$$P(X|w_i) = \frac{1}{(2\pi)^{\frac{1}{2}}|\Sigma|^{\frac{1}{2}}} \cdot \exp\{-\frac{1}{2}(X-\mu_i)^T\Sigma^{-1}(X-\mu_i)\}$$

By Bayes theory, $P(w_i|X) = \dfrac{P(X|w_i)\cdot P(w_i)}{P(X)} \propto P(X|w_i)$, suppose $P(w_i) = c_i$

Discrimation function: $g_i(x) = \ln[P(X|w_i)\cdot P(w_i)]$

$$= \ln P(X|w_i) + \ln P(w_i)$$

$$= -\frac{1}{2}\ln|\Sigma| - \frac{1}{2}(x-\mu_i)^T\Sigma^{-1}(x-\mu_i) + \ln P(w_i) + C$$

$$g_1(x) - g_2(x) = \ln\left[\frac{P(w_1)}{P(w_2)}\right] - \left[\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1) - \frac{1}{2}(x-\mu_2)^T\Sigma^{-1}(x-\mu_2)\right]$$

$$= -\frac{1}{2}\left(2(\mu_2-\mu_1)^T\Sigma^{-1}x + \mu_1\Sigma^{-1}\mu_1 - \mu_2\Sigma^{-1}\mu_2\right) + \ln\left(\frac{P(w_1)}{P(w_2)}\right)$$

$(x-\mu_i)^T\Sigma^{-1}(x-\mu_i)$

$= x^T\Sigma^{-1}x - 2\mu_i^T\Sigma^{-1}x + \mu_i\Sigma^{-1}\mu_i$

$$= (\mu_1-\mu_2)^{-1}\Sigma^{-1}x - \frac{1}{2}\left[\mu_1\Sigma^{-1}\mu_1 + \mu_2\Sigma^{-1}\mu_2\right) + \ln\left[\frac{P(w_1)}{P(w_2)}\right]$$

$$\triangleq \boxed{w^Tx + w_0 = 0}$$ 决策边界