

# Complex Systems Made Simple

## Project final report



### Generative coupled model for Urban configuration optimisation

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#### Abstract

We describe an hybrid agent-based model coupling a cellular automata and a dynamic network, for the simulation of urban growth. Heterogeneous aspects of urban systems are taken into account in the sense that morphologic structure but also functional properties of the city are implied in the evolution. Classic measures of description and performances for generated configuration are used to classify and explore generated patterns, and we also propose an economic evaluation of the structure using sensitivity of segregation models to spatial configuration. We can apply the model to a real-world situation, proposing an optimisation of the repartition of activities in a zoning context.

## 1 Introduction

Recent progress in many disciplines linked directly or indirectly to Urban Planning can be interpreted as the development of a “new urban science” as BATTY stands in [7]. From agent-based modeling in quantitative geography, which achievements are reviewed in [15], and for which the best example of promising results are the series of Simpop models ([29]), to other approaches of the field that PORTUGALI present as “Complexity theories of cities” ([27]) that can involve scientists such as physicists as HAKEN on information theory ([13]) or architects as HILLIER with Space Syntax theory ([16]), the field is broad but is strongly consistent through the common view of urban systems as complex systems.

In that context, Cellular Automata models (CA), in which agents (cells) are fixed and evolve according to the state of neighbour cells, have been broadly studied and used. Their use in urban planning, for reproduction of existing urban forms and land-use patrterns, was already described by WHITE and ENGELEN in [38] and was later analysed ([5, 8]) and synthetised ([6]) by BATTY. A re-

cent review of possible application of CA models is done in [17]. The possible variations of models types and applications is quite broad . For example, a microeconomic CA for simulation of sprawl is developped in [10]. Measures for sustainable development of a fast growing region in China are investigated in [40] through a “linguistic” CA, that is that the rule are in real time updated thanks to interactions with the real population. Land-use modeling approaches are one essential point, as was the first CA model by WHITE . The two-dimensionnal dimension of the automaton is not mandatory, as the example of the 1D CA model proposed in [25] that allows to show discontinuities in settlement patterns and the strong path-dependance of these. We propose to build a model in the frame of CA modeling of urban systems.

Our model is adapted from the work of MORENO & al. ([22, 23]), that proposed to integrate network effects in a CA model for modeling urban morphology. Their aim was to test the effects of physical proximity on urban shape, introducing the modeling of urban mobility through a network which evolution is coupled with the evolution of the urban shape. We generalise the model,

allowing to take into account functional properties of the urban environment. The concept of considering heterogeneous urban activities was introduced in [37] and developed in [35], but was at our knowledge never considered from the point of view of physical accessibility and the impact on sprawl patterns.

## 2 Model description

### 2.1 Agents and rules

The world is a square lattice  $(L_{i,j})_{1 \leq i,j \leq N}$  constituted of cells that can be occupied or not (that will be denoted by a function  $\delta(i,j,t)$ , time being defined on a standard time scale  $\mathbb{T}$  which definition is introduced in [12], and for which we will take  $\tau\mathbb{N}$ ,  $\tau > 0$  to simplify). Among these fixed agents evolves an euclidian network which agents are roads:  $N(t) = (V(t), E(t))$  where  $V$  finite part of the world (set of points). At initial time, no cell is occupied and the initial network is fixed:  $N(0) = (V_0, E_0)$ . In order to translate functional mechanisms in the growth of the city, we suppose that a part of initial nodes are city centers  $C_0$  for each an activity is defined by  $a : C_0 \rightarrow [1; a_{max}]$ .

In all generality, we suppose that there exists a fixed number of function defined on cells and in time, that we call explicative variables, noted  $(d_k)_{1 \leq k \leq K}$ . Corresponding weights  $(c_k)_{1 \leq k \leq K} \in [0, 1]^K$  are parameters of the model. These will determine the value of cells  $v(i, j, t)$ . In practice, these variables will be:  $d_1$  the density in a neighborhood of radius  $r$  (parameter) around the cell;  $d_2$  the distance to the nearest road;  $d_3$  the distance to the nearest town center through the network; and  $d_4$  the “accessibility” of activities, defined as, with  $d_A(a)$  distance to nearest center with activity  $a$  through the network, and  $p_A \in [1; +\infty[$ ,  $d_4 = \|(d_A(a))_{1 \leq a \leq a_{max}}\|_{p_A} = \left(\frac{1}{a_{max}} \cdot \sum_{a=1}^{a_{max}} d_A(a)^{p_A}\right)^{1/p_A}$ .

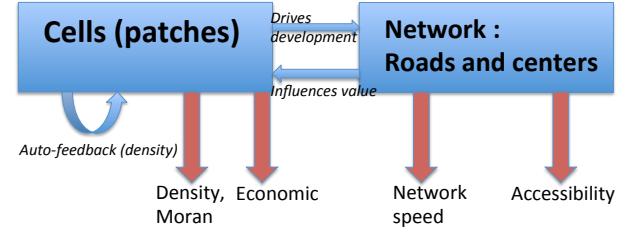
Giving these settings and variables, the evolution of the system for one time step is the following:

- New cell values are calculated by  $v(i, j, t) = \sum_{k=1}^K c_k \cdot \frac{(a_k^{max} - d_k)}{(a_k^{max} - d_k^{min})}$
- $N$  new cells chosen as the  $N$  cells with the best value are built in a random order
- For each new cell built, if  $d_2 > d_s$  fixed parameter (distance over which a new road is needed), the cell is connected to the network by a new road connecting perpendicularly with the network

Note that the process is stochastic since it depends on the random order on which the new cell are built. Sensitivity analysis and explorations will determine the influence of

respective parameters and the influence of that stochastic aspect. The growth is stopped after a fixed amount of time fixed by experience, so the final structure does not fill totally the world, what would make no sense. A good compromise has to be taken between “unfinished” structures and stationnary result totally biased by bord effects.

Figure 1 shows a “flowchart” describing the working of the agent-based model, especially what are the feedbacks between agents.



**Fig. 1 | Flowchart of the model.** Blue arrows represent interactions and feedbacks, whereas red arrows are evaluation functions.

### 2.2 Evaluation functions

Once a structure has been generated, one need to quantify its properties, either to classify it, or to compare it to other structures in an optimisation purpose. Therefore we need evaluation functions, that can be both objective quantifications or fitness functions for structures.

**Morphology** We qualify the morphologic structure of a configuration by projecting on a plan corresponding to the following indicators:

- The integrated local density, calculated by integration with a norm-p of the densities in circular neighborhood of each cell (with same radius parameter as for the run). We have, with  $p_d \in [1; +\infty[$

$$D(t) = \left[ \frac{1}{\sum_{i,j} \delta(i,j,t)} \cdot \sum_{\delta(i,j,t) \neq 0} d_1(i,j,t)^{p_d} \right]^{1/p_d}$$

- The Moran index, defined in [34] and used a lot in quantitative geography such as in [18], is used to quantify the polycentric character of the distribution of populated cells. It takes values between -1 and 1, a value close to 1 signifying a strong monocentric distribution, zero is for totally random distribution and -1 for a “chessboard-like” distribution (one cell over two is occupied). Polycentric distribution will have small positives values according to the size and respective distance of centers.

The space is decomposed into  $M$  regular areas (a grid in practice, which size has to be at the intermediate scale between cell size and world size, that is roughly  $1 \ll M \ll N$ ), and we denote by  $(P_i)_{1 \leq i \leq M}$  the occupied cells in each area and  $d_{ij}$  the distance between centroids of  $i$  and  $j$ . The Moran index of spatial auto-correlation is defined as

$$M(t) = \frac{M}{(\sum_{i=1}^M (P_i - \bar{P})^2)} \cdot \frac{\sum_{i \neq j} \frac{1}{d_{ij}} \cdot (P_i - \bar{P})(P_j - \bar{P})}{\sum_{i \neq j} \frac{1}{d_{ij}}}$$

**Network performance** Because of the way it evolves, the only loops in the network are the initial ones. Therefore, it has no sense to evaluate robustness or clustering coefficient of the network. However, since the generated network is supposed to simulate the mobility network, we can evaluate as suggested in [3] its performance through its relative speed, i. e. the quantity of detours it obliges to make. We define it, considering only travels from occupied patches to nearest center, with  $p_n \in [1; +\infty[$ ,  $d_{ij \rightarrow c}$  the distance from  $i$  to  $c$  through the network,  $d(ij, c)$  the euclidian distance and  $c_{min}^{ij} = \text{argmin}_{c \in C_0} (d_{ij \rightarrow c})$ ,

$$S(t) = \left[ \frac{1}{\sum_{i,j} \delta(i, j, t)} \cdot \sum_{\delta(i, j, t) \neq 0} \left( \frac{d_{ij \rightarrow c_{min}^{ij}}}{d(ij, c_{min}^{ij})} \right)^{p_n} \right]^{1/p_n}$$

**Functional accessibility** The global accessibility is, as for density, the integration of local accessibilities on all cells, that is simply with  $p_{GA} \in [1, +\infty[$ ,

$$A(t) = \left[ \frac{1}{\sum_{i,j} \delta(i, j, t)} \cdot \sum_{\delta(i, j, t) \neq 0} d_4(i, j, t)^{p_{GA}} \right]^{1/p_{GA}}$$

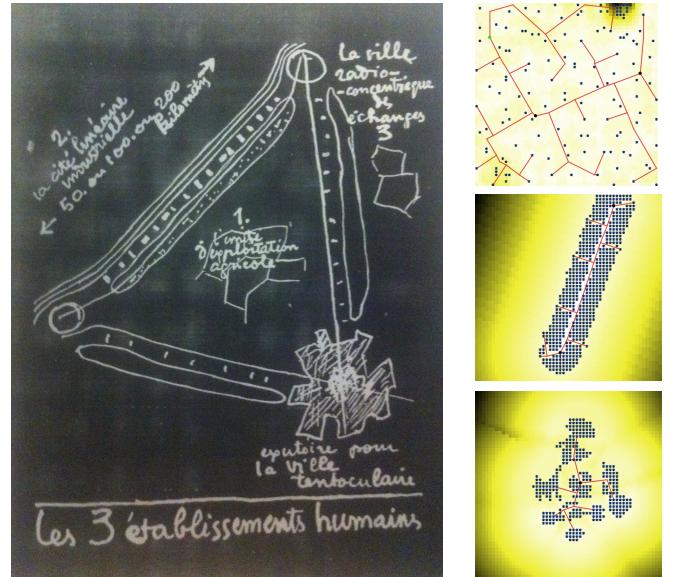
**Economic performance** It has been shown by BANOS in [2] that the SCHELLING segregation model, a basic model for socio-economic segregation introduced in [32], was highly sensitive to the spatial structure in which one can embed it (the segregation laws are in that case also influenced by spatial proximity). That justify the use of such a model as an evaluation function for the spatial structure regarding “economic performance”, in the sense of how much does the structure influence segregation. We implement a model for residential dynamics based on the model proposed in [9]. The output function is a segregation index calculated on residential patterns obtained from a distribution of constructed patches. Detailed description of the model is provided in supplementary material S1.

### 3 Results

The model was implemented in NetLogo 5.2 ([39]). Plots and charts are managed with R after export into standardised data ([31]). Primary treatment of GIS Data (for most hand vectorization of simple raster data) is handled with QGIS ([30]). See supplementary material S2 for implementation details needed for exact reproducibility.

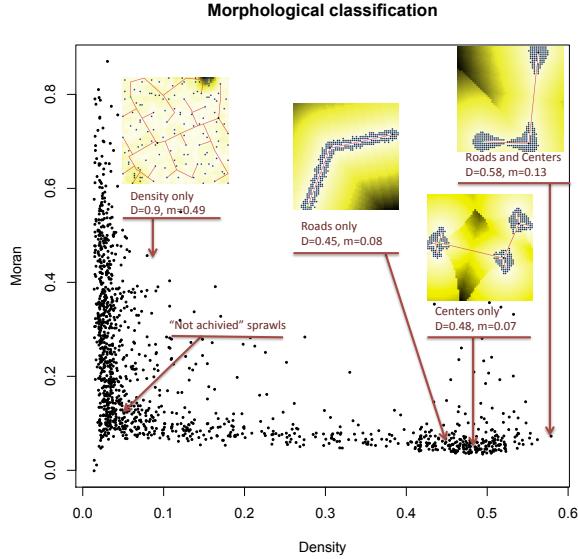
#### 3.1 Generation of urban patterns

**Typical patterns** Different configurations of the weights allow to generate very different structures. The obtained patterns for extreme values of parameters (e. g. with only influence of density on values, or of distance to centres) were quite the one expected as there is in such case less emergent behavior. It is striking to note that some characteristic patterns exactly correspond to the structures that LE CORBUSIER has analysed when trying to build a typology of the *3 établissements humains* (human settlements) as it is summed up by MANGIN in [24] (see figure 2 for details).

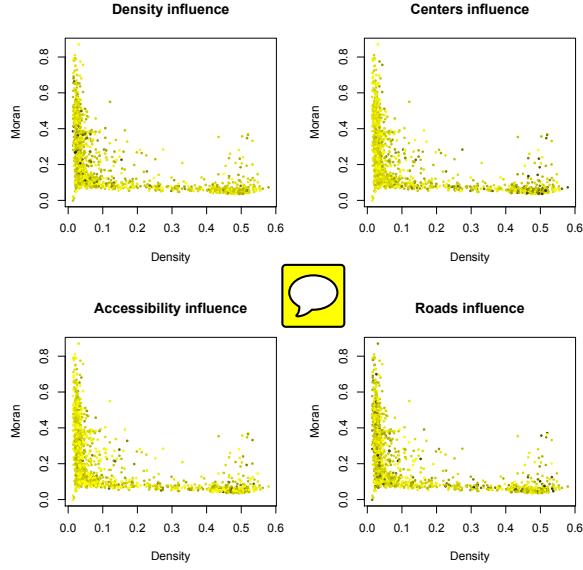


**Fig. 2 | Typical patterns reproduce Le Corbusier analysis of “human settlements”.** When trying to theorize urban planning by first analysing “by hand” the form of cities, LE CORBUSIER has enlightened 3 types of settlements, that are radioactive cities, linear cities along communication roads and the rural settlements. We were able to reproduce this typology by playing with values of weights (first only  $d_1 = 1$ , second only  $d_2 = 1$  and third  $d_1$  and  $d_3$  of equal weights). Source for drawing: [24]





**(a)** **(b)** Projection in the plan and some structures associated to some particular points. Three type of structures are very close and may sometimes not be distinguish through our simple indicators. The area close to zero corresponds to “unfinished” growth, that is structures that occupy only a small part of the world, that does not allow to compare directly with structure of bigger magnitude.



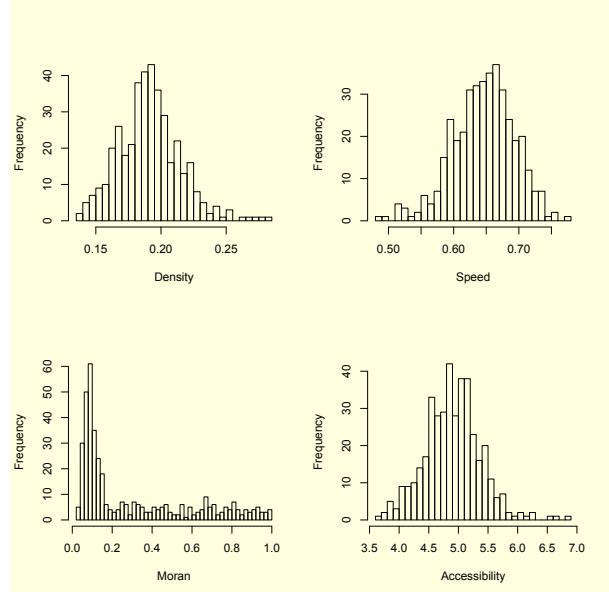
**(b)** Influence of each parameter on the location of structure. Color darkness corresponds to the relative weight of the parameter in each case. If the previous plot showed that extreme cases were located at expected places, this one shows that dark point are quite uniformly distributed, what means that some combinations created some “chaos” in the location of structures.

**Fig. 3 | Morphological classification of structures.**

**Classification of structures** Thanks to the morphological indicators, we were able to proceed to a description of morphological classes that can be obtained with any values of the parameters. The projection of a configuration in the plan  $Vect(D(t_f), M(t_f))$  allows to un-

derstand how the structure are different and where they are situated in a certain “classification”. Fig. 3 sums up the results of that classification. For extreme values of parameters, the results were quite the one expected, but some combinations seem to project the structures at unexpected places, what we can take as a sort of emerging chaos in the global system.

### 3.2 Sensitivity analysis and exploration of the parameter space



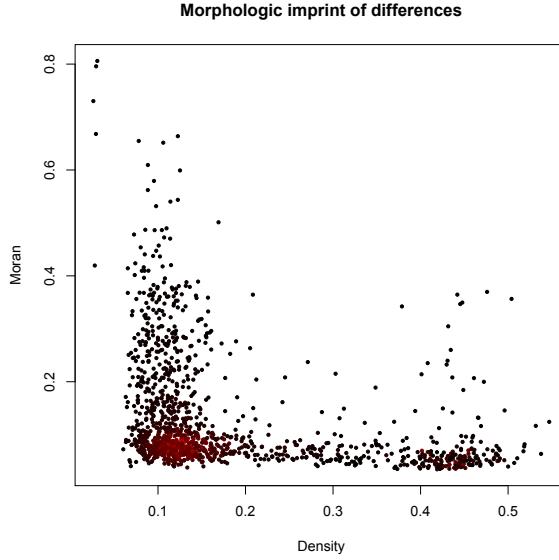
**Fig. 4 | Statistical distribution of outputs for a sample point in the parameter space.** It corresponds to equal weights (of value 1) to each variable, for which the distribution is calculated on around 500 repetitions for random initial configurations. Many other samples showed same standard deviations. The shape of distributions translates the small sensitivity of the model to spatial configuration, since it covers at most one tenth of the range of output functions (0.1/1 for density, 1/10 for accessibility e. g.).



**Sensitivity to initial conditions** To determine the magnitude of repetitions useful to get meaningful values of output functions and to ensure some validity in the results, it is necessary to investigate the sensitivity of the model to initial conditions of the spatial configuration. Indeed, if the conclusions that can be drawn in a particular case are significantly modified by small changes in the spatial configuration, one will differently apply the model than in a case where abstract topology of activities has more influence: one will obviously elaborate totally different exploration heuristics to proceed to optimisation.

Therefore, we launched large number of simulations with same parameter values but different initial configurations, in order to get statistical distributions of outputs at fixed parameters. Standard deviations were calculated for samples of the parameter space (around 20 samples)

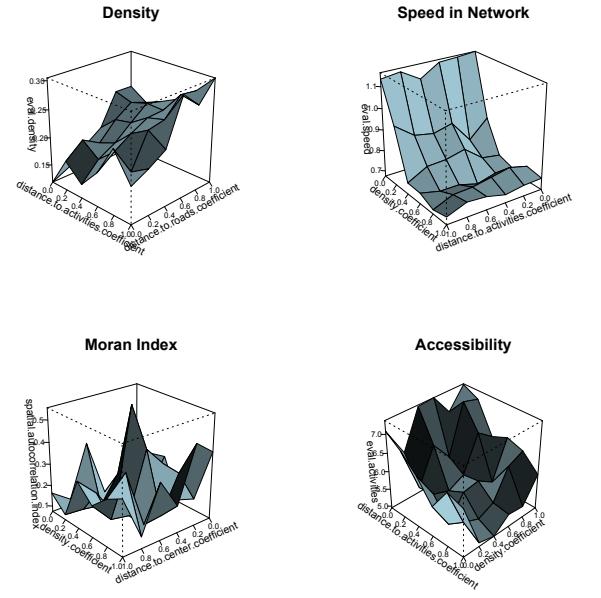
and the distributions were each time of same amplitude and same deviation. Fig. 3 shows these distributions for the sample corresponding to equal weights to each parameter. Given the numerical values of deviations (each time around 0.1 times the amplitude of the function), we can conclude that outputs are significantly less sensitive to spatial positions than to topology of activities and parameters values (as the exploration of parameter space results show in the following). That justify the small number of repetition needed to have reasonable values.



**Fig. 5 | Morphological projection of symmetric differences between configurations for both update types.** Each point corresponds to many realisations (5) for a given value of parameters. Each time, both structures for continuous ( $N = 1$ ) and sequential ( $N = 20$ ) are generated, symmetric difference is computed and projected in the morphologic plan. Points are colored according to their “significance”, defined by the product of local density of points in the cloud (closely clustered points correspond to more frequent configurations) by the relative size of the configuration (large configurations correspond to large difference and are therefore more significant). The wide spread shows a strong sensitivity. An other plot of superposed imprints for each type of update showed quite disjoint sets, what confirms this sensitivity. It is however relativised by the concentration here of significant points, which means that side effects are “constant”. Therefore taking a compromise between two types of updates should give stable and reasonable results.

**Sensitivity to update type** The generated structures seem to be quite sensible to the type of update that is done at each time step, i. e. to the number of cells that are built at each time step. That comes from the fact that at each time step, houses are built simultaneously without update of patches values. It is a consistent view since real value functions should have a certain response time that we can suppose of the magnitude of  $\tau$ . However this paradigm can be limited and it is necessary to find a compromise value for  $N$  that leads to reasonable con-

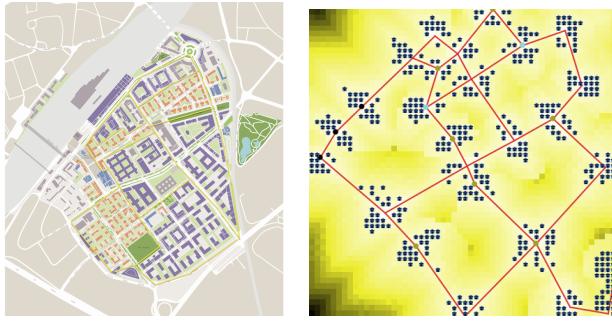
figurations. We can explore a grid of the main parameter space (weights of variables) and at each time generate patterns corresponding to a “continuous” update ( $N = 1$ ) and a “sequential” one ( $N = 20$  in our case), calculate the symmetric difference  $\mathcal{D} = \{(i, j) | \delta^{\text{cont}}(i, j, t_f^{\text{cont}}) \neq 0\} \Delta \{(i, j) | \delta^{\text{seq}}(i, j, t_f^{\text{seq}}) \neq 0\}$ , and plot the corresponding sets in the morphological plan. Results are shown in figure 5, including a colored representation of significance of points. We conclude that the model is highly sensitive, but that bord effect stay the same so finding a good compromise by experience (giving “consistent” structure) should solve the problem.



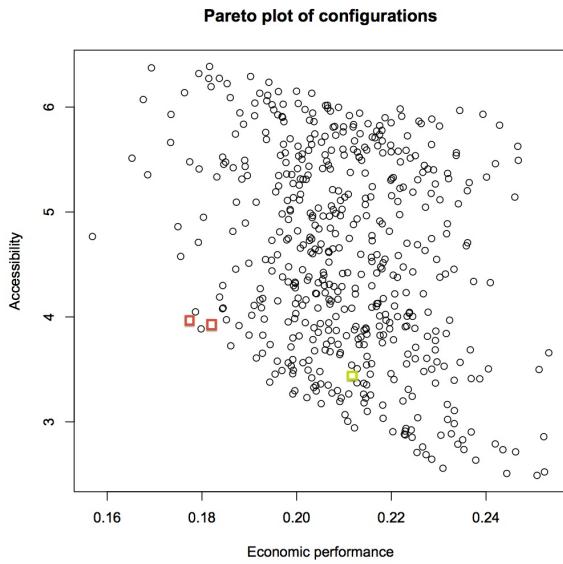
**Fig. 6 | Surface of evaluation functions depending on two parameters.** We chose some combinations of parameters that seemed to be interesting. We can extract the ranges of outputs and observe their regularities. The chaotic behavior of Moran index and accessibility confirm the existence of chaotic mechanisms in the system. Speed appears to be regular, what implies that network spread is not so far from linear regarding these parameters. Other combinations gave chaotic behavior.

**Exploration of the parameter space** We explored a grid of the 4-dimensional space of the “main” parameters, the relative weights of variables, to evaluate the global behavior of outputs when parameters vary. Other parameters such as  $r$  or  $d_s$  were not included in the exploration because they have real proxy that we can reasonably fix (around 5 cells seemed in general to generate the structures closer to the real world, what is significant whatever the scale of application, as the size of a district, it gives 50m, characteristic size of blocs, of the city 500m size of a district and of the system of cities 5km characteristic distance between cities). Step was 0.2 for each parameters, with 5 repetitions for each combinations, what lead to

the calculation of around 3200 configurations. Examples of surfaces defined on the space of two parameters are shown in figure 6. Economic evaluation was not plotted for efficiency reasons. However, since it depends directly on morphology and network and that it is not probable that the function would exactly smooth the irregularities, we can conclude that the shape of its surface would present same patterns of irregularity. The exploration allows first to appreciate the regularity of the outputs, what is essential to understand the internal mechanisms of the model that appear in some point as being somehow chaotic and non-linear. Secondly, it gives the range of outputs, allowing to state the robustness regarding initial conditions as we stand before.



(b) Left: existing masterplan for the district as it has been realised. Areas correspond to colored zones, and actual activities correspond to the color. Station, not mentioned is in the upper left (see center positions in model); Right: example of obtained configuration, color correspond to possible distribution of activities.



(b) Pareto plot of configuration in the plan economic performance/accessibility. Green point corresponds to the real situation, that is in fact not so far from the Pareto frontier so is a quite good solution. Red points that are Pareto-optimal solution with a good compromise between the two objectives, correspond to situation where activities are totally mixed.

**Fig. 7 | Concrete application.**

### 3.3 Concrete application

Since we have shown that outputs on which we want to optimise, such as accessibility to activities or network performance, are significantly more sensible to initial configuration of activities than to spatial distribution, we can apply the model to the optimisation of activities repartition, given an existing spatial structure. That situation can occur in a planning problem, then one has to decide possible landuses of predefined zones.

The concrete situation will consist on the planning from scratch of a new district, where attribution of activities to areas has to be determined (2 types of activities, residential or tertiary). Transportation infrastructure is already implanted and the station will act as center with a fixed third activity. For each area, centroid is a center and the initial network consists on main roads existing on the empty district. Configuration is imported from GIS shapefile so the application can be done on other cases, as soon as one has required files (see S2). The real world situation is the district of Massy Atlantis, France, which have just been built and planned (2012) and for which we would like to study if a more optimal planning may have been done. We fix a value of parameters translating the situation, that is : high influence of activities ( $c_4 = 1$ ) because agents will want to have accessible home, workplace and station; medium influence of density ( $c_1 = 0.7$ ) because the houses have to fill reasonably the available areas; no influence of roads since we suppose that the initial network is already built and has a few influence because of the scale; and no influence of centers because centers are here abstract entities representing all area. We then launch the calculation of configuration for all possibilities of distribution of activities in the areas (exponential on the number of free centers, 510 calculations in our case, what gives around 2500 runs with repetitions) and plot configuration in the plan of economic performance and accessibility (most significant fitness functions in our case). Results are shown in figure 7.

We plot the configuration in the plan corresponding to the two objectives and we naturally obtain a Pareto-front of “optimal solutions”. We observe that the real configuration is not so far to be optimal; although it favors more accessibility than economic performance. By examining two optimal points that are good compromises, we see that the corresponding combinations of activities are one where they are maximally mixed (totally heterogeneous repartition). That is an interesting fact and may be a clue towards evidences for more mixed landuses in planning. We conclude that the application is efficient, since it allows to propose an optimal planning regarding these criteria in such a situation. It could be generically used by planners in similar situations (staying very careful on the conditions of application).

## 4 Discussion

The reproduction of existing urban facts (in the sense of the morphological structures) and the possible applications showed that the implemented model may be useful for evidence-based decision-making in urban planning. However, many aspects can be discussed as they raise crucial questions and would need further explorations.

**Scale of the model** One ambiguity of the model is that it can be applied at several scales and that its agents have not necessarily correspondance in the real world. Indeed, as we saw in results and applications, it can be used at the macro scale of a system of cities (settlement patterns), the meso scale of the city (patterns for one city) or the micro scale of the district (concrete application). Without going into an ontologic debate on the quantity of abstraction of object, it can obviously be pointed as a huge issue. However, we argue that this multi-scale application is legitimate. First, a large number of urban systems and associated processes have been shown to be scale-free as **Pumain** shows in [28], what implies that scaling laws can be present in our model and that it is therefore applicable at any scale. Furthermore, urban form have been qualified to have fractal properties ([4]) what would strengthen that paradigm. Following **Barthelemy** in [19] when he insists on the fact that the failure of most multi-agent urban models is that they don't focus on the "dominating" physical process but integrate many aspect that have in reality no influence on the emergent properties of the system, we suppose here to have isolated at least "good" proxies of dominating processes.

**Local application** When the model is considered at a meso or micro scale, one could object that it neglects too much phenomena such as economic exchanges by considering the system as a "closed" system (it is formally closed but not "closed in reality", since just the new constructions can be seen as exchanges with the outside - the precise critical would be that in and out flows are too much simplified). As economic models considering the global complex network of cities such as in [1] have also reproduced well existing patterns in urban systems, it is obvious that the equivalence of approaches and the compromise that need to be made in each case have to be further investigated. It joins the fundamental issue, already implicitly raised in the previous point, of the existence of a minimal dimension for a generalised representation of urban systems. Concretely in our case, attractivity of centers is a proxy for economic processes, and although the model does not directly include economic considerations, they are underlying, confirming that urban processes are strongly interdependant. The problem

would be to understand how general the dependance is and if a generalised minimalist formulation can be constructed. Speculations towards that have been proposed by HAKEN in [13] with the notion of semantic information that would be linked to properties of attractors of dynamical systems. This theory has however neither been quantified, confirmed or invalidated.

**Quantitative calibration** The question of validity of the model is also linked to the need for a fine quantitative calibration on real patterns. Calibration of such models is still an issue, since on the one hand calibration on errors on proxies for output function does not directly influence the spatial patterns but on the other hand calibration on spatial patterns is too constraining because it eludes "similar" patterns. Some work has been done on tries for such calibrations such as in [21] but the results are not satisfying. To go in that direction, we would first need to apply the model with finer grain for the spatial resolution, i. e. on a very large world. Computational issues can appear, even though we optimised processes in the implementation (see supplementary material S2, e. g. single calculation and caching of network shortest paths allow a huge gain in computational complexity). But the increase in size can also cause methodological issues, since the amount of details in patterns can become too huge and bring more noise in the indicators, biasing the results. Therefore, an additional operation could be a solution, that is the extraction of the morphological enveloppe of the generated pattern, following the innovative method proposed in [33, 11]. Other ways to deal with noise such as gaussian smooth should also be investigated.

**Complex coupling with economic model** The way we proceed to economic evaluation consists in a simple coupling with an other agent-based model. It could be a direction of research to look towards a complex coupling (in the sense of **VARENNE** in [36]) between the two models. Indeed, considering the sprawl at an other time scale implies simultaneous evolution of inhabitants, rents of buildings and values of terrain. Of course such a coupling would lead to a completely different model that would need to be explored again and that would be applicable to totally different situations (as evaluation of long-term rent policies for social diversity for example).

## Conclusion

We have proposed and explored an heterogeneous CA model to simulate urban growth patterns, using also a new way to evaluate economic properties of spatial patterns. It has shown to be able to reproduce characteristic

urban facts of a possible typology of “human settlements” (cf 3.1). Furthermore the model was applicable to a concrete situation giving an “optimal” (in the Pareto sense) solution to a planning problem in a zoning context. The results go in the direction of evidences for a “mixed-use” city, on which litterature is still poor and that could be the object of future work. That paradigm is proposed by urbanists such as Mangin in [20] (concept of the “passing city”) and would need to be confirmed by quantitative results.

Beyond these technical facts useful for planning purposes, we argue that our work gives a vision of the current state “quantitative urbanism”, confirming the view of PORTUGALI in [26], in the sense that the conception and application of such models is in fact very subtle and can lead to many misunderstanding and contradictory results. Here the link between model and application is in that frame, and can become obviously wrong just by an application with other parameters. However, we stand that a better quantitative comprehension of urban systems begins to be actual and that evidence-based solution to planning problems are real, at the opposite of the vision of LEFEBVRE in [14] when he stood that scientific approaches could never translate or predict the mechanisms of a city.

## Supplementary materials

### S1 Description of coupled ABM for economic evaluation.

**S2** Core source code of the NetLogo implementation of the model, sample of data used for applications, source code for generation of plots and charts.

- [1] Claes Andersson, Alexander Hellervik, Kristian Lindgren, Anders Hagson, and Jonas Tornberg. Urban economy as a scale-free network. *Physical Review E*, 68(3):036124, 2003.
- [2] Arnaud Baroas. Network effects in schelling’s model of segregation: new evidences from agent-based simulation. *Environment and Planning B: Planning and Design*, 39(2):393–405, 2012.
- [3] Arnaud Baroas and Cyrille Genne-Grandpierre. Towards new metrics for urban road networks: Some preliminary evidence from agent-based simulations. In *Agent-based models of geographical systems*, pages 627–641. Springer, 2012.
- [4] M. Batty and P. Longley. *Fractal Cities: A Geometry of Form and Function*. Academic Press, London, 1994.
- [5] Michael Batty. Cellular automata and urban form: a primer. *Journal of the American Planning Association*, 63(2):266–274, 1997.
- [6] Michael Batty. *Cities and Complexity: Understanding Cities with Cellular Automata, Agent-based Models, and Fractals*. MIT Press, 2007.
- [7] Michael Batty. *The New Science of Cities*. MIT Press, 2013.
- [8] Michael Batty and Yichun Xie. Possible urban automata. *Environment and Planning B*, 24:175–192, 1997.
- [9] Itzhak Benenson. Multi-agent simulations of residential dynamics in the city. *Computers, Environment and Urban Systems*, 22(1):25–42, 1998.
- [10] Geoffrey Caruso, Gilles Vuidel, Jean Cavailhes, Pierre Frankhauser, Dominique Peeters, and Isabelle Thomas. Morphological similarities between dbm and a microeconomic model of sprawl. *Journal of Geographical Systems*, 13:31–48, 2011.
- [11] Pierre Frankhauser and Cécile Tannier. A multi-scale morphological approach for delimiting urban areas. In *9th Computers in Urban Planning and Urban Management conference (CUPUM'05)*, University College London, 2005.
- [12] Boris Golden, Marc Aiguier, and Daniel Krob. Modeling of complex systems ii: A minimalist and unified semantics for heterogeneous integrated systems. *Applied Mathematics and Computation*, 218(16):8039–8055, 2012.
- [13] Herman Haken and Juval Portugali. The face of the city is its information. *Journal of Environmental Psychology*, 23(4):385–408, 2003.
- [14] Lefebvre Henri. *Le droit à la ville*. Anthropos, 1968.
- [15] Alison J Heppenstall, Andrew T Crooks, and Linda M See. *Agent-based models of geographical systems*. Springer, 2012.
- [16] Bill Hillier, Adrian Leaman, Paul Stansall, and Michael Bedford. Space syntax. *Environment and Planning B: Planning and Design*, 3(2):147–185, 1976.
- [17] Sanna Iltanen. Cellular automata in urban spatial modelling. In *Agent-based models of geographical systems*, pages 69–84. Springer, 2012.

- [18] Florent Le Néchet and Anne Aguilera. Déterminants spatiaux et sociaux de la mobilité domicile-travail dans 13 aires urbaines françaises : une approche par la forme urbaine, à deux échelles géographiques. In *ASRDLF 2011*, SCHOELCHER, Martinique, July 2011. <http://asrdlf2011.com/>.
- [19] R. Louf and M. Barthelemy. Modeling the polycentric transition of cities. *ArXiv e-prints*, September 2013.
- [20] David Mangin. *La ville franchisée: formes et structures de la ville contemporaine*. Éditions de la Villette Paris, 2004.
- [21] Cláudia Maria de Almeida, Michael Batty, Antonio Miguel Vieira Monteiro, Gilberto Câmara, Britaldo Silveira Soares-Filho, Gustavo Coutinho Cerqueira, and Cássio Lopes Pennachin. Stochastic cellular automata modeling of urban land use dynamics: empirical development and estimation. *Computers, Environment and Urban Systems*, 27(5):481–509, 2003.
- [22] Diego Moreno, Dominique Badariotti, and Arnaud Banos. Un automate cellulaire pour expérimenter les effets de la proximité dans le processus d'étalement urbain : le modèle raumulus. *Cybergeo : European Journal of Geography*, 2009.
- [23] Diego Moreno, Arnaud Banos, and Dominique Badariotti. Conception d'un automate cellulaire non stationnaire à base de graphe pour modéliser la structure spatiale urbaine: le modèle remus. *Cybergeo: European Journal of Geography*, 2007.
- [24] Philippe Panerai and David Mangin. Projet urbain. 1999.
- [25] Dominique Peeters and Mark Rounsevell. Space time patterns of urban sprawl, a 1d cellular automata and microeconomic approach. *Environment and Planning B: Planning and Design*, 36:968–988, 2009.
- [26] Juval Portugali. Complexity theories of cities: Achievements, criticism and potentials. In *Complexity Theories of Cities Have Come of Age*, pages 47–62. Springer, 2012.
- [27] Juval Portugali, Han Meyer, Egbert Stolk, and Ekim Tan. *Complexity theories of cities have come of age: an overview with implications to urban planning and design*. Springer, 2012.
- [28] Denise Pumain. Scaling laws and urban systems. *Santa Fe Institute, Working Paper n° 04-02*, 2:26, 2004.
- [29] Denise Pumain. Multi-agent system modelling for urban systems: The series of simpop models. In *Agent-based models of geographical systems*, pages 721–738. Springer, 2012.
- [30] QGIS Development Team. *QGIS Geographic Information System*. Open Source Geospatial Foundation, 2009.
- [31] R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2013.
- [32] Thomas C Schelling. Models of segregation. *The American Economic Review*, 59(2):488–493, 1969.
- [33] Cécile Tannier, Gilles Vuidel, and Pierre Frankhauser. Délimitation d'ensembles morphologiques par une approche multi-échelle. In J.-C. Foltête, editor, *Actes des huitièmes Rencontres de Théo Quant*, page 14, Besançon, France, 2008. <http://thema.univ-fcomte.fr/theoq/>.
- [34] Yu-Hsin Tsai. Quantifying urban form: compactness versus' sprawl'. *Urban Studies*, 42(1):141–161, 2005.
- [35] Jasper van Vliet, Jelle Hurkens, Roger White, and Hedwig van Delden. An activity-based cellular automaton model to simulate land-use dynamics. *Environment and Planning-Part B*, 39(2):198, 2012.
- [36] Franck Varenne, Marc Silberstein, et al. *Modéliser & simuler. Epistémologies et pratiques de la modélisation et de la simulation, tome 1*. 2013.
- [37] Roger White. Modeling multi-scale processes in a cellular automata framework. In *Complex artificial environments*, pages 165–177. Springer, 2006.
- [38] Roger White and Guy Engelen. Cellular automata and fractal urban form: a cellular modelling approach to the evolution of urban land-use patterns. *Environment and planning A*, 25(8):1175–1199, 1993.
- [39] U. Wilensky. Netlogo. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL., 1999.
- [40] Fulong Wu. A linguistic cellular automata simulation approach for sustainable land development in a fast growing region. *Computers, Environment and Urban Systems*, 20:367–87, 1996.