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Second-order Control of Complex Systems with Correlated Synthetic Data

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Abstract

Generation of hybrid synthetic data resembling real data to some criteria is an important methodological and thematic issue in most disciplines which study complex systems. Interdependencies between constituting elements, materialized within respective relations, lead to the emergence of macroscopic patterns. Being able to control the dependance structure and level within a synthetic dataset is thus a source of knowledge on system mechanisms. We describe in this paper a methodology consisting in the generation of synthetic datasets on which correlation structure is controlled. The method is applied in a first example on financial time-series and allows to understand the role of interferences between components at different scales on performances of a predictive model. A second application on a geographical system is then proposed, in which the weak coupling between a population density model and a network morphogenesis model allows to simulate territorial configurations. The calibration on morphological objective on European data and intensive model exploration unveils a large spectrum of feasible correlations between morphological and network measures. We demonstrate therein the flexibility of our method and the variety of possible applications.

Keywords: Synthetic Data; Statistical Control; Correlations; Financial Time-series; Land-use Transportation Interactions

Introduction

Developing methods to study complex systems, such as simulation models or data-mining techniques, often requires testbeds and benchmarks to ensure expected properties. The use of synthetic data, in the sense of statistical populations generated randomly under constraints of proximity of patterns to a studied system, is a widely used methodology tackling this issue. This approach is used in several disciplines related to complex systems such as therapeutic evaluation [1], territorial science [2, 3], machine learning [4] or bio-informatics [5].

Generation of synthetic datasets can consist in data disaggregation by producing a microscopic population with fixed macroscopic properties [6]. The creation of synthetic populations for microsimulation models is a typical example where empirical statistical distributions are reproduced [7]. In data extensive contexts, several methods have been developed and improved for a better reproduction of margin distributions [8].

Synthetic datasets can also be generated at the same scale than the targeted real dataset, with a broad range of realism levels and corresponding constraints on the generated data [9]. For example, [10] show that some datamining techniques such

as decision trees can be inverted to produce datasets capturing complex non-linear patterns.

The constraints of proximity to reality of synthetic dataset will depend on expected applications. They range for example from a strong statistical fit on given indicators, to weaker assumptions of similarity on aggregated patterns. In the case of systems where emergence plays a central role, a microscopic property does not directly imply given macroscopic patterns, and synthetic datasets may have to capture some of these. This approach therein becomes part of the complex systems simulation toolbox. Indeed, with the rise of new computational paradigms [11], data (simulated, measured or hybrid) shape our understanding of complex systems. Methodological tools for data-mining, modeling and simulation, including the generation of synthetic data, are therefore crucial to be developed.

Synthetic data and dependency structures

Reproducing data patterns at the first order, in the sense of distribution moments, is broadly used and understood. A targeted average will be easily reproduced. Similarly, marginals are fitted when generating synthetic population. However, higher orders of data structure are more difficult to include in synthetic data generation methods. At the second order, this corresponds to a control of the covariance structure between generated variables.

Some specific examples where interdependency structure is controlled can be found. [12] investigates the sensitivity of discrete choices models to the distributions of inputs and to their dependency structure. [13] develop a generic framework to generate synthetic micro-data from heterogeneous aggregated data sources, which in particular can include second-order effects in the models considered. [14] propose to reconstruct multi-dimensional synthetic data using copulas, which capture the dependency structure between marginal distributions. It is also possible to interpret complex networks generative models [15] as the production of an interdependence structure for a system, contained within link topology. Most methods yielding a high level of accuracy on synthetic covariance structure depend on sampling or data reconstruction methods, and need therefore large datasets.

Synthetic data and socio-spatial systems

Synthetic data with a spatial dimension, in the sense of spatial coordinates of generated data points, or more complicated spatial structures, require proper methods and paradigms. Such approaches have been proposed in disciplines such as geostatistics or Earth sciences. [16] describe a method to generate cross-correlated random spatial fields using Fourier transforms. [17] introduce a multilevel sampling technique to produce correlated random fields. Concrete applications of such spatial synthetic data include atmospheric circulation models [18], rainfall-runoff simulations [16], or engineering [19].

In the case of socio-spatial systems, these kind of methods is less developed. Simulation approaches to spatialized social systems are already well studied by disciplines such as geosimulation [20], urban analytics [21] or theoretical and quantitative geography [22]. The use of synthetic data in these contexts is however systematically reduced to the generation of synthetic populations within agent-based models or

microsimulation models, applied for example to mobility [23], land-use transport interaction models [3], or demography microsimulation models [13].

While several examples of stylized models initialized on synthetic configurations can be found in the literature, such as the first Simpop model [24] to simulate the dynamics of settlements at a macroscopic scale, or the SimpopNet model [25] for the co-evolution of cities and transportation networks, these are run on a single stylized synthetic configuration. There is to the best of our knowledge very few examples of works coupling a synthetic data generator with a model at an other scale than the microscopic scale of the population.

Recently, a systematic control of the effects of the initial spatial configuration on the behavior of simulation models was proposed by [26]. The aim is to be able to distinguish proper effects due to intrinsic model dynamics from particular effects due to the geographical structure of the case study. [27] introduce a method to generate realistic social networks associated to a synthetic population in the geographical space. Such results are essential for the validation of conclusions obtained with modeling and simulation practices in quantitative geography. Being able to generate correlated synthetic configurations of territorial systems is thus an important development remaining to be investigated.

Proposed approach

This literature review on different aspects of synthetic data generation unveils at least two gaps: (i) a lack of attention on controlling covariance structures when generating synthetic data; and (ii) an absence of such methods applied to the study of socio-spatial systems at aggregated scales. As spatio-temporal dependencies structures are essential in driving the dynamics of such systems [28], the combination of these two aspects appears as an unexplored research problem.

We propose in this paper to study the generation of correlated synthetic data, and more particularly in the case of socio-spatial systems. We introduce here a generic methodology taking into account the dependance structure for the generation of synthetic datasets, more precisely by controlling the average of correlation matrices. It is suited to be applied on cases where microscopic data is not available and system similarity is expected on aggregated indicators.

We investigate thus the question of how to generate correlated synthetic data at aggregated levels, where constraints on macroscopic indicators are fulfilled and correlation structure is controlled. We focus on this problem in the particular case of socio-spatial systems, but keep in mind the genericity of the approach.

Our contribution is twofold: (i) we implement a generation of spatial synthetic data for socio-spatial systems, which to the best of our knowledge has never been done in that context; (ii) the method introduced is generic, and we illustrate it with an application to financial time-series.

The rest of the paper is organized as follows. The generic method to generate correlated synthetic data is formally described. We then apply it to a generative model of territorial configurations, composed by the sequential coupling of a reaction-diffusion model for population density with a road network generation model, and study the produced correlation patterns. We then illustrate the genericity of the method by applying it to financial time-series.

Method Formalization

Domain-specific methods aforementioned are too broad to be summarized into a same formalism. We propose therefore here a rather generic and model-agnostic framework, focused on the control of correlations structures in synthetic data.

Let \vec{X}_I a multidimensional stochastic process (that can be indexed e.g. with time in the case of time-series, but also space, or discrete set abstract indexation). We assume given a real dataset $\mathbf{X} = (X_{i,j})$, interpreted as a set of realizations of the stochastic process. We propose to generate a statistical population $\tilde{\mathbf{X}} = \tilde{X}_{i,j}$ such that

- 1 a given criteria of proximity to data is verified, i.e. given a precision ε and an indicator f , we have

$$\|f(\mathbf{X}) - f(\tilde{\mathbf{X}})\| < \varepsilon \quad (1)$$

- 2 level of correlation is controlled, i.e. given a matrix R fixing correlation structure (symmetric matrix with coefficients in $[-1, 1]$ and unity diagonal), we have

$$\hat{\text{Var}} \left[(\tilde{X}_i) \right] = \Sigma R \Sigma \quad (2)$$

where the standard deviation diagonal matrix Σ is estimated on the synthetic population.

The second requirement will generally be conditional to parameter values determining generation procedure, either generation models being simple or complex (R itself is a parameter). Formally, synthetic processes are parametric families $\tilde{X}_i[\vec{\alpha}]$.

We propose to apply the methodology on very different examples, both typical of complex systems: territorial systems and financial high-frequency time-series. We illustrate the flexibility of the method, and claim to help building interdisciplinary bridges by methodology transposition and reasoning analogy. In the first case, morphological calibration of a population density distribution model allows to respect real data proximity. Correlations of urban form with transportation network measures are empirically obtained by exploration of coupling with a network morphogenesis model. The control is in this case indirect as feasible space is empirically determined. In the second case, proximity to data is the equality of signals at a fundamental frequency, to which higher frequency synthetic components with controlled correlations are superposed. It follows a logic of hybrid data for which hypothesis or model testing is done on a more realistic context than on purely synthetic data.

Correlated population density and road network

We now apply the method to territorial systems of human settlements, in the particular case here of population distribution in correlation with road network. In this application, simulation appears as a crucial step to implement the method.

Territorial configuration model

We propose in our case to generate territorial systems summarized in a simplified way as a spatial population density $d(\vec{x})$ and a transportation network $n(\vec{x})$. Correlations we aim to control are correlations between urban morphological measures and network measures. The question of interactions between territories and networks is already well-studied [29] but remains highly complex and difficult to quantify [30]. A dynamical modeling of implied processes should shed light on these interactions [31]. We develop in that frame a simple coupling (i.e. without any feedback loop) between a density distribution model and a network morphogenesis model.

Density model

We use a model D similar to aggregation-diffusion models [32] to generate a discrete spatial distribution of population density. A generalization of the basic model is proposed in [33], providing a calibration on morphological objectives (entropy, hierarchy, spatial auto-correlation, mean distance) against real values computed on the set of 50km sized grid extracted from European density grid [34]. We recall here rapidly the processes in the model. An square grid of width W , initially empty, is represented by population $(P_i(t))_{1 \leq i \leq W^2}$. At each time step, until the total population reaches a fixed parameter P_m ,

- total population is increased of a fixed number N_G (growth rate), following a preferential attachment such that

$$\mathbb{P}[P_i(t+1) = P_i(t) + 1 | P(t+1) = P(t) + 1] = \frac{(P_i(t)/P(t))^\alpha}{\sum(P_j(t)/P(t))^\alpha} \quad (3)$$

- a fraction β of population is diffused to four closest neighbors is operated n_d times

The two opposite processes of urban concentration and urban sprawl are captured by the model, what allows to reproduce with a good precision a large number of existing morphologies regarding macroscopic urban form indicators.

Network model

On the other hand, we are able to generate a planar transportation network by a model N , at a similar scale and given a density distribution. Because of the conditional nature to the density of the generation process, we will first have conditional estimators for network indicators, and secondly natural correlations between network and urban shapes should appear as processes are not independent. The nature and modularity of these correlations as a function of model parameters are still to determine by exploration of the coupled model.

The heuristic network generation procedure is the following :

- 1 A fixed number N_c of centers that will be first nodes of the network si distributed given density distribution, following a similar law to the aggregation process, i.e. the probability to be distributed in a given patch is $\frac{(P_i/P)^\alpha}{\sum(P_j/P)^\alpha}$. Population is then attributed according to Voronoi areas of centers, such that a center cumulates population of patches within its extent.

- 2 Centers are connected deterministically by percolation between closest clusters : as soon as network is not connected, two closest connected components in the sense of minimal distance between each vertices are connected by the link realizing this distance. It yields a tree-shaped network.
- 3 Network is modulated by potential breaking in order to be closer from real network shapes. More precisely, a generalized gravity potential between two centers i and j is defined by

$$V_{ij}(d) = \left[(1 - k_h) + k_h \cdot \left(\frac{P_i P_j}{P^2} \right)^\gamma \right] \cdot \exp \left(-\frac{d}{r_g(1 + d/d_0)} \right) \quad (4)$$

where d can be euclidian distance $d_{ij} = d(i, j)$ or network distance $d_N(i, j)$, $k_h \in [0, 1]$ a weight to modulate role of populations, γ giving shape of the hierarchy across population values, r_g characteristic interaction distance and d_0 distance shape parameter.

- 4 A fixed number $K \cdot N_L$ of potential new links is taken among couples having greatest euclidian distance potential ($K = 5$ is fixed).
- 5 Among potential links, N_L are effectively realized, that are the one with smallest rate $\tilde{V}_{ij} = V_{ij}(d_N)/V_{ij}(d_{ij})$. At this stage only the gap between euclidian and network distance is taken into account : \tilde{V}_{ij} does indeed not depend on populations and is increasing with d_N at constant d_{ij} .
- 6 Planarity of the network is imposed by creating nodes at possible intersections formed by new links.

We insist on the fact that the network generation procedure is entirely heuristic and result of thematic assumptions (connected initial network, gravity-based link creation) combined with trial-and-error during first explorations. Other model types could be used as well, such as biological self-generated networks [35], local network growth based on geometrical constraints optimization [36], or a more complex percolation model than the initial one that would allow the creation of loops for example. We could thus in the frame of a modular architecture, in which the choice between different implementations of a functional brick can be seen as a meta-parameter [37], choose network generation function adapted to a specific need (as e.g. proximity to real data, constraints on output indicators, variety if generated forms, etc.).

Parameter space for the coupled model is constituted by density generation parameters $\vec{\alpha}_D = (P_m/N_G, \alpha, \beta, n_d)$ (we study for the sake of simplicity the rate between population and growth rate instead of both varying, i.e. the number of steps needed to generate the distribution) and network generation parameters $\vec{\alpha}_N = (N_C, k_h, \gamma, r_g, d_0)$. We denote $\vec{\alpha} = (\vec{\alpha}_D, \vec{\alpha}_N)$.

Urban form and network structure are quantified by numerical indicators in order to modulate correlations between these. Morphology is defined as a vector $\vec{M} = (r, d, \varepsilon, a)$ giving spatial auto-correlation (Moran index), mean distance, entropy and hierarchy (see [38] for a precise definition of these indicators). Network measures $\vec{G} = (c, l, s, \delta)$ are with network denoted (V, E)

- Mean centrality c defined as average *betweenness-centrality* (normalized in $[0, 1]$) on all links.

- Mean path length l given by $\frac{1}{d_m} \frac{2}{|V| \cdot (|V|-1)} \sum_{i < j} d_N(i, j)$ with d_m normalization distance taken here as world diagonal $d_m = \sqrt{2}N$.
- Mean network speed [39] which corresponds to network performance compared to direct travel, defined as $s = \frac{2}{|V| \cdot (|V|-1)} \sum_{i < j} \frac{d_{ij}}{d_N(i, j)}$.
- Network diameter $\delta = \max_{ij} d_N(i, j)$.

We study the cross-correlation matrix $\text{Cov}[\vec{M}, \vec{G}]$ between morphology and network. We estimate it on a set of n realizations at fixed parameter values $(\vec{M}[D(\vec{\alpha})], \vec{G}[N(\vec{\alpha})])_{1 \leq i \leq n}$ with standard unbiased estimator. We estimate correlation with associated Pearson estimator.

Results

The coupling of generative models is done both in a formal and operational way. We indeed loosely couple independent implementations. The OpenMOLE software [40] for model exploration offers a proper framework for this. Its modular workflow language allows to compose model tasks and integrate these into diverse numerical experiments. For the population density generation, we use the `scala` implementation provided by [33]. The network generation model is implemented in NetLogo [41], which offers a good compromise between performance and interactive model validation and exploration. The two models are coupled with a specific OpenMOLE script. Source code is available at <https://github.com/JusteRaimbault/CityNetwork/tree/master/Models/Synthetic>.

Results

The study of the density model alone is developed in [33]. It is in particular calibrated on European density grid data, on 50km width square areas with 500m resolution for which real indicator values have been computed on whole Europe. Furthermore, a grid exploration of model behavior yields feasible output space in reasonable parameters bounds (roughly $\alpha \in [0.5, 2]$, $N_G \in [500, 3000]$, $P_m \in [10^4, 10^5]$, $\beta \in [0, 0.2]$, $n_d \in \{1, \dots, 4\}$). The reduction of indicators space to a two dimensional plan through a Principal Component Analysis (variance explained with two components $\simeq 80\%$) allows to isolate a set of output points that covers reasonably precisely real point cloud. It confirms the ability of the model to reproduce morphologically the set of real configurations.

With a fixed population density, the conditional exploration of network generation model parameter space suggest a good flexibility on global indicators \vec{G} , together with good convergence properties. In order to illustrate synthetic data generation method, the exploration has been oriented towards the study of cross-correlations.

Given the large relative dimension of the parameter space, an exhaustive grid exploration is not possible. We use a Latin Hypercube sampling procedure with bounds given above for $\vec{\alpha}_D$ and for $\vec{\alpha}_N$, we take $N_C \in [50, 120]$, $r_g \in [1, 100]$, $d_0 \in [0.1, 10]$, $k_h \in [0, 1]$, $\gamma \in [0.1, 4]$, $N_L \in [4, 20]$. For number of model replications for each parameter point, less than 50 are enough to obtain confidence intervals at 95% on indicators of width less than standard deviations. For correlations a hundred give confidence intervals (obtained with Fisher method) of size around 0.4, we take thus $n = 80$ for experiments. Simulation results are available on the dataverse at <http://dx.doi.org/10.7910/DVN/UIHBC7>.

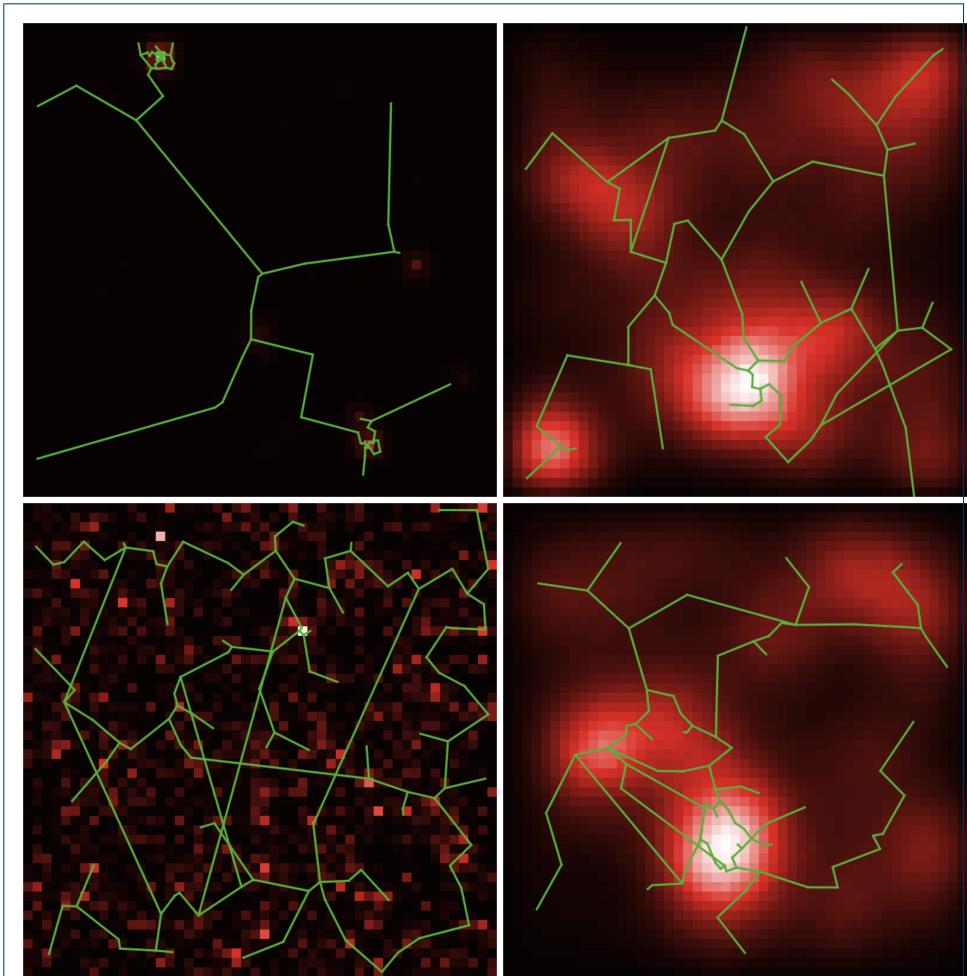
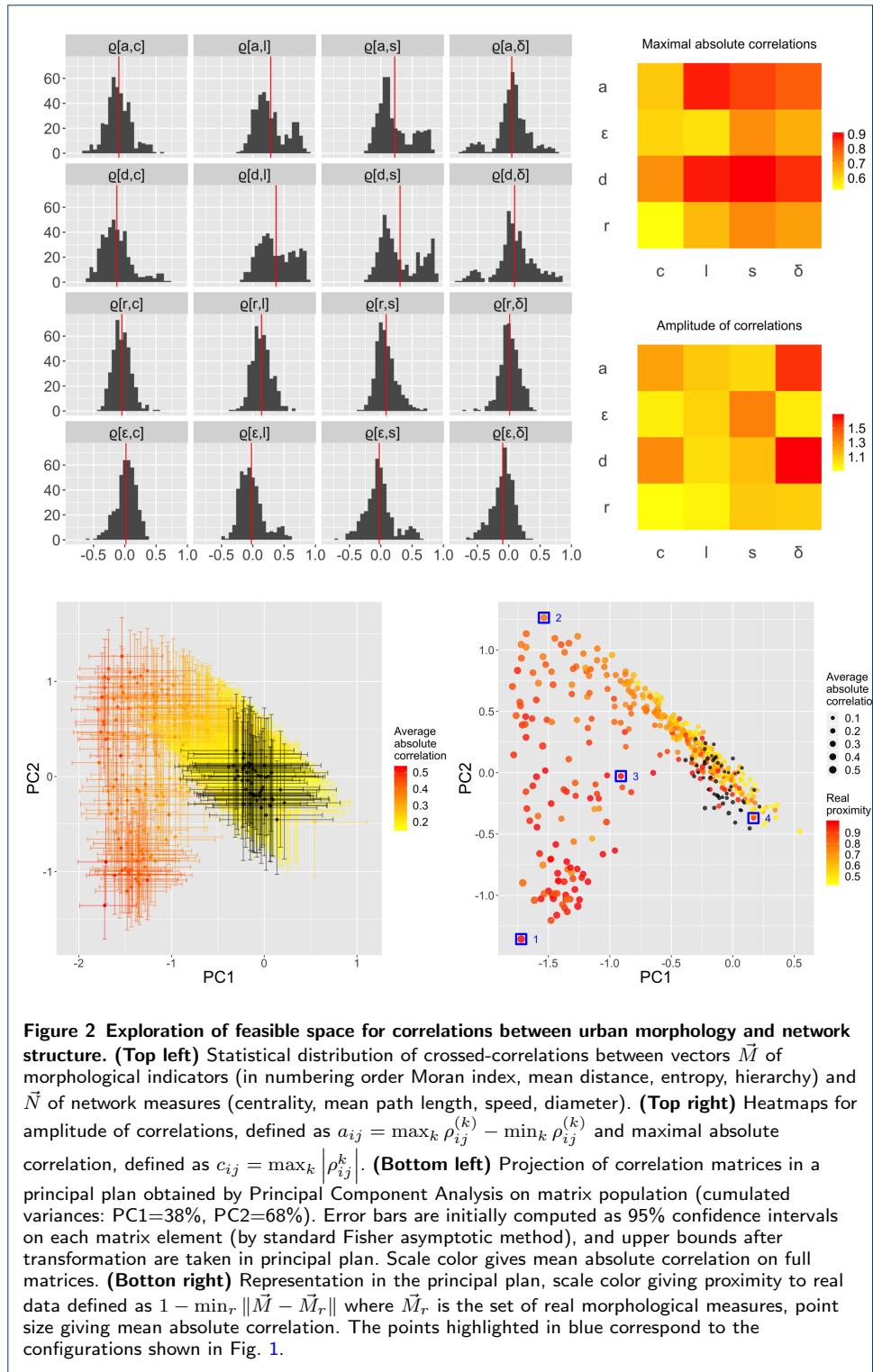


Figure 1 Configurations obtained for parameters giving the four emphasized points in Fig. 2, in order from left to right and top to bottom. We recognize polycentric city configurations (2 and 4), diffuse rural settlements (3) and aggregated weak density area (1). See appendix for exhaustive parameter values, indicators and corresponding correlations. For example \bar{d} is highly correlated with \bar{l}, \bar{s} ($\simeq 0.8$) in (1) but not for (3) although both correspond to rural environments ; in the urban case we observe also a broad variability : $\rho[\bar{d}, \bar{c}] \simeq 0.34$ for (4) but $\simeq -0.41$ for (2), what is explained by a stronger role of gravitation hierarchy in (2) $\gamma = 3.9, k_h = 0.7$ (for (4), $\gamma = 1.07, k_h = 0.25$), whereas density parameters are similar.

We show in Fig. 1 examples of generated territorial configurations. This visualization and some values of associated correlations already suggest that the method application yields a broad spectrum of generated correlation patterns.

Fig. 2 gives details of experiment results. Regarding the subject of correlated synthetic data generation, we can summarize the results as follows:

- Empirical distributions of correlation coefficients between morphology and network indicators are not simple and some are bimodal (for example $\rho_{46} = \rho[r, \bar{l}]$ between Moran index and mean path length).
- it is possible to modulate up to a relatively high level of correlation for all indicators, maximal absolute correlation varying between 0.6 and 0.9. Amplitude of correlations varies between 0.9 and 1.6, allowing a broad spectrum of values. Point cloud in principal plan has a large extent but is not uniform:



it is not possible to modulate at will any coefficient as they stay themselves correlated because of underlying generation processes. A more refined study at higher orders (correlation of correlations) would be necessary to precisely understand degrees of freedom in correlation generation.

- Most correlated points are also the closest to real data, what confirms the intuition and stylized fact of a strong interdependence in reality.
- Concrete examples taken on particular points in the principal plan show that similar density profiles can yield very different correlation profiles.

This case study could be refined by extending correlation control method. A precise knowledge of N behavior (statistical distributions on an exhaustive grid of parameter space) conditional to D would allow to determine $N^{<-1>}|D$ and have more latitude in correlation generation. We could also apply specific exploration algorithms to reach exceptional configurations realizing an expected correlation level, or at least to obtain a better knowledge of the feasible space of correlations [42].

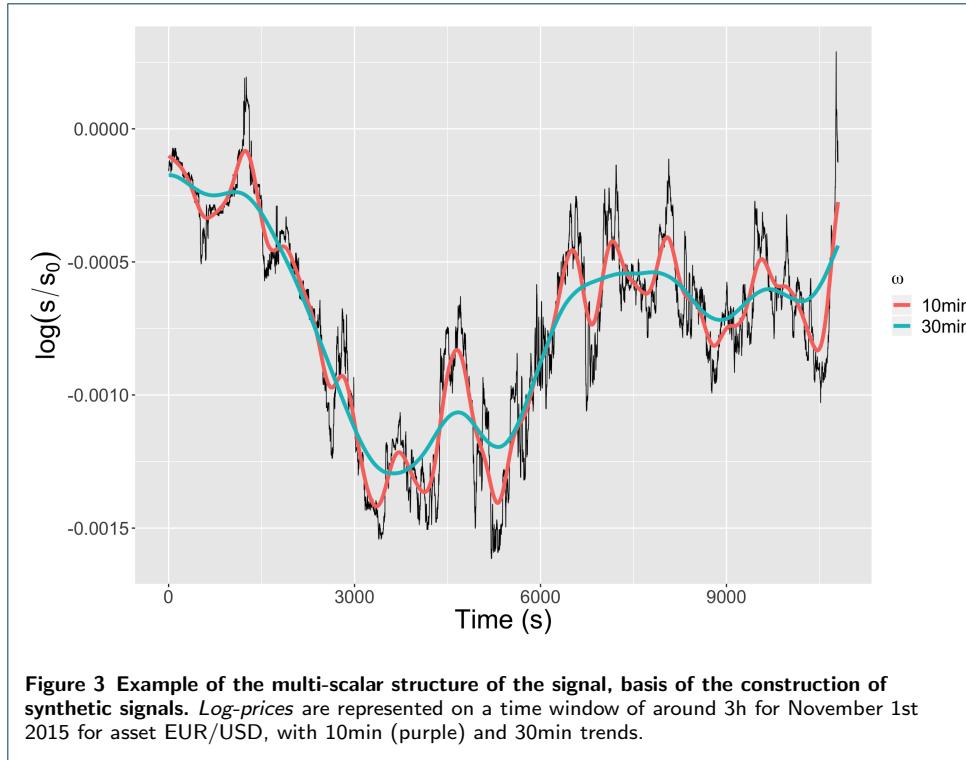
Correlated financial time-series

We also apply the method to a totally different type of system, namely financial complex systems. Financial time-series are heterogeneous, multi-scalar and non-stationary [43]. Correlations are broadly explored in that field. For example, Random Matrix Theory allows to distinguish signal from noise for a correlation matrix computed for a large number of asset with low-frequency signals [44]. Similarly, Complex Network Analysis on networks constructed from correlations, with methods such as Minimal Spanning Tree [45] or more refined topologically constrained network generation methods [46], provide reconstructions of economic sector structure. At high frequencies, the precise estimation of interdependence parameters in the framed of fixed assumptions on asset dynamics, has been extensively studied from a theoretical point of view aimed at refinement of models and estimators [47]. Theoretical results must be tested on synthetic datasets as they ensure a control of most parameters in order to check that a predicted effect is indeed observable all things being otherwise equal. Empirical confirmation of the improvement of estimators is obtained on a synthetic dataset at a fixed correlation level.

We consider a network of assets $(X_i(t))_{1 \leq i \leq N}$ sampled at high-frequency (typically 1s). We use a multi-scalar framework (used e.g. in wavelet analysis approaches [48] or in multi-fractal signal processing [49]) to interpret observed signals as the superposition of components at different time scales : $X_i = \sum_{\omega} X_i^{\omega}$. We denote by $T_i^{\omega} = \sum_{\omega' \leq \omega} X_i^{\omega'}$ the filtered signal at a given frequency ω . A recurrent problem in the study of complex systems is the prediction of a trend at a given scale. It can be viewed as the identification of regularities and their distinction from components considered as random. For the sake of simplicity, we represent such a process as a trend prediction model at a given temporal scale ω_1 , formally an estimator $M_{\omega_1} : (T_i^{\omega_1}(t'))_{t' < t} \mapsto \hat{T}_i^{\omega_1}(t)$ which aims to minimize error on the real trend $\|T_i^{\omega_1} - \hat{T}_i^{\omega_1}\|$. In the case of autoregressive multivariate estimators, the performance will depend among other parameters on respective correlations between assets. It is thus interesting to apply the method to the evaluation of performance as a function of correlation at different scales. We assume a Black-Scholes dynamic for assets [50], i.e. $dX = \sigma \cdot dW$, with W Wiener process. Such a dynamic model allows an easy modulation of correlation levels.

Data generation

We can straightforward generate \tilde{X}_i such that $\text{Var}[\tilde{X}_i^{\omega_1}] = \Sigma R \Sigma$ (with Σ estimated standard deviations and R fixed correlation matrix) and verifying $X_i^{\omega \leq \omega_0} = \tilde{X}_i^{\omega \leq \omega_0}$.



This means in practice that the data proximity indicator is the identity of components at a lower frequency than a fundamental frequency $\omega_0 < \omega_1$. We use therefore the simulation of Wiener processes with fixed correlation. Indeed, if $dW_1 \perp\!\!\!\perp dW_1^{\perp\!\!\!\perp}$ (and $\sigma_1 < \sigma_2$ indicatively, assets being interchangeable), then

$$W_2 = \rho_{12}W_1 + \sqrt{1 - \frac{\sigma_1^2}{\sigma_2^2} \cdot \rho_{12}^2} \cdot W_1^{\perp\!\!\!\perp} \quad (5)$$

is such that $\rho(dW_1, dW_2) = \rho_{12}$. Signals for other components can be constructed the same way by Gram orthonormalization. We isolate the component at the desired frequency ω_1 by filtering the signal, i.e. $\tilde{X}_i^{\omega_1} = W_i - \mathcal{F}_{\omega_0}[W_i]$ (with \mathcal{F}_{ω_0} low-pass filter with cut-off frequency ω_0). We reconstruct then the hybrid synthetic signals by taking

$$\tilde{X}_i = T_i^{\omega_0} + \tilde{X}_i^{\omega_1} \quad (6)$$

The method is tested on an example with two assets from foreign exchange market (EUR/USD and EUR/GBP), on a six month period from June 2015 to November 2015. Data was obtained from <http://www.histdata.com/>. The data cleaning procedure, starting from original series sampled at a frequency around 1s, consists in a first step to the determination of the minimal common temporal range (missing sequences being ignored, by vertical translation of series, i.e. $S(t) := S(t) \cdot \frac{S(t_n)}{S(t_{n-1})}$ when t_{n-1}, t_n are extremities of the “hole” and $S(t)$ value of the asset, what is

equivalent to keep the constraint to have returns at similar temporal steps between assets). We study then *log-prices* and *log-returns* [51], defined by $X(t) := \log \frac{S(t)}{S_0}$ and $\Delta X(t) = X(t) - X(t-1)$. Raw data are filtered at a maximal frequency $\omega_m = 10\text{min}$ (which will be the maximal frequency for following treatments) for concerns of computational efficiency. As time-series are then sampled at $3 \cdot \omega_m$ to avoid aliasing, a day of size 86400 for 1s sampling is reduced to a much smaller size of 432. We use a non-causal gaussian filter of total width ω . We fix the fundamental frequency $\omega_0 = 24\text{h}$ and we propose to construct synthetic data at frequencies $\omega_1 = 30\text{min}, 1\text{h}, 2\text{h}$. We show in Fig. 3 an example of signal structure at the scales ω_m and $\omega_1 = 30\text{min}$, compared with the non-filtered raw signal.

It is crucial to consider the interference between ω_0 and ω_1 frequencies in the reconstructed signal: the correlation which is indeed estimated is

$$\rho_e = \rho \left[\Delta \tilde{X}_1, \Delta \tilde{X}_2 \right] = \rho \left[\Delta T_1^{\omega_0} + \Delta \tilde{X}_1^\omega, \Delta T_2^{\omega_0} + \Delta \tilde{X}_2^\omega \right] \quad (7)$$

Assuming to be in the reasonable limit $\sigma_1 \gg \sigma_0$ (fundamental frequency small enough), that $\text{Cov} \left[\Delta \tilde{X}_i^{\omega_1}, \Delta X_j^\omega \right] = 0$ for all $i, j, \omega_1 > \omega$ and that returns are centered at any scale, we can develop the previous expression to compute the correction on effective correlation due to interferences. We obtain at the first order the expression of effective correlation given by

$$\rho_e = [\varepsilon_1 \varepsilon_2 \rho_0 + \rho] \cdot \left[1 - \frac{1}{2} (\varepsilon_1^2 + \varepsilon_2^2) \right] \quad (8)$$

what corresponds to the correlation that we can effectively simulate in synthetic data.

Correlation is in practice estimated with a Pearson estimator, the covariances being corrected for bias, i.e.

$$\hat{\rho}[X1, X2] = \frac{\hat{C}[X1, X2]}{\sqrt{\hat{\text{Var}}[X1] \cdot \hat{\text{Var}}[X2]}} \quad (9)$$

where

$$\hat{C}[X1, X2] = \frac{1}{(T-1)} \sum_t X_1(t) X_2(t) - \frac{1}{T \cdot (T-1)} \sum_t X_1(t) \sum_t X_2(t) \quad (10)$$

and

$$\hat{\text{Var}}[X] = \frac{1}{T} \sum_t X^2(t) - \left(\frac{1}{T} \sum_t X(t) \right)^2 \quad (11)$$

The generated synthetic data are then used to test a toy model. We propose in particular to investigate the predictive power of a very simple linear model.

The tested predictive model M_{ω_1} is a simple *ARMA* for which parameters $p = 2, q = 0$ are fixed (as we do not create lagged correlation, we do not expect large orders of auto-regression as these kind of processes have short memory for real data; furthermore smoothing is not necessary as data are already filtered). It is however applied in an adaptive way, in the sense that given a time window T_W , we estimate for any t the model on $[t - T_W + 1, t]$ in order to predict signals at $t + 1$.

Experiments are implemented in the R language, using in particular the MTS [52] library for time-series models. Cleaned data and source code are available on an open git repository at <https://github.com/JusteRaimbault/SyntheticAsset>.

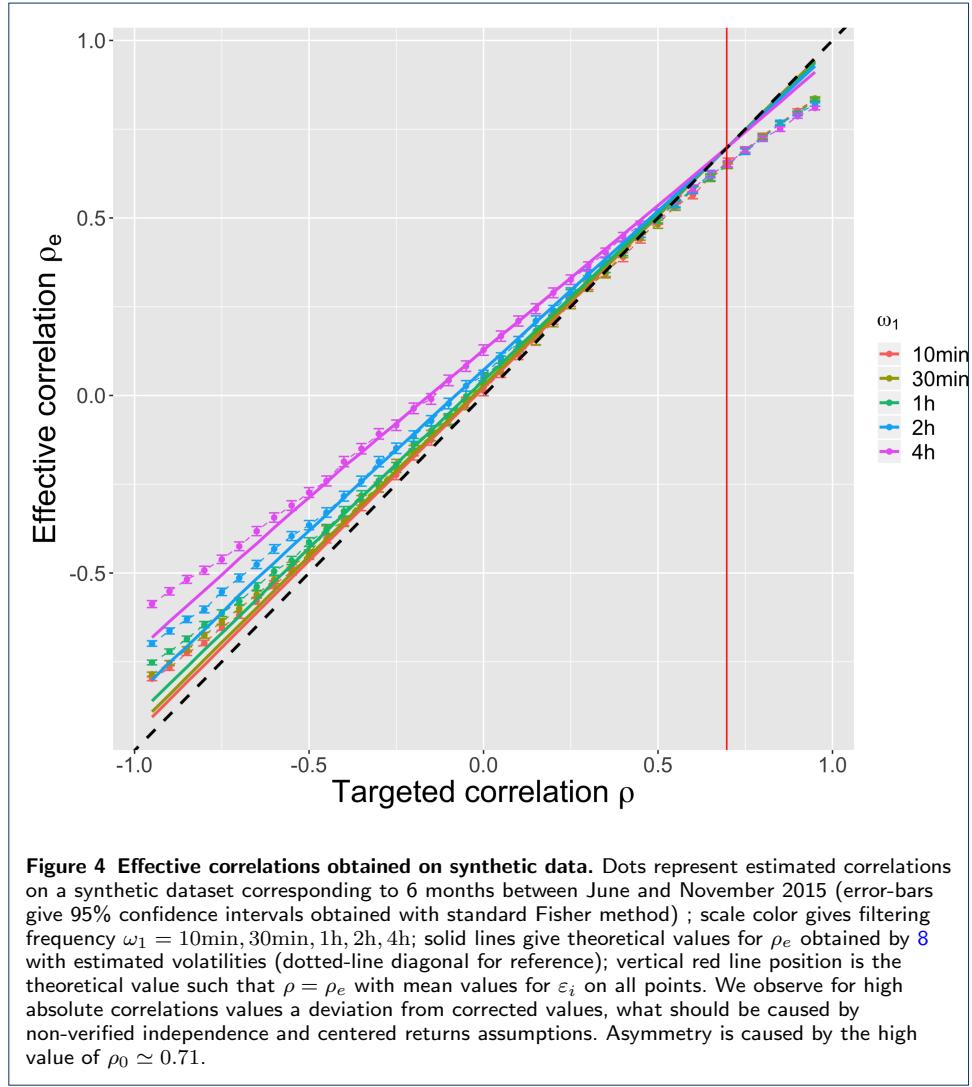
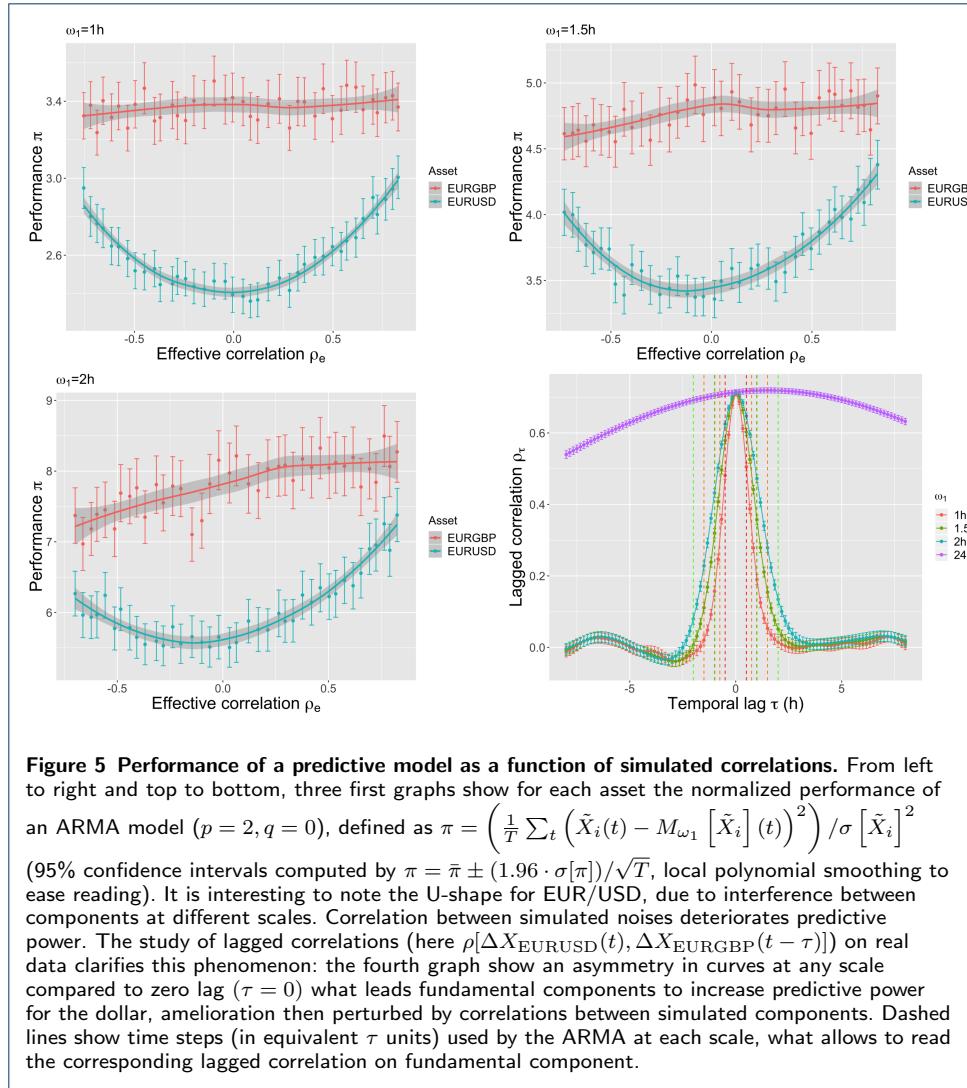


Fig. 4 shows the effective correlations computed on synthetic data. For standard parameter values (for example $\omega_0 = 24\text{h}$, $\omega_1 = 2\text{h}$ and $\rho = -0.5$), we find $\rho_0 \simeq 0.71$ et $\varepsilon_i \simeq 0.3$ what yields $|\rho_e - \rho| \simeq 0.05$. We observe a good agreement between observed ρ_e and values predicted by Equation 8 in the interval $\rho \in [-0.5, 0.5]$. On the contrary, for larger absolute values, a deviation increasing with $|\rho|$ and as ω_1 decreases : it confirms the intuition that when frequency decreases and becomes



closer to ω_0 , interferences between the two components are not negligible anymore and invalidate independence assumptions for example.

Application

We apply then the predictive model described above to synthetic data, in order to study its mean performance as a function of correlation between signals. Results for $\omega_1 = 1h, 1h30, 2h$ are shown in Fig. 5. The a priori counter-intuitive result of a maximal performance at vanishing correlation for one of the assets confirms the role of synthetic data to better understand system mechanisms : the study of lagged correlations shows an asymmetry in the real data that we can understand at a daily scale as an increased influence of EUR/GBP on EUR/USD with a rough two hours lag. The existence of this *lag* allows a “good” prediction of EUR/USD thanks to fundamental component. This predictive power is perturbed by added noises in a way that increases with their correlation. The more noises correlated are, the more the model will take them into account and will make false predictions because

of the Markovian character of simulated brownian (note that the model used has theoretically no predictive power at all on pure brownians).

This case study on a *toy-model* illustrates the relevance of using simulated synthetic data. Further developments can be directed towards the simulation of more realistic data (presence of consistent *lagged correlation* patterns, more realistic models than Black-Scholes) and apply it on more operational predictive models.

Discussion

Contributions

We investigated in this paper the possibility of generating synthetic data at a macroscopic level with a controlled correlation structure. The generic method we introduce can be applied to any complex system, where the proximity to real data is measured on aggregated indicators. The method was designed more particularly for socio-spatial systems. We show in the case of transportation network and territories, by exploring a weak coupled model for population density and road network generation, that varying model parameters yield a broad output space of effective correlations. Two configurations with the same first order indicator values can capture very different underlying correlations. This means that future applications to the study of upstream models to the sensitivity of spatial initial configuration, such as the one done by [26], should be made possible.

We postulate that the method can be also applied in other fields where similar constraints can be of interest.

Future work

Starting from the second example which was limited to data generation, we propose examples of direct applications that should give an overview of the range of possibilities. The calibration of the network generation component at given density, on real data for transportation network, is a potential development. It would be relevant typically on road networks given the shape of generated networks, what should be possible using OpenStreetMap open data which have a reasonable quality for Europe [53], with however some adjustments required on the generation procedure in order to avoid edge effects due its restrictive frame. This should theoretically allow to unveil parameter sets reproducing accurately existing configurations both for urban morphology and network shape. It could be then possible to derive a “theoretical correlation” for these, as an empirical correlation is according to some theories of urban systems not computable as a unique realization of stochastic processes is observed. Because of non-ergodicity of urban systems [54], there are strong chances that involved processes are different across different geographical areas (or from an other point of view that they are in an other state of meta-parameters, i.e. in an other regime) and that their interpretation as different realizations of the same stochastic process makes no sense, the impossibility of covariation estimation following. By attributing a synthetic dataset similar to a given real configuration, we would be able to compute a sort of *intrinsic correlation* proper to this configuration. As territorial configurations emerge from spatio-temporal interdependences between components of territorial systems, this intrinsic correlation emerges the same way, and its knowledge gives information on these interdependences and thus on relations between territories and networks.

As already mentioned, most of simulation models need an initial state generated artificially as soon as model parametrization is not done completely on real data. An advanced model sensitivity analysis implies a control on parameters for synthetic dataset generation, seen as model meta-parameters [26]. In the case of a statistical analysis of model outputs it provides a way to operate a second order statistical control.

We studied in the first example stochastic processes in the sense of random time-series, whereas time did not have a role in the second case. We can suggest a strong coupling between the two model components (or the construction of an integrated model) and to observe indicators and correlations at different time steps during the generation. In a dynamical spatial models we have because of feedbacks necessarily propagation effects and therefore the existence of lagged interdependences in space and time [28]. It would drive our field of study towards a better understanding of dynamical correlations.

We were limited to the control of first and second moments of generated data, but we could imagine a theoretical generalization allowing the control of moments at any order. However, as shown by the geographical example, the difficulty of generation in a concrete complex case questions the possibility of higher orders control when keeping a consistent structure model and a reasonable number of parameters. The study of non-linear dependence structures as proposed in [55] is in an other perspective an interesting possible development.

Conclusion

Our overall approach enters a particular epistemological frame. On the one hand the multidisciplinary aspect, and on the other hand the importance of empirical component through computational exploration methods, make this approach typical of Complex Systems science, as it is recalled by the roadmap for Complex Systems having a similar structure [56]. It combines transversal research questions (horizontal integration of disciplines) with the development of heterogeneous multi-scalar approaches which encounter similar issues as the one we proposed to tackle (vertically integrated disciplines). The combination of empirical knowledge obtained from data mining, with knowledge obtained by modeling and simulation is generally central to the conception and exploration of multi-scalar heterogeneous models. Results presented here is an illustration of such an hybrid paradigm.

We proposed an abstract method to generate synthetic datasets in which correlation structure is controlled, but the empirical data required can be sparse or targeting macroscopic aggregated criteria. Its implementation in two very different fields shows its flexibility and the broad range of possible applications. More generally, it is crucial to favorise such practices of systematic validation of computational models by statistical analysis, in particular for agent-based models for which the question of validation remains an open issue.

Availability of data and material

All data and code used in this study, including simulation results, are openly available on git repositories and on the dataverse repository, following the links provided in main text.

Competing interests

The author declares to have no competing interests.

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Authors' contributions

JR designed the study, did the analysis and wrote the paper.

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