× at equilibrium (linked to reversible) .
× very large systems
× to determine mean values .
× in the steady states .
× for static observables .

$\mathring{P}(x,t)$: probability that the particle is at position x at time t.

- normalized: $\forall t, \ \sum_x \mathring{P}(x,t) = 1$.
- initial condition: $\mathring{P}(x,0) = \delta_{x,0}$ (Kronecker δ) $= \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases}$.

- Master equation for $P(x,t)$

$$\mathring{P}(x,t+\delta_t) = \mathring{D}\mathring{P}(x-\delta_x,t) + \mathring{D}\,\mathring{P}(x+\delta_x,t) + (1-2\mathring{D})\mathring{P}(x,t).$$

discrete time, discrete space master eq.

"Good" object → B_c. In continuous → proba densities.

Prob. density $P(x,t)$: [DEF]

$\delta_x P(x,t) =$ Probability of being in $[x-\frac{\delta_x}{2}; x+\frac{\delta_x}{2}]$ at time t.

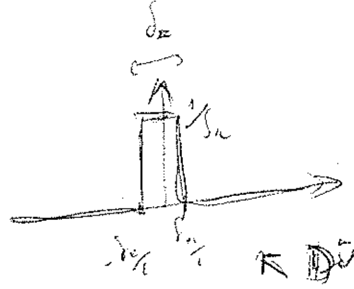
→ $\mathring{P}(x,t)$.

so $P(x,t) = \dfrac{\mathring{P}(x,t)}{\delta_x}$.

→ norm.: $\sum_x \delta_x P(x,t) = 1$

$$\boxed{\int dx\, P(x,t) = 1}$$

→ Initial condition: $\dfrac{\mathring{P}(x,0)}{\delta_x} = \dfrac{\delta_{x,0}}{\delta_x}$ →



← Dirac in the limit.

$P(x,0) = \delta(x)$: Dirac delta "function".

Rappel: $\int dx\, f(x)\delta(x-x_0) = f(x_0)$.) proper def. ($\langle \delta, f \rangle = f(0)$).

$\int dx\, f(x) P(x,0) = f(0)$.

↳ mean value of observable f with initial distribution.

Physical interpretation: → extremely sharped around a value. ... message ...

Integration of dirac & derivation:

Exercise: determine $A_{x_0}(x) = \int_{-\infty}^{x} dx'\, \delta(x'-x_0)$.

Consitently of hypothesis? → eq only in $\hat{x}$.

OK iff $-1-\zeta=-3\zeta$, i.e. $\zeta=\frac{1}{2}$.

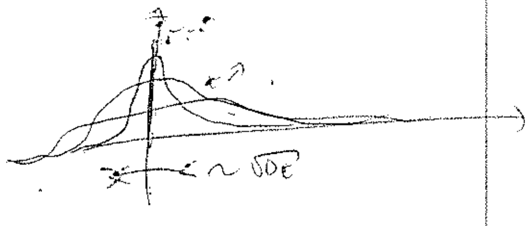
$$\boxed{\hat{P}(\hat{x})+\hat{x}\hat{P}'(\hat{x})=2D\,\hat{P}''(\hat{x})}$$ Recover the same scaling again!

Solving by variation of constants: $\hat{P}(\pm\infty)=0$.

$$\boxed{\hat{P}(\hat{x})=\frac{1}{\sqrt{4\pi D}}\exp\left(-\frac{1}{2}\frac{\hat{x}^2}{2D}\right)}$$

$\int d\hat{x}\,\hat{P}(\hat{x})=1$.

$\forall t,\ \int dx\, P(x,t)=1$.



<u>Solution</u>:

$$P(x,t)=\frac{1}{\sqrt{4\pi D t}}\exp\left(-\frac{1}{2}\frac{x^2}{2Dt}\right)$$

$\lim_{t\to 0} P(x,t)=\delta(x)$.

Let put many random walkers with interactions.

→ add Force or potential.

<u>Next goal</u>: put forces & interactions: $x_{t+\delta_t}=x_t+\mathring{\eta}_t+\mathring{F}(x_t)$.

or rather $\dot{x}_t=\frac{x_{t+\delta_t}-x_t}{\delta t}=\frac{\mathring{\eta}_t}{\delta_t}+\frac{\mathring{F}(x_t)}{\delta_t}$.

$\eta(t)$? properties?

<u>Mean and variance of the noise</u>?

$\dot{x}(t)=\eta(t)$, "$\lim_{\tau\to 0}\frac{\mathring{\eta}_t}{\delta_t}$".

What is the distribution of the continuous noise $\eta(t)$?

- <u>Mean</u>: $\langle\eta(t)\rangle=\langle\partial_t x\rangle=\partial_t\langle x\rangle=0$
  from ergodicity & distribution.

- <u>"Variance"</u> 2nd moment: $\langle\eta(t_1)\eta(t_2)\rangle=$ ~~$\langle\eta(t_1)\eta(t_2)\rangle$ only~~ $=R(t_1,t_2)$

Let's focus on $C(t_2,t_1)=\langle[x(t_2)-x(t_1)]^2\rangle$ $t_2>t_1$

One uses that $P(x_2 t_2|x_1 t_1)=\frac{1}{\sqrt{4\pi D(t_2-t_1)}}e^{-\frac{1}{2}\frac{(x_2-x_1)^2}{2D(t_2-t_1)}}$ use <u>absence</u> of memory of evolution

$$C(t_1,t_2)=\int dx_1\,dx_2\,(x_2-x_1)^2P(x_2,t_2|x_1,t_1)=2D|t_2-t_1|.$$

$C(t_2,t_1)=2D\cdot|t_2-t_1|$

- $C(t_2,t_1)=C(t_2-t_1)$.

$x(t_2)-x(t_1)=\int_{t_1}^{t_2}dt\,\underbrace{\dot{x}(t)}_{\eta(t)}$ → $C(t_2-t_1)=\left\langle\left(\int_{t_1}^{t_2}dt\,\eta(t)\right)^2\right\rangle=\left\langle\int_{t_1}^{t_2}dt\int_{t_1}^{t_2}dt'\,\eta(t)\eta(t')\right\rangle$

$=\iint dt\,dt'\,R(t'-t)$