# **Artificial Intelligence**

Week 6: Learning for Game Playing

COMP30024

April 9, 2021



#### Adversarial Search

- Minimax enumerates all outcomes, selects the best one inefficient. Two ideas:
- Prune search graph: ignore regions of state space that cannot lead to optimal or complete solutions.
  - Provably ignore suboptimal subgraphs.
- Approximate the expected utility for σ through heuristic evaluation function f(σ) to eliminate simulation.
  - Probably ignore suboptimal subgraphs.



## Noughts and Crosses

- Branching factor b = 9, maximum goal depth is m = 9.
- Naively number of possible games  $\leq 9! = 362880$ .
  - Most games terminate before 9 plys, many identical under board symmetry.
- Complete enumeration of game tree, exact minimax play (TC  $\mathcal{O}(b^m)$ , SC  $\mathcal{O}(bm)$ ) possible.

## Noughts and Crosses

- Book learning exact lookup from game tree obtained via minimax. Provably optimal but slowest possible.
- 2. Efficient  $\alpha$ - $\beta$  ordering:.  $\alpha$ - $\beta$  prevents enumeration of suboptimal regions of state space. Generate optimal successor at each move to maximize efficiency, Approximate optimal ordering via the killer move heuristic:
  - Use iterative deepening to search a small depth below current move, use best recorded path to inform move ordering.
- 3. Learning weights  $w_i$  of evaluation function  $f(\sigma) = g\left(\sum_i w_i x_i\right)$ . Use f to evaluate the utility for  $\sigma$  at depth limit. Not optimal but lowest time, space complexity.

Games with larger game trees (Chess, Go, etc.) need approximate evaluation.

Maintain a set of counters  $\{c_1, \dots c_9\}$  of pseudocounts of state visits. For each game, record all visited states. At the end of the game, for each visited state  $c_i$ :

- $c_i \rightarrow c_i + \Delta$ , if win.
- $c_i \rightarrow c_i \Delta$ , if loss.
- $c_i \rightarrow c_i$ , if tie.

For new games, probability of placing token in square i is

$$p_i = \frac{c_i}{\sum_{j=1}^9 c_j}$$

Number of possible states in noughts and crosses?

- Naïvely  $3^9 = 19683$  states (9 squares, 3 possible states for each square).
  - Note number of possible games > number of unique states as a game considers state ordering.
- How to do better? Note game finishes after 9 moves (5 by X, 4 by O).
  - ▶ How many ways to place first X on board?  $9 = \binom{9}{1}$ .
  - ► How many ways to place first *O* on board?  $72 = \binom{9}{1} \times \binom{8}{1}$ .
  - ► How many ways to place second X on board?  $\binom{9}{2} \times \binom{7}{1}$ .
  - ► How many ways to place second O on board?  $\binom{9}{2} \times \binom{7}{2}$ .
  - ► How many ways to place third X on board?  $\binom{9}{3} \times \binom{6}{2}$ .
  - ► How many ways to place third O on board?  $\binom{9}{3} \times \binom{6}{3}$ , etc.
  - ▶ Sum all up to get 6046
- This can be reduced a bit more via symmetry.

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  - Number of possible states larger likely no/little data collected on vast majority of positions, leading to uninformed play.