Formulas:

$\overline{Q} = f(L,K)$

$AP_L = \frac{f(L,K)}{L}$ $\quad MP_L = \frac{\partial f(L,K)}{\partial L}$

$AP_K = \frac{f(L,K)}{K}$ $\quad MP_K = \frac{\partial f(L,K)}{\partial K}$

$MRTS_{LK} = \frac{MP_L}{MP_K}$

Perfect substiture

$f(L,K) = aL + bK$

Perfect complement

$f(L,K) = \min\{aL, bK\}$

Perfect Cobb-Douglas

$F(L,K) = AL^{\alpha}K^{\beta}$

$\Rightarrow MRTS_{LK} = \frac{\alpha}{\beta}\left(\frac{K}{L}\right)$

Constant Returns to scale: $f(\lambda L, \lambda K) = \lambda Q$

Increasing Returns to scale: $f(\lambda L, \lambda K) > \lambda Q$

Decreasing Returns to scale: $f(\lambda L, \lambda K) < \lambda Q$

Cobb Douglas

$f(L,K) = AL^{\alpha}K^{\beta}$

$f(\lambda L, \lambda K) = \lambda^{\alpha+\beta} AL^{\alpha}K^{\beta}$

$\Rightarrow \lambda^{\alpha+\beta} f(L,K)$

$\alpha + \beta > 1$: Increasing

$\alpha + \beta < 1$: Decreasing

$C(Q) = VC(Q) + F$

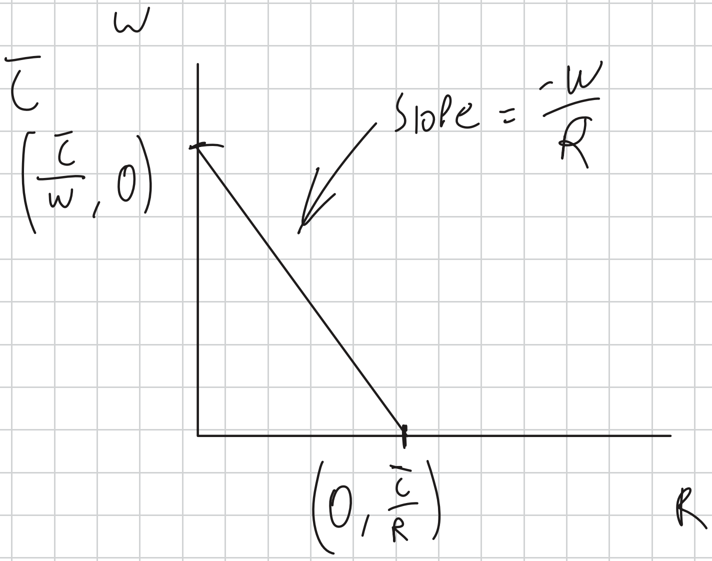
Cannot be Recovered (sunk)

Can be Recovered (Avoidable)

$f(L,K) = Q = AL^{\alpha}K^{\beta}$

Isocost

$wL + RK = \overline{C}$



Cost minimization:

Find (L,K) on Iso Quant that lies on Isocost that's closest to origin

Interior Solutions - to show optimality

$MRTS_{LK} = \frac{w}{R} \Rightarrow \frac{MP_L}{MP_K} = \frac{w}{R} \Rightarrow \frac{MP_L}{w} = \frac{MP_K}{R}$

Problem: $f(L,K) = 2L^{\frac{1}{2}}K^{\frac{1}{2}}$, $w = 9$, $R = 4$, $\overline{Q} = 50$

$f(L,K) = \overline{Q} = 50 = 2L^{\frac{1}{2}}K^{\frac{1}{2}}$

$25 = L^{\frac{1}{2}}K^{\frac{1}{2}}$

$MRTS_{LK}: \frac{\frac{1}{2}}{\frac{1}{2}}\left(\frac{K}{L}\right) = \frac{MP_L}{MP_K} = \frac{\left(K^{\frac{1}{2}}\right)\left(\frac{1}{\sqrt{L}}\right)}{\left(L^{\frac{1}{2}}\right)\left(\frac{1}{\sqrt{K}}\right)} = \frac{9}{4}$

$4\left(\frac{\sqrt{K}}{\sqrt{L}}\right) = 9\left(\frac{\sqrt{L}}{\sqrt{K}}\right)$

$4K = 9L$

$L = \frac{4K}{9}$

$9\left(\frac{4K}{9}\right) + 4K = \overline{C}$

$8K = \overline{C}$

$\overline{C} = 300$

$2\left(\frac{2\sqrt{K}}{3}\right)\left(\sqrt{K}\right) = 50$

$\frac{2K}{3} = 25$

$K = 37.5$

Cost minimization - corner solutions
may only occur if
$MRTS_{LK}$ is Always Greater or lesser
than $\frac{w}{r}$ for all of L, K

$\frac{MP_L}{MP_K} > \frac{w}{r}$, it's only L

$\frac{MP_L}{MP_K} < \frac{w}{r}$, it's only K

Cost functions

$AC(Q) = \frac{C(Q)}{Q}$

$L^* = \frac{\alpha}{\alpha+\beta}\left(\frac{1}{w}\right)Q$

$MC(Q) = \frac{\partial C(Q)}{\partial Q}$

Cost minimization yields $(L(Q), K(Q))$

$C(Q) = W \cdot L(Q) + RK(Q)$

Efficient scale for costs

$AC = MC$ or $AC'(Q) = 0$

Minimum Average Cost

- Plug 0 for Q

$AC(0) =$ min. cost

$f(L,K) = 4\sqrt{L}\sqrt{K}$, $W=10$, $R=15$, $\bar{K}=25$

$C(L,K) = \bar{C} = WL + RK$

$\bar{C} = WL + 15(25)$

$\bar{C} = WL + 375$

Say, we want $Q=100 = f(L,K)$

$f(L,K) = 20\sqrt{L}$

$Q = 100 = 20\sqrt{L}$

$5 = \sqrt{L}$

$L = 25$

So $10(25) + 375$

$250 + 375$

$= 625$

$\frac{\partial x}{\partial P_x} \gtrless 0$ can show about SE vs IE effect (Giffen Good)

$C(\lambda Q) \overset{?}{\gtreqless} \lambda C(Q)$

Do AC ↑ or ↓ when Q↑

(Economies of scale or Diseconomies)

$AC(Q)\downarrow$ as $Q\uparrow$ $AC(Q)\uparrow$ as $Q\uparrow$

$\pi(Q) = P(Q) \cdot Q - C(Q)$

profit maximization at $MR = MC$

So

$\frac{d\pi}{dQ} = P'(Q) \cdot Q + P(Q) - C'(Q)$

see $\frac{d\pi}{dQ} = 0$ so $P(Q) + QP'(Q) = C'(Q)$

If > 0, produce more

< 0, produce less

Price taking firms

$\pi = Q^* P - C(Q)$ Also $\pi = Q^*(P - AC(Q^*))$

$\frac{\partial \pi}{\partial Q} = P - C'(Q)$

Optimal quantity can occur at

$\frac{\partial \pi}{\partial Q} = 0$, Hence $P = C'(Q)$

-or $Q = 0$, and $\frac{\partial \pi}{\partial Q} < 0$

Quantity Rule

Derive Q^* such that $P = MC(Q^*)$

such that

$\pi^* = \pi(Q^*) = P \cdot Q^* - C(Q^*)$

If > 0, optimal Quantity is Q^*

If < 0, optimal quantity is 0 so shutdown

If $= 0$, Either $Q = 0$ or Q^* is fine

If fixed costs exist, then

$C(Q) = \underset{\substack{\uparrow \\ \text{- Variable} \\ \text{- Fixed}}}{TAC(Q)} + \underset{\substack{\uparrow \\ \text{sunk cost}}}{SC}$

$\boxed{\text{Note: we Don't care About SC}}$

$\underset{\substack{\uparrow \\ \text{Average Avoidable cost}}}{AAC(Q)} = \frac{TAC(Q)}{Q}$ By Definition, If $Q = 0$, then Avoidable costs are 0

Problem: $P = 8$, $C(Q) = 200 + 4Q + \frac{Q^2}{50}$

$TAC(Q) = 200$

$\pi^* = PQ^* - [TAC(Q)]$

$MC = C'(Q)$

$8 = 4 + \frac{2Q}{50}$

$200 = 2Q$

$Q = 100$

$AAC(Q) = \frac{\text{avoidable cost}(Q)}{Q}$

$\Rightarrow \frac{C(Q) - SC}{Q}$

to find min do

$\frac{\partial AAC(Q)}{\partial Q} = 0$

we can find optimal Quantity By $Q(P) \leftarrow$ supply function

Note Qty. Rule is $P = MC(Q)$

Problem: $200 + 4Q + \frac{Q^2}{50}$, 200 is SC

$P = MC(Q) \Rightarrow C'(Q) = 4 + \frac{Q}{25}$

$$Q(P) = \begin{cases} (P-4)25 & \text{If } P > 4 \\ 0 & \text{If } P < 4 \end{cases}$$

$$\text{AKA} \quad Q(P) = \begin{cases} MC^{-1}(Q) & \text{If } P > MC(Q) \text{ or } P \geq AAC_{min} \\ 0 & (\text{Find } P \text{ when } Q(P) = 0) \end{cases}$$

free entry market equilibrium

$P = AAC_{min}$

$Q = Q_d(AAC_{min})$

$N = \frac{Q}{Q_{firm}(AAC_{min})}$

$Q^d(P) = \sum_i Q_i^d(P)$

$Q^S_{LR}(P) = \begin{cases} 0, & P < AC_{min} \\ \infty, & P = AC_{min} \end{cases}$

zero econ profit

when $Q^S(P) = Q^D(P)$

is equilibrium