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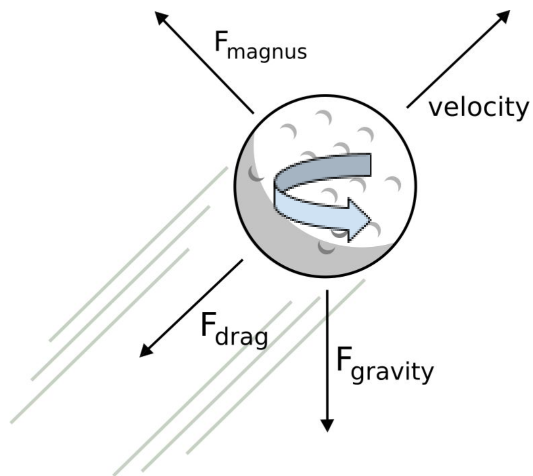
Golf Ball Projectile Motion

Introduction

The purpose of our term project was to determine the optimum angle to strike a golf ball for maximum range. We decided to analyze the motion of the golf ball as, using the kinematics equations with gravity as a constant acceleration, the hypothesized optimum angle would be 45 degrees. This is not the case under real-life situations as most shots from professional golfers don't exceed over 15 degrees, but still travel very far. We wanted to determine what causes this dramatic decrease for the optimum angle, by factoring in more real-life situations to the golf ball such as Drag Force and Magnus Effect. By varying the initial angle of the ball, we can determine the greatest distance and then compare it to the distance of the ball with an initial angle of 45 degrees. This will demonstrate the effects of a non-constant acceleration on a spinning projectile. We hypothesize that the maximum range will occur at an angle between 10 and 13 degrees due to the indirect impact of angle on drag and Magnus effect. Drag and Magnus depend on the velocity which will differ depending on the angle used; Vx and Vy vary with the angle. A larger angle means a greater Drag force and greater Magnus force. A smaller angle would reduce the effects of the two forces allowing the ball to fly farther.

Model

The motion of the golf ball is divided into three parts: the acceleration, the velocity and its position. The golf ball is shot with an initial linear velocity V, an angular velocity ω, and an initial angle θ. This creates two forces on the ball including the force of gravity which is Drag and Magnus. The Drag force always acts opposite to the direction of motion and the Magnus acts perpendicular to the direction of the angular velocity and linear velocity of the ball.



The Drag Force is directly proportional to the velocity of the ball and is given by the equation:

$$Fd = Cd\,\rho\,A\,V^2$$

$$Fd_x = Fd\,cos(\theta)$$

$$Fd_y = Fd\,sin(\theta)$$

- *Equation obtained from Computational Physics 2nd Edition (p 45)*

Where Cd is equal to the drag coefficient of a sphere which will be approximated at 0.5 (Computational Physics 45), and will remain constant during the motion. Another assumption made will be that ρ, the density of the fluid (air), will remain constant as well, at a pressure of 1.00 atm and temperature of 25℃, and equal to 1.184 kg/m^3 (*EngineersEdge).* The cross-sectional area of the Golf Ball will be calculated based on the average diameter of the golf ball which is equal to 42.7mm (0.0427m) (Werner 3), therefore the area will be estimated at $0.00143m^2$.

The Magnus effect of the ball is equal to the cross product of the angular velocity and linear velocity of the golf ball and is given by the equation:

$$Fm = So\,(\omega \times V)$$

- *Equation obtained from Berglund and Street (p 9)*

Where So is equal to the Magnus coefficient of the ball. However, we will be making an approximation that the angular velocity (ω) remains constant during the motion of the ball and its direction is perpendicular to the 'x' and 'y' plane of the ball. This will assume a nearly perfect shot, ignoring outcomes like hook and slice, in which the ball curves due to the curve of the shot. It will then be approximated that So = (Soω)/mass = $0.22s^{-1}$ (Based on Computational Physics 45 and Werner 12). After doing the cross product with the components of the velocities, the components of the Magnus force are equal to:

So after doing the cross product, the components of the Magnus force are equal to:

$$Fm_x = So\,(-V_y)$$

$$Fm_y = So\,(V_x)$$

Reminder: the 'x' and 'y' components of the velocity will depend on the angle.

The angle of the velocity vector with respect to the horizontal will change depending on the position of the ball during its flight.

The resultant forces acting on the golf ball in component form is then represented by:

$$Fx = ma_x = -Fd_x + Fm_x$$

$$Fy = ma_y = -Fd_y + Fm_y - mg$$

After dividing by the mass we get the set of differential equations we will have to solve for which are:

$$\frac{dVx}{dt} = -\frac{Fd_x}{m} + \frac{Fm_x}{m}$$

$$\frac{dVy}{dt} = -\frac{Fd_y}{m} + \frac{Fm_y}{m} - g$$

- *The mass of the ball will be 0.04593 kg (Arif, Elert)*

Numerical Method

To solve these differential equations, we will use Euler's method of numerical analysis, to determine the velocity and position of the ball at a given time. Euler's method employs a function that changes with time, in our case, the acceleration of the ball which changes throughout its motion. Euler's involves taking very small increments of time (dt) and evaluating the function at that instance. The function for the acceleration is the derivative of the velocity of the ball; to determine the velocity at a given time, the previous velocity of the ball will be added to the value of the acceleration function at that time multiplied by the small time interval. In the same manner, Euler will be applied with the velocity of the ball and multiplied by a small time interval to determine the position. This gives the equations:

$$Vx_{i+1} = Vx_i + a_x * dt$$

$$Vy_{i+1} = Vy_i + a_y * dt$$

$$x_{i+1} = x_i + Vx_i * dt$$

$$y_{i+1} = y_i + Vy_i * dt$$

- *Equation taken from notes by Jean-François Brière*

With the components of the velocity after a given time, we apply the Pythagorean theorem to determine the magnitude of the velocity vector. Then apply arctan of the y component of the velocity over the x-component of the velocity to determine the new angle with respect to the horizontal.

$$V^2 = Vx^2 + Vy^2$$

$$\theta = tan^{-1}(Vy/Vx)$$

Verlet's was also used after the optimum angle was found to plot the trajectory of the graphs of optimum angle and 45 degrees. Verlet's is used when solving for a value from a second order derivative, in the case of the golf ball, the acceleration is the second order derivative of the position. It is a more refined version of Euler's method but requires at least two values to be known which is why Euler's is applied in advance. The x and y positions of the ball will then be given by:

$$x_{i+1} = 2x_i - x_{i-1} + x_i * dt^2$$

$$y_{i+1} = 2y_i - y_{i-1} + y_i * dt^2$$

- *Equation taken from notes by Jean-François Brière*

The program will then run through a loop to determine the x and y positions until the y position is found to be less than zero. We will take the x-position at the same instance to be our final position, not factoring in bounce or rolling.

To solve for the optimum angle, a brute force method will be used. The program will run a loop incrementing the initial angle by 0.017453 (0.10 degrees) each time until it reaches 1.5708 (π/2). The final position will be compared with the final position of the previous loop and if the final position is larger, it will be saved as the new final position and the initial angle of that loop will be the new optimum angle. This method will also produce a graph of the final position of the ball over the initial angle. This will demonstrate the relationship we are examining where small angles produce greater distances.

Results

Once the program is run we are given the following message:

"The angle in degrees to launch the golf ball at for maximum distance is 22.600

The maximum distance travelled by the ball in meters is 101.177

The maximum distance travelled by the ball in meters when the angle is 45 degrees is 83.327"

All graphs shown were produced by the java program and are created at the same time once it is run.

Under a model with no resistance or Magnus effect, the expected result is to receive a maximum distance when a projectile is shot at 45 degrees. However, as the figures below demonstrate, under a more realistic model including these two effects a 45-degree initial angle is no the optimal solution. The program demonstrates that an angle of roughly 22.6 degrees results in the furthest distance. When shot at an initial velocity of 70 metres per second, the golf ball was launched at a distance of 101.177 metres, 17.85 metres farther than if it was launched at a 45-degree angle with the same initial speed.

*Figure 1: Left shows the trajectory of the golf ball at the optimum hitting angle with a maximum y distance ≈24.376 metres and a maximum x ≈*101.177*. Right shows the trajectory at a hitting angle of 45 degrees with a maximum y distance ≈48.0448 metres and a maximum x ≈*83.327.

- The optimal solution graph has fragments above the horizontal. These are a product of some array values not being reassigned resulting in this strange zig-zag like pattern in the graph. These fragments do not have any bearing on the rest of the values. A slight fix in the code gives us the wanted graph with removed fragments:

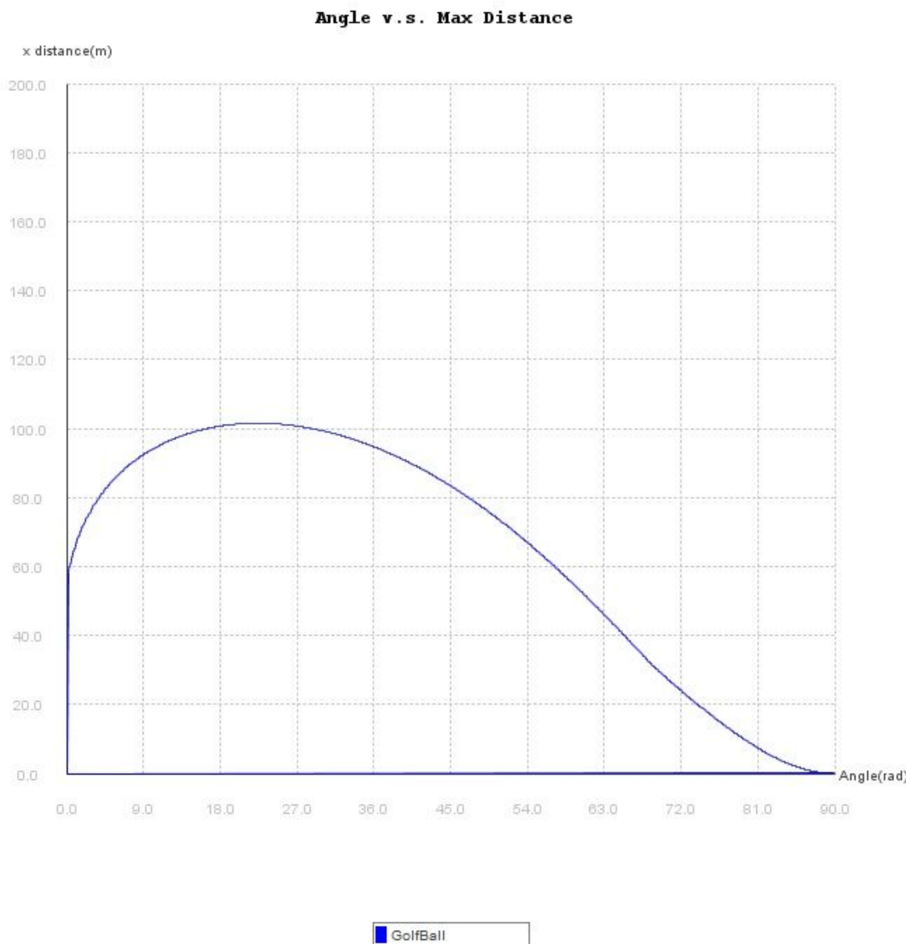


Figure 2: Demonstrates the initial angle, from 0 degrees to 90 degrees, and its correlation with its maximum distance.

This graph portrays the relationship the hitting angle has with maximum horizontal distance. The graph demonstrates that the maximum x distance happens between 19 and 27 degrees, the max point being (22.6,101.177).

Discussion

The values found were compared with the values for angle and distance from *Computational Physics* and the average angles and distances of balls in *Forbes*. With an initial speed of 70 m/s, the optimum angle from *Computational Physics* was found to be 9 degrees with a distance of 215m. The *Forbes* article details that the optimum angles fall under 10-12 degrees and travel nearly 200 yards or about 183 metres. The difference is due to the added effects of dimples on golf balls, which was neglected in this simulation. The added effects of dimples cause the ball to travel twice as far and for the optimum angle to be reduced by half. Our hypothesized optimal angle was based on a dimpled ball and therefore does not match with the simulations results. However, the graphs obtained follow the same almost parabolic motion with a slow increment at first and a more sudden drop after, like the scientific articles researched (Werner). This doesn't change with initial velocity either. Using the Figure 2 graph, a more refined method of optimization could have been used to determine the optimum angle as well, like the golden-section search to find the max value of the graph.

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