0.1 Prologue

Below, I make reference to the following free lecture notes. If you feel you are missing some prerequisites, everything is in these lecture notes.

http://www.physique.usherbrooke.ca/tremblay/cours/phy-892/N-corps.pdf

Many of these lectures are on YouTube

https://www.youtube.com/playlist?list=PL9IKDS79pLpNJS9KLrAZU0zw_Tin4E6ed

0.2 Lecture 1 (30 minutes) Second quantization

Chapter 81 : Handeling many-interacting particles : Second quantization

81.1 Fock space : Creation-annihilation operators

Number operator

81.2 Change of basis

87.2.1 Position and momentum basis

87.2.2 Wave functions

81.3 One-body operators

81.4 Two-body operators

0.3 Lecture 2 (45 minutes) Time-ordered product, Green functions

Chapter 83 Perturbation theory (interaction representation)

$$e^{-\beta\widehat{K}} = e^{-\beta\widehat{K}_0}\widehat{U}(\beta) \tag{1}$$

$$\widehat{U}(\beta) \equiv T_\tau\left[e^{-\int_0^\beta \widehat{K}_1(\tau)d\tau}\right] \tag{2}$$

$$\widehat{K}_1(\tau) \equiv e^{\widehat{K}_0\tau}\widehat{K}_1 e^{-\widehat{K}_0\tau}. \tag{3}$$

Chapter 29 Matsubara Green's function

84.1 Photoemission and fermion correlation functions

$$\frac{\partial^2\sigma}{\partial\Omega\partial\omega} \propto \sum_{mn} e^{-\beta K_m}\langle m|\, c^\dagger_{\mathbf{k}_{||}}\, |n\rangle \langle n|\, c_{\mathbf{k}_{||}}\, |m\rangle\, \delta\left(\omega - (K_m - K_n)\right) \tag{4}$$

29.1 Definition of the Matsubara Green function

$$\mathcal{G}_{\alpha\beta}(\tau) = -\left\langle T_\tau c_\alpha(\tau)\, c^\dagger_\beta(0)\right\rangle \tag{5}$$

$$= \quad -\left\langle c_\alpha(\tau)\, c^\dagger_\beta(0)\right\rangle\theta(\tau) + \left\langle c^\dagger_\beta(0)\, c_\alpha(\tau)\right\rangle\theta(-\tau). \tag{6}$$

29.3 Antiperiodicity and Fournier expansion

$$\mathcal{G}_{\alpha\beta}(ik_n) = \int_0^\beta d\tau e^{ik_n\tau}\mathcal{G}_{\alpha\beta}(\tau) \tag{7}$$

$$\mathcal{G}_{\alpha\beta}(\tau) = T\sum_n e^{-ik_n\tau}\mathcal{G}_{\alpha\beta}(ik_n) \tag{8}$$

29.5 Lehman representation

$$\mathcal{G}_{\mathbf{k}}(ik_n) = \int \frac{d\omega}{2\pi}\frac{A_{\mathbf{k}}(\omega)}{ik_n - \omega} \tag{9}$$

$$A_{\mathbf{k}}(\omega) \equiv \sum_{n,m}\frac{1}{Z}\left(e^{-\beta K_m} + e^{-\beta K_n}\right)\langle n| c_{\mathbf{k}} |m\rangle \langle m| c_{\mathbf{k}}^\dagger |n\rangle 2\pi\delta(\omega - (K_m - K_n)) \tag{10}$$

29.8 Green function for $U = 0$

$$\mathcal{G}_{\mathbf{k}}(ik_n) = \frac{1}{ik_n - \zeta_{\mathbf{k}}} \tag{11}$$

29.2 Time-ordering operator in practice

$$\left\langle T_\tau \psi(\tau_1)\psi^\dagger(\tau_3)\psi(\tau_2)\psi^\dagger(\tau_4)\right\rangle = -\left\langle T_\tau \psi^\dagger(\tau_3)\psi(\tau_1)\psi(\tau_2)\psi^\dagger(\tau_4)\right\rangle \tag{12}$$

0.4 Lecture 3 (45 minutes) Spectral weight, Self-energy, Quasiparticles

84.4 Spectral weight and how it is related to $\mathcal{G}_{\mathbf{k}}(ik_n)$ and to photoemission

$$\frac{\partial^2\sigma}{\partial\Omega\partial\omega} \propto A_{\mathbf{k}}(\omega) f(\omega) \tag{13}$$

29.6 Obtaining the spectral weight from $\mathcal{G}_{\mathbf{k}}(ik_n)$, the problem of analytic continuation

$$\mathcal{G}_{\mathbf{k}}(ik_n) = \int \frac{d\omega'}{2\pi}\frac{A_{\mathbf{k}}(\omega')}{ik_n - \omega'} \tag{14}$$

$$G^R_{\mathbf{k}}(\omega) = \int \frac{d\omega'}{2\pi}\frac{A_{\mathbf{k}}(\omega')}{\omega + i\eta - \omega'} \tag{15}$$

The notion of self-energy, what it means, what it hides
(20 minutes) Chapter 17 Self-energy

$$A(\mathbf{k};\omega') = \frac{2\Gamma}{(\omega - \widetilde{\varepsilon}_{\mathbf{k}})^2 + \Gamma^2} \tag{16}$$

$$G^R(\mathbf{k},\omega) = \frac{1}{\omega - \widetilde{\varepsilon}_{\mathbf{k}} + i\Gamma}. \tag{17}$$

$$G^R(\mathbf{k},\omega) = \frac{1}{\omega + i\eta - \varepsilon_{\mathbf{k}} - \Sigma^R(\mathbf{k},\omega)} = \frac{1}{G_0^R(\mathbf{k},\omega)^{-1} - \Sigma^R(\mathbf{k},\omega)}. \tag{18}$$

With the simple approximation that we did for the self-energy,

$$\Sigma^R(\mathbf{k},\omega) = \widetilde{\varepsilon}_{\mathbf{k}} - \varepsilon_{\mathbf{k}} - i\Gamma, \tag{19}$$

$$G^R(\mathbf{k},\omega)^{-1} = G_0^R(\mathbf{k},\omega)^{-1} - \Sigma^R(\mathbf{k},\omega) \tag{20}$$

$$G^R(\mathbf{k},t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \frac{1}{\omega - \widetilde{\varepsilon}_{\mathbf{k}} + i\Gamma} = -i\theta(t)\, e^{-i\widetilde{\varepsilon}_{\mathbf{k}} t - \Gamma t} \tag{21}$$

$$|\langle \mathbf{k}|\ \psi(t)\rangle|^2 = \left|G^R(\mathbf{k},t)\right|^2 = \theta(t)\, e^{-2\Gamma t}. \tag{22}$$

18.3 Importance of poles of $G^R_{\mathbf{k}}$, Dyson's equation

$$\mathcal{G}_{\mathbf{k}}(ik_n) = \mathcal{G}^0_{\mathbf{k}}(ik_n) + \mathcal{G}^0_{\mathbf{k}}(ik_n)\,\Sigma_{\mathbf{k}}(ik_n)\,\mathcal{G}_{\mathbf{k}}(ik_n) \tag{23}$$

$$G^R_{\mathbf{k}\uparrow}(\omega)^{-1} = G^{(0)R}_{\mathbf{k}\uparrow}(\omega)^{-1} - \Sigma^R_{\mathbf{k}\uparrow}(\omega) \tag{24}$$

85.3 A few properties of the self-energy

$$\operatorname{Im}\Sigma^R_{\mathbf{k}\uparrow}(\omega) < 0 \tag{25}$$

31.3 Some experimental results
31.4 Quasiparticules

$$A(\mathbf{k},\omega) \approx 2\pi Z_{\mathbf{k}} \frac{1}{\pi} \frac{-Z_{\mathbf{k}} \operatorname{Im}\sum^R(\mathbf{k},\omega)}{(\omega - E_{\mathbf{k}} + \mu)^2 + \left(Z_{\mathbf{k}} \operatorname{Im}\sum^R(\mathbf{k},\omega)\right)^2} + inc \tag{26}$$

31.5 Fermi liquid interpretation

$$\operatorname{Im}\Sigma^R_{\mathbf{k}\uparrow}(\omega) \sim \omega^2 + (\pi T)^2 \tag{27}$$