GENERATIVE COUPLED MODEL FOR URBAN CONFIGURATION OPTIMIZATION

A. Banos, R. Doursat and J. Raimbault [*†‡§]

Abstract. We describe an hybrid agent-based model coupling a cellular automata and a dynamic network, for the simulation of urban growth. Heterogeneous aspects of urban systems are taken into account in the sense that morphologic structure but also functional properties of the city are implied in the evolution. Classic measures of description and performances for generated configuration are used to classify and explore generated patterns, and we also propose an economic evaluation of the structure using sensitivity of segregation models to spatial configuration. We can apply the model to a real-world situation, proposing an optimization of the repartition of activities in a zoning context.

Keywords. Agent-based modeling, Cellular automata, Bi-objective Pareto optimization, Evidence-based urbanism, Urban morphogenesis.

1 Introduction

Recent progress in many disciplines linked directly or indirectly to Urban Planning can be interpreted as the development of a "new urban science" as Batty states in [1]. From agent-based modeling in quantitative geography, which achievements are reviewed in [2], and for which the best example of promising results are the series of Simpop models ([3]), to other approaches of the field that Portugali present as "Complexity theories of cities" ([4]) that can involve scientists such as physicists as Haken on information theory ([5]) or architects as Hillier with Space Syntax theory ([6]), the field is broad but is strongly consistent through the common view of urban systems as complex systems.

In that context, Cellular Automata models (CA), in which agents (cells) are fixed and evolve according to the state of neighbor cells, have been broadly studied and used. Their use in urban planning, for reproduction of existing urban forms and land-use patterns, was already described by White and Engelen in [7] and was later analysed ([8, 9]) and synthesized ([10]) by Batty. A recent review of possible application of CA models is done in [11]. The possible variations of models types and applications is quite broad . For example, a microeconomic CA for simulation of sprawl is developed in [12]. Measures for sustainable development of a fast growing region in China are investigated in [13] through a "linguistic" CA, that is that the rule are in real time updated thanks to interactions with the real population. Land-use modeling approaches are one essential point, as was the first CA model by White . The two-dimensionnal aspect of the automaton is not mandatory, as the example of the 1D CA model proposed in [14] that allows to show discontinuities in settlement patterns and the strong path-dependence of these. We propose to build a model in the frame of CA modeling of urban systems.

Our model is adapted from the work of Moreno & al. ([15, 16]), that proposed to integrate network effects in a CA model for modeling urban morphology. Their aim was to test the effects of physical proximity on urban shape, introducing the modeling of urban mobility through a network whose evolution is coupled with the evolution of the urban shape. We generalize the model, allowing to take into account functional properties of the urban environment. The concept of considering heterogeneous urban activities was introduced in [17] and developed in [18], but was at our knowledge never considered from the point of view of physical accessibility and the impact on sprawl patterns.

We describe formally the model in a first section and detail the indicator functions used to quantify the generated patterns. We then propose internal and external validations by sensitivity analysis and reproduction of typical urban patterns. We are then able to apply the model to a concrete case, proposing an bi-objective optimiza-

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tion heuristic for functional configuration, with objective functions relevant indicators that we have studied in the validation section.

2 Model description

2.1 Agents and rules

The world is a square lattice $(L_{i,j})_{1 \leq i,j \leq N}$ constituted of cells that can be occupied or not (that will be denoted by a function $\delta(i,j,t)$, time being defined on a standard time scale $\mathbb{T}$ which definition is introduced in [19], and for which we will take $\tau\mathbb{N}$, $\tau > 0$ to simplify). Among these fixed agents evolves an euclidian network whose agents are roads: $N(t) = (V(t), E(t))$ whith V finite part of the world (set of points). At initial time, no cell is occupied and the initial network is fixed: $N(0) = (V_0, E_0)$. In order to translate functional mechanisms in the growth of the city, we suppose that a part of initial nodes are city centers C_0 for each an activity is defined by an integer-valued function $a : C_0 \to \{1, \ldots, a_{max}\}$. The initial network and centers can be either generated at random (following consistent specifications) or following a user-defined configuration.

In all generality, we suppose that there exists a fixed number of functions defined on cells and in time, that we call explicative variables, noted $(d_k)_{1 \leq k \leq K}$. Corresponding weights $(c_k)_{1 \leq k \leq K} \in [0,1]^K$ are parameters of the model. These will determine the value of cells $v(i,j,t)$. In practice, these variables will be: d_1 the density in a neighborhood of radius r (parameter) around the cell; d_2 the distance to the nearest road; d_3 the distance to the nearest town center through the network; and d_4 the "accessibility" of activities, defined as, with $d_A(a)$ distance to nearest center with activity a through the network, and $p_A \in [1; +\infty[$, $d_4 = \|(d_A(a))_{1 \leq a \leq a_{max}}\|_{p_A} = \left(\frac{1}{a_{max}} \cdot \sum_{a=1}^{a_{max}} d_A(a)^{p_A} \right)^{1/p_A}$.

Giving these settings and variables, the evolution of the system for one time step is the following:

- New cell values are calculated by $v(i,j,t) = \frac{1}{\sum_{k=1}^{K} c_k} \cdot \sum_{k=1}^{K} c_k \cdot \frac{(d_k^{max} - d_k)}{(d_k^{max} - d_k^{min})}$. It corresponds to a weighted average of the normalized heterogeneous functions d_k, for which each influence is controlled by the weight c_k.

- N new cells chosen as the N cells with the best value are built in a random order.

- For each new cell built, if $d_2 > d_s$ fixed parameter (distance over which a new road is needed), the cell is connected to the network by a new road connecting perpendicularly with the network

Note that the process is stochastic since it depends on the random order on which the new cell are built. Sensitivity analysis and explorations will determine the influence of

respective parameters and the influence of that stochastic aspect. The growth is stopped after a fixed amount of time fixed by experience, so the final structure does not fill totally the world, what would make no sense. A good compromise has to be taken between "unfinished" structures and stationary result totally biased by bord-effects.

Figure 1 shows a "flowchart" describing the working of the agent-based model, especially what are the feedbacks between agents.
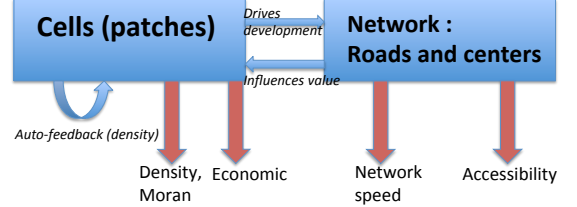

Figure 1: Flowchart of the model. Blue arrows represent interactions and feedbacks, whereas red arrows are evaluation functions.

2.2 Evaluation functions

Once a structure has been generated, one need to quantify its properties, either to classify it, or to compare it to other structures in an optimization purpose. Therefore we need evaluation functions, that can be both objective quantifications or fitness functions for structures. The following proposed measures take into account all explicative variables, since their distribution that is an emerging feature is not supposed to be known before development and are therefore essential to monitor, at the exception of the distance to roads (d_2) on which we keep always control precisely because of the network development rule (fixed distance d_s over which a road is built).

Morphology We qualify the morphologic structure of a configuration by projecting on a plane corresponding to the following indicators:

- The integrated local density, calculated by integration with a norm-p of the densities in circular neighborhood of each cell (with same radius parameter as for the run). We have, with $p_d \in [1; +\infty[$

$$D(t) = \left[\frac{1}{\sum_{i,j} \delta(i,j,t)} \cdot \sum_{\delta(i,j,t) \neq 0} d_1(i,j,t)^{p_d} \right]^{1/p_d}$$

- The Moran index, defined in [20] and used a lot in quantitative geography such as in [21], is used to quantify the polycentric character of the distribution of populated cells. It takes values between -1

and 1, a value close to 1 corresponding to a strong monocentric distribution, zero is for totally random distribution and -1 for a "chessboard-like" distribution (one cell over two is occupied). Polycentric distribution will have small positives values according to the size and respective distance of centers. The space is decomposed into M regular areas (a grid in practice, which size has to be at the intermediate scale between cell size and world size, that is roughly $1 \ll M \ll N$), and we denote by $(P_i)_{1 \leq i \leq M}$ the occupied cells in each area and d_{ij} the distance between centroids of i and j. The Moran index of spatial auto-correlation is defined as

$$M(t) = \frac{M}{\sum_{i \neq j} \frac{1}{d_{ij}}} \cdot \frac{\sum_{i \neq j} \frac{1}{d_{ij}} \cdot (P_i - \bar{P})(P_j - \bar{P})}{(\sum_{i=1}^{M} (P_i - \bar{P}))^2}$$

We recognize the normalized ratio between a modified covariance (weighting the pairwise correlations by the distance between centroids) and the variance of the distribution.

Network performance Because of the way it evolves, the only loops in the network are the initial ones. Therefore, it has no sense to evaluate robustness or clustering coefficient of the network. However, since the generated network is supposed to simulate the mobility network, we can evaluate as suggested in [22] its performance through its relative speed, i. e. the quantity of detours it obliges to make. We define it, considering ony travels from occupied patches to nearest center, with $p_n \in [1; +\infty[$, $d(ij, c)$ the euclidian distance and $c_{min}^{ij} = argmin_{c \in C_0}(d_{ij \to c})$,

$$S(t) = \left[\frac{1}{\sum_{i,j} \delta(i,j,t)} \cdot \sum_{\delta(i,j,t) \neq 0} \left(\frac{d_3(i,j,t)}{d(ij, c_{min}^{ij})} \right)^{p_n} \right]^{1/p_n}$$

Functional accessibility The global accessibility is, as for density, the integration of relative local accessibilities on all cells. We consider the relative accessibility in order to be able to compare configuration of different sizes through a normalized measure That is simply with $p_{GA} \in [1, +\infty[$,

$$A(t) = \left[\frac{1}{\sum_{i,j} \delta(i,j,t)} \cdot \sum_{\delta(i,j,t) \neq 0} \left(\frac{d_4(i,j,t)}{max_{i,j} d_4} \right)^{p_{GA}} \right]^{1/p_{GA}}$$

Economic performance It has been shown by Banos in [23] that the Schelling segregation model, a basic model for socio-economic segregation introduced in [24], was highly sensitive to the spatial structure in which one can embed it (the segregation laws are in that case also influenced by spatial proximity). That justify the use of such a model as an evaluation function for the spatial structure regarding "economic performance", in the sense

of how much does the structure influence segregation. We implement a model for residential dynamics based on the model proposed in [25]. The output function is a segregation index calculated on residential patterns obtained from a distribution of constructed patches. Detailed description of the model is provided in supplementary material S1.

3 Results

The model was implemented in NetLogo 5.2 ([26]). Plots and charts are managed with R after export into standardized data ([27]). Primary treatment of GIS Data (for most hand vectorization of simple raster data) is handled with QGIS ([28]). See supplementary material S2 for implementation details needed for exact reproducibility.

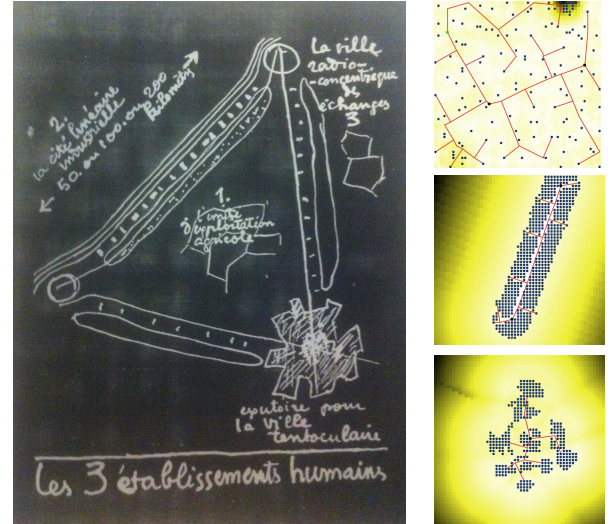## 3.1 Generation of urban patterns : external validation of the model

Figure 2: Typical patterns reproduce Le Corbusier analysis of "human settlements" When trying to theorize urban planning by first analyzing "by hand" the form of cities, Le Corbusier has enlighten 3 types of settlements, that are radio-concentric cities, linear cities along communication roads and the rural settlements. We were able to reproduce this typology by playing with values of weights (first only $d_1 = 1$, second only $d_2 = 1$ and third d_1 and d_3 of equal weights). Source of drawing: [29]

Typical patterns We launch the model on random initial configurations, with randomly distributed centers whose activities are also random but are globally balanced between all possible activities. The initial network is created through progressive connexion of connected components (what always terminates since the number

of connected components in the network is then strictly decreasing). Different configurations of the weights allow to generate very different structures. The obtained patterns for extreme values of parameters (e. g. with only influence of density on values, or of distance to centers) were quite the one expected as there is in such case less emergent behavior. It is striking to note that some characteristic patterns exactly correspond to the structures that LE CORBUSIER has analyzed when trying to build a typology of the *3 Établissements humains* (human settlements) as it is summed up by MANGIN in [29] (see figure 2 for details).

Classification of structures   Thanks to the morphological indicators, we were able to proceed to a description of morphological classes that can be obtained with any values of the parameters. The projection of a configuration in the plan $Vect(D(t_f), M(t_f))$ allows to understand how the structure are different and where they are situated in a certain "classification". Fig. 3 sums up the results of that classification.

For extreme values of parameters, the results were quite the one expected, but some combinations seem to project the structures at unexpected places, what we can take as a sort of emerging chaos in the global system.

3.2   Sensitivity analysis and exploration of the parameter space : internal validation

Sensitivity to initial conditions   To determine the magnitude of repetitions useful to get meaningful values of output functions and to ensure some validity in the results, it is necessary to investigate the sensitivity of the model to initial conditions of the spatial configuration. Indeed, if the conclusions that can be drawn in a particular case are significantly modified by small changes in the spatial configuration, one will differently apply the model than in a case where abstract topology of activities has more influence: one will obviously elaborate totally different exploration heuristics to proceed to optimisation.

Therefore, we launched large number of simulations with same parameter values but different initial configurations, in order to get statistical distributions of outputs at fixed parameters. Standard deviations were calculated for samples of the parameter space (around 20 samples) and the distributions were each time of same amplitude and same deviation. Fig. 4 shows these distributions for the sample corresponding to equal weights to each parameter. Given the numerical values of deviations (each time around 0.1 times the amplitude of the function), we can conclude that outputs are significantly less sensitive to spatial positions than to topology of activities and parameters values (as the exploration of parameter space results show in the following). That justify the small number of repetition needed to have reasonable values.
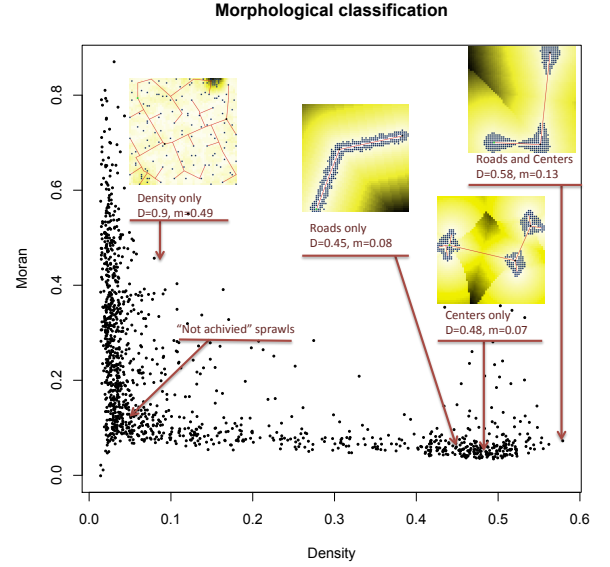
(a) Projection in the plan and some structures associated to some particular points. Three type of structures are very close and may sometimes not be distinguish through our simple indicators. The area close to zero corresponds to "unfinished" growth, that is structures that occupy only a small part of the world, that does not allow to compare directly with structure of bigger magnitude.
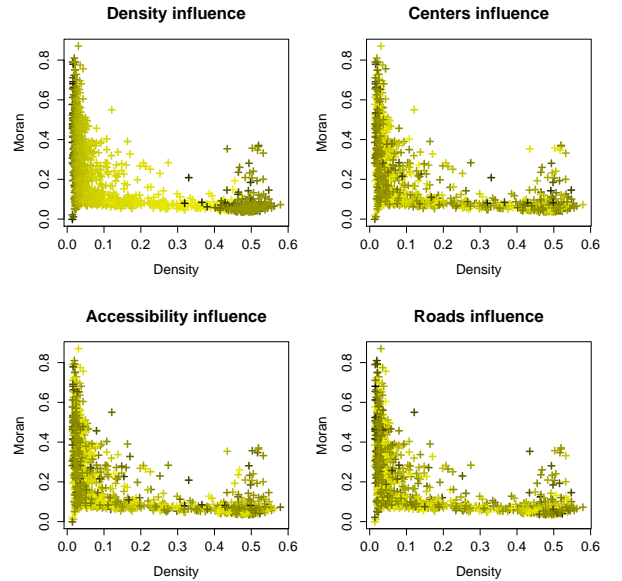
(b) Influence of each parameter on the location of the structure in the morphological plane. Color darkness correspond to the relative weight of the parameter in each case. If the previous plot showed that extreme cases were located at expected places, this one shows that dark point are quite uniformly distributed for the role of centers, accessibility and roads. That means that some combinations created some "chaos" in the location of structures. Only density weight gives one predictable result: the fact that the cloud of points in low densities corresponds to great influence of density in growth. However, the could at the right of the plot shows that interaction between weights quickly limits that predictability.

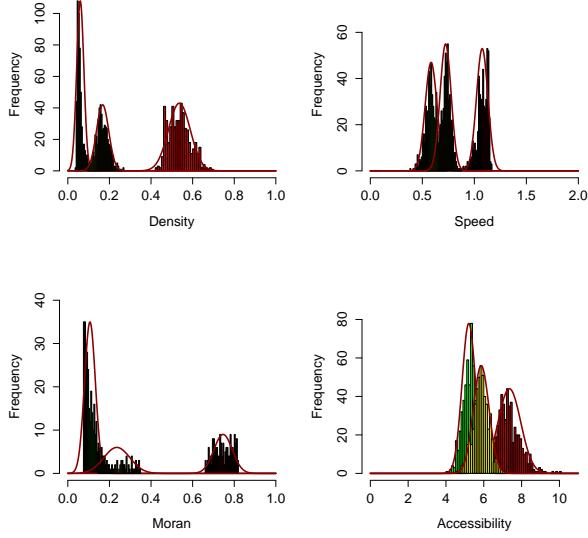Figure 3: Morphological classification of structures.

Figure 4: Statistical distribution of outputs for sample points in the parameter space . For many combination of weights (c_k), we calculated the distribution of these 4 outputs on around 500 repetitions for random initial configurations. We represent for each output three points of the parameter space that give quite disjoint distributions, to show how we are in those case able to compare them and draw conclusions from the comparison. At each time, we plot the Gaussian fit (red curves), that is generally close to the distribution. The shape of distributions translates the small sensitivity of the model to spatial configuration, since it covers at most one tenth of the range of output functions (0.1/1 for density, 1/10 for accessibility e. g.). That confirms the consistency of the model since we expected the output to be quite independent of initial positions and that their comparison could lead to statistically consistent results. That allows also to fix the number of repetition needed to obtain a certain level of statistical significance in the outputs of the model : for example, under assumption of normal distribution for an indicator and its mean, with $\sigma = 0.2$ supposed constant and estimated, we need $n = \frac{2\sigma \cdot 1.96}{0.1}^2 \simeq 60$ repetitions to have a 95% confidence interval of length 0.1 on the calculated mean.
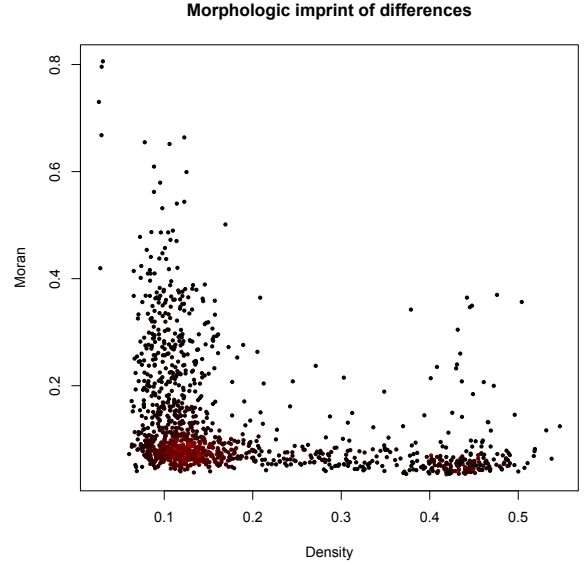
Figure 5: Morphological projection of symmetric differences between configurations for both update types. Each point corresponds to many realizations (50) for a given value of parameters. Each time, both structures for continuous $(N = 1)$ and sequential $(N = 20)$ are generated, symmetric difference is computed and projected in the morphologic plan. Points are colored according to their "significance", defined by the product of local density of points in the cloud (closely clustered points correspond to more frequent configurations) by the relative size of the configuration (large configurations correspond to large difference and are therefore more significant). The wide spread shows a strong sensitivity. An other plot of superposed imprints for each type of update showed quite disjoint sets, what confirms this sensitivity. It is however relativized by the concentration here of significant points, which means that side effects are "constant". Therefore taking a compromise between two types of updates should give stable and reasonable results.

Sensitivity to update type The generated structures seem to be quite sensible to the type of update that is done at each time step, i. e. to the number of cells that are built at each time step. That comes from the fact that at each time step, houses are built simultaneously without update of patches values. It is a consistent view since real value functions should have a certain response time that we can suppose of the magnitude of τ. However this paradigm can be limited and it is necessary to find a compromise value for N that leads to reasonable configurations. We can explore a grid of the main parameter space (weights of variables) and at each time generate patterns corresponding to a "continuous" update ($N = 1$) and a "sequential" one ($N = 20$ in our case), calculate the symmetric difference $\mathcal{D} = \{(i,j)|\delta^{cont}(i,j,t_f^{cont}) \neq 0\}\Delta\{(i,j)|\delta^{seq}(i,j,t_f^{seq}) \neq 0\}$, and plot the corresponding sets in the morphological plan. Results are shown in figure 5, including a colored representation of significance of points. We conclude that the model is highly sensitive, but that bord-effect stay the same so finding a good compromise by experience (giving "consistent" structure) should solve the problem.

Density   **Speed in Network**
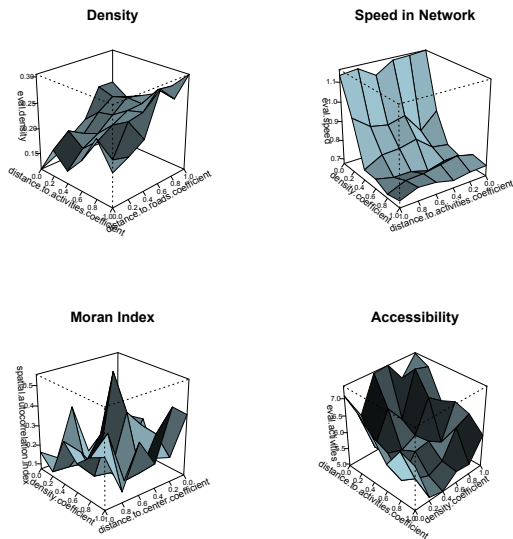
Moran Index   **Accessibility**

Figure 6: Surface of evaluation functions depending on two parameters. We chose some combinations of parameters that seemed to be interesting. We can extract the ranges of outputs and observe their regularities. The chaotic behavior of Moran index and accessibility confirm the existence of chaotic mechanisms in the system. Speed appears to be regular, what implies that network spread is not so far from linear regarding these parameters. Other combinations gave chaotic behavior.

**Exploration of the parameter space** We explored a grid of the 4-dimensional space of the "main" parameters, the relative weights of variables, to evaluate the global behavior of outputs when parameters vary. Other parameters such as r or d_s were not included in the exploration because they have real proxy that we can reasonably fix (around 5 cells seemed in general to generate the structures closer to the real world, what is significant whatever the scale of application, as the size of a district, it gives 50m, characteristic size of blocs, of the city 500m size of a district and of the system of cities 5km characteristic distance between cities). Step was 0.2 for each parameters, with 5 repetitions for each combinations, what lead to the calculation of around 3200 configurations. Examples of surfaces defined on the space of two parameters are shown in figure 6. For each measure, we chose to plot against 2 parameters out of the 4 possible dimensions, whose were supposed to have more influence and for which the surfaces were the most revealing. Economic evaluation was not plotted for efficiency reasons. However, since it depends directly on morphology and network and that it is not probable that the function would exactly smooth the irregularities, we can conclude that the shape of its surface would present same patterns of irregularity. The exploration allows first to appreciate the regularity of the outputs, what is essential to understand the internal mechanisms of the model that appear in some point as being somehow chaotic and non-linear. Secondly, it gives the range of outputs, allowing to state the robustness regarding initial conditions as we stand before.

3.3   Concrete application

Since we have shown that outputs on which we want to optimize, such as accessibility to activities or network performance, are significantly more sensible to initial configuration of activities than to spatial distribution, we can apply the model to the optimization of activities repartition, given an existing spatial structure. That situation can occur in a planning problem, then one has to decide possible land-uses of predefined zones.

The concrete situation will consist on the planning from scratch of a new district, where attribution of activities to areas has to be determined (2 types of activities, residential or tertiary). Transportation infrastructure is already implanted and the station will act as center with a fixed third activity. For each area, centroid is a center and the initial network consists on main roads existing on the empty district. Configuration is imported from GIS shapefile so the application can be done on other cases, as soon as one has required files (see S2). The real world situation is the district of Massy Atlantis, France, which have just been built and planned (2012) and for which we would like to study if a more optimal planning may have been done. We fix a value of parameters translating the situation, that is : high influence of activities ($c_4 = 1$) because agents will want to have accessible home, workplace and station; medium influence of density ($c_1 = 0.7$) because the houses have to fill reasonably the available ar-

eas; no influence fo roads since we suppose that the initial network is already built and has a few influence because of the scale; and no influence of centers because centers are here abstract entities representing a all area. We then launch the calculation of configuration for all possibilities of distribution of activities in the areas (exponential on the number of free centers, 510 calculations in our case, what gives around 2500 runs with repetitions) and plot configuration in the plan of economic performance and accessibility (most significant fitness functions in our case). Results are shown in figure 7.
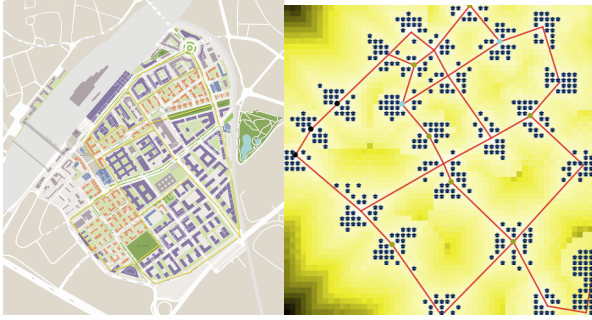
(a) Left: existing masterplan for the district as it has been realized. Areas correspond to colored zones, and actual activities correspond to the color. Station, not mentioned is in the upper left (see center positions in model); Right: example of obtained configuration, color correspond to possible distribution of activities.
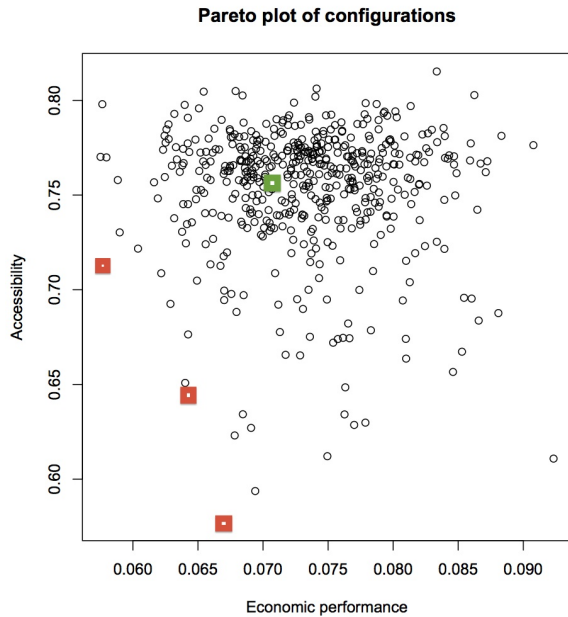
(b) Pareto plot of configuration in the plan economic performance/accessibility. Green point corresponds to the real situation, that is in fact not so far from the Pareto frontier so is a mean solution in quality ($E = 0.07, A = 0.76$). Red points that are some of the Pareto-optimal solutions with a good compromise between the two objectives, correspond to situation where activities are totally mixed.

Figure 7: Concrete application.

We plot the configuration in the plan corresponding to the two objectives and we naturally obtain a Pareto-front of "optimal solutions". We observe that the real configuration is not so far to be optimal; although it favors more accessibility than economic performance. By examining two optimal points that are good compromises, we see that the corresponding combinations of activities are one where they are maximally mixed (totally heterogeneous repartition). That is an interesting fact and may be a clue towards evidences for more mixed land-uses in planning. We conclude that the application is efficient, since it allows to propose an optimal planning regarding these criteria in such a situation. It could be generically used by planners in similar situations (staying very careful on the conditions of application).

4 Discussion

The reproduction of existing urban facts (in the sense of the morphological structures) and the possible applications showed that the implemented model may be useful for evidence-based decision-making in urban planning. However, many aspects can be discussed as they raise crucial questions and would need further explorations.

Scale of the model   One ambiguity of the model is that it can be applied at several scales and that its agents have not necessary correspondence in the real world. Indeed, as we saw in results and applications, it can be used at the macro scale of a system of cities (settlement patterns), the meso scale of the city (patterns for one city) or the micro scale of the district (concrete application). Without going into an ontological debate on the quantity of abstraction of object, it can obviously be pointed as a huge issue. However, we argue that this multi-scale application is legitimate. First, a large number of urban systems and associated processes have been shown to be scale-free as PUMAIN shows in [30], what implies that scaling laws can be present in our model and that it is therefore appliable at any scale, since qualitative properties should remained unchanged (quantitative evolution of relations should depend on the underlying exponents of the scaling laws). Furthermore, urban form have been qualified to have fractal properties ([31]) what would strengthen that paradigm. Following BARTHELEMY in [32] when he insists on the fact that the failure of most multi-agent urban models is that they don't focus on the "dominating" physical process but integrate many aspect that have in reality no influence on the emergent properties of the system, we suppose here to have isolated at least "good" proxies of dominating processes.

Local application   When the model is considered at a meso or micro scale, one could object that it neglects too much phenomena such as economic exchanges by considering the system as a "closed" system (it is formally

closed but not "closed in reality", since just the new constructions can be seen as exchanges with the outside - the precise critical would be that in and out flows are too much simplified). As economic models considering the global complex network of cities such as in [33] have also reproduced well existing patterns in urban systems, it is obvious that the equivalence of approaches and the compromise that need to be made in each case have to be further investigated. It joins the fundamental issue, already implicitly raised in the previous point, of the existence of a minimal dimension for a generalized representation of urban systems. Concretely in our case, attractivity of centers is a proxy for economic processes, and although the model does not directly include economic considerations, they are underlying, confirming that urban processes are strongly interdependent. The problem would be to understand how general the dependance is and if a generalized minimalist formulation can be constructed. Speculations towards that have been proposed by Haken in [5] with the notion of semantic information that would be linked to properties of attractors of dynamical systems. This theory has however neither been quantified, confirmed or invalidated.

Quantitative calibration The question of validity of the model is also linked to the need for a fine quantitative calibration on real patterns. Calibration of such models is still an issue, since on the one hand calibration on errors on proxies for output function does not directly influence the spatial patterns but on the other hand calibration on spatial patterns is too constraining because it eludes "similar" patterns. Some work has been done on tries for such calibrations such as in [34] but the results are not satisfying. To go in that direction, we would first need to apply the model with finer grain for the spatial resolution, i. e. on a very large world. Computational issues can appear, even though we optimized processes in the implementation (see supplementary material S2, e. g. single calculation and caching of network shortest paths allow a huge gain in computational complexity). But the increase in size can also cause methodological issues, since the amount of details in patterns can become too huge and bring more noise in the indicators, biasing the results. Therefore, an additional operation could be a solution, that is the extraction of the morphological envelope of the generated pattern, following the innovative method proposed in [35, 36]. Other ways to deal with noise such as gaussian smooth should also be investigated.

Complex coupling with economic model The way we proceed to economic evaluation consists in a simple coupling with an other agent-based model. It could be a direction of research to look towards a complex coupling (in the sense of Varenne in [37]) between the two models. Indeed, considering the sprawl at an other time scale

implies simultaneous evolution of inhabitants, rents of buildings and values of terrain. Of course such a coupling would lead to a completely different model that would need to be explored again and that would be appliable to totally different situations (as evaluation of long-term rent policies for social diversity for example).

5   Conclusion

We have proposed and explored an heterogeneous CA model to simulate urban growth patterns, using also a new way to evaluate economic properties of spatial patterns. It has shown to be able to reproduce characteristic urban facts of a possible typology of "human settlements" (cf 3.1). Furthermore the model was applicable to a concrete situation giving an "optimal" (in the Pareto sense) solution to a planning problem in a zoning context. The results go in the direction of evidences for a "mixed-use" city, on which literature is still poor and that could be the object of future work. That paradigm is proposed by urbanists such as Mangin in [29] (concept of the "passing city") and would need to be confirmed by quantitative results.

Beyond these technical facts useful for planning purposes, we argue that our work gives a vision of the current state "quantitative urbanism", confirming the view of Portugali in [38], in the sense that the conception and application of such models is in fact very subtle and can lead to many misunderstanding and contradictory results. Here the link between model and application is in that frame, and can become obviously wrong just by an application with other parameters. However, we stand that a better quantitative comprehension of urban systems begins to be actual and that evidence-based solution to planning problems are real, at the opposite of the vision of Lefebvre in [39] when he stood that scientific approaches could never translate or predict the mechanisms of a city.

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Appendices

Appendix A : Description of the agent-based model for economic evaluation

Purpose of use

We want to propose a model similar in behavior to Schelling's segregation model ([24]), based on the work by Benenson on residential dynamics ([25]). The question of the influence of the underlying spatial structure on the output of the model appears as crucial. Banos showed in [23], unlike previous results, that the network structure add strong effects on the convergence speed of the model and on the final result.

Therefore, our approach is to proceed to a simple coupling that consists in evaluating the spatial structure by measures on the final configuration given by the dynamical economic agent-based model. We are able to define a "segregation potential", in the sense of the segregation induced by the spatial structure embedded in a Schelling's model.

Description of the model

We are still located in the lattice world, in which cells values, houses positions and network are fixed.

A new variable is associated to houses (i. e. occupied cells), which is the rent $r(i, j, t)$. Mobile agents are households $h_k \in H$ that have a wealth function $w(k, t)$ and occupy the house $(i(k), j(k))$.

We begin with no occupied house, initial rents are a mean value m.

At each time step:

- Already present households may decide to move to an other inoccupied house, depending on the condition $\frac{v(i(k), j(k))}{v_{max}} \cdot \frac{w(k,t) - r(i(k), j(k), t)}{max_k(w(k,t) - r(i(k), j(k), t))} < \epsilon$, where ϵ is a threshold parameter fixed. They occupy the better house possible according to $\frac{v}{r}$. If no house is available, they die (leave the city)

- N_i new immigrant come, with a random wealth $w \sim \mathcal{N}(m, \sigma)$ with σ parameter small regarding m, and occupy best free houses (maximal number of immigrant is fixed by available houses).

- Rents are updated, set to the mean within a given radius r_e of wealth of households and rents of surrounding houses (gentrification effect).

- Wealth are not updated considering the supposed small time scale.

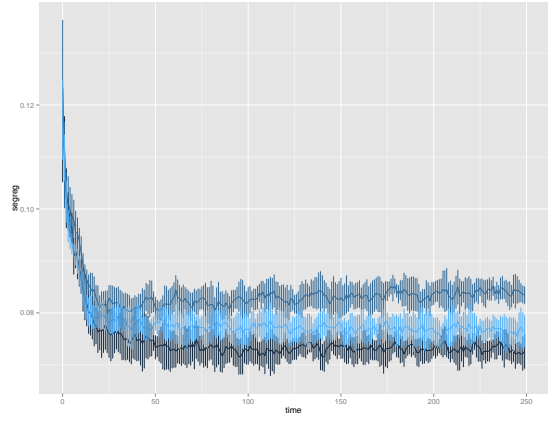### Convergence and evaluation of segregation

After a certain number of time steps (around 100 in practice) the system freezes concerning indicators of cumulated wealth and segregation index, as it is explained in the literature. Figure 1 shows the convergence of segregation index and mean rent of houses on many repetitions of the model for different values of parameters.

We are then able to calculate the segregation index on the frozen state, and defined by the classic spatial diversity index calculated on households:

$$d = \frac{1}{2max_h(w(h))} \cdot \frac{\sum_{h' \neq h} \frac{\left| w(h) - w(h') \right|}{d(h, h')}}{\sum_{h' \neq h} \frac{1}{d(h, h')}}$$
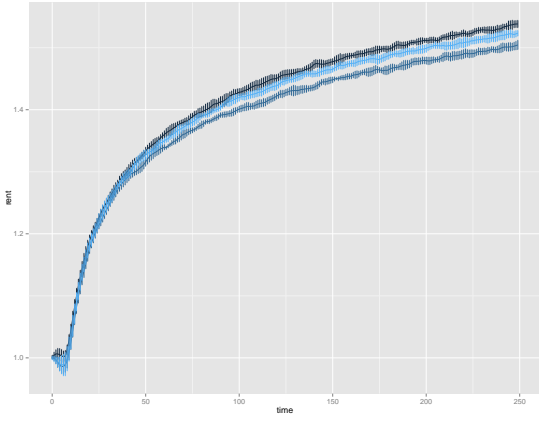
(a) Time-series of rents.

(b) Time-series of segregation indexes.

Figure 8: Convergence of the model. For each plot, each three curves correspond to a configuration of parameters. The values are calculated on 20 repetitions of the economic ABM, and the calculation is repeated on 100 initial spatial configuration, for which the error bars of distribution are plotted for each point.

Appendix B : Source code of the implementation of the model

Core source code of the NetLogo implementation of the model, sample of data used for applications and source code for generation of plots and charts are available online at http://github.com/JusteRaimbault/UrbanMorphogenesis.