Models for the Co-evolution of Cities and Networks

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1 Introduction

2 Specification of co-evolution models

This section extends the logic of integrating a system of cities with a transportation network, which has been pursued in a static way for network behavior in the interaction model developed and explored in section **??**, to propose a *macroscopic model of co-evolution for systems of cities.*

2.1 Rationale

This first approach relies in a direct extension of the interaction model within a system of cities described in chapter **??**, at a macroscopic scale with an ontology typical to systems of cities. For the sake of simplicity, we still stick to an unidimensional description of cities by their population.

Concerning network growth, we propose also to stay at a relatively aggregated and simplified level, allowing to test growth heuristics at different levels of abstraction. In order to be flexible on model mechanisms, diverse processes can be taken into account, such as direct interactions between cities, intermediate interactions through the network, the feedback of network flows and a demand-induced network growth.

Empirical characteristics emphasized by [**?**] for the French railway network suggest the existence of feedbacks of network use, or of flows traversing it, on its persistence and its development, whose properties have evolved in time: a first phase of strong development would correspond to an answer to a high need of coverage, followed by a reinforcement of main link and the disappearance of weakest links.

The coupling between cities and the network will be achieved by the intermediate of flows between cities in the network: these capture the interactions between cities and have simultaneously an influence on the network in which they flow.

2.2 Model description

The urban system is characterized by populations $\mu_i(t)$ and the network $\mathbf{G}(t)$, to which can be associated a distance matrix $d_{ij}^G(t)$. Flows between cities ϕ_{ij} follow the expression given in **??** with network distance. The same way, the evolution of populations follows the specifications of the base model. The Fig. 1 shows the structure of the model.

Concerning the network, we assume that it evolves following the equation

$$\mathbf{G}(t+1) = F(\mathbf{G}(t), \phi_{ij}(t)) \tag{1}$$

such that the assignment of flows within the network and a local variation of its elements is possible. We propose in a first time to consider patterns linked to distance only, and to specify a relation on an abstract network as

$$d_{ij}^G(t+1) = F(d_{ij}^G(t), \phi_{ij}(t)) \tag{2}$$

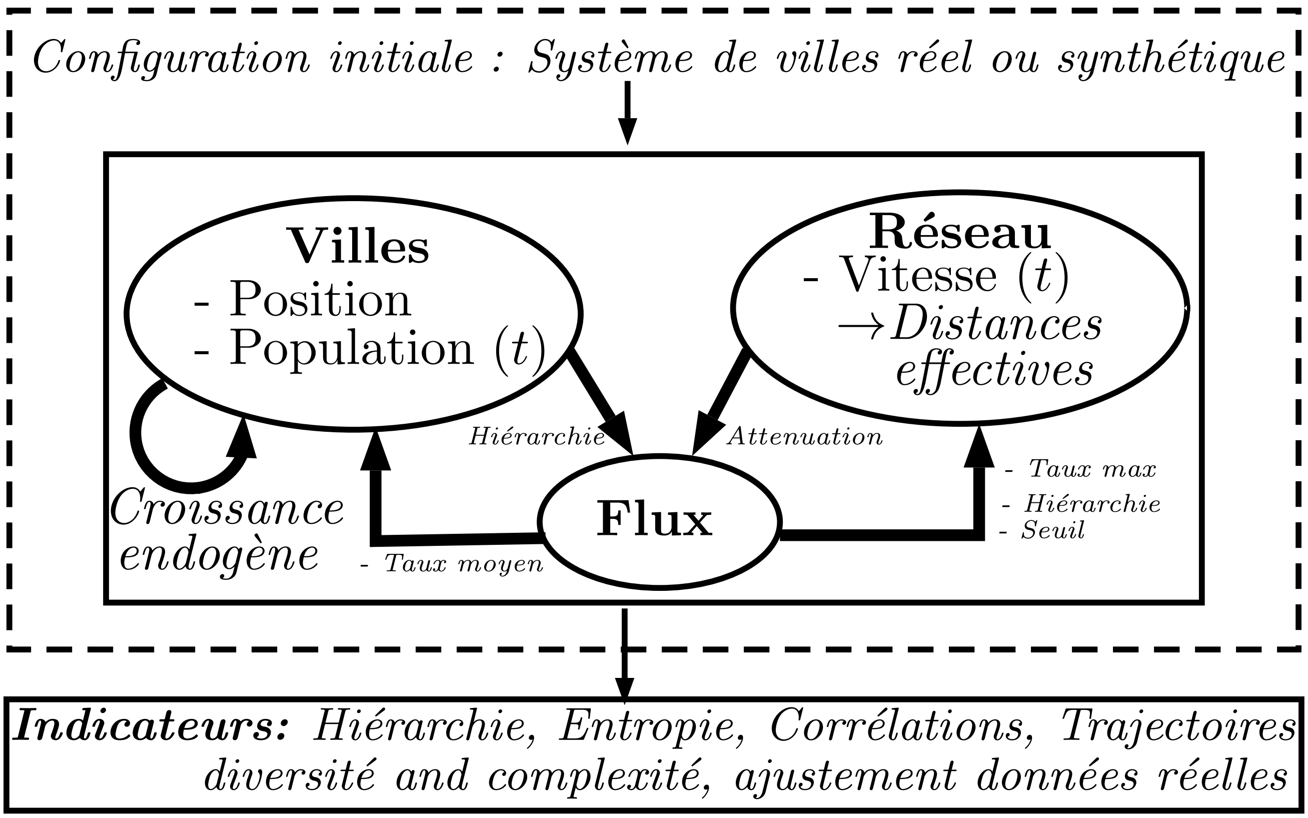

Figure 1: **Abstract representation of the model.** Ellipses correspond to main ontological elements (cities, network, flows), whereas arrows translate processes for which associated parameters are given. The model is described in its broader ecosystem of initialisation and output indicators.

i.e. an evolution of the distance matrix only. In this spirit, we keep an interaction model strictly at a macroscopic scale, since a precise spatialization of the network would imply to take into account a finer scale that includes the local shape of the network which determines shortest paths.

Following a thresholded feedback heuristic, given a flow ϕ in a link, we assume its effective distance to be updated by:

$$d(t+1) = d(t) \cdot \left(1 + g_{max} \cdot \left[\frac{1 - \left(\frac{\phi}{\phi_0}\right)^{\gamma_s}}{1 + \left(\frac{\phi}{\phi_0}\right)^{\gamma_s}}\right]\right) \tag{3}$$

with γ_s a hierarchy parameter, ϕ_0 the threshold parameter and g_{max} the maximal growth rate at each step. This auto-reinforcement function can be interpreted the following way: above a limit flow ϕ_0, the travel conditions improve, whereas they deteriorate below. The hierarchy of gain is given by γ_s, and since $\frac{1-\left(\frac{\phi}{\phi_0}\right)^{\gamma_s}}{1+\left(\frac{\phi}{\phi_0}\right)^{\gamma_s}} \to_{\phi\to\infty} -1$, g_{max} is the maximal distance gain. This function is similar to the one used by [?][1].

2.3 Indicators

3 Results

3.1 Implementation

The coupling of the interaction model to a finer representation of the network (for example an encoding of the whole network structure) makes the full integration into an OpenMole plugin more difficult, as it was done for the model studied in **??**. We need here an *ad hoc* implementation. The use of a workflow as a mediator for coupling is an interesting solution but which is realistic only for a weak coupling as in **??**. One of the issues that the meta-modeling library for OpenMole that is currently being developed around OpenMole will have to tackle is the possibility to allow strong coupling (for example in the sense of a dynamical coupling during the evolution of the simulation) of heterogeneous components in a transparent way, in order to benefit from the advantages of different languages or of already existing implementations.

We choose here a full implementation with NetLogo, for the simplicity of coupling between components. A particular care is taken for the duality of network representation, both as a distance matrix and as a physical network, in order to facilitate the extension to physical network heuristics.

3.2 Exploration on a synthetic system of cities

The model is first tested and explored on synthetic city systems, in order to understand some of its intrinsic properties. In this case, we consider the model with an abstract network as specified above, i.e. without spatial description of the network and with evolution rules acting directly on d^G_{ij} given the previous specifications.

A synthetic city system is generated following the heuristic used in the previous section: (i) N_S cities are randomly distributed in the euclidian plan; (ii) populations are attributed to cities following an inverse power law, with a hierarchy parameter α_S and such that the largest city has a population equal to P_{max}, i.e. following $P_i = P_{max} \cdot i^{-\alpha_S}$.

To simplify, several meta-parameters are fixed: the number of cities is fixed at $N_S = 30$, the maximal population at $P_{max} = 100000$ and the maximal network growth to $g_{max} = 0.005$. Final time is fixed at $t_f = 30$, what corresponds to distances divided approximatively by 5^2, in order to comply to an empirical

[1] Which uses $\Delta d = \Delta t \left[\frac{\phi^\gamma}{1+\phi^\gamma} - d\right]$. This function yield similarly a threshold effect, since the derivative vanishes at $\phi^* = \left(\frac{d}{1-d}\right)^{1/\gamma}$, but it can not be adjusted.

[2] Indeed, we can compute that the minimal multiplicative factor for distance is $(1-g_{max})^{t_f}$, what gives for these values $(1-0.05)^{30} \simeq 0.214$, i.e. a division by 5 of the travel time.

constraint: this corresponds to the evolution of the travel time between Paris and Lyon from around ten hours at the beginning of the century to two hours today, showed for example by [?]. We also neglect network effects at the second order by taking $w_N = 0$.

We explore a grid in the parameter space α_S, ϕ_0, γ_s, w_G, d_G, γ_G. We use the indicators introduced in **??** to quantify model behavior in the parameter space. We describe the results for $\alpha_S = 1$, what is the closest to existing city systems (in comparison to 0.5 and 1.5, see the systematic review of the rank-size law estimations done by [?]).

L'évolution de la centralité de proximité moyenne dans le temps est visualisée en Fig. **??** (haut) pour $w_G = 0.001$, et à (γ_G, ϕ_0) variables. Le comportement n'est pas sensible à d_G (voir graphique complet en **??**). Cette évolution témoigne d'une transition en fonction du niveau de hiérarchie : lorsque celui-ci décroit, on observe l'émergence de trajectoires où la centralité moyenne croît dans le temps, ce qui correspond à des situations où l'ensemble des villes bénéficie en moyenne d'accroissements d'accessibilité.

Concerning the entropy of populations, for which the temporal trajectory is shown in Fig. **??** (bottom), all parameters give a decreasing entropy, i.e. a behavior of convergence of cities trajectories in time[3].

Looking at the complexity of accessibility trajectories, we observe for values of $\phi_0 > 1.5$ a maximum of complexity as a function of interaction distance d_G, stable when w_G and γ_G vary (see also the exhaustive plots in Fig. **??**, Appendix **??**). This intermediate scale can be interpreted as producing regional subsystems, large enough for each to develop a certain level of complexity, et isolated enough to avoid the convergence of trajectories over the whole system. We reconstruct therein a spatial non-stationarity, typically observed in **??**, and rejoin the concept of the ecological niche[4] localized in space: the emergent subsystems that are relatively independent, are good candidates to contain processes of co-evolution. The emergence of this intermediate scale can be compared to the modularity of the French urban system showed by [?].

Finally, the behavior of rank correlations for accessibility reveals that the interaction distance systematically increases the number of hierarchy inversions, what corresponds in a sense to an increase in overall system complexity. The hierarchy parameter diminishes this correlation, what means that a more hierarchical organization will impact a larger number of cities in the qualitative aspects of their trajectories. This effect is similar to the "first mover advantage" showed by [?], which unveils a path dependency and an advantage to be rapidly connected to the network: in our case, the modifications in the hierarchy correspond to cities that benefit from their positioning in the network.

[3] Indeed, the entropy for the population variable gives the dispersion of the distribution of populations, and thus its decrease translate a trend to concentrate in time.

[4] As it was already described in **??**, an ecological niche in the sense of [?] corresponds to the relatively independent ecosystem in which there is co-evolution between the species.