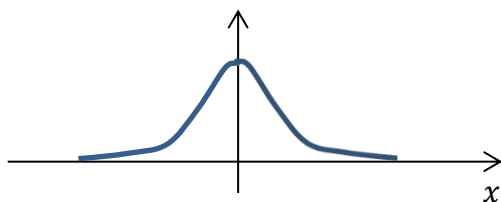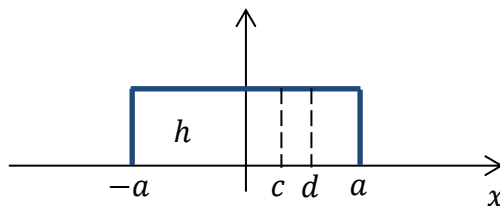# Homework Set 1

Problem 1 (Probability density function)

| (A) | (B) |

a) Figure (A) shows a Cauchy probability density function whose density is given by $\frac{1}{K}\frac{1}{1+x^2}$. What is the value of K for the density to be a probability density function? What are the mean and variance of the distribution?

b) Figure (B) shows a uniform probability density function. What is the height h of the density function? What are the mean and variance of the distribution? What is the probability that x lies in the interval between the vertical lines marked by c and d?

c) Consider two fair dice with six sides marked with the usual numbers 1 through 6. What is the probability that a throw of the dice results in a score of 7? What is the mean of the numbers that arise when two dice are thrown? What is the variance?

Problem 2 (Sample statistics and confidence intervals)
In characterizing the noise in an amplifier, which is normally distributed, we have the following noise voltages in micro Volts (μV):

-0.4326 -1.6656   0.1253   0.2877 -1.1465   1.1909   1.1892 -0.0376   0.3273   0.1746

a) Estimate the mean of the noise voltages and the variance of the mean.

b) Calculate the 95% and 99% confidence intervals of the mean (of noise voltages).

c) How confident are we that the noise voltage at any time lies between $1\mu V$ and $1.1\mu V$?

Problem 3 (Calculus)
Let $f(x,y) = 3x^2 + y^2 - xy - 11x$

a) Find $\frac{\partial f}{\partial x}$, the partial derivative of f with respect to x. Also find $\frac{\partial f}{\partial y}$.

b) Find the pair $(x,y) \in \mathcal{R}^2$ that minimizes f.

c) Show that the pair (x,y) you found in b. is a minimizer instead of a maximizer.

Problem 4 (Vector Norms)
Compute the 0, 1, 2, and ∞ norms for $[3\ \text{-}1\ 3\ 5\ 0\ 2\ ]^\text{T}$.

Problem 5 (t-Test)

Consider the "French fries" data, which reports the number of fries in a small bag for a sample of 10 McDonald's bags and 5 Burger King bags. The sample data are:

McDonald's: 35 39 43 52 53 55 34 37 45 40

Burger King: 51 55 52 43 44

a) Determine a 90% confidence interval for the difference in population means.

b) Find out if the two means are different with significance level of 0.01 and 0.05.