

# A systematic comparison of interaction models for systems of cities

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CCS 2018

*Session Infrastructure, Planning and Environment*

Thessaloniki

September 24th 2018

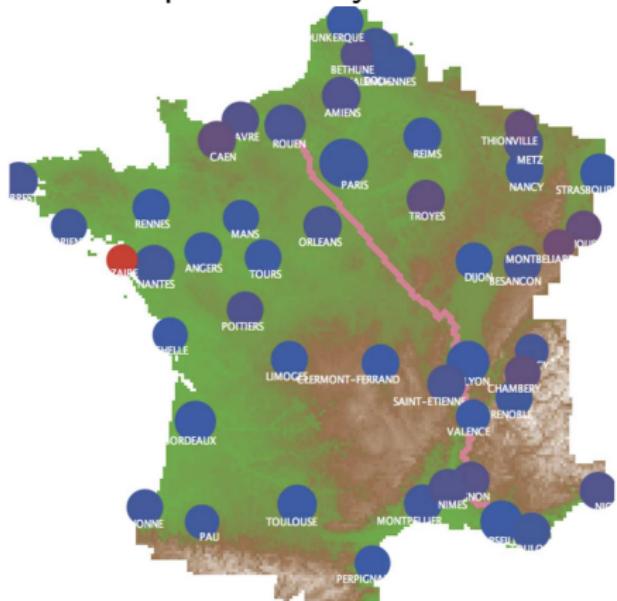
# The long time memory of urban systems



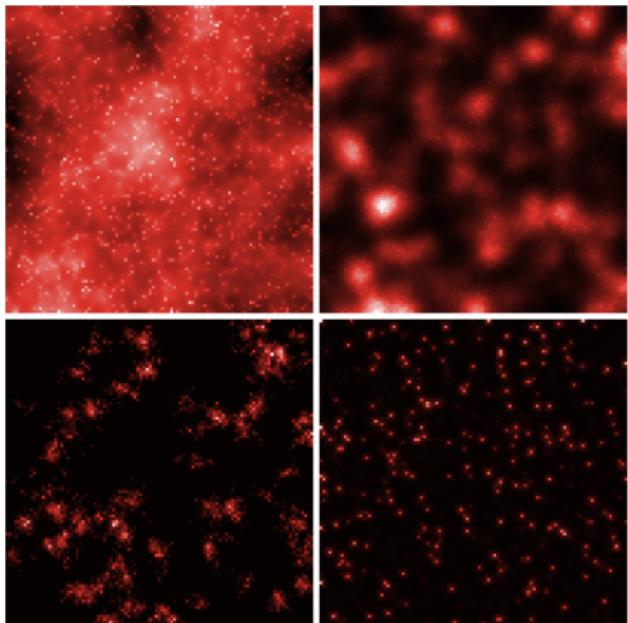
*Source: Wikipedia*

# Modeling urban growth

## Macroscopic urban system



## Mesoscopic territorial systems

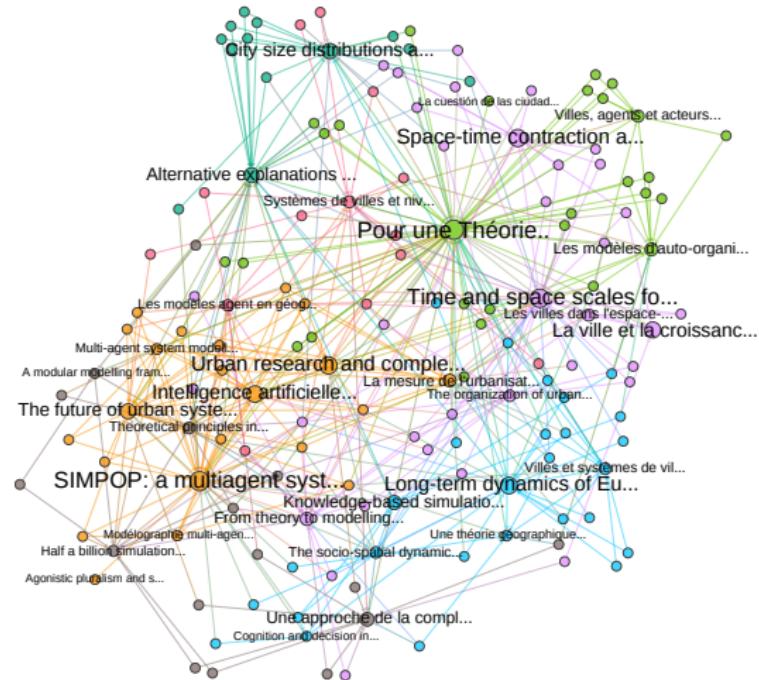


Rambault, J. (2018). Indirect evidence of network effects in a system of cities. *Environment and Planning B: Urban Analytics and City Science*, 2399808318774335.

Rambault, J. (2018). Calibration of a density-based model of urban morphogenesis. *PloS one*, 13(9), e0203516.

# An evolutionary urban theory

From [Pumain, 1997] to [Pumain, 2018]: systems of cities as co-evolutive systems in which interactions are crucial



[Raimbault, 2017] Citation network analysis of core publications in the evolutionary urban theory

# Towards a systematic model comparison

- several models in this context have been introduced, but never compared (e.g. in terms of explanatory power)
- multidimensionality of urban systems and the potential complementarity between very different processes

**Research objective :** *Benchmark interaction models for systems of cities developed in the frame of the evolutionary urban theory, on several comparable systems of cities.*

Comparison of three approaches based on the evolutionary urban theory [Pumain, 1997] capturing different dimensions of urban systems:

- The Favaro-Pumain model for the diffusion of innovation  
[Favaro and Pumain, 2011]
- The Marius model family based on economic exchanges  
[Cottineau, 2014]
- An interaction model including physical transportation networks  
[Raimbault, 2018b]

# Description of models

## Network interaction model

- Endogenous growth
- Interactions inducing growth through gravity potential
- Static physical network taken into account (geographical shortest path with topography)

## Favaro-Pumain model

- Endogenous growth
- Innovation emerge and diffuse in cities
- Growth rates adapted according to utility of innovation and level of adaptation

## Marius model

- Cities produce economic goods
- Economic exchanges are estimated according to gravity flows
- Populations grow depending on final economic balances

## Dataset

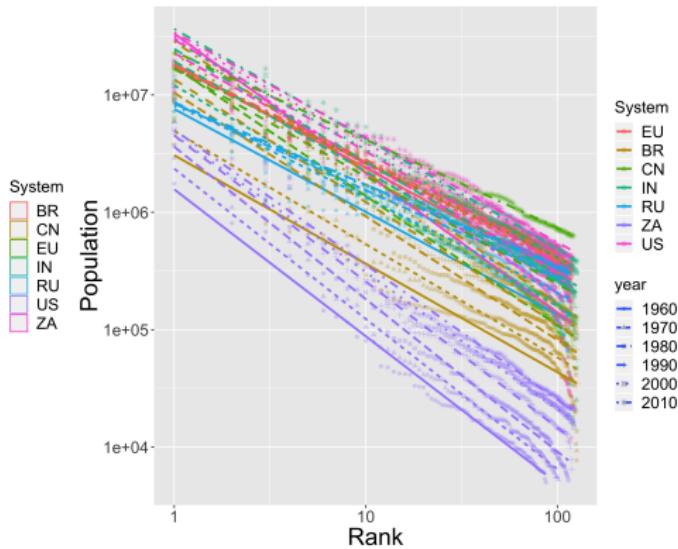
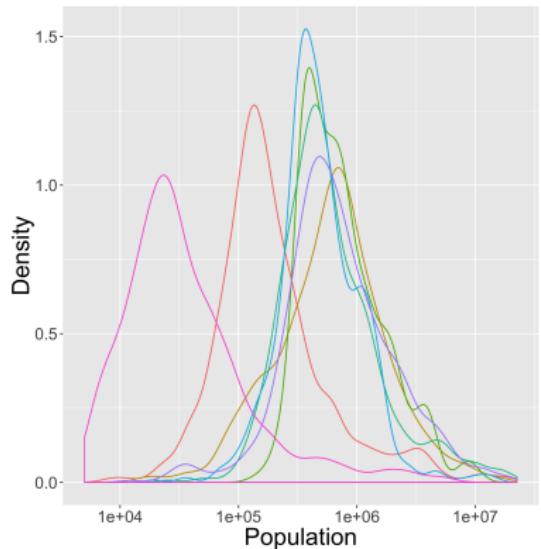
Harmonized dataset, explored in particular by [Pumain et al., 2015] (built in the context of Geodiversity ERC): Urban systems for Europe, United States, Brazil, China, India, South Africa, Russia, consistent and comparable between 1960 and 2010 each 10 years.

## Data preprocessing

- Remove small cities and medium-sized cities ([Adam, 2006] for definition of medium-sized) for scalability of simulations
- Shortest paths matrices (direct and taking into account topography, using the EEA world DEM) computed *a priori* and cached (*no evolution of the network*)

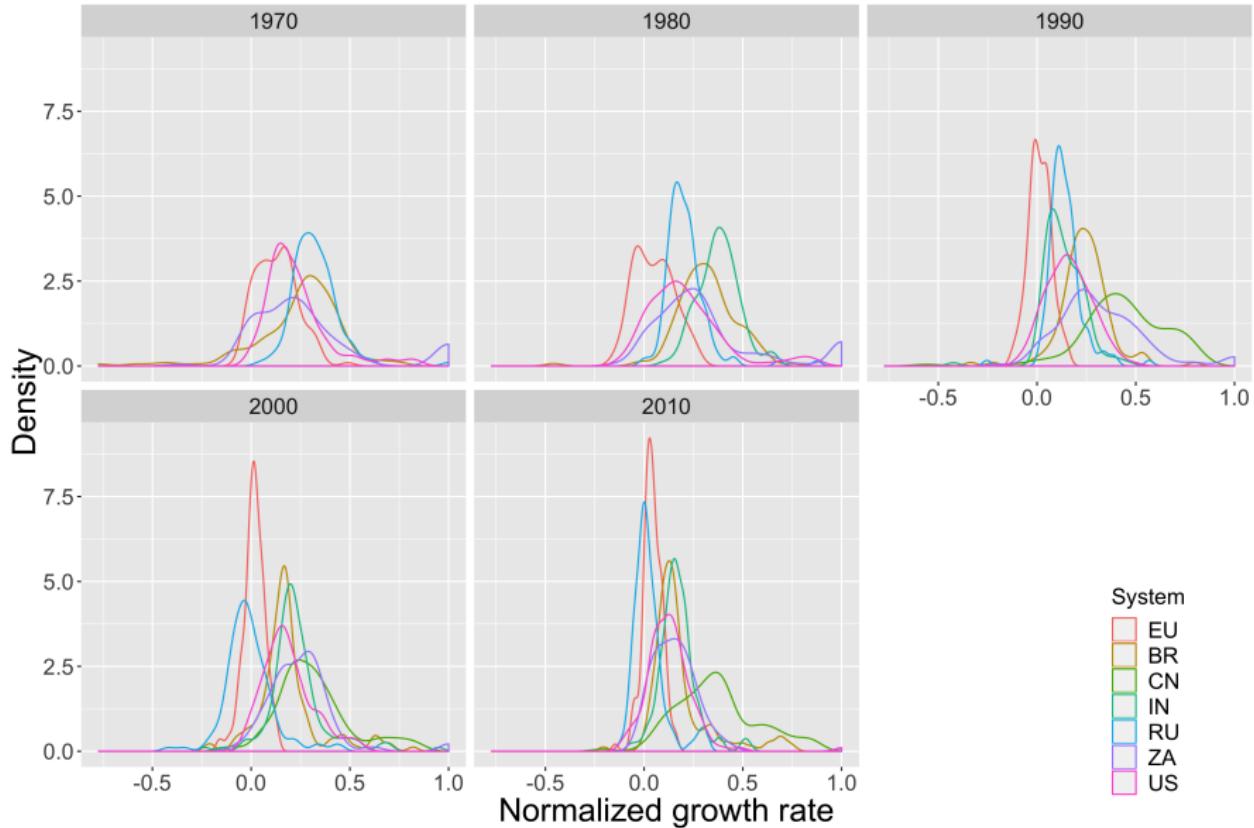
# Stylized facts

*Population distributions mainly log-normal (75% with KS-test, all with AIC); consistent hierarchies in time.*



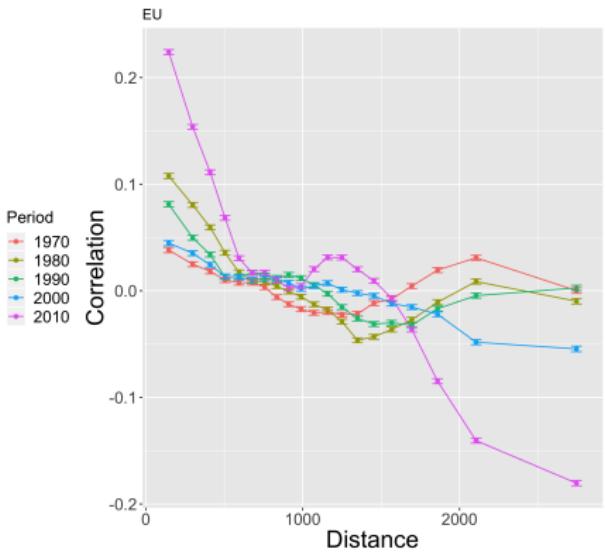
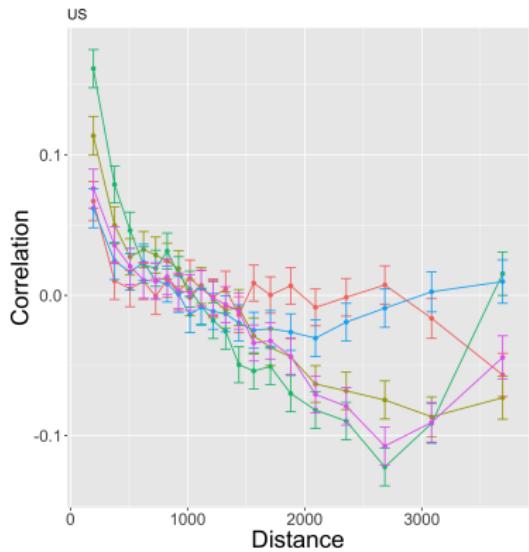
# Stylized facts

*Growth rates distributions evolve considerably in time*



# Stylized facts

*Correlation as a function of distance: long-range correlations in several systems*



## Model calibration

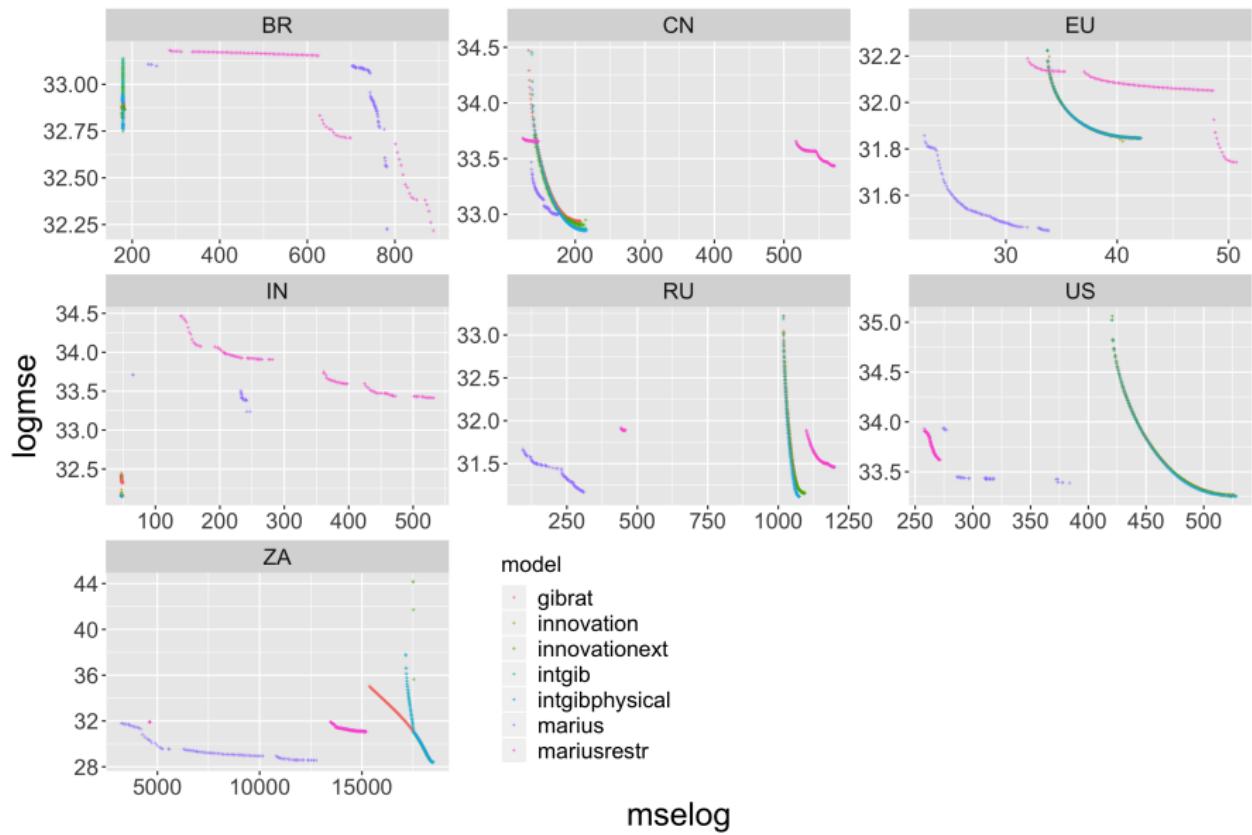
Bi-objective calibration of 7 models for 7 city systems → use of genetic algorithms on grid, made smooth with the OpenMOLE software  
<https://next.openmole.org/>



*OpenMOLE: (i) embed any model as a black box; (ii) transparent access to main High Performance Computing environments; (iii) model exploration and calibration methods.*

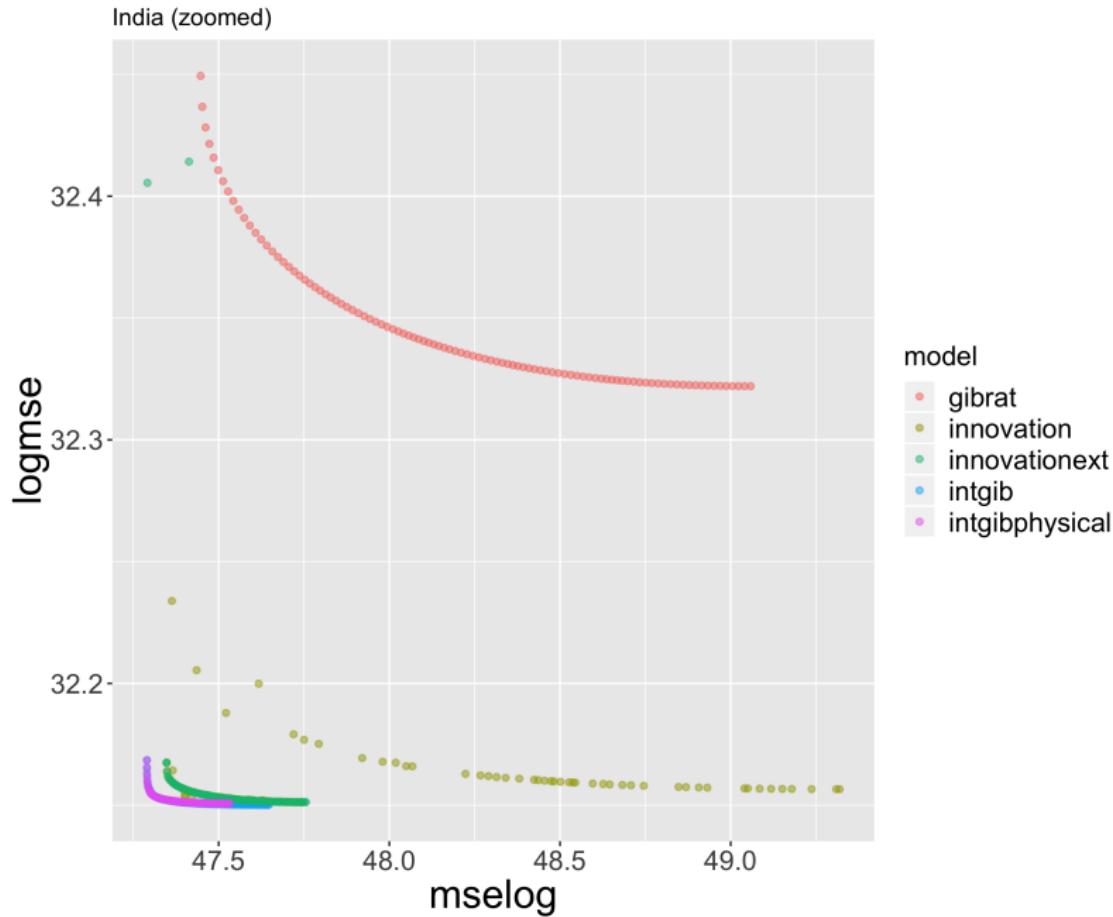
Come to the Satellite on Wednesday, and apply to the summer school !  
(<https://exmodelo.org/>)

# Comparison of models on all systems

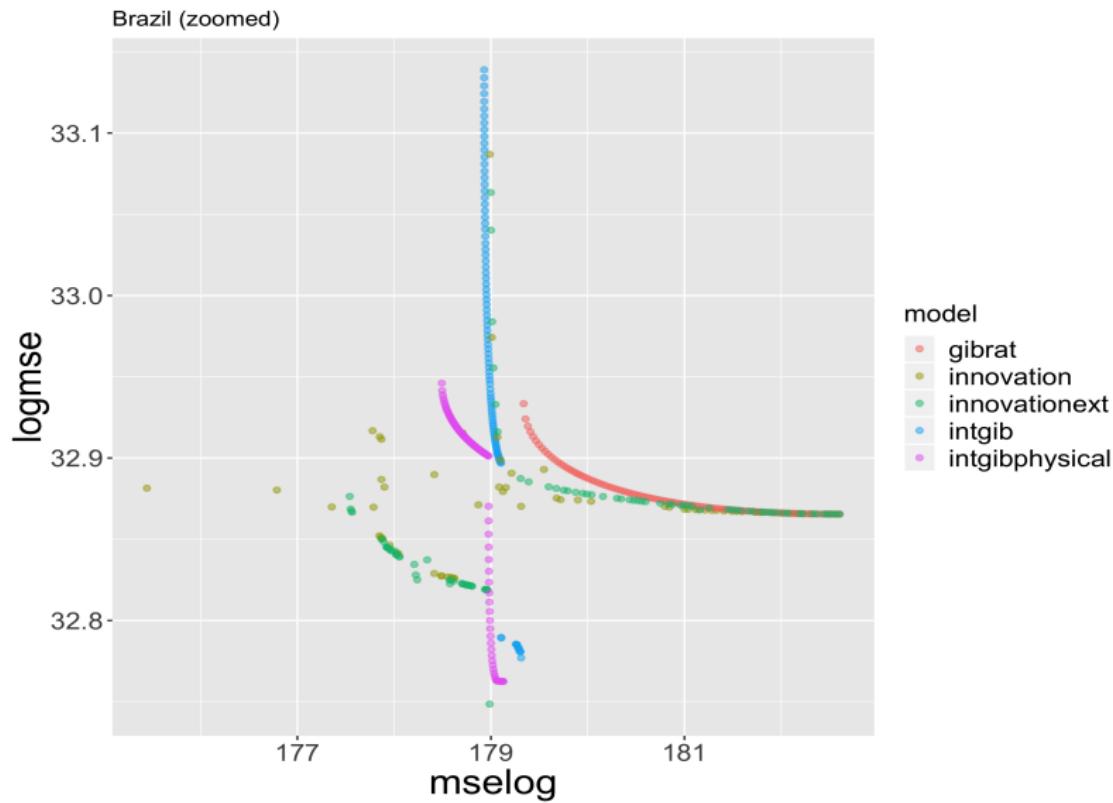


*Systems driven by economic exchanges: EU, US, RU, ZA*

# Indian urban system: direct interactions

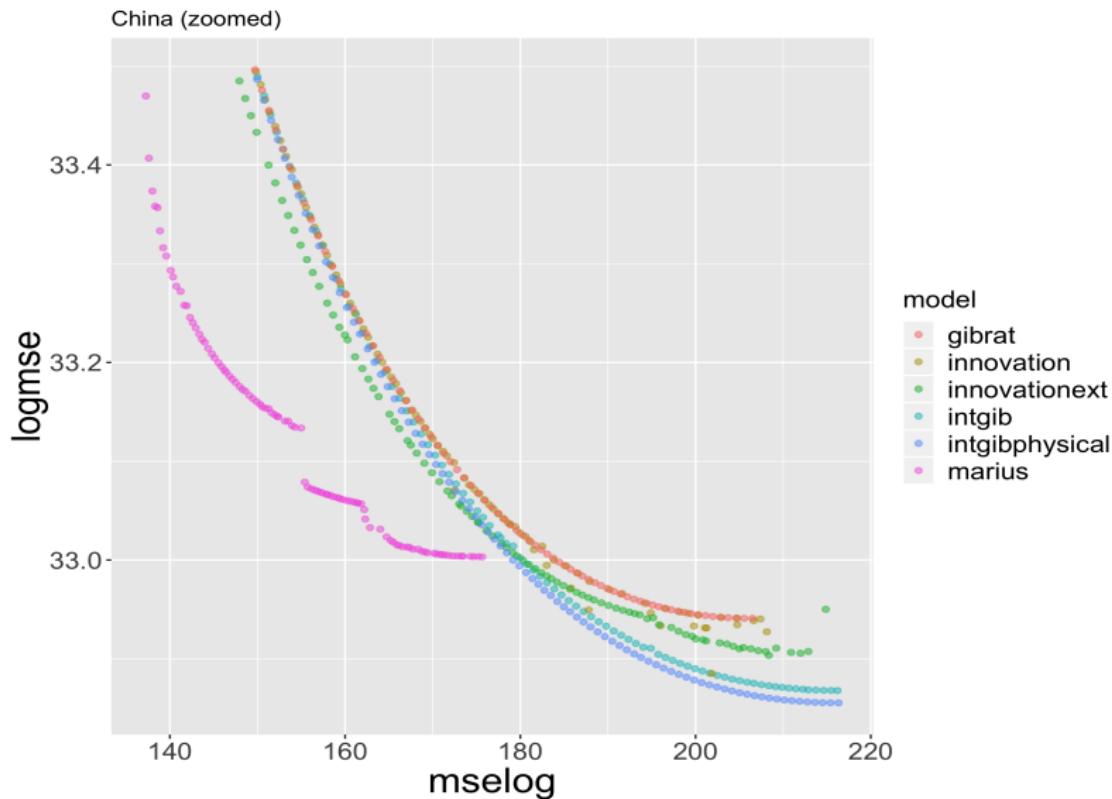


# Brazilian urban system: multiple factors



*Importance of topography; innovation processes mostly.*

# China: a tight competition



*No clear best model: other processes in play ? (strong top-down planning)*

## Results: empirical AIC

*Comparing Gibrat and Network model by correcting for the number of parameters [Raimbault, 2018b]: additional parameters actually improve the fit (but not always)*

System	$\Delta AIC$ (log)	$\Delta BIC$ (log)	$\Delta AIC$ (mse)	$\Delta BIC$ (mse)
ZA	33.82658	29.40496	-838.07184	-842.49347
CN	20.67713	16.78273	13.40767	9.51327
BR	-263.27003	-267.71575	-407.55220	-411.99792
IN	-85.01949	-89.24206	-188.16703	-192.38960
RU	50.60535	46.24233	104.94806	100.58503

## Implications

- Complementarity of economic, innovation and direct interaction processes
- High dimensionality of urban systems

## Developments

- Still points on robustness to be investigated: influence of stochasticity, convergence of GA, saturation of parameters, number of cities.
- Towards integrated models coupling these different components ?
- Compare on synthetic systems of cities with appropriate indicators [Raimbault, 2018c].
- Calibrate at the second order (correlations), non-stationary in time.
- More elaborated method to compare models in a “fair” way (correcting for additional parameters, open question for models of simulation): model reduction ?

# Conclusion

- First step towards systematic benchmarks and multi-modeling. **Need for more systematic model exploration.**
- Model integration and multi-scalarity ? **Need for more integrated models.**
- Multiple perspectives on urban systems ? **Need for more interdisciplinarity.**

## Related works

Raimbault, J. (2018). Indirect evidence of network effects in a system of cities. *Environment and Planning B: Urban Analytics and City Science*, 2399808318774335.  
<https://halshs.archives-ouvertes.fr/halshs-01788559>

Raimbault, J. (2018). Modeling the co-evolution of cities and networks. *Forthcoming in Handbook of cities and networks, Rozenblat C., Niel Z., eds.* arXiv:1804.09430.

Raimbault, J. (2018). Caractérisation et modélisation de la co-évolution des réseaux de transport et des territoires (Doctoral dissertation, Université Paris 7 Denis Diderot).  
<https://halshs.archives-ouvertes.fr/tel-01857741>

**Open repository** at <https://github.com/JusteRaimbault/UrbanGrowth>

**Acknowledgments:** thanks to the *EGI* for access to the infrastructure.

# Reserve Slides

## Models settings

→ Work under Gibrat independence assumptions, i.e.

$\text{Cov}[P_i(t), P_j(t)] = 0$ . If  $\vec{P}(t+1) = \mathbf{R} \cdot \vec{P}(t)$  where  $\mathbf{R}$  is also independent, then  $\mathbb{E}[\vec{P}(t+1)] = \mathbb{E}[\mathbf{R}] \cdot \mathbb{E}[\vec{P}](t)$ . Consider expectancies only (higher moments computable similarly)

→ With  $\vec{\mu}(t) = \mathbb{E}[\vec{P}(t)]$ , we generalize this approach by taking  
 $\vec{\mu}(t+1) = f(\vec{\mu}(t))$

# Network model

Direct network interaction model [Raimbault, 2018b]:

Let  $\vec{\mu}(t) = \mathbb{E}[\vec{P}(t)]$  cities population and  $(d_{ij})$  distance matrix

Model specified by

$$f(\vec{\mu}) = r_0 \cdot \mathbf{Id} \cdot \vec{\mu} + \mathbf{G} \cdot \mathbf{1} + \mathbf{N}$$

with

- $G_{ij} = w_G \cdot \frac{V_{ij}}{\langle V_{ij} \rangle}$  and  $V_{ij} = \left( \frac{\mu_i \mu_j}{\sum \mu_k^2} \right)^{\gamma_G} \exp(-d_{ij}/d_G)$
- $N_i = w_N \cdot \sum_{kl} \left( \frac{\mu_k \mu_l}{\sum \mu} \right)^{\gamma_N} \exp(-d_{kl,i})/d_N$  where  $d_{kl,i}$  is distance to shortest path between  $k, l$  computed with slope impedance ( $Z = (1 + \alpha/\alpha_0)^{n_0}$  with  $\alpha_0 \simeq 3$ )

# Innovation diffusion

Favaro-Pumain model [Favaro and Pumain, 2011]:

- 1) Diffuse innovations according to

$$\delta_{c,i,t} = \frac{\sum_j p_{c,j,t-1}^{s_c} \exp(-\lambda_s d_{ij})}{\sum_c \sum_j p_{c,j,t-1}^{s_c} \exp(-\lambda_s d_{ij})}$$

- 2) Update population with  $G_{ij}$  (see network model) such that

$$V_{ij} = \frac{p_i p_j}{(\sum_k p_k)^2} \exp(-\lambda_m d_{ij} \prod_c \delta_{c,i}^{\phi_c})$$

with  $\phi_c = \sum_i p_{i,c} / \sum_{i,c} p_{i,c}$

- 3) Introduce innovation with utility  $s_{c+1} = g_0 \cdot s_c$  in a randomly chosen city with a hierarchy parameter  $\alpha_l$ , if global adoption share  $\phi_c$  is larger than a threshold  $\theta_l$ . Initial utility  $s_0$  is a parameter. New innovation has an initial penetration rate  $r_l$  in the city.

# Economic exchanges

Marius model [Chérel et al., 2015]:

Initial wealth as a power law of population (exponent  $\alpha_W$ )

- 1) Update supply and demands as superlinear functions of population (exponents  $\alpha_S, \alpha_D$ )
- 2) Exchange goods according to a gravity potential of interaction (distance decay  $d_M$ ), supplies and demands; update wealth accordingly
- 3) Update population such that population difference is a power law of wealth difference (economic multiplier  $e_M$  and exponent  $\alpha_P$ )

## Benchmarked models

- ① Gibrat model: 1 param.  $r_0$
- ② Direct interaction model (geographical distance): 4 param.  
 $r_0, w_G, \gamma_G, d_G$
- ③ Physical network interaction model (topographical distance: 4 param.  $r_0, w_G, \gamma_G, d_G$ )
- ④ Innovation diffusion model (simplified): 4 param.  $r_0, w_I, \lambda_s, \lambda_m$   
(other parameters at default values from  
[Favaro and Pumain, 2011])
- ⑤ Innovation diffusion model (full): 9 param.  
 $r_0, w_I, \lambda_s, \lambda_m, s_0, g_0, r_I, \alpha_I, \theta_I$
- ⑥ Restricted Marius model: 4 param.  $e_M, \alpha_S, \alpha_D, d_M$
- ⑦ Marius model: 6 param.  $e_M, \alpha_S, \alpha_D, d_M, \alpha_W, \alpha_P$

# Model calibration

Genetic algorithm NSGA2:

- Around 40000 generations (4e6 model runs)
- Population  $\mu = 100$
- Convergence tested with hypervolume relative variation

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*Environment and Planning B: Urban Analytics and City Science*,  
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Unveiling co-evolutionary patterns in systems of cities: a systematic exploration of the simpopnet model.  
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