

## S2 Text : Semi-analytical analysis of the simplified model

### Partial Differential Equation

We propose to derive the PDE in a simplified setting. To recall the configuration given in main text, the system has one dimension, such that  $x \in \mathbb{R}$  with  $1/\delta x$  cells of size  $\delta x$ , and we use the expected values of cell population  $p(x, t) = \mathbb{E}[P(x, t)]$ . We furthermore take  $n_d = 1$ . Larger values would imply derivatives at an order higher than 2 but the following results on the existence of a stationary solution should still hold.

Denoting  $\tilde{p}(x, t)$  the intermediate populations obtained after the aggregation stage, we have

$$\tilde{p}(x, t) = p(x, t) + N_g \cdot \frac{p(x, t)^\alpha}{\sum_x p(x, t)^\alpha}$$

since all populations units are added independently. If  $\delta x \ll 1$  then  $\sum_x p^\alpha \simeq \int_x p(x, t)^\alpha dx$  and we write this quantity  $P_\alpha(t)$ . We furthermore write  $p = p(x, t)$  and  $\tilde{p} = \tilde{p}(x, t)$  in the following for readability.

The diffusion step is then deterministic, and for any cell not on the border ( $0 < x < 1$ ), if  $\delta t$  is the interval between two time steps, we have

$$\begin{aligned} p(x, t + \delta t) &= (1 - \beta) \cdot \tilde{p} + \frac{\beta}{2} [\tilde{p}(x - \delta x, t) + \tilde{p}(x + \delta x, t)] \\ &= \tilde{p} + \frac{\beta}{2} [(\tilde{p}(x + \delta x, t) - \tilde{p}) - (\tilde{p} - \tilde{p}(x - \delta x, t))] \end{aligned}$$

Assuming the partial derivatives exist, and as  $\delta x \ll 1$ , we make the approximation  $\tilde{p}(x + \delta x, t) - \tilde{p} \simeq \delta x \cdot \frac{\partial \tilde{p}}{\partial x}(x, t)$ , what gives

$$(\tilde{p}(x + \delta x, t) - \tilde{p}) - (\tilde{p} - \tilde{p}(x - \delta x, t)) = \delta x \cdot \left( \frac{\partial \tilde{p}}{\partial x}(x, t) - \frac{\partial \tilde{p}}{\partial x}(x - \delta x, t) \right)$$

and therefore at the second order

$$p(x, t + \delta t) = \tilde{p} + \frac{\beta \delta x^2}{2} \cdot \frac{\partial^2 \tilde{p}}{\partial x^2}$$

Substituting  $\tilde{p}$  gives

$$\begin{aligned} \frac{\partial^2 \tilde{p}}{\partial x^2} &= \frac{\partial^2 p}{\partial x^2} + \frac{N_G}{P_\alpha} \cdot \frac{\partial}{\partial x} \left[ \alpha \frac{\partial p}{\partial x} p^{\alpha-1} \right] \\ &= \frac{\partial^2 p}{\partial x^2} + \alpha \frac{N_G}{P_\alpha} \left[ \frac{\partial^2 p}{\partial x^2} p^{\alpha-1} + (\alpha - 1) \left( \frac{\partial p}{\partial x} \right)^2 p^{\alpha-2} \right] \end{aligned}$$

By supposing that  $\frac{\partial p}{\partial t}$  exists and that  $\delta t$  is small, we have  $p(x, t + \delta t) - p(x, t) \simeq \delta t \frac{\partial p}{\partial t}$ , what finally yields, by combining the results above, the partial differential equation

$$\delta t \cdot \frac{\partial p}{\partial t} = \frac{N_G \cdot p^\alpha}{P_\alpha(t)} + \frac{\alpha \beta (\alpha - 1) \delta x^2}{2} \cdot \frac{N_G \cdot p^{\alpha-2}}{P_\alpha(t)} \cdot \left( \frac{\partial p}{\partial x} \right)^2 + \frac{\beta \delta x^2}{2} \cdot \frac{\partial^2 p}{\partial x^2} \cdot \left[ 1 + \alpha \frac{N_G p^{\alpha-1}}{P_\alpha(t)} \right] \quad (1)$$

Initial conditions should be specified as  $p_0(x) = p(x, t_0)$ . To have a well-posed problem similar to more classical PDE problems, we need to assume a domain and boundary conditions. A finite support is expressed by  $p(x, t) = 0$  for all  $t$  and  $x$  such that  $|x| > x_m$ .

## Stationary solution for density

The non-linearity and the integral terms making the equation above out of the scope for analytical resolution, we study its behavior numerically in some cases. Taking a simple initial condition  $p_0(0) = 1$  and  $p_0(x) = 0$  for  $x \neq 0$ , we show that on a finite domain, density  $d(x, t)$  always converge to a stationary solution for large  $t$ , for a large set of values of  $(\alpha, \beta)$  with fixed  $N_G = 10$  ( $\alpha \in [0.4, 1.5]$  varying with step 0.025 and  $\log \beta \in [-1, -0.5]$  with step 0.1). We show in Fig. 1 the corresponding trajectories on a typical subset. The variation of the asymptotic distribution as a function of  $\alpha$  and  $\beta$  are not directly visible, as they depend on very low values of the outward flows at boundaries. We give in Fig. 2 their behavior, by showing the value of the maximum of the distribution.

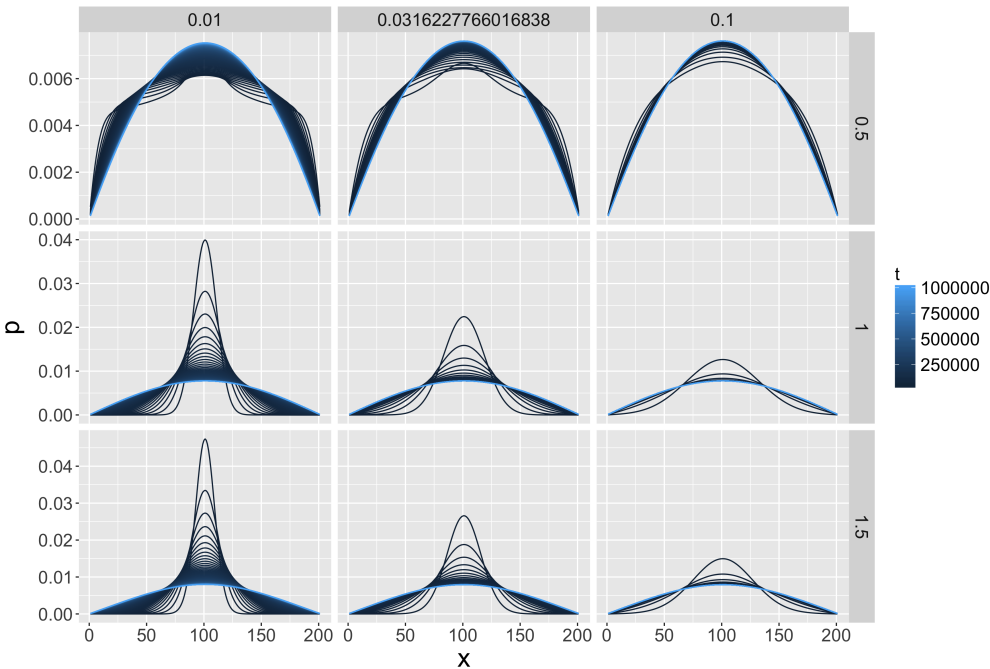


Figure 1

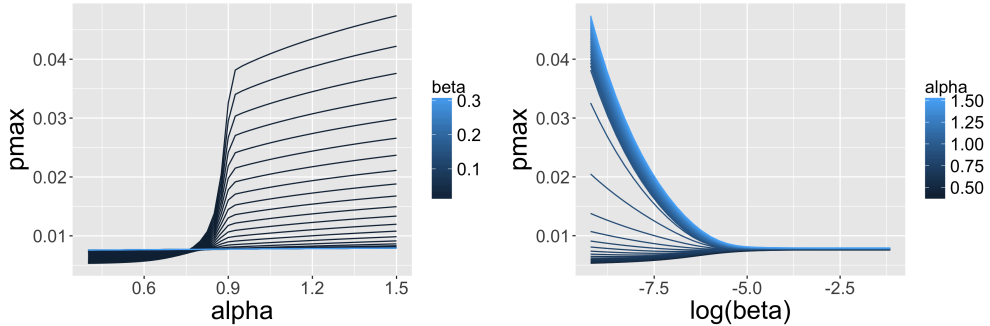


Figure 2

## Randomness and bifurcations

The previous analyses were done on a deterministic version of the system. How can randomness influence the trajectory ? On some configurations strong path-dependency can be shown.