MATH 3250: Final Project

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Table of Contents

Table of Figures

Abstract

Kudzu is a menacing invasive species that overtakes through any stretch of vegetation and area that it is introduced to. With Kudzu's spectacular growth probabilities—through seed germination and dispersal, along with vegetative spread—the issue regarding this species is understanding the best way to efficiently and effectively eradicate it, but if not eradication, at least manage it. To eradicate the species, awareness of how it grows becomes the most important factor to attack it the right way. To undergo this process, a Leslie matrix for its seed germination or dispersal was developed along with an added model for proper harvesting levels of these seeds, a pre-existing competing species model was included to understand how the plant invades the native populations it surrounds and how a culling factor of the Kudzu species could affect that relationship, and a pre-existing exponential probability growth model to establish probabilities of how fast and how far this plant can grow in specific time frames. From these models and simulating some results, it was illustrated that it would take approximately 6.4% harvesting of the 3+ year old plants and their seeds in order to minimize seed germination and dispersal levels and potentially cause a decline in that type of growth. From the competing species model, we see that Kudzu grows more than twice as fast as the species it surrounds and it would take about 70-80% culling levels, over about 5 consistent periods to effectively see a decline in growth within the population, which makes management more of a possibility than full eradication. From the probabilistic growth model, given Kudzu's dense and rapid growth, Kudzu has much higher probabilities of growing densely in a small section of area before moving and spreading itself to vast distances in thin layers. Overall, under these circumstances, eradication is not as profitable, ecologically or economically, as culling is not very effective, chemical pesticides are harmful to people, animals, and surrounding vegetation, and harvesting the seeds alone does not get to the root of the problem. As a result, management of Kudzu and recycling these harvested plants could potentially solve this issue or make it more beneficial than harmful in terms of use as a biofuel, food, medicine, and even livestock feed over eradication itself, establishing a pathway for future studies and research to explore these ideas.

Introduction

Invasive species are very interesting in the world of biology, as they can completely uproot the harmony of an environment or population of native species simply by being different than them. Although this is an oversimplification of what invasive species are, it lays the groundwork of what science attempts to achieve: understanding the unknown. With interest in such an area of biology, a search of invasive species brought upon the amazing discovery of the dangerous, yet beautiful, Kudzu plant and developed the idea for the project ahead.

Figure 1: Kudzu plants spread over a vast expanse of vegetation

Kudzu—with Pueraria montana var. lobata being its scientific name—originated in China, Japan, and the Indian subcontinent, but was brought over to America in the late 1800s for many uses, such as erosion control, its general attractiveness—given its beautiful purple flowers that bloom in late summer (only on vertical growing vines, not on horizontal growing vines) (refer to Figure 2), and for livestock feed purposes [1]. Although using this plant for these purposes turned out to be a poor decision because in the blink of an eye it can destroy hundreds of acres of native vegetation simply by its spectacular growth properties, costing upwards of $500 million per year to deal with in this regard [4]. What makes Kudzu such a dangerous invasive species is based on how it grows in all directions, and establishes itself on pretty much any surface (refer to Figure 1)[1]. It grows almost one foot per day, mostly by vegetative spread simply with developing more nodes as it finds bare soil that it reaches along its war path and by growing through seed spread [1], and given how it is densely packed in its growth development, tens of thousands of plants can grow on a one-acre section of land [12]. Although, seeds from this invasive species cannot develop on its horizontal growing vines—only on its vertical growing vines--as they do not reach enough sunlight, but regardless, they are not as profitable in growth as they tend to only produce about 1 to 2 viable seeds that need to fall properly below the ground layer and grow effectively [1] along with sexual reproduction being rare in the invasive range as per Bentley et. al [6].

Figure 2: Kudzu flower

Beyond Kudzu’s main growth mechanisms, it has many more functions that improve its ability to grow so quickly and effectively. Kudzu leaves can shift their orientation to maximize water retention and photosynthesis which also assists their growth capabilities. Moreover, Kudzu populations are able to completely swallow acres of vegetation by covering them from receiving any sunlight and being able to cover whole trees stunting their grow or destroying them simply by the weight of the vines growing on them [1]. Additionally, these plants have nitrogen-fixing properties that allow them to grow in low nitrogen soils which simultaneously steals nutrients and resources that help other plants grow, further developing their destruction of local vegetation and boosting their growth abilities [1]. These and more functions will be discussed later in this paper, but these give a taste of how dangerous this plant can truly be.

Much like with any invasive species, the main goal is to combat them and allow local populations to thrive without the competition, but it is even more urgent and prominent with Kudzu because of how fast it spreads and what it does to every environment it invades. As a result, the goal of this paper is to understand how Kudzu grows and takes over in order to find the best methods of eradication based on environmental/biological, economic, and growth factors.

Model Formulation

Kudzu reproduces by two possible methods: vegetative (asexual) reproduction and seed germination. As stated above, seed germination has a low probability of producing new Kudzu plants given that so few seeds are made from the vertical growing vines and the fact that they may be dispersed or not dispersed effectively in fertile soil for their development. Given this, we can develop a Leslie matrix to model its growth over time as the seeds deposited in the seed bank below the vertical vines may take several years to germinate and vertical growing Kudzu vines do not flower until their third year, which makes this method of growth and spread have a low probability. On the other hand, Kudzu can grow asexually through vegetative spread at a rate of about 30cm per day, which is extremely fast and given that it can develop new nodes wherever there is bare soil, an exponential growth model could potentially be appropriate here, given that production rate is relatively constant and that that plants that exist live forever and don't die off, unless eradicated. Overall, a mixture of these two models will allow for a better understanding of ways that we can minimize that growth and possibly maximize eradication efforts.

LESLIE MATRIX FOR SEED GERMINATION

(BASED FROM [1])

Seed germination is not a significant factor in the growth of Kudzu, but it does attribute to its development and as a result, we will develop a model for it. A Leslie matrix seemed to be the appropriate method to display seed germination tendencies as it develops the plant's survival probabilities at specific age groups and the fecundity that they experience as well. Therefore, we will take the following approach:

Given that seeds are not formed until a Kudzu plant is in its third year of growth [1], we will split the matrix into 3 ages: age 1 plants, age 2 plants, and age 3+ plants (3 and older). Assuming no outside intervention, such as the use of pesticides or weeding (eradication methods), we develop these probabilities shown in the matrix below. Also, given that the probability of actually finding soil and growing properly due to where they fall or if certain animals disperse them is present, that will factor into the probability of survival as well [9]. Additionally, in a study done by Geerts et. al, only 9% of seeds actually germinated in germination trials, with a minimal number of seedlings actually found in the area(s) of study [9]. This prior information was included with the following information to complete the concept which developed the Leslie matrix as a whole: when a Kudzu plant produces seeds, they produce 3-10, but only 1-2 are viable and that is only after physical scarification, so the fecundity of those 3+ age plants is quite small and given the information from the article quoted above, let's say it is about 10%. Therefore, using the hypothetical maximized amount of seed production which includes the probability of physical scarification happening along with other environmental factors playing a role in the efficiency of seed dispersal and growth, we present the matrix below.

- Given these stipulations, a preliminary Leslie matrix could be:

0	**0**	**0.10**
0.90	**0**	**0**
0	**0.98**	**0.98**

The quotation below further explains how these probabilities render suitable values:

Despite Kudzu's minimal ability to produce viable seeds for sexual reproduction purposes, this is further established by the fact that in areas of South Africa, a study on seed dispersal found that seed spread is choked out by the lack of pollination, insect predation on seeds, and animal, or natural, dispersal methods. Typically, seed spread in these areas occurs by wind, water, and certain animals, but many of the animals in these areas—baboons, vervet monkeys, guinea fowls, and bird varieties—tend to avoid utilisation and dispersal of these Kudzu seeds, which again lessens the ability of seed production to occur in these Kudzu populations. This gives another dimension of explanation for this lack of seed production and Kudzu's ability or need to do so.

As a result, from studies done on South African populations of Kudzu which flower more often than in the USA, which shows how minimal seed germination attributes to Kudzu growth and I assume that the probability of survival at each age is not given that environmental and physical factors may play into them being destroyed, but it is not very likely given that they grow so efficiently and in many ways.

EXPONENTIAL GROWTH MODEL (ASEXUAL REPRODUCTION (VEGETATIVE SPREAD))

(BASED ON HORIZONTAL GROWTH)

Given the ability if Kudzu to grow over any stretch of vegetation or environment in which it is established [4], this traversal-type growth is where the heart of this plant's growth lies. Although these vines from the Kudzu plant can grow on vertical structures, we will simplify the models and simulations discussed to be focused upon the ground-level, or horizontal growth capabilities of this plant. In this regard, two separate models will be presented to deal with the complexity of the Kudzu plant's growth patterns, one based on its probability of growing over a certain distance over time as developed by Liu et al. [7] based on the growth tendencies of another invasive species by the name of B. rubostriata and revisited by Harron et. al [3]; the other is developed upon the idea of a competing species model [10] as Kudzu competes for survival against other plants and other types of vegetation that are native to the area in which the invasive Kudzu resides. The assumptions and notions under which these models came to fruition are as follows:

Assume we start with one Kudzu plant in the population (fully-grown adult—3+ years old). We take the growth rate to be given by the following: up to 30cm a day [5], and anywhere from 10 to 30m a season, with up to 30 vines growing from a single root [9] and once again, tens of thousands of these plants having the ability to thrive on only one-acre of land [12]. The reason that we are not dealing with a carrying capacity because Kudzu can grow in any environment and under almost any conditions while being able to outcompete the other species in the environment for energy and resources [3]. Furthermore, given how Kudzu competes with the plants around it for resources and space, or outcompetes them, we will incorporate competing species modelling into our overall modelling components. As a result, the two models and their stipulations follow below:

COMPETING SPECIES MODEL (based on models from Dobrushkin [10])

$$dx/dt = x(r_1 - a_1x - b_2y)$$

$$dy/dt = y(r_2 - a_2y - b_1x)$$

The parameters from our differential competing species equations are represented as follows:

- r_i = maximum per capita growth rate for species i
- a_i = intra-specific competition for species i
- b_i = inter-specific competition for species i
- x is the population of Kudzu plants in a certain stretch of area and y is the population of the other species in a certain stretch of area (given that Kudzu can grow in such vast proportions, it was decided that we should study the effects of this competition in a smaller scale or degree).
- We must note that anything denoted by the subscript "1", represents the Kudzu population and the subscript "2" represents the population of the competing species in the area.

- Further assumptions: 2 species in the area, we start with an adult Kudzu plant (3+ seasons old), Kudzu grows 10-30m per season, that the other species can grow asexually, and that Kudzu can outgrow almost any species it competes with.

Given that Kudzu is able to outgrow any vegetation it encounters, r_1 will be much higher than r_2, given its nitrogen fixing capacity, ability to grow in all directions, and the fact that it grows over anything in its path [3] [4]. Also, through Kudzu's growth capabilities, intra-specific and interspecific competition that affects it will not be as highly effectual as in other species, so those parameters will tend to have much lower values than b_2 (interspecific competition based on the side of the competing species in the area), but a_2 (intra-specific competition within the other species in the area) completely depends on the type of plant or vegetation which we are discussing. We must note though, that each parameter must be greater than zero—non-negative—given that growth rates and rates of competitiveness, biologically, cannot be negative as that does not makes any sense. Finally, understanding that this plant grows even more ferociously in the US and other places outside of its native area of growth gives further motivation to incorporate this competing species model into our modelling of this plant [9]. A sensitivity analysis for this model will be dealt with in the results section of this paper.

BASED ON PROBABILISTIC GROWTH MODEL

(from [3] in accordance with [7])

- $P(x) = exp^{(A-Bx)}$

The parameters from the model above are designated as follows:
P(x) represents the probability of growth over a certain time frame (five years in the study [3]), x represents the distance that Kudzu grows between two set locations (in meters), and A and B are parameters that match the probability of growth/spread over a certain distance in the given time frame.

This model was brought to my attention by Harron et al. [3] and they based this model off a five-year time frame, but I have decided to keep that time-frame more open and fluid in order to more closely study its patterns and how eradication methods can be intensified. I agree with the exponential growth model proposed for how Kudzu grows as its ferocious and nearly unstoppable growth tendencies allow us to assume that there is essentially no carrying capacity—or a very large one at the very least—but I have decided to improve it to develop the idea over different time periods keeping the location/distance measure consistent with this previous model. The parameters are said to be in-line with growth/spread tendencies, but we can easily link this to the competitive species model to figure out how to further establish these parameters, by adding in those environmental and other species variability in growth.

- A and B are growth parameters that are specifically based on the following idea from an article by Liu et. al:

"For simplicity purpose, we use a spread rate model with the following form to predict the invasion of *B. rubostriata*. Px=eA−Bx, where *P(x)* denotes the probability that *B. rubostriata* spreads from one parcel to another. Spread is assumed to be a function of the distance between the two parcels, *x*; *A* and *B* are parameters chosen to match the calculated probabilities with the observed speed of spread of *B. rubostriata.* Specifically, the values of *A* and *B* are chosen so that the probability of *B. rubostriata* spreading to a new parcel half a mile away in a year is equal to 0.9 while the probability of *B. rubostriata* spreading to a new parcel 10 miles away is 0.0005 (Cook, 2013). *B. rubostriata* becomes ubiquitous almost immediately after reaching a new area. Therefore, we do not model the intensity of *B. rubostriata* explicitly in our simulation." [7]

What this model attempts to do is to illustrate or allows us to understand how much growth we should expect in a certain time frame, which not only helps to predict growth tendencies, but is integral for eradication purposes in terms of funding, time, and type of method that will be necessary to manage this plant. Also, it can be manipulated based on specific factors, such as harvesting and the use of pesticides, to understand how growth can be stunted as well, or how well eradication efforts are doing based on expected growth from this model.

Overall, as Harron et. al generalized their model to the invasive species discussed above that has similar tendencies, we can generalize this model—for simplicity—to our case of Kudzu. This model will be tested later on using a sensitivity analysis, where we will figure out different parameter values and how they affect the model.

Results

Results for seed germination model

MATLAB code that determines the eigenvalues, eigenvectors and the stable age distribution can be found in the appendix.

From this MATLAB code output, we see that the largest eigenvalue is 1.0587, which means that even given the low fecundity of this plant, we see that its ability to grow vegetatively and survive amongst other species and in many conditions, allows its seed germination capabilities to thrive nonetheless. Furthermore, after finding the dominant eigenvector given by $(0.0937, 0.0797, 0.9924)^T$, which corresponds to the largest positive eigenvalue described above, we were able to find the stable age distribution which is given by $(0.0804, 0.,0684, 0.8512)^T$. With this stable age distribution, we can see that the eldest plants (age 3 and older) make up the majority of the population where Kudzu resides—more than 85% of it—which illustrates the fact that the most resilient plants of the species dominate the population which further develop the overall populations growth capabilities and how invasive they really are to surrounding vegetation and plants.

Harvesting in terms of seed germination

Despite seed germination and reproduction not being very profitable in comparison to vegetative spread in Kudzu populations, we could still develop a model upon which harvesting is included to show the possibility of the management of seed spread along with using this harvested section of plant for food or even medicinal purposes discussed in more detail in the discussion section.

> $harvest := \langle 0, 0.90, 0 | 0, 0, 0.98 | 0.10 - 0.10 \cdot h, 0, 0.98 - 0.98 \cdot h \rangle;$

$$harvest := \begin{bmatrix} 0 & 0 & 0.10 - 0.10\,h \\ 0.90 & 0 & 0 \\ 0 & 0.98 & 0.98 - 0.98\,h \end{bmatrix}$$

> $CP := CharacteristicPolynomial(harvest, \mathbf{lambda});$

$$CP := -0.98\,\lambda^2 + 0.98\,\lambda^2\,h + \lambda^3 - 0.088200 + 0.088200\,h$$

Figure 3: Harvesting matrix and characteristic polynomial function from seed germination model

Looking at figure 3 above, we see how the harvesting would be applied to 3+ year old plants and their fecundity (seed reproduction probability) given that affecting Kudzu plants of that age would be most effective since they are the strongest and most resilient of the plants in the population and help the most in growth overall, along with the fact that they are the ones that produce the seeds. From solving this matrix and characteristic polynomial, we got the result that the level of seed germination and the general plant population could potentially decline if we harvest about 6.5+% of them in each season, which will help to be used for beneficial purposes, but it develops the idea that harvesting may assist in regards to minimizing the seed reproduction tendencies of these Kudzu populations. Although, this matrix does not describe how this would affect vegetative spread and no data or parameters from outside sources were available, so this model of harvesting and the original matrix itself are simulated versions of what we could expect if we had undergone these processes in this manner. Nonetheless, this is a promising result in terms of eradication and management possibilities in future studies.

Results for competing species model

Maple code for determining the fixed points and stability of the differential equations given above for the competing species model can be found in the Appendix.

From the output of the Maple code, we see that the fixed points are given by the following:

fixedpts := {x = 0, y = 0}, {x = 0, y = r2/a2}, {x = r1/a1, y = 0}, {x = (-b1*r2 + r1*a2)/(a1*a2 - b1*b2), y = (-b2*r1 + r2*a1)/(a1*a2 - b1*b2)}

Interpreting these, we see that for the first fixed point, when the Kudzu population is at zero and the competing species population is also at zero which means that neither of the species exist in the area. When we find the eigenvalues for this first fixed point and solve for them, r1 and r2 must be negative for them to exist, which is not possible, so this point would not be stable regardless of its existence, and so if we introduced either species to the environment, they would grow away from the zero point, or that population of that plant would flourish without the notion of ever coming back to the non-existent level.

As for the second and third fixed points, we see that the equilibrium values occur in situations where one population exists and reaches its carrying capacity given that the other species does not exist within that area, which is what we expect to happen. After solving for the eigenvalues and their stability conditions, these points uphold stability when each parameter is greater than zero and when (r1*a2)/r2 < b1—for the second fixed point—and when (r2*a1)/r1 < b2—for the third fixed point. What these final conditions refer to is when the rate of growth of one species multiplied by the intraspecific competition rate in the other species divided by the growth rate of the second species is less than the interspecific competition established by the first species, which essentially means that when the ratio of the growth rates of the two species—scaling by intraspecific competition rates—is less than the intraspecific competition level of one or the other, than the populations will remain at this fixed point once they reach it.

Finally, for the fourth fixed point, Maple produced very complex eigenvalue solutions and was unable to provide output for the solutions to the differential equations—which establish the stability conditions—although when using a characteristic polynomial function to solve for those stability conditions, we see that it deals with a fluid situation that depends on the level of intraspecific and interspecific competition on the growth rates of the other species population, as shown by the Maple output in the Appendix of this report.

Sensitivity Analysis

Sensitivity analysis for the competing species model

Based on this equilibrium point: {x = (-b1*r2 + r1*a2)/(a1*a2 - b1*b2), y = (-b2*r1 + r2*a1)/(a1*a2 - b1*b2)}

As discussed prior, Kudzu is able to outcompete nearly any species it is fighting space for, and given this, we will set some stipulations on how these parameters interact with one another: r1 must be much greater than r2, b1 will be much greater than b2, a1 will be quite small simply due to the fact that Kudzu can grow almost anywhere and on anything which minimizes the intraspecific competition, and a2 is arbitrary depending on the competing plant. Also, each of these rates are dependent upon a certain time period, but in this case, we will be developing these rates upon per week growth rates and base them on 5m by 5m grids of growth. We must note, that Kudzu can grow up to 30cm per day, or 0.3m per day.

Let r1 = 0.5, r2 = 0.1, b1 = 0.8, b2 = 0.05, a1 = 0.05, a2 = 0.1:

Given these parameter values:

x = 0.857 and y = 0.571

These values show us that Kudzu grows 1.5 times as fast as the competing species in the area under this fixed point, which is what we expected.

As for the differential equations themselves, let's say that we have twice as many Kudzu plants in a 5m by 5m stretch of area as its relative competing species and we use the growth rates from above. Also, with these population proportions, r1 must be greater than (-a1*x – b1*y) in order to experience positive growth rates, and r2 must be greater than (-a2*y – b2*x) in order to also have a positive growth rate. Given these stipulations, we can plot how the competing species interact, by referring to figure 4 below:

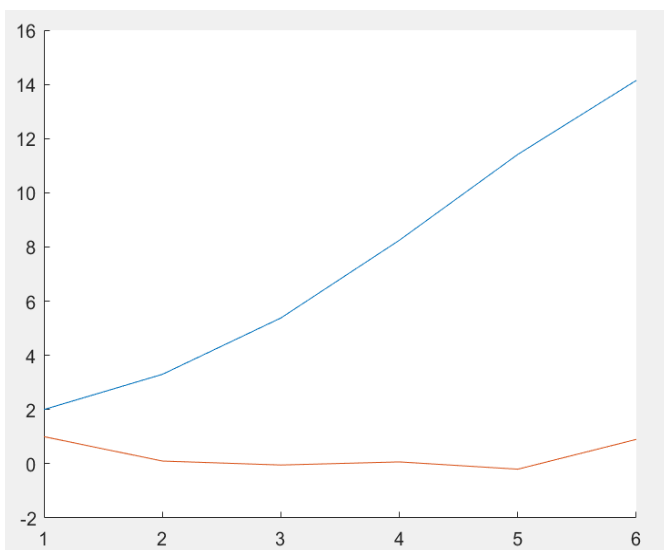

Figure 4: Kudzu population growth line (blue) and its competing species' growth line (orange) over 5 seasons or time periods

From figure 4 above, we model the growth of these two populations over 5 time periods and see how the Kudzu population vastly outgrows its competing species in that time frame, given the parameters described in the output which can be found in the Appendix under the

MATLAB code heading. With Kudzu's high growth rate and heavy interspecific competition, this is the result that we would expect to see in a typical Kudzu population as it invades a native population of vegetation. Therefore, this shows Kudzu's invasiveness without any human intervention, but we can also model how culling may affect it, as shown in figure 5 below, where many different levels of culling of the Kudzu plants are shown.

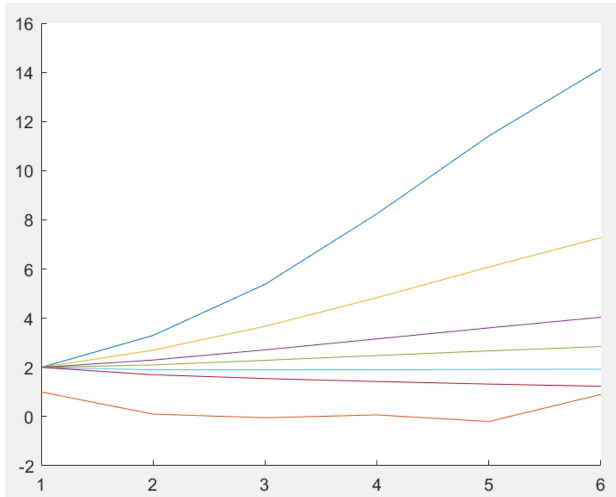

Figure 5: Kudzu populations with no culling (top dark blue line), 30% culling (yellow line), 50% culling (purple line), 60% culling (green line), 70% culling (light blue line), 80% culling (red line), and the original model line for the competing species (bottom orange line), over 5 seasons or time periods.

Figure 5 illustrates many different percentages, or levels, of culling these plants, starting from culling 30% of the Kudzu population, to culling 80% of it and how the growth is affected. From the plots, we see that culling does affect the growth rate of the Kudzu population, but only around 70-80% culling levels over 5 consistent seasons actually cause the population to start declining. Moreover, this is a positive result as it shows that eradication and management under the right conditions is possible, but the magnitude under which it has to be done may be too costly or take up too many resources, without even taking into account regrowth of the population and other possible factors affecting this process, such as how it would affect the surrounding vegetation and the methods to actually perform on the population to achieve this result. Therefore, culling may not be the best route of action for eradication, but it can help to manage the population and allow Kudzu to be used for other purposes, once again, discussed in following sections.

Sensitivity analysis for the probability growth model

From the Harron et. al paper on a case of study of Kudzu growth in Oklahoma, the parameters of A and B we stated to be set based on the probability of growth over a certain distance over a five-year period. The probability of growing 30m over a five-year time period was 90%, and the probability of growing 1610m (1 mile) is 0.05% [3]. Although, I am attempting to see how these same growth rates would fair in a shorter time frame, from year to year (proposed season) [1]. Also, we can use the fact that 1 to 2 plants most likely exist within a square foot of land [1]—which equals approximately 0.09 square meters—and in a 5m by 5m grid, we'll experience approximately 55 Kudzu plants (5/0.09 = 55) at full growth capacity. Also, in a separate article, it is stated that a Kudzu vine can grow up to 9.5-30.5m in a season [1]. Factoring in all of this information, let's assume that the probability of growing 30m in a season will be about 20% of what the five-year time period states, so:

P(distance|time-frame) = exp{A – Bx}

1) P(30m|one year) = 0.9*0.2 = 0.18 = 18%

2) P(1610m|one year) = 0.0005*0.2 = 0.0001 = 0.01%

We'll now use these values to set preliminary parameter values for A and B:

EQUATION 1 (x = 30)

0.18 = exp{A – B*(30)}

ln(0.18) = A – B*(30)

A = ln(0.18) + B*(30)

EQUATION 2 (x = 1610)

0.0001 = exp{A – B*(1610)}

ln(0.0001) = A – B*(1610)

B = (A – ln(0.0001))/1610

When we solve this system of equations, we get A = -1.57 and B = 0.00475, where we can use these values to see the probability that the Kudzu plant covers certain distances using our model based on a one-year time frame, as shown in figure 6.

For example, the probability of growing 1m to the probability of growing 1000m in a season would be shown by the following—coded in MATLAB:

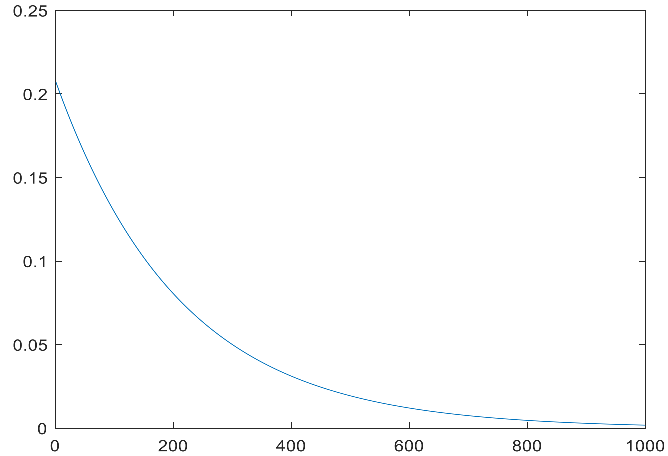

Figure 6: Probability growth model of growth over a season based on 1m to 1000m

Figure 3 displays the exponential growth tendencies of Kudzu in a different light, not based on how they grow, but in how the probability drops off very quickly as the distance from a central node increases. This further develops the idea of the densely-packed growth of Kudzu rather than Kudzu growing rapidly but in thin layers, which establishes that idea of it being able to overcrowd an area, cover and overhaul trees with how heavy and dense it can grow on them, and shadowing plants from sunlight, also displayed in figures 7, 8, and 9.

Kudzu growth simulations

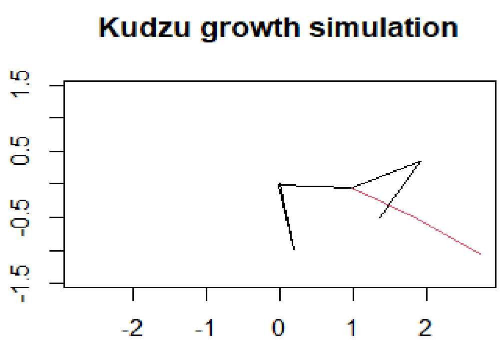

Figure 7: Kudzu growth simulation after 5 time periods

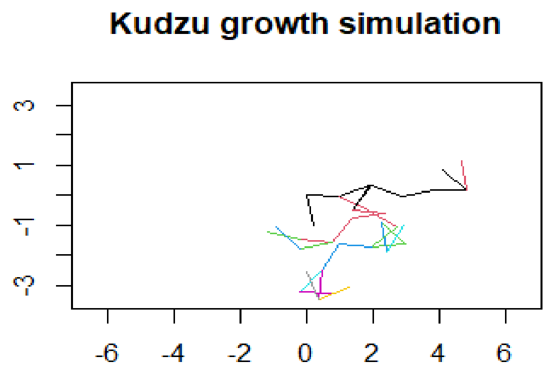

Figure 8: Kudzu growth simulation after 10 time periods

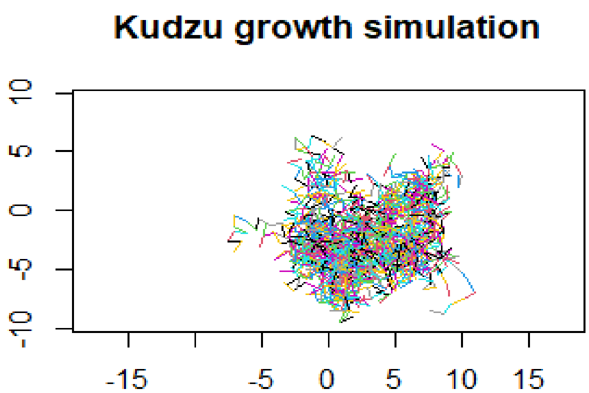

Figure 9: Kudzu growth simulation after 25 time periods

R code for these figures can be found in the Appendix of this paper.

These plots from figures 7, 8, and 9 represent what Kudzu growth would potentially look like, given the randomness in direction in which it could grow through the use of sinusoidal functions in the codes, and how Kudzu can become extremely dense and overtake certain areas of land in which it resides. Also, the different colours shown in each plot represent the multitude of vines that could possibly grow out of one root crown, up to 30 vines in some cases [5].

Discussion

From the models and results displayed above, Kudzu eradication is a more daunting task than many can believe, although there are options available with varying drawbacks. Frye et. al have developed a study that displayed the effects of leaf cutting and root crown drilling on Kudzu plants which had shown poor results in terms of killing the plants off. They used 50% and 75% levels of leaf-and-shoot clipping alongside root crown drilling, but that did not disturb the above-ground biomass at all, which leads to the idea that physical management may not be as profitable in eradication on its own, as one might hope, suggesting that chemical controls would be needed to supplement these bio-controls to possibly affect these Kudzu populations [13]. Although, in another paper by Frye et. al, only 75% leaf cutting was done and this resulted in shorter primary vines and reductions in internode distances, in which leaf damage can decrease the Kudzu vines' ability to maximize light exposure, along with internode distance reduction leading to possible decrease in Kudzu plants' ability to form roots at stem nodes, reducing the ability of it to vegetatively reproduce [14]. Even though eradication is not on the table in this regard, for management purposes, this is very promising, despite potentially having high costs to perform these processes. Moving forward with chemical controls, in a paper written by Dr. James H. Miller, he discusses different pesticides and spraying methods, and comes to a conclusion that Tordon 101 Mixture applied at two gallons per acre, or Tordon Kapplied at one gallon per acre using perpendicular passes along those infected Kudzu areas is the most cost-effective method to help with the eradication of Kudzu, but ultimately states that pesticides are harmful to surrounding vegetation, animals, and humans which develops more drawbacks then benefits to be able to be used effectively enough to rid an area of Kudzu [15]. Given these ideas, and the models of growth shown above, eradication of Kudzu may not be the best course of action to deal with it, but rather finding beneficial uses for it could help to manage its growth and make it useful given its abundance in nature through its growth capabilities.

Developing the idea of management of Kudzu and other uses for this abundantly growing plant, seem much more feasible as a way to deal with it as illustrated by our simulations in prior sections of this paper. Furthermore, with the result we reached upon the harvesting level in the sensitivity analysis of the seed germination model, we modelled when the seed germination levels and general levels of the Kudzu plant populations decline, which gives one the ability to undergo harvesting procedures with the expected results of managing it after harvesting only about 6.4% of the 3+ year-old plants and their seeds. With this in mind, not only can we manage the Kudzu populations through this, but this harvesting material can be used for many beneficial purposes to make Kudzu more useful than pest-like. In a study from Glass and Al-Hamdani on Kudzu as a source of livestock feed for ruminants—mostly cattle—they discussed how given its relatively good sources of crude protein, at a level of 24.46%, and a total digestible nutrient (TDN) level of 55.99%, Kudzu plants—the leaves more than the stems—are in the recommended and required ranges for ruminant feed by the National Research Council, which establishes this idea that harvested vines and leaves can be used in this manner, not only solving a management problem for Kudzu, but this is economically and ecologically beneficial overall [12]. Moreover, in a study done by Sage et. al, they found that the total non-structural carbohydrate levels (TNC) range from 20-68%, within Kudzu roots, and a range of 1-9 t ha^{-1} of fermentable carbohydrates in the form of glucose, sucrose and starch, which compete with levels of these values in sugar cane and maize for example [4]. In referring to this, these researchers found that because of these levels, Kudzu could potentially be a source of biofuel in the future, along with its uses in paper production, medicine, and in culinary aspects in many dishes in Asia [4][16]. With this in mind, Kudzu's immense growth ability and potential could make it an abundant source of biofuel if harvested in a cost-effective way and illustrates that this invasive species could be more beneficial rather than harmful and could solve fuel issues in many areas of business and production in our society and economy moving forward.

Overall, as we have modelled and discussed above, Kudzu is a very interesting invasive species and is quite troublesome in terms of eradication completely, but management is a definite possibility given what is shown in our models and especially through the harvesting seed analysis from the seed germination model. Given Kudzu's amazing growth abilities this plant illustrated by the models developed above, Kudzu populations can potentially be better understood and can purport future studies to establish it more as a helpful part of our society rather than as a destructive monster to vegetation and the people that surround it.

References

1)

"New Yorkinvasive Species (IS) Information." *New York Invasive Species Information*, http://nyis.info/invasive_species/kudzu/.

2)

Pueraria Montana Var. Lobata (Kudzu), https://www.cabi.org/isc/datasheet/45903.

3)

Harron, Paulina, et al. "Predicting Kudzu (Pueraria Montana) Spread and Its Economic Impacts in Timber Industry: A Case Study from Oklahoma." *PLOS ONE*, vol. 15, no. 3, 2020, https://doi.org/10.1371/journal.pone.0229835.

4)

Sage, Rowan F., et al. "Kudzu [Pueraria Montana (Lour.) Merr. Variety Lobata]: A New Source of Carbohydrate for Bioethanol Production." *Biomass and Bioenergy*, vol. 33, no. 1, 2009, pp. 57–61., https://doi.org/10.1016/j.biombioe.2008.04.011.

5)

"Kudzu." *Ontario.ca*, https://www.ontario.ca/page/kudzu.

6)

Bentley, Kerin E., and Rodney Mauricio. "High Degree of Clonal Reproduction and Lack of Large-Scale Geographic Patterning Mark the Introduced Range of the Invasive Vine, Kudzu (Pueraria Montana Var. Lobata), in North America." *American Journal of Botany*, vol. 103, no. 8, 2016, pp. 1499–1507., https://doi.org/10.3732/ajb.1500434.

7)

Liu, Lirong, and Brian Piper. “Predicting the Total Economic Impacts of Invasive Species: The Case of b. Rubostriata (Red Streaked Leafhopper).” *Ecological Economics*, vol. 128, 2016, pp. 139–146., https://doi.org/10.1016/j.ecolecon.2016.04.014.

8)

Yang, Xuefang, et al. “Differences in Growth and Herbivory Damage of Native and Invasive Kudzu (Peuraria Montana Var. Lobata) Populations Grown in the Native Range.” *Plant Ecology*, vol. 215, no. 3, 2014, pp. 339–346., https://doi.org/10.1007/s11258-014-0304-4.

9)

Geerts, Sjirk, et al. “Lack of Human-Assisted Dispersal Means Pueraria Montana Var. Lobata (Kudzu Vine) Could Still Be Eradicated from South Africa.” *Biological Invasions*, vol. 18, no. 11, 2016, pp. 3119–3126., https://doi.org/10.1007/s10530-016-1226-y.

10)

“Mathematica Tutorial under the Terms of the GNU General Public License (GPL) for the Second Course. Part III: Competing Species.” *MATHEMATICA Tutorial, Part 2.3: Competing Species*, https://www.cfm.brown.edu/people/dobrush/am34/Mathematica/ch3/species.html.

11)

A discussion about the R code used for the simulation of the Kudzu plant growth was undergone between Prof. Georges Monette and I in order for me to gain a better insight on it.

12)

Glass, Dylan, and Safaa Al-Hamdani. “Kudzu Forage Quality Evaluation as an Animal Feed Source.” *American Journal of Plant Sciences*, vol. 07, no. 04, 2016, pp. 702–7, https://doi.org/10.4236/ajps.2016.74063.

13)

Frye, Matthew J., et al. “Response of Kudzu (Pueraria Montana Var. Lobata) Seedlings and Naturalized Plants to Simulated Herbivory.” *Invasive Plant Science and Management*, vol. 5, no. 4, Dec. 2012, pp. 417–26, https://doi.org/10.1614/IPSM-D-12-00001.1.

14)

Frye, M. J., and J. Hough-Goldstein. “Plant Architecture and Growth Response of Kudzu (Fabales: Fabaceae) to Simulated Insect Herbivory.” *Environmental Entomology*, vol. 42, no. 5, Oct. 2013, pp. 936–41, https://doi.org/10.1603/en12270.

15)

Miller, James H. *Chapter 28 Kudzu Eradication and Management*. www.srs.fs.usda.gov/pubs/ja/ja_miller006.pdf. In: Hoots, Diane; Baidwin, Juan&a, camps., eds. 1996. Kudzu the vine to love or hate. Kodak, TN: Suntop Press: 137-149,.

16)

“Pueraria Montana (Kudzu) | North Carolina Extension Gardener Plant Toolbox.” *Plants.ces.ncsu.edu*, plants.ces.ncsu.edu/plants/pueraria-montana/?msclkid=320c445dad1311ec9c03c392b6d990cf. Accessed 23 Apr. 2022.

Appendix

<u>MATLAB</u>

<u>Code describing the seed germination model</u>

Input

```
% SEED DISPERSAL AND GROWTH MODELLING
seed = [0,0,0.10;0.9,0,0;0,0.98,0.98]
[V,D] = eig(seed)
% The largest positive eigengvalue from our "seed" matrix turned out to be
% 1.0587, which means that Kudzu, given these survival probabilities and
% probability of seed germination, the population will in fact grow and not
% decay given seed germination or seed growth capabilities.

% To show the stable age distribution:

% Given that D(:,3) gives us our largest positive eigenvalue, we will
% normalize V(:,3) to find the stable age distribution of these Kudzu
% plants (divide the dominant vector by the sum of its parts):

stable_age_kudzu = V(:,3)/sum(V(:,3))
% The vector given shows that more than 85% of the population of Kudzu in a
% given area will be dominated by plants that are age 3 or older, which
% makes sense as those would be the plants that are ready to develop seed
% and so they will have develop strength in their stalks and root crowns,
% while having many vines grown out of them up to that point which protects
% them from many environmental factors and such.
```

Output

MATLAB

```
MATH_3250_PROJECT_MODEL_SEED

seed =

         0         0    0.1000
    0.9000         0         0
         0    0.9800    0.9800


V =

  -0.0312 + 0.2270i  -0.0312 - 0.2270i   0.0937 + 0.0000i
   0.7143 + 0.0000i   0.7143 + 0.0000i   0.0797 + 0.0000i
  -0.6367 - 0.1786i  -0.6367 + 0.1786i   0.9924 + 0.0000i


D =

  -0.0393 + 0.2859i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0393 - 0.2859i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0587 + 0.0000i


stable_age_kudzu =

    0.0804
    0.0684
    0.8512
```

Code describing plot for probabilistic growth model

```
% Growth over one season (time period)
A = -1.57
B = 0.00475
for i = 1:1000
P(i) = exp(A - B*i)
end
plot(P)
```

Code describing plot for competing species model (no culling)

```
%% Plotting the competing species model %%
%kudzu = x*(r1 - a1*x - b1*y)
%comp = y*(r2 - a2*y - b2*x)
% Kudzu is represented by the blue line
clear;
x(1) = 2;
y(1) = 1;

r1 = [0,0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2];
```

```
r2 = [0,0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2];
a1 = [0,0.01,0.02,0.03,0.04,0.05,0.06,0.07,0.08,0.09,0.1];
a2 = [0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1];
b2 = [0.5,0.6,0.7,0.8,0.9,1,1.1,1.2,1.3,1.4,1.5];
b1 = [0.05,0.1,0.15,0.2,0.25,0.3,0.35,0.4, 0.45,0.5];
% kudzu(1) = 2
% comp(1) = 1
% Given r1 = 1.2, r2 = 0.4, a1 = 0.05, b1 = 0.8, a2 = 0.1, and b2 = 0.05

for i = 1:5
    x(i + 1) = x(i)*(r1(10) - a1(6)*x(i) - b1(1)*y(i))
    y(i + 1) = y(i)*(r2(7) - a2(2)*y(i) - b2(1)*x(i))
    % x(i + 1) = x(i) + kudzu(i)
    % y(i + 1) = y(i) + comp(i)
end
i = [1:6]
hold on
plot(i,x)
plot(i,y)
hold off
```

Code describing plot for competing species model (with added Kudzu culling term)

(**Note: This code was plotted over the original one for the following culling terms (rewriting the code and running it again): 0.3*x(i), 0.5*x(i), 0.6*x(i), 0.7*x(i), and 0.8*x(i))**

```
%% Plotting the same model but with a culling term for Kudzu %%
%kudzu = x*(r1 - a1*x - b1*y)
%comp = y*(r2 - a2*y - b2*x)
% Kudzu is represented by the blue line
clear;
x(1) = 2;
y(1) = 1;

r1 = [0,0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2];
r2 = [0,0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2];
a1 = [0,0.01,0.02,0.03,0.04,0.05,0.06,0.07,0.08,0.09,0.1];
a2 = [0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1];
b2 = [0.5,0.6,0.7,0.8,0.9,1,1.1,1.2,1.3,1.4,1.5];
b1 = [0.05,0.1,0.15,0.2,0.25,0.3,0.35,0.4, 0.45,0.5];
% kudzu(1) = 2
% comp(1) = 1
% Given r1 = 2, r2 = 1.2, a1 = 0.05, b1 = 0.05, a2 = 0.1, and b2 = 0.5
% The plot will show 30-80% harvesting levels, but the final code will only
% show the 80% culling term (-0.8*x(i)) which can be changed to anything
% necessary for modelling in any case.
for i = 1:5
    x(i + 1) = x(i)*(r1(10) - a1(6)*x(i) - b1(1)*y(i)) - 0.8*x(i)
    y(i + 1) = y(i)*(r2(7) - a2(2)*y(i) - b2(1)*x(i))
    % x(i + 1) = x(i) + kudzu(i)
    % y(i + 1) = y(i) + comp(i)
end
i = [1:6]
hold on
plot(i,x)
```

```
% plot(i,y)
hold off
```

MAPLE

Competing species model codes

```
> restart:
> with(LinearAlgebra):
> eq1 := x·(r1 − a1·x − b1·y);
```

$$eq1 := x\,(-a1\,x - b1\,y + r1) \quad \textbf{(1)}$$

```
> eq2 := y·(r2 − a2·y − b2·x);
```

$$eq2 := y\,(-a2\,y - b2\,x + r2) \quad \textbf{(2)}$$

```
> fixedpts := solve({eq1 = 0, eq2 = 0}, {x, y});
```

$$fixedpts := \{x = 0, y = 0\}, \left\{x = 0, y = \frac{r2}{a2}\right\}, \left\{x = \frac{r1}{a1}, y = 0\right\}, \left\{x = \frac{a2\,r1 - b1\,r2}{a1\,a2 - b1\,b2}, y = \frac{r2\,a1 - b2\,r1}{a1\,a2 - b1\,b2}\right\} \quad \textbf{(3)}$$

```
> J := Matrix(2, 2, [[diff(eq1, x), diff(eq1, y)], [diff(eq2, x), diff(eq2, y)]]);
```

$$J := \begin{bmatrix} -2\,a1\,x - b1\,y + r1 & -x\,b1 \\ -y\,b2 & -2\,a2\,y - b2\,x + r2 \end{bmatrix} \quad \textbf{(4)}$$

```
> J00 := subs(fixedpts[1], J);
```

$$J00 := \begin{bmatrix} r1 & 0 \\ 0 & r2 \end{bmatrix} \quad \textbf{(5)}$$

```
> eigval00 := Eigenvalues(J00);
```

$$eigval00 := \begin{bmatrix} r2 \\ r1 \end{bmatrix} \quad \textbf{(6)}$$

```
> solve({eigval00[1] < 0, eigval00[2] < 0});
```

$$\{r1 < 0, r2 < 0\} \quad \textbf{(7)}$$

```
> J01 := subs(fixedpts[2], J);
```

$$J01 := \begin{bmatrix} -\frac{b1\,r2}{a2} + r1 & 0 \\ -\frac{r2\,b2}{} & -r2 \end{bmatrix} \quad \textbf{(8)}$$

$$J01 := \begin{bmatrix} -\frac{r2\,b2}{a2} & -r2 \end{bmatrix} \quad (8)$$

> eigval01 := Eigenvalues(J01);

$$eigval01 := \begin{bmatrix} -r2 \\ \frac{r1\,a2 - b1\,r2}{a2} \end{bmatrix} \quad (9)$$

> solve({eigval01[1] < 0, eigval01[2] < 0, r1 > 0, a1 > 0, b1 > 0, r2 > 0, a2 > 0, b2 > 0});

$$\left\{0 < a1, 0 < a2, 0 < b2, 0 < r1, 0 < r2, \frac{r1\,a2}{r2} < b1\right\} \quad (10)$$

> J10 := subs(fixedpts[3], J);

$$J10 := \begin{bmatrix} -r1 & -\frac{r1\,b1}{a1} \\ 0 & -\frac{b2\,r1}{a1} + r2 \end{bmatrix} \quad (11)$$

> eigval10 := Eigenvalues(J10);

$$eigval10 := \begin{bmatrix} -r1 \\ \frac{r2\,a1 - b2\,r1}{a1} \end{bmatrix} \quad (12)$$

> solve({eigval10[1] < 0, eigval10[2] < 0, r1 > 0, a1 > 0, b1 > 0, r2 > 0, a2 > 0, b2 > 0});

$$\left\{0 < a1, 0 < a2, 0 < b1, 0 < r1, 0 < r2, \frac{r2\,a1}{r1} < b2\right\} \quad (13)$$

> J11 := subs(fixedpts[4], J);

$$J11 := \left[\left[-\frac{2\,a1\,(-b1\,r2 + r1\,a2)}{a1\,a2 - b1\,b2} - \frac{b1\,(-b2\,r1 + r2\,a1)}{a1\,a2 - b1\,b2} + r1, -\frac{(-b1\,r2 + r1\,a2)\,b1}{a1\,a2 - b1\,b2}\right],\right.$$
$$\left.\left[-\frac{(-b2\,r1 + r2\,a1)\,b2}{a1\,a2 - b1\,b2}, -\frac{2\,a2\,(-b2\,r1 + r2\,a1)}{a1\,a2 - b1\,b2} - \frac{b2\,(-b1\,r2 + r1\,a2)}{a1\,a2 - b1\,b2} + r2\right]\right] \quad (14)$$

> eigval10 := Eigenvalues(J10);

$$eigval10 := \begin{bmatrix} -r1 \\ \frac{r2\,a1 - b2\,r1}{a1} \end{bmatrix} \quad (12)$$

> solve({eigval10[1] < 0, eigval10[2] < 0, r1 > 0, a1 > 0, b1 > 0, r2 > 0, a2 > 0, b2 > 0});

$$\left\{0 < a1,\ 0 < a2,\ 0 < b1,\ 0 < r1,\ 0 < r2,\ \frac{r2\,a1}{r1} < b2\right\} \quad (13)$$

> J11 := subs(fixedpts[4], J);

$$J11 := \left[\left[-\frac{2\,a1\,(-b1\,r2 + r1\,a2)}{a1\,a2 - b1\,b2} - \frac{b1\,(-b2\,r1 + r2\,a1)}{a1\,a2 - b1\,b2} + r1,\ -\frac{(-b1\,r2 + r1\,a2)\,b1}{a1\,a2 - b1\,b2}\right],\right. \quad (14)$$
$$\left.\left[-\frac{(-b2\,r1 + r2\,a1)\,b2}{a1\,a2 - b1\,b2},\ -\frac{2\,a2\,(-b2\,r1 + r2\,a1)}{a1\,a2 - b1\,b2} - \frac{b2\,(-b1\,r2 + r1\,a2)}{a1\,a2 - b1\,b2} + r2\right]\right]$$

> # eigval11 := Eigenvalues(J11);

> # solve({eigval11[1] < 0, eigval11[2] < 0, r1 > 0, a1 > 0, b1 > 0, r2 > 0, a2 > 0, b2 > 0});

> *# The method of solving for the stability conditions from eigenvalues and eigenvectors turned out to not be able to provide a solution, and so the Charatersitic Polynomial was used instead below:*

> CharacteristicPolynomial(J11, **lambda**);

$$\lambda^2 + \frac{(a1\,r1\,a2 + a2\,r2\,a1 - a1\,b1\,r2 - a2\,b2\,r1)\,\lambda}{a1\,a2 - b1\,b2} + \frac{(-b2\,r1 + r2\,a1)\,(-b1\,r2 + r1\,a2)}{a1\,a2 - b1\,b2} \quad (15)$$

> solve({ (a1·r1·a2 + a2·r2·a1 − a1·b1·r2 − a2·b2·r1)/(a1·a2 − b1·b2) > 0, (−b2·r1 + r2·a1)(−b1·r2 + r1·a2)/(a1·a2 − b1·b2) > 0, r1 > 0, a1 > 0, b1 > 0, r2 > 0, a2 > 0, b2 > 0, a2·r1 > b1·r2, a1·a2 > b1·b2, r2·a1 > b2·r1 });

$$\left\{0 < a2,\ 0 < b1,\ 0 < b2,\ 0 < r2,\ r1 < \frac{r2\,a1}{b2},\ \frac{b1\,b2}{a2} < a1,\ \frac{b1\,r2}{a2} < r1\right\} \quad (16)$$

Seed germination model codes (harvesting)

```
> restart:
> with(LinearAlgebra):
> seed := ⟨0, 0.90, 0|0, 0, 0.98|0.10, 0, 0.98⟩;
```

$$seed := \begin{bmatrix} 0 & 0 & 0.10 \\ 0.90 & 0 & 0 \\ 0 & 0.98 & 0.98 \end{bmatrix} \quad (1)$$

```
> Eigenvalues(seed);
```

$$\begin{bmatrix} -0.0393459005541615 + 0.285941016220846\,I \\ -0.0393459005541615 - 0.285941016220846\,I \\ 1.05869180110832 + 0.\,I \end{bmatrix} \quad (2)$$

```
> # This vector of eigenvalues shows that this population is growing given that its largest positive
    eigenvalue is larger than 1, so harvesting could be a possible solution to slowing down this growth.
> # Now we are going to figure out the amount that we can harvest of this plant population in order to
    manage it or slow its growth.  Here we assume that we are only harvesting the 3+ year old plants, as
    they are the ones that produce the seeds and are the important plants in the growth tendencies of the
    whole population of Kudzu plants (by harvesting the plant, we are reducing its fecundity and its
    survival probability):
> harvest := ⟨0, 0.90, 0|0, 0, 0.98|0.10 - 0.10·h, 0, 0.98 - 0.98·h⟩;
```

$$harvest := \begin{bmatrix} 0 & 0 & 0.10 - 0.10\,h \\ 0.90 & 0 & 0 \\ 0 & 0.98 & 0.98 - 0.98\,h \end{bmatrix} \quad (3)$$

```
> CP := CharacteristicPolynomial(harvest, lambda);
```

$$CP := -0.98\,\lambda^2 + 0.98\,\lambda^2\,h + \lambda^3 - 0.088200 + 0.088200\,h \quad (4)$$

```
> # The level at which the population is growing or not declining is given when the dominant eigenvalue is
    greater than or equal to 1, as a result, we are going to solve for the portion, h, that we can harvest this
    population to see what it'll take to cause it to decline:
> solve(subs(lambda = 1, CP) = 0, h);
```

$$0.06384572177 \quad (5)$$

```
> # Therefore, this result shows us that we could potentially halt seed reproduction and minimize some of
    the Kudzu growth by harvesting just over 6.4% of 3+ year old Kudzu plants within a Kudzu
    population, and that allows us to understand that one type of growth can be managed in this situation.
    Also, it gives us an idea of how much we could harvest for food or medicinal purposes before we slow
    down the population growth altogether.  To note, this matrix is simply a simulation of what we might
    expect without the use of any real data or paraemters, as they were not available for this study, but
    this shows promising resutls nonetheless.
```

R [11]

Input

#' Kudzu grows in all directions and can grow up to 30 cm per day, with up

#' to around 30 vines growing from a single root crown. Kudzu's traversal

#' growth ability allows it to grow horizontally across vast areas of land

#' finding new areas of soil to develop new roots in.

#' %%%%%%%%%%%(https://www.ontario.ca/page/kudzu)%%%%%%%%%%%%%%%

#'

#' We will try to model it's horizontal growth first with a spatial, or

#' lattice graphical, model and then move onto how it grows vertically.

#'

#'

#' Creating the simulation for horizontal growth:

Matrix starting with zeros

Poisson distribution (changing parameter)

```
grow <- function(plant = list(), nbuds = .1){
if(length(plant) == 0) return(list(matrix(0,ncol = 4, nrow = 1)))
  buds <- plant[[length(plant)]]
  newbuds <- matrix(0, ncol = 4, nrow = 0)
  for(bud in 1:nrow(buds)){
    new <- 1 + rpois(1, nbuds)
    for(newbud in 1:new){
      # dir <- rbind(c(-1,0), c(0,-1), c(1,0), c(0,1))[sample(1:4,1),]
      theta <- 2*pi*runif(1)
      dir <- c(cos(theta), sin(theta))
      newbuds <- rbind(newbuds, c(buds[bud,c(3,4)], buds[bud,c(3,4)] + dir))
    }
  }
  c(plant, list(newbuds))
}
```

```
plot.plant <- function(plant){
  extreme_x <- max(sapply(plant, function(mat){max(abs(mat[,3]))}))
  extreme_y <- max(sapply(plant, function(mat){max(abs(mat[,4]))}))
  plot(c(-extreme_x,extreme_x), c(-extreme_y,extreme_y), type = "n", asp = 1,
       xlab = NA ,ylab = NA, main = "Kudzu growth simulation")
  for(i in 1:length(plant)){
    mat <- plant[[i]]
    for(i in 1:nrow(mat)){
      xs <- mat[i,c(1,3)]
      ys <- mat[i,c(2,4)]
      lines(xs,ys, col = i)
    }
  }
}

plant <- grow() # initializes the plot
n = 5 # n can be any number of iterations, it is just five here
for(i in 1:n){
plant <- grow(plant, .3)
}
plot.plant(plant)
```

Output

Refer to “Kudzu growth simulations” section above.