

$$\begin{aligned}
\mathcal{F}\{h[n]\} &= \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k} \\
&= \frac{1}{M+1} \left(\sum_{k=0}^M e^{-j\omega k} \right) \\
&= \frac{1}{M+1} \left(\sum_{k'=-M/2}^{M/2} e^{-j\omega(k'+M/2)} \right) \\
&= \frac{e^{-j\omega M/2}}{M+1} \left(\sum_{k'=-M/2}^{M/2} e^{-j\omega k'} \right) \\
&= \frac{e^{-j\omega M/2}}{M+1} \left(\frac{(e^{-j\omega})^{-M/2} - (e^{-j\omega})^{M/2+1}}{1 - e^{-j\omega}} \right) \\
&= \frac{e^{-j\omega M/2}}{M+1} \left(\frac{e^{j\omega M/2} - e^{-j\omega(M/2+1)}}{1 - e^{-j\omega}} \right) \\
&= \frac{e^{-j\omega M/2}}{M+1} \left(\frac{e^{j\omega M/2} - e^{-j\omega(M/2+1)}}{1 - e^{-j\omega}} \right) \left(\frac{e^{j\omega/2}}{e^{j\omega/2}} \right) \\
&= \frac{e^{-j\omega M/2}}{M+1} \left(\frac{e^{j\omega(M+1)/2} - e^{-j\omega(M+1)/2}}{e^{j\omega/2} - e^{-j\omega/2}} \right) \\
&= \frac{e^{-j\omega M/2}}{M+1} \left(\frac{e^{j\omega(M+1)/2} - e^{-j\omega(M+1)/2}}{e^{j\omega/2} - e^{-j\omega/2}} \right) \left(\frac{j2}{j2} \right) \\
&= \frac{e^{-j\omega M/2}}{M+1} \left(\frac{\sin(\omega(M+1)/2)}{\sin(\omega/2)} \right).
\end{aligned}$$