# Worldwide estimation of parameters for a simple reaction-diffusion model of urban growth

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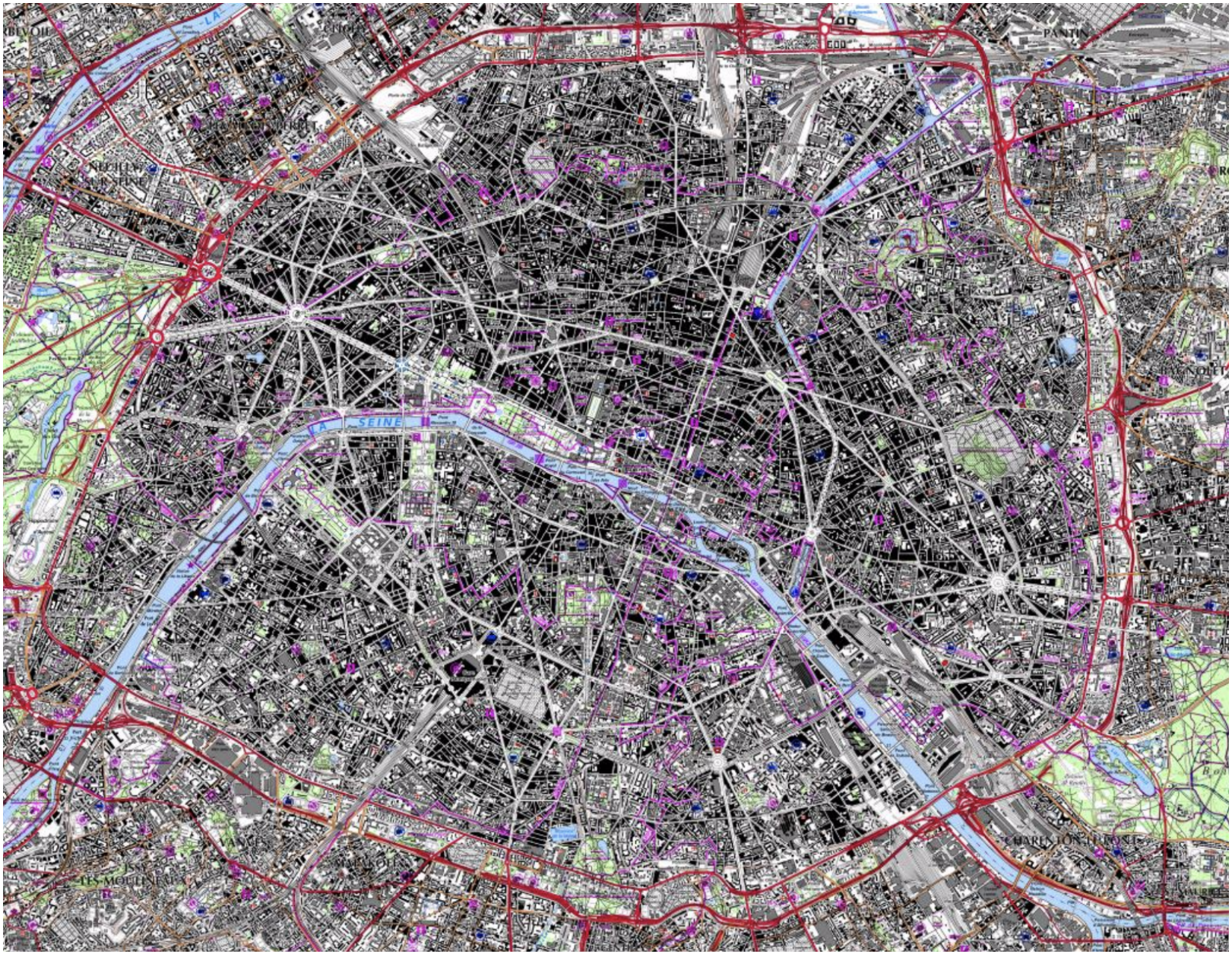
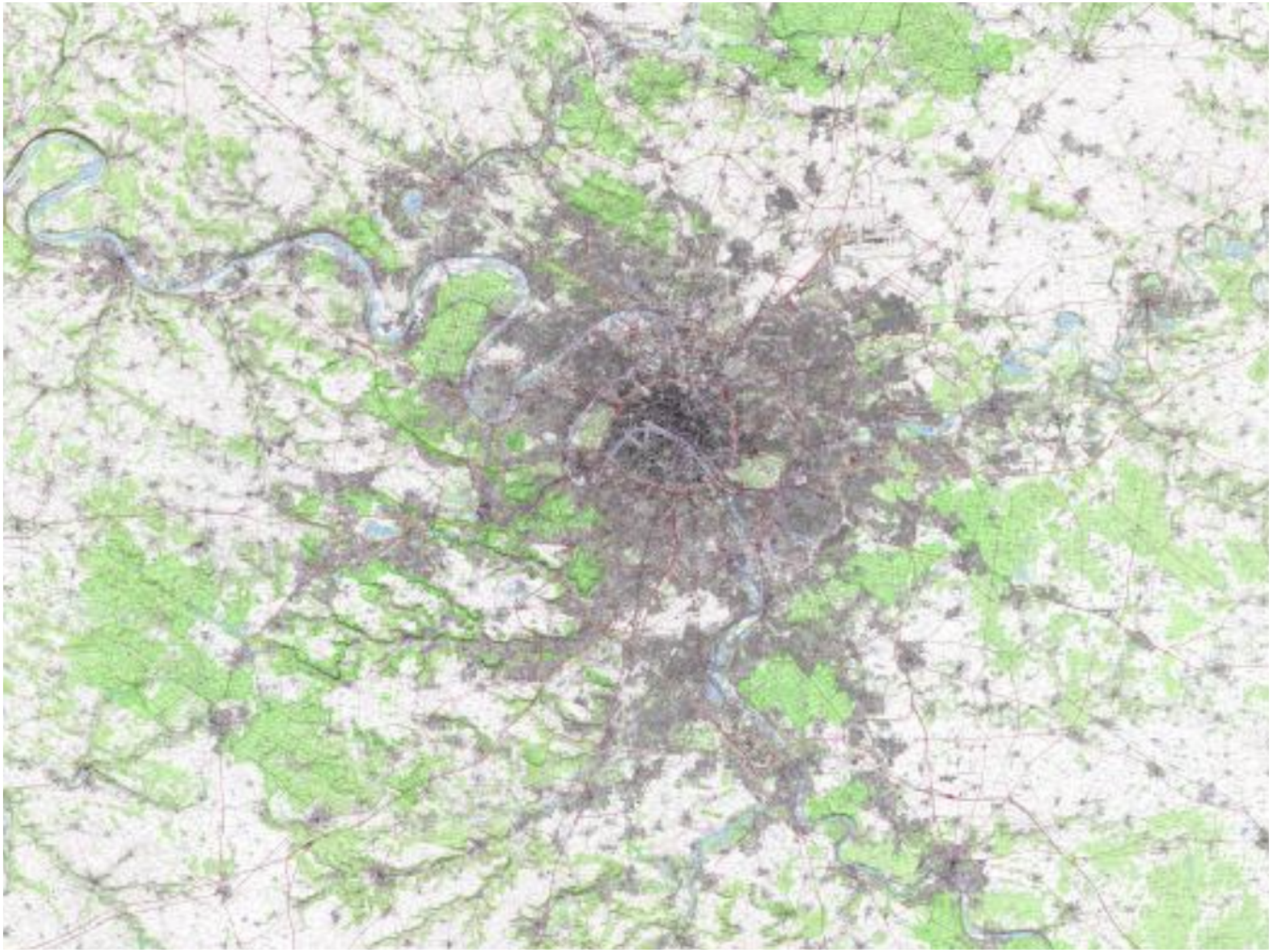
International Land-Use Symposium 2019
3.1: Fractals and Multi-fractals
5th December 2019

# Complex processes of Urban Morphogenesis



# Complex processes of Urban Morphogenesis



# Modeling urban growth and land-use change

[Engelen and White, 2008] calibration/validation + map comparison
[Zhou et al., 2018] urban form / emissions
$\rightarrow$*At the crossroad between Urban Simulation and Artificial Life, few models try to integrate and explain the link between Urban Form and Function*
$\rightarrow$*Importance of parcimonious, stylized models: modeling as a tool to understand processes*

# Contribution

Explore simple models to capture morphogenesis based on abstract representation of urban processes; test their ability to reproduce existing urban systems.

# A simple Reaction-diffusion model

$\rightarrow$ Crucial role of the interplay between concentration forces and dispersion forces [Fujita and Thisse, 1996] in keeping Urban Systems at the border of chaos

$\rightarrow$ Potentiality of aggregation mechanisms (such as Simon model) to produce power laws [Dodds et al., 2017]

$\rightarrow$ Link with Reaction-diffusion approaches in Morphogenesis [Turing, 1952]

$\rightarrow$ Extension of a DLA-type model introduced by [Batty, 1991], with simple abstract processes of population aggregation and diffusion

Raimbault, J. (2018). Calibration of a density-based model of urban morphogenesis. PloS one, 13(9), e0203516.

# Model formalization

$\rightarrow$ Grid world with cell populations $\left(P_i\left(t\right)\right)_{1\leq i\leq N^2}$.

$\rightarrow$ At each time step:

1. Population growth with exogenous rate $N_G$, attributed independently to a cell following a preferential attachment of strength $\alpha$
2. Population is diffused $n_d$ times with strength $\beta$

$\rightarrow$ Stopping criterion: fixed maximal population $P_m$.

$\rightarrow$ Output measured by morphological indicators: Moran index, average distance, rank-size hierarchy, entropy.

# Measuring urban morphology

1. Rank-size slope $\gamma$, given by $\ln\left(P_{\tilde{i}}/P_0\right) \sim k + \gamma \cdot \ln\left(\tilde{i}/i_0\right)$ where $\tilde{i}$ are the indexes of the distribution sorted in decreasing order.
2. Entropy of the distribution:

$$\mathscr{E} = \sum_{i1}^{M} \frac{P_i}{P} \cdot \ln \frac{P_i}{P} \tag{1}$$

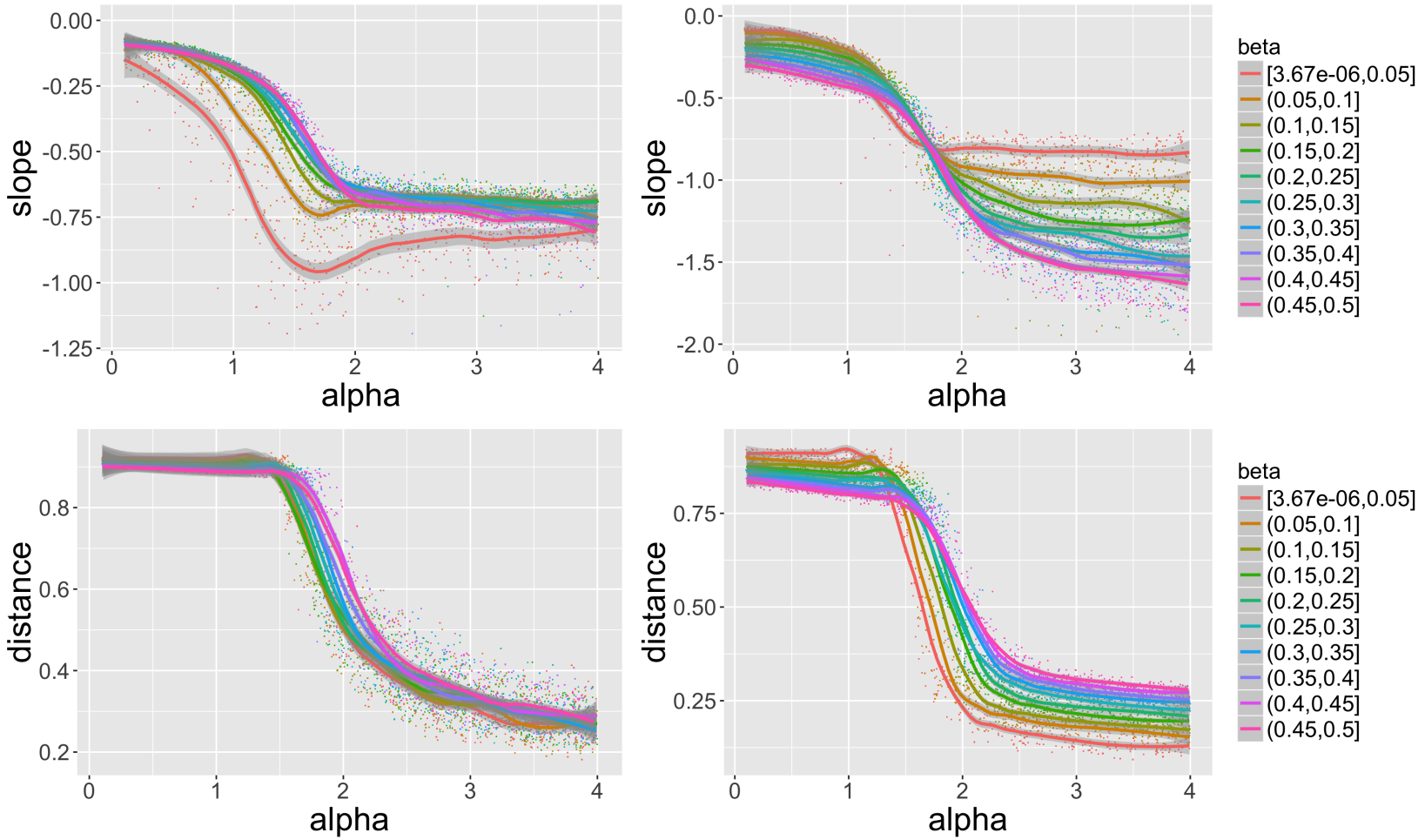
   $\mathscr{E} = 0$ means that all the population is in one cell whereas $\mathscr{E} = 0$ means that the population is uniformly distributed.
3. Spatial-autocorrelation given by Moran index, with simple spatial weights given by $w_{ij} = 1/d_{ij}$

$$I = M \cdot \frac{\sum_{ij/} w_{ij} \left(P_i - \bar{P}\right) \cdot \left(P_j - \bar{P}\right)}{\sum_{ij/} w_{ij} \sum_i \left(P_i - \bar{P}\right)^2}$$

4. Mean distance between individuals

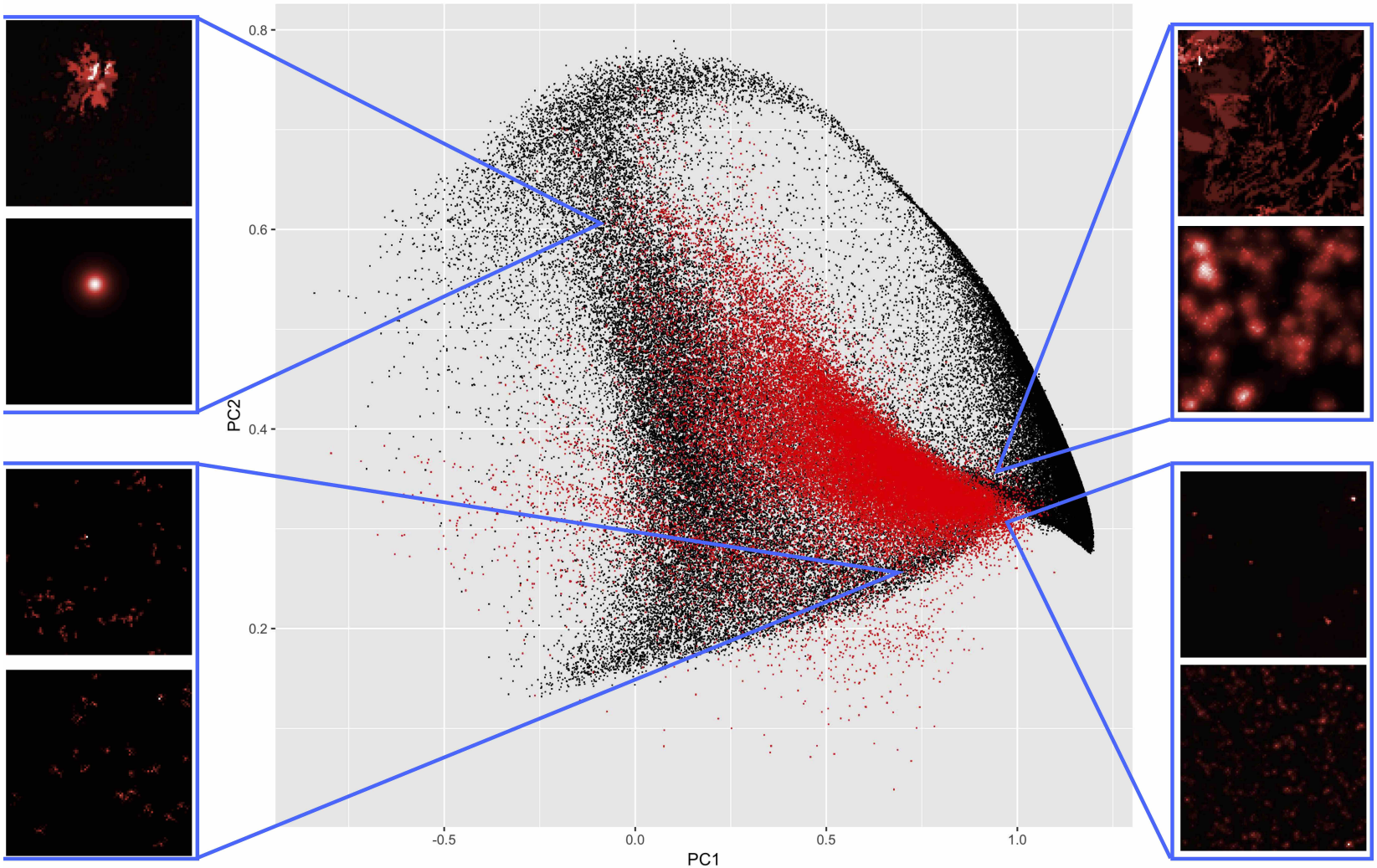
$$\bar{d} = \frac{1}{d_M} \cdot \sum_{i<j} \frac{P_i P_j}{P^2} \cdot d_{ij}$$

# Model behavior (synthetic data))




*Phase transitions of indicators unveiled by exploration of the parameter space (80000 parameter points, 10 repetitions each)*

# Model application on a spatial grid




*Brute force calibration by exploring the parameter space. Reproduction of most existing configuration in the morphological sense (here in principal plan).*

# GHSL database

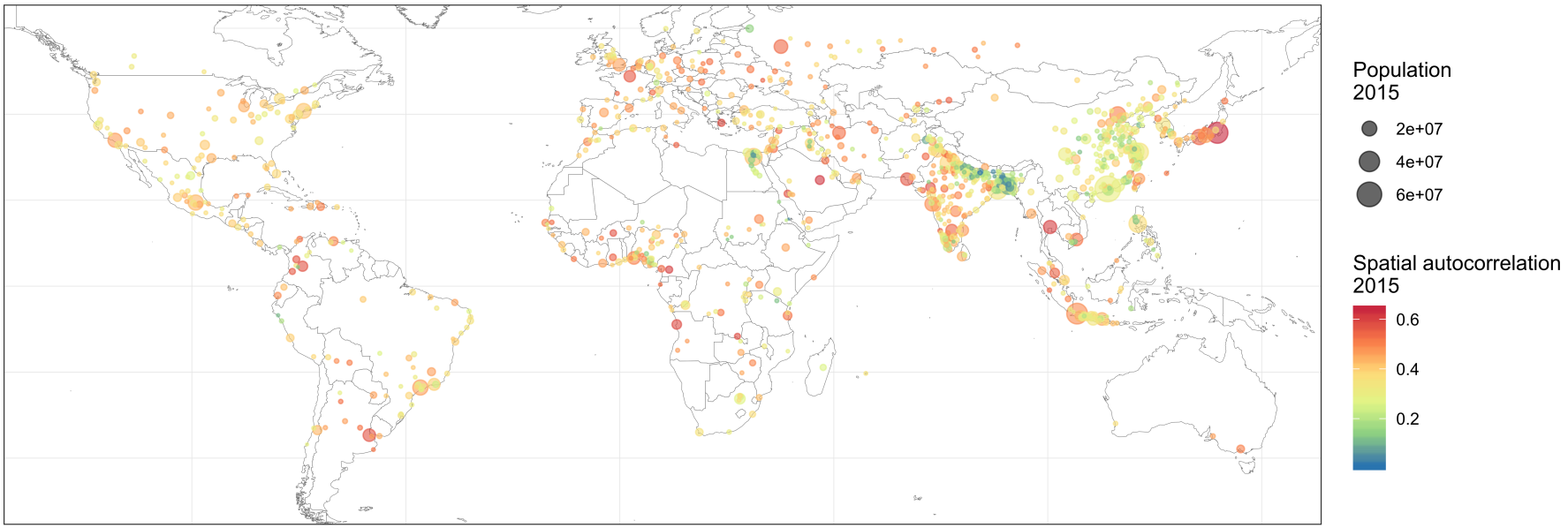
→ Application on worldwide urban areas using the GHSL population grid

→ Dynamical calibration on three successive time windows (1975-1990, 1990-2000, 2000-2015)

→ Extraction of neighborhood (spatial span times 1.5) of 1000 larger urban areas (2015 population)

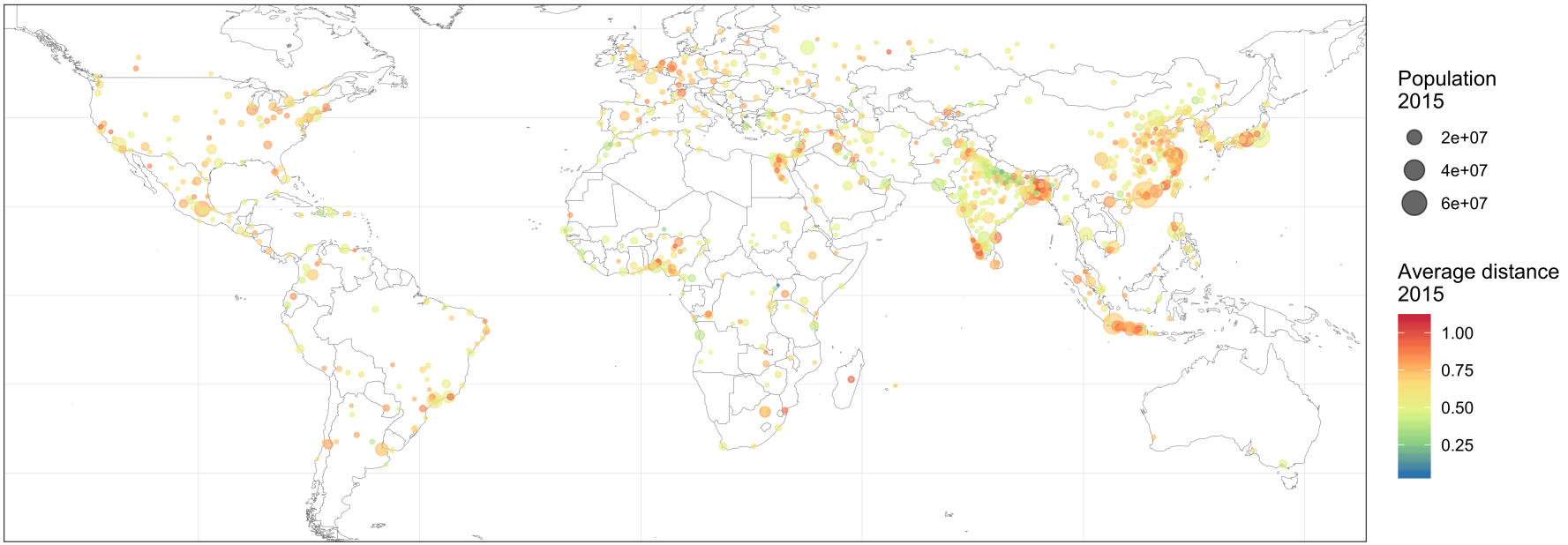
→ Computation of morphological indicators on corresponding population grids

# Global indicators distribution: Moran



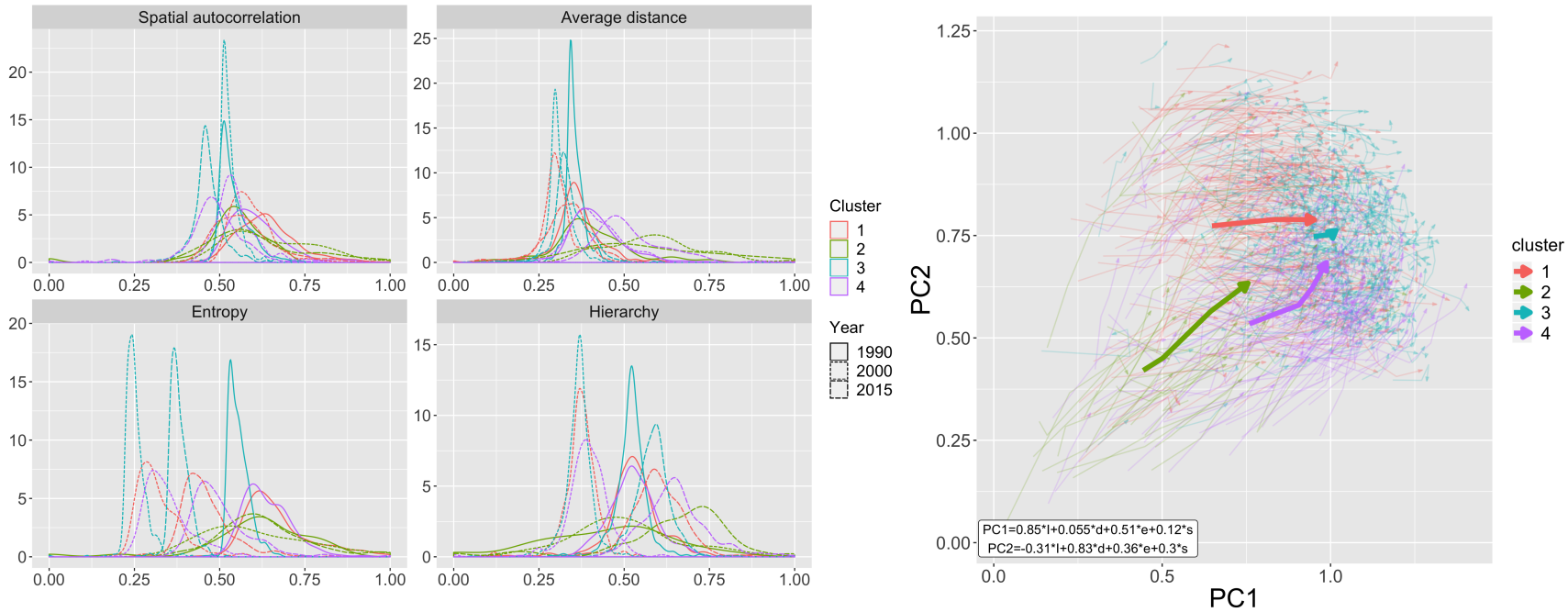

*Regional patterns for Moran index*

# Global indicators distribution: average distance



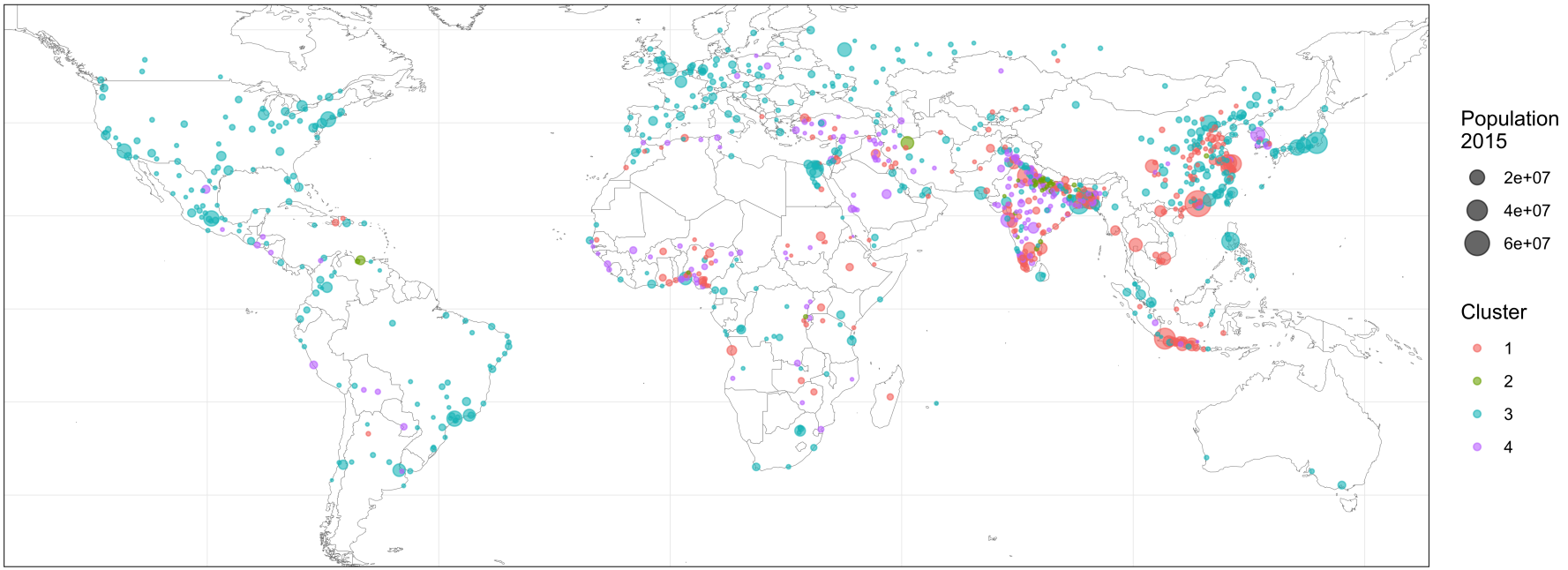

*Complementarity of average distance indicator*

# Clustering morphological trajectories

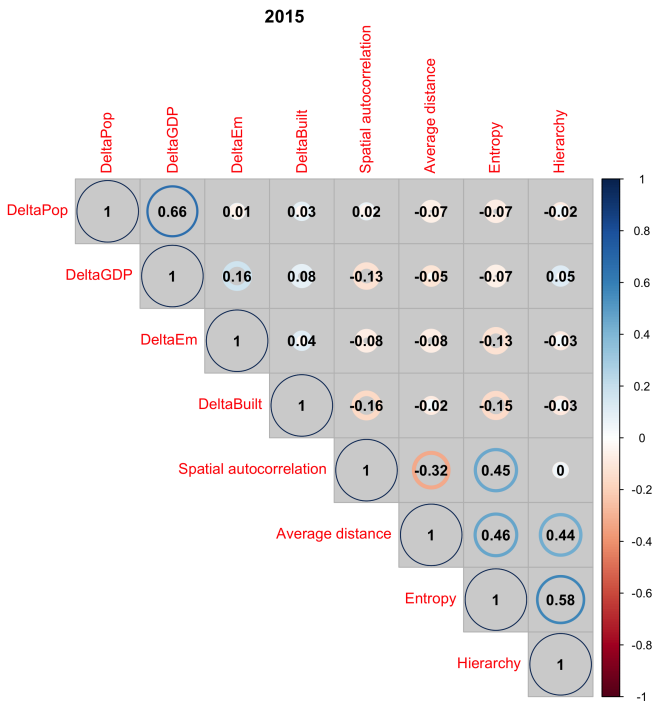


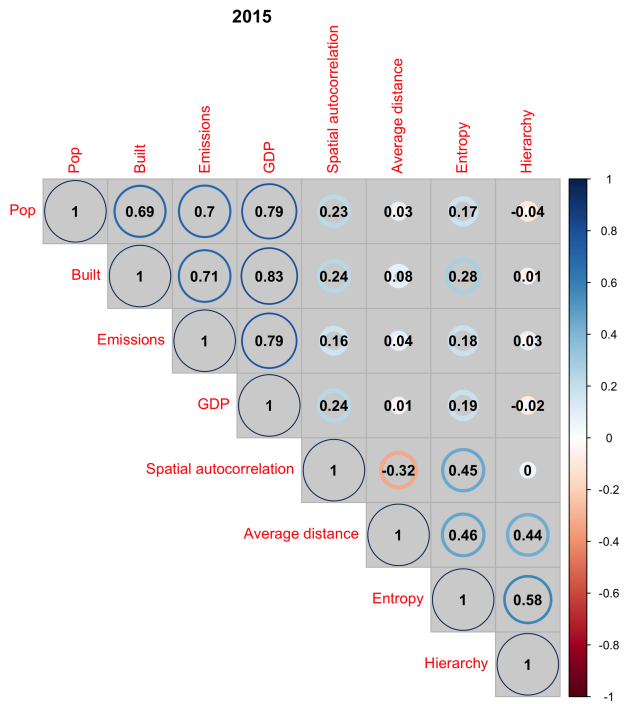

*Disjoint distributions and temporal trajectories: model application is relevant*

# Distribution of clusters



# Correlations



# Calibration method

# Implementation

*Performance constraints: simulate N mesoscopic morphogenesis models in parallel (macroscopic interactions are efficient as based on matrices)*

$\rightarrow$ model implemented in `scala` and integrated within a broader library (including implementations of [**?**] [Raimbault, 2018]
[**?**] [**?**])

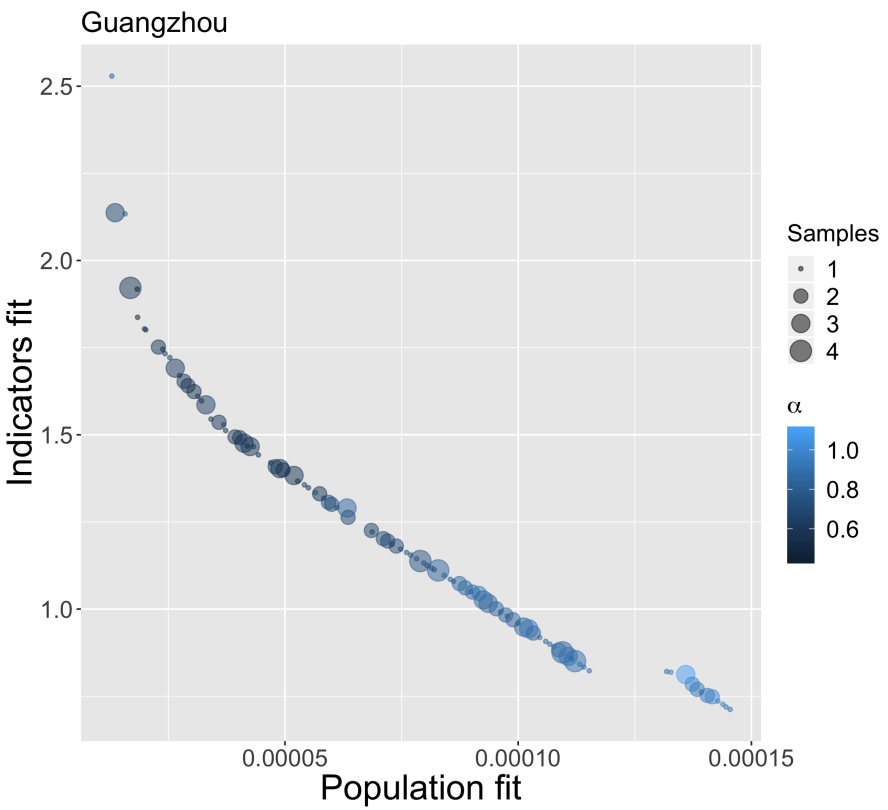
*Large number of parameters and output indicators*

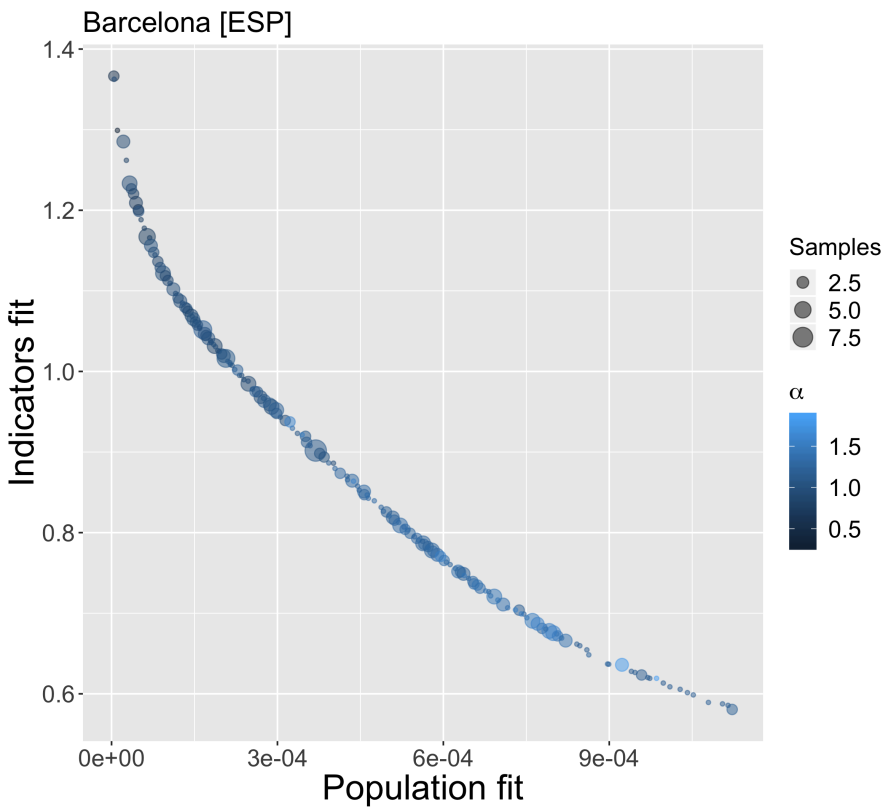
$\rightarrow$ integration into the OpenMOLE model exploration open source software [Reuillon et al., 2013]


*Enables seamlessly (i) model embedding; (ii) access to HPC resources; (iii) exploration and optimization algorithms*

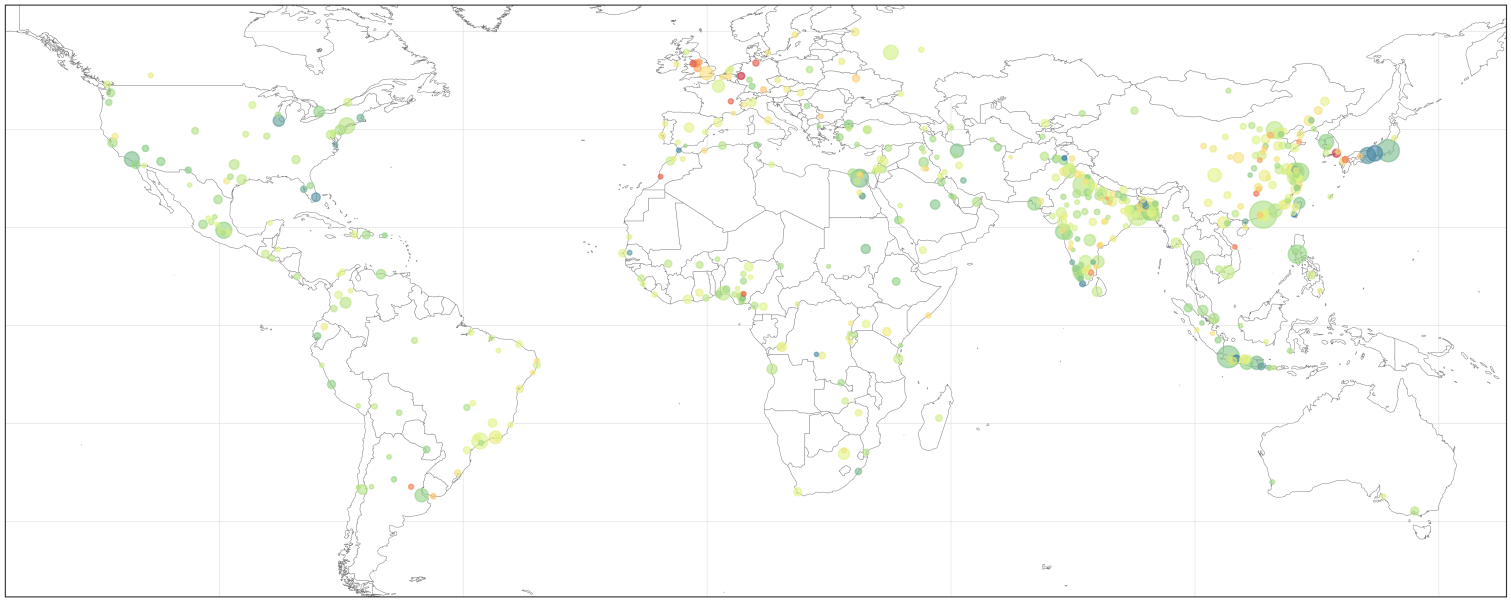
**Come to the satellite *New Methods and Epistemologies to Explore Simulation Models* tomorrow afternoon in LHN-TR+05**

# Examples of results



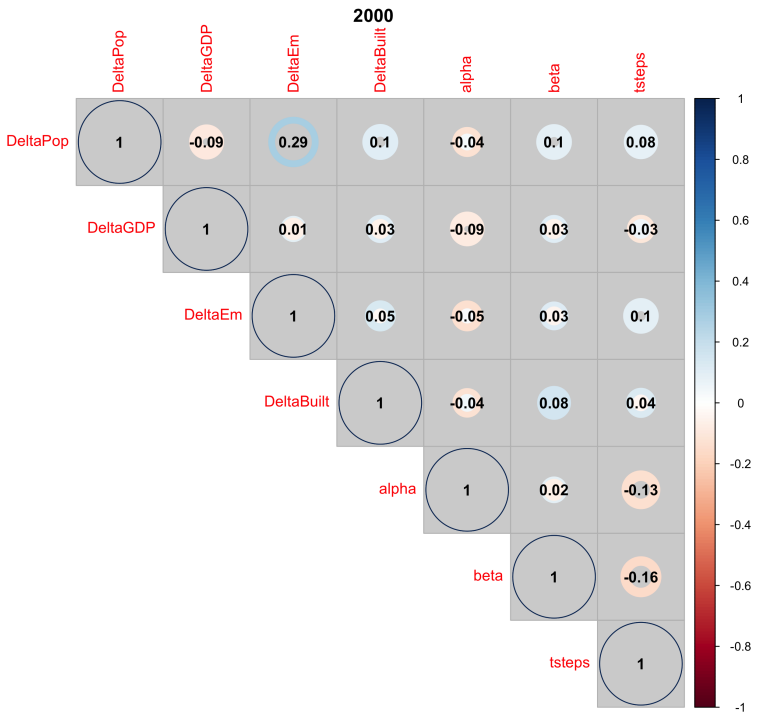



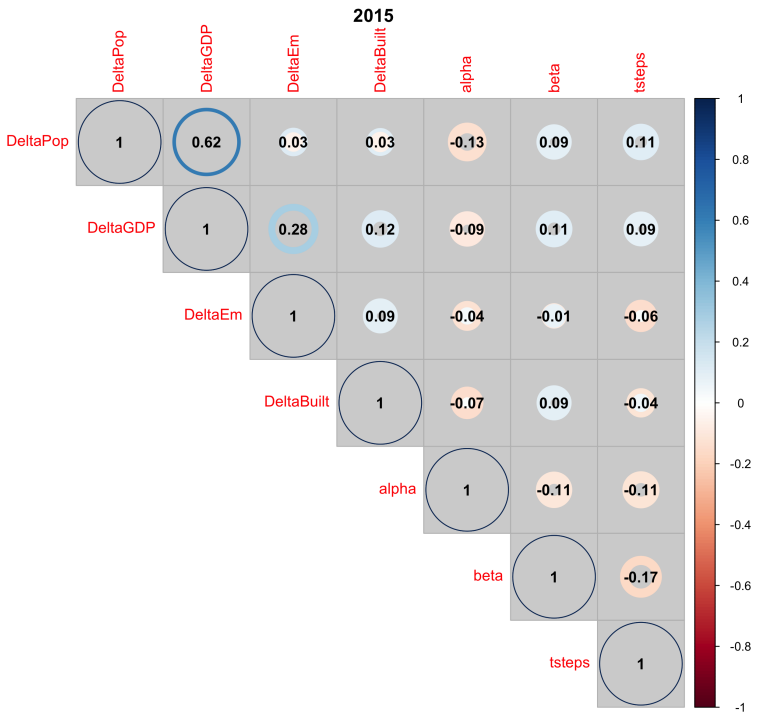
# Adjusted parameters



# Trajectories in phase diagrams

# Correlations



**Implications**

→ This rather simple model reproduces most of existing urban forms in Europe for both population distribution and road network : which intrinsic dimension to the urban system and its morphological aspect ?

→ Ability to reproduce static correlations and a variety of dynamical lagged correlation regimes suggests that the model captures some of the processes of co-evolution

**Developments**

→ Towards a dynamical calibration ? Need of dynamical data

→ Investigate the link between spatial non-stationarity and non-ergodicity through simulation by the model

→ Compare network generation in a “fair” way (correcting for additional parameters, open question for models of simulation)

# Towards policy applications

**More realistic models?**

→ Introducing more concrete ontologies, economic processes [Bonin and Hubert, 2014], qualitative differentiation [Bonin and Hubert, 2012] governance processes [Le Néchet and Raimbault, 2015]

→ Possible bridges with Land-use change models/Land-use Transport models [Wegener and Fürst, 2004], with systems of cities models [Pumain and Reuillon, 2017]

**More data-driven models?**

→ Work in progress: calibration of the reaction-diffusion model on world urban areas with the Global Human Settlements Layer database

→ Link with sustainability indicators: GHG emissions, economics, etc. [Raimbault, 2019]

→ Study models on hybrid synthetic data [Raimbault et al., 2019]: systematic conclusions for policies

# Conclusion

$\rightarrow$ A novel model of urban morphogenesis at the mesoscopic scale systematically explored: **need for more coupling and comparison of models.**

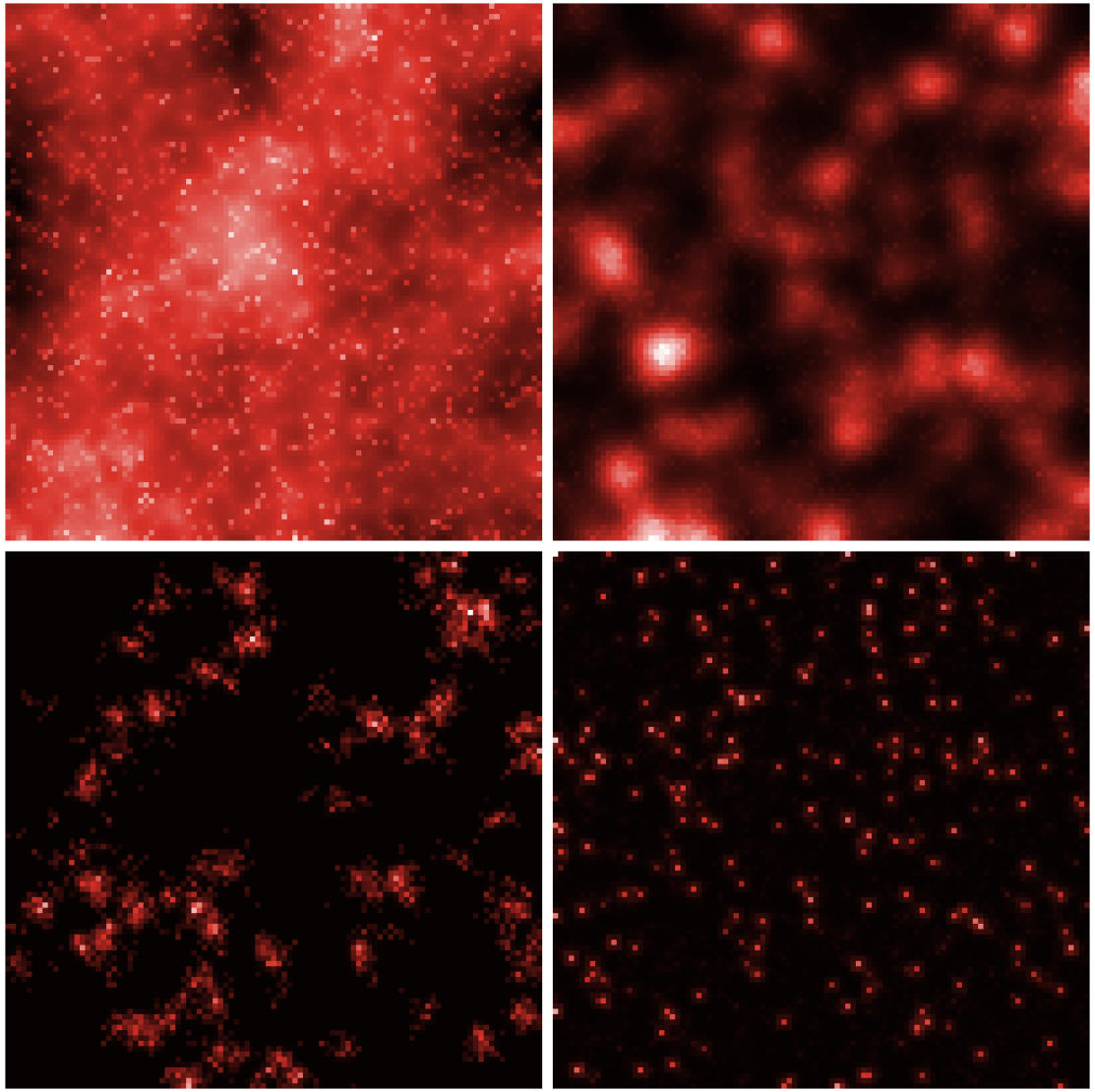
$\rightarrow$ At the macro scale of the system of cities? **Need for multi-scale models.**

$\rightarrow$ With more refined urban characteristics and other dimensions ? **Need for more interdisciplinarity.**

- Code, data and results available at
`https://github.com/JusteRaimbault/CityNetwork`
- Acknowledgments: Thanks to the *European Grid Infrastructure* and its *National Grid Initiatives* (*France-Grilles* in particular) to give the technical support and the infrastructure.

# Reserve Slides

# Generating Population Distributions



*Examples of generated territorial shapes*

# Path-dependence and frozen accidents

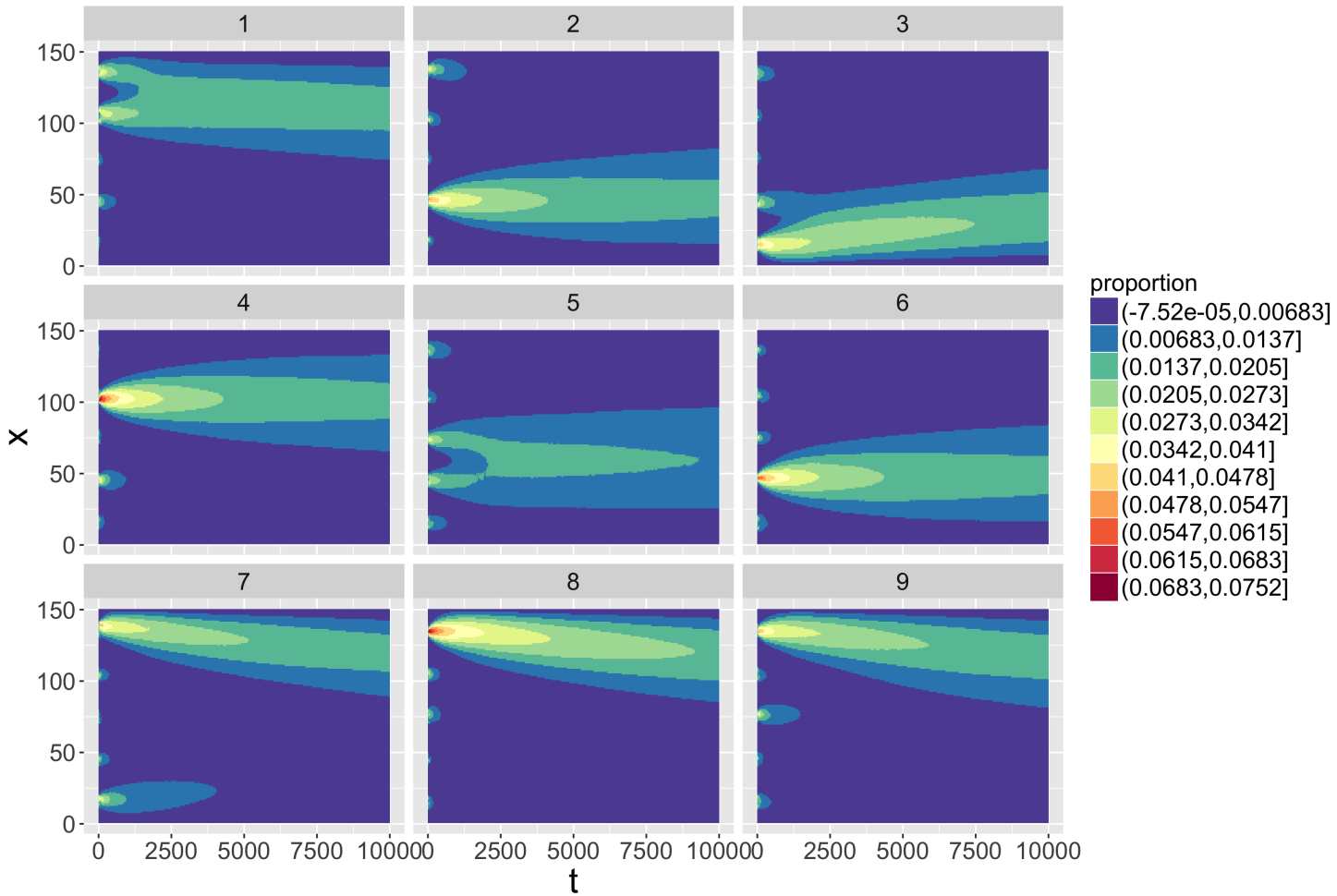
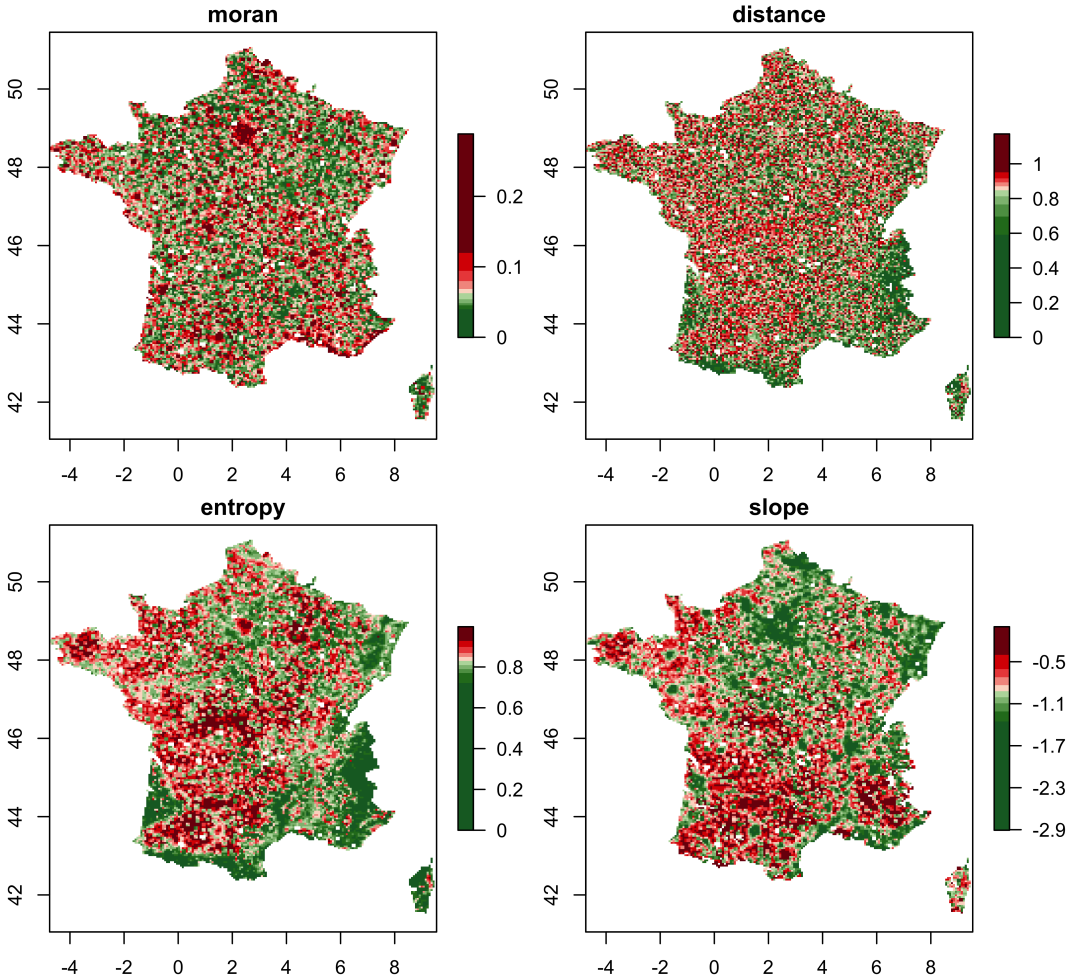
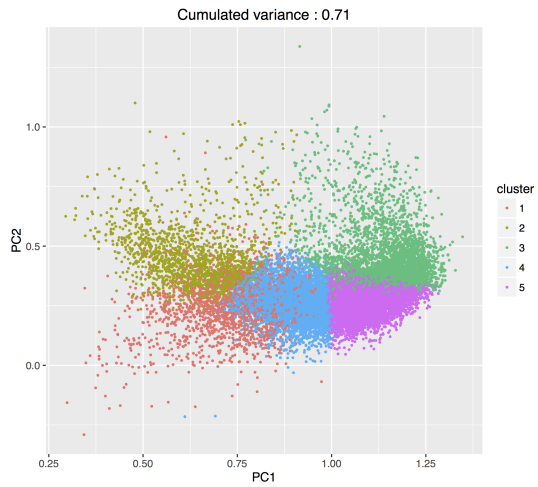
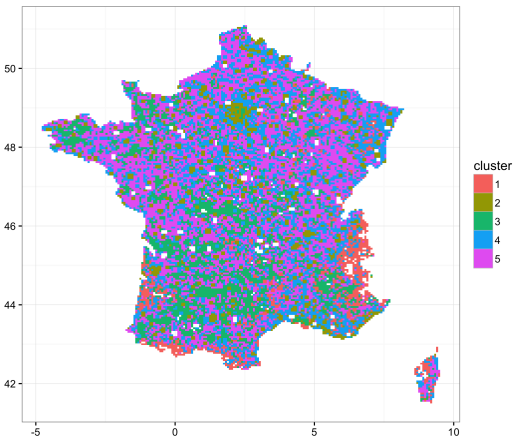



*Illustration of path-dependence in a simplified one-dimensional version of the model: cell trajectories in time for 9 independent repetitions from the same initial configuration.*

# Empirical Data (moving window)



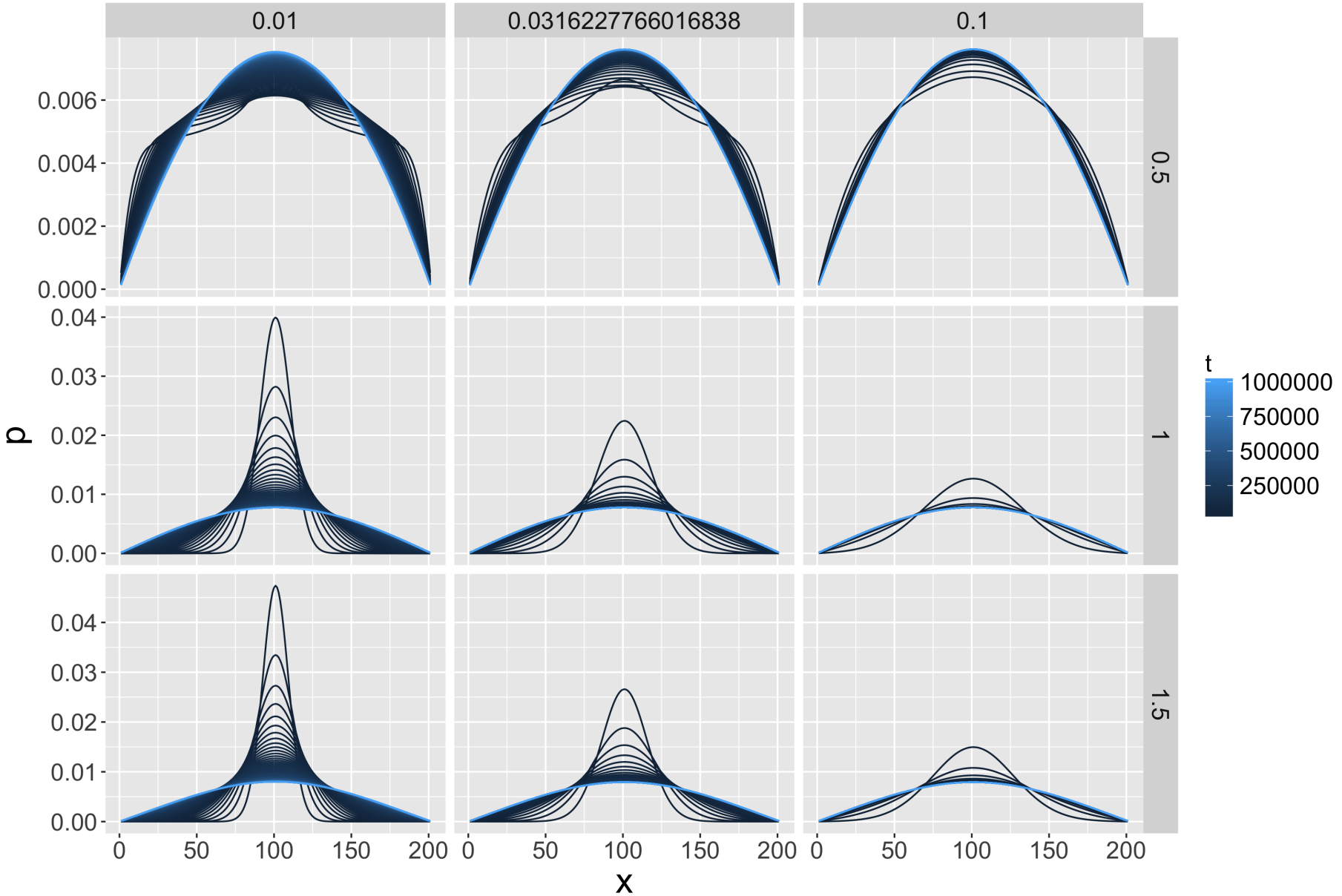




*Computation of morphological indicators on population density data for Europe (shown here on France), morphological classification.*

The one-dimensional model verifies the PDE :

$$\delta t \cdot \frac{\partial p}{\partial t} = \frac{N_G \cdot p^{\alpha}}{P_{\alpha} t} + \frac{\alpha\beta\left(\alpha-1\right)\delta x^2}{2} \cdot \frac{N_G \cdot p^{\alpha-2}}{P_{\alpha}\left(t\right)} \cdot \left(\frac{\partial p}{\partial x}\right)^2 + \frac{\beta \delta x^2}{2} \cdot \frac{\partial^2 p}{\partial x^2} \cdot \left[1 + \alpha \frac{N_G p^{\alpha-1}}{P_{\alpha t}}\right] \tag{2}$$

# Stationary behavior of 1D model



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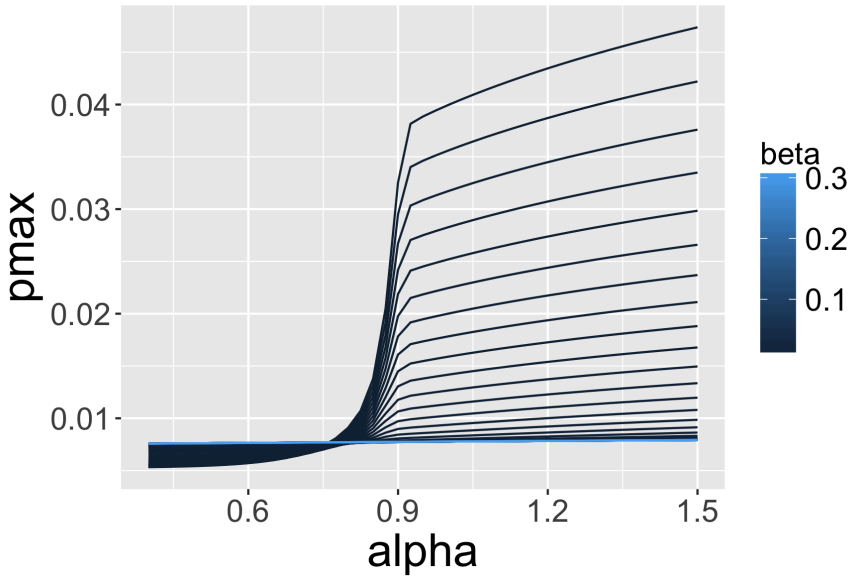
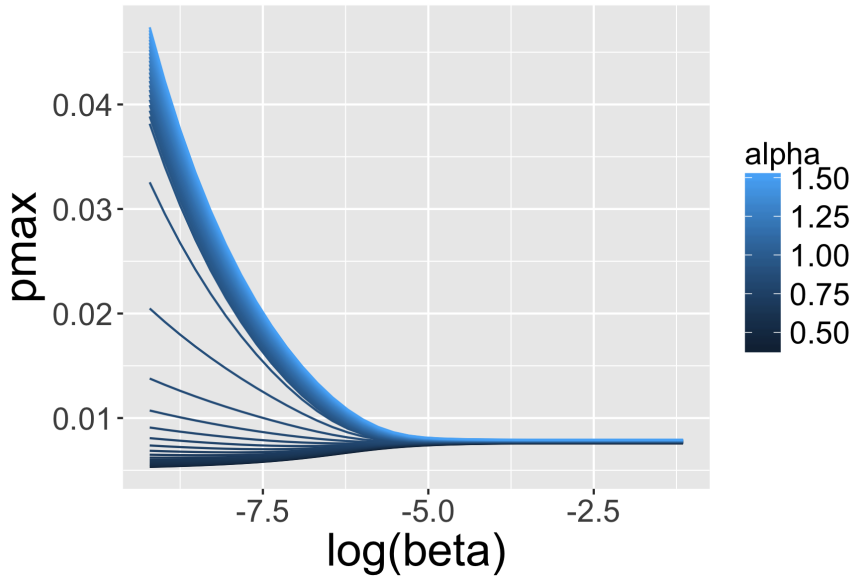
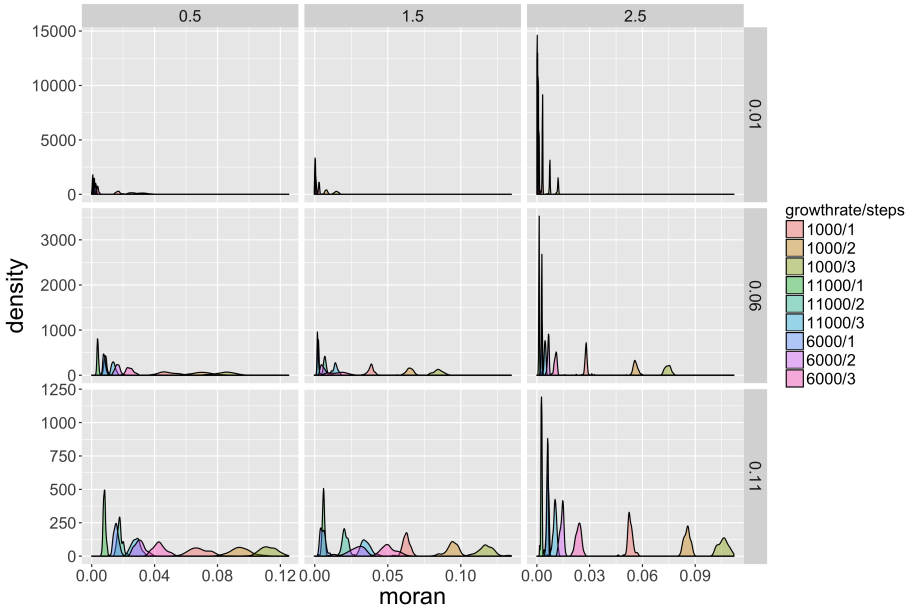
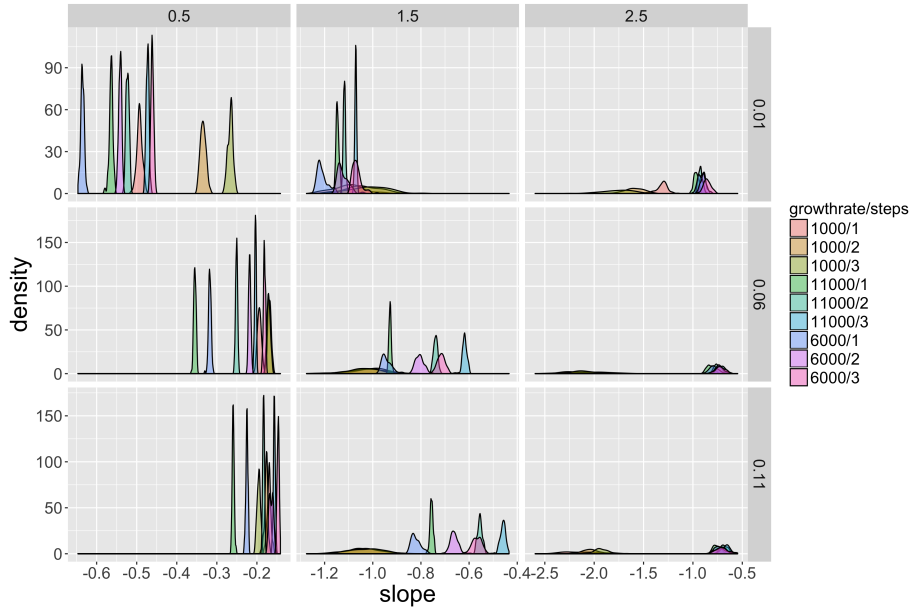


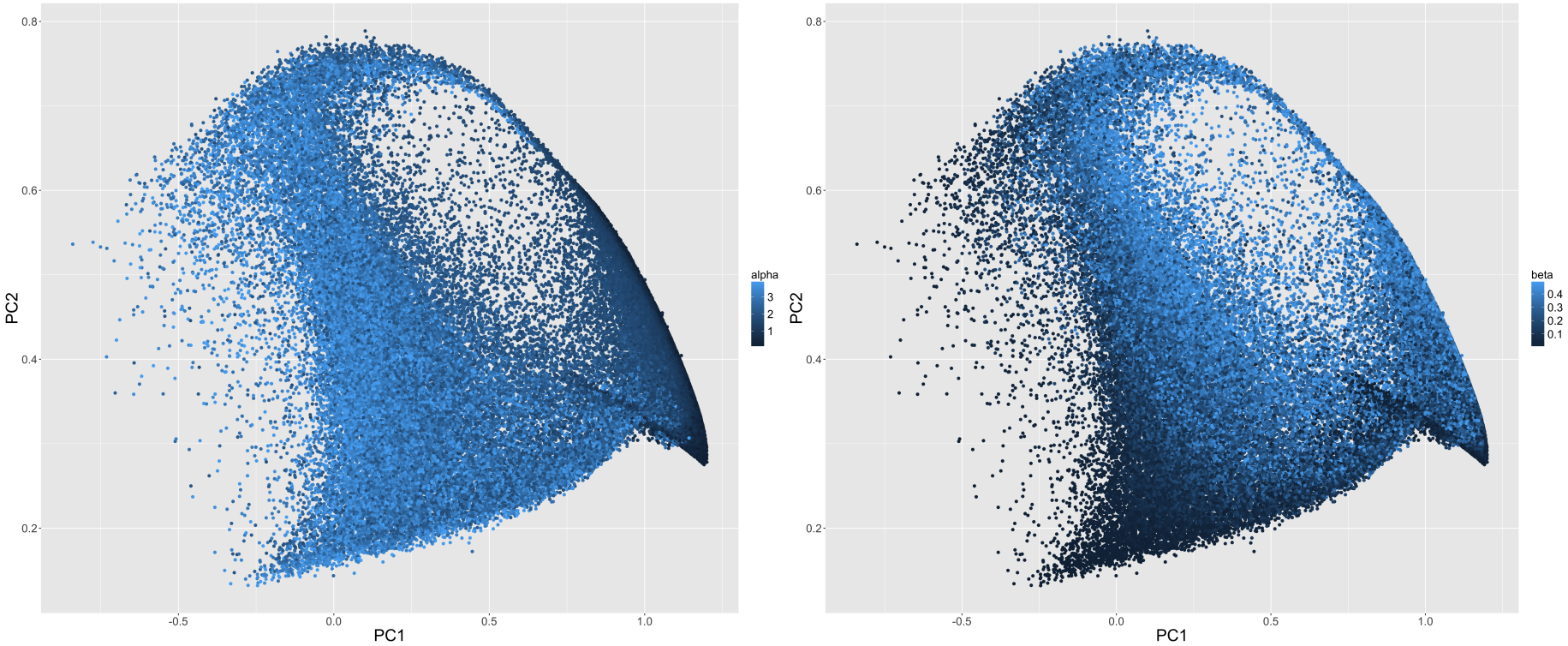
# Model behavior : Convergence

## Large number of repetitions show good convergence properties



# Model behavior



# Empirical indicators computation

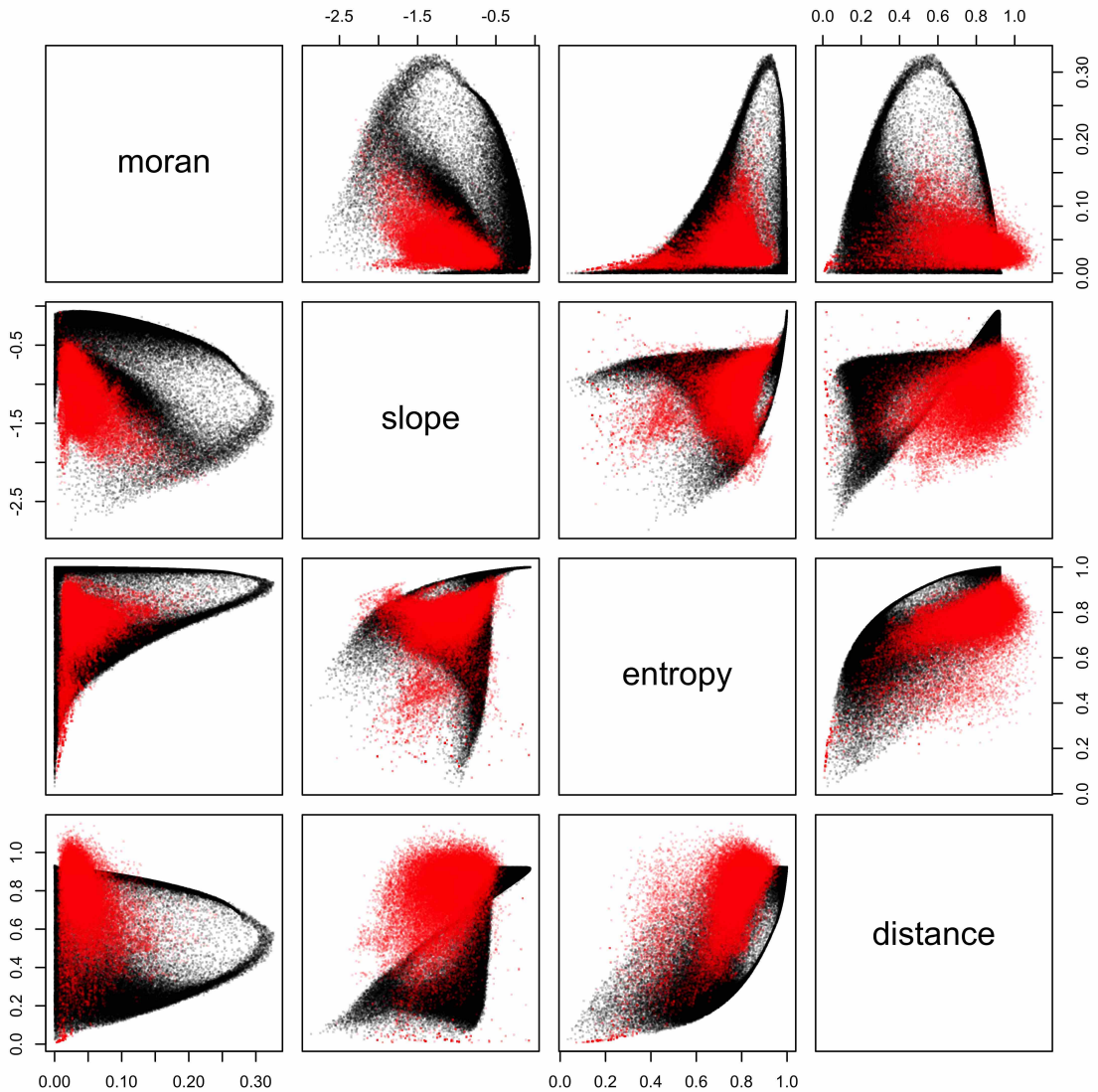
→ Eurostat population density raster (100m, simplified at 500m resolution)

→ Overlapping (10km offset) squares of 50km side : equivalent to smoothing, removes window shape effect. Not very sensitive to window size (tested with 30km and 100km)

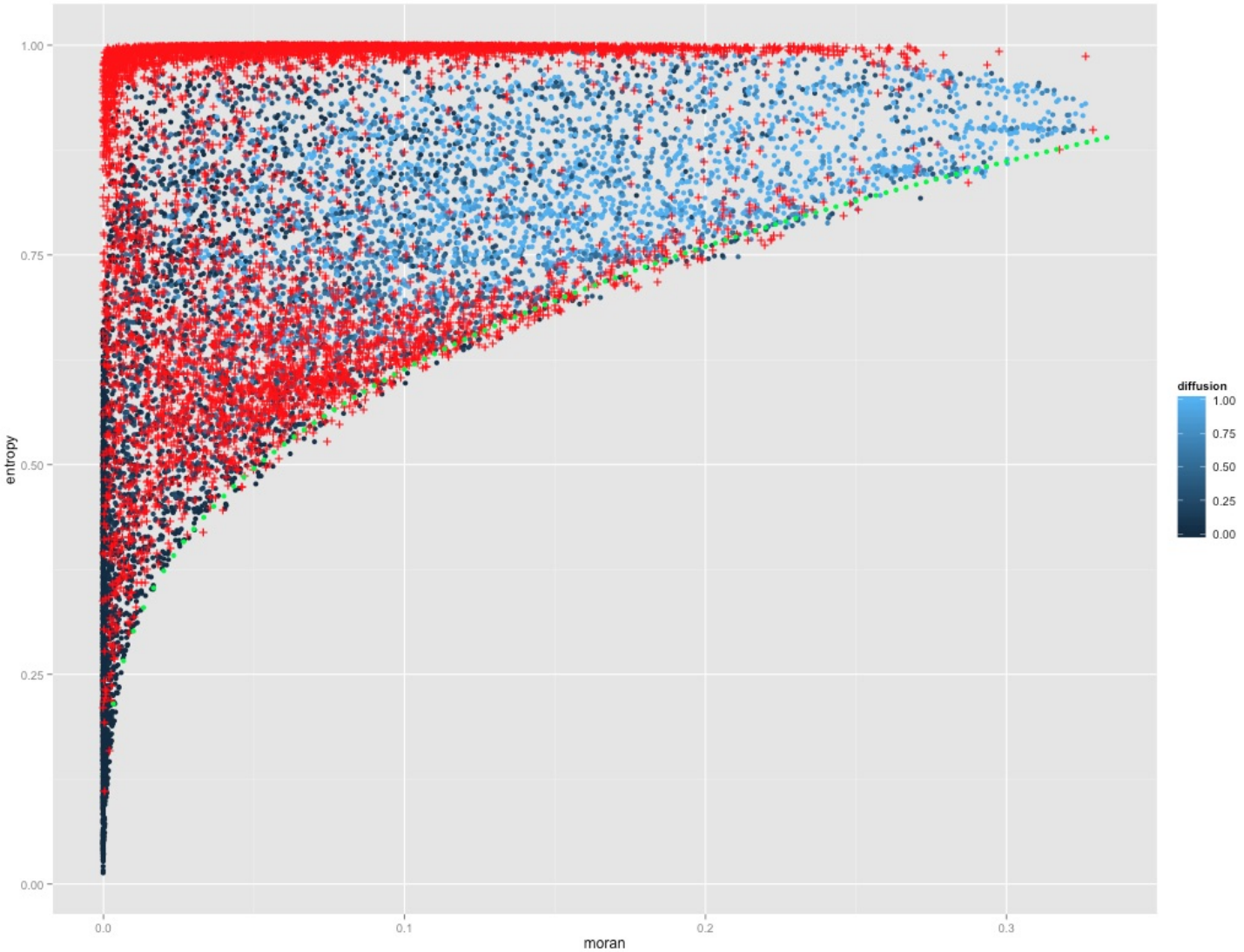
→ Indicators computed using Fast Fourier Transform Convolution

→ Classification using repeated k-means ; number of clusters taken at transition in clustering coefficient.

# Model calibration: all indicators



# Model Targeted Exploration




*Potentialities of targeted model explorations: here feasible space using Pattern Space Exploration algorithm [Chérel et al., 2015].*

*Proposition of an interdisciplinary definition*

**Meta-epistemological framework of imbricated notions:**
Self-organization $\supsetneq$ Morphogenesis $\supsetneq$ Autopoiesis $\supsetneq$ Life

**Properties:**

- Architecture links form and function
- Emergence strength [Bedau, 2002] increases with notion depth, as bifurcations [Thom, 1974]

**Definition of Morphogenesis :** *Emergence of the form and the function in a strongly coupled manner, producing an emergent architecture [Doursat et al., 2012]*

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