R08631036 熊顥鋒

1.

(a) False, suppose we are estimating σ^2, $\hat{\sigma}^2 = \frac{1}{n}\sum(X_i-\bar{X})^2$ and $E(\hat{\sigma}^2) = \frac{n-1}{n}\sigma^2$ (biased estimator)

$Var(\hat{\sigma}^2) = \frac{2\sigma^4(n-1)}{n^2}$ as n (sample size) increases, $\hat{\sigma}^2$ concentrates to σ^2, becoming more and more consistent.

(b) False, suppose we are estimating μ, X is unbiased estimator of μ and has distribution of $N(\mu, \sigma^2)$. But, as the sample size increases, since we set the distribution fixed, it won't concentrate or change. Therefore, it is not consistent.

(c) True (d) True (e) True

2.

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{X}$$

$$\Rightarrow \hat{\beta}_0 \xrightarrow{P} \bar{y} - b_1\bar{X} = b_0 \#$$

3.

(a)

$$\hat{y}_i = \hat{\gamma}_1 w_i + u_i = \hat{\gamma}_1(X_i+\varepsilon_i) + u_i$$

$$Q_n = \sum u_i^2 = \sum[\hat{y}_i - \hat{\gamma}_1(X_i+\varepsilon_i)]^2$$

$$\frac{\partial Q_n}{\partial \gamma_1} = -2\sum[\hat{y}_i - \hat{\gamma}_1(X_i+\varepsilon_i)](X_i+\varepsilon_i) = 0$$

$$\Rightarrow \sum(\hat{y}_iX_i + \hat{y}_i\varepsilon_i - 2\hat{\gamma}_1X_i^2 - \cancel{2\hat{\gamma}_1X_i\varepsilon_i}^{\,0}) = 0$$

$$\Rightarrow \hat{\gamma}_1 = \frac{\sum(\hat{y}_i)(X_i+\varepsilon_i)}{2\sum X_i^2} \Rightarrow \operatorname*{plim}_{i\to\infty}\hat{\gamma}_1 = \lim_{i\to\infty}\frac{\sum X_iy_i + \sum\varepsilon_iy_i}{2\sum X_i^2}$$

(b)

Underestimate, unconsistent

Since $\beta_1 = \frac{\sum X_iy_i}{\sum X_i^2}$ and $\gamma_1 = \frac{1}{2}\frac{\sum X_iy_i}{\sum X_i^2} + \frac{1}{2}\frac{\sum\varepsilon_iy_i}{\sum X_i^2}$

This part should be small, so it won't let $\gamma_1 \to \beta_1$.

(c) $\hat{\gamma}_1 = \dfrac{\Sigma (X_i + \varepsilon_i) y_i}{2 \Sigma X_i^2}$. As $\sigma_x^2 \to \infty$, $\hat{\gamma}_1$ should converge close to 0

because the denominator is second order relative to the numerator

(d) $\hat{\gamma}_1$ should $\to \infty$ since all the other variables stay unchanged.

4.

(a) Yes, if $\operatorname{cov}(X_1, X_2) = 0$ #

(b)

$$\tilde{\alpha}_1 = \left(\frac{1}{n} \Sigma X_{1i} X_{1i}'\right)^{-1} \left(\frac{1}{n} \Sigma X_{1i} y_i\right)$$

$$= \beta_1 + \left(\frac{1}{n} \Sigma X_{1i} X_{1i}'\right)^{-1} \left[\left(\frac{1}{n} \Sigma X_{1i} X_{2i}\right) \beta_2 + \cancel{\left(\frac{1}{n} \Sigma X_{1i} u_i\right)}^{\to 0} \right]$$

$$= \beta_1 + \frac{\sigma_{X_1 X_2}}{\sigma_{X_1 X_1}} \beta_2 \to \text{overestimate.} \ \#$$

(c)

By definition in slide, $\sqrt{n}(\tilde{\alpha}_1 - \beta_1) \to N\left(0, E(X_1^2)^{-1} \sigma_{X_1 X_2} E(X_1^2)^{-1}\right)$

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5.

$$RDR' = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \end{pmatrix} \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1(k+1)} \\ d_{21} & d_{22} & \cdots & d_{2(k+1)} \\ \vdots & \vdots & \ddots & \vdots \\ d_{(k+1)1} & d_{(k+1)2} & \cdots & d_{(k+1)(k+1)} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ & \vdots & \end{pmatrix}$$

$$= \begin{pmatrix} d_{21} & d_{22} & \cdots & d_{2(k+1)} \\ d_{31} & d_{32} & \cdots & d_{3(k+1)} \\ d_{41} & d_{42} & \cdots & d_{4(k+1)} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ & \vdots & \end{pmatrix} = \begin{pmatrix} d_{22} & d_{23} & d_{24} \\ d_{32} & d_{33} & d_{34} \\ d_{42} & d_{43} & d_{44} \end{pmatrix}$$

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