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AP Calc AB Pd. 5
Problem Set 24

1. Point moving on $y = \frac{1}{1+x^2}$ such that $\frac{dx}{dt} = 2\ \text{cm/hr}$.

Find the value of $\frac{dy}{dt}$ @ $x = -2$.

$f(x) = (1+x^2)^{-1}$

$a(x) = x^{-1}$ $\qquad$ $b(x) = 1+x^2$

$a'(x) = -x^{-2}$ $\qquad$ $b'(x) = 2x\frac{dx}{dt}$

$f'(x) = -(1+x^2)^{-2} \cdot 2x\frac{dx}{dt}$

$= -2x(x^2+1)^{-2}\frac{dx}{dt}$

$= \frac{-2x\frac{dx}{dt}}{(x^2+1)^2}$

$\frac{dy}{dt} = \frac{-2x\frac{dx}{dt}}{(x^2+1)^2}$

$\frac{dy}{dt}$ @ $x = -2$, $\frac{dx}{dt} = 2\text{cm/hour}$

$\Rightarrow \frac{dy}{dt} = \frac{-2(-2)(2\ \text{cm/hr})}{(-2^2+1)^2} = \frac{8\ \text{cm/hr}}{25}$

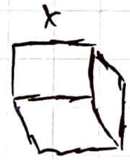
$\boxed{= \frac{8}{25}\ \text{cm/hr}}$

2. Edges of a cube are shrinking @ $\frac{1}{14}$ cm/hour. How fast is the volume changing when the edge length is 7cm?

Find $\frac{dy}{dt}$ @ $x = 7$

$\frac{dx}{dt} = \frac{1}{14}$ cm/hour

x



vol = y

$y = x^3$

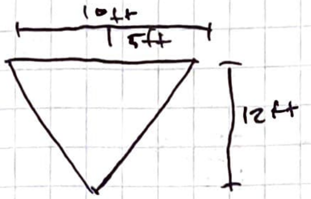
$\Rightarrow \frac{dy}{dt} = 3x^2\frac{dx}{dt}$

@ $x = 7$, $\Rightarrow \frac{dy}{dt} = 3(7)^2(\frac{1}{14})$

$= 3(49)(\frac{1}{14})$

$= \frac{3 \cdot 7 \cdot 7}{2 \cdot 7} = \boxed{\frac{21}{2}\ \text{cm}^3\text{/hour}}$

3. Conic tank ⟹ 10 ft across the top filling @ 10ft^3/hour. Find rate of change
12ft deep.
@ 8 ft.



$$\frac{r}{h} = \frac{5}{12} \Rightarrow r = \frac{5h}{12}$$

$$r^2 = \frac{25h^2}{144}$$

$$V = \frac{\pi}{3}\left(25h^2/144\right) \cdot h$$

$$\frac{dV}{dt} = \frac{25\pi}{144 \cdot 3} \cdot 3h^2 \frac{dh}{dt}$$

$$= \frac{25\pi \cdot \cancel{3}}{144 \cdot \cancel{3}} \cdot (8)^2(10)$$

$$= \frac{25\pi}{144} \cdot 640 = \frac{25 \cdot 40\pi}{9} = \boxed{\frac{1000\pi}{9}}$$

4. $y^2 = 2 + xy$

a. Show that $\frac{dy}{dx} = \frac{y}{2y-x}$

$$2y\frac{dy}{dx} = \cancel{0} + x\frac{dy}{dx} + y$$

$$xy \Rightarrow x\frac{dy}{dx} + y$$

$$\frac{dy}{dx}(2y-x) = y$$

$$\boxed{\frac{dy}{dx} = \frac{y}{2y-x}}$$

b. Find point where the slope of the tangent line is $\frac{1}{3}$.

$$\frac{y}{2y-x} = \frac{1}{3}$$

$y=1$, $y=-1$

$2y-x=3$	$2y-x=-3$
$2(1)-x=3$	$-2-x=-3$
$-x=1$	$-x=-1$
$x=-1$	$x=1$

$\boxed{(-1,1) \text{ and } (1,-1)}$

c. x/y functions of time. $t=5$, $y=3$ and $\frac{dy}{dt} = 6$ $\frac{\text{inches}}{\text{min}}$. Find $\frac{dx}{dt}$ @ $t=5$

$y^2 = 2 + xy \Rightarrow 3^2 = 2 + x(3)$ $\quad 9 = 2 + 3x \quad 7 = 3x \quad x = \frac{7}{3}$

$$2y\frac{dy}{dt} = \frac{dy}{dt}x + \frac{dx}{dt}y \Rightarrow @\ t=5 \Rightarrow 2(3)(6) = 6\left(\tfrac{7}{3}\right) + \frac{dx}{dt}(3)$$

$$= 36 = 14 + 3\frac{dx}{dt}$$

$$22 = 3\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{22}{3} \frac{\text{inches}}{\text{min}}$$

$$\boxed{\frac{dx}{dt} = \frac{22}{3} \text{ inches/min @ } t=5}$$

5. 15 ft ladder.

a.

x moves away from building @ $1/2$ ft/second

y = ~~len~~ len $(\overline{oy})$. Find rate of change when x is 9 ft away.

$y =$ $\quad \frac{dx}{dt} = \frac{1}{2}$ $\quad 9^2 + y^2 = 225$

$y^2 = 144$

$y = 12$

$x^2 + y^2 = 225$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ $\quad = @\ x = 9\text{ ft}$

$18(1/2) + 2y\frac{dy}{dt}$

$9 + 24(\frac{dy}{dt}) = 0$

$\frac{dy}{dt} = \frac{-9}{24} = \boxed{\frac{-3}{8}\text{ ft/second}}$

b. A = area of Oxy. Find R.O.C of A when x = 9 ft.

$\frac{dx}{dt} = \frac{1}{2}$ ft/second. $\quad x = 9\text{ft}, y = 12\text{ft}.$

$A = \frac{1}{2}(x)(y)$ $\quad A = \frac{1}{2}(9 \cdot 12) = 9 \cdot 6 = 54\text{ ft}^2$

$A\frac{da}{dt} = \frac{1}{2}(\frac{dx}{dt}y + \frac{dy}{dt}x)$

$54\frac{da}{dt} = \frac{1}{2}((12(\frac{1}{2}) + 9(-\frac{3}{8}))$

$\frac{da}{dt} = \frac{1}{54} \cdot \frac{1}{2}(6 - \frac{27}{8})$ $\quad 6 \cdot 8 = 48$ $\quad 48 - 27 = 21$

$= \frac{1}{108}(\frac{21}{8}) = \frac{7}{36 \cdot 8} = \frac{7}{288}$ ft/sec.

c. θ is measure in radians of $\angle$ oxy. $x = 9$ $\quad f(x) = \tan^{-1}(x)$ $\quad \theta = \tan^{-1}(12/9) = \tan^{-1}(4/3)$

$f'(x) = 1/x^2+1$

$x = 9 \Rightarrow$
$y = 12$
$\Rightarrow \frac{dx}{dt} = \frac{1}{2}$
$\frac{dy}{dt} = \frac{-3}{8}$

$\theta = \tan^{-1}(y/x)$

$\theta\frac{d\theta}{dt} = \left(\frac{\frac{dy}{dt}x - \frac{dx}{dt}y}{x^2}\right) \cdot \frac{1}{\tan^{-1}(\frac{y}{x})+1}$ $\Rightarrow$ $\frac{\frac{dy}{dt}x - \frac{dx}{dt}y}{x^2}$

$\tan^{-1}(12/9)\frac{d\theta}{dt} = \left(\frac{\frac{-3 \cdot 9}{8} - 6}{81}\right)\left(\frac{1}{\tan^{-1}(12/9)+1}\right)$

$\frac{d\theta}{dt} = \left(\frac{-\frac{27}{8} - \frac{54}{8}}{81}\right)\left(1/\tan^{-1}(12/9)+1\right) = -\frac{1}{8}\left(\frac{1}{\tan^{-1}(\frac{12}{9})+1}\right)$

wrong?? $\Rightarrow$ $\frac{4}{?} - 1/15$ $\frac{\text{rad}}{\text{sec}}$