SIGNALS AND SYSTEMS I
Computer Assignment 3

Exercises

For Exercises 1-3, use the discrete system described by the difference equation,

$$y(n) - 2r\cos(\omega_0)y(n-1) + r^2 y(n-2) = r\sin(\omega_0)x(n-1)$$

where $x(n)$ is the system's input, $y(n)$ is the system's output and

$$r = 0.9 \text{ and } \omega_0 = \pi/7.$$

1. Draw

   a) a Direct Form I block diagram of the system.
   b) a Direct Form II block diagram of the system.
   c) a transposed Direct Form II block diagram of the system.

   Generate state equations for your Direct Form II and transposed Direct Form II block diagrams.

2. Using a *for* loop or a *while* loop, write programs that implement your Direct Form I block diagram and your state equations for your Direct Form II and transposed Direct Form II block diagrams. Using all three programs, calculate the first 51 outputs of the system's impulse response. Plot the input and outputs using the **stem**, **title** and **subplot** functions. (You should generate 4 plots on 1 page.)

3. Using your three programs, calculate the first 51 outputs of the system's zero input response (ZIR) where the systems initial conditions are

   $y(-1) = -r^{-1}\sin(\omega_0)$ and $y(-2) = -r^{-2}\sin(2\omega_0)$ Direct Form I
   $q_1(0) = 0$ and $q_2(0) = r^{-2}$ Direct Form II.
   $q_1(0) = 0$ and $q_2(0) = r\sin(\omega_0)$ Transposed Direct Form II.

   Plot your results using the **stem**, **title** and **subplot** functions. (You should generate 3 plots on 1 page.) Compare (**max**(**abs**(*difference*))) these results with your results in Exercise 2. (They should be almost identical.)

Convolution

In general, a lumped linear time invariant system can be described by a linear Nth order constant coefficient difference equation of the form

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

where $x(n)$ is the system's input and $y(n)$ is the system's output. If the system is nonrecursive (the output is not a function of the output at other times), then the difference equation can be written as

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) .$$

This nonrecursive system's impulse response, $h(n)$, can be determined by letting $x(n) = \delta(n)$ which implies that

$$h(n) = \sum_{k=0}^{M} b_k \delta(n-k) = b_n .$$

As a result, difference equations of nonrecursive systems are typically written as

$$y(n) = \sum_{k=0}^{M} h(k) x(n-k)$$

which is the convolution sum for a system with a finite impulse response (FIR) of length M+1.

Because nonrecursive systems do not have feedback, Direct Form I and Direct Form II implementations of nonrecursive systems are identical. Often, Direct Form implementations of convolution are implemented on array processors or digital signal processors (DSPs). These processor are typically optimized to perform vector matrix operations. As a result, convolution is often expressed as

$$y(n) = \sum_{k=0}^{M} h(k) x(n-k) = \mathbf{h}^T \mathbf{x}(n)$$

where

$$\mathbf{h}^T = [h(0) \quad h(1) \quad \cdots \quad h(M)] \text{ and } \mathbf{x}(n) = [x(n) \quad x(n-1) \quad \cdots \quad x(n-M)]^T$$

Exercises

For Exercises 4-6, use the discrete system described by the finite impulse response, $h(n)$, where

$h(0) = h(8) = -0.07568267$ $\quad h(2) = h(6) = 0.09354893$ $\quad h(4) = 0.4$

$h(1) = h(7) = -0.06236595$ $\quad h(3) = h(5) = 0.30273069$

4. Draw

   a) a Direct Form block diagram of the system.
   b) a transposed Direct Form II block diagram of the system.

   Indicate the number of delays (memory registers), adds and multiplies required to calculate each output sample.

5. Using the **circshift** function and vector multiplication, write a program for each of the block diagram that you drew in Exercise 4. Using these programs, calculate the system's first 51 outputs when the system's input, $x(n)$, is

   a) $x(n) = \delta(n)$.
   b) $x(n) = u(n)$.
   c) $x(n) = \cos(0.2\pi n)u(n)$
   d) $x(n) = \cos(0.2\pi n)u(n) + \cos(0.7\pi n)u(n)$

   Plot the inputs and outputs using the **stem**, **title** and **subplot** functions. (You should generate 12 plots on 4 pages, that is, three plots per page.)

6. Using MATLAB's built-in **conv** function, calculate the system's first 51 outputs when the system's input, $x(n)$, is

   a) $x(n) = \delta(n)$.
   b) $x(n) = u(n)$.
   c) $x(n) = \cos(0.2\pi n)u(n)$
   d) $x(n) = \cos(0.2\pi n)u(n) + \cos(0.7\pi n)u(n)$

   Plot the inputs and outputs using the **stem**, **title** and **subplot** functions. (You should generate 8 plots on 2 pages, that is, four plots per page.)