18-240: Structure and Design of Digital Systems

HW0 Solutions [19 problems, 64 points]

Covers lectures L0 – L1 *Due: 5 September 2023*

Drill Problems [32 points]

1. [4 points, Lecture 1] Construct a truth table for the following functions:

A	B	C	P	Q	R	S
0	0	0	1	1	0	0
0	0	1	1	0	1	1
0	1	0	1	1	1	0
0	1	1	1	0	0	0
1	0	0	1	0	1	1
1	0	1	0	1	1	0
1	1	0	0	1	1	1
1	1	1	0	1	0	1

2. [3 points, Lecture 1] For this truth table, what are F, G, and H in canonical SOP form?

A	B	C	F	G	H
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	0	1

$$F = A'BC' + A'BC + AB'C$$
$$G = A'B'C' + A'B'C + A'BC' + AB'C'$$
$$H = A'B'C + A'BC' + AB'C' + ABC$$

3. [3 points, Lecture 1] For the truth table in Question 2, what are F, G, and H in canonical POS form?

$$F = (A+B+C)\cdot(A+B+C')\cdot(A'+B+C)\cdot(A'+B'+C)\cdot(A'+B'+C')$$
$$G = (A+B'+C')\cdot(A'+B+C')\cdot(A'+B'+C)\cdot(A'+B'+C')$$
$$H = (A+B+C)\cdot(A+B'+C')\cdot(A'+B+C')\cdot(A'+B'+C)$$

4. [3 points, Lecture 1] Write F, G and H from problem #2 as a sequence of minterms.

$$F = m_2 + m_3 + m_5$$
$$G = m_0 + m_1 + m_2 + m_4$$
$$H = m_1 + m_2 + m_4 + m_7$$

You could also write them using sigma notation if you wished (i.e. $F = \Sigma m(2,3,5)$).

5. [3 points, Lecture 1] Write F, G and H from problem #3 as a sequence of maxterms.

$$F = M_0 \cdot M_1 \cdot M_4 \cdot M_6 \cdot M_7$$
$$G = M_3 \cdot M_5 \cdot M_6 \cdot M_7$$
$$H = M_0 \cdot M_3 \cdot M_5 \cdot M_6$$

6. [2 points, Lecture 0] Minimize F from problem #2 using theorems, laws and axioms of Boolean algebra.

$F = A'BC' + A'BC + AB'C$	Given
$= A'B(C'+C) + AB'C$	L8
$= A'B(1) + AB'C$	L5D
$= A'B + AB'C$	L1

7. [4 points, Lecture 0] Minimize G from problem #3 using theorems, laws and axioms of Boolean algebra. Note that you must start with the POS form, not the SOP form.

$G = (A+B'+C')\cdot(A'+B+C')\cdot(A'+B'+C)\cdot(A'+B'+C')$	Given
$= (A+B'+C')\cdot(A'+B+C')\cdot(A'+B'+C)\cdot(\mathbf{A'+B'+C'})\cdot(\mathbf{A'+B'+C'})$	L3
$= (A+B'+C')\cdot(\mathbf{A'+B'+C'})\cdot(A'+B+C')\cdot(A'+B'+C)\cdot(A'+B'+C')$	L6
$= (\mathbf{B'+C'+A})\cdot(\mathbf{B'+C'+A'})\cdot(A'+B+C')\cdot(A'+B'+C)\cdot(A'+B'+C')$	L6D
$= ((\mathbf{B'+C'})+(\mathbf{A\cdot A'}))\cdot(A'+B+C')\cdot(A'+B'+C)\cdot(A'+B'+C')$	L8D
$= ((B'+C')+0)\cdot(A'+B+C')\cdot(A'+B'+C)\cdot(A'+B'+C')$	L5
$= (\mathbf{B'+C'})\cdot(A'+B+C')\cdot(A'+B'+C)\cdot(A'+B'+C')$	L1D
$= (B'+C')\cdot(A'+B+C')\cdot(A'+B'+C)\cdot(\mathbf{A'+B'+C'})\cdot(\mathbf{A'+B'+C'})$	L3
$= (B'+C')\cdot(A'+B+C')\cdot(\mathbf{A'+B'+C'})\cdot(A'+B'+C)\cdot(A'+B'+C')$	L6
$= (B'+C')\cdot(\mathbf{A'+C'+B})\cdot(\mathbf{A'+C'+B'})\cdot(A'+B'+C)\cdot(A'+B'+C')$	L6D twice
$= (B'+C')\cdot(\mathbf{A'+C'})\cdot(A'+B'+C)\cdot(A'+B'+C')$	L8D, L5, L1D on 2nd two terms
$= (B+C)\cdot(A'+B)\cdot(\mathbf{A'+B'})$	repeat L8D, L5, L1D on last two terms

8. [2 points, Lecture 0] Using Boolean algebra, show that $x'\oplus y' + ((xy)'\cdot y)' = 1$

$x'\oplus y' + [(xy)'\cdot y]'$	Given
$x'\oplus y' + (\mathbf{xy})'' + \mathbf{y}'$	L12
$x'\oplus y' + \mathbf{xy} + y'$	L4
$\mathbf{x'y'' + x''y'} + xy + y'$	Definition of XOR
$x'\mathbf{y} + \mathbf{x}y' + xy + y'$	L4, twice
$x'y + \mathbf{xy} + \mathbf{xy'} + y'$	L6D
$x'y + \mathbf{x} + y'$	L10
$\mathbf{yx'} + x + y'$	L6
$\mathbf{x} + \mathbf{y} + y'$	L11
$x + 1$	L5D
1	L2D

9. [2 points, Lecture 0] Using Boolean algebra, show that $x' \oplus xy = x' + y$

$x' \oplus xy$	Given
$x''xy + x'(xy)'$	Definition of XOR
$\mathbf{x}xy + x'(xy)'$	L4
$\mathbf{x}y + x'(xy)'$	L3
$xy + x'(\mathbf{x}' + \mathbf{y}')$	L12
$xy + \mathbf{x}'\mathbf{x}' + \mathbf{x}'\mathbf{y}'$	L8
$xy + \mathbf{x}' + x'y'$	L3
$xy + \mathbf{x}'$	L9D
$\mathbf{yx} + x'$	L6
$\mathbf{y} + x'$	L11D
$\mathbf{x}' + \mathbf{y}$	L6D

10. [4 points, Lecture 1] Rewrite the following equations as canonical SOP and POS equations, expressed as fully written-out terms. In other words, the results should be in terms of the input variables.

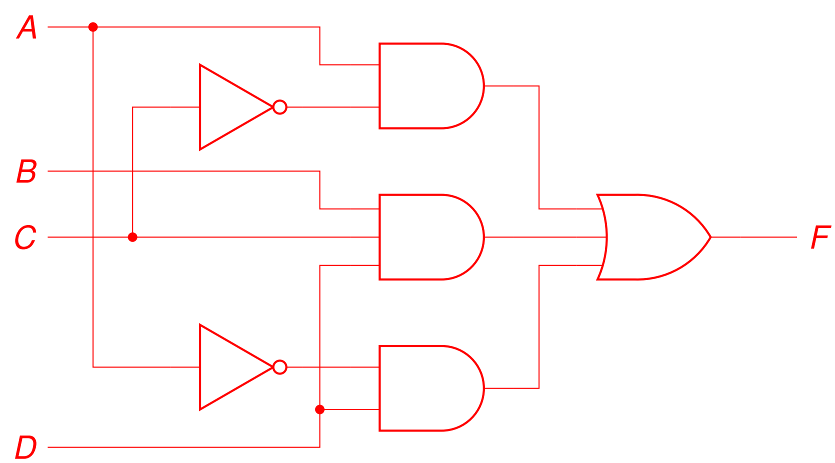
$$F(A,B,C) = \Sigma m(1,3,4,7)$$
$$G(A,B,C,D) = \Pi M(0,3,4,5,9,12)$$

$$F = A'B'C + A'BC + AB'C' + ABC$$
$$F = (A+B+C)\cdot(A+B'+C)\cdot(A'+B+C')\cdot(A'+B'+C)$$
$$G = (A+B+C+D)\cdot(A+B+C'+D')\cdot(A+B'+C+D)\cdot$$
$$(A+B'+C+D')\cdot(A'+B+C+D')\cdot(A'+B'+C+D)$$
$$G = A'B'C'D + A'B'CD' + A'BCD' + A'BCD + AB'C'D'$$
$$+ AB'CD' + AB'CD + ABC'D + ABCD' + ABCD$$

11. [1 point, Lecture 1] Find the canonical SOP form of $F(A,B,C) = A'C + AC' + B'C + BC$

$$\begin{aligned} F &= A'B'C + A'BC + AB'C' + ABC' + AB'C + A'B'C + ABC + A'BC \\ &= A'B'C + A'BC + AB'C' + ABC' + AB'C + ABC \end{aligned}$$

12. [1 point, Lecture 0] Draw a circuit realization (i.e. a schematic) of $F = AC' + BCD + A'D$



Non-Drill Problems [32 points]

13. [4 points, Ethics Lab]

    You were presented with four concepts that a responsible engineer should be aware of:

    (a) Honesty / transparency

    (b) Bias / fairness

    (c) Accountability

    (d) Reliability

    Honesty and Accountability seem pretty similar. Write a paragraph or so explaining what each of these two concepts are and why they are different – and how to tell the difference.

    You will be graded on the clarity of your writing as well as the correctness of your answer.

    Honesty refers to the act of telling customers, consumers, and your team about the reality and limitations of your technology. It also ensures people have a correct understanding of the product and can then have a sense of "informed consent" about its use and risks.

    Accountability means fulfilling expectations made by the promises and guarantees of the technology such that systems are created in such a way that they fulfill their intended function. Importantly, it also means being compliant with relevant laws and rights; as well as taking responsibility if something goes wrong.

    One can be honest and transparent while still trying to dodge responsibility and skirting the law. The fallout from not being honest can often result in reputational damage and a breach of trust. But, the fallout from accountability often results in fines and other governmental penalties.

14. [6 points, Lecture 0] Use the definition of XOR, the facts that XOR commutes and associates (if you need this), and all the non-XOR axioms and theorems you know to prove this distributive rule:

$$A \cdot (B \oplus C) = (A \cdot B) \oplus (A \cdot C)$$

Let's start with the left side:

$A \cdot (B \oplus C)$	Given
$A \cdot (BC' + B'C)$	Definition of XOR
$ABC' + AB'C$	L8

Now, switch your attention to the right side:

$(A \cdot B) \oplus (A \cdot C)$	Given
$(A \cdot B) \cdot \overline{(A \cdot C)} + \overline{(A \cdot B)} \cdot (A \cdot C)$	Definition of XOR
$AB \cdot (A' + C') + (A' + B') \cdot AC$	L12, twice
$AB \cdot (A' + C') + AC \cdot (A' + B')$	L6
$ABA' + ABC' + ACA' + ACB'$	L8, twice
$BAA' + ABC' + CAA' + ACB'$	L6, twice
$B \cdot 0 + ABC' + C \cdot 0 + ACB'$	L5, twice
$0 + ABC' + 0 + ACB'$	L2, twice
$ABC' + 0 + ACB' + 0$	L6D, twice
$ABC' + ACB'$	L1D, twice
$ABC' + AB'C$	L6

Hey, look! That's the same as the left side. Therefore, multiplicative distribution works for XOR. This turns out to be the only situation where distribution works with XOR.

15. [6 points, Lecture 0] Charles Peirce showed, in 1880, that the NAND function forms a complete set of logic gates. In other words, it is possible to implement any Boolean expression using nothing more than 2-input NAND gates. Is the same true for NOR? Show whether this is true in a very convincing fashion.

Boolean algebra defines three operations: AND, OR and NOT. If Peirce is correct, then we need to be able to accomplish those three operations with nothing but NOR gates.

The NOR gate's operation is: $\overline{(a+b)}$

So, the question comes down to defining AND, OR and NOT using $\overline{(a+b)}$ forms

NOT: $a' = \overline{(a+a)}$ using L3D. Therefore, NOT can be accomplished with a NOR gate, if the input signal goes to both inputs of the NOR gate.

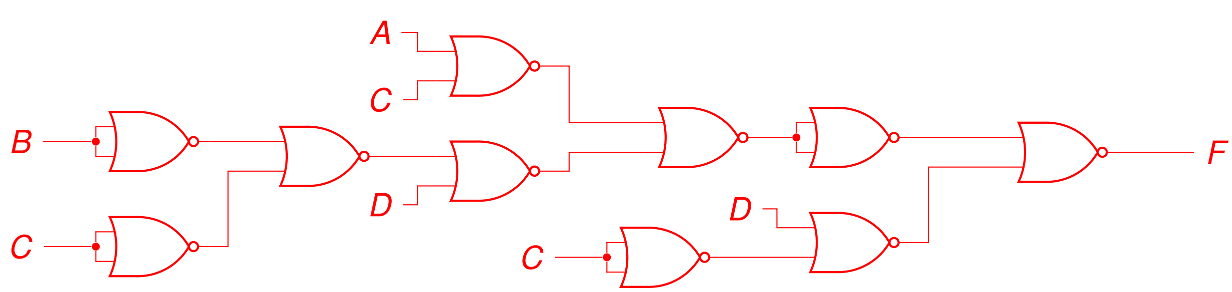
OR: $a+b=\overline{\overline{(a+b)}}$ using L4. Since we can already do the outside NOT using a NOR gate (see previous paragraph), we can create an OR gate by inverting the output of the NOR gate. Thus, a and b should be connected to the inputs of a NOR gate, the output of which is then inverted with another NOR gate.

AND: $a \cdot b = \overline{(a' + b')}$ using De Morgan's Law (L12). Therefore, we can create an AND gate by inverting each input to a NOR gate. The inversion is done with separate NOR gates with the inputs tied together.

16. [4 points, Lecture 0] Draw a schematic of $F = [(A \cdot C)' \cdot (B' \cdot C' + D') \cdot (C + D')]'$ using only 2-input NOR gates.

While it is possible to simplify the equation first, that wasn't really the point of the problem. Instead, I recognized the NAND gates in the circuit and did the DeMorgan equivalencies to transform them to NOR gates.

Beware of the 3-input gate transformations! Note also that (luckily) inverters can be replaced with 2-input NOR gates whose inputs are connected.



17. [2 points, Lecture 1] For an n-input, m-output combinational circuit, how many rows are there in the truth table? How many columns on the right side? How many columns on the left side?

There are 2^n rows (input combinations), m columns on the right side and n columns on the left

18. [4 points, Lecture 1] Compliments to those who can complement. Show the complement of the following function in canonical POS form:

$$F(A,B,C,D,E) = ABC'DE' + A'BC'DE' + AB'C'D'E' + AB'C'D'E + AB'CD'E + AB'CD'E' + A'BCD'E'$$

This expression is the OR of a bunch of product terms, each of which contains all the input variables. Therefore, each of these is a minterm. Therefore, the given equation is in canonical SOP form.

I'm going to write it in simplified notation.

$$F(A,B,C,D,E) = \Sigma m(26,10,16,17,21,20,12)$$

Note that I showed the minterms in the same order as the given equation. Often the values are sorted.

$\overline{F}$ can be written as an SOP formula with "all the other minterms."

$$\overline{F}(A,B,C,D,E) = \Sigma m(0,1,\ldots,\text{except for } 10,12,16,17,20,21,26)$$

But, we want $\overline{F}$ in POS format. That equation will have a Π instead of Σ and will, once again, reverse the values in the list. POS is a list of maxterms, which are all the terms that aren't minterms.

So, the list of maxterms for $\overline{F}$ will be the same list as the first equation we wrote.

$$\overline{F}(A,B,C,D,E) = \Pi M(26,10,16,17,21,20,12) \quad = \Pi M(10,12,16,17,20,21,26)$$

19. [6 points, Lecture 1] Draw a hierarchical logic diagram of the logic described below. Make your drawing in a manner similar to Figure 1.7 in the *LDUSV* book. Label all the interconnections inside the instantiations of modules **a** and **b**, and also inside module **top**.