A Discrepancy-based Framework to Compare Robustness between Multi-Attribute Evaluations

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Complex systems

Multi-attribute Evaluation in Complex Socio-technical Systems

Systematic multi-objective nature of problems in design of Complex Industrial Systems [Marler and Arora, 2004] and in the study of Complex Natural Systems [Newman, 2011]

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→ Territorial systems as typical examples : e.g. sustainable urban design [Souami, 2012], multi-criteria decision-making for transportation infrastructures [Bavoux et al., 2005]

Robustness of Evaluations

Towards a Generic Robustness Framework

Research Objective : *Investigate a generic data-driven approach to Robustness in Multi-attribute evaluations of Complex Socio-Technical Systems*

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Theoretical Framing

Assumptions

Objectives as Spatial Integrals

Linearly Aggregated Objectives

Formal Description (I)

Territorial Systems $S_i = (\mathbf{X}_i, \mathbf{Y}_i) \in \mathscr{X}_i \times \mathscr{Y}_i$ with $\mathscr{X}_i = \prod_k \mathscr{X}_{i,k}$

$$(\mathscr{X}, \mathscr{Y}) \underset{def}{=} \left(\prod \tilde{\mathscr{X}_c}\right) \times \left(\prod \tilde{\mathscr{Y}_c}\right) = \left(\prod_{\mathscr{X}_{i,k} \in \mathscr{D}_{\mathscr{X}}} \mathbb{R}^{p^X_{i,k}}\right) \times \left(\prod_{\mathscr{Y}_{i,k} \in \mathscr{D}_{\mathscr{Y}}} \mathbb{R}^{p^Y_{i,k}}\right)$$

Objectives : H_c space of real-valued functions on $(\tilde{\mathscr{X}_c}, \tilde{\mathscr{Y}_c})$, such that for all $h \in H_c$:

1. h is "enough" regular (tempered distributions e.g.)
2. $q_c = \int_{(\tilde{\mathscr{X}_c}, \tilde{\mathscr{Y}_c})} h$ is a function describing the "urban fact" (the indicator in itself)

Formal Description (II)

Integral approximation theorem gives upper bound on error, linked to data discrepancy [Niederreiter, 1972][Varet, 2010]

$$\left\| \int h_c - \frac{1}{n_{i,c}} \sum_l h_c(\vec{X}_{i,c,l}) \right\| \leq K \cdot |||h_c||| \cdot D_{i,c}$$

which propagates to the linear aggregation

$$\left\| \int \sum w_{i,c} h_c - \frac{1}{n_{i,c}} \sum_l w_{i,c} h_c(\vec{X}_{i,c,l}) \right\| \leq K \sum_c |w_{i,c}| \, |||h_c||| \cdot D_{i,c}$$

Formal Description (III)

A relative *Robustness Ratio* can thus be defined between two evaluations :

$$R_{i,i'} = \frac{\sum_c w_{i,c} \cdot D_{i,c}}{\sum_c w_{i',c} \cdot D_{i',c}} \tag{1}$$

Implementation on Synthetic Data

Metropolitan Segregation

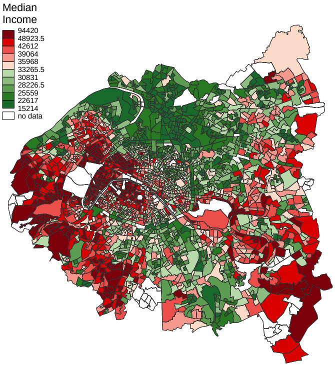
Metropolitan Segregation on Ile-de-France, Insee income data (2011)

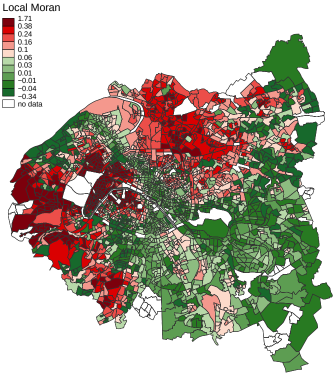
Indicators :

- Spatial autocorrelation Moran index, defined as weighted normalized covariance of median income by $\rho = \frac{N}{\sum_{ij} w_{ij}} \cdot \frac{\sum_{ij} w_{ij} \left(X_i - \bar{X}\right)\left(X_j - \bar{X}\right)}{\sum_i \left(X_i - \bar{X}\right)^2}$
- Dissimilarity index (close to Moran but integrating local dissimilarities rather than correlations), given by $d = \frac{1}{\sum_{ij} w_{ij}} \sum_{ij} w_{ij} \left| \tilde{X}_i - \tilde{X}_j \right|$ with $\tilde{X}_i = \frac{X_i - \min(X_k)}{\max(X_k) - \min(X_k)}$
- Complementary of the entropy of income distribution that is a way to capture global inequalities $\varepsilon = 1 + \frac{1}{\log(N)} \sum_i \frac{X_i}{\sum_k X_k} \cdot \log\left(\frac{X_i}{\sum_k X_k}\right)$

Metropolitan Segregation

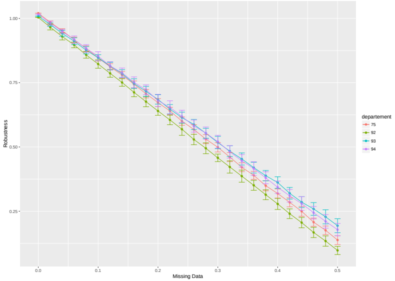
Example of Segregation maps

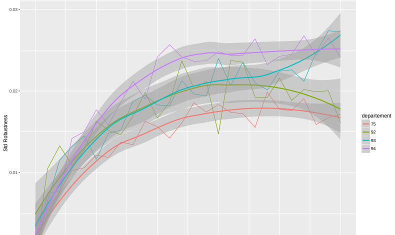




Metropolitan Segregation

Framework Application : sensitivity to missing data





Applicability

→ Application to decision-making procedures : adding robustness as a dimension ?

→ Availability of raw data

→ Assumptions validity ranges : some indicators may difficultly be viewed as spatial integrals (as some accessibility measures [Kwan, 1998]

Further Developments

$\rightarrow$ Application to existing open frameworks (e.g. [Tivadar et al., 2014])

$\rightarrow$ More general formulation, first to non-linear aggregation (e.g. for Lipschitzian functions [Dragomir, 1999])

Conclusion

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- All code available at
https://github.com/JusteRaimbault/RobustnessDiscrepancy

- Paper preprint available at http://arxiv.org/abs/

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