Review Test Submission: A short quiz on interpolation

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Course	Computational Science I - MATH-2072U/CSCI-2072U - 201901 - 70861.201901.XLIST
Test	A short quiz on interpolation
Started	3/8/19 8:39 AM
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Due Date	3/8/19 12:30 PM
Status	Completed
Attempt Score	1 out of 5 points
Time Elapsed	5 minutes out of 15 minutes
Instructions	You will not be able to take this quiz more than once or backtrack when taking the test. You may use your notes, Python and Internet resources.
Results Displayed	All Answers, Submitted Answers, Correct Answers, Feedback

Question 1

1 out of 1 points

Suppose we have n+1 distinct interpolation nodes and n+1 function values. We compute a polynomial interpolant of the lowest possible order.
Which of the following statements is true?

Selected Answer: ✓ d.
The maximal order of the interpolant is n. The order might be less, depending on the function values.

Answers:
a. The order of the interpolant will be n, regardless of the function values.

b.
The maximal order of the interpolant is n+1. The order might be less, depending on the function values.

c. The order of the interpolant will be n+1, regardless of the function values.

✓ d.
The maximal order of the interpolant is n. The order might be less, depending on the function values.

Response Feedback: Indeed, a polynomial of order n has n+1 coefficients and can be used to fit n+1 function values. If the interpolation data happen to fit on a lower order polynomial, the interpolant will have lower order.

Question 2

0 out of 1 points

Suppose we are given n+1 interpolation nodes and n+1 function values and use the Vandermonde system to find an interpolant.
In some code, we need to compute interpolants many times.
Which of the following statements is true?

Selected Answer: b.
Each time we compute an interpolant, the order of complexity depends on the interpolation data.

Answers: a.
It is much faster to interpolate the same set of function values on many different sets of nodes than to interpolate many different sets of function values on the same set of nodes.

b.
Each time we compute an interpolant, the order of complexity depends on the interpolation data.

c.
Whenever we deal with a different set of nodes or a different set of function values, we have to solve a dense matrix-vector problem, at the cost of $O(n^3)$ FLOPS.

d.
It is much faster to interpolate many different sets of function values on the same set of nodes than to interpolate the same set of function values on many different sets of nodes.

Response Feedback: Since the Vandermonde matrix depends only on the nodes, we can compute its LU-decomposition once and then use forward and backward substitution for subsequent interpolations. This costs only $O(n^2)$ FLOPS. However, if the nodes change, we need to recompute the Vandermonde matrix and its decomposition, at the cost of $O(n^3)$ FLOPS.

Question 3

0 out of 1 points

Suppose we compute the interpolating polynomial through three points (x_0,y_0), (x_1,y_1) and (x_2,y_2). It so happens that these three points lie on a straight line. Which of the following statements is true?

Selected Answer: c. There will be infinitely many solutions to the associated linear system.

Answers: a. There will be no solution to the associated linear system.

b.
There will be a unique solution to the associated linear system but it will not correspond to an interpolating polynomial.

c. There will be infinitely many solutions to the associated linear system.

d.
We will find a unique solution to the associated linear system and the second order coefficient of the interpolant will be zero (up to numerical accuracy).

Response Feedback: The interpolation problem will still have a unique solution and this solution corresponds to the interpolant of lowest order, in this case one.

Question 4

0 out of 1 points

The Vandermonde system is a matrix-vector equation, the solution to which is the vector of

coefficients of the interpolant. Which of the following statements is true?

Selected Answer: c.
The Vandermonde matrix is dense and depends both on the interpolation nodes and on the function values.

Answers: a.
The Vandermonde matrix is dense (meaning that it does not have many zero elements) and depends only on the interpolation nodes. The right hand side depends only on the function values.

b.
The Vandermonde matrix is sparse (meaning that is has many zero elements) and depends only on the interpolation nodes. The right hand side depends only on the function values.

c.
The Vandermonde matrix is dense and depends both on the interpolation nodes and on the function values.

d. The Vandermonde matrix is dense and depends only on the function values.

Response Feedback: The matrix can have at most one row with mostly zeros so it is not sparse. It is determined by the nodes, while the right hand side is determined only by the function values.

Question 5

0 out of 1 points

Suppose we measure the temperature (in degrees Celcius) as a function of height (in metres) and find the following data:

height	temperature
1	20.10
100	17.34
1000	15.09
5000	5.103
10000	-2.044

In these measurements, the last digit of the temperature is uncertain.
We want to use the Vandermonde system to find an interpolating polynomial of order 4.
Which of the following statements is true?

Selected Answer: b.
Because the Vandermonde matrix is only 5 by 5, it is well-conditioned and we can accurately compute the coefficients.

Answers: a.
The condition number of the Vandermonde matrix for these data is large, so we have to use LU-decomposition to accurately compute the coefficients of the interpolant.

b.
Because the Vandermonde matrix is only 5 by 5, it is well-conditioned and we can accurately compute the coefficients.

c.
The condition number of the Vandermonde matrix for this problem is so big, that we may not be able to compute any significant digits of the coefficients.

d.
The condition number of the Vandermonde matrix for this problem is large because the temperatures, measured in Celcius, give large numbers.

Response Feedback: The condition number is very large, mainly because the nodes, i.e. the heights in metres, have large numerical values. This can compromise the accuracy of the

coefficients of the interpolant, independent of the algorithm used to solve the linear system.

Friday, March 8, 2019 8:44:27 AM EST

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