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# Strategic design of public bicycle sharing systems with service level constraints

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ABSTRACT

This study addresses the strategic planning of public bicycle sharing systems with service level considerations. In considering the interests of both users and investors, the proposed model attempts to determine the number and locations of bike stations, the network structure of bike paths connected between the stations, and the travel paths for users between each pair of origins and destinations. A small example is created to illustrate the proposed model. Sensitivity analysis is also performed to gain better insights into knowing how several important parameters affect the design of the system.



## 1. Introduction

Public bicycle systems, also known as bicycle sharing systems, have been introduced as part of the urban transportation system to extend the accessibility of public transportation systems to final destinations. By integrating with other public transportation systems and providing free or affordable access to bicycles for city travel, the public bicycle systems are used to lessen the use of automobiles for short trips inside the central business district, thereby alleviating traffic congestion and reducing noise/air pollution. Public bicycle systems are viewed as an innovative inner-city transportation mode to meet many commuters' needs and to integrate them with other public transit systems. The idea is that the commuters can take the bicycles whenever they need them and leave them behind when they reach their destinations.

Since public bicycles were first introduced in Amsterdam in the 1960s (the so-called white bicycle plan), public bicycle systems have been promoted in urban cities around the world such as Paris, Barcelona, Berlin, Montreal, Salt Lake City, and so on. The success of the public bicycle systems heavily depends on the network of bike paths and the locations of bike stations where the bikes can be picked up and returned. However, there are relatively few studies in the literature that focus on the strategic design of these public bicycle systems. Seeking to fill this gap is the underlying motivation of this paper. In what follows, we therefore address the network design problem related to the bike paths and the facility location problem associated with the bike pick-up/drop-off stations in considering the strategic design of a public bicycle system in an urban area.

The optimal design of such a system requires an integrated view of the travel costs of users, the facility costs of bike stations, the setup costs of bicycle lanes, as well as the service level, which is measured by the coverage range of both the origins and destinations and the availability rate of pick-up bike requests at stations. This study aims to develop a mathematical model that provides such an integrated view. The proposed model attempts to determine the number and locations of bike

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stations in the system, the network structure of bike paths connected between the stations, and the travel paths for users between each pair of origins and destinations. The design decisions need to strike a balance or find an appropriate trade-off between the overall cost and the service levels. The main concerns in this model are the long-term decisions on facility investments for bike stations, the setup cost for the construction of bicycle lanes, the bike inventory costs, and the path travel costs for bike users.

The bike stations are used to distribute and collect the public bikes. The user traffic is consolidated into a number of bike stations where the public bikes can be picked up, and then from there the users can travel to another bike station to drop off the bikes. Therefore, the bike stations play roles that are similar to the hubs in typical hub-and-spoke systems. The proposed model can be viewed as a hub location model that takes the coverage level into consideration. We first review the literature related to hub location problems. Hub location problems have constituted one of the important classic facility location problems due to the use of hub and spoke networks in transportation and telecommunications systems. Hub and spoke network systems serve the demands of each origin/destination via a smaller set of links between origins/destinations and hubs, and between pairs of hubs, rather than serving demand with direct links. The hub location problem involves determining the hub facilities and determining the links to connect origins, destinations and hubs. Since the early work of O'Kelly (1986), the hub location problem has been applied to design an air transportation network (Aykin, 1995; O'Kelly, 1998; Adler, 2005), to locate a hub location for express common carriers (Lin and Chen, 2004, 2008), to design parcel service networks (Ernst and Krishnamoorthy, 1999; Wasner and Zapfel, 2004), and to locate inter-modal freight hubs (Racunica and Wynter, 2005), among other applications. Extensions of basic hub problems include models with flow dependent cost discounts (O'Kelly and Bryan, 1998), models that allow direct origin–destination paths (Aykin, 1995), models that consider capacity constraints on hub facilities (Yang, 2008; Rodriguez et al., 2007), models with competition considerations (Marianov et al., 1999; Skorin-Kapov, 1998 and Adler, 2005), and models that locate hub arcs (Campbell et al., 2005a,b). Solution procedures for hub location problems include the use of linear programming approaches (Ebery et al., 2000), Lagrangian relaxation-based approaches (Aykin, 1994), dual ascent methods (Mayer and Wagner, 2002), enumerative algorithms (Ernst and Krishnamoorthy, 1998) and heuristics (Klincewicz, 1992, 2002). Since the hub location problems are difficult to solve exactly, there are a larger number of heuristics used to tackle the many types of hub location problems proposed in the literature for various applications. Campbell et al. (2002) present extensive surveys of applications and solution procedures for the hub location problem.

The service level proposed in this study is measured by both the coverage range of the origins and destinations and the availability rate of pick-up bike requests at stations. It is for this reason that related studies on maximal covering problems are also reviewed here. The maximal covering model was first formulated by Church and ReVelle (1974) and has been used in a variety of applications. A demand location is "covered" if a facility is located within a given distance $S$ of the location. The maximal covering problem identifies $P$ locations that maximize the amount of demand within a distance $S$ of at least one facility. In the standard maximal covering problem, $P$ is an input parameter. However, a fixed-charge version of the model can be formulated to optimize the possible values for Mirchandani and Francis (1990) and Daskin (1995) present surveys of applications and solution procedures for the maximal covering problem.

Some studies have also applied the concept of covering problems to measure the level of service. Logistics systems design models that take into account the service level based on coverage performance considerations include the work of Nozick and Turnquist (2000), Nozick (2001), and Lin et al. (2006). The public transit system design models that take into consideration coverage performance considerations include the work of Bruno et al. (2002), Murray (2003), Wu and Murray (2005), and Matisziw et al. (2006).

To ensure that the hub network can effectively handle the traffic through hubs and uncertain demands, several performance constraints may be included in the hub location models. However, these constraints are most common in telecommunications systems and logistics systems. Logistics systems design models that include demand uncertainty and performance constraints include the works by Cole (1995), Nozick and Turnquist (1998, 2000, 2001), Shen et al. (2003), Miranda and Garrido (2004), and Lin et al. (2006). The model used in this study can be viewed as a hub location model with performance constraints.

This study makes the following contributions: (1) Although there are some studies related to bicycle systems, most of them focus on safety issues. In our review of the related literature, we do not find any studies that address the network design of bike lanes and the problems related to the location of the bike stations in public bicycle systems. This paper therefore develops a mathematical model for the strategic design of a public bicycle system. This has so far not been proposed in the literature. (2) The model is designed from a practical point of view, because it considers both the users' and the investors' interests. The service level provided to users is measured by the demand coverage level, availability of bikes at stations, and travel costs, while the bike inventory costs, the setup costs for bike stations and bike lanes are also considered from the point of view of the investor. (3) A contrived example is created to test the proposed model. Sensitivity analysis is also conducted to gain better insights into, and understandings of, the properties of the proposed model.

The remainder of this article is organized as follows. In Section 2, the problem definition is presented and a mathematical model introduced to formulate the strategic design of the public bicycle system. In Section 3, a contrived network is created to illustrate the proposed model. Sensitivity analysis is also performed to gain insights into the problem. Finally, some concluding remarks are discussed in Section 4.

## 2. Model formulation

### 2.1. Problem description

The problem can be summarized as follows. Given a set of origins, destinations, candidate sites of bike stations, and the stochastic travel demands from origin to destination with known parameters, we would like to determine where to locate the bike stations, where to build the bicycle lanes, and what paths should be used for users from each origin to each destination. Since the public bicycle system is designed to be integrated with other public transportation systems by providing the final linkage to the destination, the trips of users from origins to destinations consist of three segments: (1) the user walking from an origin to a bike station to pick-up a bicycle, (2) the user riding the bicycle from the pick-up bike station to a bike station close to the destination to drop off the bicycle, and (3) the user walking from the drop-off bike station to the destination. The public bicycle system is designed to be integrated with other public transportation systems. One important feature of a public bicycle system that is integrated with other public transportation systems is that it enables one-way trips to be made, and therefore its design should be such that it facilitates one-way use. Since users have to pick-up and drop off bicycles at bike stations, paths that only visit a single bike station are not feasible. The general structure of the public bicycle system addressed in this study is represented in Fig. 1. The existing street network structure between the rental stations is not explicitly considered in this study. We only consider direct links between stations because this avoids all the complications associated with assigning riding bicycle traffic on the built bicycle lanes of the existing street network. However the results of this model need further treatment in guiding bicycle lane investments.

It is crucial for the success of the system that users find bike stations in convenient locations and in sufficient numbers. The systems need sufficient stations for users to pick-up a bicycle close to their origins and to drop it off close to their destinations. Existing examples show that the bike stations should not be located more than 300–500 m from important traffic origins and destinations. On the other hand, it is also important for the success of the system that the system guarantee the availability of a bicycle. Each rental station must carry enough bicycles to increase the possibility that each user can find a bicycle when needed. Therefore, measures of service quality in the system include both the availability rate (i.e., the proportion of pick-up requests at a bike station that are met by the bicycle stock on hand) and the coverage level (the fraction of total demand at both origins and destinations that is within some specified time or distance from the nearest rental station). A trip is considered to be covered only if both its origin and destination are covered. Fewer rental stations result in lower overall bike inventory costs, but also lower coverage of demand. By contrast, a network with more stations also allows shorter travel trips between origins/destinations and stations, thus potentially decreasing total travel costs. However, additional costs of constructing and operating the stations will be incurred due to the larger number of stations. Thus there is a basic trade-off between the number of bike stations and the users' travel costs. Based on the network structure, we are ready to present the mathematical model for the system.

### 2.2. Mathematical model

To formulate this problem, the following symbols, variables, and parameters are first introduced.

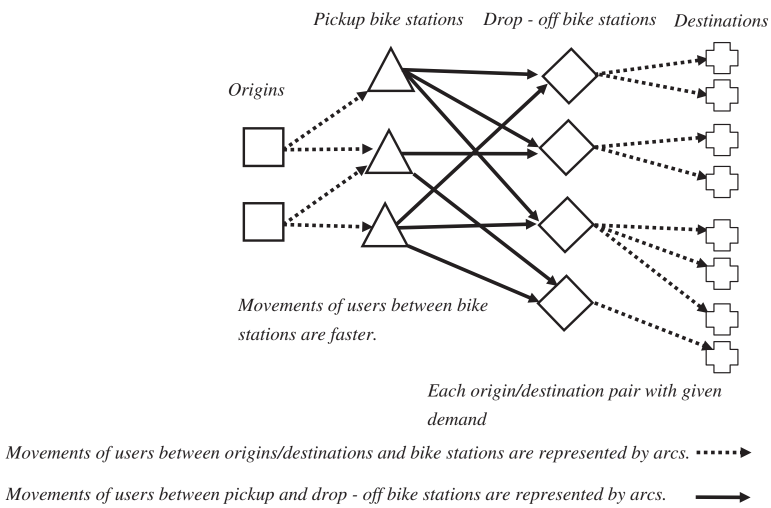


**Fig. 1.** Network structure of public bicycle systems.

*Subscripts and sets*

$i \in I$ denotes the origins
$j \in J$ denotes the destinations
$k, l \in K$ denotes the potential bike pick-up/drop-off stations

*Input parameters*

$\lambda_{ij}$ is the yearly mean travel demand from origin $i$ to destination $j$
$\sigma^2_{ij}$ is the daily variance of travel demand from origin $i$ to destination $j$
T is the number of days per year (used to convert daily demand)
$h$ is the annual bike holding cost
$\tau$ is the replenishment lead time of bikes at bike stations in days
$\rho$ is the desired availability rate of request bikes at bike stations
$z_\rho$ is the standard normal deviate such that $P(z < z_\rho) = \rho$
$d_{ik}$ is the distance from origin $i$ to bike station $k$
$d_{kl}$ is the distance from bike station $k$ to bike station $l$
$d_{kj}$ is the distance from bike station $k$ to destination $j$
$f_k$ is the fixed cost of locating a bike station at $k$
$c_{kl}$ is the construction cost of constructing a bicycle lane from bike stations $k$ to $l$; it is equal to 0 if there already exists a bicycle lane between station $k$ and station $l$
$q_{ik}$ equals 1 if a bike station located at candidate site $k$ cannot cover demand at origin $i$, and is 0 otherwise
$q_{jl}$ equals 1 if a bike station located at candidate site $l$ cannot cover demand at destination $j$, and is 0 otherwise
$\alpha$ is the unit traveling cost on links from origins to bike stations for a person
$\beta$ is the unit traveling cost on links from pick-up bike stations to drop-off bike stations for a person
$\gamma$ is the unit traveling cost on links from bike stations to destinations for a person
$\delta$ is the unit penalty cost for uncovered demands at origins and destinations

*Decision variables*

$X_k$ equals 1 if bike station $k$ is opened and 0 otherwise
$Y_{iklj}$ equals 1 if the demand from origin $i$ to destination $j$ travels through bike station $k$ and $l$ in sequence; and 0 otherwise
$Z_{kl}$ equals 1 if a bicycle lane is required to be connected between bike stations $k$ and $l$; and 0 otherwise

Based on the notation, the following mathematical model can be formulated:

$$\begin{aligned} \min \alpha \sum_{i\in I}\sum_{k\in K} d_{ik} \sum_{l\in K}\sum_{j\in J} Y_{iklj}\lambda_{ij} + \beta \sum_{k\in K}\sum_{l\in K} d_{kl} \sum_{i\in I}\sum_{j\in J} Y_{iklj}\lambda_{ij} + \gamma \sum_{l\in K}\sum_{j\in J} d_{lj} \sum_{i\in I}\sum_{k\in K} Y_{iklj}\lambda_{ij} + \sum_{k\in K} f_k X_k + \sum_{k\in K}\sum_{l\in K} c_{kl} Z_{kl} \\ + \delta\left(\sum_{i\in I}\sum_{k\in K} q_{ik} \sum_{l\in K}\sum_{j\in J} Y_{iklj}\lambda_{ij} + \sum_{j\in J}\sum_{l\in K} q_{jl} \sum_{k\in K}\sum_{i\in I} Y_{iklj}\lambda_{ij}\right) + \sum_{k\in K} \frac{h\tau}{T} \sum_{i\in I}\sum_{j\in J}\sum_{l\in K} Y_{iklj}\lambda_{ij} \\ + h z_\rho \sum_{k\in K} \sqrt{\tau \sum_{i\in I}\sum_{j\in J}\sum_{l\in K} Y_{iklj}\sigma^2_{ij}} \end{aligned} \tag{1}$$

such that

$$\sum_{k\in K}\sum_{l\in K\neq k} Y_{iklj} = 1 \quad \forall i \in I,\ \forall j \in J \tag{2}$$

$$2Z_{kl} \leqslant X_k + X_l \quad \forall k \in K,\ \forall l \in K\neq k \tag{3}$$

$$Z_{kk} = 0 \quad \forall k \in K \tag{4}$$

$$Y_{iklj} \leqslant Z_{kl} \quad \forall i \in I,\ \forall k \in K,\ \forall l \in K\neq k,\ \forall j \in J \tag{5}$$

$$Y_{iklj} = \{0,1\} \quad \forall i \in i,\ \forall k \in K,\ \forall l \in K\neq k,\ \forall j \in J \tag{6}$$

$$X_k = \{0,1\} \quad \forall k \in K \tag{7}$$

$$Z_{kl} = \{0,1\} \quad \forall k \in K,\ \forall l \in K \tag{8}$$

The objective function (1) contains eight terms. The first term is the sum of travel costs on links from origins to pick-up bike stations. The second term is the sum of travel costs on links between bike stations. The third term is the sum of travel costs on links from drop-off bike stations to destinations. The fourth term is the sum of the setup costs for the bike stations.

The fifth term is the sum of the setup costs for the bicycle lanes. The sixth term is the sum of the penalty costs for uncovered demands. The seventh term is the bicycle stock costs. The eighth term is the bike safety stock costs. The model minimizes the total overall costs of the eight terms.

We assume that the daily travel demand from origin $i$ to destination $j$ is normally distributed with mean $\lambda_{ij}/T$ and standard deviation $\sigma_{ij}$. Therefore, the demand for bikes at station $k$ during the replenishment lead time $\tau$ has a mean defined by $\frac{\tau}{T}\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}\lambda_{ij}Y_{iklj}$ and the standard deviation $\sqrt{\tau\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}Y_{iklj}\sigma_{ij}^2}$. The station must carry enough inventory to ensure a low probability $(1-\rho)$ of being out of bikes during the replenishment lead time, $\tau$. The inventory level needed at station $k$ then comprises the bicycle cycle stock $\frac{\tau}{T}\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}\lambda_{ij}Y_{iklj}$ plus the safety stock $z_\rho\sqrt{\tau\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}Y_{iklj}\sigma_{ij}^2}$. Therefore, the bicycle cycle stock cost is the bicycle stock held at all stations $\sum_{k\in K}\frac{\tau}{T}\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}\lambda_{ij}Y_{iklj}$ times the holding cost $h$, and the bike safety stock cost is the safety stock held at all stations $z_\rho\sum_{k\in K}\sqrt{\tau\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}Y_{iklj}\sigma_{ij}^2}$ times the holding cost $h$. The inventory level is conservatively calculated since the bikes dropped off are not taken into account in determining the number of bicycles available for reuse. To make the best use of the available bikes, the drop-off bikes should be accounted for as being available for reuse. The demand for dropping off bikes at station $k$ during the replenishment lead time $\tau$ has a mean defined by $\frac{\tau}{T}\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}\lambda_{ij}Y_{ilkj}$ and the standard deviation $\sqrt{\tau\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}Y_{ilkj}\sigma_{ij}^2}$. If we ignore the riding trip duration (that implies the drop-off bikes are immediately available for reuse), the net demand for pick-up bikes at station $k$ during the replenishment lead time $\tau$ has a mean defined by $\frac{\tau}{T}\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}\lambda_{ij}(Y_{iklj}-Y_{ilkj})$ and the standard deviation $\sqrt{\tau\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}(Y_{iklj}+Y_{ilkj})\sigma_{ij}^2}$. The inventory level needed at station $k$ to ensure the desired availability $(\rho)$ then comprises the bicycle cycle stock $\frac{\tau}{T}\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}\lambda_{ij}(Y_{iklj}-Y_{ilkj})$ plus the safety stock $z_\rho\sqrt{\tau\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}(Y_{iklj}+Y_{ilkj})\sigma_{ij}^2}$. Therefore, the bicycle cycle stock cost is the bicycle stock held at all stations $\sum_{k\in K}\frac{\tau}{T}\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}\lambda_{ij}(Y_{iklj}-Y_{ilkj})$ times the holding cost $h$, and the bicycle safety stock cost is the safety stock held at all stations $z_\rho\sum_{k\in K}\sqrt{\tau\sum_{i\in I}\sum_{j\in J}\sum_{l\in K}(Y_{iklj}+Y_{ilkj})\sigma_{ij}^2}$ times the holding cost $h$. Note that the total bicycle cycle inventory level is indeed zero in this case. The inventory level is dramatically underestimated since the bike trip riding durations are not taken into account in determining the number of bicycles available for reuse. The exact estimation of the bike inventory level at rental stations requires further investigation.

Constraint (2) ensures that the whole of the demand is satisfied. Constraint (3) and (4) ensures that only the pair of bike stations that are both opened may need to build a bicycle lane connecting them. Constraint (5) ensures that the users can travel the path only by bicycle when the bicycle lanes are built. Constraint (6) ensures that the routing variables are nonnegative. Constraints (8) and (9) are the integrality requirements for the location variables of the bicycle stations and bicycle lanes.

Based on the mathematical formulation, the model is an integer nonlinear program. Suppose that we consider a network with $n$ candidate bike stations and $m$ pairs of travel demand from origins to destinations. The number of decision variables is estimated in Table 1 and the number of constraints is listed in Table 2.

## 3. An illustrative example

### 3.1. Data Settings

A small example, as shown in Fig. 2, was created to illustrate the proposed model. The network consists of four bus stations, two mass rapid transit (MRT) stations, and six office buildings. 11 candidate sites are considered for the bike stations. Among them, there are six sites near the bus/MRT stations and five sites near the office buildings. The travel demands are derived between the bus/MRT stations and office buildings. Therefore, there are 72 pairs of travel demands. The public bicycle system is designed to integrate the public transportation systems and provide access to final destinations. A commuter walks from one of the bus/MRT stations to the closest bike station and picks up a bike. Then, he/she rides the bike to the second bike station and returns the bike. Finally, he/she walks from the second bike station to the office building. The reverse direction, from the office buildings to the bus/MRT stations, also derives travel demand. The origins/destinations demand matrix is shown in Table 3 and the distance matrices are shown in Tables 4 and 5. We assume the coefficient of variation of daily demand to be 0.3. The coverage distance is assumed to be 300 m and the unit penalty cost for uncovered demand is deemed to be NTD 100 per trip. We assume that users can walk at a pace of 4 km per hour and that they can ride a bicycle

**Table 1**
The number of decision variables for P(1).

| Decision variables | Number of variables |
|---|---|
| $X_k$ (integer) | $n$ |
| $Z_{kl}$ (integer) | $n^2$ |
| $Y_{iklj}$ (integer) | $mn^2$ |
| Total integer variables | $mn^2 + n^2 + n$ |

**Table 2**
The number of constraints.

| Constraints | Number of constraints |
|---|---|
| (2) | $m$ |
| (3) | $n(n-1)$ |
| (4) | $n$ |
| (5) | $mn(n-1)$ |
| (6) | $mn(n-1)$ |
| (7) | $n$ |
| (8) | $n^2$ |
| Total | $(m+1)(n^2-n)+m$ |

Note that the total number of constraints did not take account of constraints (6)–(8) since they are binary constraints.

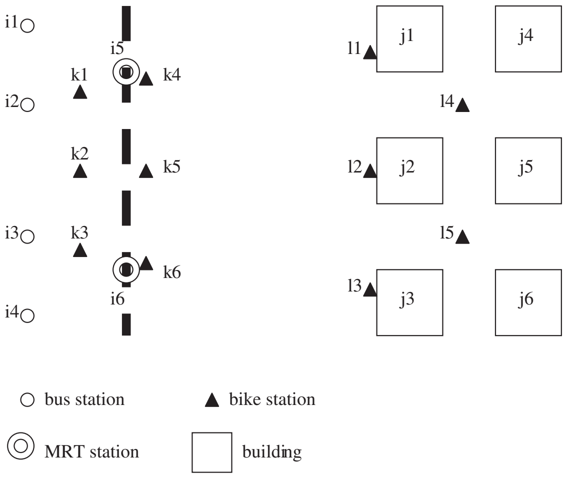


**Fig. 2.** Location sites for the illustrative example.

**Table 3**
Travel demands.

| | *i*1 | *i*2 | *i*3 | *i*4 | *i*5 | *i*6 | *j*1 | *j*2 | *j*3 | *j*4 | *j*5 | *j*6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *i*1 | 0 | 0 | 0 | 0 | 0 | 0 | 10,000 | 15,000 | 10,000 | 15,000 | 10,000 | 15,000 |
| *i*2 | 0 | 0 | 0 | 0 | 0 | 0 | 20,000 | 25,000 | 20,000 | 25,000 | 20,000 | 25,000 |
| *i*3 | 0 | 0 | 0 | 0 | 0 | 0 | 10,000 | 15,000 | 10,000 | 15,000 | 10,000 | 15,000 |
| *i*4 | 0 | 0 | 0 | 0 | 0 | 0 | 20,000 | 25,000 | 20,000 | 25,000 | 20,000 | 25,000 |
| *i*5 | 0 | 0 | 0 | 0 | 0 | 0 | 30,000 | 30,000 | 35,000 | 30,000 | 35,000 | 30,000 |
| *i*6 | 0 | 0 | 0 | 0 | 0 | 0 | 40,000 | 40,000 | 45,000 | 40,000 | 45,000 | 40,000 |
| *j*1 | 10,000 | 20,000 | 10,000 | 20,000 | 30,000 | 40,000 | 0 | 0 | 0 | 0 | 0 | 0 |
| *j*2 | 15,000 | 25,000 | 15,000 | 25,000 | 30,000 | 40,000 | 0 | 0 | 0 | 0 | 0 | 0 |
| *j*3 | 10,000 | 20,000 | 10,000 | 20,000 | 35,000 | 45,000 | 0 | 0 | 0 | 0 | 0 | 0 |
| *j*4 | 15,000 | 25,000 | 15,000 | 25,000 | 30,000 | 40,000 | 0 | 0 | 0 | 0 | 0 | 0 |
| *j*5 | 10,000 | 20,000 | 10,000 | 20,000 | 35,000 | 45,000 | 0 | 0 | 0 | 0 | 0 | 0 |
| *j*6 | 15,000 | 25,000 | 15,000 | 25,000 | 30,000 | 40,000 | 0 | 0 | 0 | 0 | 0 | 0 |

at 20 km per hour. The time value of users is NTD 800 per hour, which is about the average labor cost. Therefore, the unit walking cost is NTD 0.2 per meter per trip and the unit bicycle riding cost is NTD 0.04 per meter per trip (the ratio of the unit walking cost to the unit bicycle riding cost is equal to 5). The fixed cost of building a bicycle lane is NTD 100 per meter times the distance between two stations, and the fixed costs of bicycle stations range from NTD 4 million to NTD 8 million. We assume that the bicycle stocks are replenished every day. This means that the parameter T used to convert daily demand is 365. The unit bicycle holding cost is NTD 2000 per year and the availability rate of bicycles requested at stations is 99%.

### *3.2. Test results*

The illustrative problem was solved by a Branch and Bound solver of a commercial optimization software, LINGO 11.0, on a notebook computer (Intel 1.83 GHz Core Duo and 1 GB of memory) with a Microsoft Windows XP operating system. Fig. 3

**Table 4**
The distance matrix from origins/destinations to stations (or from stations to origins/destinations).

| | *k*1 | *k*2 | *k*3 | *k*4 | *k*5 | *k*6 | *l*1 | *l*2 | *l*3 | *l*4 | *l*5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *i*1 | 200 | 300 | 600 | 300 | 500 | 700 | 2400 | 2500 | 2600 | 2500 | 2600 |
| *i*2 | 100 | 200 | 500 | 300 | 400 | 600 | 2400 | 2450 | 2550 | 2450 | 2550 |
| *i*3 | 500 | 200 | 100 | 600 | 400 | 300 | 2550 | 2450 | 2400 | 2550 | 2450 |
| *i*4 | 600 | 300 | 200 | 700 | 500 | 300 | 2600 | 2500 | 2400 | 2600 | 2500 |
| *i*5 | 200 | 300 | 500 | 5 | 400 | 600 | 2002 | 2202 | 2502 | 2202 | 2502 |
| *i*6 | 500 | 300 | 200 | 600 | 400 | 5 | 2502 | 2202 | 2002 | 2502 | 2202 |
| *j*1 | 2302 | 2402 | 2702 | 2002 | 2202 | 2502 | 5 | 300 | 600 | 300 | 600 |
| *j*2 | 2402 | 2302 | 2402 | 2202 | 2002 | 2202 | 300 | 5 | 300 | 300 | 300 |
| *j*3 | 2702 | 2402 | 2302 | 2502 | 2202 | 2002 | 600 | 300 | 5 | 600 | 300 |
| *j*4 | 2800 | 2800 | 2900 | 2400 | 2600 | 2700 | 350 | 400 | 700 | 300 | 600 |
| *j*5 | 2800 | 2750 | 2800 | 2400 | 2550 | 2400 | 400 | 350 | 400 | 300 | 300 |
| *j*6 | 2900 | 2800 | 2800 | 2700 | 2600 | 2400 | 700 | 400 | 350 | 600 | 300 |

**Table 5**
The distance matrix from stations to stations.

| | *k*1 | *k*2 | *k*3 | *k*4 | *k*5 | *k*6 | *l*1 | *l*2 | *l*3 | *l*4 | *l*5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *k*1 | 0 | 150 | 300 | 150 | 200 | 400 | 2300 | 2400 | 2700 | 2600 | 2700 |
| *k*2 | 150 | 0 | 150 | 210 | 150 | 210 | 2400 | 2300 | 2400 | 2600 | 2600 |
| *k*3 | 300 | 150 | 0 | 400 | 200 | 150 | 2700 | 2400 | 2300 | 2700 | 2600 |
| *k*4 | 150 | 210 | 400 | 0 | 155 | 310 | 2000 | 2200 | 2500 | 2200 | 2500 |
| *k*5 | 200 | 150 | 200 | 155 | 0 | 155 | 2200 | 2000 | 2200 | 2400 | 2400 |
| *k*6 | 400 | 210 | 150 | 310 | 155 | 0 | 2500 | 2200 | 2000 | 2500 | 2200 |
| *l*1 | 2300 | 2400 | 2700 | 2000 | 2200 | 2500 | 0 | 160 | 320 | 160 | 300 |
| *l*2 | 2400 | 2300 | 2400 | 2200 | 2000 | 2200 | 160 | 0 | 160 | 160 | 160 |
| *l*3 | 2700 | 2400 | 2300 | 2500 | 2200 | 2000 | 320 | 160 | 0 | 300 | 160 |
| *l*4 | 2600 | 2600 | 2700 | 2200 | 2400 | 2500 | 160 | 160 | 300 | 0 | 160 |
| *l*5 | 2700 | 2600 | 2600 | 2500 | 2400 | 2200 | 300 | 160 | 160 | 160 | 0 |

illustrates the solution. The system design yielded a recommendation of eight bicycle stations located at *k*1, *k*4, *k*6, *l*1, *l*2, *l*3, *l*4, and *l*5, and 30 bicycle lanes lying in between these stations. All travel demands are routed on the corresponding shortest travel cost path, given the open stations and bicycle lanes. Those users from the six bus/MRT stations merely walk to the nearest open bicycle station to pick-up a bicycle, ride to the open station nearest to their respective destinations, drop off the bicycle at the station, and then walk to their destinations. For example, the users traveling from origin *i*1 to destination *j*1 walk to station *k*1 to pick-up a bicycle, ride to station *l*1, drop off the bicycle at station *l*1, and then walk to destination *j*1. Those users from the six bus/MRT stations who travel to destination *j*5, however, may use different drop-off stations. The users from the upper three origins (namely, *i*1, *i*2 and *i*5) drop off bicycles at station *l*4 and then walk to destination *j*5.

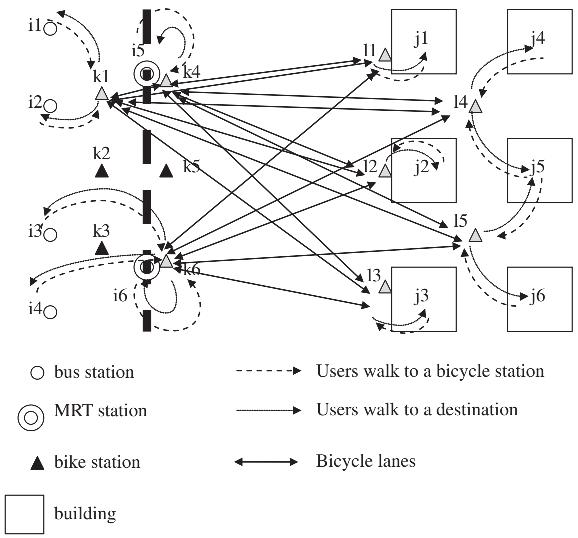


**Fig. 3.** Network design and routing choices for the illustrative example.

The users from the lower three origins (namely, *i*3, *i*4 and *i*6) drop off bicycles at station *l*5 and then walk to destination *j*5. For those users who travel from the six office buildings to the bus/MRT stations, the routing choices are identical to those of the users who travel from the bus/MRT stations, except that they move in the reverse direction.

If we only consider the pick-up demands at rental stations and do not take the drop-off bikes into account for reuse, the bicycle cycle stocks held at stations *k*1, *k*4, *k*6, *l*1, *l*2, *l*3, *l*4 and *l*5 are 575, 521, 1260, 356, 411, 384, 589 and 616, respectively. The bicycle safety stocks held at stations *k*1, *k*4, *k*6, *l*1, *l*2, *l*3, *l*4 and *l*5 are 56, 54, 83, 44, 48, 46, 57 and 58, respectively. The overall bike inventory level for the system is approximately 5158, which is overestimated as mentioned in Section 2. If we consider both the pick-up and drop-off demands at rental stations but ignore the riding trip duration, the bicycle cycle stocks held at stations *k*1, *k*4, *k*6, *l*1, *l*2, *l*3, *l*4 and *l*5 are all zero. The bicycle safety stocks held at stations *k*1, *k*4, *k*6, *l*1, *l*2, *l*3, *l*4 and *l*5 are 79, 76, 117, 63, 67, 46, 57 and 58, respectively. The overall bike inventory level for the system is approximately 629, which is underestimated as mentioned in Section 2. In addition, the value of the parameter T used to convert the replenishment lead time demand affects the inventory level significantly. If we replenish the bike inventory at rental stations twice a day (once after the morning peak period and once after the evening peak period), the total inventory level drops off dramatically.

### 3.3. Sensitivity analysis

The proposed design model described in Section 2 provides several parameters that are significant levers affecting the solution:

1. the fixed cost of locating a bike station ($f$);
2. the unit penalty cost for uncovered demands at origins and destinations ($\delta$);
3. the construction costs for building a bicycle lane ($c$ matrix);
4. the ratio of the unit walking cost to the unit bicycle riding cost $\left(\frac{\alpha}{\beta} \text{ and } \frac{\gamma}{\beta}\right)$ which represents different bicycle riding speeds.
5. the availability rate of bicycles requested at stations ($\rho$).

To illustrate how these parameters affect the solution, we first change the fixed facility costs to higher values to identify a network design with fewer bicycle stations, and to lower values to identify a network design with more bicycle stations. Second, we change the value of the unit penalty cost to zero to observe the change in routing choices and the network design. Third, we setup different bicycle lane unit costs to see the change in the network design. Fourth, we use two sets of $\frac{\alpha}{\beta}$ and $\frac{\gamma}{\beta}$ ratios to represent the situation where the bicycle riding speed is relatively lower or higher. At last, we change the availability rate to a lower value to identify a network design with more bicycle stations.

Fig. 4 illustrates the network design and routing choices when the costs of setting up involve extremely high fixed costs ranging from NTD 8 million to NTD 16 million. In comparison with the network design of the above illustrative example, the solution yields a bicycle sharing network with only seven bicycle stations located at *k*4, *k*6, *l*1, *l*2, *l*3, *l*4 and *l*5, and 20 bicycle lanes situated between these stations. Although the number of open stations decreases, all users still pick-up and drop off bicycles at the open stations that are within the coverage distance. As bicycle station *k*1 is closed, the users from origins *i*1

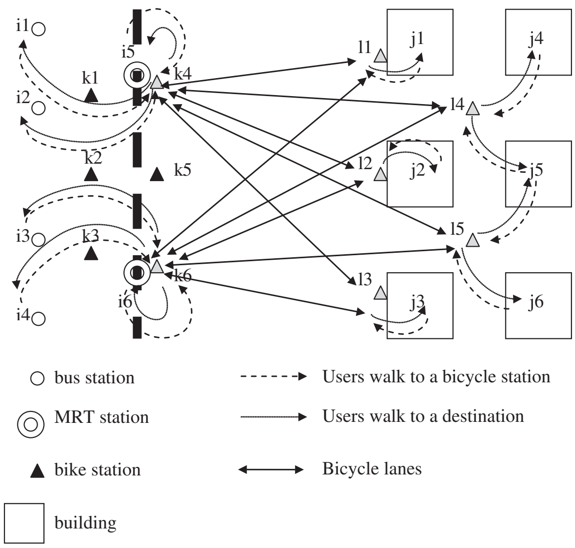


**Fig. 4.** Network design and routing choices while setting extremely high fixed station costs.

and $i2$ pick-up bicycles at station $k4$ instead and their route choices change. For example, the users traveling from $i1$ to $j3$ pick-up a bicycle at station $k4$ instead of $k1$ and switch from routes $i1$, $k1$, $l3$ and $j3$ to routes $i1$, $k4$, $l3$ and $j3$ since station $k1$ is no longer available.

Fig. 5 illustrates the network design and routing choices when the costs of setting up involve lower fixed costs that range from NTD 500,000 to NTD 1 million. In comparison with the network design of the above illustrative example, the solution yields a bicycle sharing network with nine bicycle stations located at $k1$, $k3$, $k4$, $k6$, $l1$, $l2$, $l3$, $l4$ and $l5$, and 40 bicycle lanes situated between these stations. As bicycle station $k3$ is opened, the users from origins $i3$ and $i4$ pick-up bicycles at station $k3$ instead of picking up bicycles at station $k6$ and their route choices change. For example, the users traveling from $i3$ to $j1$ pick-up a bicycle at station $k3$ instead of $k6$ and switch from routes $i3$, $k6$, $l1$ and $j1$ to routes $i3$, $k4$, $l1$ and $j1$ since station $k3$ is available now.

Fig. 6 illustrates the network design while setting the unit penalty cost of uncovered demand to zero. Compared to the network design of the illustrative example, the solution yields a network design very different from that of the illustrative example, which consists of only seven bicycle stations located at $k1$, $k3$, $k4$, $k6$, $l1$, $l2$, and $l3$, and 24 bicycle lanes situated between these stations. Without including the penalty costs of uncovered demand, the respective demands for bicycles from and to $j4$, $j5$ and $j6$ are not within a reasonable walking distance. This demonstrates that it is important to include the coverage service level in the public bicycle sharing system design.

When the model setup involves higher unit bicycle lane costs, the solution yields a bicycle sharing network with fewer bicycle stations (only seven stations) and fewer bicycle lanes in comparison with the network design of the illustrative example. The network design and routing choices are identical to those for setting up higher fixed station costs as shown in Fig. 4. When the unit bicycle lane costs are lower, the solution yields a bicycle sharing network with more bicycle stations and more bicycle lanes in comparison with the network design of the illustrative example. The network design and routing choices are identical to those with lower fixed station costs as shown in Fig. 5. This illustrates that it is important to consider both the setup costs of stations and bicycle lanes.

When a higher ratio of the unit walking cost to the unit bicycle riding cost (setting the $\frac{\alpha}{\beta}$ and $\frac{\gamma}{\beta}$ ratios with a value of 10) is assumed to represent the scenario of faster bicycle riding speeds, the solution yields a bicycle sharing network with more bicycle stations and more bicycle lanes in comparison with the network design of the illustrative example. The network design and routing choices are identical to those where lower fixed station costs are assumed as shown in Fig. 5. When a lower ratio of the unit walking cost to the unit bicycle riding cost (setting the $\frac{\alpha}{\beta}$ and $\frac{\gamma}{\beta}$ ratios with a value of 2.5) is assumed, which represents the scenario of lower bicycle riding speeds, the solution yields a bicycle sharing network with fewer bicycle stations and fewer bicycle lanes in comparison with the network design of the illustrative example. The network design and routing choices are identical to those with lower fixed station costs as shown in Fig. 4. This indicates that the inclusion of user costs has a significant impact on the network design as does the user riding speed.

Finally, we change the availability rate to 70% to identify a network with more bicycle stations. In comparison with the network design of the above illustrative example, the solution yields a bicycle sharing network with more bicycle stations and more bicycle lanes as shown in Fig 5. The lower the availability rate, the fewer bicycles that need to be stocked at stations. This is similar to a setup with lower fixed station costs.

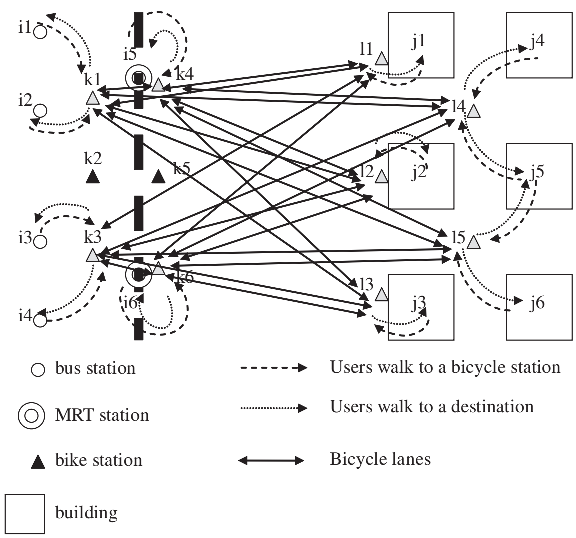


**Fig. 5.** Network design and routing choices while setting extremely low fixed station costs.

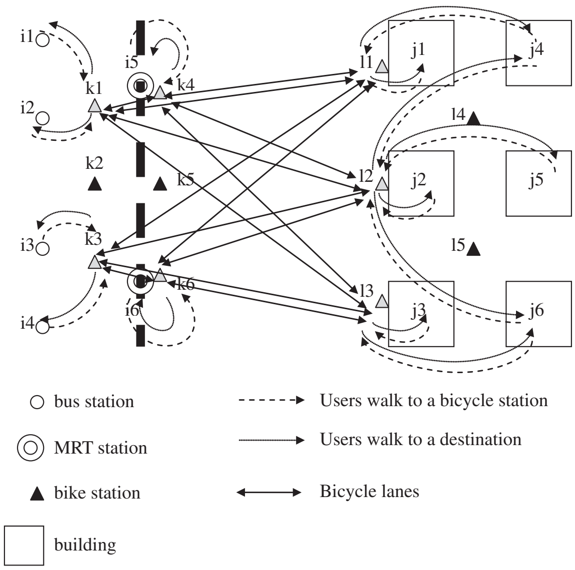


**Fig. 6.** Network design and routing choices while setting the unit penalty for uncovered demand to zero.

## 4. Summary and concluding remarks

Public bicycle systems have attracted a great deal of attention in recent years, having been used as a new inner-city transportation mode that can be integrated with existing public transit systems in many cities. This study considers both the user's and the investor's point of view in addressing the design of public bicycle systems. The level of service provided to users is measured by the demand coverage level and travel costs, while the setup costs for bike stations and bike lanes are considered in the case of the investor. The optimal design of the public bicycle sharing system requires an integrated view that encompasses the travel costs of users, the facility costs of bike stations, the setup costs of bicycle lanes, as well as the service level. This paper has developed a mathematical model that provides such an integrated view. A small network is created to illustrate the proposed model, and sensitivity analysis is conducted to test how several important parameters affect the network design and routing choices. The contributions of the paper to the literature are as follows: (1) with most of the studies related to bicycle systems in the literature having focused on promotion policy and safety issues (Martens, 2007; Aultman-Hall and Kaltenecker, 1999), we have not found any study that has addressed the network and facility location design problem for public bicycle systems from the perspective of strategic planning; (2) the literature related to public transportation has mostly focused on bus, railway, or airline services. While some studies have considered bicycles as one of the potential access modes (Rietveld, 2000; Debrezion et al., 2009), to the authors' knowledge, there are no studies that address the public bicycle transportation system. This paper first addresses the design of the public bicycle sharing system and formulates the problem as a mathematical model.

Future research would be useful in at least the following directions. First, the existing street network structure is not explicitly considered in the current model formulation. It would therefore be useful in a practical application to include how the bicycle lanes can be created on the existing street network. Second, the travel demands may vary over a day (or a replenishment lead time). It would therefore be helpful to develop a formal model incorporating demand variation and to evaluate the influence of demand variation on the system design and routing choices. Third, the calculation of the bike inventory level at rental stations is conservative since the reuse is not accounted for. It would be helpful to develop a more accurate estimate.

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