S2 Text : Semi-analytical analysis of the simplified model

Partial Differential Equation

We propose to derive the PDE in a simplified setting. To recall the configuration given in main text, the system has one dimension, such that $x \in [0;1]$ with $1/\delta x$ cells of size δx, and we use the expected values of cell population $p(x,t) = \mathbb{E}[P(x,t)]$. We furthermore take $n_d = 1$. Larger values would imply derivatives at an order higher than 2 but the following results on the existence of a stationary solution should still hold.

Denoting $\tilde{p}(x,t)$ the intermediate populations obtained after the aggregation stage, we have

$$\tilde{p}(x,t) = p(x,t) + N_g \cdot \frac{p(x,t)^\alpha}{\sum_x p(x,t)^\alpha}$$

since all populations units are added independently. If $\delta x \ll 1$ then $\sum_x p^\alpha \simeq \int_x p(x,t)^\alpha dx$ and we write this quantity $P_\alpha(t)$. We furthermore write $p = p(x,t)$ and $\tilde{p} = \tilde{p}(x,t)$ in the following for readability.

The diffusion step is then deterministic, and for any cell not on the border ($0 < x < 1$), if δt is the interval between two time steps, we have

$$\begin{aligned} p(x, t+\delta t) &= (1-\beta) \cdot \tilde{p} + \frac{\beta}{2} \left[\tilde{p}(x-\delta x, t) + \tilde{p}(x+\delta x, t)\right] \\ &= \tilde{p} + \frac{\beta}{2} \left[(\tilde{p}(x+\delta x, t) - \tilde{p}) - (\tilde{p} - \tilde{p}(x-\delta x, t))\right] \end{aligned}$$

Assuming the partial derivatives exist, and as $\delta x \ll 1$, we make the approximation $\tilde{p}(x+\delta x, t) - \tilde{p} \simeq \delta x \cdot \frac{\partial \tilde{p}}{\partial x}(x,t)$, what gives

$$(\tilde{p}(x+\delta x, t) - \tilde{p}) - (\tilde{p} - \tilde{p}(x-\delta x, t)) = \delta x \cdot \left(\frac{\partial \tilde{p}}{\partial x}(x,t) - \frac{\partial \tilde{p}}{\partial x}(x-\delta x, t)\right)$$

and therefore at the second order

$$p(x,t+\delta t) = \tilde{p} + \frac{\beta \delta x}{2}$$

Characteristic stationary distance

Randomness and bifurcations