The Derivation of Kinetic Friction Force Differentials with Kinematic Formulae.

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The goal of this equation is to find the difference between the theorized acceleration of the object compared to the observed acceleration, in order to derive the acceleration loss attributed to kinetic friction.

$$v_{loss} = v_{obs} - v_{theo}$$

V_{obs} is derived from the previous proof, whereas v_{theo} is derived using kinematics. Acceleration can be replaced by gravity, and initial velocity can be omitted.

$$v_{theo}^2 = v_i^2 + 2ad \mid a = g \cdot sin\theta$$

$$v_{theo}^2 = v_i^2 + 2(g \cdot sin\theta)d$$

$$v_{theo} = \sqrt{2(g \cdot sin\theta)d}$$

Using the same kinematic equation as before, the acceleration due to kinetic friction can be derived.

$$v_{loss}^2 = 2a_k d$$

$$a_k = \frac{v_{loss}^2}{2d}$$

By substituting the initial equation, the final equation can be derived for acceleration due to kinetic friction.

$$a_k = \frac{(v_{obs} - \sqrt{2(g \cdot sin\theta)d})^2}{2d}$$

Using $F = ma$, the equation above can be set equal to force rather than acceleration.

$$F_{kinetic} = m\frac{(v_{obs} - \sqrt{2(g \cdot sin\theta)d})^2}{2d}$$

Since any loss in velocity during the accelerative phase can be nearly entirely attributed to kinetic friction, it can be deduced that the difference in between values would be equal to that of kinetic friction.

	Theo V0	Mag V0	Mag Fk	nonMag V0	nonMag Fk	V0 % diff	Fk % diff
Test #1	3.193m/s	0.923m/s	0.11N	0.564m/s	0.147N	48.197%	29.109%
Test #2	3.193m/s	0.957m/s	0.106N	0.527m/s	0.151N	57.996%	34.835%
Test #3	3.193m/s	0.991m/s	0.103N	0.489m/s	0.155N	67.882%	40.508%
Test #4	3.193m/s	0.923m/s	0.11N	0.564m/s	0.147N	48.197%	29.109%
Test #5	3.193m/s	0.957m/s	0.106N	0.602m/s	0.143N	45.569%	29.269%
Test #6	3.193m/s	1.09m/s	0.094N	0.639m/s	0.139N	52.234%	38.43%
Averages	3.193m/s	0.973m/s	0.105N	0.564m/s	0.147N	53.346%	33.544%