

# Urban Morphogenesis and the co-evolution of Transportation Networks and Territories

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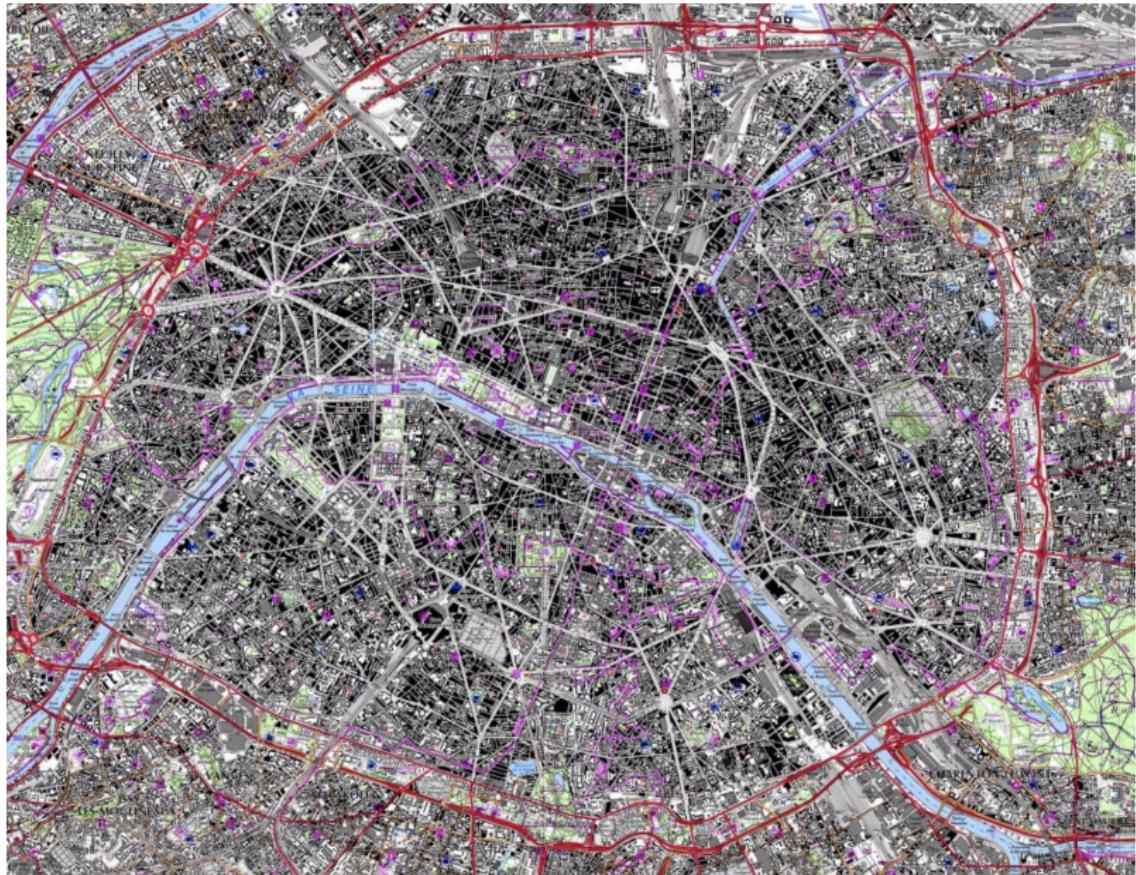
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Séminaire Equipe Paris

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# Complex processes of Urban Morphogenesis



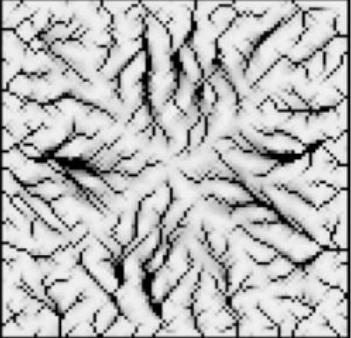
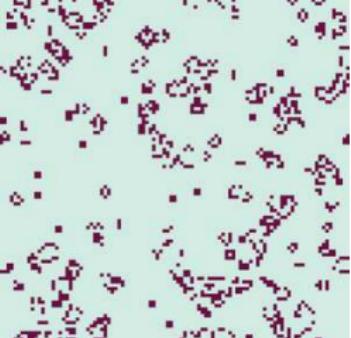
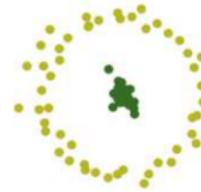
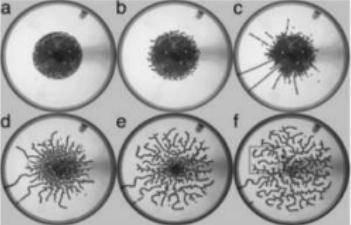
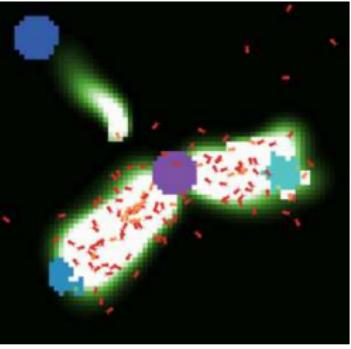
Source: Geoportal

# Complex processes of Urban Morphogenesis



*Source: Geoportail*

# What is Morphogenesis ?

	Physical	Biological	Engineered
Non Functional			
Functional			

Sources (in order by column). Ants, Erosion, Game of Life: NetLogo Library ; Arbotron [Jun and Hübler, 2005]; Industrial design [Aage et al., 2017]; Swarm chemistry [Sayama, 2007]

# Defining Morphogenesis

*Construction of an interdisciplinary definition in [Antelope et al., 2016]*

## Properties:

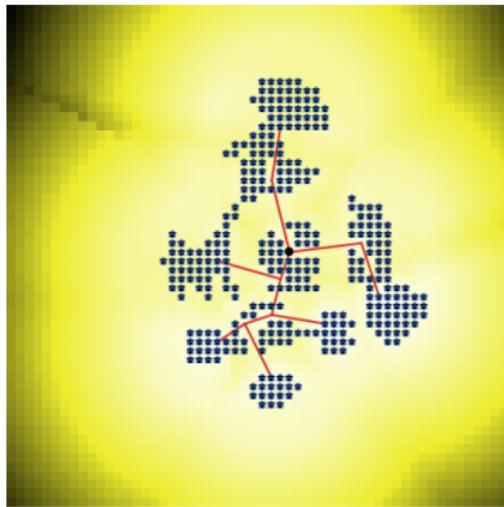
- Architecture links form and function
- Emergence strength [Bedau, 2002] diminishing with depth, whereas bifurcations increase [Thom, 1974]

**Meta-epistemological framework of imbricated notions:**

Self-organization  $\supseteq$  Morphogenesis  $\supseteq$  Autopoiesis  $\supseteq$  Life

**Definition of Morphogenesis :** *Emergence of the form and the function in a strongly coupled manner, producing an emergent architecture* [Doursat et al., 2012]

# Which models for Urban Morphogenesis ?



- At the crossroad between *Urban Simulation* and *Artificial Life*, few models try to integrate and explain the link between *Urban Form and Function*
- Importance of *parcimonious models*

*Example: a basic hybrid model based on elementary processes for density and network*  
*[Raimbault et al., 2014]*

**Research Objective :** Explore simple models to capture morphogenesis based on abstract representation of urban processes; test their ability to reproduce existing urban systems

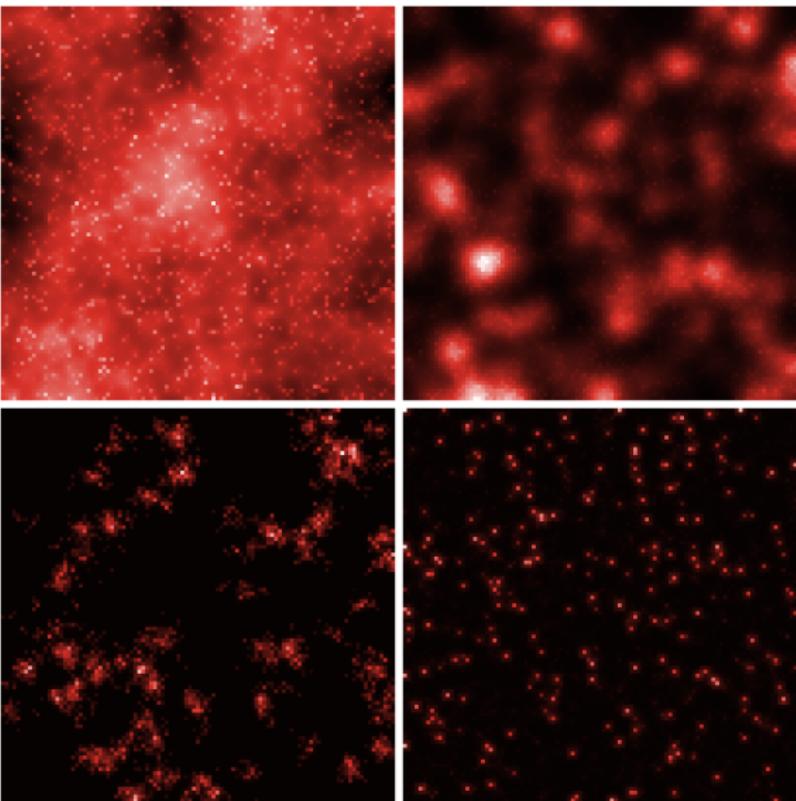
## A simple Reaction-diffusion model

- Crucial role of the interplay between concentration forces and dispersion forces [Fujita and Thisse, 1996] in keeping Urban Systems at the border of chaos
- Potentiality of aggregation mechanisms (such as Simon model) to produce power laws [Sheridan Dodds et al., 2016]
- Link with Reaction-diffusion approaches in Morphogenesis [Turing, 1952]
- Extension of a DLA-type model introduced by [Batty, 1991], with simple abstract processes of population aggregation and diffusion

# Model Formalization

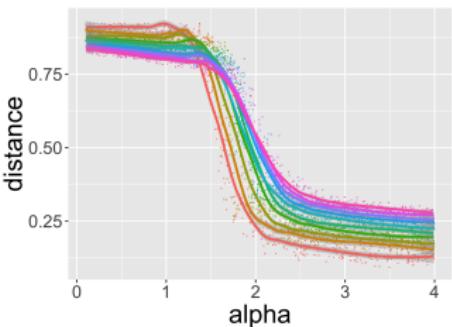
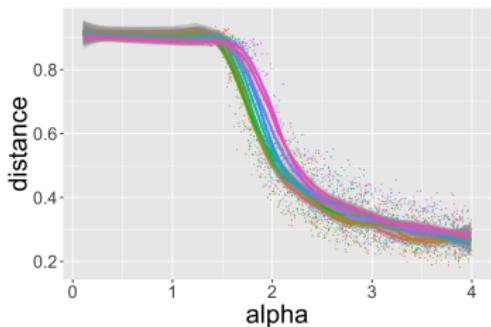
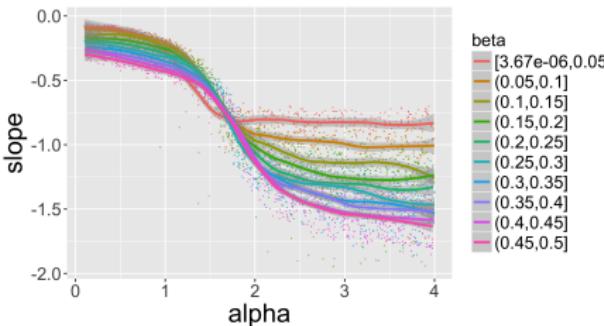
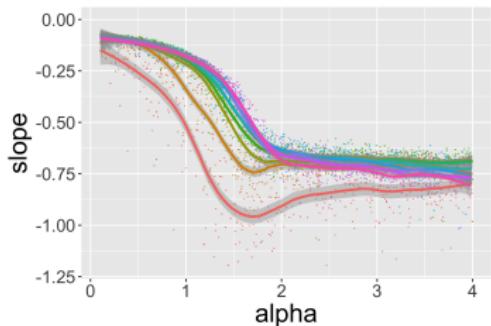
- Grid world with cell populations  $(P_i(t))_{1 \leq i \leq N^2}$ .
- At each time step:
  - ① Population growth with exogenous rate  $N_G$ , attributed independently to a cell following a preferential attachment of strength  $\alpha$
  - ② Population is diffused  $n_d$  times with strength  $\beta$
- Stopping criterion: fixed maximal population  $P_m$ .
- Output measured by morphological indicators: Moran index, average distance, rank-size hierarchy, entropy.

# Generating Population Distributions



*Examples of generated territorial shapes*

# Model behavior



beta

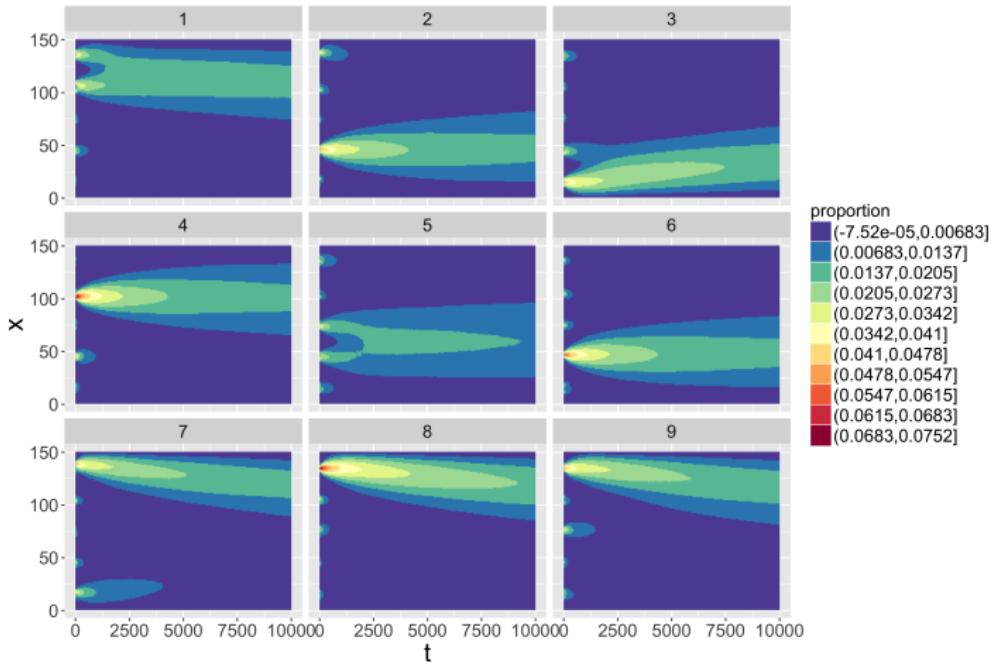
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- (0.4, 0.45]
- (0.45, 0.5]

beta

- [3.67e-06, 0.05]
- (0.05, 0.1]
- (0.1, 0.15]
- (0.15, 0.2]
- (0.2, 0.25]
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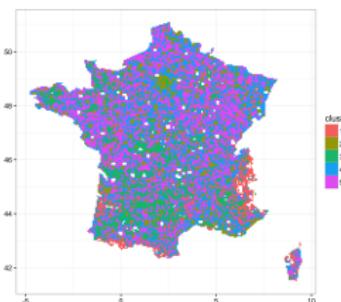
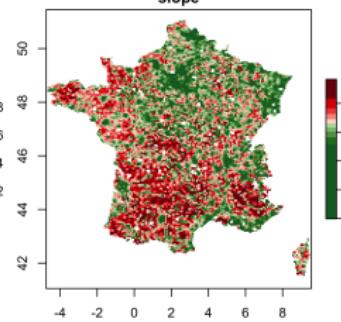
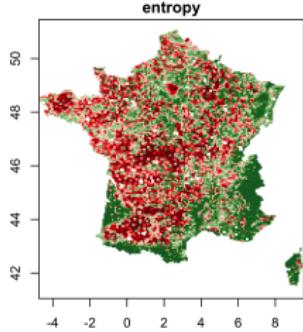
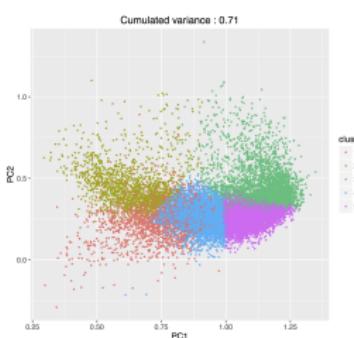
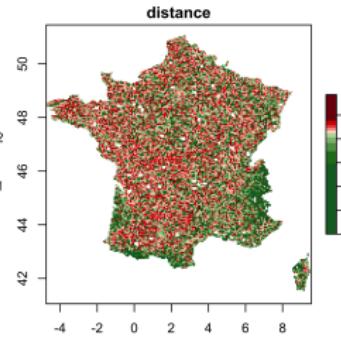
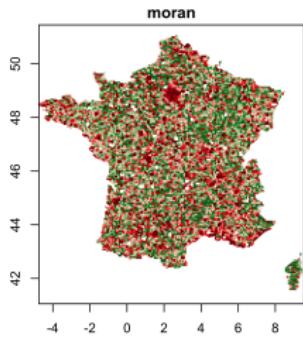
*Phase transitions of indicators unveiled by exploration of the parameter space (80000 parameter points, 10 repetitions each)*

# Path-dependence and frozen accidents



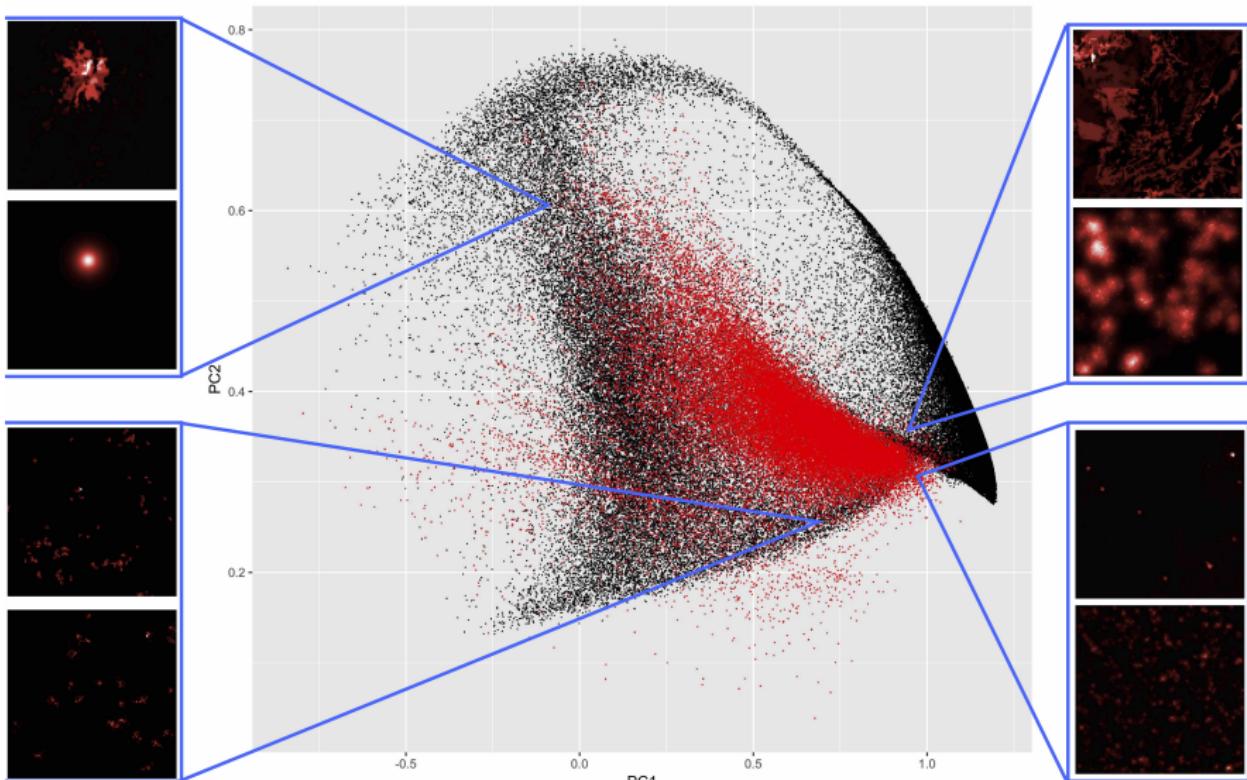
*Illustration of path-dependence in a simplified one-dimensional version of the model: cell trajectories in time for 9 independent repetitions from the same initial configuration.*

# Empirical Data for Calibration



*Computation of morphological indicators on population density data for Europe (shown here on France), morphological classification.*

# Model Calibration



*Brute force calibration by exploring the parameter space. Reproduction of most existing configuration in the morphological sense (here in principal plan).*

# Including more complex processes ?

*Which ontology to include more complex functional properties ?*

- Territorial systems as the strong coupling between territories and (potential and realized) networks [Dupuy, 1987]
- Networks convey functional notions of centralities and accessibility, among others

# Interactions between Networks and Territories

*Complex co-evolutive processes between Territories and Transportation Networks*



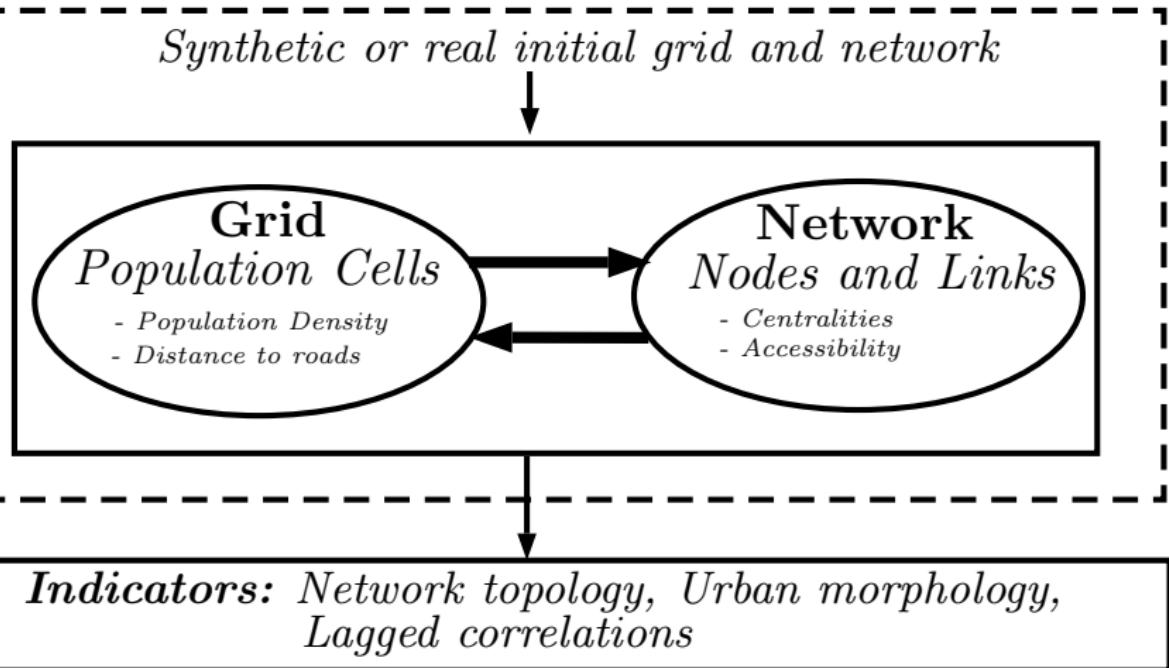
*Expanding HSR network in China and ambiguous effects (Source : fieldwork survey)*

# A Morphogenesis Model of co-evolution

- Coupled grid population distribution and vector transportation network, following the core of [Raimbault et al., 2014]
- Local morphological and functional variables determine a patch-value, driving new population attribution through preferential attachment ; combined to population diffusion (aggregation-diffusion processes studied in [Raimbault, 2017a])
- Network growth is also driven by morphological, functional and local network measures, following diverse heuristics corresponding to different processes (multi-modeling)

*Local variables and network properties induce feedback on both, thus a strong coupling capturing the **co-evolution***

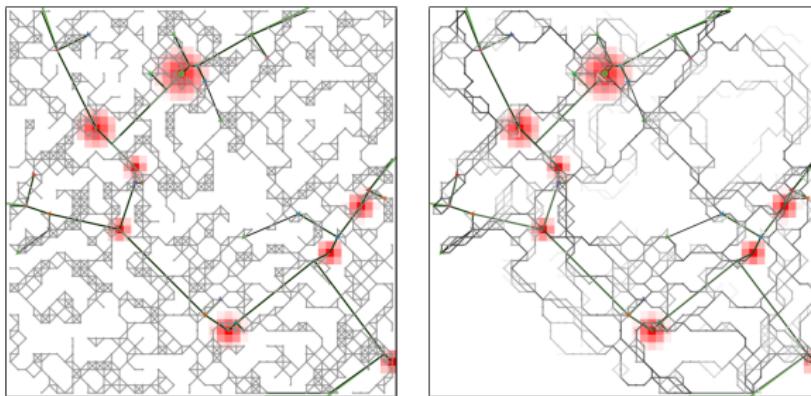
# Model : Specification



# Network Generation

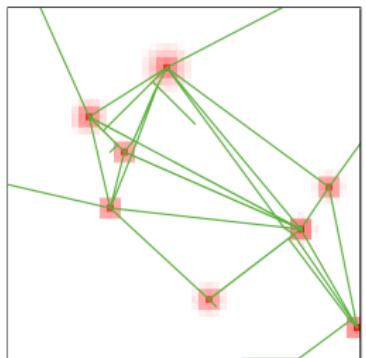
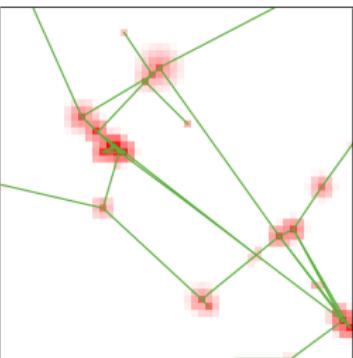
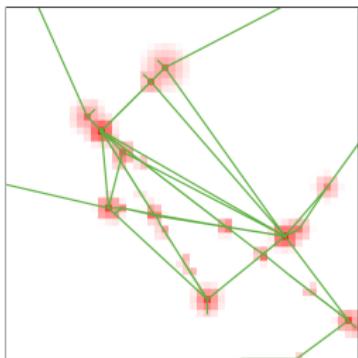
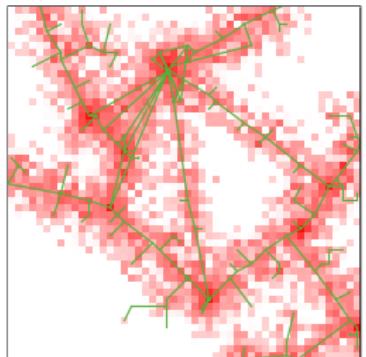
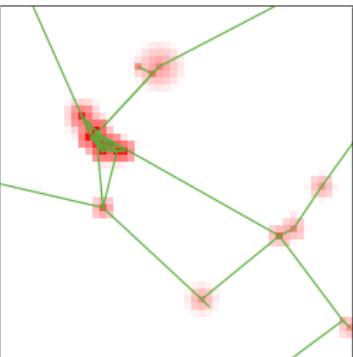
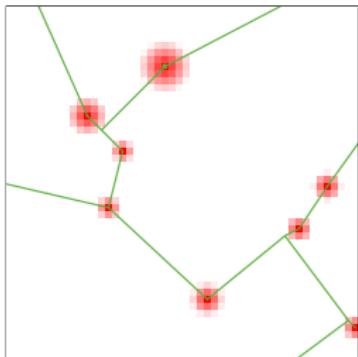
At fixed time steps :

- ① Add new nodes preferentially to new population and connect them
- ② Variable heuristic for new links, among: nothing, random, gravity-based deterministic breakdown, gravity-based random breakdown (from [Schmitt, 2014]), cost-benefits (from [Louf et al., 2013]), biological network generation (based on [Tero et al., 2010])



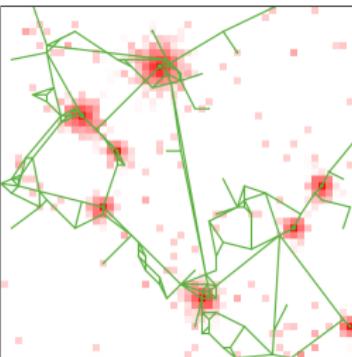
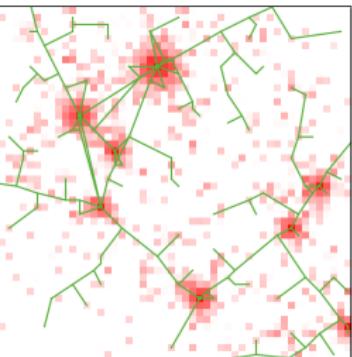
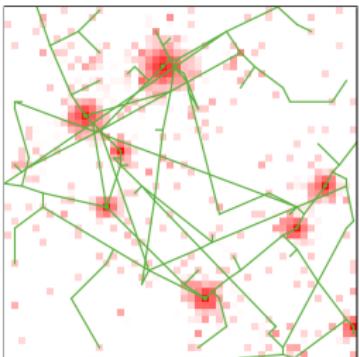
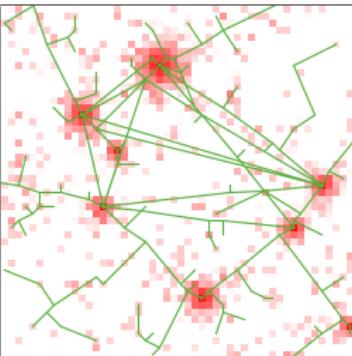
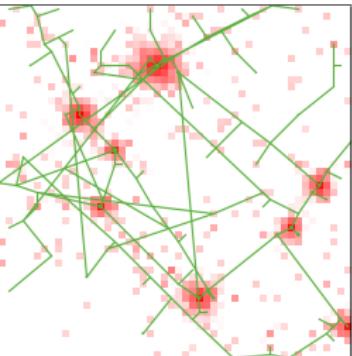
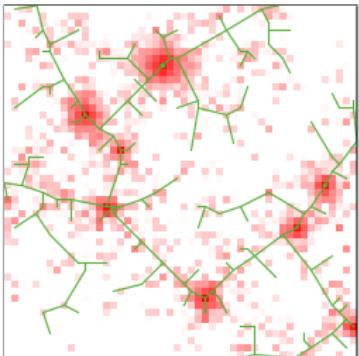
*Intermediate stage for biological network generation*

# Generated Urban Shapes: Urban Form



*In order: setup; accessibility driven; road distance driven; betweenness driven; closeness driven; population driven.*

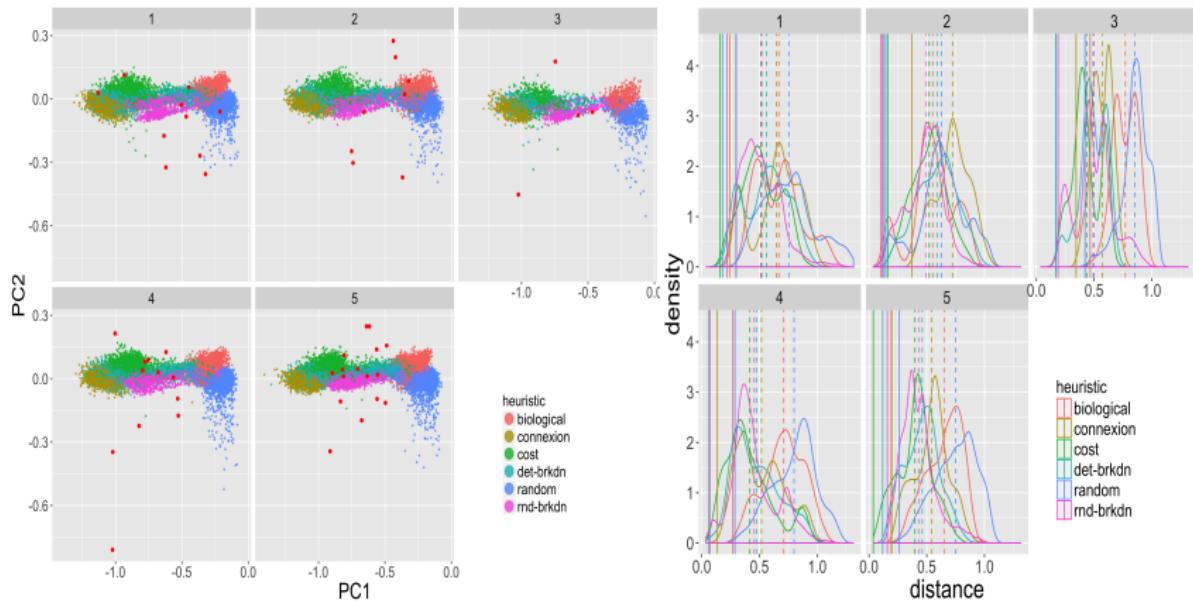
# Generated Urban Shapes: Network



*In order: connection; random; deterministic breakdown; random breakdown; cost-driven; biological.*

# Results : Network Heuristics

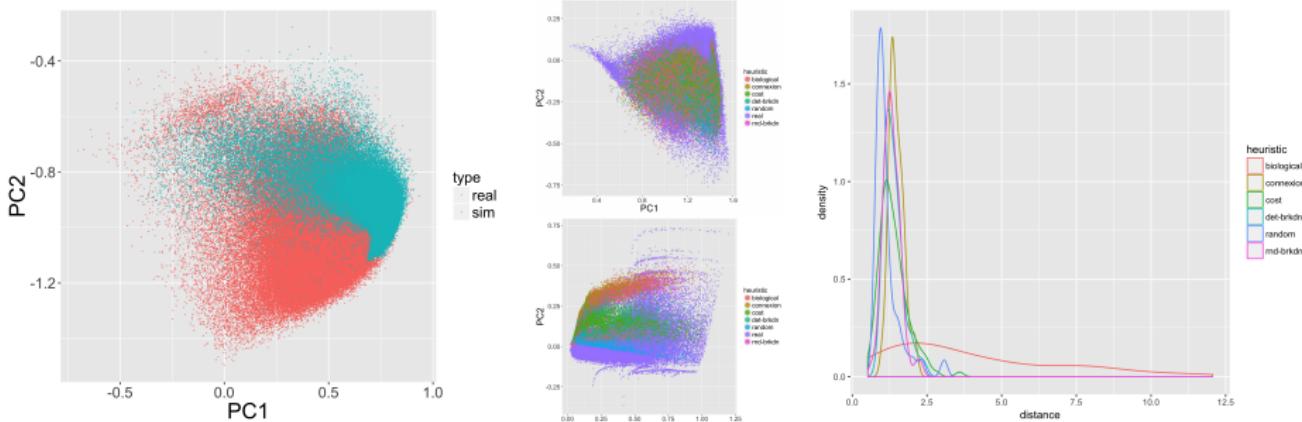
*Comparison of feasible space for network indicators with fixed density*



(Left) Feasible spaces by morphological class and network heuristic; (Right) Distribution of distances to topologies of real networks

# Results : Calibration

Calibration (model explored with OpenMole [Reuillon et al., 2013],  $\sim 10^6$  model runs) at the first order on morphological and topological objectives, and on correlations matrices.

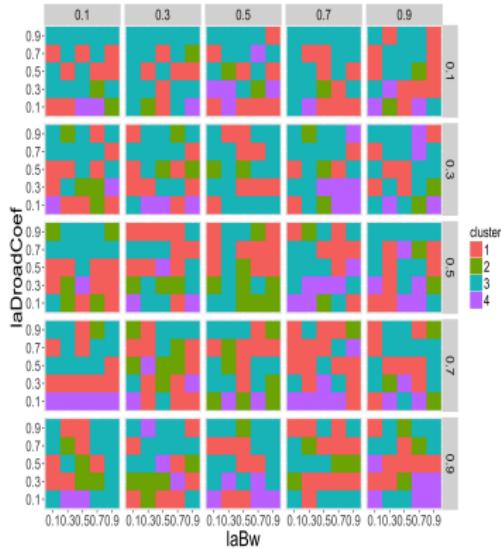
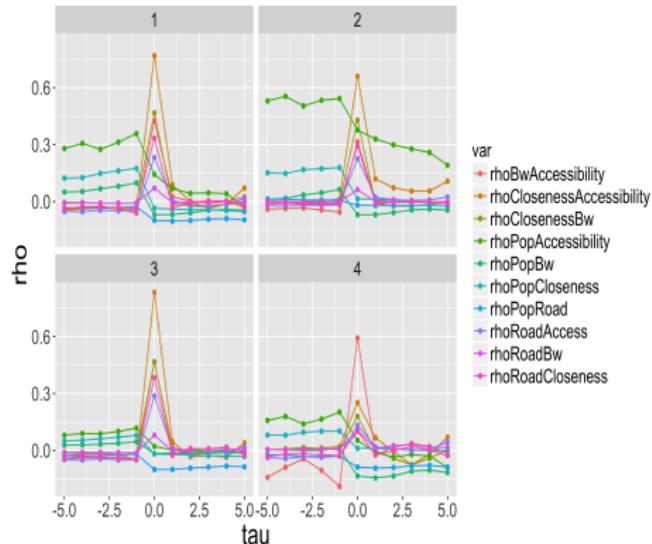


(Left) Full indicator space; (Middle) Morphological and Topology, by network heuristic;  
(Right) Distance distribution for cumulated distance for indicators and correlations.

# Results : Causality Regimes

*Unsupervised learning on lagged correlations between local variables unveils a diversity of causality regimes*

→ Link between co-evolution regime and morphogenetic properties of the urban system



(Left) Lagged correlation profiles of cluster centers; (Right) Distribution of regimes across parameter space

# Discussion

## Implications

- This rather simple model reproduces most of existing urban forms in Europe for both population distribution and road network : which intrinsic dimension to the urban system and its morphological aspect ?
- Ability to reproduce static correlations and a variety of dynamical lagged correlation regimes suggests that the model captures some of the processes of co-evolution

## Developments

- Towards a dynamical calibration ? Need of dynamical data
- Investigate the link between spatial non-stationarity and non-ergodicity through simulation by the model
- Compare network generation in a “fair” way (correcting for additional parameters, open question for models of simulation)

# Conclusion

- Code, data and results available at  
<https://github.com/JusteRaimbault/CityNetwork>
- Paper on arXiv
- Acknowledgments : We thank the *European Grid Infrastructure* and its *National Grid Initiatives* (*France-Grilles* in particular) to give the technical support and the infrastructure.

# Reserve Slides

# Morphogenesis concepts

- **Morphogenesis and Self-Organisation** : when does a system exhibit an architecture ? Insights from Morphogenetic Engineering [?]. Architecture : the relation between the form and the function ?
- **Scales, Units and Boundaries** From local interactions to global information flow (Holland's *signal and boundaries* [Holland, 2012]: morphogenesis as the development of Complex Adaptive Systems ?)
- **Symmetry and Bifurcations** : on quantitative becoming qualitative. René Thom's *theory of catastrophes* [Thom, 1974]
- **Life and Death** : link with autopoiesis and cognition [Bourgine and Stewart, 2004] ; co-evolution of subsystems as an alternative definition ? In psychology, attractors of the mind.

# Catastrophe Theory

A system is viewed as its internal state  $X_w$ , where  $w \in W$  is a control parameter.

Catastrophe set  $K \subset W$  is where the system endures phase transition.

Thom classified possible topologies for  $K$  depending on the dimension of  $W$ .

# Modeling Urban Morphogenesis

- [Makse et al., 1998] correlated growth;
- [Murcio et al., 2015] multi-scale migration and percolation;
- [Bonin et al., 2012] qualitative differentiation of urban function;
- [Achibet et al., 2014] procedural model at the micro-scale

## Model classification : PDE

The one-dimensional model verifies the PDE :

$$\delta t \cdot \frac{\partial p}{\partial t} = \frac{N_G \cdot p^\alpha}{P_\alpha(t)} + \frac{\alpha \beta (\alpha - 1) \delta x^2}{2} \cdot \frac{N_G \cdot p^{\alpha-2}}{P_\alpha(t)} \cdot \left( \frac{\partial p}{\partial x} \right)^2 + \frac{\beta \delta x^2}{2} \cdot \frac{\partial^2 p}{\partial x^2} \cdot \left[ 1 + \alpha \right] \quad (1)$$

# Morphological indicators

- ① Rank-size slope  $\gamma$ , given by  $\ln(P_{\tilde{i}}/P_0) \sim k + \gamma \cdot \ln(\tilde{i}/i_0)$  where  $\tilde{i}$  are the indexes of the distribution sorted in decreasing order.
- ② Entropy of the distribution:

$$\mathcal{E} = \sum_{i=1}^M \frac{P_i}{P} \cdot \ln \frac{P_i}{P} \quad (2)$$

$\mathcal{E} = 0$  means that all the population is in one cell whereas  $\mathcal{E} = 0$  means that the population is uniformly distributed.

- ③ Spatial-autocorrelation given by Moran index, with simple spatial weights given by  $w_{ij} = 1/d_{ij}$

$$I = M \cdot \frac{\sum_{i \neq j} w_{ij} (P_i - \bar{P}) \cdot (P_j - \bar{P})}{\sum_{i \neq j} w_{ij} \sum_i (P_i - \bar{P})^2}$$

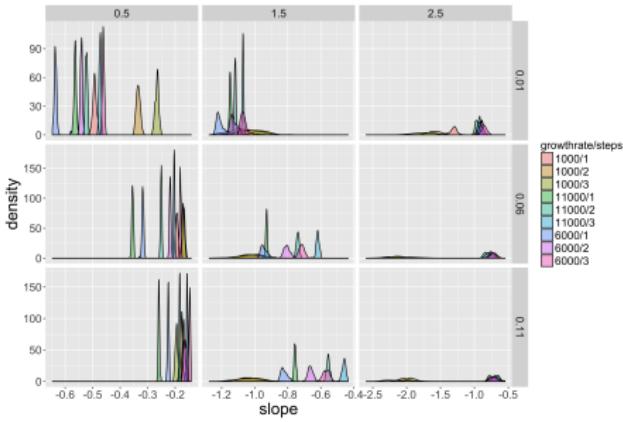
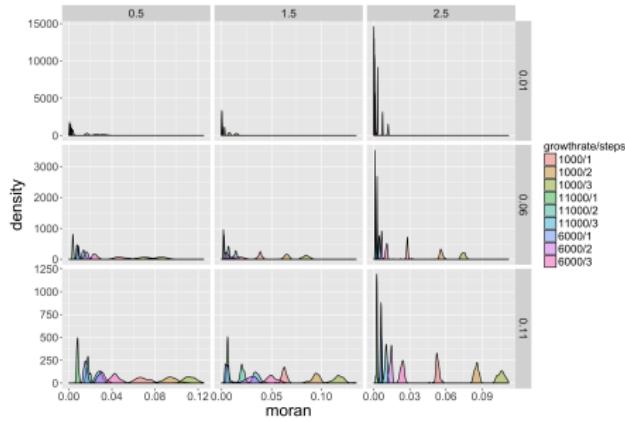
- ④ Mean distance between individuals

$$\bar{d} = \frac{1}{d_M} \cdot \sum_{i < j} \frac{P_i P_j}{P^2} \cdot d_{ij}$$

where  $d_M$  is a normalisation constant

# Model behavior : Convergence

Large number of repetitions show good convergence properties



## Empirical indicators computation

- Eurostat population density raster (100m, simplified at 500m resolution)
- Overlapping (10km offset) squares of 50km side : equivalent to smoothing, removes window shape effect. Not very sensitive to window size (tested with 30km and 100km)
- Indicators computed using Fast Fourier Transform Convolution
- Classification using repeated k-means ; number of clusters taken at transition in clustering coefficient.

# Defining co-evolution

No clear definition of co-evolution in the literature : [Bretagnolle, 2009] distinguishes “reciprocal adaptation” where a sense of causality can clearly be identified, from co-evolutive regimes

[?] identifies multiple causality regimes in a simple strongly coupled growth model → to be put in perspective with a theoretical definition of co-evolution based on the conjunction of Morphogenesis and the Evolutive Urban Theory, summarised by [Raimbault, 2017b]

# Modeling Co-evolution

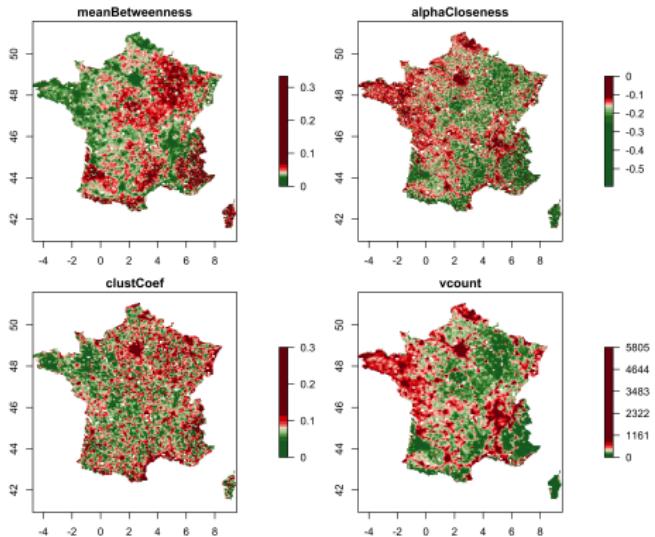
[Baptiste, 2010] system dynamics with evolving capacities

[Wu et al., 2017] population diffusion and network growth

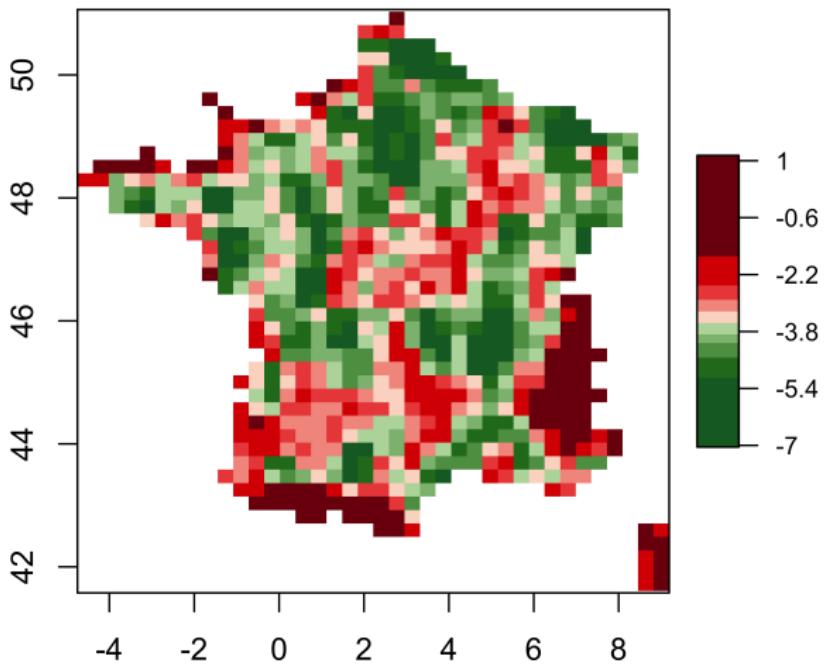
[Blumenfeld-Lieberthal and Portugali, 2010] and [Schmitt, 2014] : random potential breakdown for network growth.

[Barthélemy and Flammini, 2009] geometrical network growth model making network topology co-evolve with vertex density

# Empirical Data : network indicators



# Empirical Data : correlations



Network Topology measured by:

- Betweenness and Closeness centralities: average and hierarchy
- Accessibility (weighted closeness)
- Efficiency (network pace relative to euclidian distance)
- Mean path length, diameter

# Model specification

Patch utility given by  $U_i = \sum_k w_k \cdot \tilde{x}_k$  with  $\tilde{x}_k$  normalized local variables among population, betweenness and closeness centrality, distance to roads, accessibility ; aggregation done with probability  $(U_i / \sum_k U_k)^\alpha$  ; diffusion among neighbors  $n_d$  times with strength  $\beta$

## Network Generation :

Adding a fixed number  $n_N$  of new nodes : for patches such that  $d_r < d_0$ , probability to receive a node is

$$p = P/P_{max} \cdot (d_M - d)/d_M \cdot \exp\left(-((d_r - d_0)/\sigma_r)^2\right)$$

Nodes connected the shortest way to existing network.

## General model parameters :

- Patch utility weights  $w_k$
- General network generation parameters: growth time steps  $t_N$ , maximal additional links

- ① Gravity potential given by

$$V_{ij}(d) = \left[ (1 - k_h) + k_h \cdot \left( \frac{P_i P_j}{P^2} \right)^{\gamma} \right] \cdot \exp \left( -\frac{d}{r_g(1 + d/d_0)} \right)$$

- ②  $k \cdot N_L$  links are selected with lowest  $V_{ij}(d_N)/V_{ij}(d_{ij})$ , among which  $N_L$  links with highest (lest costly) are realized
- ③ Network is planarized

# Biological Network generation

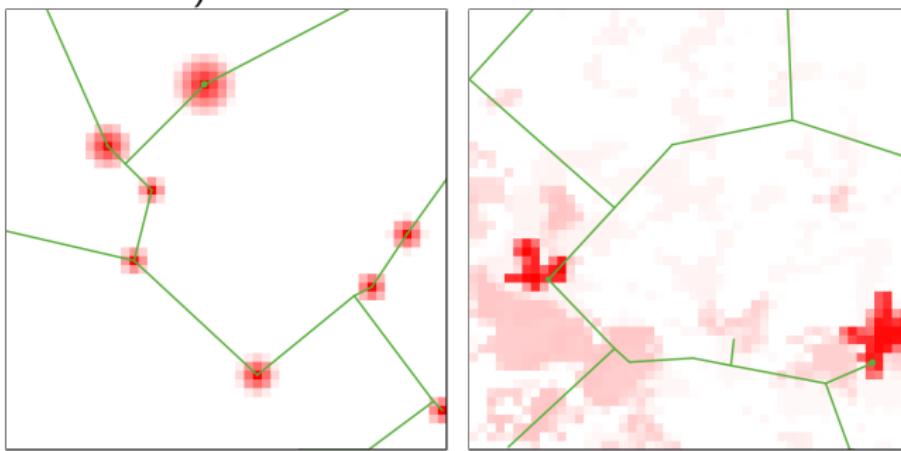
Adding new links with biological heuristic:

- ① Create network of potential new links, with existing network and randomly sampled diagonal lattice
- ② Iterate for  $k$  increasing ( $k \in \{1, 2, 4\}$  in practice) :
  - Using population distribution, iterate  $k \cdot n_b$  times the slime mould model to compute new link capacities
  - Delete links with capacity under  $\theta_d$
  - Keep the largest connected component
- ③ Planarize and simplify final network

# Model setup

**Synthetic setup:** rank-sized monocentric cities, simple connection with border nodes to avoid border effects

**Real setup:** Population density raster at 500m resolution (European Union, from Eurostat)

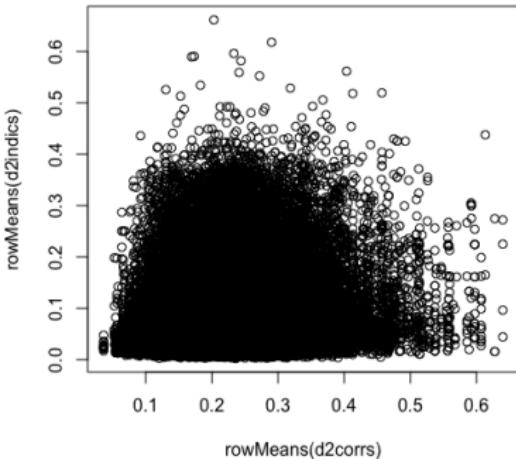
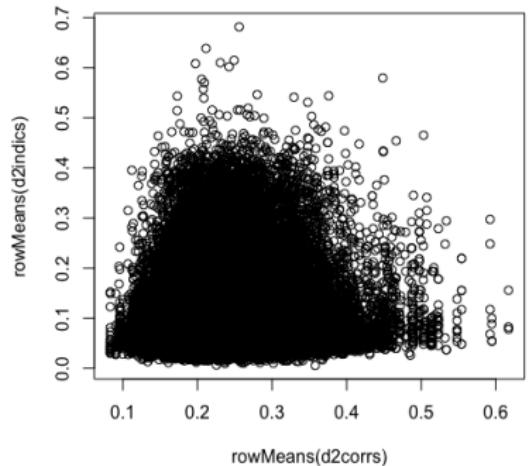


**Stopping conditions:** fixed final time; fixed total population; fixed network size.

# Calibration Method

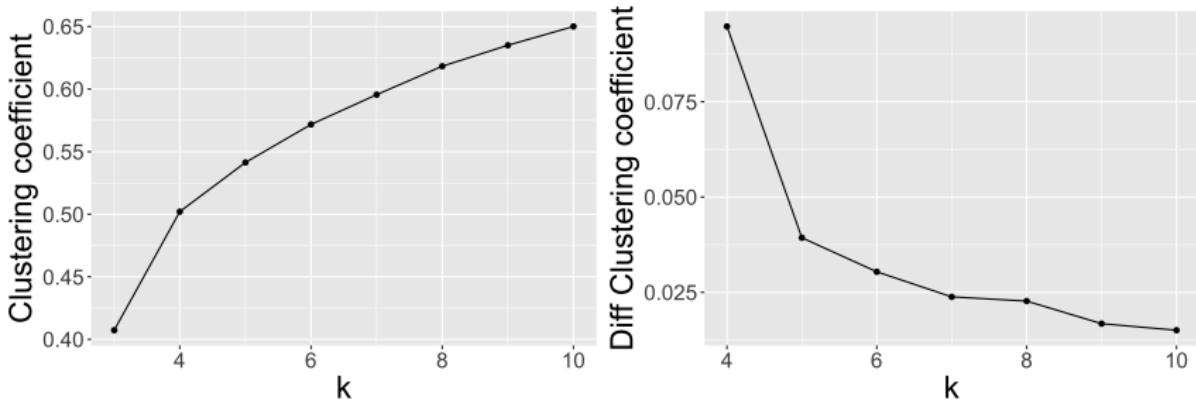
- Brute force exploration of a LHS sampling, 10 repetitions of the model for each parameter point.
- For each simulated point, closest in indicator space (euclidian distance for normalized indicators) among real points are selected.
- Among these, point with lowest distance to correlation matrix are taken.

# Calibration : optimal points



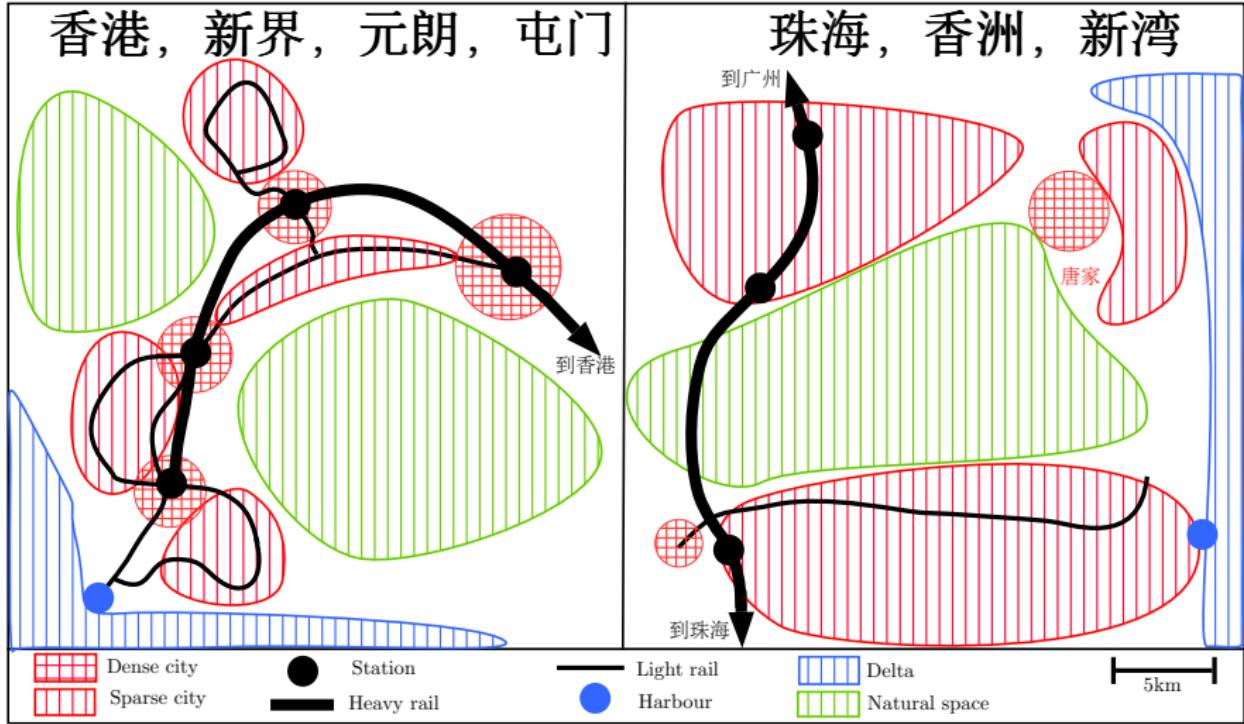
*Pareto plots of distance to indicators and distance to correlation matrices, for a given simulated configuration and all real points.*

# Causality regimes: clustering



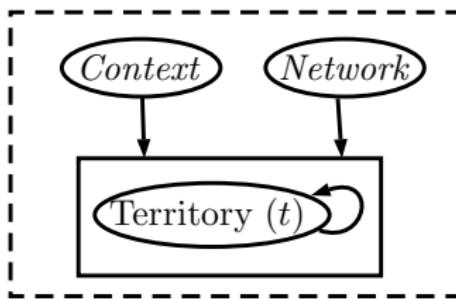
*Clustering coefficient (left) and its derivative (right) as a function of number of clusters*

# Implementations of TOD

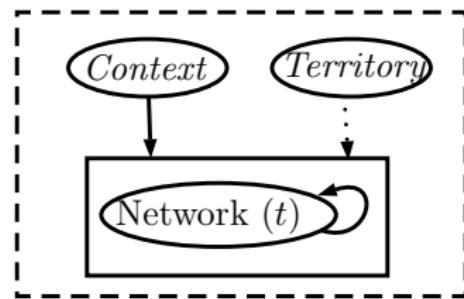


# Co-evolution Models

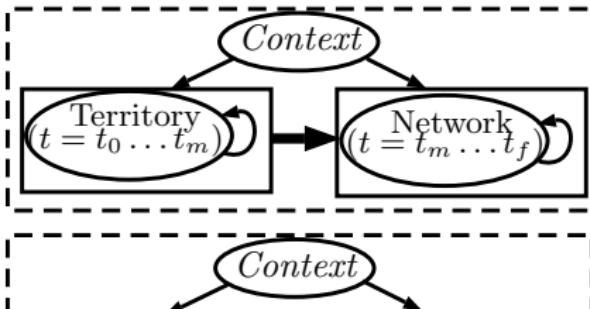
*LUTI*



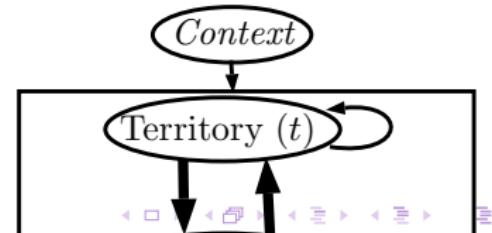
*Network Growth*



*Weak Coupling*



*Strong Coupling / Co-evolution*



# Citation Network

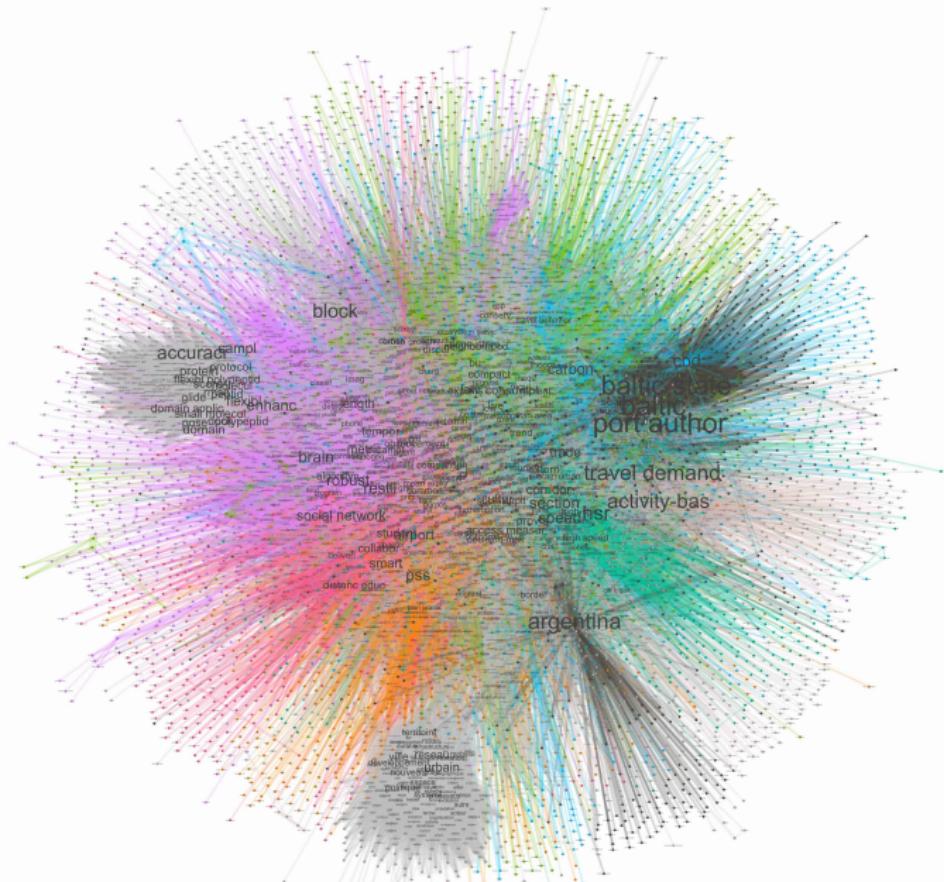


# Citation Network Properties

For the core (hence full) subnetwork :

- Size  $V = 3510$ , Mean degree  $\bar{d} = 2.53$  and density  $\gamma = 0.0013$ , weakly connected.
- 13 communities, directed modularity [Nicosia et al., 2009] 0.66 (null model gives  $0.0005 \pm 0.0051$  on  $N = 100$  bootstraps)
- Content : LUTI (18%), Urban and Transportation Geography (16%), Infrastructure Planning (12%), TOD (6%), Spatial Networks (17%), Accessibility studies (18%)

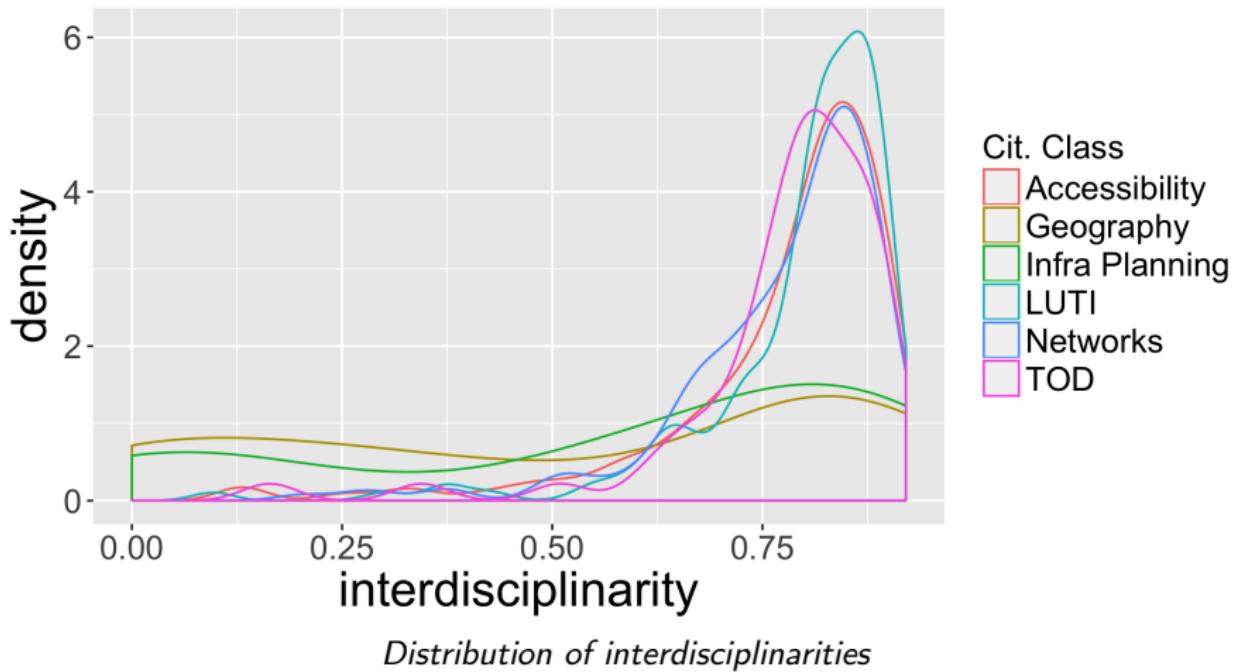
# Semantic Network



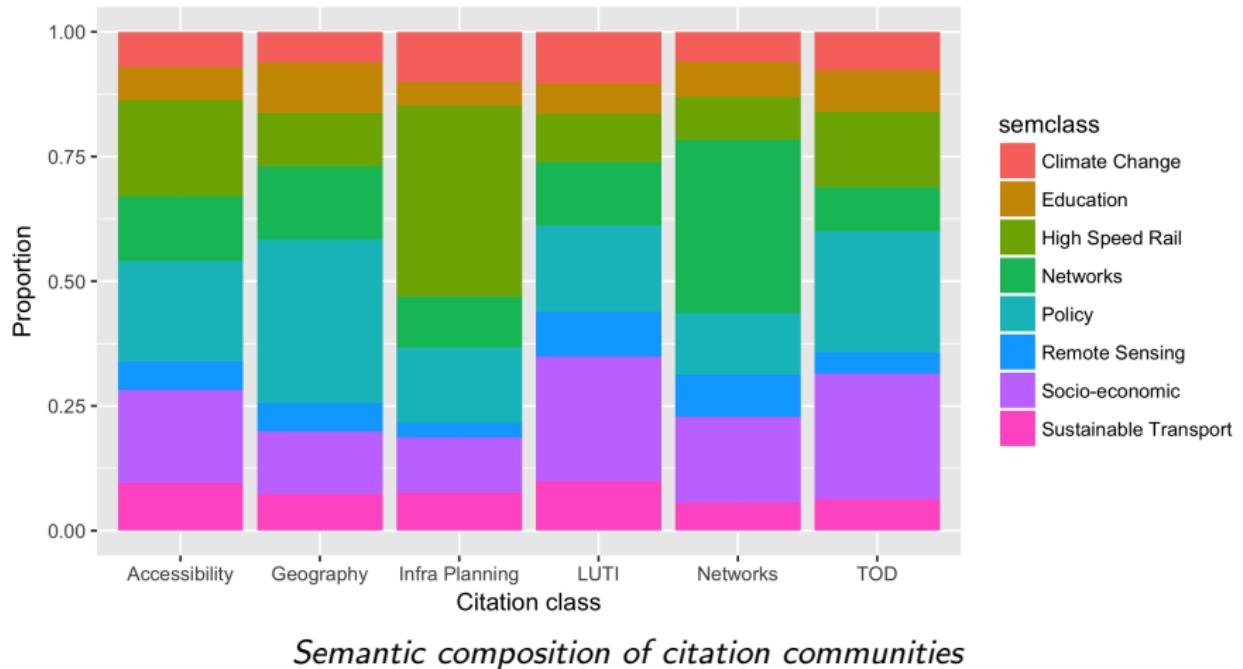
# Semantic communities

Name	Size	Weight	Keywords
Networks	820	13.57%	social network, spatial network, res
Policy	700	11.8%	actor, decision-mak, societi
Socio-economic	793	11.6%	neighborhood, incom, live
High Speed Rail	476	7.14%	high-spe, corridor, hsr
French Geography	210	6.08%	système, développement, territoire
Education	374	5.43%	school, student, collabor
Climate Change	411	5.42%	mitig, carbon, consumpt
Remote Sensing	405	4.65%	classif, detect, cover
Sustainable Transport	370	4.38%	sustain urban, travel demand, activi
Traffic	368	4.23%	traffic congest, cbd, capit
Maritime Networks	402	4.2%	govern model, seaport, port author
Environment	289	3.79%	ecosystem servic, regul, settlement
Accessibility	260	3.23%	access measur, transport access, urb
Agent-based Modeling	192	3.18%	agent-bas, spread, heterogen
Transportation planning	192	3.18%	transport project, option, cba
Mobility Data Mining	168	2.49%	human mobil, movement, mobil phone
Health Geography	196	2.49%	healthcar, inequ, exclus
Freight and Logistics	239	2.06%	freight transport, citi logist, modali
Measuring	166	1.0%	score, sampl, metric

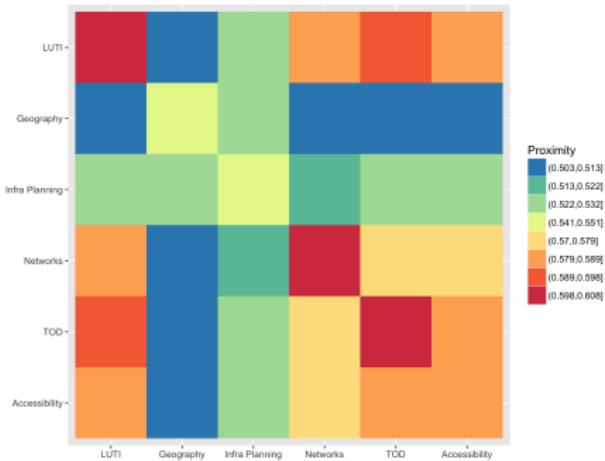
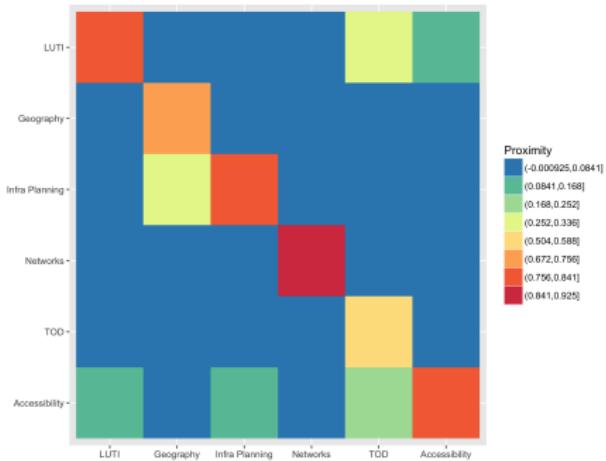
# Interdisciplinarity patterns



# Interdisciplinarity patterns



# Community Proximities



*Proximities between citation communities (Left) and semantic communities (Right)*

# Macroscopic Model Rationale

**Rationale :** extend an interaction model for system of cities by including physical network as an additional carrier of spatial interactions (see [Raimbault, 2016] for developed theoretical context)

- Work under Gibrat independence assumptions, i.e.  $\text{Cov}[P_i(t), P_j(t)] = 0$ . If  $\vec{P}(t+1) = \mathbf{R} \cdot \vec{P}(t)$  where  $\mathbf{R}$  is also independent, then  $\mathbb{E}[\vec{P}(t+1)] = \mathbb{E}[\mathbf{R}] \cdot \mathbb{E}[\vec{P}](t)$ . Consider expectancies only (higher moments computable similarly)
- With  $\vec{\mu}(t) = \mathbb{E}[\vec{P}(t)]$ , we generalize this approach by taking  $\vec{\mu}(t+1) = f(\vec{\mu}(t))$

# Macroscopic Model Formulation

Let  $\vec{\mu}(t) = \mathbb{E}[\vec{P}(t)]$  cities population and  $(d_{ij})$  distance matrix

Model specified by

$$f(\vec{\mu}) = r_0 \cdot \mathbf{Id} \cdot \vec{\mu} + \mathbf{G} \cdot \mathbf{1} + \mathbf{N}$$

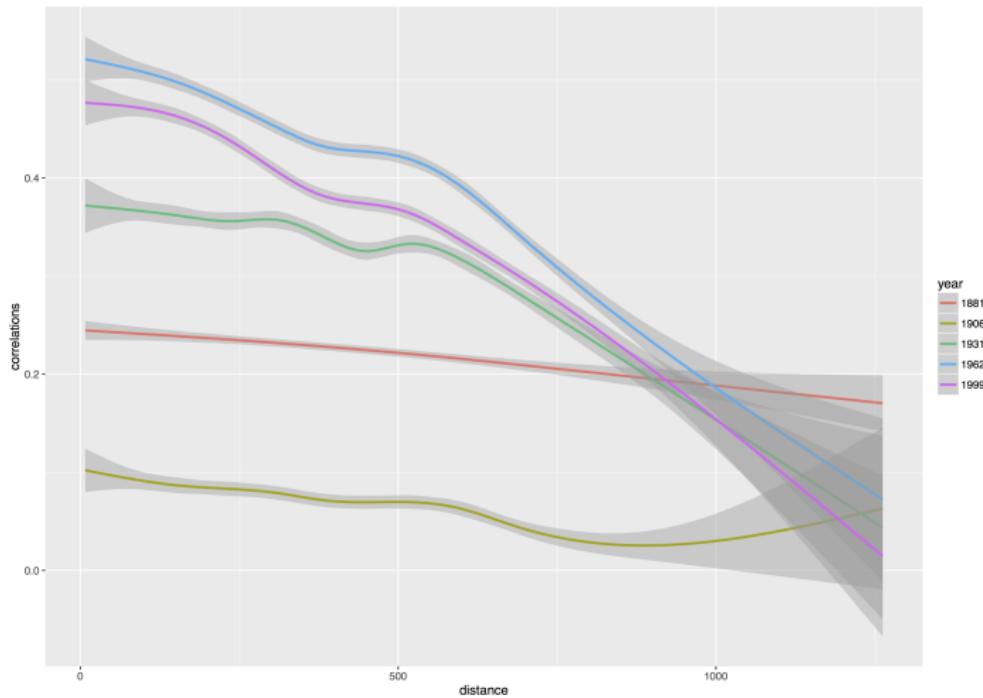
with

- $G_{ij} = w_G \cdot \frac{V_{ij}}{\langle V_{ij} \rangle}$  and  $V_{ij} = \left( \frac{\mu_i \mu_j}{\sum \mu_k^2} \right)^{\gamma_G} \exp(-d_{ij}/d_G)$
- $N_i = w_N \cdot \sum_{kl} \left( \frac{\mu_k \mu_l}{\sum \mu} \right)^{\gamma_N} \exp(-d_{kl,i})/d_N$  where  $d_{kl,i}$  is distance to shortest path between  $k, l$  computed with slope impedance ( $Z = (1 + \alpha/\alpha_0)^{n_0}$  with  $\alpha_0 \simeq 3$ )

# Data : stylized facts

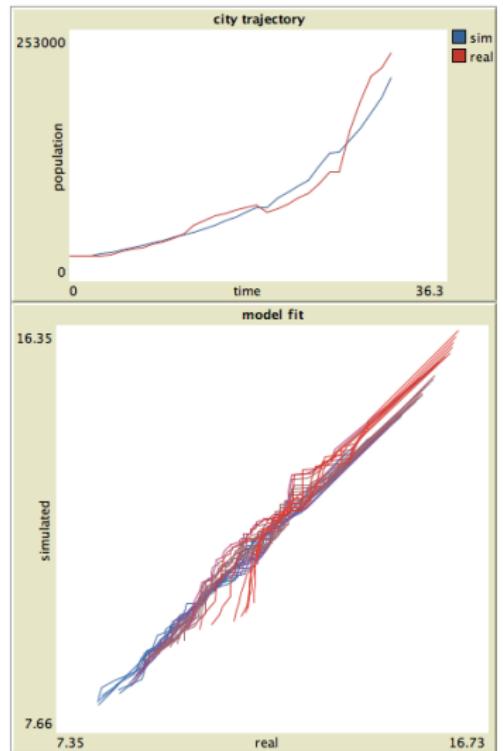
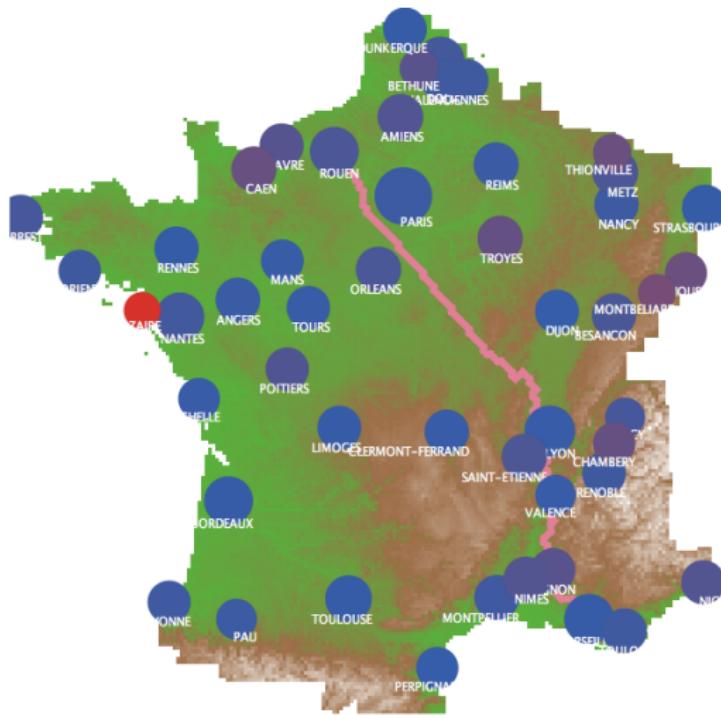
Population data for French-cities (Pumain-INED database : 1831-1999)

*Non-stationarity of log-returns correlations function of distance*



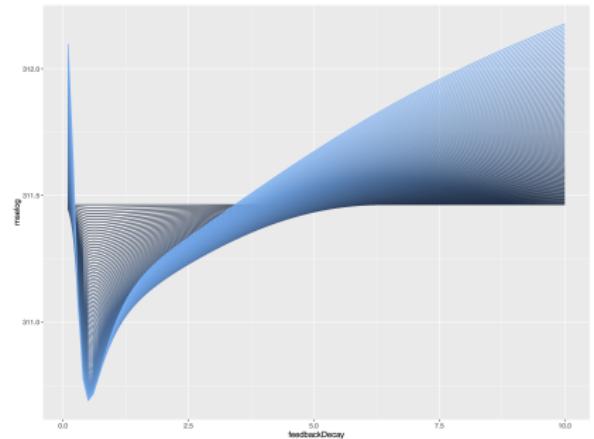
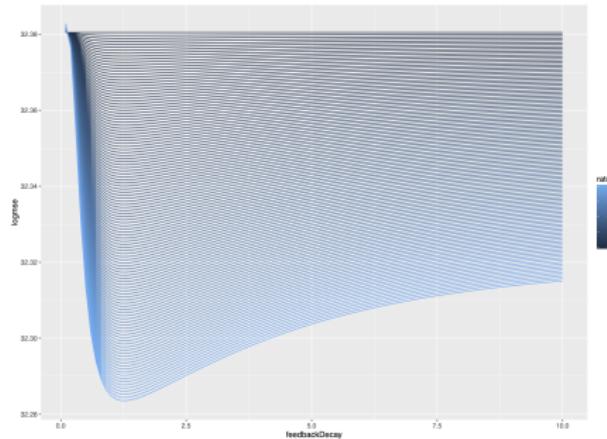
# Geographic abstract network

*Physical transportation network abstracted through a geographical shortest path network*



# Results : model exploration

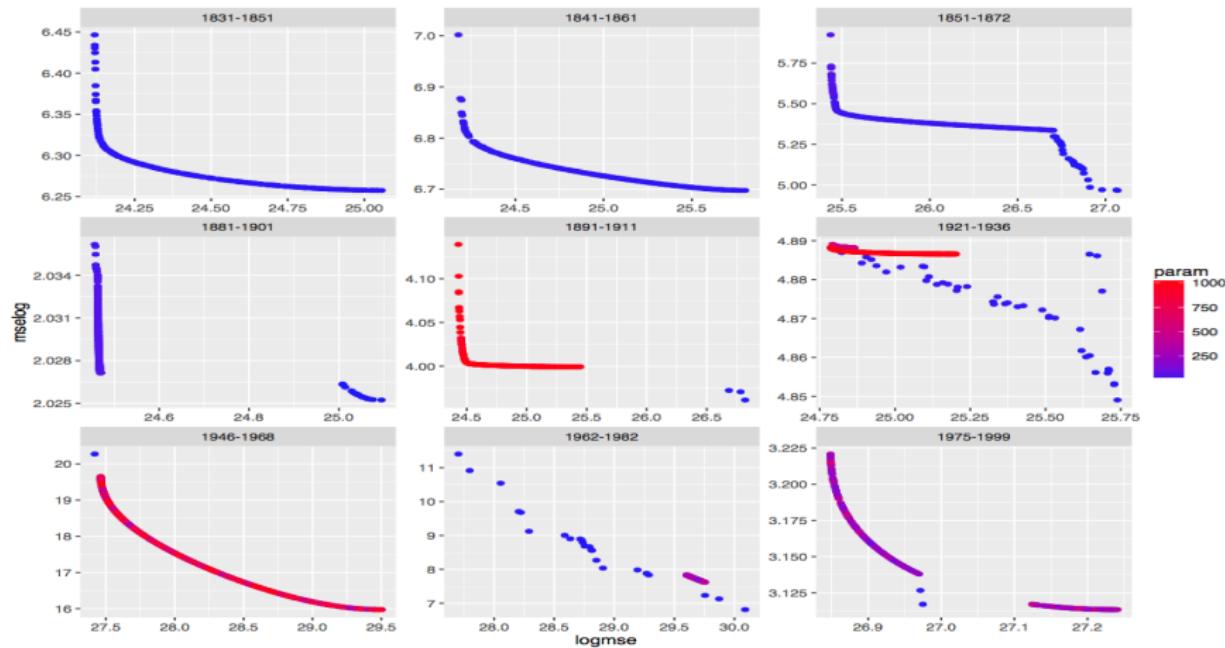
*Evidence of physical network effects : fit improve through feedback at fixed gravity*



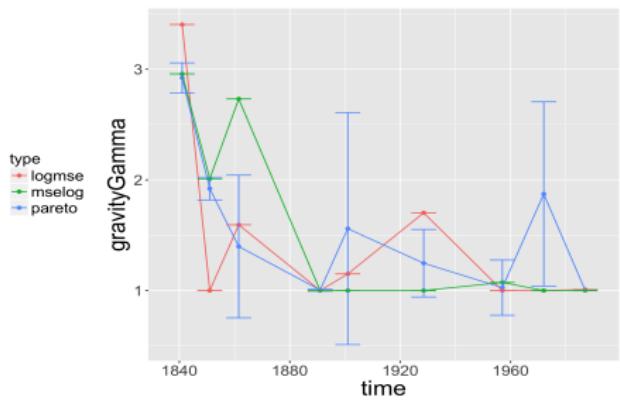
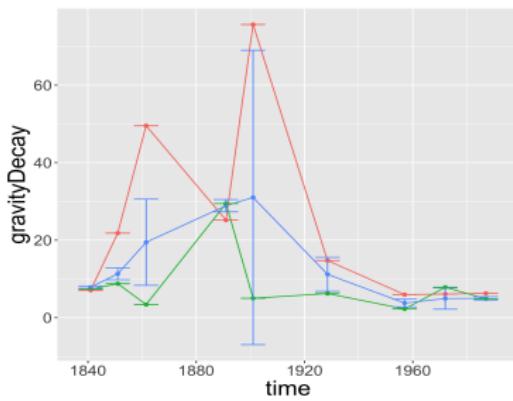
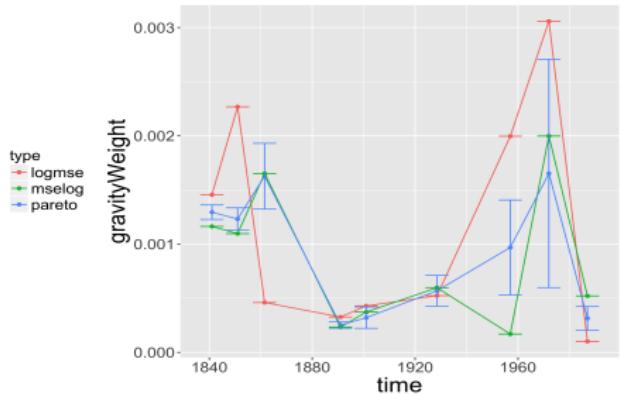
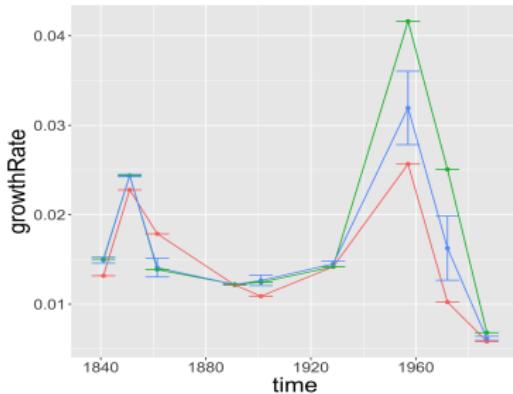
# Results : model calibration

Model calibration using GA on computation grid, with software OpenMole [Reuillon et al., 2013]

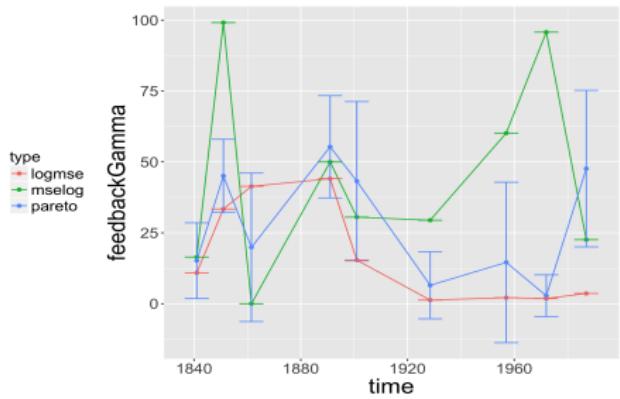
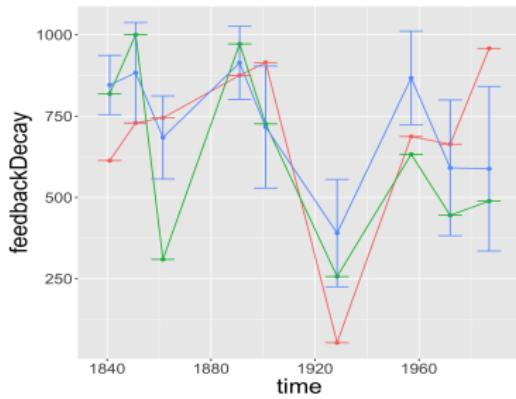
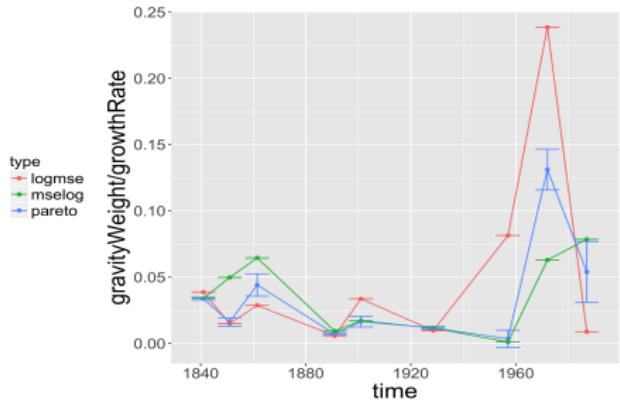
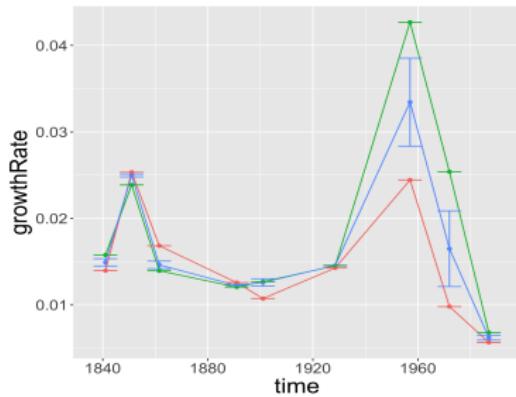
*Pareto front for full model calibration, objectives MSE and MSE on logs*



# Results : non-stationary gravity model calibration



# Results : non-stationary full model calibration



## Quantifying overfitting : Empirical AIC

*Not clear nor well theorized how to deal with overfitting in models of simulation.* **Intuitive idea :** Approximate gain of information by approaching models of simulation by statistical models.

Let  $M_k^* = M_k[\alpha_k^*]$  computational models heuristically fitted to the same dataset. With  $S_k \simeq M_k^*$ , we suppose that  $\Delta D_{KL}(M_k^*, M_{k'}^*) \simeq \Delta D_{KL}(S_k, S_{k'})$  if fits of  $S_k$  are negligible compared to fit difference between computational models and models have same parameter number.

**Application**  $M_1$  : gravity only model with ( $r_0 = 0.0133, w_G = 1.28e-4, \gamma_G = 3.82, d_G = 4e12$ ) ;  $M_2$  : full model with ( $r_0 = 0.0128, w_G = 1.30e-4, \gamma_G = 3.80, d_G = 8.4e14, w_N = 0.603, \gamma_N = 1.148, d_N = 7.474$ )

# Empirical AIC values

Table: Empirical AIC results.

Résultats de l'AIC empirique.

Modèle Statistique	Ajustement pour $M^{(1)}$	Ajustement pour $M^{(2)}$	$\Delta$
Polynomial	0.01438	0.01415	1
Log-polynomial	0.01565	0.01435	1
Polynomial Généralisé	0.01415	0.01399	1

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