Time Series Analysis - Introduction

ECON 722

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Introduction

- History - popular in early 90 s, making comeback now.
- What is a Time Series? Y_t
  - The main difference between time series econometrics and cross-section is in dependence structure. Cross section econometrics mainly deals with i.i.d. observations, Y_i, while in time series each new arriving observation is stochastically depending on the previously observed, Y_t.
- The dependence is our best friend and a great enemy.
  - On one side, the dependence screw up your inferences (CLT you learned was for i.i.d. data). On the other side, the dependence allow us to do more by exploiting it. For example, we can make forecasts (which are almost non-sense in cross-section).

Introduction

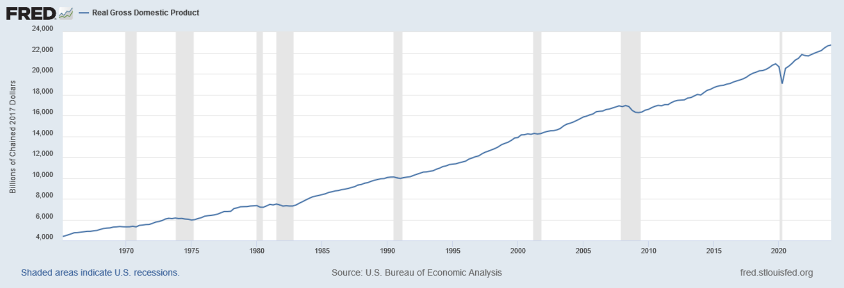
- We are going to ask questions like:
  - What are the characteristics of time series data?
  - Which type of questions can we ask/answer?
  - Which type of questions we cannot answer? Or can we?
- We can roughly divide time series into macro and finance related stuff.
  - Macro Time series mostly focuses on means. Often limited by small number of observations available over long horizon (e.g. 20 years monthly is T=300).
  - Financial data usually high-frequency over short period of time. This allows us to model volatility and higher moments.

Introduction

- Which type of questions can we ask/answer?
    - In the first part of the class we will be using Regression Models for *Forecasting*.
    - Forecasting and estimation of causal effects are quite different objectives (more on this later).
    - In the second half of the class we will discuss if and when we can talk about "causality".
- Forecasting:
    - It is very important in macro and finance.
    - Omitted variable bias isn't a problem.
    - External validity is paramount: the model estimated using historical data must hold into the (near) future
    - Intepretation of the coefficients is not the goal.

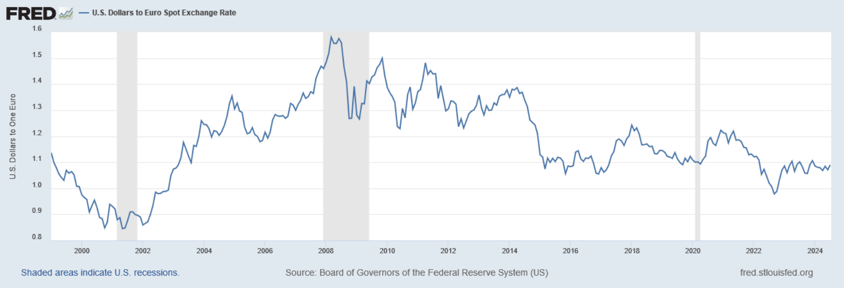
Some Examples of Time Series Data

Real GDP, 1965:1-2024:1, Quarterly (GDP is only quarterly), Seasonally adjusted, billions of chained 2017 dollars. From FRED.



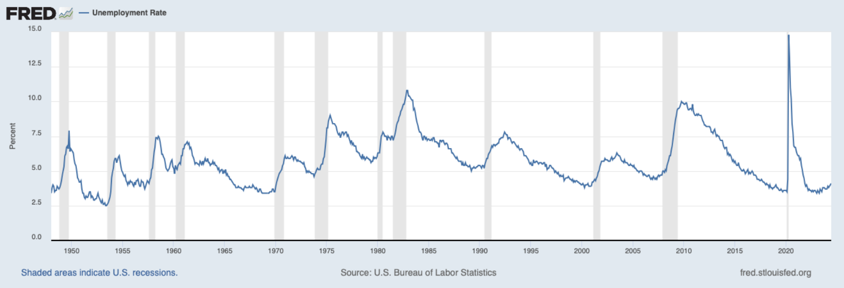
Some Examples of Time Series Data

US Dollars to Euro Spot Exchange rate. Not Seasonally Adjusted. 1999:01:04-2024:07:12. From FRED.



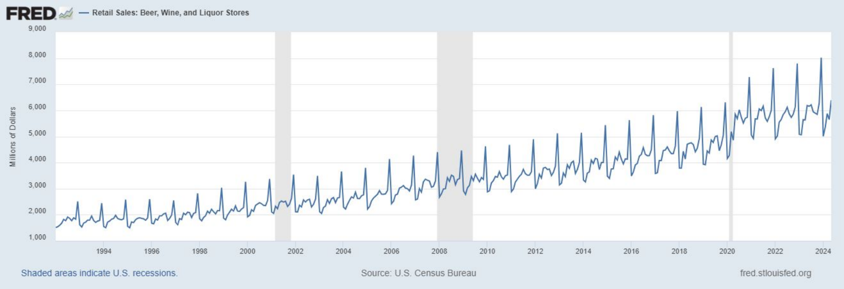
Some Examples of Time Series Data

Unemployment Rate (in %). Seasonally Adjusted. 1948:01-01 - 2024:06-01. From FRED.



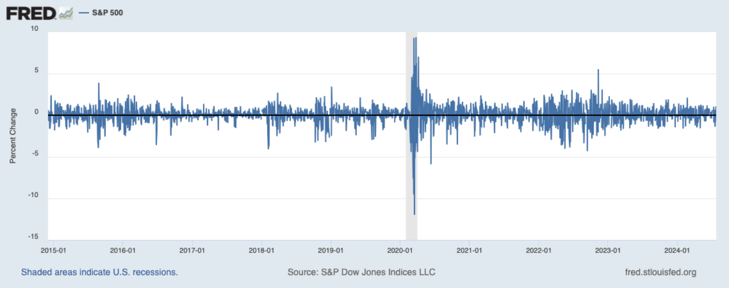
Some Examples of Time Series Data

- U.S. Liquor Retail Sales: Beer, Wine, and Liquor Stores
- Millions of Dollars, Not Seasonally Adjusted
- 1992-01-01 to 2024-05-01, Monthly



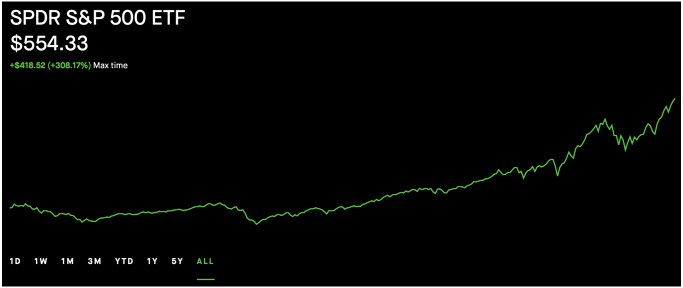
Some Examples of Time Series Data

- SP-500 returns
- Percentage Change, Not Seasonally Adjusted
- 2014-12-01 to 2024-07-23, Daily



Some Examples of Time Series Data

SPDR S&P 500 ETF (Ticker: SPY). Not Seasonally Adjusted.
2000:01-30 - 2024:07-21. From Robinhood



Components of any Time Series

There are **three** main components in any Time Series:

Trend
Part of a series' movement that corresponds to long-term, slow evolution

Seasonality
Part of a series' movement that repeats each year

Cycles
A catch-all phrase for various forms of dynamic behavior that link the present to the past and the future to the present

Trends and Breaks

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- **Stochastic trends** are non predictable (e.g unit roots, random walk)

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$$y_t = c + \beta_1 t + \beta_2 t^2$$

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- Quadratic Trend

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- Exponential Trend

$$y_t = ce^{\beta_1 t} \text{ or } \log(y_t) = \log(c) + \beta_1 t$$

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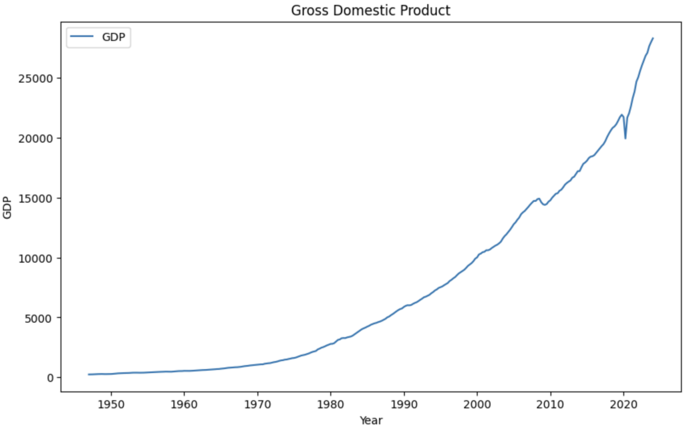
There are other kinds of trends but those three approximate any trend fairly well!

Adding an explanatory variable that looks like a trend will explain the same property.

Looking Closer at GDP

Follow the Module 1 Python code file to graph this using the data 'GDP.csv'



How do we estimate a Deterministic Trend?

The model we have in mind is

$$GDP_t = c + \beta t + u_t$$

We can run this using any statistical software using some OLS command and we get the following

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    GDP   R-squared:                       0.865
Model:                            OLS   Adj. R-squared:                  0.865
Method:                 Least Squares   F-statistic:                     1973.
Date:                Tue, 23 Jul 2024   Prob (F-statistic):          1.03e-135
Time:                        19:45:10   Log-Likelihood:                -2884.3
No. Observations:                 309   AIC:                             5773.
Df Residuals:                     307   BIC:                             5780.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Constant   -4763.0379    311.942    -15.269      0.000   -5376.853   -4149.223
Time          77.8620      1.753     44.422      0.000      74.413      81.311
==============================================================================
Omnibus:                       34.120   Durbin-Watson:                   0.005
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               42.573
Skew:                           0.902   Prob(JB):                     5.69e-10
Kurtosis:                       3.236   Cond. No.                         355.
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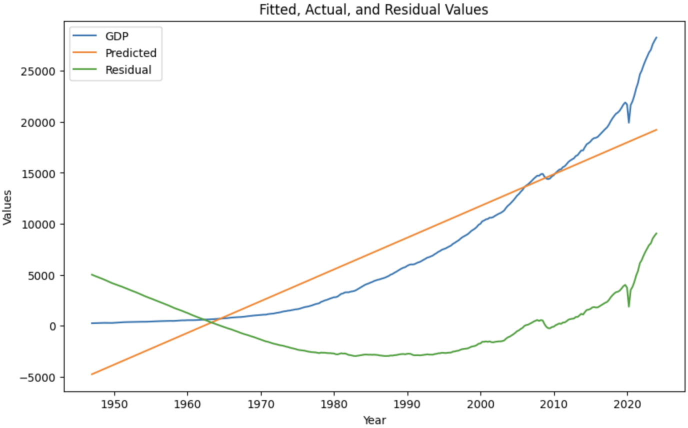
- Trend explains most of GDP variation. Typical of most time series'.
- R^2 is really high. Is this a good regression? Why or why not?
- But how do we know if this is truly deterministic trend? We will spent A LOT of time talking about this and the difference between deterministic and stochastic trends

How do we estimate a Deterministic Trend?



- Look at fitted values! (What do we learn?)
- Which pattern do we want the residuals to have?

How do we estimate a Deterministic Trend?

Let us now try a quadratic deterministic trend model!

$$GDP_t = c + \beta_1 t + \beta_2 t^2 + u_t$$

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Model:                            OLS   Adj. R-squared:                  0.994
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Date:                Tue, 23 Jul 2024   Prob (F-statistic):               0.00
Time:                        20:19:05   Log-Likelihood:                -2395.0
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Df Residuals:                     306   BIC:                             4807.
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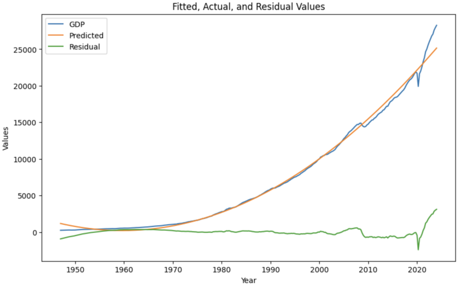

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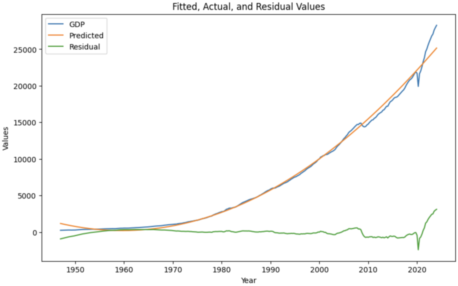

- What can you say from the output? What about from the graph of the residuals?

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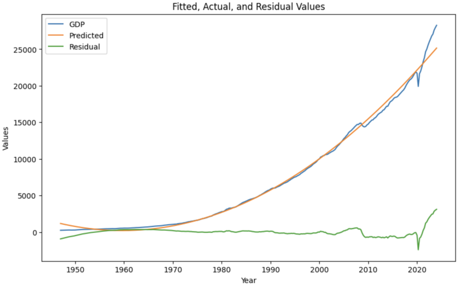

- What can you say from the output? What about from the graph of the residuals?
- **In general we would like the residuals to be inside the interval in the graph.**

How do we estimate a Deterministic Trend?

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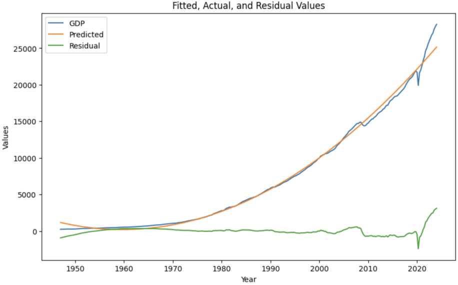

Deterministic Trends do not have an economics interpretation, yet it is incredibly powerful for forecasting.

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- The model with the highest R^2 is not always the best model for out of sample forecast.
- Most ICs attempt to find the model with the smallest out-of-sample 1-step-ahead mean squared error.

$$MSE = \frac{\sum_{t=1}^{T} e_t^2}{T} \text{ where } e_t = y_t - \hat{y}_t = \hat{u}_t$$

R^2 and Information Criteria

Note that

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum e_t^2}{\sum \left(y_t - \bar{y}\right)^2}$$

Where TTS depends only on the data.

$$\boxed{\text{Smallest MSE} \Leftrightarrow \text{highest } R^2}$$

We could look at the model with smallest MSE. We said that R^2 increases even if we include irrelevant variables so that we needed to correct for the degrees of freedom (d.f.). The same is true for MSE.

Information Criteria

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- Schwartz Information Criteria (SIC)

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Between two models, pick the one with the smallest AIC and SIC.

Breaks

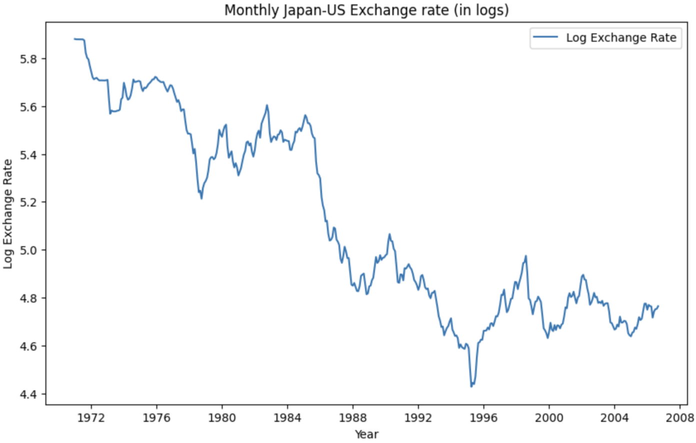
Breaks

Often we may want to model a break in the mean or a break in trend.

- We can use dummy variables to model that.
- Break in the intercept or break in the slope of the trend? (think of an example)
- If we know the date of the break (new law, big event..) then we can impose that break. **We should test for significance of that break**.
- If we do not know the date of the break we can test for a break at unknown time.

Broken Trend

Some time we may be interested in estimating a “ broken trend”.Often there are reason to expect that there is a structural break at a specific date.

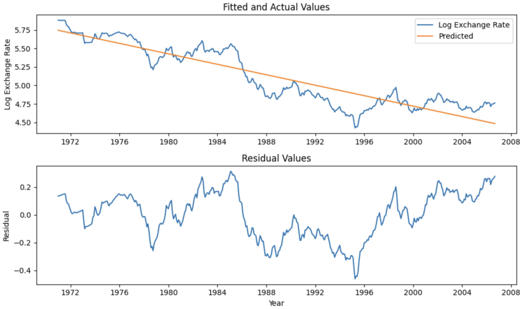


Broken Trend

As before, we can of course estimate a linear trend

$$yen_t = c + \beta t + u_t$$

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 Log_ER   R-squared:                       0.831
Model:                            OLS   Adj. R-squared:                  0.830
Method:                 Least Squares   F-statistic:                     2096.
Date:                Tue, 23 Jul 2024   Prob (F-statistic):          7.93e-167
Time:                        20:54:44   Log-Likelihood:                 165.73
No. Observations:                 429   AIC:                            -327.5
Df Residuals:                     427   BIC:                            -319.3
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Constant       5.7447      0.016    361.595      0.000       5.713       5.776
Time          -0.0029   6.43e-05    -45.786      0.000      -0.003      -0.003
==============================================================================
Omnibus:                       27.346   Durbin-Watson:                   0.027
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               17.741
Skew:                          -0.368   Prob(JB):                     0.000140
Kurtosis:                       2.328   Cond. No.                         494.
==============================================================================
```



It looks like there is a break in the trend around 1986. Before mid 1986 the trend is flatter and it is decreasing after that!

Broken Trend

To model this we use Dummy variables!

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A **dummy variable** is a variables that only take the values of 0 and 1.
Define D_t as follows

$$D_t = \begin{cases} 0 \text{ if } t < \text{December 1985} \\ 1 \text{ if } t > \text{December 1985} \end{cases}$$

Let's see what model we are estimating if we include this variable in this regression.

$$yen_t = c + \beta_1 D_t + \beta_2 t + u_t$$

Broken Trend

Clearly, before 1986, $D_t = 0$

$$yen_t = c + \beta_2 t + u_t$$

However, after 1986, $D_t = 1$

$$yen_t = (c + \beta_1) + \beta_2 t + u_t$$

Ergo, the coefficient on the dummy determines how the intercept has changed before and after 1986!

Broken Trend

```
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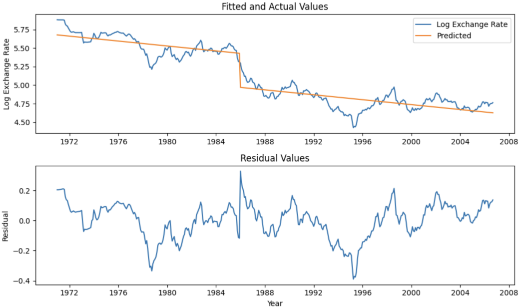
==============================================================================
Dep. Variable:                 Log_ER   R-squared:                       0.917
Model:                            OLS   Adj. R-squared:                  0.917
Method:                 Least Squares   F-statistic:                     2354.
Date:                Tue, 23 Jul 2024   Prob (F-statistic):          5.67e-231
Time:                        20:59:32   Log-Likelihood:                 318.55
No. Observations:                 429   AIC:                            -631.1
Df Residuals:                     426   BIC:                            -618.9
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Constant       5.6766      0.012    489.406      0.000       5.654       5.699
Post_85       -0.4583      0.022    -21.038      0.000      -0.501      -0.415
Time          -0.0014   8.68e-05    -15.912      0.000      -0.002      -0.001
==============================================================================
Omnibus:                       30.970   Durbin-Watson:                   0.091
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               35.794
Skew:                          -0.661   Prob(JB):                     1.69e-08
Kurtosis:                       3.507   Cond. No.                         979.
==============================================================================
```



- How many data points are used to estimate the mean in each time period?
- What does this tell us about how many data points we need in each period?

Broken Trend

We can use the same idea to allow for the possibility of a change in the slope. Use an "interaction" term $(D_t \cdot t)$

$$yen_t = c + \beta_1 D_t + \beta_2 t + \beta_3 (D_t \cdot t) + u_t$$

Before 1986, $D_t = 0$

$$yen_t = c + \beta_2 t + u_t$$

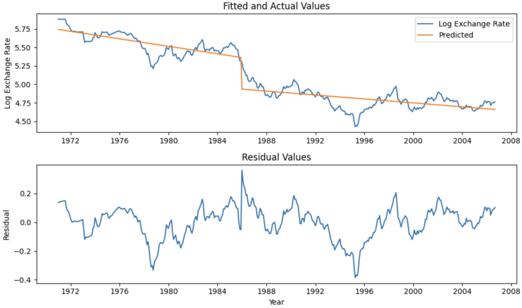
After 1986, $D_t = 1$

$$yen_t = (c + \beta_1) + (\beta_2 + \beta_3)\, t + u_t$$

The coefficient on the dummy determines how the intercept has changed before and after 1986, the coefficient on the interactive term determines how the slope has changed before and after 1986.

Broken Trend

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 Log_ER   R-squared:                       0.922
Model:                            OLS   Adj. R-squared:                  0.922
Method:                 Least Squares   F-statistic:                     1679.
Date:                Tue, 23 Jul 2024   Prob (F-statistic):          3.64e-235
Time:                        21:03:39   Log-Likelihood:                 332.34
No. Observations:                 429   AIC:                            -656.7
Df Residuals:                     425   BIC:                            -640.4
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Constant       5.7417      0.017    345.225      0.000       5.709       5.774
Post_85       -0.6069      0.035    -17.313      0.000      -0.676      -0.538
Time          -0.0021      0.000    -13.120      0.000      -0.002      -0.002
Time_Post_85   0.0010      0.000      5.312      0.000       0.001       0.001
==============================================================================
Omnibus:                       35.302   Durbin-Watson:                   0.092
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               43.676
Skew:                          -0.671   Prob(JB):                     3.28e-10
Kurtosis:                       3.800   Cond. No.                     2.27e+03
==============================================================================
```



Testing for Breaks

Why is important to detect breaks?

- If a break occurs in the population regression model in the sample, then the OLS will estimate a relationship that holds 'on average'. The average combines the two periods!

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  - Depending on the location and the size of the break the estimated 'average' relationship can be very different from the true population line.
  - For this reason it is important that we test the presence of breaks in time series.

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$$y_t = c + \beta_1 D_t + \beta_2 t + \beta_3 (D_t \cdot t) + u_t$$

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- We want to test the null hypothesis of no break. What is the null? Which test can we use? **Answer: Let's try the Chow's Test!**

Operationalizing the Chow's Test

Consider the model

$$yen_t = c + \beta_1 D_t + \beta_2 t + \beta_3 D_t \cdot t + u_t$$

We want to test the hypothesis

$$H_0 : \beta_2 = \beta_3 = 0$$

$$H_a : \text{otherwise}$$

The null hypothesis of joint insignificance of D can be run as an F-test with k and $N_1 + N_2 - 2k$ degrees of freedom where N_1 is $card(t_1, ..., t^*)$ and N_2 is $card(t^* + 1, ..., T)$ and k parameters.

$$F = \frac{(RSS_C - (RSS_1 + RSS_2))/k}{(RSS1 + RSS_2)/(N_1 + N_2 - 2k)}$$

Note: The same result can be achieved with dummy variables!

Operationalizing the Chow's Test

```python
# Run a Chow test to test for structural break

# Define the two subsamples
er_pre = er[er['DATE'] <= '1985-12-31']
er_post = er[er['DATE'] > '1985-12-31']

# Run the regression for the two subsamples
model_pre = sm.OLS(er_pre['Log_ER'], er_pre[['Constant', 'Time']])
results_pre = model_pre.fit()

model_post = sm.OLS(er_post['Log_ER'], er_post[['Constant', 'Time']])
results_post = model_post.fit()

# Compute the sum of squared residuals for the two subsamples
SSR_pre = np.sum(results_pre.resid**2)
SSR_post = np.sum(results_post.resid**2)

# Compute the total sum of squared residuals
SSR_total = np.sum((er['Log_ER'] - np.mean(er['Log_ER']))**2)

# Compute the Chow test statistic
Chow = ((SSR_total - (SSR_pre + SSR_post)) / 2) / ((SSR_pre + SSR_post) / (len(er) - 4))
Chow
```

[17] ✓ 0.0s Python

```
2518.029081609119
```

Clearly, we find that we reject the null hypothesis! The same conclusion was reached through the use of dummy variables.

Breaks at Unknown Time

- More often we don't know when the break occurred but we suspect that it was sometime between date t_0 and t_1. This is a much more interesting question!
- The Chow test can be modified to to handle this by testing for breaks at all possible dates between t_0 and t_1 and then taking the largest of the resulting F-tests.
- This is called the **Quandt test** (or sup-Wald test)
- Because the Quandt test is the *largest* of an individual F-test, its distribution is not the same as the individual tests but it will have its own distribution.

Digression: Use of Dummy Variables

Dummy variables are very useful. Once you understand the role of the dummy variables and the interactive terms, we can apply the same principle to many questions:

- Test if the demand is on average different at different times of the years (i.e. seasonality, more examples later).
- Test if the elasticity of demand is different at different times of the year (car sales example etc).
- Isolate specific times of the year

Remember the question before. How many data points do we use in each of the periods we isolate?

Seasonality

Seasonality

- Seasonality is a pattern that repeats every year.
- From a micro point of view, seasonality comes from links of technology, preferences and institutions to the calendar. (examples?).
- We will only look at deterministic seasonality = repetition is exact and predictable.
- Seasonality is a very typical component of Time Series.

How do we deal with Seasonality?

1. If we are interested in forecasting non seasonal fluctuations we may want to remove seasonality and work with seasonally adjusted series.

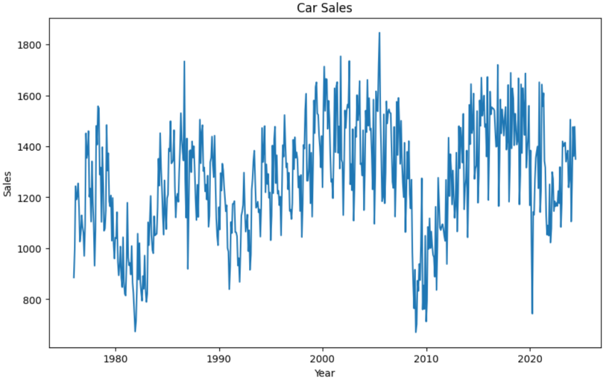
   > Do we really want to do this? In general we want to forecast all variations in the series.

2. We can take seasonality into account in our forecast and model seasonality.

   > Often data are already seasonally adjusted (SA), be careful when you download data which kind of data you really want. Data that is SA is passed through a complicated filter.

How to Model Seasonality

- To model seasonality, we use dummy variables!
- To motivate, let us use the cars.csv dataset which spans from 1967:1 to 2024:6
- It looks like the sales of cars are very different.



Reminder: Dummy Variable Trap

- If you run your regression with the constant, you need to define one less dummy than categories (in the example, 4 seasons $\implies$ 3 dummy variables).
- Alternatively, you can define the same number of dummies as categories and then run the regression without the constant.

Intercept and Slope in a Dummy Model

We formulate a pure seasonal dummy model w/ one explanatory variable (for now)

$$cars_t = c + \beta_1 Q_1 + \beta_2 Q_2 + \beta_3 Q_3 + \beta_4 price_t + u_t$$

Clearly from here, we know that

$$\mathbb{E}(cars_t | winter) = c + \beta_1 + \beta_4 price$$
$$\mathbb{E}(cars_t | spring) = c + \beta_2 + \beta_4 price$$
$$\mathbb{E}(cars_t | summer) = c + \beta_3 + \beta_4 price$$
$$\mathbb{E}(cars_t | fall) = c + \beta_4 price$$

The intercept is different for each season. What does this mean in economic terms?

Some Results

```
                            OLS Regression Results
==============================================================================
Dep. Variable:              Car Sales   R-squared:                       0.191
Model:                            OLS   Adj. R-squared:                  0.185
Method:                 Least Squares   F-statistic:                     34.00
Date:                Tue, 20 Aug 2024   Prob (F-statistic):           1.69e-25
Time:                        15:28:13   Log-Likelihood:                -3906.6
No. Observations:                 582   AIC:                             7823.
Df Residuals:                     577   BIC:                             7845.
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Constant     826.2518     46.051     17.942      0.000     735.804     916.700
Q1           -11.4374     23.436     -0.488      0.626     -57.467      34.592
Q2           140.0223     23.435      5.975      0.000      93.995     186.050
Q3            69.5385     23.556      2.952      0.003      23.272     115.805
Car Price      2.9879      0.330      9.048      0.000       2.339       3.637
==============================================================================
Omnibus:                        2.671   Durbin-Watson:                   0.858
Prob(Omnibus):                  0.263   Jarque-Bera (JB):                2.658
Skew:                          -0.165   Prob(JB):                        0.265
Kurtosis:                       2.973   Cond. No.                         768.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

For a given car price, the average sales are higher in the second and third quarters than in the fourth quarter.

Some Extensions

- What is the economic intuition of our estimates?
- What other variable/s would you include in this model?
- Do you think that differences in the average demand for cars across seasons is the only kind of seasonality we need to take into account?

Interaction Terms

- It may also be that not only is the sales higher in Q2 for a given price but also that the demand for cars is more sensitive to the interest rate in the different seasons!
- We want to model possible shifts in the slope of the regression line for different seasons
- We can do this by looking at the interaction of the dummy variables with the price variable

Interactive Dummy Variables

Consider this simple model with intercept and slope dummy

$$cars_t = c + \beta_1 Q1 + \beta_2 Q_2 + \beta_3 Q_3 + \beta_4 p_t + \beta_5 (Q_1 \cdot p_t) + \beta_6 (Q_2 \cdot p_t) + \beta_7 (Q_3 \cdot p_t) + u_t$$

```
                            OLS Regression Results
==============================================================================
Dep. Variable:              Car Sales   R-squared:                       0.195
Model:                            OLS   Adj. R-squared:                  0.185
Method:                 Least Squares   F-statistic:                     19.84
Date:                Tue, 20 Aug 2024   Prob (F-statistic):           7.26e-24
Time:                        16:13:23   Log-Likelihood:                -3905.1
No. Observations:                 582   AIC:                             7826.
Df Residuals:                     574   BIC:                             7861.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Constant     748.0431     89.883      8.322      0.000     571.503     924.583
Q1           142.5440    123.148      1.157      0.248     -99.332     384.420
Q2           279.7676    124.202      2.253      0.025      35.822     523.713
Q3            72.6679    126.668      0.574      0.566    -176.122     321.457
Car Price      3.5895      0.679      5.283      0.000       2.255       4.924
Q1_Price      -1.1878      0.932     -1.274      0.203      -3.019       0.644
Q2_Price      -1.0761      0.939     -1.146      0.252      -2.921       0.769
Q3_Price      -0.0206      0.960     -0.021      0.983      -1.907       1.865
==============================================================================
Omnibus:                        2.873   Durbin-Watson:                   0.850
Prob(Omnibus):                  0.238   Jarque-Bera (JB):                2.879
...
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

What is the underlying model for each season?

Forecasting with Seasonality

- Given that we are only looking at deterministic seasonality, forecasting with seasonal dummy variables is very easy! (i.e. we know exactly how the dummy variable looks in the future)
- You can predict what the value for the dummy will be next quarter and you can use the estimated values for your parameters to forecast.
- We can re-estimate using only part of our sample and see how we are doing in forecasting.
- We don't have a good R^2 so most likely we will not do too well in forecasting.

Cycles

Cycles

- Cycles include any sort of dynamics that is not captured by trend and seasonality.
- This includes dynamic, persistence, and any way in which the present is linked to the past or the future.

Cycles

- We can explain this as thinking that the error terms/data are positively correlated with each other over time.

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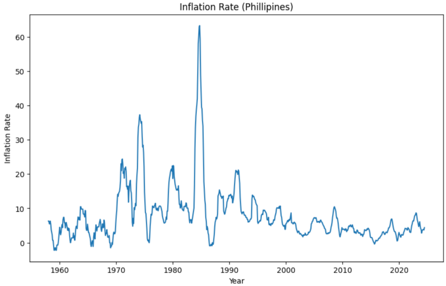
- The economy is subject to *shocks*! These shocks move the economy up or down in the period that occurs, then persist for a period of time.
- The shocks are represented by the values of the error terms.
- Think of technology shocks slowly moving to different sectors.

Stationarity

- Before modeling cycles, we need to define when a time series is *stationary* since we can only deal (for the most part) with data that is stationary.
- Think about a time series. We observe a part of the path of the series. In theory, a time series begins in the infinite past and continues in the infinite future and we only observe a small part of it.
- Since we only observe a finite period, we would like the series to be stable over all periods.
- **Stationarity** is then very important!

Autocorrelation

When we think of cycles, we often think about *autocorrelation*, the correlation of a series with itself lagged.



	Inflation Rate	Change in the Inflation Rate
Lag 1	0.983451	0.443811
Lag 2	0.952210	0.311938
Lag 3	0.910634	0.233348
Lag 4	0.861315	0.186828

- The inflation rate is **highly serially correlated** since $\rho(1) = 0.98$
- Last month's inflation rate contains much information about this month's inflation. Moreover, the plot is dominated by multiyear swings but there are still surprise movements!

How do we model cycles?

- The best way to capture the fact that things move slowly is to include a lagged dependent variable as an explanatory variable (ala $AR(1)$). *Why the name?*
- I could also add two lags $\implies AR(2)$
- Or we can use *MA* or *ARMA* models! We will learn to see which model/s are best when
- You can really only talk about *AR* terms when you have stationary data. If you don't (unit roots) you have to transform the data so they are stationary (i.e. through differencing)

Cycles - in Practice

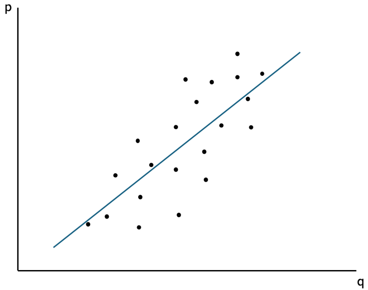
- Most time series will need **at least** an $AR(1)$, often not more than an $AR(2)$
- Once you add lags and other variables, the coefficients may become insignificant. Intuition?
- R^2 will increase significantly!
- AR terms are your best friends when it comes to forecasting.
- They are at the core of time series analysis.
- **Always** look at the residuals or the ACF or PACF of the residuals to make sure there is no serial correlation in the errors. If there is, add one more lag.

Other Topics

- What if there are breaks? $\rightarrow$ Testing
- How do we forecast and evaluate the forecasts?
- What if there is autocorrelation in the variance instead of the mean $\rightarrow$ ARCH and GARCH
- Extension to multivariate $\rightarrow$ VAR and SVAR
- What if the data is non stationary? $\rightarrow$ Unit Root, Unit Root Testing, Cointegration, VECM
- Estimation in the Frequency Domain
- Kalman Filter
- ... We will see how much time we have!

Structural vs Time Series Models

Simple Intuition



- If we regress p on q, the estimated regression line will be the blue line. Is this the *demand curve* or the *supply curve*?

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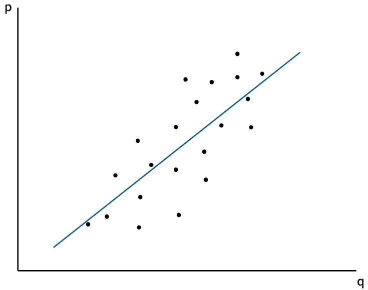

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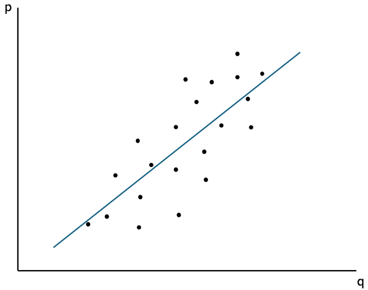

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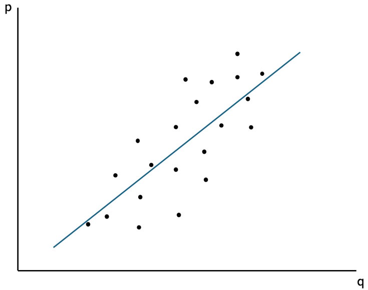

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- The estimated slope is not the slope of the demand function but a combination of the true slope of demand and supply
- The estimator is not correct, it is biased! The slope is not estimating the elasticity. There is a simultaneity bias!
- The prices are a **combined** effect of supply and demand, because both are price contingent but also influence prices *simultaneously!*

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- Suppose you are interested in the *price elasticity of demand*.
    - If you set prices based on the expectation that you will have an increase in demand...
    - You will not be estimating the elasticity by simply regressing demand on prices
- If we really want to estimate the slope of the demand function, we need to make sure that the RHS variable is "exogenous/predetermined" (e.g. say weather)

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- But if we only care about **forecasting** prices and/or quantities, then we could just regress q on p or vice-versa and get good forecasts even if we can't exactly estimate the demand function

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- What you do is a combination of these two which introduces difficulties in interpreting your coefficients.

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- If you are not sure, then DO NOT give too much weight on the estimates of price elasticity, as a bias is probably there!
- We cannot always give a **causal** interpretation unless we are sure that p is exogenous
- This will not affect the quality of your forecasts!