R08631036 熊顥鋒

1.

$$\sum_{i=1}^{n} \hat{r}_{i,1} \cdot x_{i,1} = \sum \hat{r}_{i,1} \cdot (\hat{r}_{i,1} + \bar{x}_1) \qquad \bar{x}_1 \cdot \sum \hat{r}_{i,1} = \bar{x}_1 \cdot 0 = 0$$

$$= \sum \hat{r}_{i,1}^{2} + \cancel{\sum \hat{r}_{i,1} \cdot \bar{x}_1}$$

Law of Large Numbers.

$$= \sum \hat{r}_{i,1}^{2}$$

#

2.

(a)

$$Var(\hat{\beta}_1) = Var\left(\frac{\sum_1^n \hat{r}_{i,1} \cdot y_i}{\sum_1^n \hat{r}_{i,1}^2}\right) = \frac{\sum_1^n \hat{r}_{i,1}^2 \cdot Var(y_i)}{\left(\sum_1^n \hat{r}_{i,1}^2\right)^2} = f(x_i) \cdot \frac{1}{\sum_1^n \hat{r}_{i,1}^2}$$

Becomes a function of x_i

$\Rightarrow \hat{\beta}_1$ is biased #

(b)

$$E(\hat{\beta}_1) = \frac{\sum \hat{r}_{i,1} \cdot E(y_i)}{\sum_1^n \hat{r}_{i,1}^2} = \frac{\sum \hat{r}_{i,1} \cdot (\cancel{\beta_0}^{0} + \beta_1 \cancel{x_{i1}}^{1} + \cancel{\beta_2 x_{i2}}^{0} + \beta_3 x_{i3})}{\sum \hat{r}_{i,1}^2}$$

$$= \beta_1 + \beta_3 \cdot \frac{\sum \hat{r}_{i,1} \cdot x_{i3}}{\sum \hat{r}_{i,1}^2} \neq \beta_1 \Rightarrow \text{biased}$$

(c)

$E(\hat{\beta}_1) =$ same procedure as (b) $= \beta_1 \Rightarrow$ unbiased

3.

No, because the target y is not given, $\beta = (X^T X)^{-1} X^T y$ can't be calculated.

4.

$$SST = \sum_1^n (y_i - \bar{y})^2$$

$$= \sum \left[(y_i - \hat{y}_i) + (\hat{y}_i - \bar{y}) \right]^2$$

$$= \underbrace{\sum (y_i - \hat{y}_i)^2}_{SSE} + \underbrace{\sum (\hat{y}_i - \bar{y})^2}_{SSR} + 2\sum (y_i - \hat{y}_i)(\hat{y}_i - \bar{y})$$

so it turns out to prove if this term equals to 0

$$\Rightarrow \sum (y_i - \hat{y}_i)(\hat{y}_i - \bar{y})$$

$$= \sum \left[\hat{y}_i (y_i - \hat{y}_i) - \bar{y}(y_i - \hat{y}_i) \right]$$

$$= \sum \hat{y}_i u_i - \sum \bar{y} u_i \neq 0$$

since $\hat{y}_i$ doesn't intercept with $\bar{y}$, the summation can't be zero.