A comparison of simple models for urban morphogenesis

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Abstract

The spatial distribution of population and activities within urban areas, or urban form at the mesoscopic scale, is the outcome of multiple antagonist processes. We propose in this paper to benchmark different models of urban morphogenesis, to systematically compare the urban forms they can produce. Different types of approaches are included, such as a reaction-diffusion model, a gravity-based model, and correlated percolation. Applying a diversity search algorithm, we estimate the feasible space of each model within a space of urban form indicators, in comparison of empirical values for worldwide urban areas. We find a complementarity of the different types of processes, advocating for a plurality of urban models.

Introduction

Understanding the dynamics of cities is an increasing issue for sustainability, since the proportion of the world population expected to live in cities will grow to a large majority in the next decades, and that cities combine both positive and negative externalities on most aspects. Their complexity implies that quantitative and qualitative predictions are not relevant, but planners can *invent future cities* [1], what requires though a knowledge of key urban processes which can be acted upon. In that context, the growth of *urban form* in its different definition and scales, is essential [2]. Considering urban form at a mesoscopic scale, i.e. roughly the scale of urban areas, it can be understood as the spatial distribution of activities. More particularly the distribution of population density has a strong impact on commuting, energy consumption and emissions [3]. Being able to link microscopic processes with the growth of different types of urban form is thus important for a long term planning of sustainable urban systems.

Urban modeling at the mesoscopic scale is the subject of diverse approaches and disciplines. Intra-urban urban economic models, building on classic works such as the Alonso-Mills-Muth model or the Fujita-Ogawa model, propose models linking land-use with land and building markets, which are spatially explicit to different degrees [4]. Transportation and Urban Planning also have a long history in urban dynamics models, including Land-use transport interaction models [5]. Spatial interaction models can also be used in a similar manner to study urban dynamics and as a by-product urban form [6]. Cellular automata models of urban growth are also a privileged approach to study the growth of urban form from a data-driven perspective [7].

At the interface of physics, artificial life and quantitative geography, a few approaches propose simple models to explain the growth of urban form, and generally rely on an unidimensional description of urban form, namely the distribution of population or of the built environment. In that context, the correlated percolation model introduced by [8] was a precursor. Such models can rely on abstract physical processes but also on agent behavior, such as in the Sugarscape model which according to [9] can be considered as a model for human settlements. [10] use migration between cities at multiple scales to simulate urban growth. Diffusion-limited aggregation (DLA) is an other approach transferred from physics to urban modeling [11] and has shown relevant to reproduce fractal urban structures and urban migration processes [12]. [13] combines DLA with percolation to obtain more realistic urban forms. Closer to the idea of urban morphogenesis, [14] proposes a reaction-diffusion model to capture fundamental urban growth processes. [15] describes an urban growth model based on geographical processes, namely an aggregation of population driven by spatial interaction. All these works have in common to model urban growth in synthetic settings, at a mesoscopic scale, considering population distribution only, and in a stylized way. They furthermore consider diverse processes, remaining simple in their structure although they lead to the emergence of a complex behavior. We will in this paper focus on such models, referring to them as *models of urban morphogenesis*.

Exhibiting models with a few number of parameters and processes is useful from an explanative viewpoint, when these can reproduce real world configurations. Having multiple concurrent models which include diverse, complementary or contradictory processes, is furthermore useful for the construction of integrated urban theories, since concurrent explanations can be benchmarked, compared and possibly integrated into multi-modeling approaches. This plurality in urban modeling is intrinsic to a literature with multiple disciplines focusing on a same object of study [16].

We propose thus in this paper to benchmark several simple models of urban morphogenesis, in order to understand the potentialities of some of these models to exhibit a complex behavior and reproduce existing urban forms, and compare them in a systematic way. More precisely, our contributions are the following:

(i) we integrate four different models (correlated percolation, reaction-diffusion, gravity and exponential mixture) into a single software framework; (ii) we compute measures of urban form for urban areas worldwide; (iii) we apply a novelty search algorithm to the models in order to determine their feasible morphological space, and compare these to real urban form values. This contributes to a general understanding of the complementarity of urban models, more particularly for urban morphogenesis at this scale.

The rest of this paper is organized as follows: we first describe the models benchmarked and the quantitative measures used for urban form; we describe empirical values of urban morphology indicators for urban areas worldwide; these are then compared to the feasible space of each model obtained with a diversity search algorithm. We finally discuss the implications of these results for theories of urban morphogenesis and possible developments.

Materials and methods

Urban morphogenesis models

We study and compare four different models of urban morphogenesis. We consider a population grid of size $N = W \times H$ (not necessarily square), each cell being characterized by its population P_i with $1 \leq i \leq N$.

Gravity-based model

Following the so-called "first law of geography", entities in space have interaction patterns which can be described with spatial interaction models [17], including the gravity model. [15] proposed an urban growth model including this process within an iterative growth with population aggregation processes, extending a more simple model introduced by [18]. We generalize this model by adding (i) a hierarchy parameter regarding population aggregation and (ii) seeding multiple initial sites to allow the emergence of polycentric urban forms.

Formally, an initial grid is seeded with $P_0^{(G)}$ sites with population 1, randomly selected. Then, iteratively, one unit of population is added to each cell at each time step with a probability proportional to

$$p_i \propto \frac{\sum_{j \neq i} P_j^{\gamma_P^{(G)}} \cdot d_{ij}^{-\gamma_D^{(G)}}}{\sum d_{ij}^{-\gamma_D^{(G)}}} \tag{1}$$

where probabilities are rescaled such that the cell with the larger value has a probability of $g^{(G)}$. This last parameter allows modifying the speed of growth. The model is stopped when a total population $P_M^{(G)}$ is reached.

Reaction-diffusion

In his attempt to understand embryogenesis, Alan Turing proposed to use chemical partial differential equations (PDEs) to model morphogenesis, introducing the nowadays famous reaction-diffusion equations [19]. In such systems, chemical substances react together and diffuse in space, leading to the emergence of complex geometrical patterns. The concept of morphogenesis has been since well used in urban studies [20], but very few models have actually implemented reaction-diffusion equations, [21] being a notable exception. [14] proposes to capture the fundamental processes of agglomeration economies (positive externalities) leading to aggregation and of congestion (negative externalities) leading to sprawl, as an "aggregation-diffusion" model of urban morphogenesis. The model yields indeed in certain limits reaction-diffusion PDEs. Formally, starting from an empty grid, $N^{(R)}$ units of population are added at each time step, and attributed independently to cells with a probability proportional to $P_i^{\alpha^{(R)}}$ (probabilities are rescaled to obtain a probability distribution over all cells). Population is then diffused in space $d^{(R)}$ times with a strength $\beta^{(R)}$. The model is stopped when a maximum population $P_M^{(R)}$ is reached.

Correlated percolation

The first two models presented are iterative and can in theory be used dynamically. Other approaches, closer to procedural modeling [22], do not simulate the progressive growth of population. They can however capture processes at play in the growth of urban form. The correlated percolation model described by [8] integrates for example clustering processes in cities. A method to generate a spatial field exhibiting long range correlations was introduced for problems in physics by [23]. It is combined to a monocentric density profile in [8] to produce urban forms. In practice, a correlated field p_i is generated by (i) generating a random spatial field; (ii) compute its spatial Fourier transform; (iii) introduce a correlation by multiplying it with a spectral density function with a power-law exponent $\alpha^{(C)}$; (iv) retrieve a long-range correlated spatial field by taking the inverse Fourier transform. This field is combined to a density field ρ_i to determine a binary value for the cell: it is populated if $p_i > \theta_i$ with $\rho_i = \int_{-\infty}^{\theta_i} d\mathbb{P}(p_i)$. We generalize the initial model by taking a polycentric density field with

$$\rho_i(\vec{x}) = \sum_{j=1}^{n^{(C)}} \exp\left(\|\vec{x} - \vec{x}_j\|/r_j\right) \tag{2}$$

where the kernel centers are chosen at random and kernel sizes r_j are taken such that kernel populations follow a rank-size law of hierarchy $\beta^{(C)}$ and the largest kernel has a fixed size $r_0^{(C)}$.

Kernel mixtures

Finally, to provide some kind of null model to understand the advantages of each approach compared to a simple description of population distribution, we also include urban forms generated as kernel mixtures. We consider in particular exponential mixtures [24], where population density is written as previously in Eq. 2. Parameters for this model are the number of kernels $n^{(E)}$, the rank-size hierarchy $\alpha^{(E)}$, the size of the largest kernel r_0^E. Contrary to previous models in which total population had an influence by controlling the speed of growth, density can here be rescaled arbitrarily (morphological indicators used are not changed through rescaling, see below), and we set the maximal density for one kernel to one.

Measures of urban form

Quantitative measures of urban form are multiple and depend on the scale considered [25]. [26] for example introduces measures for buildings at the district scale. The field of Landscape Ecology has its own metrics similar to urban form measures [27]. For the scale we consider and considering population distribution only, metrics of urban form have been proposed for example to quantify sprawl [28]. These can be related to fractal approaches to urban form [29]. The effective dimension when applied to real cities is reasonably low [30], and a few complementary indicators can be used. We thus follow [14] and consider urban form measures which are: (i) Moran index to capture spatial autocorrelation and the existence of centers; (ii) average distance between individuals which captures a level of aggregation; (iii) distribution entropy (aspatial) to capture the uniformity of the distribution; and (iv) rank-size slope which captures the hierarchy of population distribution.

Results

Implementation

The models are implemented in `scala` and integrated into the `spatialdata` library for spatial sensitivity analysis [31]. The library is bundled as an OpenMOLE plugin for the numerical experiments. OpenMOLE is an open source software for model exploration and validation [32] combining model embedding with state-of-the-art exploration methods (including for example sensitivity analysis, design of experiments, calibration with genetic algorithms) and a transparent distribution of computations on high performance computing environments. In our case, we use its workflow system and an integrated algorithm to determine the feasible space of models.

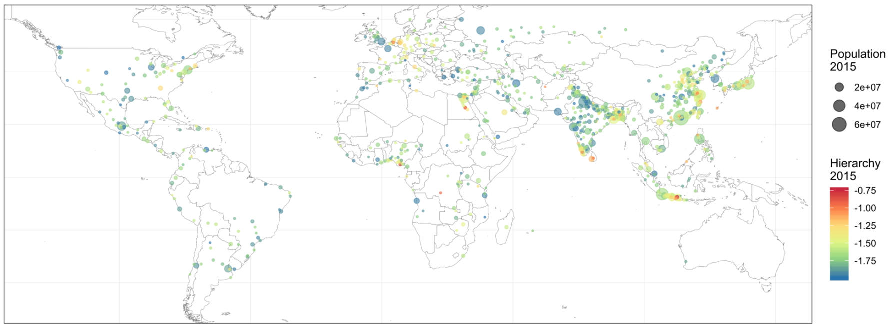
Empirical data

Generated urban forms

Feasible morphological spaces

Discussion

[33]



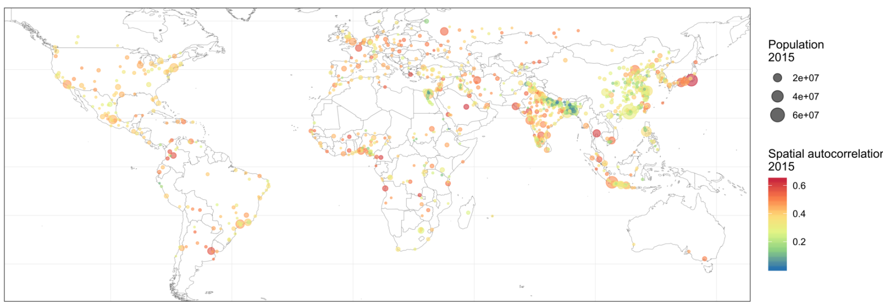


Fig 1. .

Conclusion

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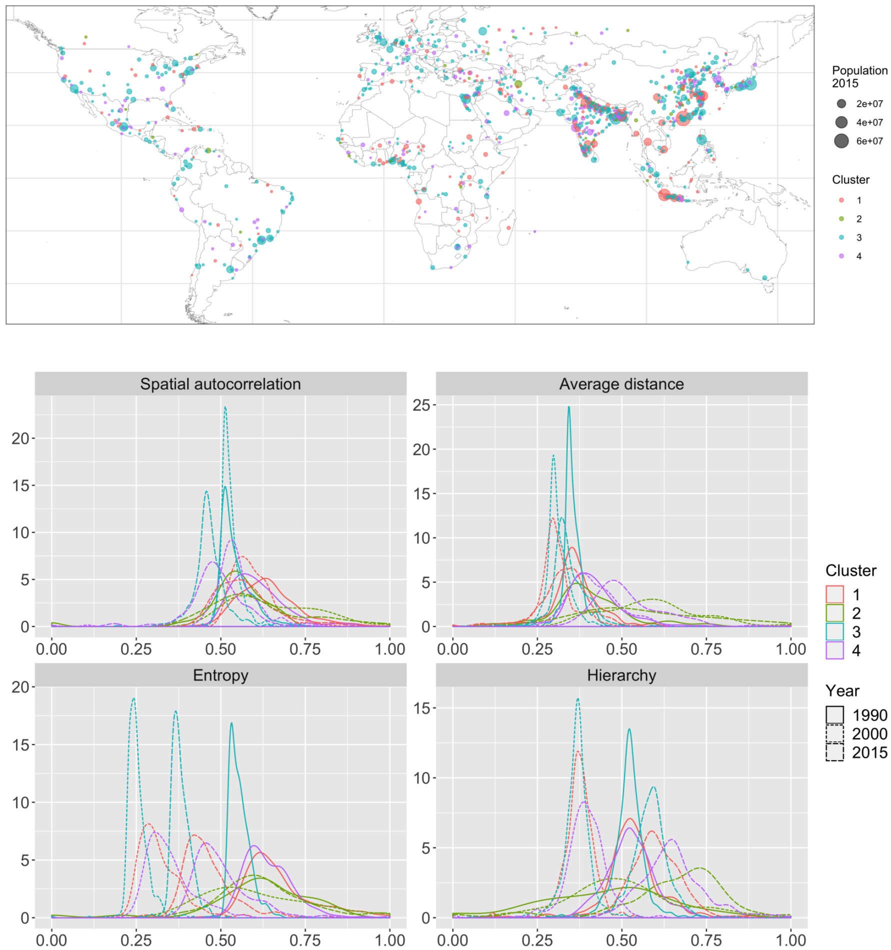

Fig 2. .

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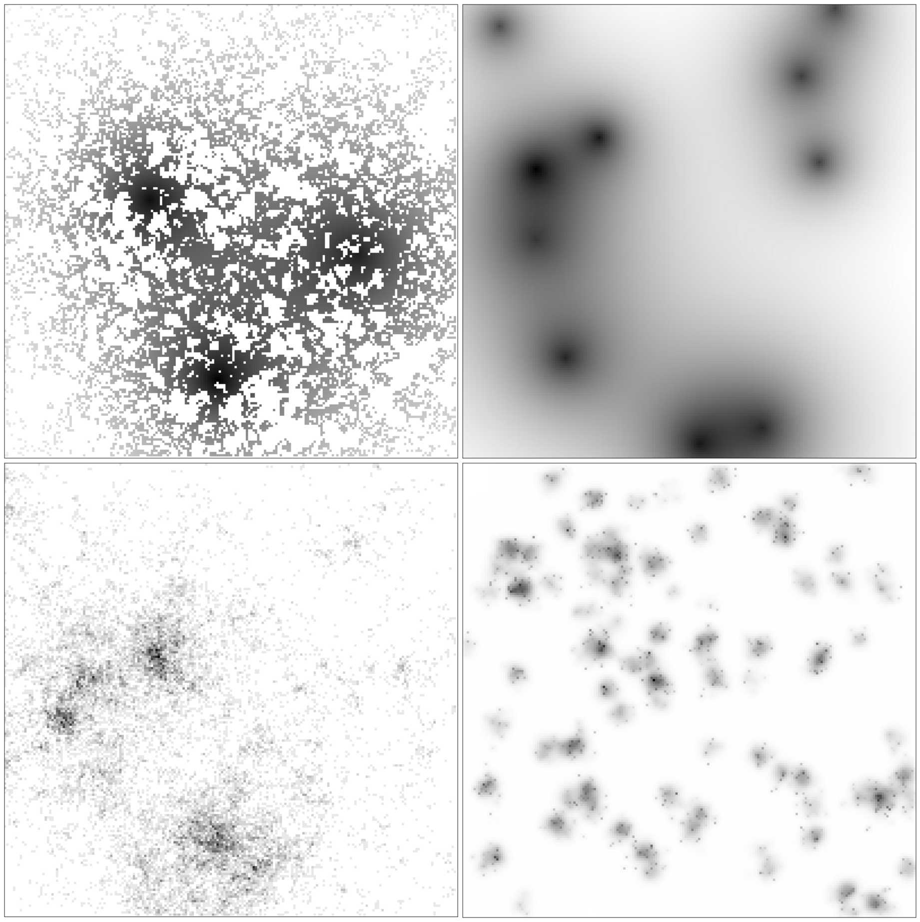

Fig 3. .

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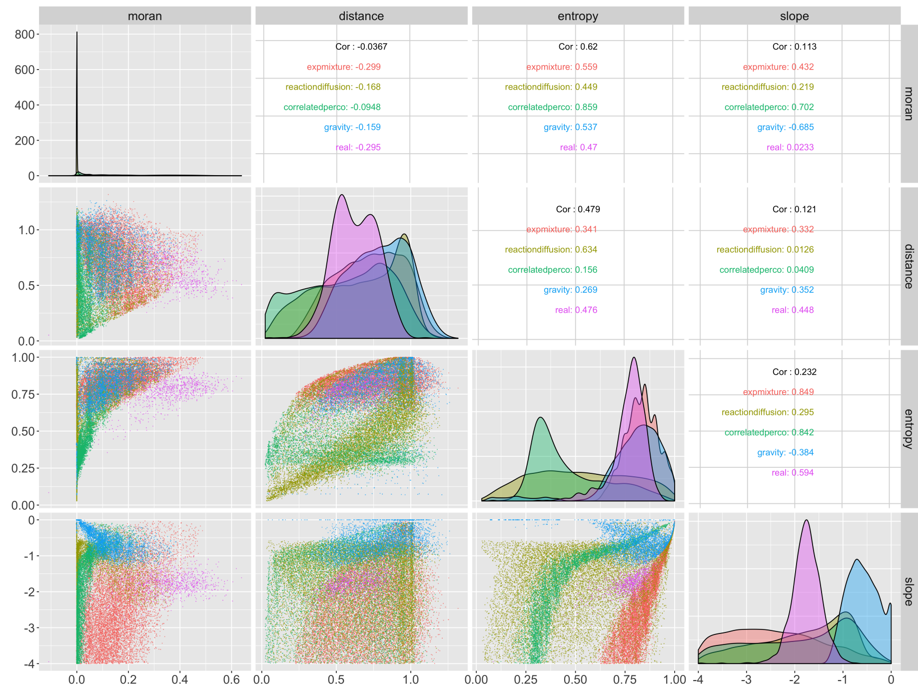

Fig 4. **.**

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