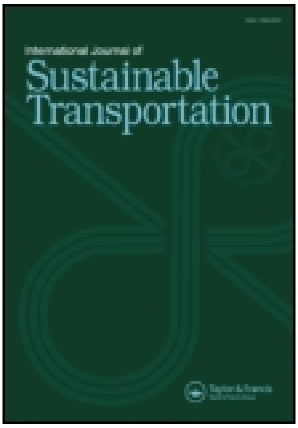

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Large-Scale Vehicle Sharing Systems: Analysis of Vélib'

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ABSTRACT
A quantitative analysis of the pioneering large-scale bicycle sharing system, *Vélib'* in Paris, France is presented. This system involves a fleet of bicycles strategically located across the network. Users are free to check out a bicycle from close to their origin and drop it off close to their destination to complete their trip. The analysis provides key insights on the functioning of such systems and serves to inform policy makers in other urban communities wanting to explore bicycle-sharing systems. This article studies the Vélib' system from several aspects, including system characteristics, utilization patterns, the connection between public transit and bicycle-sharing systems, and flow imbalances between stations. Since flow from one station to another is seldom matched by flows in the reverse direction, the bicycle fleet can become spatially imbalanced over time. This leads to lower levels of service for users who must seek alternate stations to park or check out vehicles. Using a stochastic characterization of demand and a model developed in prior work, fleet-management strategies to deal with this flow asymmetry are presented. Reliability metrics using this characterization show the performance of the system and help identify stations with capacity bottlenecks. Utilization rates also suggest that close coupling of transit and vehicle-sharing systems are beneficial.
Key Words: bicycle-sharing program, fleet management, public transit accessibility

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1. SETTING

The draft strategic plan of the United States (U.S.) Department of Transportation (DOT) seeks to diversify the transportation landscape in the U.S. (USDOT 2010) through new livable community initiatives. With coordinated policies and investment strategies across federal entities in housing and the environment, the strategic plan refocuses the goal of providing safe, clean, and sustainable transport. Amongst several initiatives that aim to reduce automobile dependence, the draft strategic plan also places greater credence on pedestrian and bicycle modes. These federal-level initiatives mirror the efforts of some local DOT's who grapple with increasing congestion and mobility problems. Though transit has long been preferred by transportation planners as a solution to congestion woes, data on traveler choices in the U.S. shows it to be a losing proposition with just 1.5% of all trips and 4.7% of all commute trips (USDOT 2001). A more recent approach to reduce automobile dependence in urban areas is through the use of shared-vehicle programs.

Shared-vehicle programs involve a network of strategically located stations that host a fleet of vehicles (bicycles, cars, or electric vehicles). In its most flexible form, users making a trip can check out a vehicle close to their origin and return it close to their destination. A shared-vehicle system can be construed as an individual mode (for short trips) or as a vital segment of an intermodal route (for longer trips). In the latter case, it serves as a vital "last-mile" connection, the lack of which dissuades potential riders. In this role, shared-vehicle systems increase transit accessibility. They are strongly aligned with integrated transit systems explored in the past that also aim to increase the catchment area of transit (Rodier et al. 2004; Shaheen and Rodier 2006; Transportation Research Board 2005).

The emphasis on livability at the federal level, growing urban mobility problems at the local level, and the potential benefits of shared vehicles in alleviating some of the problems have lead to an increased interest in these systems for the U.S. The widespread adoption of such innovations in the urban transport sector is hindered to some extent by the uncertainty surrounding the response of the traveling public to such systems. Though bicycle sharing systems exist in over 100 cities (Britton 2008), in the U.S. context, local transportation agencies and policymakers have limited precedent to aid in developing such systems for their communities. Several papers have studied the shared-vehicle system through the prism of marketing aspects (Haines and Skinner 2005), policy considerations (Shaheen, Schwartz, and Wipyewski 2004), environmental concerns (Katzev 2003), technology challenges (Transportation Research Board 2005), growth trends (Shaheen and Cohen 2007; Shaheen, Guzman, and Zhang, 2010), political issues (DeMaio and Gifford 2004), and user response (Shaheen, Meyn, and Wipyewski 2003). Quantitative studies have focused on state prediction using time series models (Borgnat et al. 2009, 2011; Kaltenbrunner et al. 2010), the use of data mining methods such as clustering (Froehlich, Neumann, and Oliver 2009) for short term prediction, and fleet management (Barth and Todd 1999; Fan, Machemehl, and Lownes 2008; Kek et al. 2009; Nair and Miller-Hooks 2011) . The focus of this article is on system configuration and operational aspects. Using trip information from the pioneering Vélib' bicycle-sharing system in Paris, France, key system design aspects are highlighted. The system accounts for as many as 120,000 trips daily (Erlanger 2008) and is considered very successful.

For users, shared bicycles offer increased travel utility through flexibility and cost. Users are free to choose their departure time, routes, and destinations. Compared to other modes, these systems are attractively priced. For the Vélib' system, a nominal annual membership fee ($37.50 as of July, 2010) entitles a member to an unlimited number of free trips that last 30 minutes or less for a year. To ensure circulation, trips lasting longer are charged for every additional half hour accrued. Vélib' also offers daily or weekly passes. Additionally, transit fare passes (Passe Navigo) also work on the Vélib', though there are no preferential fares for transit customers using Vélib'.

Increased flexibility of shared bicycles places an exceptional logistical challenge on Vélib' operators who need to ensure future short-term demand for vehicles and parking slots are met. Since flow from one station to another is seldom equal to flow in the opposing direction, the bicycle fleet can become spatially imbalanced over time. To meet near-future demand, operators must then redistribute vehicles between stations to correct this asymmetry. Since future demand is not known exactly and is highly variable (as shown in later sections), the challenges faced by operators are amplified. There is limited literature on fleet management for shared vehicles. A variety of approaches have been studied including approaches that employ simulation (Barth and Todd 1999; Kek, Cheu, and Chor 2006; Kek and Cheu 2005), user-assisted relocation (Barth, Todd, and Xue 2001), mixed-integer programming (Kek et al. 2009), and multi-stage stochastic programming with recourse (Fan et al. 2008). In prior work by the authors (Nair 2010; Nair and Miller-Hooks 2011), a mixed-integer chance-constrained program is used to generate redistribution plans to meet a target reliability level. Should resources in the system be insufficient to meet the desired levels of reliability, the model generates partial redistribution plans that utilize existing resources at lower levels of service. While generating redistribution plans is not the focus of this article, the chance-constrained program provides a probabilistic characterization of the system. Such a characterization quantifies key system performance measures that elucidate the efficiency of stations in dealing with uncertain demand. These measures allow for a quantitative, reliability-based analysis that is performed on the Vélib' system.

The objective of this article is to provide empirical evidence of the usage patterns of large-scale shared-vehicle systems and derive quantitative metrics that can be used to analyze the system. This article makes the following contributions. Key attributes of the Vélib' bicycle sharing program pertaining to system configuration and utilization are presented. The public transit–Vélib' connection is explored. Flow patterns based on observed data and the extent of flow asymmetries between stations are presented. The demand for bicycles and parking slots is characterized probabilistically. Using the chance-constrained fleet-management model from prior work (Nair 2010; Nair and Miller-Hooks 2011) described above, several reliability-based performance measures are quantified to provide insights into the workings of a successful bicycle-sharing system. Such a comprehensive quantitative analysis of shared-vehicle systems has not been previously presented in the literature.

The article is organized as follows: Section 2 presents the key design characteristics of the system pertaining to configuration, usage, and transit–Vélib' interaction. Operational considerations are presented next in section 3, and section 4 presents results of simulation analysis and reliability indicators for system performance.

2. SYSTEM DESIGN CHARACTERISTICS

As one of the largest bicycle-sharing programs in the world, Vélib' has a fleet of 20,000 bicycles spread across 1,450 stations. Figure 1 shows the map of all stations in the Paris region. All subsequent analysis of the system is based on trip data for a four-month period from March 1, 2009, through July 9, 2009, provided by the operator, JCDeaux. Due to the proprietary nature of the dataset, the scale of some descriptive statistics are not presented, though patterns suggested by the data are

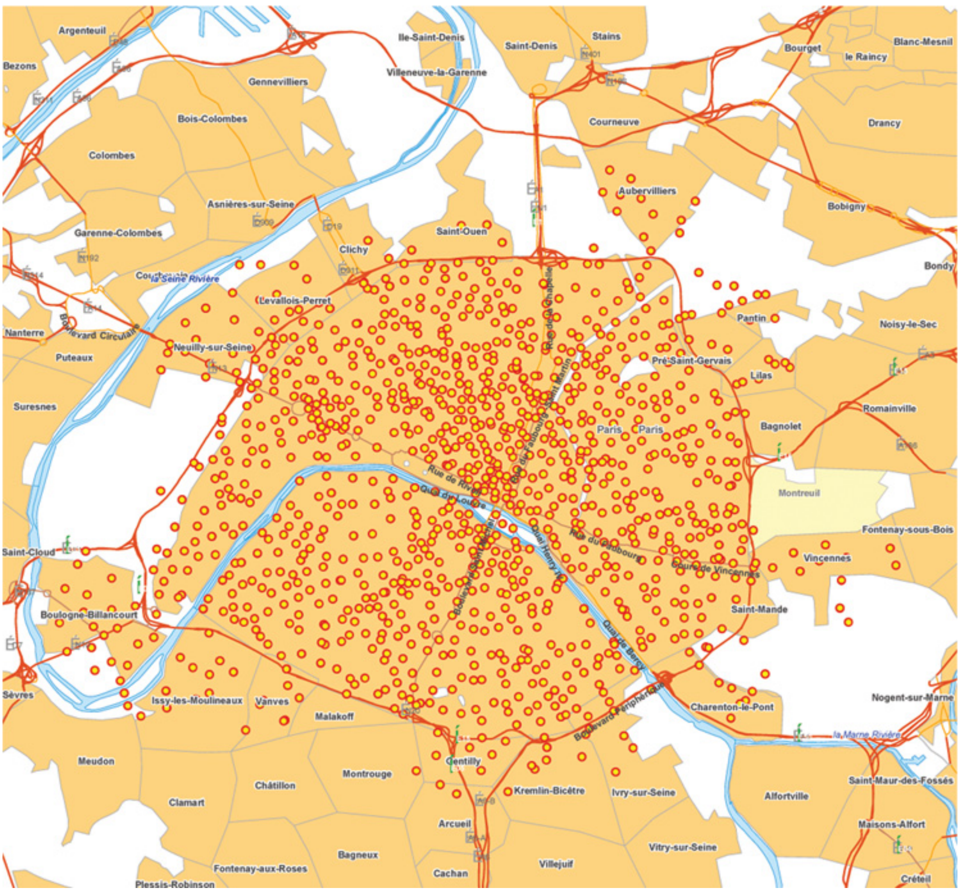

Figure 1. Spatial distribution of Vélib' stations in Paris, France. (Figure appears in color online.)

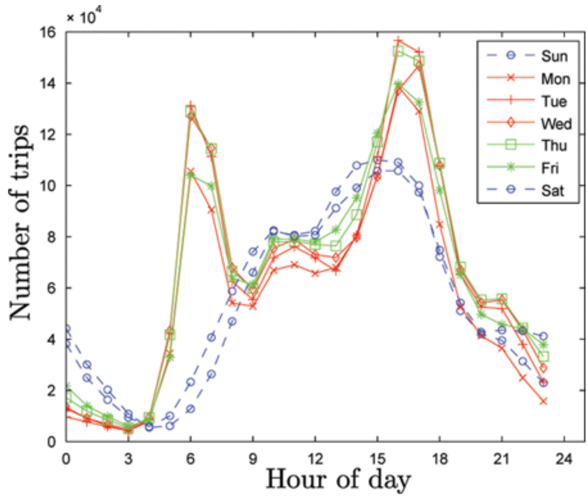

Figure 2. Temporal distribution of trips. (Figure appears in color online.)

highlighted. The system logged 10,392,808 trips during this period, at an average of 79,945 trips per day. The temporal distribution of trips over the course of a day is plotted in Figure 2, showing that usage on a weekday follows familiar travel patterns with two peaks. The morning rush hour is shorter in duration than the evening one. The weekend use is void of commuter peaks and shows a different pattern.

The system is designed for a quick turnaround of bicycles and the pricing structure discourages long-term rentals. Figure 3(a) shows the percentage of trips completed in less than 30 and 45 minutes. While most stations allow a free ride time of 30 minutes, a subset of stations, termed Vélib'+, that are located in areas of low

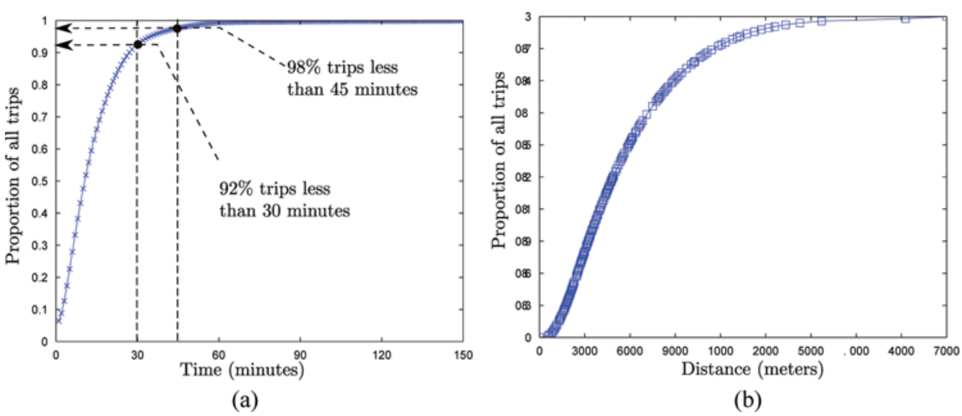

Figure 3. Cumulative distribution of travel characteristics: (a) Travel times, and (b) Travel distance. (Figure appears in color online.)

bicycle accessibility (in high altitude or in the outskirts of the city), allow an additional 45 minutes of travel time. This allows riders sufficient time to access these stations and provides incentives to users to move bicycles to these stations. Trips that last longer than either 30 minutes for a Vélib' station and 45 minutes for a Vélib'+ station incur charges. As inferred from Figure 3(a), this overage charge is paid only by a small proportion of users. A similar of trip-duration pattern has been observed for another system in Lyons, France (Jensen et al. 2010).

2.1. Transit–Vélib' Interaction

The Paris Métro is one of the busiest transit systems in the world. The system consists of 16 underground lines and 300 stations. With a high density of stations within the urban core of Paris, it offers good transit accessibility and high levels of service. For transit users, the Vélib' offers a potential segment of transit-based intermodal trips. The configuration of the metro system and Vélib' are juxtaposed to evaluate the effectiveness of modal transfers between the two systems. Accessibility is used as a proxy for the ease with which users can transfer between the systems. The nearest transit stop to each Vélib' station is determined. Figure 4 shows the distribution of distances of Vélib' stations and their corresponding closest transit stops. With the majority of the Vélib' stations being well within the pedestrian catchment area for transit (the zone defined by the maximum distance a transit user would walk to reach transit services) considered to be around 400 meters, Vélib' appears to be deeply coupled with the Paris Métro. By evaluating the utilization of Vélib' stations for the study period, the effectiveness of placing bicycle-sharing stations close to transit stops hypothesized herein can be confirmed.

From the trip data, the stations are ranked based on utilization. The 20-top ranked stations are reported in Table 1 for incoming flows and Table 2 for outgoing flows. It can be seen that highly ranked Vélib' stations typically have spatially

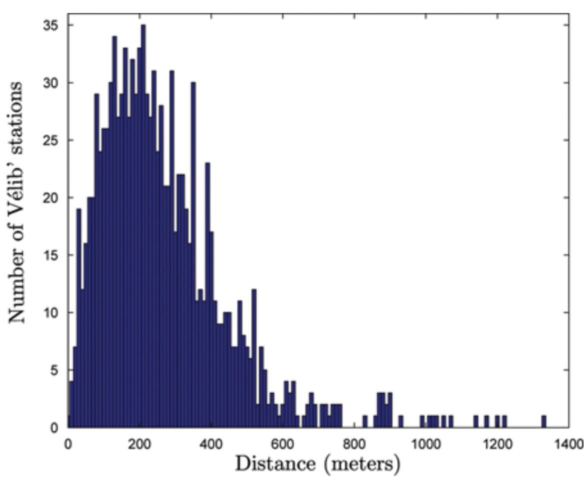

Figure 4. Distribution of distances from Vélib' stations to closest transit stops. (Figure appears in color online.)

Table 1 Top 20 Vélib' stations with highest inflows with distance to closest public transit stop.

Rank	Vélib' station	Closest metro stop (line)	Distance (m)
1	Beaubourg Rambuteau	Rambuteau (m11)	139.87
2	Faubourg du Temple Place de la Republique	Republique (m3-5-8-9-11)	118.59
3	Saint Paul Pavée	Saint-Paul (m1)	79.00
4	Quai de la Loire	Jaures (m2-5-7bis)	61.06
5	Hotel de Ville	Hotel de Ville (m1-11)	84.83
6	Les Halles Saint Eustache	Chatelet - Les Halles (rA-B-D)/m (1-4-7-11-14)	173.93
7	Traversiere	Ledru-Rollin (m8)	97.69
8	Bastille Richard Lenoir	Bastille (m1-5-8)	59.35
9	Faidherbe Chaligny	Faidherbe - Chaligny (m8)	132.59
10	Turenne Bretagne	Filles du Calvaire (m8)	67.32
11	Francs Bourgeois	Saint-Paul (m1)	443.80
12	Boulevard Voltaire	Voltaire (m9)	131.04
13	Tolbiac Nationale	Olympiades (m14)	55.92
14	Bastille	Bastille (m1-5-8)	81.18
15	Lobau	Hotel de Ville (m1-11)	83.84
16	Rivoli Saint Denis	Chatelet (m1-4-7-11-14)	145.02
17	Gare de Lyon Van Gogh	Gare de Lyon (m1-14)(rA-D)	44.68
18	Crozatier	Ledru-Rollin (m8)	113.67
19	Jacques Bonsergent	Jacques Bonsergent (m5)	138.03
20	Odeon Quatre Vents	Odeon (m4-10)	26.13

proximate transit stops and services. The exception to the rule being the Vélib' station 'Francs Bourgeois' located in the Marais district that houses several hotels.

Further evidence of the importance of the close coupling of transit and Vélib' is shown in Figure 5. Stations are represented as segments along the circumference with the size of the segment proportional to flows (both incoming and outgoing) handled by a particular station. Therefore, a large segment implies a Vélib' station with high utilization measured as the total number of incoming and outgoing trips. Wider bands indicate greater flows. The stations are further arranged in clockwise direction in descending utilization. The major Vélib' stations, as depicted by the size of the segment on the circumference, can be seen to have close correspondence to transit stops (denoted by "M"). Though not presented here, the data also reveals the existence of secondary Vélib' stations that serve as a buffer to major ones when they are full.

3. OPERATIONAL FLEET MANAGEMENT

This section of the article studies the operational characteristics of the system, with a focus on the management of the fleet of bicycles. By completing trips from one station to another, users shift bicycle inventory from one portion of the

Table 2. Top 20 Vélib' stations with highest outflows with distance to closest public transit stop.

Rank	Vélib' station	Closest metro stop (line)	Distance (m)
1	Beaubourg Rambuteau	Rambuteau (m11)	139.87
2	Faubourg Du Temple Place de la Republique	Republique (m3-5-8-9-11)	118.59
3	Saint Paul Pavee	Saint-Paul (m1)	79.00
4	Quai de la Loire	Jaures (m2-5-7bis)	61.06
5	Hotel de Ville	Hotel de Ville (m1-11)	84.83
6	Halles Saint Eustache	Chatelet - Les Halles (rA-B-D)/m (1-4-7-11-14)	173.93
7	Traversiere	Ledru-Rollin (m8)	97.68
8	Bastille Richard Lenoir	Bastille (m1-5-8)	59.35
9	Faidherbe Chaligny	Faidherbe - Chaligny (m8)	132.59
10	Turenne Bretagne	Filles du Calvaire (m8)	67.32
11	Francs Bourgeois	Saint-Paul (m1)	443.80
12	Boulevard Voltaire	Voltaire (m9)	131.04
13	Tolbiac Nationale	Olympiades (m14)	55.91
14	Bastille	Bastille (m1-5-8)	81.18
15	Gare de Lyon Van Gogh	Gare de Lyon (m1-14)(rA-D)	44.68
16	Rivoli Saint Denis	Chatelet (m1-4-7-11-14)	145.02
17	Lobau	Hotel de Ville (m1-11)	83.83
18	Crozatier	Ledru-Rollin (m8)	113.67
19	Jacques Bonsergent	Jacques Bonsergent (m5)	138.03
20	Odeon Quatre Vents	Odeon (m4-10)	26.13

network to another. For operators, this presents an exceptional logistical problem, since they must initiate corrective actions to ensure that short-term future demands are met. Shared-bicycle users expect to find a bicycle at a station close to their origin and available parking docks at station close to their destination. In the event of a 'stock out' when there is an absence of either bicycles or spaces, users need to seek out the nearest alternate station with sufficient resources. This process involves a loss in level-of-service, since travel times are increased and repeated stock-outs harm the perception of the system. The operator therefore needs to initiate operational fleet management strategies to ensure that adequate resources are available to meet *most* short-term demand scenarios. It could potentially be cost-prohibitive to satisfy all demand needs, thus operators need to maintain adequate inventories such that a predetermined level-of-service is met. For policymakers and transportation agencies developing new systems, the extent of operational cost and associated personnel is difficult to forecast. The purpose of this section is two-fold. First, the extent of flow asymmetries in the Vélib' system is highlighted. Second, a fleet management model proposed in the literature (Nair and Miller-Hooks 2011) is employed to generate fleet redistribution plans that correct imbalances.

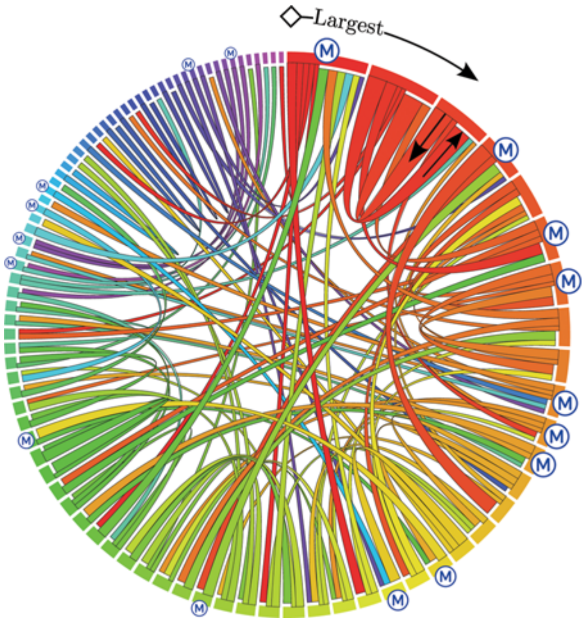

Figure 5. Average inter-station flows exceeding 10 trips/day; Vélib' stations within 100 meters of a Métro station denoted by M. (Figure appears in color online.)

3.1. Flow Asymmetry

The general mobility patterns of urban residents are reflected in the use of the Vélib' system. There is considerable directionality in movement of bicycles in both the spatial and temporal dimensions. As an example, Figure 6 highlights the sources and destinations of Vélib' traffic for a single Vélib' station during two different time periods. The width of connecting segments represents the flow as a proportion of total incoming (or outgoing) flows for that time period. In contrast to Figure 5, the trips between two stations are color coded for comparison between time periods and not arranged according to utilization. Several observations are worth noting. By comparing Figures and, inflows and outflows for the same period are from different stations. Figures and show flows during the evening rush hour with flows from a greater variety of stations than in the morning. Figure also shows two heavily utilized routes denoted by the two thick ribbons connect the segments.

3.2. Fleet Management

As demonstrated, flow from one station to another is seldom equal to flow in the opposing direction, and the bicycle fleet can become spatially imbalanced. To meet near-future demand, operators must then redistribute vehicles to correct this asymmetry. Redistribution plans can be based on historical information for services at particular stations. The two types of demand for bicycles and parking slots are related. The increase in one implies a reduction in the other. Users checking

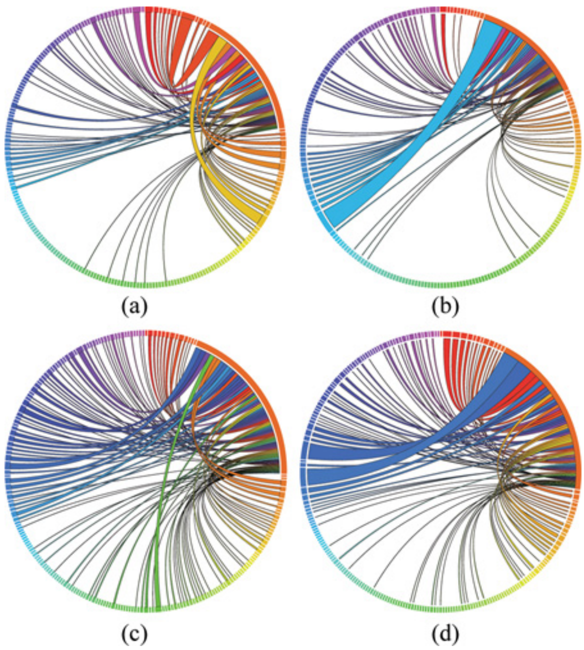

Figure 6. Flows to and from Les Halles Vélib'station: (a) Average 8–9 a.m. inflows, (b) Average 8–9 a.m. outflows, (c) Average 4–5 p.m. inflows, and (d) Average 4–5 p.m. outflows. (Figure appears in color online.)

out bicycles free up parking spaces for other users to return them. Therefore, at the station-level, the task of the operator is to maintain the vehicle inventory within acceptable limits. Too few bicycles leads to unserviced demand, as does having too many bicycles at a station. During the process of redistribution, resources from stations having excess bicycles are transferred to those approaching the "stock-out" condition. This action balances the fleet across all the stations in the network.

Three aspects limit the extent to which redistribution can be carried out. First, the task of redistributing vehicles implies operational costs for the operator. Redistribution activities can only be done judiciously to minimize operational costs. Second, during high-demand scenarios, the resources in the system could potentially be inadequate to meet future demand. Third, no amount of redistribution can address the shortfall. In these cases, there is a drop in system performance. Therefore, the operator's goal is to meet *most* future demand scenarios.

To determine what constitutes most demand scenarios, the demand for services is probabilistically characterized based on historical information. This allows operators to quantify the state of the system in a reliability measure and allows the operator to set target levels-of-service that need to be met using redistribution. A chance-constrained, mixed-integer program developed in prior work by the authors (Nair and Miller-Hooks 2011) is described briefly next. This framework allows for a probabilistic characterization of the state of the system.

Given (1) the system configuration (stations, capacities, fleet size), (2) current inventory levels at each station, (3) relocation costs, and (4) a probabilistic characterization of demand at each station, the Vélib' operator seeks to find a least-cost fleet redistribution plan such that *most* near future demand scenarios are satisfied.

The Vélib' system is described as a network of n stations with each station i having capacity C_i and bicycle inventory V_i. The fixed cost of relocating vehicles between station i and j is denoted by a_{ij}, with a penalty δ to move each additional vehicle. The operator has real-time awareness of the system and had perfect information on the state of the system. Each station i experiences two types of demand, one for bicycles, denoted by ξ_i^1, and one for spaces, denoted by ξ_i^2. Both types of demands are random variables with known probability distributions. Section 4.1 demonstrates how these probability distributions can be estimated. Since the two types of demand exhibit duality, it is more convenient to work with the "net" demand $\xi_i = \xi_i^1 - \xi_i^2$. Note that ξ_i^1 and ξ_i^2 represent the latent demand process and not individual realizations. Therefore, they can take values that exceed capacity. Let x_{ij} denote a binary decision variable indicating if vehicles are moved from i to j in anticipation of future demand and y_{ij} denote the number of bicycles moved. Additionally, define w_i and z_i as phantom resources that serve to relax the level-of-service when demand exceeds supply. These phantom resources are only used with a large penalty γ and represent the demands for vehicles and spaces that are not met. The optimal fleet redistribution plan can written as a stochastic optimization problem (Nair 2010; Nair and Miller-Hooks 2011) as

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} \left(a_{ij} x_{ij} + \delta y_{ij} \right) + \sum_{i=1}^{n} \gamma (w_i + z_i), \tag{1}$$

$$\text{s.t.} \quad \mathbb{P} \begin{pmatrix} V_i + \sum_{j=1}^{n} (y_{ji} - y_{ij}) + w_i \geq \xi_i, & i = 1, \cdots, n \\ C_i - V_i + \sum_{j=1}^{n} (y_{ij} - y_{ji}) + z_i \geq -\xi_i, & i = 1, \cdots, n \end{pmatrix} \geq p, \tag{2}$$

$$\sum_{j=1}^{n} y_{ij} \leq V_i \quad i = 1, \cdots, n, \tag{3}$$

$$\sum_{j=1}^{n} y_{ji} \leq C_i - V_i \quad i = 1, \cdots, n, \tag{4}$$

$$y_{ij} \leq M \cdot x_{ij} \quad i = 1, \cdots, n, j = 1, \cdots, n, \tag{5}$$

$$y_{ij}, w_i, z_j \in \mathbb{Z}_+ \quad i = 1, \cdots, n, j = 1, \cdots, n, \tag{6}$$

$$x_{ij} \in [0, 1] \quad i = 1, \cdots, n, j = 1, \cdots, n. \tag{7}$$

The objective (1) represents the fixed cost for relocating vehicles, the cost of moving additional vehicles, and the penalty costs for utilizing phantom resources.

Fixed cost of redistribution between two stations can be based on distance. The operator seeks to minimize the total cost of redistribution. The probabilistic level-of-service constraint (2) states that the redistribution plan must result in inventories that satisfy p proportion of all demand scenarios in the planning horizon. If available resources are insufficient, then this constraint is relaxed using phantom resources. There are capacity constraints (3) that limit the number of vehicles relocated out of a station to be no greater than the vehicles available at the start of the planning period. Similarly, there are capacity constraints (4) for slots at a station stating that the number of vehicles relocated to a station does not exceed the number of slots available. Constraints (5) relates the decision variables. All decision variables are non-negative integer valued (6), except x_{ij} which is binary (7).

The presence of the probabilistic level-of-service constraint potentially makes the feasible region non-convex. Several techniques that exploit the integral property of the random variable ξ_i can be used to overcome the non-convexity, details of which are presented in Nair (2010)Nair and Miller-Hooks (2011). For this paper, a failure apportionment procedure is utilized that transforms the joint chance-constraint into a set of linear constraints. The main idea is as follows. The level-of-service constraint (2) specifies allowable failure of the system as $1 - p$. If this system-wide failure can be distributed among individual stations in a manner that the system-wide level-of-service is met, then we can decompose the joint chance-constraint to station specific constraints. One method to distribute acceptable failure is using the Boole-Bonferroni inequality, and then apportioning the failure probabilities equally across all stations. Using this approach and expressing the constraints using the inverse marginal distributions allows the constraint (2) to be written as

$$V_i + \sum_{j=1}^{n}(y_{ji} - y_{ij}) + w_i \geq F_{\xi_i}^{-1}\left(\frac{1+p_i}{2}\right) \quad i = 1, \cdots, n, \tag{8}$$

$$-\left[C_i - V_i + \sum_{j=1}^{n}(y_{ij} - y_{ji}) + z_i\right] \leq F_{\xi_i}^{-1}\left(\frac{1-p_i}{2}\right) \quad i = 1, \cdots, n. \tag{9}$$

The resulting program is a linear mixed-integer program that be solved for each time period to generate redistribution plans. If resources are inadequate (too many bicycles are in use, or station capacities are insufficient to handle demand), the program generates *partial* redistribution plans that aim to provide the best service with available resources, though shy of the target level-of-service.

The efficacy of generating redistribution strategies using stochastic information of demand has been demonstrated in Nair (2010)Nair and Miller-Hooks (2011). Here we extend the modeling framework to characterize the system that relates system configuration, current state, and offered level-of-service. One advantage of using this framework for analysis of bicycle-sharing systems is that the model makes no assumptions on user behavior, only quantifying probabilistically patterns of flow. The system configuration is expressed as a vector of station capacities C_i

and the current state of the system captured in vehicle inventory at each station V_i. The demand is probabilistically represented by the distribution of ξ_i. Two level-of-service measures can be derived. The probability that the current number of bicycles at a station i, and the number of free parking slots can satisfy demand during the planning period can be written as

$$p_i = \mathbb{P}(-(C_i - V_i) \leq \xi_i \leq V_i) \quad i = 1, \cdots, n. \tag{10}$$

Here, p_i is termed the component-level reliability, since it measures the reliability level at the Vélib' station level. The lower bound represents the number of 'net' spaces needed to satisfy a particular realization and is the negative of the vacant spaces inventory, while the upper bound represents the "net" number of bicycles needed to meet demand. If demands for bicycles and spaces are assumed to be independent across stations, the system-wide reliability measure can be expressed as

$$p = \mathbb{P}(-(C_i - V_i) \leq \xi_i \leq V_i, i = 1, \cdots, n) = \prod_{i=1}^{n} p_i. \tag{11}$$

While conceptually important, for large systems like Vélib', the system reliability p can be very small, since it involves the product of component-level measures which are themselves very small. This may introduce numerical issues due to truncation in floating-point arithmetic. In this case, an average component reliability measure can be used to aggregate component-level measures. To calculate both reliability measures, the distribution of ξ_i must be determined. If sufficient historical data on vehicle check outs and returns are available, it can be employed to construct the distribution. However, such an approach confounds two aspects: (1) The observed demand process is not the "true" demand process, since extreme demand scenarios are not reflected in historical data; (e. g., users wanting to check out vehicles from an empty station are not represented in observed data); and (2) data on system utilization includes vehicles that are moved by the operator. These actions are done in response to the "true" demand process and have to be excluded from impacting the distribution of ξ_i. The Vélib' data distinguishes trips made by users and the operator. This allows the demand distribution to be separate from any operational decisions. The first concern requires additional assumptions that aim to approximate the demand distribution.

The system constraint on capacity limits the values that ξ_i can take. Historical information will show ξ_i to strictly fall in in the interval $(-C_i, C_i)$. A theoretical distribution that fits this data is constructed. To capture the demand process that exceeds system capacity, the theoretical demand distribution is allowed to have a support that exceeds this interval. This extension assumes that the theoretical unobserved demand process follows the same distribution as the observed process as shown in Figure 7.

In addition to the system and component reliability measures, the stochastic redistribution model provides further insights into system operations. The available system resources (bicycles, or parking slots) may be insufficient to serve

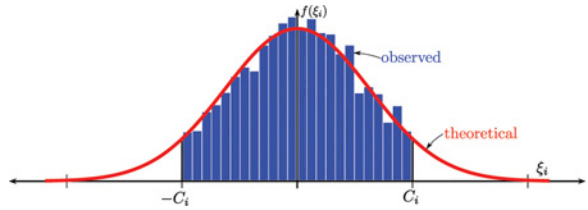

Figure 7. Theoretical unobserved demand distribution. (Figure appears in color online.)

high-demand scenarios. In these cases, the stochastic fleet redistribution model yields *partial* redistribution plans by using phantom resources w_i and z_i. The magnitude of the phantom resources indicate the shortfall of resources at a particular station. If w_i and z_i are persistently non-zero for a wide range of scenarios, the station i can be marked for capacity improvements, with the magnitude of phantom resources suggesting the extent to which capacities should be enhanced. Therefore, if an operator seeks to prioritize capacity improvements, simulation studies can be employed in conjunction with the proposed framework to determine the extent of improvements. The next section employs these concepts to Vélib'.

4. ANALYSIS

4.1. Demand Distributions

Based on historical data of 10.3 million trips, demand processes for vehicle check outs and returns are characterized for each station in the Vélib' system. The two demand processes at each Vélib' station are for bicycles and spaces. It is assumed that at a particular station demand for bicycles is independent of demand for spaces. The operator is interested in knowing the probability distribution of the number of vehicle and parking slot requests for a future time period, termed the planning period. Based on the observed trip patterns, the aggregate resource requests at each station is tabulated for various time periods. As Figure 2 shows, there are significant differences in usage patterns during the week, though weekends show different patterns. Therefore, each work week day is treated as the same. Note that no assumptions are made on trip characteristics determined by users (origins, destinations, and duration). Several distributions were fitted on the observed data on bicycle check outs and returns at each station and the demand process is closest matched by the negative binomial distribution. This distribution is related to the Poisson process, though it exhibits higher dispersion. The Poisson distribution requires the mean and variance to be equal. Negative binomial distribution has a variance that is larger than the mean. This implies that there is high variability in demand for vehicles and spaces. If a variate x follows the negative binomial distribution, then the probability of x taking a value k is given by

$$\mathbb{P}(x = k) = \binom{r + k - 1}{k} p^r (1 - p)^k, \tag{12}$$

where r and p are the parameters of the distribution. This is denoted by $x \sim NB(r, p)$. Since the two demand processes exhibit duality the distribution of the 'net' demand ξ_i is determined. The inherent assumption is that vehicle returns and withdrawals cancel each other out for short planning periods. This encodes both types of demand in one random variable. When the net demand is positive, then there is greater number of vehicle requests than returns. The redistribution plan could potentially direct bicycles to the station to meet this higher demand. When the net demand is negative, there are more requests for spaces. The operator can

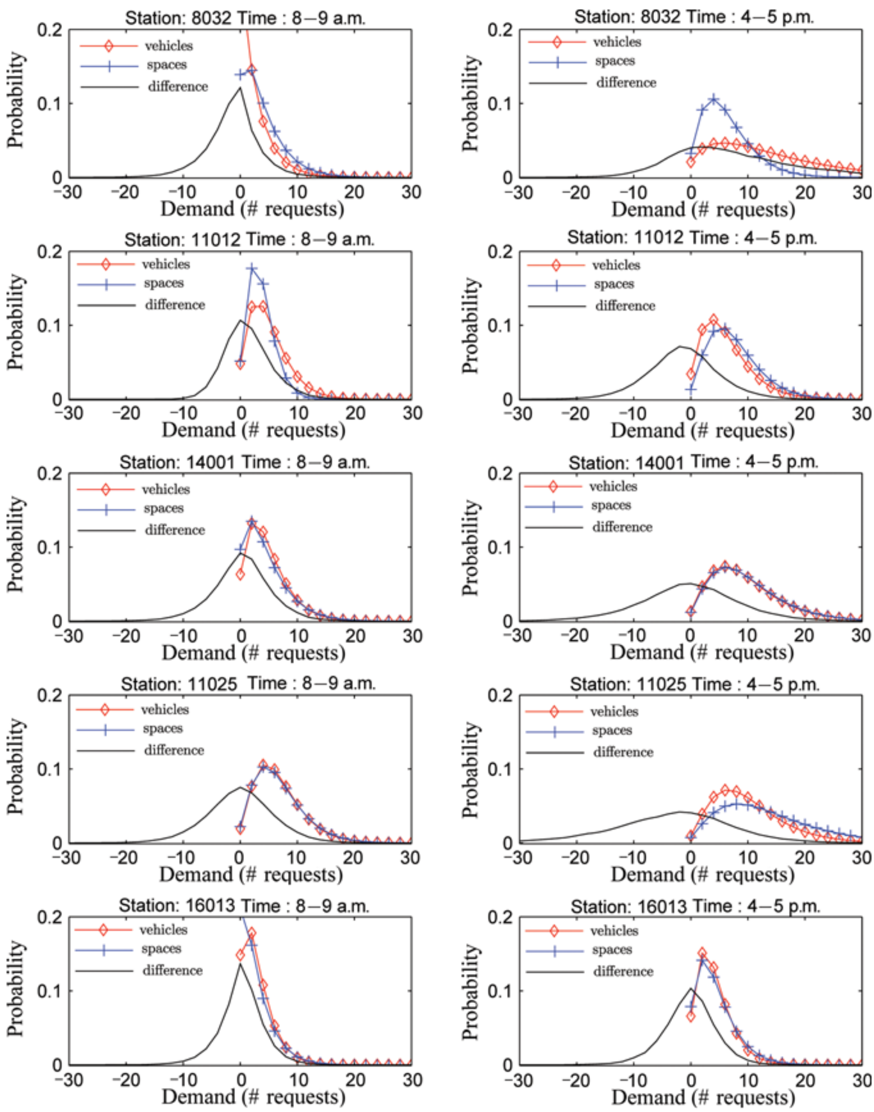

Figure 8. Theoretical demand distribution for selected *Velib'* stations. (Figure appears in color online.)

free up parking capacity by moving bicycles out of the station to other locations. For Vélib', the vehicle and demand processes were determined to be negative binomial. The difference of two negative binomially distributed random variables takes the form of a nameless four parameter distribution. It can be shown that if $x \sim NB(r_x, \ p_x)$ and $y \sim NB(r_y, \ p_y)$, then the distribution of $z = x - y$ can be expressed as:

$$\mathbb{P}(z = k) = \begin{cases} p_x^{r_x} p_y^{r_y} (1 - p_x)^k \frac{(r_x)_k}{k!} F(r_x + k, r_y; k + 1; (1 - p_x)(1 - p_y)) & k \geq 0 \\ p_x^{r_x} p_y^{r_y} (1 - p_y)^k \frac{(r_y)_{|k|}}{|k|!} F(r_x, r_y + |k|; |k| + 1; (1 - p_x)(1 - p_y)) & k < 0 \end{cases} \tag{13}$$

where

$$F(\alpha, \beta; \gamma; z) = \sum_0^\infty \frac{(\alpha)_n (\alpha)_n}{(\gamma)_n} \frac{z^n}{n!} \tag{14}$$

is the hypergeometric function (Barndorff-Nielsen, Pollard, and Shephard 2010) and $(a)_n = \Gamma(a + n)/\Gamma(a)$. There is no known closed-form solution for the inverse of this four parameter distribution, which allows for the computation of reliability. Therefore, simulation is used to numerically calculate the distribution. Figure 8 shows the fitted demand distribution for vehicles and spaces along with the simulated four parameter difference distribution for selected stations. Similar analysis on other systems like the Singapore car-sharing system (Nair and Miller-Hooks 2011), show the aggregate demand to be Skellam distributed which has lower dispersion. This indicates higher demand variability in the Vélib' system.

4.2. Reliability Measures

With the demand distributions characterized, the component reliability measures from Equation (10) are computed for all possible inventory levels. The component reliability measure for selected stations are plotted in Figure 9. The

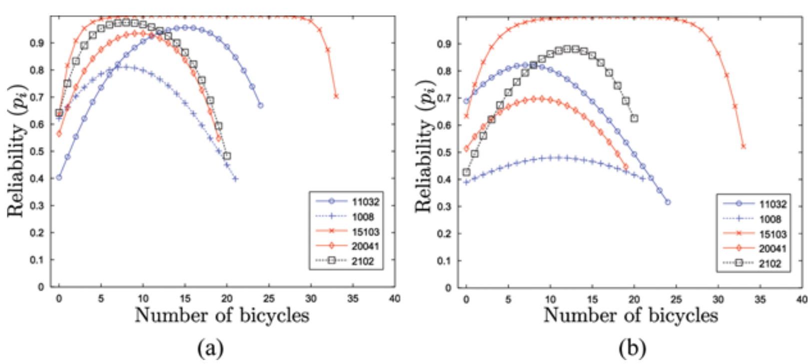

Figure 9. Component reliability p_i for selected stations and two planning periods: (a) 8–9 a.m, and (b) 4–5 p.m. (Figure appears in color online.)

Table 3. Summary of Vélib' simulation results over five days and two time periods.

Day	Period	p	Average p_i		Minimum p_i		Improvement in avg(p_i) through redistribution	Relocation cost
			No redistribution	With redistribution	No redistribution	With redistribution		
1	8–9 a.m.	0.5	0.78	0.844	0.237	0.254	0.064	98377.77
		0.7	0.78	0.84	0.237	0.254	0.06	112963.77
		0.8	0.78	0.836	0.237	0.254	0.056	123559.24
		0.9	0.78	0.831	0.237	0.254	0.05	134108.28
	4–5 p.m.	0.5	0.85	0.938	0.054	0.371	0.088	197040.06
		0.7	0.846	0.932	0.054	0.338	0.086	205577.06
		0.8	0.844	0.929	0.054	0.338	0.085	203817.72
		0.9	0.841	0.922	0.054	0.338	0.081	198245.60
2	8–9 a.m.	0.5	0.795	0.838	0.254	0.254	0.043	135791.10
		0.7	0.791	0.831	0.254	0.254	0.04	149966.10
		0.8	0.788	0.827	0.254	0.254	0.039	175499.19
		0.9	0.784	0.82	0.251	0.279	0.037	193130.72
	4–5 p.m.	0.5	0.847	0.929	0.054	0.366	0.083	287080.26
		0.7	0.844	0.922	0.054	0.356	0.078	296815.46
		0.8	0.84	0.917	0.054	0.356	0.077	297056.10
		0.9	0.834	0.909	0.054	0.356	0.074	300909.94
3	8–9 a.m.	0.5	0.796	0.824	0.269	0.279	0.028	185896.61
		0.7	0.792	0.819	0.269	0.279	0.026	221110.61
		0.8	0.789	0.814	0.269	0.279	0.025	235895.03
		0.9	0.784	0.807	0.269	0.279	0.023	279840.96
	4–5 p.m.	0.5	0.838	0.908	0.267	0.371	0.07	563506.57
		0.7	0.835	0.9	0.267	0.356	0.065	628805.26
		0.8	0.832	0.895	0.267	0.338	0.063	663414.01
		0.9	0.828	0.886	0.267	0.338	0.058	682289.76

(Continued)

Table 3. Continued.

Day	Period	p	Average p_i		Minimum p_i		Improvement in avg(p_i) through redistribution	Relocation cost
			No redistribution	With redistribution	No redistribution	With redistribution		
4	8–9 a.m.	0.5	0.794	0.814	0.237	0.279	0.02	230152.33
		0.7	0.79	0.81	0.237	0.279	0.02	265150.63
		0.8	0.787	0.806	0.237	0.279	0.019	300129.68
		0.9	0.781	0.798	0.237	0.279	0.017	361489.94
	4–5 p.m.	0.5	0.833	0.885	0.054	0.371	0.051	675432.80
		0.7	0.831	0.878	0.054	0.356	0.047	465706.61
		0.8	0.829	0.872	0.054	0.356	0.043	421459.41
		0.9	0.822	0.864	0.054	0.356	0.042	369287.47
5	8–9 a.m.	0.5	0.789	0.799	0.237	0.279	0.01	236383.41
		0.7	0.785	0.798	0.237	0.279	0.012	322601.16
		0.8	0.783	0.796	0.237	0.279	0.013	378419.25
		0.9	0.781	0.795	0.237	0.279	0.014	471935.38
	4–5 p.m.	0.5	0.824	0.872	0.338	0.371	0.048	353352.74
		0.7	0.824	0.866	0.338	0.356	0.042	319316.22
		0.8	0.825	0.863	0.338	0.356	0.038	286453.42
		0.9	0.824	0.856	0.338	0.356	0.032	257859.70

reliability curves follow an inverted-U pattern. At one end, when the station is empty, all demand for bicycles are not met leading to a drop in reliability. The reliability is not zero, since demand for returns are met. At the other end, when stations are full, all demands to return bicycles are unmet, which also reduces reliability. At intermediate inventory levels, a certain proportion of both demands are met. The extent of the reliability depends on the demand distribution. At stations with low demand levels such as station 15103, a wide range of bicycle stocks have high reliability. For the Vélib' station 1008, during the busy period of 4–5 p.m., at no level of bicycle inventory is the component reliability greater than 0.5. This implies that capacity limitations exist at this station. Note that even when stations are empty the component reliability is non-zero since the station still services demands for spaces.

4.3. Redistribution Plans

The mixed-integer program in equations (1)–(7) using the equal failure apportionment bound in equations (8) and (9) was coded in Java and solved using CPLEX 11.2. Results were imported in MATLAB for analysis. Redistribution plans for the Vélib' system are generated for various system-wide reliability levels p. Over a period of five days, redistribution is considered for two planning periods, one from 8–9 a.m. and the other from 4–5 p.m. In practice, approximately 30 relocation teams carry out relocations on an opportunistic basis. These teams employ

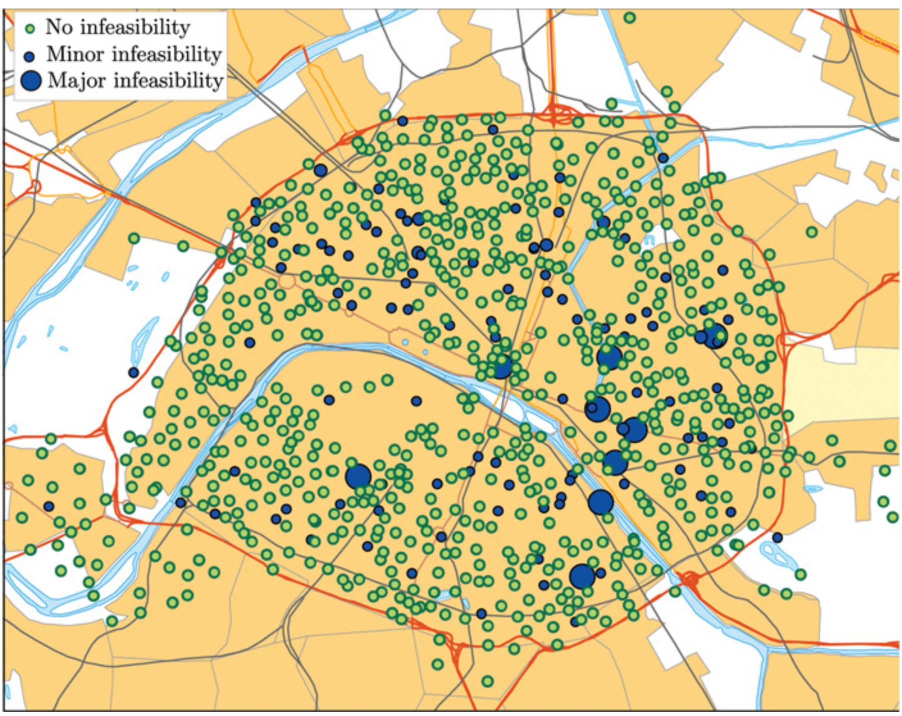

Figure 10. Stations with persistent shortages. (Figure appears in color online.)

a van capable of transporting several bicycles at a time. The model outputs where and how many bicycles to move. However, the exact pattern of movement is mostly local and not very illustrative from a systems perspective. Therefore, the focus of the analysis is on system characteristics in terms of reliability metrics. Table 3 summarizes the component reliability measures for two cases when redistribution is performed and when it is not. Redistribution is shown to improve the average component reliability in every simulated instance. Additional insights can be gleaned from the model as well by assessing the inability of stations in handling demand. When stations are unable to meet level-of-service requirements, the associated phantom variables w_i and z_i are non-zero. Stations that experience persistent low levels-of-service can be identified, and the extent of capacity improvements needed can be assessed. Figure 10 shows that the majority of stations have sufficient capacity to handle demand uncertainty. However, some key stations require capacity enhancements. Using the magnitude of the phantom resources, the operator can prioritize local improvements.

5. CONCLUSIONS

An empirical analysis of the pioneering Vélib' bicycle sharing system in Paris, France is presented. The analysis provides key insights on the functioning of such systems and serves to inform policy makers in other urban communities wanting to explore bicycle sharing systems. This article studies the Vélib' system from several aspects, including system characteristics, utilization patterns, the connection between public transit and bicycle sharing systems, and flow imbalances between stations.

The key findings of this research include the following. The usage pattern of Vélib' stations near transit stops suggests that a close coupling of transit and the bicycle sharing system can lead to higher utilization. While several factors such as visibility, availability, and location impact utilization, the causal relationship between transit proximity and Vélib' stations suggests that intermodal trips with shared-vehicle segments can provide value-addition for users. Therefore, policies that seek to integrate the two systems can be profitable. Examples of such policies include seamless fare collection, preferential fares for transit users, and prime location of shared-bicycle stations.

The need for and value of quantitative reliability metrics are demonstrated. By characterizing the demand probabilistically, the target reliability levels at the component or system level can be set and the required fleet and parking slot capacity calculated. Redistribution plans are generated based on these target levels showing the improvement in component-level reliability. The proposed measures can serve to highlight parts of the shared-bicycle network that need enhancement. Through simulation studies, portions of the network that have capacity limitations are uncovered in the case of Vélib'.

This research can be extended in several directions. Alternative characterizations of the demand processes along with associated simulation analysis frameworks can be developed. Since systems track redistributed vehicles, a framework that analyzes the costs and benefits of each redistribution action at the disaggregate level would serve to improve redistribution plans and improve the proposed model. Dynamic versions of the model, where redistribution actions in the current period take into account the benefits in several future periods, can be considered.

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