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Indirect Evidence of Network Effects in a System of Cities

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Abstract

We propose a simple model of urban growth for systems of cities, which investigate in particular the role of physical networks on interdependence of growth rates. Under the assumption of stochastic independence, a generalized non-linear formulation of recursive population growth captures spatial interactions between cities. At the second order, feedback of physical network, introduced as an abstraction of transportation network, is introduced as influencing average growth rates. Model exploration and calibration using large-scale computation and specific algorithms, yield typical characteristics of spatial interaction, such as decay distance, and their evolution in time under non-stationarity hypothesis. Furthermore, network effects are revealed by a fit improvement when adding network module.

Keywords

Urban Systems, Urban Growth, Spatial Interactions, Network Effects

Introduction

Cities are paradoxically both unsustainable and source of negative externalities, but also the best chance to reach sustainability and resilience to climate change (Glaeser 2011). The dynamics of Urban Systems at a macroscopic scale, and more precisely drivers of urban growth, are crucial to be understood to meet these potentialities. A better knowledge of how cities differentiate, interact and grow is thus a relevant topic both for policy application and from a theoretical perspective. Pumain et al. (2009) suggests that cities are incubators of social change, their fate being closely linked to the one of societies. Various disciplines have studied models of urban growth with different objectives and taking diverse aspects into account. For example, Economics are still reluctant to include spatial interactions in the models (Krugman 1998) but are extremely detailed on market processes, even for models in Economic Geography,

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whereas Geography focuses more on territorial specificities and interactions in space but will produce general conclusion with more difficulty. The example of this two disciplines shows how it is difficult to make bridges, as it needed exceptional efforts to translate from one to the other (as P. Hall did for Von Thunen work (Taylor 2016)), and therefore how it is far from evident to grasp the complexity of Urban Systems in an integrated way.

The simplest model to explain urban growth, the Gibrat model, that assumes random growth rates, has been shown by Gabaix (1999) to asymptotically produce the expected rank-size law (Zipf's law) for system of cities which is considered as one of the most regular stylized facts, at least in its generalized scaling law formulation (Nitsch 2005). Explaining urban scaling laws is closely related to the understanding of urban growth, as Bettencourt et al. (2008) suggests that these reflect underlying universal processes and that all cities are scaled version of each other. This approach however does not reflect the complex relation between economic agents for which Storper and Scott (2009) advocates. Using a bottom reconstruction of urban areas using dynamical microscopic population data, Rozenfeld et al. (2008) shows indeed that positive deviations to the rank-size law systematically exist, and that these must be an effect of spatial interaction between urban areas. Complexity approaches are good candidates to integrate these into models. Andersson et al. (2006) introduce for example a model of urban economy as a growing complex network of relations. The Evolutive Urban Theory, introduced by Pumain (1997), focuses on cities as co-evolving entities and produces explanations for growth at the system of cities level. Pumain et al. (2006) shows that scaling laws could be due to functional differentiation and diffusion of innovation between cities. The positioning regarding universality of laws is more moderate than Scaling theories, as Pumain (2012b) highlights that ergodicity can difficultly be assumed in the frame of complex territorial systems. One crucial feature of this paradigm is the importance of interactions between agents, generally the cities, to produce the emergent patterns at the scale of the system. Pumain and Sanders (2013) has investigated the advantages of Agent-based models compared to more classical equation systems, and this methodological aspect is in accordance with the theoretical positioning, as it allows to take into account the heterogeneity of possible interactions, the geographical particularities, and to naturally translate emergence between levels and render multi-scale patterns.

In this paper we aim at exploring further the assumption, central to Pumain's Evolutive Urban Theory, that spatial interactions between cities are significant drivers of their growth. More precisely, we consider both abstract interactions and flow interactions mediated through the physical networks, mainly transportation network. We extend existing models accordingly. Our contribution is twofold: (i) we show that very basic interaction models based on population only can be fitted to empirical data and that fitted parameter values are directly interpretable; and (ii) we introduce a novel methodology to quantify overfitting in models of simulation, as an extension of Information Criteria for statistical models, which applied to our calibrated models confirms that fit improvement is not only due to additional parameters, but that the extended model effectively capture more information on system processes. This will unveil network effects in an indirect way. We first review modeling approach to urban growth based on spatial interactions.

Urban Growth and Spatial Interaction First of all, we must precise that we consider only models at the macro-scale, ruling out the numerous and rich approaches at the mesoscopic scale, that include for exemple Cellular Automata models, models of Urban Morphogenesis or Land-use change models. We also naturally rule out economics models that do not include explicitly spatial interactions. Several models of Urban Growth at the macro scale have insisted on the role of space and spatial interactions. Bretagnolle et al. (2000) proposed a spatial extension of the Gibrat model. The gravity-based interaction model that Sanders (1992) use to apply concept of Synergetics to cities is also close to this idea of interdependent urban growth, contained physically in the phenomenon of migration between cities. A more refined extension with economic cycles and innovation waves was developed by Favaro and Pumain (2011), yielding a system dynamics version of the core of Simpop models (Pumain 2012a). This family of models have started with a toy-model based on economic interactions between cities as agents, that yield hierarchical patterns at the scale of the system (Sanders et al. 1997). Later, the Simpop2 model, still based on distance interaction for commercial exchanges, including successive innovation waves, unveiled structural differences between the European and the US Urban Systems (Bretagnolle and Pumain 2010). The SimpopLocal model (Pumain and Reuillon 2017) is used to show the emergence of initial settlement patterns. The Marius model (Cottineau 2014) couples population and economic growth with cities interaction, allowing to accurately reproduce real trajectories on the former Soviet Union after calibration with multi-modeling of processes.

Urban Growth and Transportation Networks Under similar assumptions of previously reviewed models, the inclusion of transportation networks has been rarely pursued, contrary to the mesoscopic scale at which relations between networks and territories have been widely studied by Luti models for example (Chang 2006). Network growth models (Xie and Levinson 2009), prolific in Economics and Physics, can not be utilized to explain urban growth. Bigotte et al. (2010) studies an optimization model for network design combining the effects of urban hierarchy and of transportation network hierarchy. Baptiste (1999) has modeled dynamical interplay between network links capacity and city growth on a subset of French city system. The SimpopNet model (Schmitt 2014) goes a step further in modeling the co-evolution between cities and transportation networks, as it allows new network links to be created in time. These examples shows the difficulty of coupling these two aspects of urban systems in models of growth, and we will for this reason take into account network effects in a simplified way as detailed further.

The rest of this paper is organized as follows : our model is introduced and formally described in next section; we then describe results obtained through exploration and calibration of the model on data for French cities, in particular the unveiling of network effects significantly influencing growth processes thanks to a novel methodology introduced. We finally discuss the implications of these results.

Model Description

Rationale Some confusion may arise when surveying at stochastic and deterministic models of urban growth. To what extent is a proposed model "complex" and is the simulation of stochasticity necessary ? Concerning Gibrat model and most of its extensions, independence assumptions and linearity produce a totally predictable behavior and thus not complex in the sense of exhibiting emergence, in the sense of weak emergence (Bedau 2002). In particular, the full distribution of random growth models can be analytically at any time Gabaix (1999), and in the case of studying only first moment, a simple recurrence relation avoids to proceed to any Monte-Carlo simulation. Under these assumptions, it is natural to work with a deterministic model, as it is done for example for the Marius model Cottineau (2014). We will work under that hypothesis, capturing complexity through non-linearity. We work on simple territorial systems assumed as regional city systems, in which cities are basic entities. The time scale corresponds to the characteristic scale associated to this spatial scale, i.e. around one or two centuries. Spatial interactions will be captured through gravity-type interactions, this simple formulation having the advantage of being simple and of capturing the first law of Tobler, namely that interaction strength fades with distance. Other approaches introduced recently perform similarly at this scale (Masucci et al. 2013).

Model description We consider on a deterministic extension of the Gibrat model, what is equivalent to consider only expectancies in time. Let $\vec{P}(t) = (P_i(t))_i$ be the population of cities in time. Under Gibrat independence assumptions, we have $\text{Cov}[P_i(t), P_j(t)] = 0$. A linear extended version would write $\vec{P}(t+1) = \mathbf{R} \cdot \vec{P}(t)$ where $\mathbf{R}$ is an independent random matrix of growth rates (identity in the original case). It yields directly thanks to the independence assumption that $\mathbb{E}\left[\vec{P}(t+1)\right] = \mathbb{E}[\mathbf{R}] \cdot \mathbb{E}\left[\vec{P}\right](t)$. We generalize this linear relation to a non-linear relation that allows to be more consistent with model interpretation and more flexible. Denoting $\vec{\mu}(t) = \mathbb{E}\left[\vec{P}(t)\right]$, we take $\vec{\mu}(t+1) = \Delta t \cdot f(\vec{\mu}(t))$. Note that in that case, stochastic and deterministic versions are not equivalent anymore, precisely because of the non-linearity, but we stick to a simple deterministic version for the sake of simplicity. The specification of the interdependent growth rate is given by

$$f(\vec{\mu}) = r_0 \cdot \mathbf{Id} \cdot \vec{\mu} + \mathbf{G}\left(\vec{\mu}\right) \cdot \vec{1} + \vec{N}\left(\vec{\mu}\right) \tag{1}$$

where $\vec{1}$ is the column vector full of ones, and $\mathbf{G} = G_{ij} = w_G \cdot \frac{V_{ij}}{<V_{ij}>}$ such that the interaction potential V_{ij} follows a gravity-type expression given by

$$V_{ij} = \left(\frac{\mu_i \mu_j}{\sum {\mu_k}^2}\right)^{\gamma_G} \cdot \exp\left(-d_{ij}/d_G\right) \tag{2}$$

The network effect term $\vec{N}$ is given by

$$N_i = w_N \cdot \sum_{kl} \left(\frac{\mu_k \mu_l}{\sum \mu}\right)^{\gamma_N} \exp\left(-d_{kl,i}\right)/d_N \tag{3}$$

where $d_{kl,i}$ is distance to shortest path between k, l computed with slope impedance ($Z = (1 + \alpha/\alpha_0)^{n_0}$ with $\alpha_0 \simeq 3$). The first component is the pure Gibrat model, that we obtain by setting the weights $w_G = w_N = 0$. The second component captures direct interdependencies between cities, under the form of a separable gravity potential such as the one used in Sanders (1992). The rationale for the third term, aimed at capturing network effects by expressing a feedback of network flow between cities k, l on the city i. Intuitively, a demographic and economic flow physically transiting through a city or in its surroundings is expected to influence its development (through intermediate stops e.g.), this effect being of course dependent on the transportation mode since a high speed line with few stops will skip most of the traversed territories. Under assumption of non-stationarity, temporal evolution of fitted parameters corresponding to this feedback should therefore contain information on the evolution of transportation modes.

Table 1. Model Parameters summary.

Parameter	Notation	Process	Interpretation	Range
Growth Rate	r_0	Endogenous growth	Growth rate	$[0, 1]$
Gravity weight	w_G	Direct interaction	Max average rate	$[0, 1]$
Gravity gamma	γ_G	Direct interaction	Level of hierarchy	$[0, +\infty]$
Gravity decay	d_G	Direct interaction	Interaction range	$[0, +\infty]$
Feedback weight	w_N	Flows effect	Max average rate	$[0, 1]$
Feedback gamma	γ_N	Flows effect	Level of hierarchy	$[0, +\infty]$
Feedback Decay	r_0	Flows effect	Network effect range	$[0, +\infty]$

Model Parameter Space

Data

Population data We work with the Pumain-INED historical database for French Cities Pumain and Riandey (1986), which give populations of *Aires Urbaines* (INSEE definition) at time intervals of 5 years, from 1831 to 1999 (31 observations in time). The latest version of the database integrates Urban Areas, allowing to follow them on long time-period, according to Bretagnole's long time cities ontology Bretagnolle (2009), that constructs a definition of cities as evolving entities which boundaries are not fixed in time.

Physical flows As stated above, this modeling exercise focuses on exploring the role of physical flows, whatever the effective shape of the network. We choose for this reason not to use real network data which is furthermore not easily available at different time periods, and physical flows are assumed to take the geographical shortest path that include terrain slope. It avoids geographical absurdities such as cities with a difficult access having an overestimated growth rate. Using the 1km resolution Digital Elevation Model openly available from *Institut Gographique National* National, we construct an impedance field of the form

$$Z = \left(1 + \frac{\alpha}{\alpha_0}\right)^{n_0}$$

We take fixed parameter values $\alpha_0 = 3$ (corresponding to approximatively the real world value of a 5% slope) and $n_0 = 3$.

A semi-parametrized model Our model is assumed as hybrid as it relies on semi-parametrization on real data. It could be possible to be a full toy-model, initial configuration and physical environment being constructed as synthetic data. As Raimbault (2016a) points out, it should even be a step in an extensive study of model, using synthetic data to unveil sensibility of dynamics to meta-parameters defining setup. This enterprise is however out of the scope of this paper, as we aim here to extract advanced stylized facts from a dataset, and we focus therefore on the semi-parametrized version of the model.

Model Evaluation

We work on an explanatory rather than an exploratory model, and indicators to evaluate model outputs are therefore not linked to a performance of trajectories or obtained final states, but to a distance to phenomenon we want to explain, i.e. the data. We use therefore the following complementary indicators :

- Logarithms of mean-square error
- Mean-square error on logarithms

Results

Stylized facts

Basic stylized facts can be extracted from such a database, as it has already been widely explored in the literature Guérin-Pace and Pumain (1990).

We show in figure 1 mean time-series correlation as a function of distance

More precisely, we consider overlapping 50 years time-windows finishing respectively in (1881,1906,1931,1962,1999) and compute on each, for each couple of cities (i, j), an estimated correlation $\hat{\rho}_{ij} = \rho\left[\Delta\tilde{P}_i, \Delta\tilde{P}_j\right]$

Model Exploration

Implementation Data preprocessing, result processing and models profiling are implemented in `R`. For performances reasons and an easier integration into the OpenMole software for model exploration described by Reuillon et al. (2013), a `scala` version was also developed. The typical question of trade-off between implementation performance and interoperability appeared quickly as an issue, as a blind exploration and calibration can difficultly provide useful thematic conclusions for that kind of model. Finding an improvement in model fit among one parameter dimension is significant if the geographical situation is visualized and the improvement is confirmed as reasonable and not an absurdity.

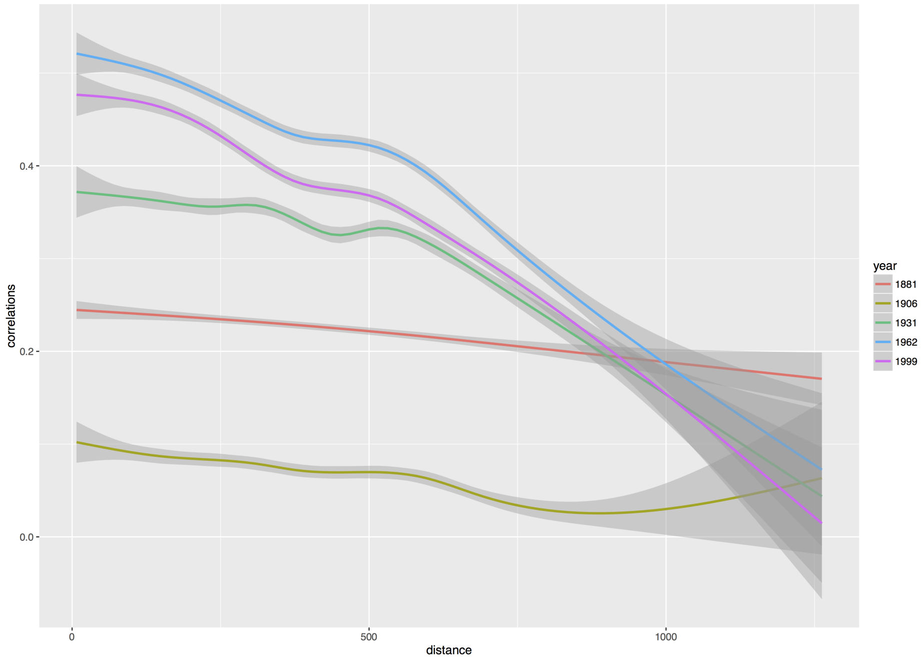

Figure 1. Time-series correlations as a function of distance. Solid line correspond to smoothed correlations, computed between each normalized population time-series, on successive periods.

Behavior Patterns

Calibrating the Gravity Model

Batty and Mackie (1972)

saturation : better for logmse == error on big cities : a priori the interaction range would change with size - as potential development for future model

We propose to interpret the distance decay parameter the following way. Let fix an arbitrary fraction α and typical ranges for a local urban system d_L and for a long range urban system d_R. Seeing d_G as the decay at which the fraction α between the migration flow of these two systems is realized, we have $d_G = \frac{d_R - d_L}{|\ln \alpha|}$

Non-stationarity in time

Calibration using Genetic Algorithms The optimization problem associated to model calibration does not present features allowing an easy solving (closed-form of a likelihood function, convexity or sparsity of the optimization problem, etc.), we must rely on alternative techniques to solve it. Brute force grid search is rapidly limited by the dimensionality curse

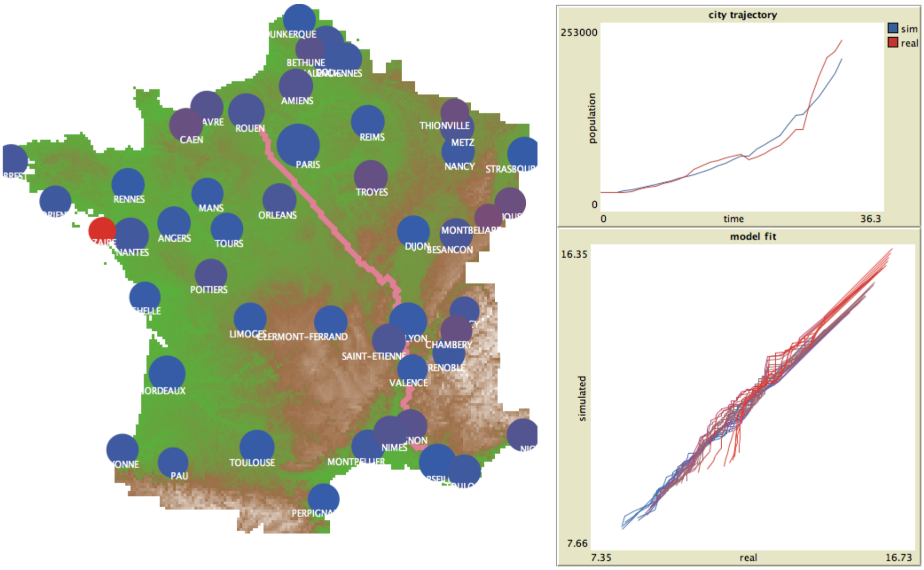

Figure 2. Example of output of the model. The graphical interface allows to explore interactively on which cities changes operate after a parameter change, what is necessary to interpret raw calibration results.

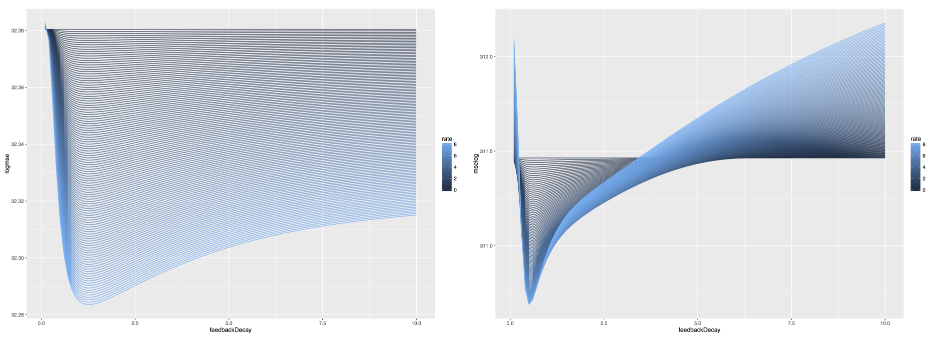

Figure 3. Evidence of network effects revealed by model exploration. Feedback with fixed gravity : first evidences of network effects ; confirmed with effect of α_0

Unveiling Network Effects

Full Model Calibration

We focus in this last experiment on quantifying the "performance" of the model, taking into account its predictive abilities, but also its structure. More precisely, we want to tackle the issue of overfitting, which has been for long recognized in Machine Learning

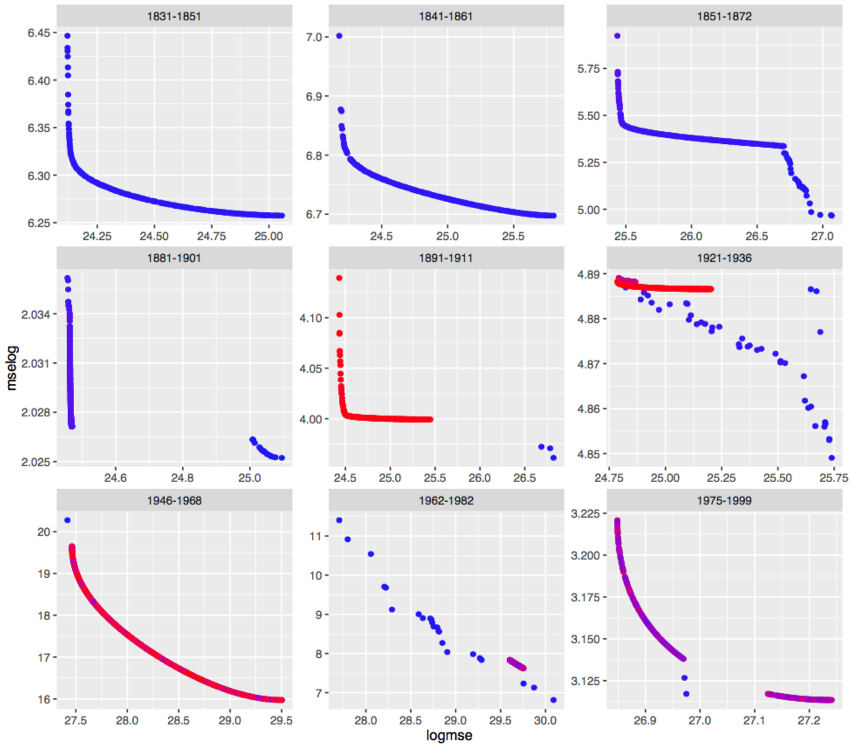

Figure 4. Calibrating the Gravity Model. Pareto-front on successive periods, with effect of distance decay parameter.

for example Dietterich (1995), but for which there is a lack of methods for models of simulation.

Quantifying overfitting : Empirical AIC Bastani et al. (2017) : model extraction - very similar, with different purpose

A large part of statistical models allow to compute tools for model comparison and selection, in particular to take into account possible overfitting due to additional degrees of freedom. The Akaike Information Criterion provides the gain in information between two models, and many extensions have been proposed since .Note that cross-validation type of methods are not suitable to our case because of the small size of the dataset. We can formalize the proposed method based on the intuitive idea of approaching models of simulation by statistical models and using the corresponding AIC under certain validity conditions. Let (X, Y) be the data and observations. Computational models are functions $(X, \alpha_k) \mapsto M^{(k)}_{\alpha_k}(X)$ mapping data values to a random variable. What is seen as data and parameters is somehow arbitrary but is separated in the formulation as corresponding dimensions will have different roles. We assume the model has been fitted to data in the

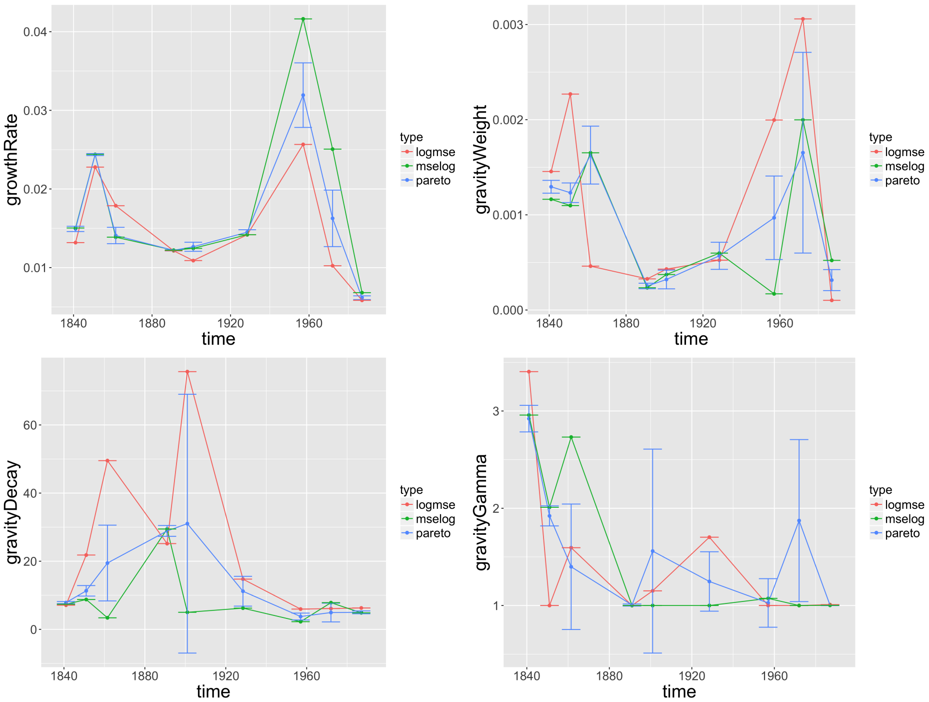

Figure 5. Interaction ranges. Values of optimal interaction range parameters.

sense that an heuristic has been used to approximate $\alpha_k^* = \text{argmin}_{\alpha_k} \left\| M_{\alpha_k}^{(k)}(X) - Y \right\|$. The gain of information between two computational models is not directly accessible and we propose an indirect way, through the fitting of statistical models. For all computational model, a large set of statistical models with similar degree of freedom are fitted.

Let S_k be the statistical models fitting best $M_{\alpha_k^*}^{(k)}(X)$. We can compute

$$\Delta D_{KL}\left(M^{(1)}|M^{(2)}\right) = \Delta D_{KL}\left(S^{(1)}|S^{(2)}\right) + \left[\Delta D_{KL}\left(S^{(2)}|M^{(1)}\right) + \Delta D_{KL}\left(S^{(1)}|M^{(2)}\right)\right]$$

Under certain assumptions, the order of magnitude of second term appears to be negligible. More precisely, with $s^{(k)} = M^{(k)} - S^{(k)}$, we have

$$\left\| \int f \log\left(\frac{S^{(k)}}{M^{(k')}}\right) \right\| = \left\| \int f \log\left(1 + \frac{s^{(k)}}{M^{(k')}}\right) \right\|$$

Discussion

Theoretical implications

- how this example illustrates well the theory

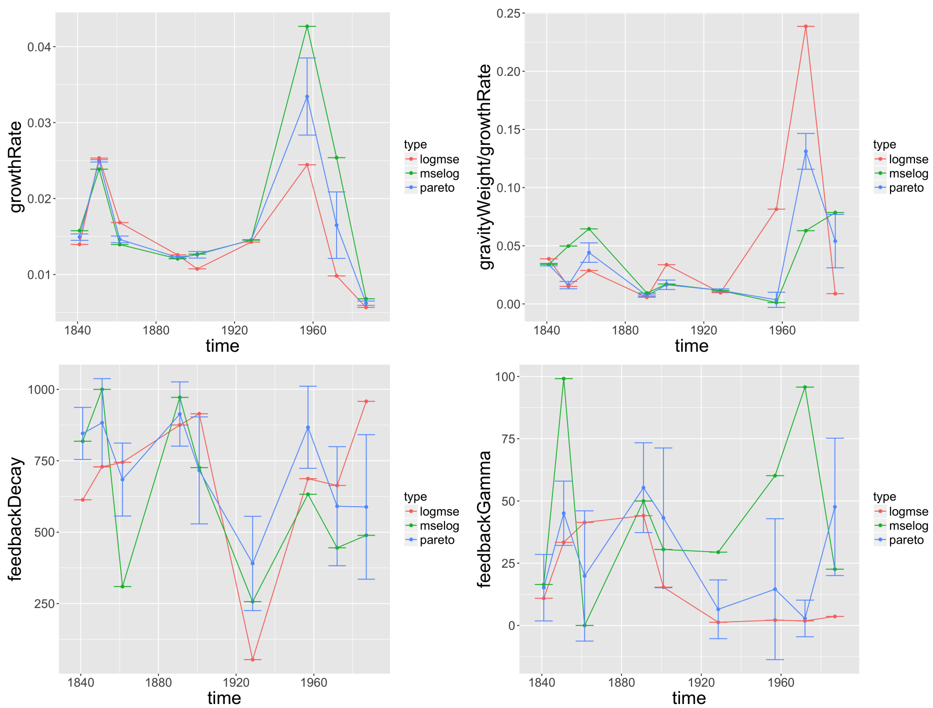

Figure 6. Pareto fronts for bi-objective calibration. Steady-state populations are obtained using Island Genetic Algorithm embedded in `OpenMole`, with parameters, and on independent time periods as detailed in text. We show corresponding parameter values for w_N, for the gravity-only model (Top) and the full model (Bottom).

We propose to support our hypothesis that *physical transportation networks are necessary to explain the morphogenesis of territorial systems* (aka *Network Necessity*) by showing on a relatively simple case that the integration of physical networks into some models effectively increase their explanative power (being careful on the precise definition of model improvement to avoid overfitting).

Methodological implications

Further developments

Specificity of the Urban System - not tested on other system of cities
The model has not yet been tested on other urban systems and other temporalities.

Towards co-evolutive models of cities and transportation networks - no network data -¿ development with train ?
Our focus on network effects remains quite limited since (i) we do not consider an effective infrastructure but abstract flows only, and (ii) we do not take into account the

possible network evolution, due to technical progresses (Bretagnolle et al. (2000)) and infrastructure growth in time. An ambitious but necessary development would be the inclusion of both in a model of co-evolution between urban growth and transportation network growth, in order to investigate to what extent the refinement of network spatial structure and network dynamics can improve the explanation of urban system dynamics. It has been shown by Raimbault (2016b) that disciplinary compartmentalization may be at the origin of the relative absence of such type of models in the literature.

- bayesian iterative formulation ? (mcmc) - how does formulation influence ? equivalence in certain cases between stoch-cov and interdependent expectancies ? - Q under which conditions we can impose a covariance structure that produces interdependencies between expectancies ? seems to be a very broader question, need to do some thinking on that.

Conclusion

References

Andersson C, Frenken K and Hellervik A (2006) A complex network approach to urban growth. *Environment and Planning A* 38(10): 1941.

Baptiste H (1999) *Interactions entre le système de transport et les systèmes de villes: perspective historique pour une modélisation dynamique spatialisée*. PhD Thesis, Centre d'études supérieures de l'aménagement (Tours).

Bastani O, Kim C and Bastani H (2017) Interpretability via Model Extraction. *ArXiv e-prints* .

Batty M and Mackie S (1972) The calibration of gravity, entropy, and related models of spatial interaction. *Environment and Planning A* 4(2): 205–233.

Bedau M (2002) Downward causation and the autonomy of weak emergence. *Principia: an international journal of epistemology* 6(1): 5–50.

Bettencourt LM, Lobo J and West GB (2008) Why are large cities faster? universal scaling and self-similarity in urban organization and dynamics. *The European Physical Journal B-Condensed Matter and Complex Systems* 63(3): 285–293.

Bigotte JF, Krass D, Antunes AP and Berman O (2010) Integrated modeling of urban hierarchy and transportation network planning. *Transportation Research Part A: Policy and Practice* 44(7): 506–522.

Bretagnolle A (2009) *Villes et réseaux de transport : des interactions dans la longue durée, France, Europe, États-Unis*. Hdr, Université Panthéon-Sorbonne - Paris I. URL http://tel.archives-ouvertes.fr/tel-00459720.

Bretagnolle A, Mathian H, Pumain D and Rozenblat C (2000) Long-term dynamics of european towns and cities: towards a spatial model of urban growth. *Cybergeo: European Journal of Geography* .

Bretagnolle A and Pumain D (2010) Comparer deux types de systèmes de villes par la modélisation multi-agents. *Qu'appelle t-on aujourd'hui les sciences de la complexité? Langages, réseaux, marchés, territoires* : 271–299.

Chang JS (2006) Models of the relationship between transport and land-use: A review. *Transport Reviews* 26(3): 325–350.

Cottineau C (2014) *L'évolution des villes dans l'espace post-soviétique. Observation et modélisations.* PhD Thesis, Université Paris 1 Panthéon-Sorbonne.

Dietterich T (1995) Overfitting and undercomputing in machine learning. *ACM computing surveys (CSUR)* 27(3): 326–327.

Favaro JM and Pumain D (2011) Gibrat revisited: An urban growth model incorporating spatial interaction and innovation cycles. *Geographical Analysis* 43(3): 261–286.

Gabaix X (1999) Zipf's law for cities: an explanation. *Quarterly journal of Economics* : 739–767.

Glaeser E (2011) *Triumph of the city: How our greatest invention makes us richer, smarter, greener, healthier, and happier*. Penguin.

Guérin-Pace F and Pumain D (1990) 150 ans de croissance urbaine. *Economie et statistique* 230(1): 5–16.

Krugman P (1998) Space: the final frontier. *The Journal of Economic Perspectives* 12(2): 161–174.

Masucci AP, Serras J, Johansson A and Batty M (2013) Gravity versus radiation models: On the importance of scale and heterogeneity in commuting flows. *Physical Review E* 88(2): 022812.

National IG (????) Bd alti, http://professionnels.ign.fr/bdalti.

Nitsch V (2005) Zipf zipped. *Journal of Urban Economics* 57(1): 86–100.

Pumain D (1997) Pour une théorie évolutive des villes. *Espace géographique* 26(2): 119–134.

Pumain D (2012a) Multi-agent system modelling for urban systems: The series of simpop models. In: *Agent-based models of geographical systems*. Springer, pp. 721–738.

Pumain D (2012b) Urban systems dynamics, urban growth and scaling laws: The question of ergodicity. In: *Complexity Theories of Cities Have Come of Age*. Springer, pp. 91–103.

Pumain D, Paulus F and Vacchiani-Marcuzzo C (2009) Innovation cycles and urban dynamics. *Complexity perspectives in innovation and social change* : 237–260.

Pumain D, Paulus F, Vacchiani-Marcuzzo C and Lobo J (2006) An evolutionary theory for interpreting urban scaling laws. *Cybergeo: European Journal of Geography* .

Pumain D and Reuillon R (2017) The simpoplocal model. In: *Urban Dynamics and Simulation Models*. Springer, pp. 21–35.

Pumain D and Riandey B (1986) Le fichier de l'ined. *Espace, populations, sociétés* 4(2): 269–277.

Pumain D and Sanders L (2013) Theoretical principles in interurban simulation models: a comparison. *Environment and Planning A* 45(9): 2243–2260.

Raimbault J (2016a) Generation of correlated synthetic data. In: *Actes des Journees de Rochebrune 2016.*

Raimbault J (2016b) Models coupling urban growth and transportation network growth: An algorithmic systematic review approach. *arXiv preprint arXiv:1605.08888* .

Reuillon R, Leclaire M and Rey-Coyrehourcq S (2013) Openmole, a workflow engine specifically tailored for the distributed exploration of simulation models. *Future Generation Computer Systems* 29(8): 1981–1990.

Rozenfeld HD, Rybski D, Andrade JS, Batty M, Stanley HE and Makse HA (2008) Laws of population growth. *Proceedings of the National Academy of Sciences* 105(48): 18702–18707.

Sanders L (1992) *Système de villes et synergétique*. Economica.

Sanders L, Pumain D, Mathian H, Guérin-Pace F and Bura S (1997) Simpop: a multiagent system for the study of urbanism. *Environment and Planning B* 24: 287–306.

Schmitt C (2014) *Modélisation de la dynamique des systèmes de peuplement: de SimpopLocal à SimpopNet.* PhD Thesis, Paris 1.

Storper M and Scott AJ (2009) Rethinking human capital, creativity and urban growth. *Journal of economic geography* 9(2): 147–167.

Taylor PJ (2016) A polymath in city studies. In: *Sir Peter Hall: Pioneer in Regional Planning, Transport and Urban Geography*. Springer, pp. 11–20.

Xie F and Levinson D (2009) Modeling the growth of transportation networks: A comprehensive review. *Networks and Spatial Economics* 9(3): 291–307.