Spring 2020 Quantitative Analysis
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Problem Set 5

Due: 4/6

Part One: Hand-Written Exercise

1. Consider the simple model:

$$y_i = \beta_0 + \beta_1 x_i + u_i,$$

where $i = 1...N$, x_i are non-random and, $u_i \sim N(0, \sigma^2)$.

   (a) Please derive the maximum likelihood estimator (MLE) of β_0 and β_1. Is it the same as the OLS estimator $\hat{\beta}_{OLS}$?

   (b) Please derive the maximum likelihood estimator (MLE) of σ^2. Is it the same as the OLS estimator $\hat{\sigma}^2_{OLS}$?

2. Verify the statement on slide 23, Lecture 5. That is, for the Probit model, show that the information equality holds.

3. Suppose the data is like below:

x	1	1	1	1	1	1	0	0	0	0	0	0
y	1	1	1	0	0	0	1	1	1	1	0	0

Please calculate $\hat{\beta}_0$ and $\hat{\beta}_1$ in Logit regression where $F(\boldsymbol{x_i}; \boldsymbol{\beta}) = G(\boldsymbol{x_i'\beta}) = \frac{1}{1+exp(-\beta_0-\beta_1 x_i)}$

Part Two: Computer Exercise

1. Please load the dataset `HMDA` in `R`, which is a cross-section data on the Home Mortgage Disclosure Act, containing 2,380 observations on 14 variables. The variable we are interested in modelling is "`deny`", an indicator for whether an applicant's mortgage application has been accepted (deny = no) or denied (deny = yes). A regressor that ought to have power in explaining whether a mortgage application has been denied is "`hirat`", the size of the anticipated total monthly loan payments relative to the the applicant's income.

   (a) Construct a Logit model with `deny` as dependent variable y, `hirat` as the independent variable x. Show the estimated coefficients and the corresponding robust standard error.

   (b) What is the probability of facing a denial on mortgage application when `hirat` = 0.2?

   (c) What is the probability of facing a denial on mortgage application when `hirat` = 0.8?

(d) Now construct a Logit model with `deny` as dependent variable y, `hirat` and `afam` as the independent variables x (Note that the variable "afam" equals 1 if the applicant is an African American and equals 0 otherwise).

(e) Fixing `hirat` $= 0.2$, please estimate the differences in `deny` between African American and non-African American.

(f) Fixing `hirat` $= 0.8$, please estimate the differences in `deny` between African American and non-African American.