

2class Logistic (Y_0, Y_1)

$X: Y_1$ if $P(Y_1|X) > P(Y_0|X)$

$X: Y_0$ if $P(Y_1|X) < P(Y_0|X)$

// Odds

$$\begin{aligned} P(Y_1|X) &\propto P(X|Y_1) \cdot P(Y_1) \\ P(Y_0|X) &\propto P(X|Y_0) \cdot P(Y_0) \end{aligned} \Rightarrow \boxed{\frac{P(Y_1|X)}{P(Y_0|X)}} = \frac{P(X|Y_1) \cdot P(Y_1)}{P(X|Y_0) \cdot P(Y_0)}$$

$a_k = \ln(P(X|Y_k) \cdot P(Y_k))$ 라고 하자.

$$P(Y_1|X) = \frac{P(X|Y_1)P(Y_1)}{P(X|Y_1)P(Y_1) + P(X|Y_0)P(Y_0)} = \frac{e^{a_1}}{e^{a_1} + e^{a_0}} = \frac{1}{1 + e^{a_0 - a_1}} \quad (1)$$

$$a_1 - a_0 = \log \left(\frac{P(X|Y_1)P(Y_1)}{P(X|Y_0)P(Y_0)} \right) = \log \left(\frac{P(Y_1|X)}{P(Y_0|X)} \right) = \log \pi = X\beta \dots (2)$$

$$\therefore (1), (2) \text{ 을 이용하여 } P(Y_1|X) = \frac{1}{1 + e^{-X\beta}}$$