18-240: Structure and Design of Digital Systems

HW0 [19 problems, 64 points]

Covers lectures L0 – L1 *Due: 5 September 2023*

Homework sets are due at 5:00PM on the due date. Upload your answers, as described below, to Gradescope by then. No late homework will be accepted. Remember, we let you drop two homework assignments over the semester.

To hand in your homework, create a PDF either by scanning your paper work, or by generating it in a software tool to start with (e.g., a word processor, LaTeX). Ensure your PDF is readable, your work legible and rotated correctly. Upload your work to Gradescope and follow the Gradescope instructions carefully to ensure your work is graded properly. Points will be deducted if you do not follow directions, for instance, by not assigning problems to pages (a five point penalty).

This homework has no other files (no SystemVerilog or other files). Therefore, you need not run the **`handin240`** script at all.

Discussions about homework in small groups are encouraged — think of this as giving hints, not solutions, to each other. However, homework must be written up individually (no copying is allowed). If you discussed your homework solutions with someone else, either as the giver or receiver of information, your write-up must explicitly identify the individuals and the manner information was shared.

If you use an AI assistant (ChatGPT, or others) for help on any of these problems, you must ensure that your answer is completely your work. Do not simply copy-n-paste any part of a ChatGPT conversation into your answer. And you must cite the AI assistant, with a thorough description of what help you received. For example, *"Conversation with ChatGPT 3.5 consisting of approximately 12 prompts asking for a thorough understanding of how to do Boolean proofs."*

You must show details of your work. There is no credit for just writing down an answer.

Drill Problems [32 points]

Drill problems are graded leniently based on your approach and effort; and not entirely on correctness. That means it is possible for you to have a perfect score on a problem whose answer is actually incorrect. Please check your work with the published solutions to verify correctness.

Drill problems are, by their nature, somewhat repetitive and often not all that interesting. My apologies for that! However, I believe that working drill problems, much like eating your broccoli, is good for you. You need to have practiced these techniques before seeing them on an exam.

1. [4 points, Lecture 1] Construct a truth table for the following functions:

$$
\begin{aligned}
P(a,b,c) &= a' + b'c' \\
Q(a,b,c) &= a'b'c' + a'bc' + ab'c + abc' + abc \\
R(a,b,c) &= \Sigma m(1,2,4,5,6) \\
S(a,b,c) &= a \oplus (b' \cdot c) + a \cdot (b \oplus c')
\end{aligned}
$$

2. [3 points, Lecture 1] For this truth table, what are F, G, and H in canonical SOP form?

A	B	C	F	G	H
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	0	1

3. [3 points, Lecture 1] For the truth table in Question 2, what are F, G, and H in canonical POS form?

4. [3 points, Lecture 1] Write F, G and H from problem #2 as a sequence of minterms.

5. [3 points, Lecture 1] Write F, G and H from problem #3 as a sequence of maxterms.

6. [2 points, Lecture 0] Minimize F from problem #2 using theorems, laws and axioms of Boolean algebra.

7. [4 points, Lecture 0] Minimize G from problem #3 using theorems, laws and axioms of Boolean algebra. Note that you must start with the POS form, not the SOP form.

8. [2 points, Lecture 0] Using Boolean algebra, show that $x' \oplus y' + ((xy)' \cdot y)' = 1$

9. [2 points, Lecture 0] Using Boolean algebra, show that $x' \oplus xy = x' + y$

10. [4 points, Lecture 1] Rewrite the following equations as canonical SOP and POS equations, expressed as fully written-out terms. In other words, the results should be in terms of the input variables.

$$F(A,B,C) = \Sigma m(1,3,4,7)$$
$$G(A,B,C,D) = \Pi M(0,3,4,5,9,12)$$

11. [1 point, Lecture 1] Find the canonical SOP form of $F(A,B,C) = A'C + AC' + B'C + BC$

12. [1 point, Lecture 0] Draw a circuit realization (i.e. a schematic) of $F = AC' + BCD + A'D$

Non-Drill Problems [32 points]

Non-Drill problems will be graded on correctness.

13. [4 points, Ethics Lab]

    You were presented with four concepts that a responsible engineer should be aware of:

    (a) Honesty / transparency

    (b) Bias / fairness

    (c) Accountability

    (d) Reliability

    Honesty and Accountability seem pretty similar. Write a paragraph or so explaining what each of these two concepts are and why they are different – and how to tell the difference.

    You will be graded on the clarity of your writing as well as the correctness of your answer.

14. [6 points, Lecture 0] Use the definition of XOR, the facts that XOR commutes and associates (if you need this), and all the non-XOR axioms and theorems you know to prove this distributive rule:

$$A \cdot (B \oplus C) = (A \cdot B) \oplus (A \cdot C)$$

15. [6 points, Lecture 0] Charles Peirce[1] showed, in 1880, that the NAND function forms a complete set of logic gates. In other words, it is possible to implement any Boolean expression using nothing more than 2-input NAND gates. Is the same true for NOR? Show whether this is true in a very convincing fashion.[2]

16. [4 points, Lecture 0] Draw a schematic of $F = [(A \cdot C)' \cdot (B' \cdot C' + D') \cdot (C + D')]'$ using only 2-input NOR gates.

17. [2 points, Lecture 1] For an n-input, m-output combinational circuit, how many rows are there in the truth table? How many columns on the right side? How many columns on the left side?

18. [4 points, Lecture 1] Compliments to those who can complement. Show the complement of the following function in canonical POS form:

$$\begin{aligned} F(A,B,C,D,E) = & ABC'DE' + A'BC'DE' + AB'C'D'E' + AB'C'D'E + \\ & AB'CD'E + AB'CD'E' + A'BCD'E' \end{aligned}$$

[1] Not a typo in the spelling of his last name. An interesting person, check out his Wikipedia page.

[2] Which is to say, put together an argument or description of why. You need not provide formal proofs.

19. [6 points, Lecture 1] Draw a hierarchical logic diagram of the logic described below. Make your drawing in a manner similar to Figure 1.7 in the *LDUSV* book. Label all the interconnections inside the instantiations of modules **a** and **b**, and also inside module **top**.

```
module a
  (input logic w, x, y,
   output logic z);

   logic d;

   xor  (d, x, y);
   nand (z, d, w);
endmodule: a

module b
  (input logic q, r, s,
   output logic t);

   logic u;

   nor (u, q, r),
       (t, u, s);
endmodule: b

module top
  (input logic d, f, w, h, i,
   output logic g);

   logic c;

   b b1 (c, d, f, g);
   a a1 (w, h, i, c);
endmodule: top
```